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The Pitcher and Batter Game

The Pitcher/Batter game poses an ideal interaction to investigate with a model. As two individuals interacting repeatedly in a zero-sum game without full information on other players' intentions, each player must approach the game knowing the other players' potential values for each outcome, but must also mix their strategies in order to not be predictable. This is compounded by the fact that each mini-interaction is slightly different, as different skill levels and past histories will affect the pitch count, which affect potential results for each player. Additionally, there is a strong influence of probability: each batter has varying skill levels, which create an array of probabilities of whether or not the batter will get a hit. To counteract this and come up with a general model, we will utilize league averages for both the batter and the hitter giving us a generic encounter. The pitcher must factor in the probability of a hit in a given situation when deciding to throw a "ball" or attempt to get a strike. In this paper, we will construct, describe, and outline our model, backing it up with real world data taken from *Fan Graphs,* a source for Major League Baseball statistics. Our model will demonstrate that there is no Nash Equilibrium in pure strategies for this game, which is why we see such an interesting interaction as both players must mix their strategy in order to avoid becoming predictable. We find that traditional knowledge appears to be accurate as to the advantaged party in each situation, and observe that in this generic case, the person temporarily advantaged will adopt a more risky strategy.

Literature Review

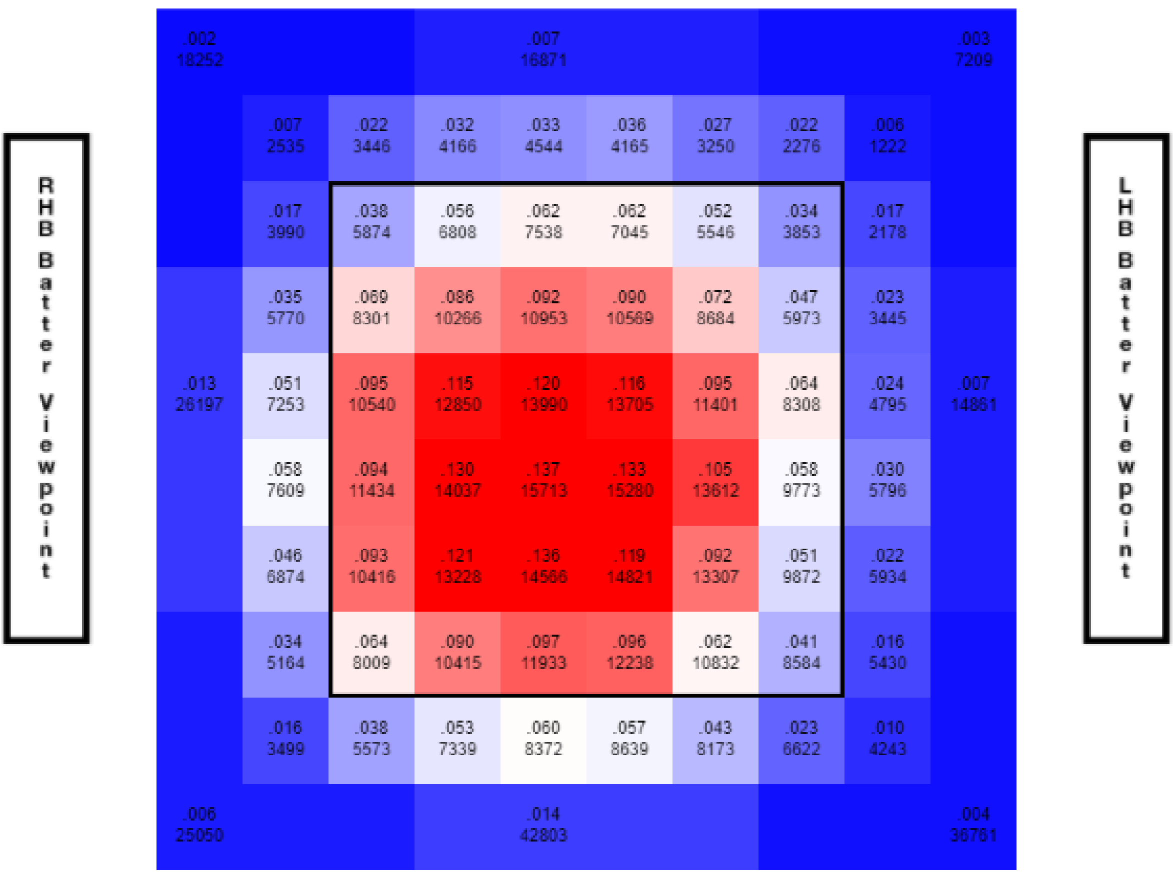
Generally in the game theoretical literature that surrounds baseball we see a very narrow focus when looking at the pitcher-batter interaction. Rather than attempt to make a cohesive argument for the whole thing through simplification, we see an attempt to add detail to a more specific event. Thus we see models such as William Spaniel's that describes the interaction between pitcher and batter with a runner on third base in which the pitcher wants to avoid throwing curveballs to not throw one past the catcher, but nonetheless maintains the same ratio in order to not be predictable for the batter (Spaniel 2012). Other investigations go beyond the minutiae of a certain play or game, such as Goldstein and Young's look at evolutionary strategies in the sphere of handedness and how over the length of baseball's existence they have altered in ratio. Their predictions found that baseball should converge to 31% of pitchers being left handed, and 27% of batters, with 11% of batters being capable of both sides, something that was observed in modern play (Goldstein and Young 1996). Finally, a more informal look at game theoretical interactions specifically in an 0-2 count is done by Drew Fairservice. He uses anecdotal evidence to suggest that even given the pitcher's advantage in this situation, he should consider throwing a fastball down the middle, something often viewed as an easy pitch to hit. He posits that the unexpectedness of the batter makes this a viable strategy, i.e. maintains a pitchers unpredictability as our game theory model will predict and show as an option. (Fairservice 2014)

