Research Methods for Political Science

Bivariate statistics:
t-tests

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Comparing means

- One-sample case (week 4)

- Two-sample case
  - Independent samples
  - Paired samples

Testing hypotheses

- Testing hypotheses
  - Null hypothesis (H_0)
  - Alternative hypothesis (H_1)

Null hypothesis

- We ask: if the null hypothesis were true, how likely would we be to collect data that is as extreme or more extreme than we have?

Dependent samples t-test

- Comparing two measurements for the same participants
- Example: thermometer score for Ahern (M=65.5, SD = 24.2) and Bruton (M=45.74, SD = 22.85). Is this *difference* statistically significant?
- N = 2570

Step 1: Assumptions

Random sampling

Level of measurement interval-ratio

Sampling distribution is normal

Step 2: stating the null hypothesis: mean difference equals 0:

H_0: $\mu_D = 0$

(H_1:$\mu_D \neq 0$)

Step 3: Selecting the Sampling Distribution and Establishing the Critical Region

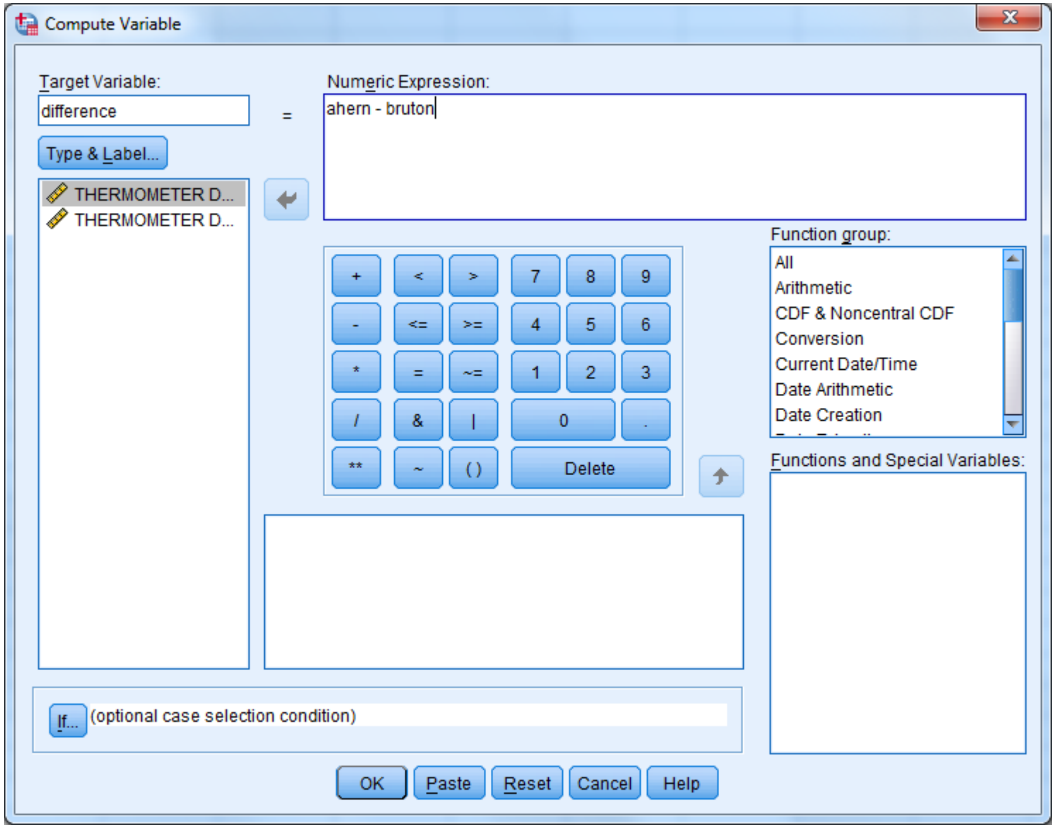
As we do not know the population standard deviation of the difference between Ahern and Bruton, we use a t-test with df = n – 1 (two-tailed). As, N = 2570, df = 2569.

