The 2D Digital Waveguide Mesh

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Presentation Outline

1. Paper Citation
2. 1D Digital Waveguide
3. Lossless Scattering Junctions
4. 2D Digital Waveguide Mesh
5. Future Work for the Final Project

Paper Citation

Van Duyne, Scott A., and Julius O. Smith. **“Physical modeling with the 2-D digital waveguide mesh.”** *Proceedings of the International Computer Music Conference (ICMC)*, 1993.

1D Digital Waveguide

- The 1D wave equation models the transverse displacement of a lossless ideal vibrating string as follows

$$u_{tt} = c^2 u_{xx}$$

- Note that the displacement $u = u(x,t)$ is a function of position x and time t, u_{tt} is the second partial derivative w.r.t. time (i.e. the **acceleration**), and u_{xx} is the second partial derivative w.r.t. longitudinal position along the string (i.e. the **curvature**)

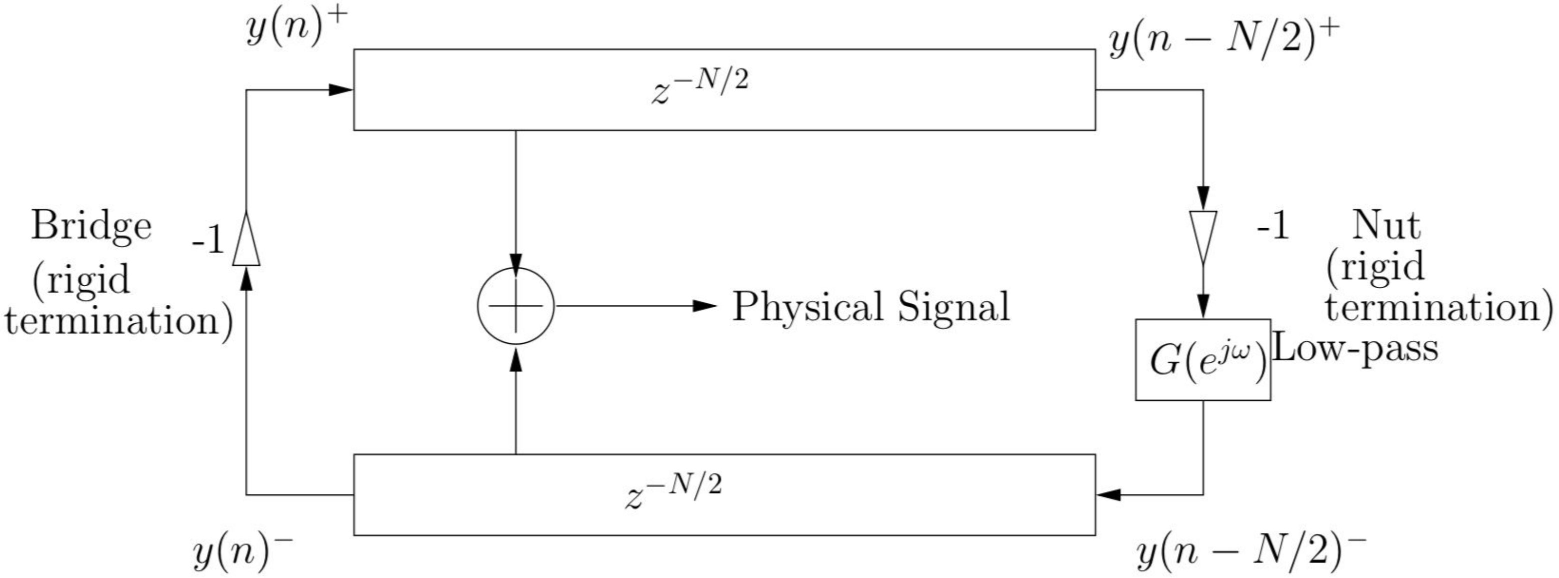
1D Digital Waveguide

- Jean d'Alembert showed in 1747 that a sum of traveling waves moving in opposite directions is a valid solution to the 1D wave equation:

$$u(x,t) = g(x - ct) + g(x + ct)$$

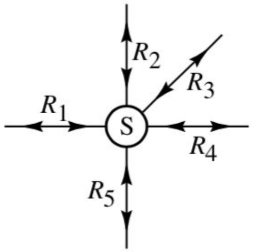
- This "traveling wave solution" may be implemented digitally using a pair of **bi-directional delay lines** to create a digital waveguide (DWG)
- We can replace *u* with either transverse force *f* or transverse velocity *v*, which is a convenient choice since ***f* = *vR* is analogous to *V* = *ir*** (Ohm's law), where *V* is voltage, *i* is current, *r* is resistance, and *R* is wave impedance

1D Digital Waveguide



Lossless Scattering Junctions

- A lossless scattering junction is a single point of **possibly varying** wave impedance where several waves intersect without any loss of energy



- Scattering junctions may be used to **model a wide variety of musical instrument phenomena**, including sound propagation in the vocal tract, the bowed action of a violin, tone holes in wind instruments, and strings coupled at the bridge of a guitar