Model Description

The model we will be using pits the two players against each other, with the pitcher as the sender and the batter as the receiver. Neither player has full information as to what the other is doing, i.e. the pitcher does not know if the batter plans on swinging at the pitch or taking the pitch, and the batter does not know if the pitcher plans on throwing a strike or throwing a ball. In order to facilitate interpretation, we are positioning the pitcher as having only these two choices, throwing a strike or throwing a ball, and being able to do both of these at will; the pitcher will never attempt to throw a strike but fail. Similarly the batter will choose to swing or take a pitch before the pitcher throws, and nothing after the pitcher's decision will impact his likelihood of doing either strategy. Furthermore, each mini-interaction will be classified as one pitch, in the context of a changing pitch count. As hitters traditionally perform differently based on the pitch count, this will effect the outcome values in each interaction, meaning that the history will play a prominent role in determining the choices of each interactor in every stage game. To further simplify the model to a generic interaction, we will utilize working MLB league averages for the various situations in our payoff models, rather than focusing on the strengths or weaknesses of any particular set of players. This will allow the analysis to be extrapolated to the general situation more easily.

Supportive Data

Using data from *Fan Graphs*, we found that as expected, the average hitter will do better swinging at strikes as opposed to balls in almost all situations. While some individuals may have quirks meaning they are more effective in certain areas that are not strikes, across the entire league we see a regression to the mean in which pitches inside the strike zone are hit at a much higher rate. Thus, our model is able to use this data to view swinging at a strike as a positive effect and a ball as a negative, with a value determined by league wide averages on these pitches. To do so, we broke the strike zone down into two broad categories, inside, and outside the zone. While this may be an oversimplification for the actual game, it serves the purpose of allowing the model to function in a generic situation.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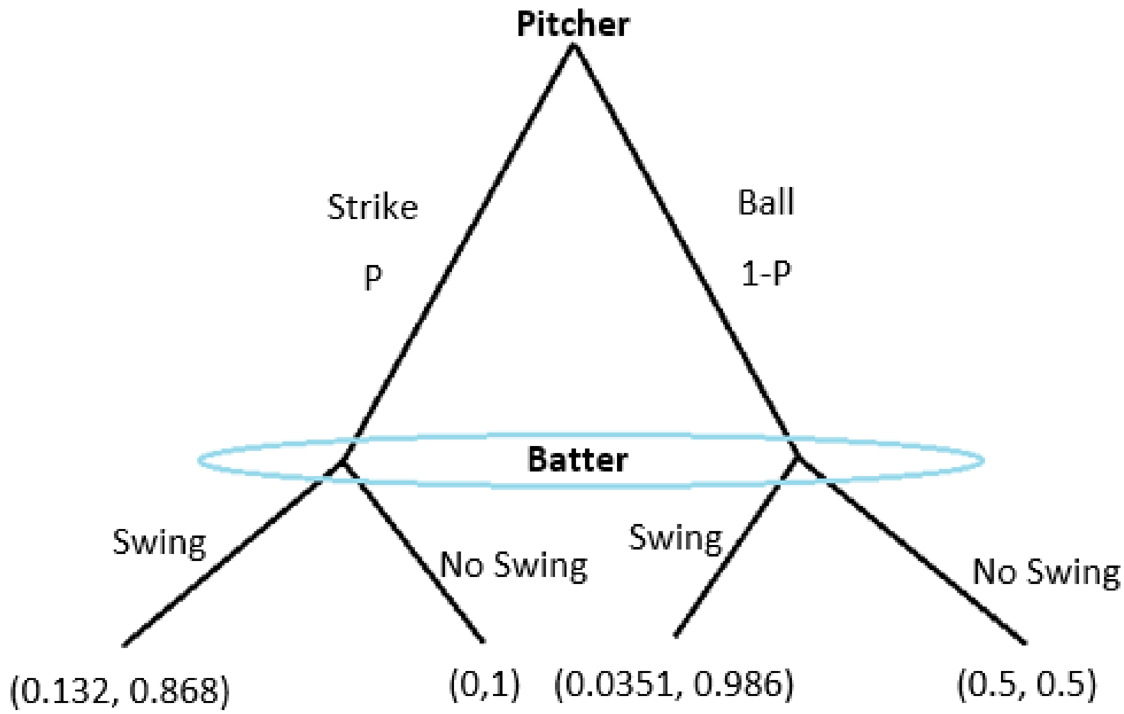
Model