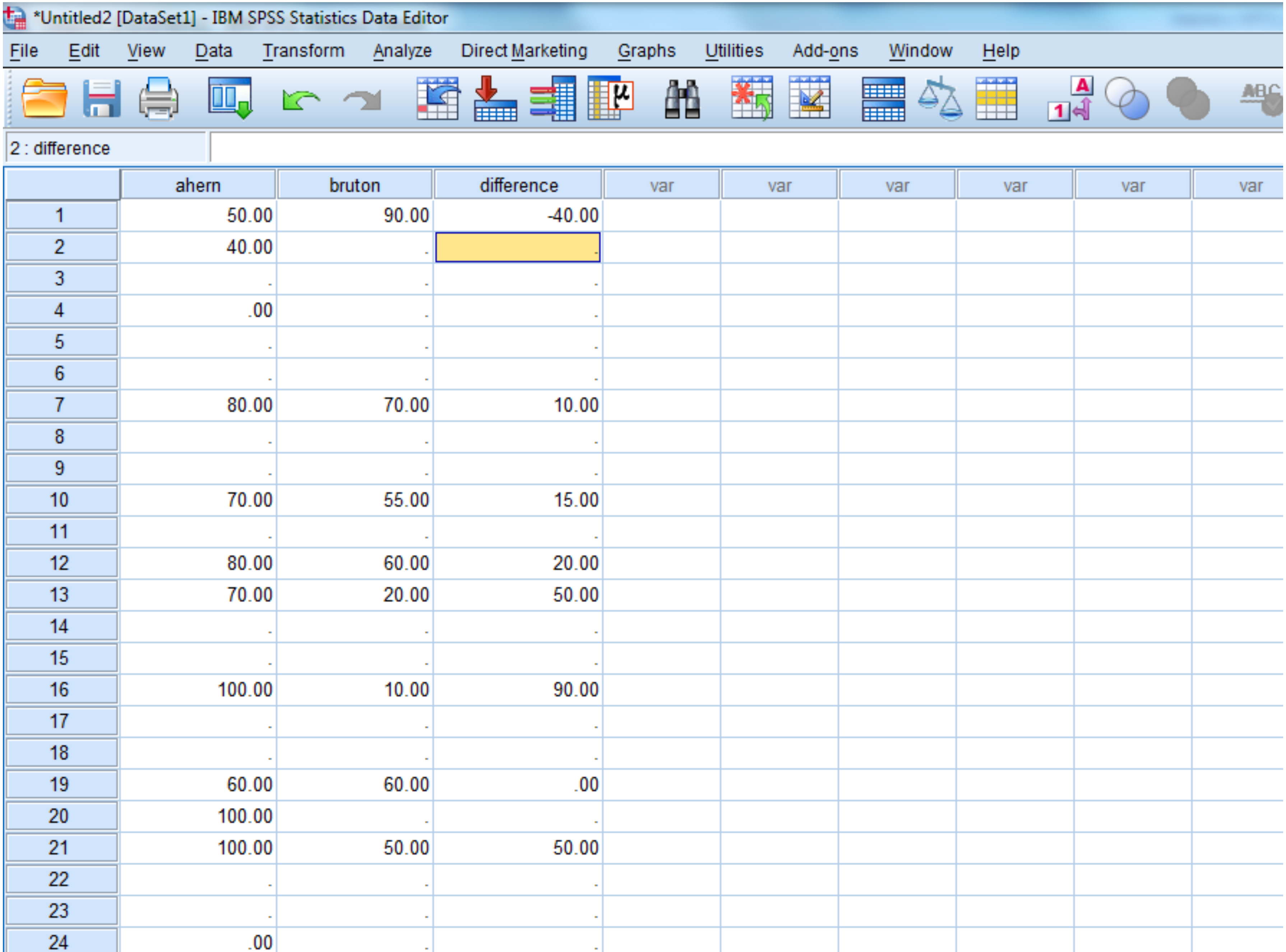
Therefore, with α = 0.05, t(critical) = 1.96

(check your understanding: why 1.96? How did we find it?)

Step 4: Computing the test statistic

First, calculate the difference between Ahern and Bruton. (SPSS: Transform … Compute)





	ahern	bruton	difference	var	var	var	var	var	var
1	50.00	90.00	-40.00						
2	40.00	.	.						
3	.	.	.						
4	.00	.	.						
5	.	.	.						
6	.	.	.						
7	80.00	70.00	10.00						
8	.	.	.						
9	.	.	.						
10	70.00	55.00	15.00						
11	.	.	.						
12	80.00	60.00	20.00						
13	70.00	20.00	50.00						
14	.	.	.						
15	.	.	.						
16	100.00	10.00	90.00						
17	.	.	.						
18	.	.	.						
19	60.00	60.00	.00						
20	100.00	.	.						
21	100.00	50.00	50.00						
22	.	.	.						
23	.	.	.						
24	.00	.	.						

