**Flight Dynamics Summary**

Cross-product  
cross(X,Y) = Z (orthogonal)  
X cross y will give you Z, which is outside the plane

Vectors  
a = ax(i) + ay(j) + az(k)  
i, j, k = components (unit vectors)  
a is represented by these components

Newton’s Laws  
F = mass \* acceleration (F=ma) OR F = m(dV/dt) (V = velocity)  
torque = inertia \* angular acceleration (T = I \* a)  
H = angular momentum = l \* ω   
T = dh/dt

Aircraft Notation  
z = cross (x,y) x points forward, z points down, y points right  
[ L M N ] = [ Roll Pitch Yaw] moments   
velocities [ U V W ] point along [ X Y Z ]   
positive yaw is to the right, positive pitch is up, positive roll is to the right

Yaw, Pitch and Roll can be summarized into matrices   
  
Yaw  
cos(γ) -sin(γ) 0  
sin(γ) cos(γ) 0  
0 0 1  
  
Pitch  
cos(Θ) 0 sin(Θ)  
0 1 0  
-sin(Θ) 0 cos(Θ)

Roll  
1 0 0  
0 cos(ɸ) -sin(ɸ)  
0 sin(ɸ) cos(ɸ)

To calculate the directional cosine matrix, multiply roll matrix \* pitch matrix \* yaw matrix

References Frames  
[XE YE ZE] = earth frame, which does not move at an arbitrary location  
[XB YB ZB] = body frame, which is fixed to the center of gravity location of the aircraft  
r = X, Y, Z = position of the aircraft relative to the frame  
angular velocities given by [p, q, r], this is omega

Deriving Equations of Motion  
Forces we want to measure are in the earth frame, and the forces are along the body frame of the aircraft so we need to convert.  
E = earth frame  
B = body frame

Start with  
ΣF = m \* (dV / dt)E= earththen the transport theorem can be used  
m \* (dV / dt)E = m \* ( ( dV / dt)B + ω B / E X VB(mass \* velocity derivative in earth frame = mass \*velocity derivitate in body frame + omega of the body relative to earth X (crossproduct) velocity in earth frame  
  
This is the acceleration that is the dV in the body frame

dU/dt = derivative  
  
This is the VB, the velocities of the aircraft  
U = along x axis  
V = along y axis  
W = along z axis  
  
This is the angular velocities ω B / E   
But the problem here is that we cannot multiply a 3x1 matrix by a 3x1 matrix  
so we need to convert the [p q r] matrix to a skew-symmetric matrix

\*

Then plug and solve for the forces on an aircraft represented by X, Y, Z  
X force = m \* (dU /dt + q \* W – R \* V)  
Y force = m \* (dV / dt + r \* U – p \* W)  
Z force = m \* (dW / dt + p \* V – q \* U)  
these are functions of p, q, and r, the acceleration, the velocity, and mass of the aircraft

Deriving Moments   
We want to measure the moments in the earth frame because this is the intertial frame  
Start with   
T = ( dH / dt )E  
then transport law again  
( dH / dt )E = ( dH / dt )B + ( ω B / E X HB )  
  
remember H = I \* ω, where ω = [ p q r ]T, T exponent is transpose, I is the inertia matrix

Solving for ( dh / dt )B   
since H = I \* ω, ( dH / dt ) = I \* ( dω / dt ) = I \* , which is the omega body frame rate of change

Where I (inertia matrix) = assuming the aircraft is symmetric, Ixz = 0, Ixy = 0

This is the skew symmetric matrix again for ω B / E

HB = \*

Plugging everything back in. dh / dt is the torque which is equal to [ L M N ]. And dh / dt for the body frame we get the cross result matrix from I \* ( dω / dt ). We have the skew symmetric matrix \* HB.

= +  \* \*

Solving for [ L M N ], the moment of the aircraft in the earth frame you get

L = Ixx \* + Ixz \* + q \* (r \* ( Izz – Iyy ) + p \* Ixz )

M = Iyy \* + p \* r ( Ixx – I xz ) + ( r2 – p2 ) \* Ixz

N = Izz \* + Ixz \* + p \* q ( Iyy – Ixx ) – q \* r \* Ixz

Deriving Rotation  
Need to solve for the yaw, pitch, and roll angles because the moment a yaw, for example, happens, the body frame changes, x and y points somewhere else. The moment direction changes, there is a change in the [ p q r ] vectors. The rotation sequence is yaw then pitch then roll. This needs to be related into an equation with angular velocity relating to changes in angles.

ω = = + R( \* + R( \* R(Θ) \*

Need to solve for [ p q r ] as a function of   
Plugging in values you get

ω = = \*