Batter

Pitcher		Swing	No Swing
	Strike	0.132, 0.868	0,1
	Ball	0.0351, 0.986	0.5, 0.5

From the earlier description of our model there are a couple of specifications to add in order to finalize it. First, we describe each interaction as a zero-sum game where there is a total of 1 value available. A hit will be characterized as worth that full one, meaning that the hitter gets 1 point and the pitcher gets 0. A strikeout will be worth one to the pitcher, meaning the inverse is true. Finally, our last possible outcome, a walk will be worth half to each. As the interaction in baseball is pitcher favored, half to each is an improvement on expectation and thus a “win” for the batter. To facilitate an understanding of the whole model, it is easier to start with backwards induction, beginning at a 3-2 count in which a ball will lead to a walk as the result, a strike a strikeout, and a hit, naturally a hit. From this point, we will work backwards using the expected value results from the 3-2 count to find those of the 3-1 and 2-2 count as proxies for the value of a strike and ball respectively there.

For a 3-2 count, given the payoffs presented, there does not appear to be any Nash equilibrium in pure strategies. If either party becomes predictable, the other will react in such a way as to make another option more viable. Thus, if we are to find an equilibrium point, it appears as if we must look to a mixed strategy Nash Equilibrium. This equilibrium point can then be used to calculate the expected values of each option in order to begin our process of reverse induction

Looking at the situation of the 3-2 count, we find that in an equilibrium, receiver indifference (the batter), is only reached when the pitcher is throwing a strike 78% of the time and a ball 22% of the time. Similarly, we find that sender indifference (the pitcher) is only reached when the batter is swinging 83.6% of the time and taking the pitch 17.4% of the time. If one looks at the expected values for each player in a 3-2 count using these numbers, we thus find that the expected value for a batter in this situation is .11 and that of a pitcher is naturally .89 given the fact of it as a zero sum game. Thus, we can use these outputs to say that in a 3-1 count, the outcome of a strike will make it a 3-2 count and so change the expected value for that result to .11 for a batter and .89 for a pitcher. In much the same vein, the value of a ball in a 2-2 count will be the same.

If we move to a 3-1 count, it appears that the hitter has the initial advantage by way of having an extra strike to maneuver with. In common baseball parlance, this is known as a "hitter's count" along with the 2-0 count. As we will see, this traditional knowledge and acceptance of this situation as batter favored is borne out by our model and provides more support to it. The data here shows a higher likelihood of a positive result from the batter when he swings at a strike as well as when he swings at a ball. This is likely because a batter in this situation understands that they have an extra margin of error, and so are willing to swing at only the best pitches. Thus, our payoffs tend to trend higher for the batter in this situation, given his worst case scenario is swinging at a ball, and even then that often leads to him simply moving on to the 3-2 count situation. The indifference conditions here result in the batter swinging at 87.6% of pitches, and the pitcher throwing strikes 92.4% of the time, with the batter swinging at more than in the full count due to the increased freedom, and the pitcher being forced to throw more strikes due to the possibilities of a walk that are severely harmful to the pitcher. The expected value from this situation results in the batter with an expected value of .14 and the pitcher with an expected value of .86. While we do see the pitcher with a much higher expected value then the batter seeming to suggest this is not a favorable situation for the batter, this cannot simply be seen in a vacuum. Rather, it must be looked at in context relative to the batters chances in a "base" situation for which we are using the full count as a stand in for. Relative to that, we see an increase in expected value for the batter and a decrease for the pitcher, aligned with the traditional view of this as a batter's count or a situation which favors the batter over the pitcher.

