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MECE 5397

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Diffusion Equation

**Abstract**

With the help of computers, complex simulations and discretizations have been able to help solve in minutes the very same problems that would have taken months to solve by hand. As an exercise in scientific computing, this project compares two different discretizations, the explicit and Crank Nicolson and some of their respective features. Some key features such as their time to compute, number of iterations to complete reveal the benefits of different discretization schemes. Next the project will conduct a brief grid convergence study where the grid will be increased in size until the change in the solution is sufficiently small. Furthermore, this project will compare the experimental graphs with the expected theoretical behavior. Included with the mathematical analysis is a history of the project via a git repository and some associated folders.

**Problem Statement**

Solve the 2D diffusion equation given the following domain and boundary conditions using the Crank Nicolson and Implicit Discretization methods.

*Rectangular Domain*

*Boundary Conditions*

*Given*

**Discretization**

Crank Nicolson

Explicit

Description of the numerical method (pseudo code included)

%% Get user inputs for N, select which discretization, select accuracy

%% Boundary Conditions

% Domain

ax = 0;

ay = 0;

bx = 2\*pi();

by = 2\*pi();

%grid

x = linspace(ax,bx,N);

y = linspace(ay,by,N);

[X,Y] = meshgrid(x,y);

h = (bx-ax)/(N+1);

%boundary conditions

u = zeros(N,N);

fb = ((by-y).^2).\*cos(y.\*pi/by);

gb = (by-y).^2.\*y;

u(1,1:N-1) = fb(1,1:N-1); %left bound

u(1,1:N-1) = gb(1,1:N-1); %right bound

toolong = fb(1) + ((x-ax)/(bx-ax)).\*(gb(1)-fb(1));

u(1:N-1,1) = toolong(1:N-1,1); %lower bound

toolong2 = fb(N) + ((x-ax)/(bx-ax)).\*(gb(N)-fb(N));

u(1:N,N) = toolong2(1:N,N); %made up boundary, couldnt figure out ghost node

%Crank Nicolson Method

%check degree of error

% explicit method

%check degree of error

% Plot Crank Nicolson

surf(X,Y,CN\_u);

% Plot Explicit Method

surf(X,Y,Exp\_u);

%% Results

fprintf('time to compute with Crank Nicolson Method is: %5.4f',value1)

fprintf('time to compute with explicit method is: %5.4f',value2)

fprintf('iterations with Crank Nicolson, value3)

fprintf(''iterations with explicit method, value3)

• Technical specifications of the computer used

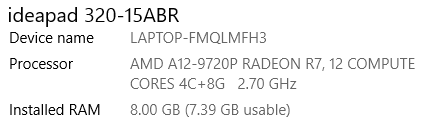


Figure 1: Computer Specifications

• Results (include graphs and comments)

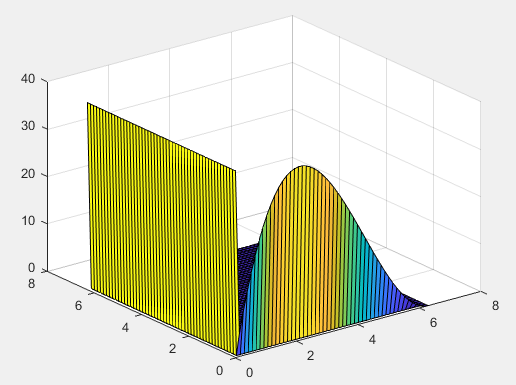


Figure 2: Mesh Boundary values

Figure 1 shows the boundary values for N=50.

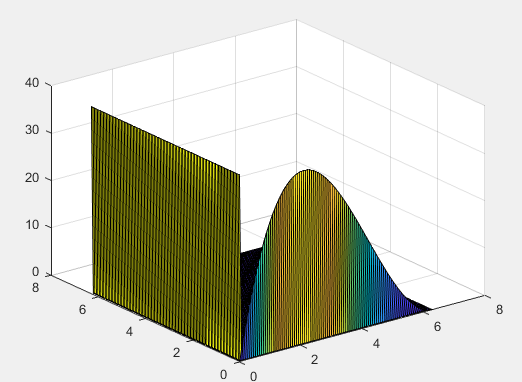


Figure 3: Boundary values N=100

This graph shows the boundary values for N=100 grid elements. Notice how the mesh was more precise than the case for N=50.