# Project 3: Cauchy point and Dogleg method

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## 1 Guideline

Here is a guideline for your project.

- The project is designed to help understand the topic, so please do not copy and paste.
- You are welcome to work in groups (no more than 3 in one group), but you should write it up for your submission/exercises on your own.
- The standard submission of project contains the following documents:
  - The code files
  - A `README` file
  - A brief writeup document for the project

In your `README` file, there are several things you need to declare:

- What language are you using to write the program;
- How to run the code;
- Contributors.

You can add additional information such as possible issues, future plan in the `README` file as well. For the writeup document, you will be answering the questions in the project. **If you are only submitting the `ipynb` file, you can include everything in it.**

## 2 Project: Dogleg method

In this project, you will write program to solve some optimization problems with the `Dogleg method` to solve the optimization problem for Rosenbrock function. The Cauchy point method has been provided in this starter kit.

### 2.1 Project Description

The `Dogleg method` is a simple algorithm to select the step for trust region method. The method first locates the leg path, if the whole leg is within the region, then choose the Newton step, otherwise choose the intersection of the leg with the region boundary.

### 2.1.1 Task 1: Implement Dogleg for the subproblem

In this task, you will implement the Dogleg method for the selection of $p_k$. The algorithm is stated as the following:

1. Locate the Newton step $p_n = -B^{-1}g$, here $g$ is gradient, check whether or not if it is within the region. If yes, then return it.
2. If it is not, then check the leg's knee $p_s = -\frac{\|g\|^2}{g^T B g} g$ is in the region or not. If yes, then you need to solve the following equation
$$\|(1-t)p_s + tp_n\|^2 = \Delta^2 \tag{1}$$
which is equivalent to solve the positive root of the following quadratic equation
$$t^2\|p_n - p_s\|^2 + 2tp_s^T(p_n - p_s) + \|p_s\|^2 - \Delta^2 = 0. \tag{2}$$
then your output would be $(1-t)p_s + tp_n$.
3. If $p_s$ is also outside, then directly we use the $-\Delta \frac{g}{\|g\|}$.

**Task**: You will implement a function called `dogleg` in your favorite programming language. Your function's signature should be like

```
function p = dogleg(x, Δ)
```
(3)

or

```
def dogleg(x, Δ):
```
(4)

returning the chosen $p$. Suppose the objective function's value, gradient. Hessian are globally defined, so you can call them inside your function (see the last part of starter kit). Assume the exact Hessian is used for the model function.

### 2.1.2 Task 2: Compare Dogleg and Cauchy point

In this task, you will use your implemented Dogleg method and the provided Cauchy point method to work on the Rosenbrock's function.

**Task**: You will be using the two methods with trust region method to solve the unconstrained optimization problem $\min_{x=(x_1,x_2)} f(x)$ for the objective function $f$ as the following function:

$$f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$$

The solution of above optimization problem is $(1, 1)$. For your implementation, you will use starting point from $\hat{x} = (1.2, -1)$ and the tolerance on gradient $\delta = 10^{-9}$, the initial and maximum radius is selected as 2.

- How many iterations does the Cauchy point method take?
- How many iterations does the Dogleg method take?
- What is the convergence type of Cauchy point method ?
- What is the convergence type of the Dogleg method?

## 3 Starter code kit

The starter code kit is in Python (MATLAB users should have no difficulty to understand.) It is up to you to following the starter code kit's style or not. As a reminder, the kit only fills out framework, there are some details you will have to write on your own.

## 4 Submission

If you prefer to use the GitHub or Bitbucket, etc. to store your code, then it is OK to simply submit the repository's link on a text file (if your repository is public), otherwise, please submit all files in a zip file through Canvas.