Lossless Scattering Junctions

- For a **series** junction (e.g. a string junction), we have the following conditions (note that *v* is the transverse velocity and *f* is the transverse force):
  - $v_1 = v_2 = \ldots = v_N$ *(analogous to* ***current*** *in a series circuit)*
  - $f_1 + f_2 + \ldots + f_N = 0$ *(analogous to* ***voltage*** *in a series circuit)*
- For a **parallel** junction (e.g. an acoustic tube junction), we have the following conditions (note that *P* is pressure and *U* is volume velocity). Look familiar?
  - $P_1 = P_2 = \ldots = P_N$ *(analogous to* ***voltage*** *in a parallel circuit)*
  - $U_1 + U_2 + \ldots + U_N = 0$ *(analogous to* ***current*** *in a parallel circuit)*

Lossless Scattering Junctions

- Combining the traveling wave relations, wave impedance relations, and series junction conditions, we can derive the **lossless scattering equations**:

$$v_J = 2\frac{\Sigma_i R_i v_i^+}{\Sigma_i R_i}$$

$$v_i^- = v_J - v_i^+$$

- Note that v_J is the junction velocity, v_i^+ are the incoming wave velocities at the junction, and v_i^- are the outgoing velocities. This equation tells us how much of the wave will **transmit** at the junction and how much will **reflect**

2D Digital Waveguide Mesh

- The 2D wave equation models the transverse displacement of a lossless ideal vibrating **membrane** as follows

$$u_{tt} = c^2 \Delta u$$
$$(\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$$

- The displacement $u = u(x,y,t)$ is a function of position x, position y, and time t

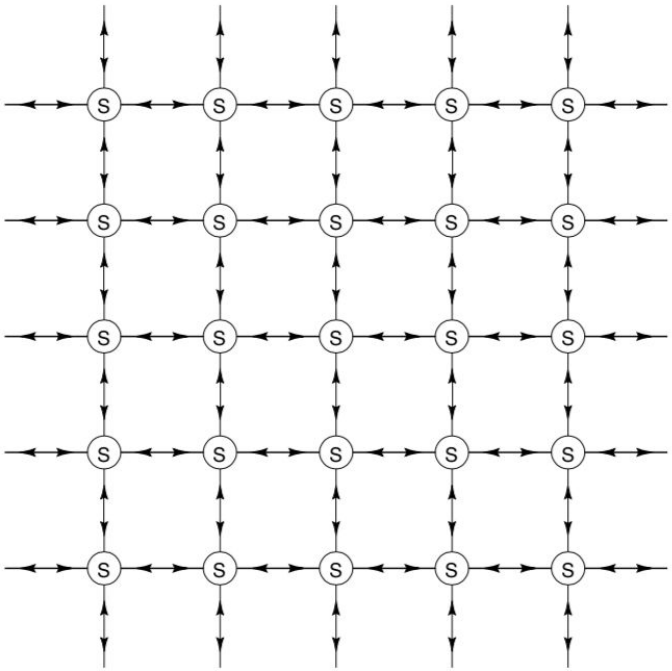
2D Digital Waveguide Mesh

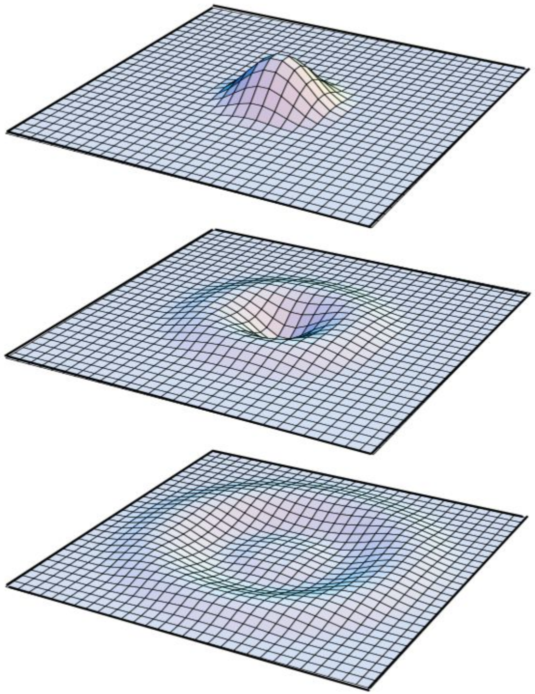
- The traveling wave solution to the 2D wave equation involves an **infinite** sum of plane waves moving in **all** directions, as shown below

$$u = \int g_\alpha(x\cos\alpha + y\sin\alpha - ct)\, d\alpha$$

- Obviously, it's not practical to assign a 1D DWG to each of these infinite number of plane waves, so an exact simulation is not feasible. **The 2D DWG mesh serves as an approximation to the traveling wave solution**.

2D Digital Waveguide Mesh





2D Digital Waveguide Mesh

- The 2D DWG mesh consists of a **network of bi-directional unit-sample delay elements** and **N 4-port scattering junctions**, where N = # rows x # columns of the mesh
- In the standard formulation of the mesh, the scattering junctions are **equal-impedance** lossless junctions
- The series and parallel junction conditions each apply in the 2D DWG mesh. Thus, we may view the mesh as a lattice of vibrating strings (series) or a lattice of acoustic tubes (parallel).

Future Work for the Final Project

- Implement the 2D DWG mesh algorithm in Matlab for a particular **use case**, such as a circular drum or a rectangular soundboard. For the implementation, I will need to consider **boundary conditions** (which I have not covered in this presentation)
- Review Van Duyne's **mathematical analysis** where he shows that the 2D DWG mesh coincides with the standard finite difference scheme for the 2D wave equation
- Show spectrum of impulse response and evidence of detuning in the higher frequencies

Questions