Descriptive Statistics

	N	Minimum	Maximum	Mean		Std. Deviation
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
difference	2570	-100.00	100.00	19.7720	.65323	33.11554
Valid N (listwise)	2570					

For dependent samples (paired samples):

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{N}}$$

Remember the null hypothesis: $\mu_D = 0$

	N	Minimum	Maximum	Mean		Std. Deviation
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
difference	2570	-100.00	100.00	19.7720	.65323	33.11554
Valid N (listwise)	2570					

Descriptive Statistics

$$t = \frac{19.77 - 0}{33.12/\sqrt{2750}} = \frac{19.77}{0.653} = 30.27$$

<u>Step 5: Making a decision</u>

t(obtained) = 30.27

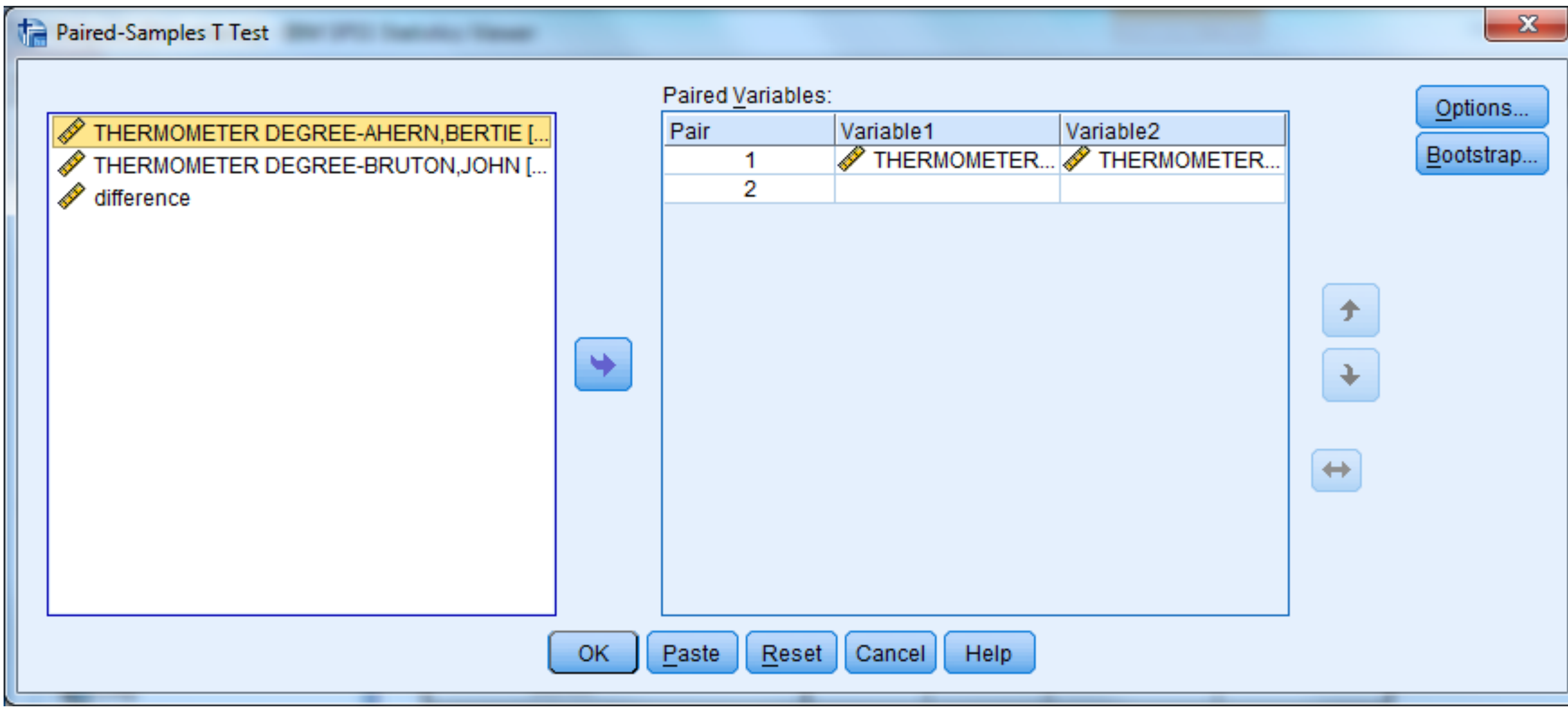
t(critical) = +/- 1.96

As t(obtained) fell in the *critical region,* we have to reject the null hypothesis.

