CSE 368: INTRODUCTION TO AI

Search: Graph search and Local search

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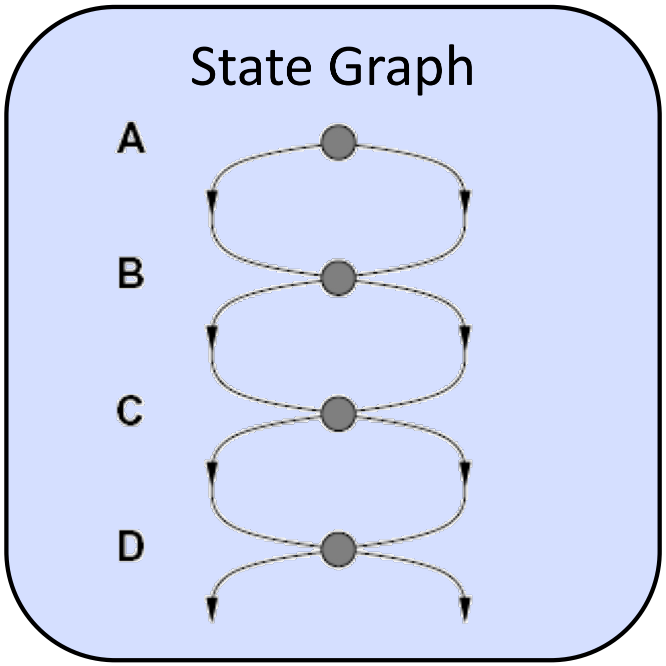
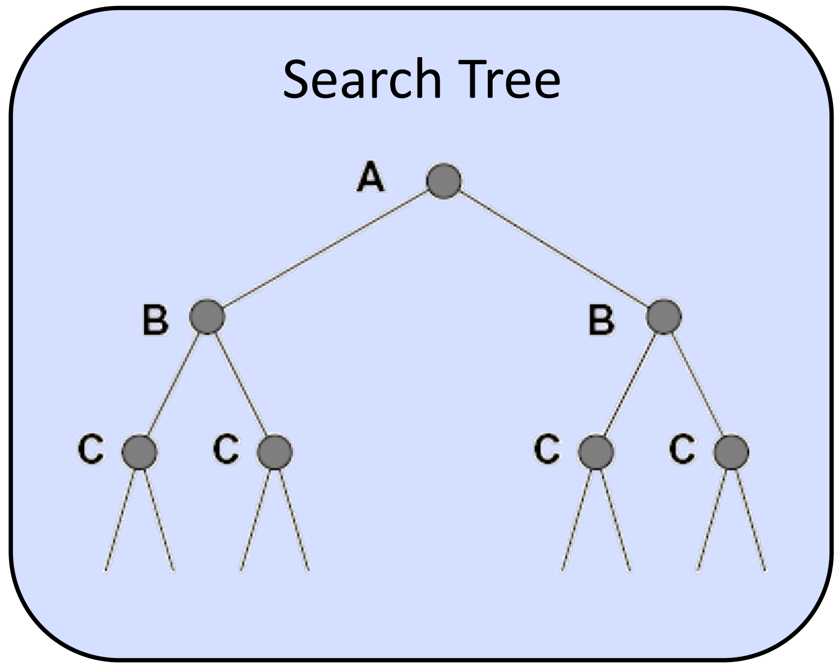
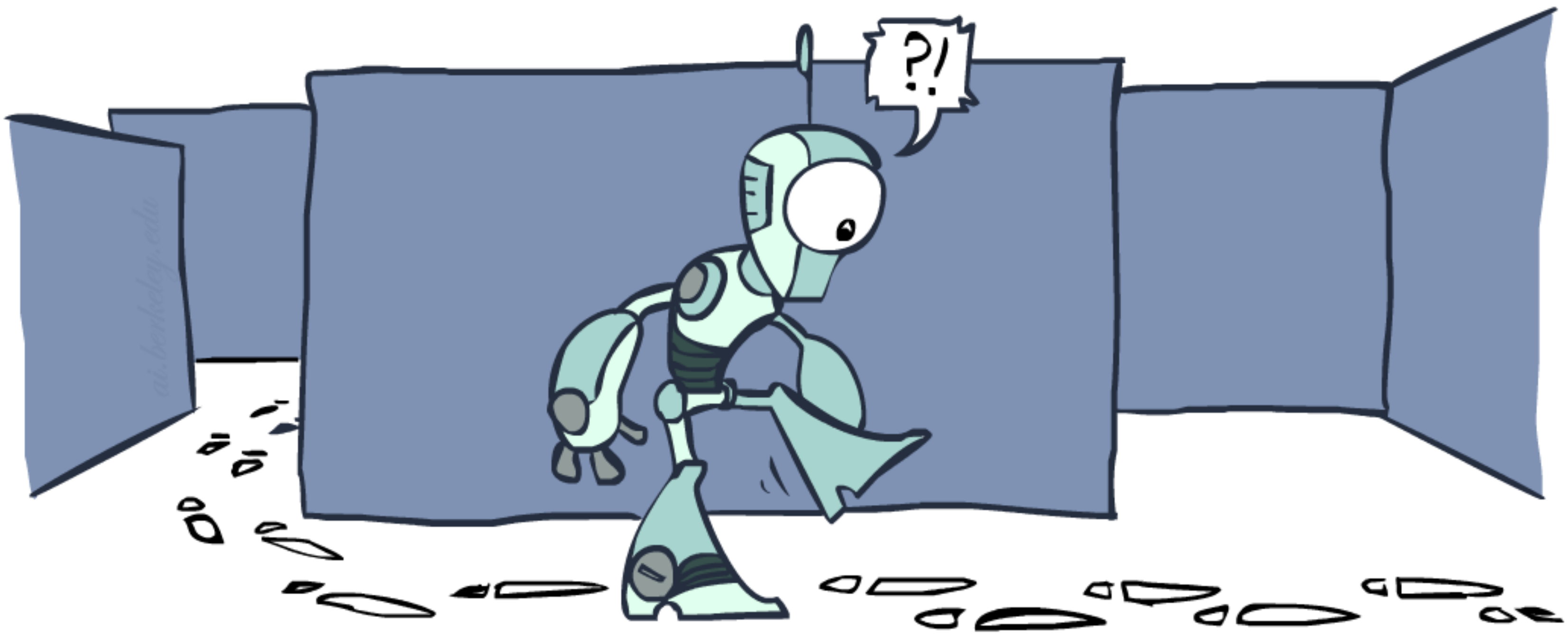
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



