Tutorial 8

Research Methods for Politcal Science A

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1 & 2 December 2020

Outline

Hypothesis Testing

In general parlance:
When claiming a causal effect we want to ensure that the mean of our sample is indeed different to that of the population.

We therefore test whether we can reject the H_0 which states that:

$$H_0\text{: } \mu = \bar{x}$$

Hypothesis Testing

However in real life we often expect a directional effect. Our dependent variable either increases or decreases our independent variable.

$$\text{H}_1\text{: } \mu > \bar{x} \text{ or}$$
$$\text{H}_1\text{: } \mu < \bar{x}$$

Hypothesis Testing

Our Hypothesis is then the alternative hypothesis H_1:

$$H_1\text{: } \mu \neq \bar{x}$$

Hypothesis Testing

When deciding whether we can reject our H_0 we choose a level of significance (α)

The most common levels are $\alpha = 5\%$ and $\alpha = 1\%$.

T-Test

To test whether we can reject our H_0 we can use a t-test.

$$t = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{n}}$$

$\bar{X}$ = sample mean

μ = population mean (hypothesised)

$\hat{\sigma}$ = sample std. deviation

n = observation

T-Test Manually

```r
set.seed(1592)
```

```r
pop <- rnorm(10000, mean = 100, sd = 15)

# Create two samples
sample1 <- sample(pop, size = 75)
```

T-Test Manually

t-test for sample 1

$\bar{X}$ = mean(sample1)

$\mu = 100$ (we hypothesis that the population mean is 100)

$\hat{\sigma}$ = sd(sample1)

$\sqrt{n} = \sqrt{75}$ = sqrt(nrow(sample1))

df = n - 1 = 74

T-Test Manually

```r
t1.sample1 <- (mean(sample1) - 100)/
              (sd(sample1) / sqrt(75))
t1.sample1
```

```
## [1] 0.09321249
```

Exercise

Calculate the t-value for a sample with 100 observations.

Testing the Null Hypothesis

```r
t.crit <- qt(p = 0.05/2, df = 74)
t.crit
```

```
## [1] -1.992543
```

```r
t1.sample1 < t.crit
```

```
## [1] FALSE
```

```r
t1.sample1 > t.crit*-1
```

```
## [1] FALSE
```

Exercise

Calculate the t-value for sample ?

Can we reject H_0?

T-Test Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

T-Test Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
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	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

T-Test

$t_{sample} = 0.09$

$t_{critical} = 1.99$

$t_{sample} < t_{critical} \rightarrow$

we fail to reject H_0

Exercise

What is the critical value, if our sample has 500 observations?

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
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26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300

T-Test in R

```r
sample2 <- sample(pop, size = 250)
t.sample <- (mean(sample2) - 100)/
              (sd(sample2) / sqrt(250))

t.sample2 <- t.test(sample2, mu = 100,
                    alternative = "two.sided")
t.sample
```

```
## [1] 0.9529825
```

```r
t.sample2
```

```
##
##  One Sample t-test
##
## data:  sample2
## t = 0.95298, df = 249, p-value = 0.3415
```

T-Test in R

Do a t-test for three samples with 50, 30 and 15 observations respectively.

Can you reject a H_0?

T-Test in R

```
sample3    <- sample(pop, size = 15)
```

T-Test in R

```
t.sample3 <- t.test(sample3, mu = 100, alternative = "two.s

t.crit <- qt(p = 0.05/2,df = 14)
```

T-Test in R

```r
t.sample3
```

```
##
##  One Sample t-test
##
## data:  sample3
## t = 0.37284, df = 14, p-value = 0.7148
## alternative hypothesis: true mean is not equal to 100
## 95 percent confidence interval:
##   94.39052 107.97012
## sample estimates:
## mean of x
##  101.1803
```

```r
t.crit
```

```
## [1] -2.144787
```