Conclusion?

Paired samples in SPSS

- Analyze ... Compare Means ... Paired-samples t-test



Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	THERMOMETER DEGREE-AHERN, BERTIE	65.5128	2570	24.20657	.47749
	THERMOMETER DEGREE-BRUTON, JOHN	45.7409	2570	22.85454	.45082

Paired Samples Correlations

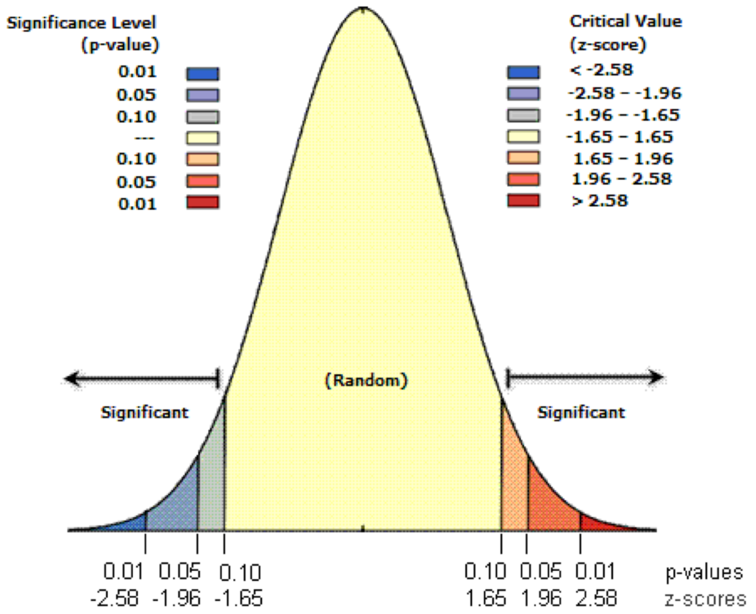
		N	Correlation	Sig.
Pair 1	THERMOMETER DEGREE-AHERN, BERTIE & THERMOMETER DEGREE-BRUTON, JOHN	2570	.011	.594

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	THERMOMETER DEGREE-AHERN, BERTIE - THERMOMETER DEGREE-BRUTON, JOHN	19.77198	33.11554	.65323	18.49108	21.05289	30.268	2569	.000

P-value

- As usual, notice the p-value. Remember that the p-value is the sum of the tail areas, i.e, p(|t|>t)
- (check your understanding: What is the connection between t-values and z-values?)



Independent samples

- Comparing means between two groups, e.g. political interest for men (M = 5, SD = 3.16, N = 10) and women (M = 5.4, SD = 2.31, N = 10) (fake data)
- Note that these two samples are *not* paired. I.e., we are asking different people. These samples are independent
- Can we say the difference in political interest is statistically significant?

Step 1: Assumptions

Random sampling

Level of measurement interval-ratio

Sampling distribution is normal

Independent observations

Step 2: stating the null hypothesis

H_0: $\mu_1 = \mu_2 \ (or\text{: } \mu_1 - \mu_2 = 0)$

(H_1: $\mu_1 \neq \mu_2$)

Step 3: Selecting the Sampling Distribution and Establishing the Critical Region

As we do not know the population standard deviation of the difference between men and women, we use a t-test with df = $N_1 + N_2 - 2$ (two-tailed). As, $N_1 = 10$, $N_2 = 10$, df = 10 + 10 – 2 = 18

Therefore, t(critical) can be looked up in Fields' table and is equal to 2.10 ($\alpha = 0.05$).

