1 Toroidal Sampling

In our experiment setting, all the polarizers, wavaplates can be rotated from 0 to 2π, thus, the parameter space becomes a $4n$ torus, given that there are one polarizer and three waveplates in each NPR section, where n is the number of NPR sections. This brings us to a question , how to sample a torus? Random sampling is a viable way, however, there is a better way which we call it toroidal search. Let's take a 2-torus for example. Assume we have two time series given by

$$\begin{aligned} \theta_1(t) &= \omega_1 t + \theta_{10} \\ \theta_2(t) &= \omega_2 t + \theta_{20} \end{aligned} \tag{1}$$

where $theta_{10}$ and $theta_{20}$ are initial parameter values, ω_1 and ω_2 are angular frequencies which are incommensurate, i.e. equation

$$m\omega_1 + n\omega_2 = 0 \tag{2}$$

doesn't have integer solution, in another word, $\frac{\omega_1}{\omega_2}$ is irrational. It is easy to proof [Wiggins] that under such conditions, $[\theta_1(t), \theta 2(t)]$ is dense on the torus. Which means using this method, it is guaranteed that one can sample any torus well enough if sampling for a long enough time.

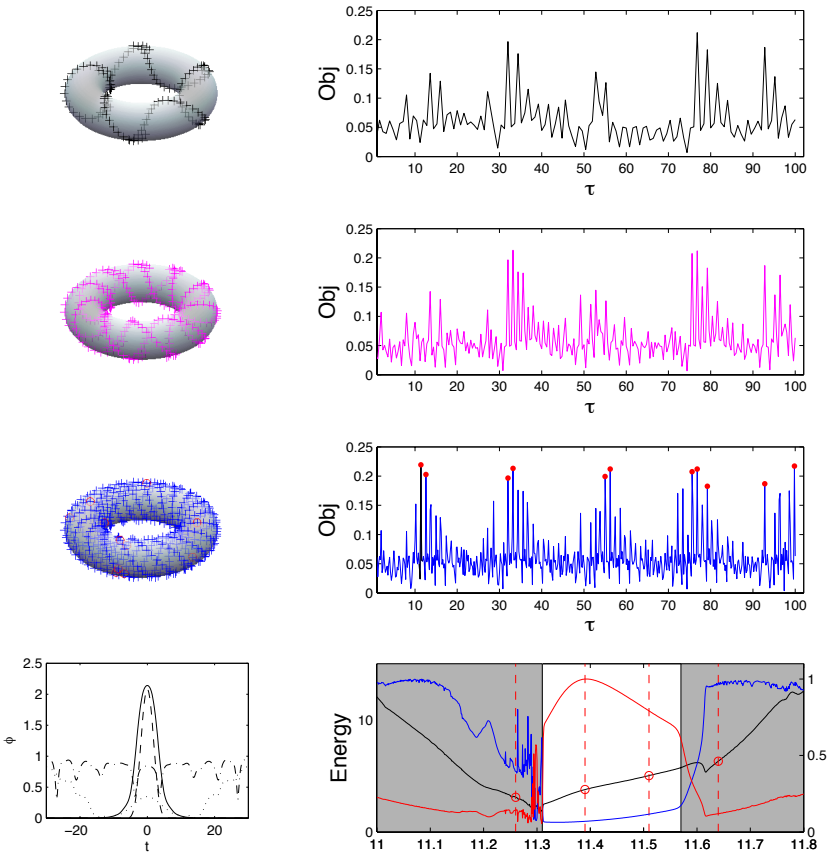

Figure 1: