Problem Set 2

PO 7001 — Quantitative Methods I

100pts total

1. If two balanced dice are rolled, what is the probability that the sum of the two numbers that appear will be odd? (4 pt)
2. Consider an experiment in which a fair coin is tossed once and a balanced die is rolled once. What is the sample space for this experiment? (4 pt)
3. Suppose that a random variable X has the uniform distribution on the integers $10, \ldots, 20$. Find the probability that X is even. (4 pt)
4. Suppose that two balanced dice are rolled, and let X denote the absolute value of the difference between the two numbers that appear. Determine and sketch the p.d.f. of X. (8 pts)
5. Suppose that a random variable X has a discrete distribution with the following probability density function:

$$f(x) = \begin{cases} \frac{c}{2^x} & \text{for } x = 0, 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

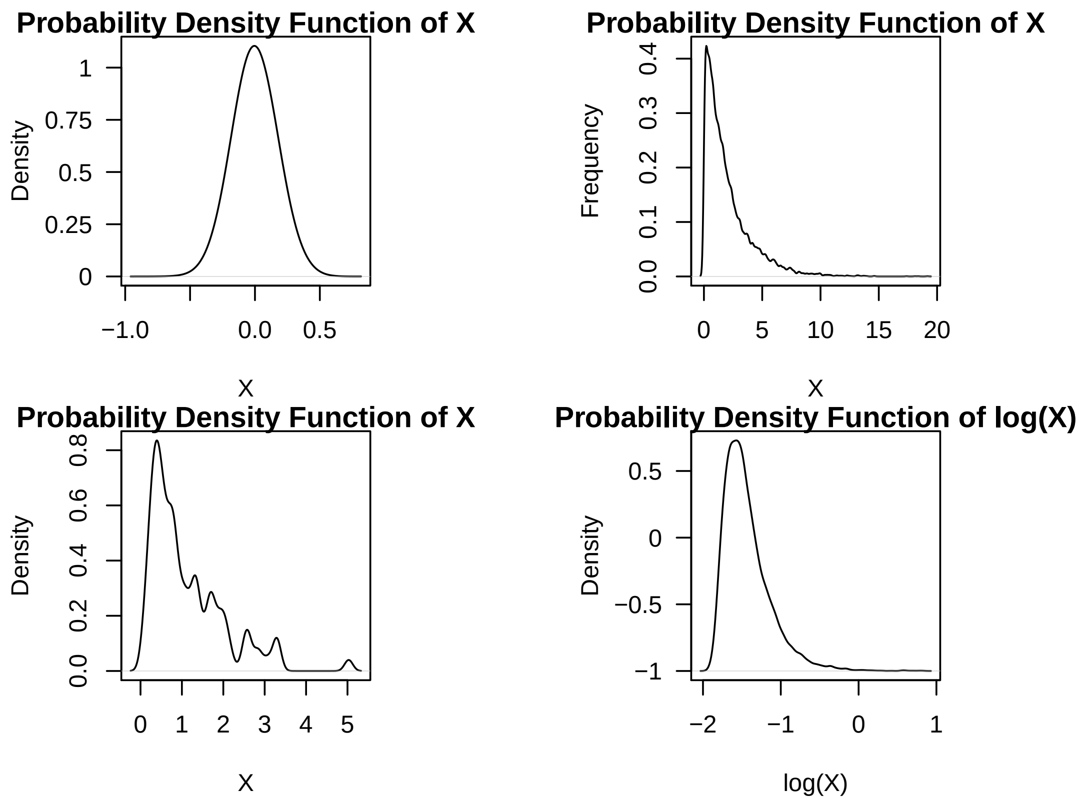
   Find the value of the constant c, if it exists. (12 pts)

6. Suppose that a random variable X has a discrete distribution with the following probability density function:

$$f(x) = \begin{cases} \frac{c}{x} & \text{for } x = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

   Find the value of the constant c, if it exists. (4 pts)

7. Suppose that a random variable X has the Bernoulli distribution with parameter $p = 0.7$. Sketch the cumulative distribution function of X. (4 pt)
8. Suppose that a coin is tossed repeatedly until a head is obtained for the first time, and let X denote the number of tosses that are required. Sketch the c.d.f. of X. (8 pts)
9. Consider the various graphs below. For each, discuss whether and how the graph may (or may not) be incorrect. Each graph may have one, several, or no mistake. (4 pt each)



10. Let $X \sim N(2, 3)$. Calculate $f(0)$, where f is the p.d.f. of X. Calculate it manually (i.e., not using R's canned functions, though you may use R's basic arithmetic tools—or a calculator). (4 pt)

11. Let $X = \{1, 12, 13\}$, $Y = \{14, 9, 7\}$ and $Z = \{2, 24, 36\}$.

    a. Calculate manually (you may use a calculator, but need to show your steps): (4 pt)

    $$Cov(X, Y)$$

    $$Cov(X, Z)$$

    b. What do you conclude about which of the two pairs ((X, Y) and (X, Z)) is more closely related? (4 pt)

12. Using R, generate 200 samples of 10000 observations each, each drawn randomly from a standard normal distribution. Calculate the covariance of every pairwise sample. Plot the frequency of these sample covariances. What is the standard deviation of these covariances? Superimpose a normal distribution on your frequency plot. (12 pts)

13. We find that two variables, X and Y, are highly correlated with $r = 0.85$. Can we conclude that X and Y were drawn from the same distribution? Justify. (4 pt)

14. Let $\mathbf{X}_1 = \{x_1, x_2, \ldots, x_n\}$ be normallly distributed random variables with mean -2 and standard deviation 1. Let $\mathbf{Y}_1 = \{y_1, y_2, \ldots, y_n\}$ be normallly distributed with mean $+2$ and standard deviation 1. Let $\mathbf{Z} = \{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n\}$, i.e., $\mathbf{Z}$ combines these two vectors into one. How is the mean of $\mathbf{Z}$ distributed? Argue why and show it empirically using R. (8pts)

15. (Bonus question): Prove that for a variable B distributed Bernoulli($p\varepsilon$), $\sigma^2 = p\varepsilon(1 - p\varepsilon)$. (1 pt)