Step 4: Computing the test statistic

For an independent-samples t-test:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)}}$$

Step 4: Computing the test statistic

Remember, that under the null hypothesis $\mu_1 - \mu_2 = 0$, thus

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{s^2{}_1}{N_1} + \frac{s^2{}_2}{N_2}\right)}}$$

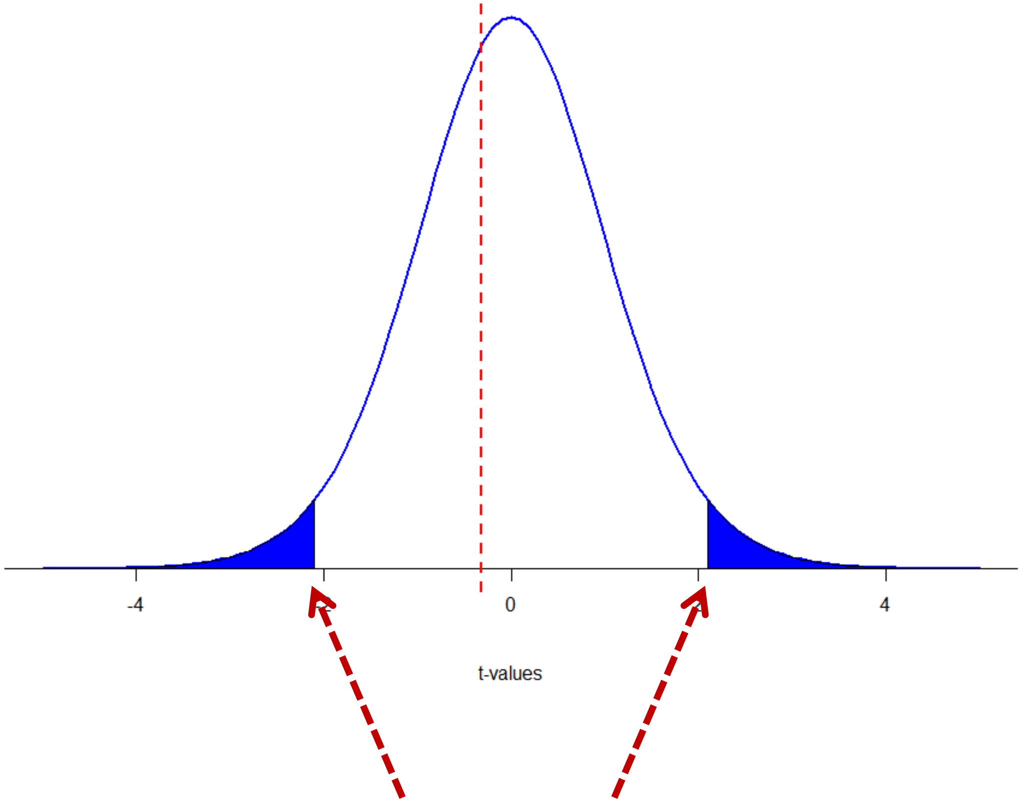
Step 4: Computing the test statistic

Remember, that under the null hypothesis $\mu_1 - \mu_2 = 0$, thus

$$t = \frac{5 \ - 5.4}{\sqrt{\left(\frac{3.16^2}{10} + \frac{2.31^2}{10}\right)}} = \frac{-0.4}{1.24} = -0.323$$

Step 5: Making a decision

t(obtained) = -0.323



t(critical) = +/- 2.10

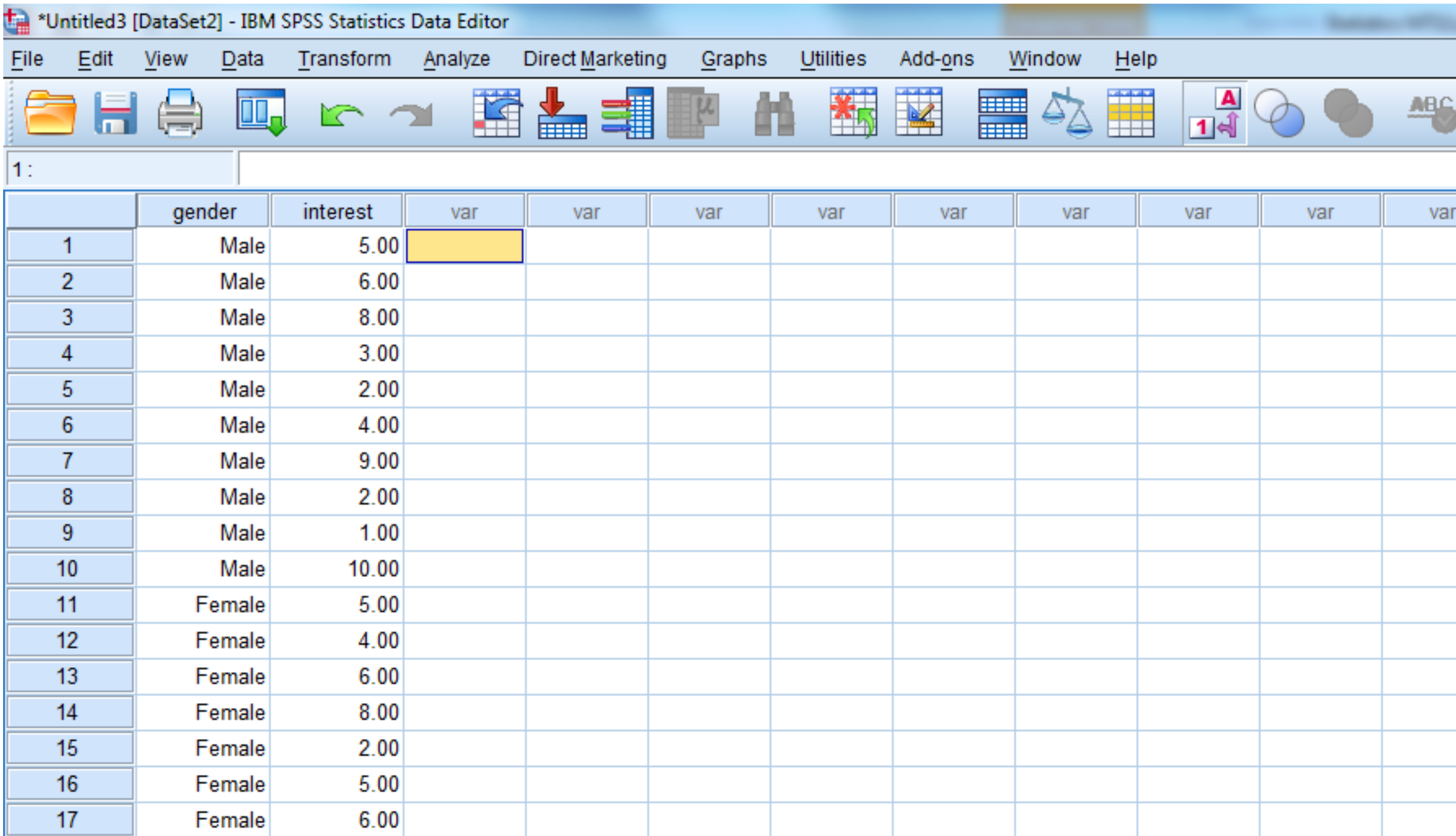
Equality of variances

- The independent samples t-test assumes that the variances of the two samples are (roughly) equal. If not, SPSS can make adjustments for us.
- Levene’s Test for Equality of Variances tests the null hypothesis that the variances of two groups are equal.
- Some objections to using it (Field 2013: 194)