Similar to the common knowledge “batter’s count” of 3-1 a situation of 2-2 is often considered a pitcher’s count that favors the sender in our game. Thus we should expect to see an increase in the expected value for the pitcher, and a decrease for the batter. Our data collected, shows that hitters tend to have a slightly lower average in this situation then in our base case scenario, but it is relatively minor as a difference. Nonetheless, the extra flexibility the pitcher gains with an extra margin of error as granted by the extra ball appears to play a large role. Despite a ball moving into the base situation, we see a drastic change in how the game is approached by both players. In finding indifference for this subgame, the pitcher will only throw a strike 40% of the time, and the batter will swing 54.7% of the time, vast swings from what we have experienced previously in the model. The pitcher tends to shy away from the strike zone much more often, using the flexibility granted to try and coax the batter into the bad result of swinging at a ball. The batter meanwhile, adapts to this by swinging much less, though still more than half the time to avert the worst case possibility of not swinging at a strike which in this situation results in an out. Due to these new constraints, this scenario finds an expected value for the batter of .064 and an expected value for the pitcher of .936. This seems to indicate a strongly pitcher favored scenario relative to the base case, as traditional knowledge seems to suggest is true in this scenario.

Rather than belabour the point with additional examples of the changes in probabilities for every potential count, we can draw some pointed evidence from the rest of the data set. It appears that in every case the common knowledge available of what is considered a “hitter’s count” (3-1, 3-0, 2-0) and what is considered a “pitcher’s count” (2-1, 2-2, 1-2, 0-2) all show expected values that lean towards the person expected to. Along with this, we see the further the pitcher gets ahead, the less likely he is to throw a strike that the batter could potentially profit of by getting a good hit on, and similarly the further behind he is, the more likely to throw a strike as he cannot risk the potential danger of walking a hitter.

Naturally, our analysis has some flaws when applied to the real world situations hitters and pitchers find themselves in. Stand out among those is the fact that pitchers are not infallible machines, at times they will attempt to throw a strike and fail, or attempt to throw a ball and throw a strike. Similarly, batters are not simply deciding and then acting. Rather seeing a pitch will give them some limited insight as to whether it is likely to be a strike or ball and they will act on this limited information. Additionally, this generic matchup between an average pitcher and average hitter is not overly informative in every situation. In many, the particular talents of a certain player will outweigh the traditional strategies and thus tilt the payoffs to favor something. Finally, it is possible that we can see an impact of shame acting in a real life scenario. For a hitter, it may reflect poorly on their teammates if they simply look at a pitch that goes by them for a strikeout, and so this could be shown in an additional penalty to utility for this outcome. Overall, the model should do an acceptable job despite these flaws.

Conclusion

As we have shown, the Pitcher/Batter interaction in a Baseball game can be simplified into a working Game Theory model. The model contains two players: The Pitcher is the Sender, and the Batter is the Receiver. Both players are attempting to optimize their outcomes, have two strategies to choose from. The Pitcher is choosing to either attempt to throw a Strike or to throw a Ball. Simultaneously, the Batter is choosing whether they should Swing or Not Swing. Neither have perfect information, and each must factor in the relative probabilities of success based on their own skill levels, and knowledge of their opponent.

Having laid this framework for our model, we have taken real Major League Baseball data from *Fan Graphs* and applied it to observe what kind of payoffs players can expect to see in repeated games. Based on this data, we see that there is no true dominant or dominated strategies for either player. In addition, there is no Nash Equilibrium in pure strategies as to do so would simply invite the other player to switch into an easily better alternative. Rather we see that in order to maintain an equilibrium, both players must be playing a mixed strategy of some kind in the subset. The particular weights in this strategy will be moderated by the situation the player finds themselves in. Overall, we find there is some truth in the conventional wisdom of the game, unlike in many circumstances in baseball's encounters with game theory throughout the years.

By breaking the Pitcher/Batter interaction down into a simple game, we are able to clearly see how each strategy will work for each player as well as the changes that will occur based on the count, which also determines the potential amount of subgames left in the future. Further analysis would delve into more specific interactions with an expanded data set in order to find more precise results.

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