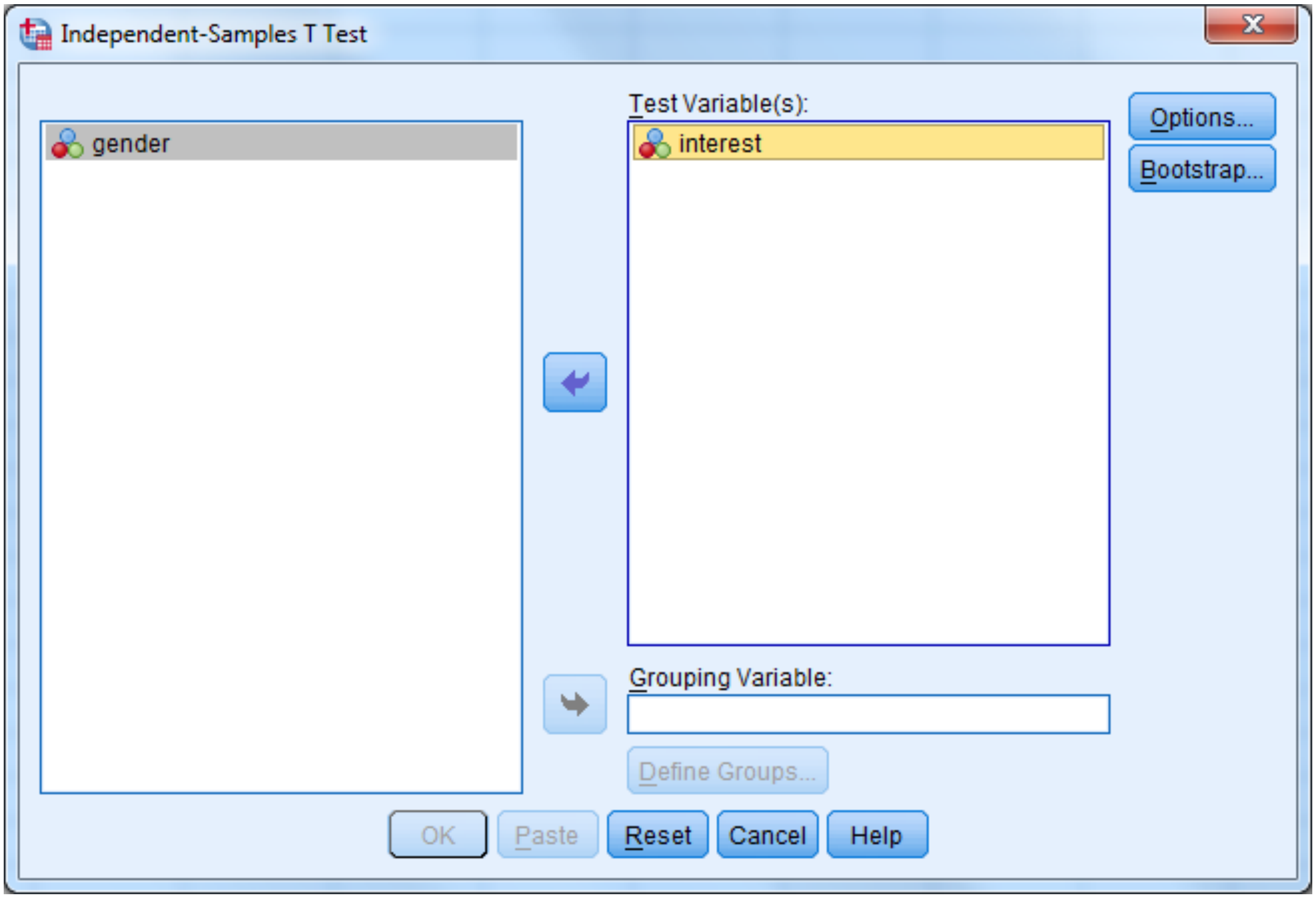
Interpreting Levene's test

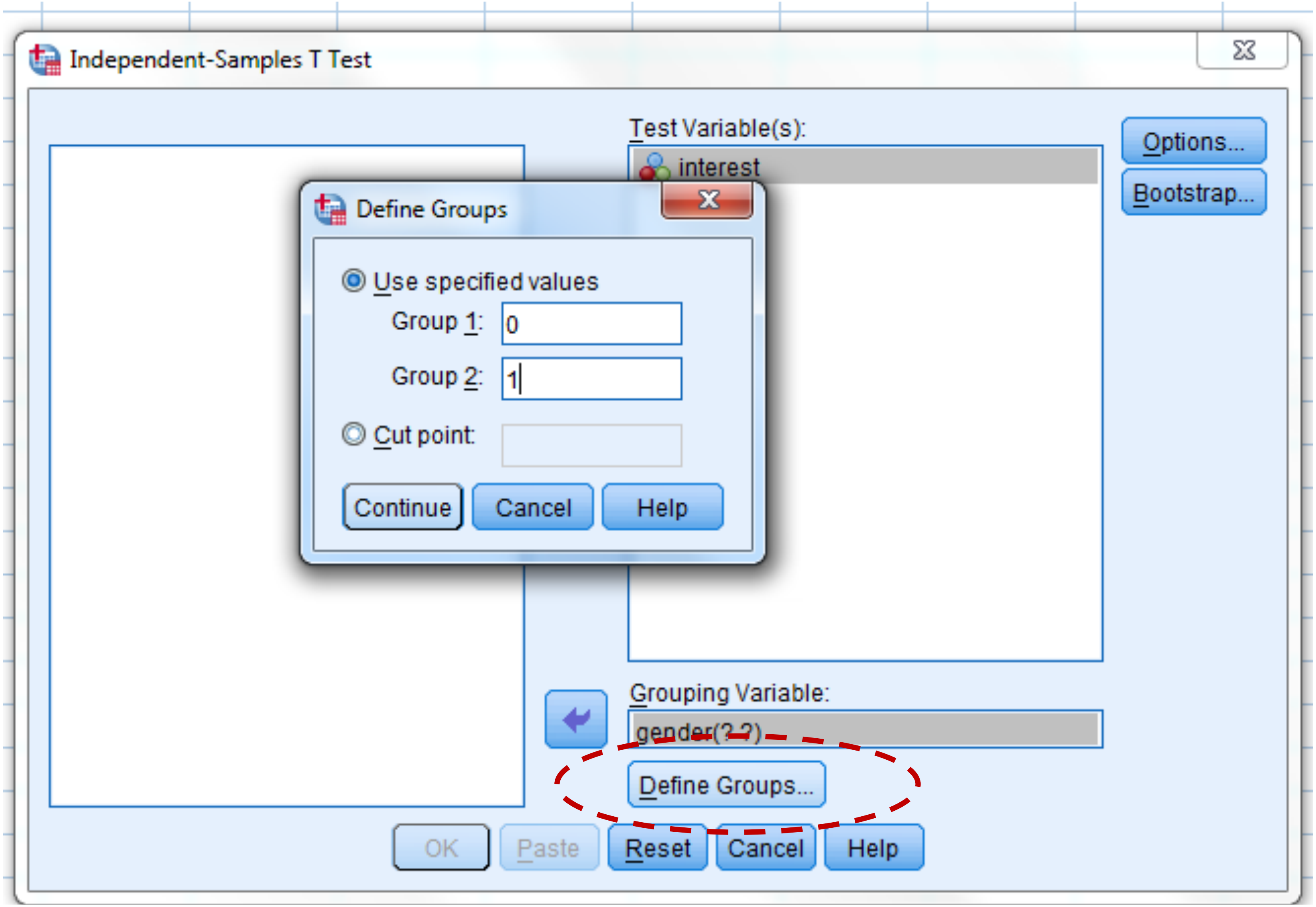
P-value	Hypothesis rejected	What does this mean?
< 0.05	Yes	Equal variances cannot be assumed, adjustments need to be made
> 0.05	No	Equal variances can be assumed, adjuments not necessary.

Independent samples t-test in SPSS



- Analyze ... Compare Means ... Independent-samples t-test




Independent-Samples T Test
Test Variable(s):
interest
Options...
Bootstrap...
Define Groups
Use specified values
Group 1:
0
Group 2:
1
Cut point:
Continue
Cancel
Help
Grouping Variable:
gender(? ?)
Define Groups...
OK
Paste
Reset
Cancel
Help

Group Statistics

	gender	N	Mean	Std. Deviation	Std. Error Mean
interest	Male	10	5.0000	3.16228	1.00000
	Female	10	5.4000	2.31900	.73333

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
interest	Equal variances assumed	1.500	.236	-.323	18	.751	-.40000	1.24007	-3.00529	2.20529
	Equal variances not assumed			-.323	16.509	.751	-.40000	1.24007	-3.02227	2.22227

Group Statistics

	gender	N	Mean	Std. Deviation	Std. Error Mean
interest	Male	10	5.0000	3.16228	1.00000
	Female	10	5.4000	2.31900	.73333

		Levene's Test for Equality of Variances	
		F	Sig.
interest	Equal variances assumed	1.500	.236
	Equal variances not assumed		

Independent Samples Test

t-test for Equality of Means						
					95% Confidence Interval of the Difference	
t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
-.323	18	.751	-.40000	1.24007	-3.00529	2.20529
-.323	16.509	.751	-.40000	1.24007	-3.02227	2.22227

Two-sided or one-sided?

- Depends on alternative hypothesis:
  - One-sided: H_1: $\mu > \mu_0$, or $\mu < \mu_0$
  - Two-sided: H_1: $\mu \neq \mu_0$