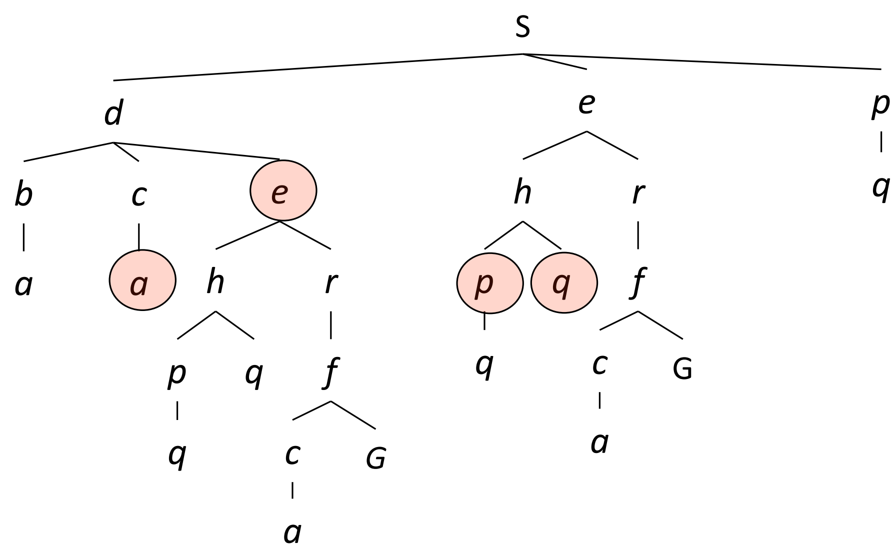


Graph Search



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

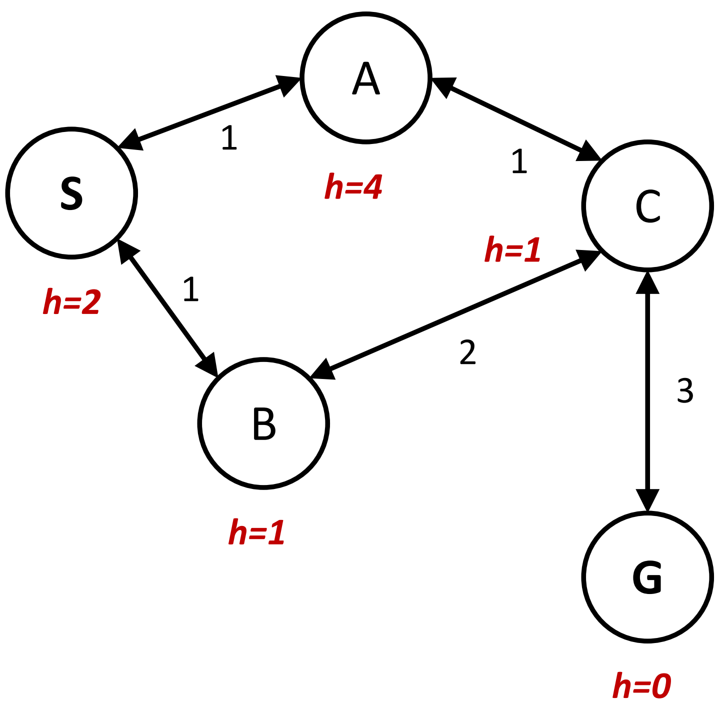


Graph Search

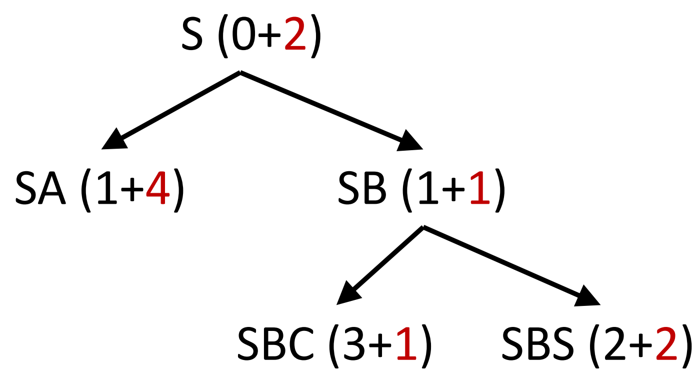
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

State space graph



Search tree

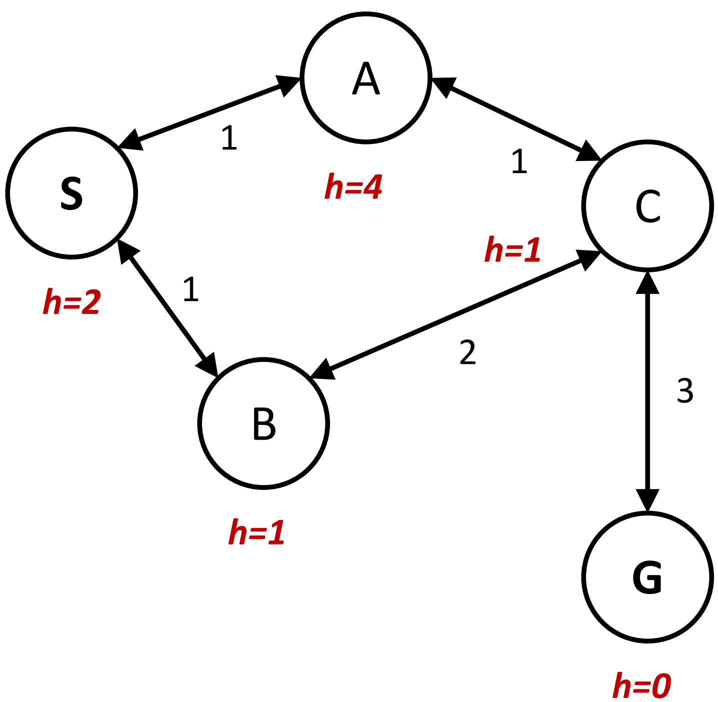


Closed set

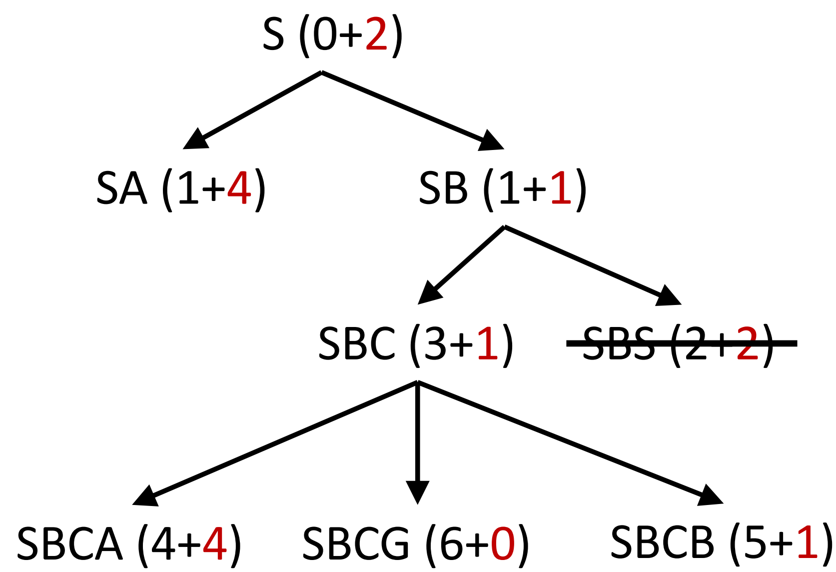
{ S B }

A* Graph Search Gone Wrong?

State space graph



Search tree

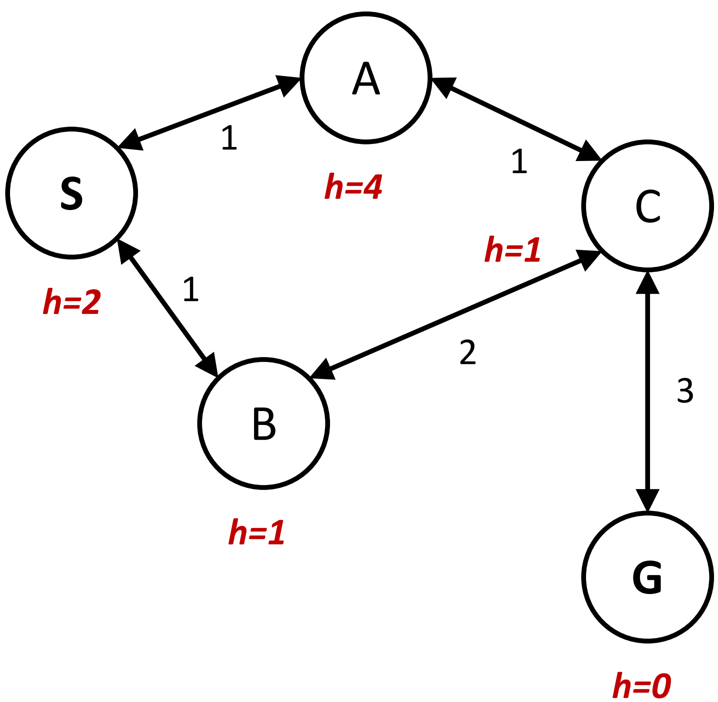


Closed set

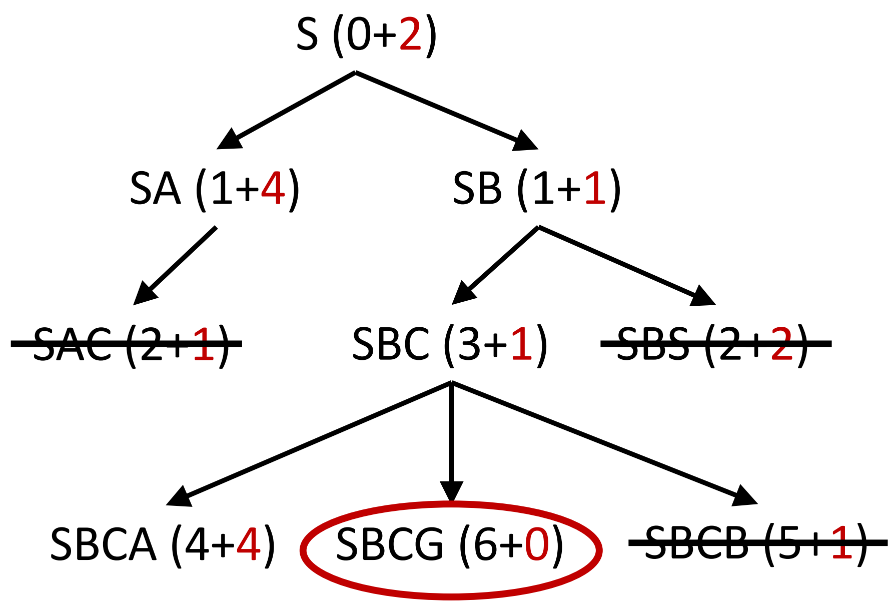
{ S B }

A* Graph Search Gone Wrong?

State space graph



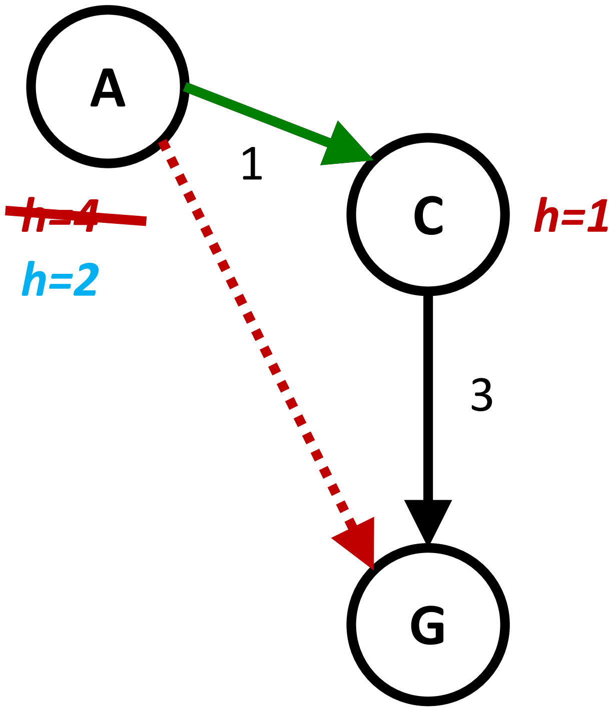
Search tree



Closed set

{ S B C A }

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost *h** from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

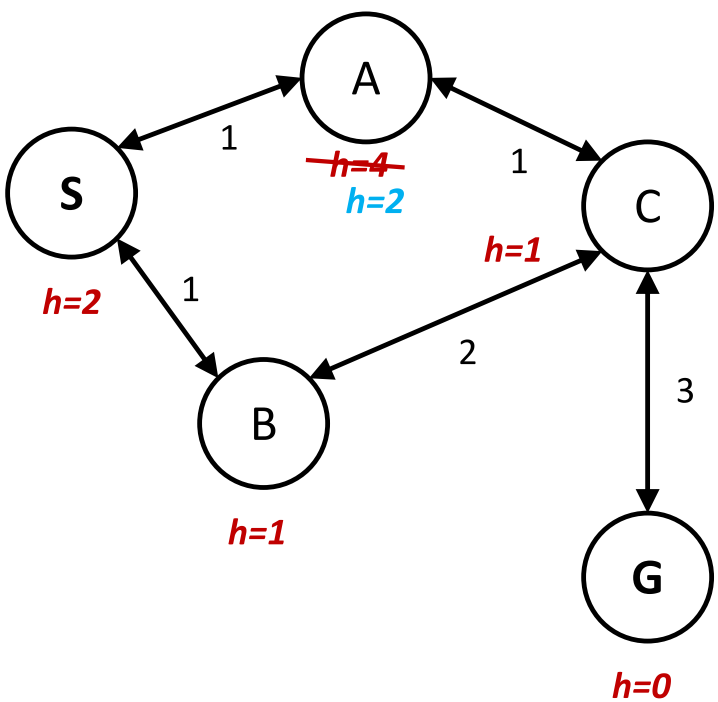
    - a.k.a. "triangle inequality": h(A) ≤ cost(A to C) + h(C)
    - Note: true cost *h** necessarily satisfies triangle inequality
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

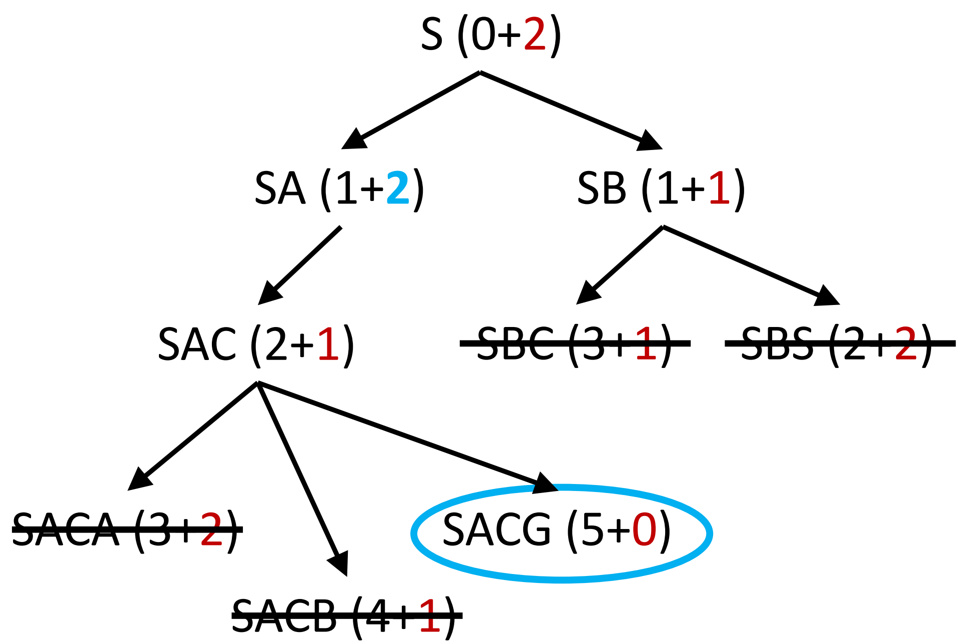
  - A* graph search is optimal

A* Graph Search with Consistent Heuristic

State space graph



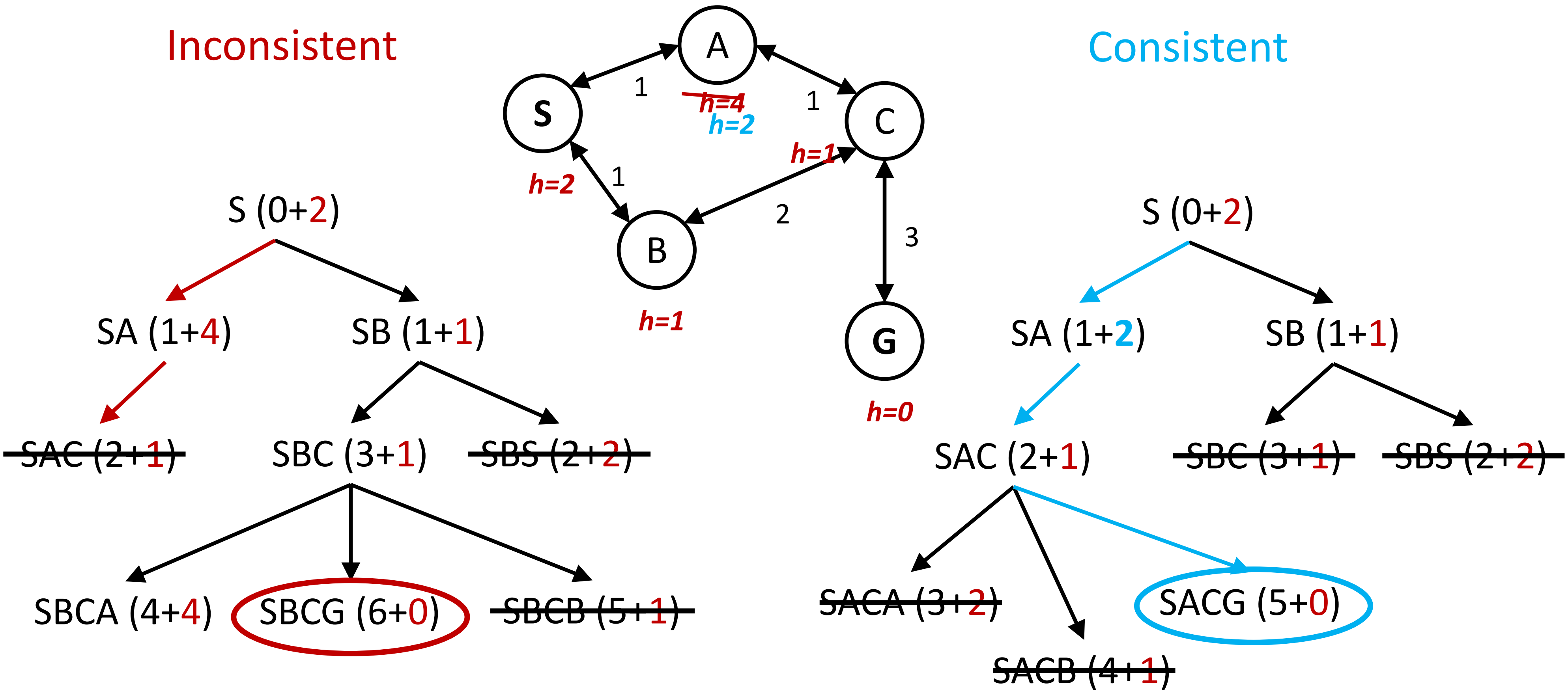
Search tree



Closed set

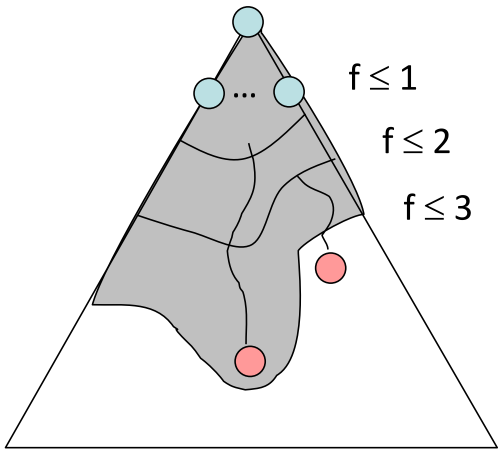
{ S B A C }

Consistency => non-decreasing f-score



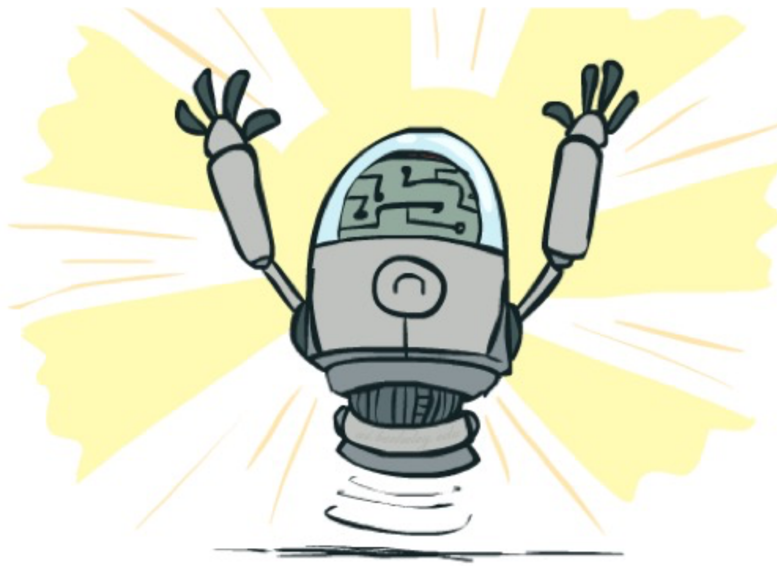
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

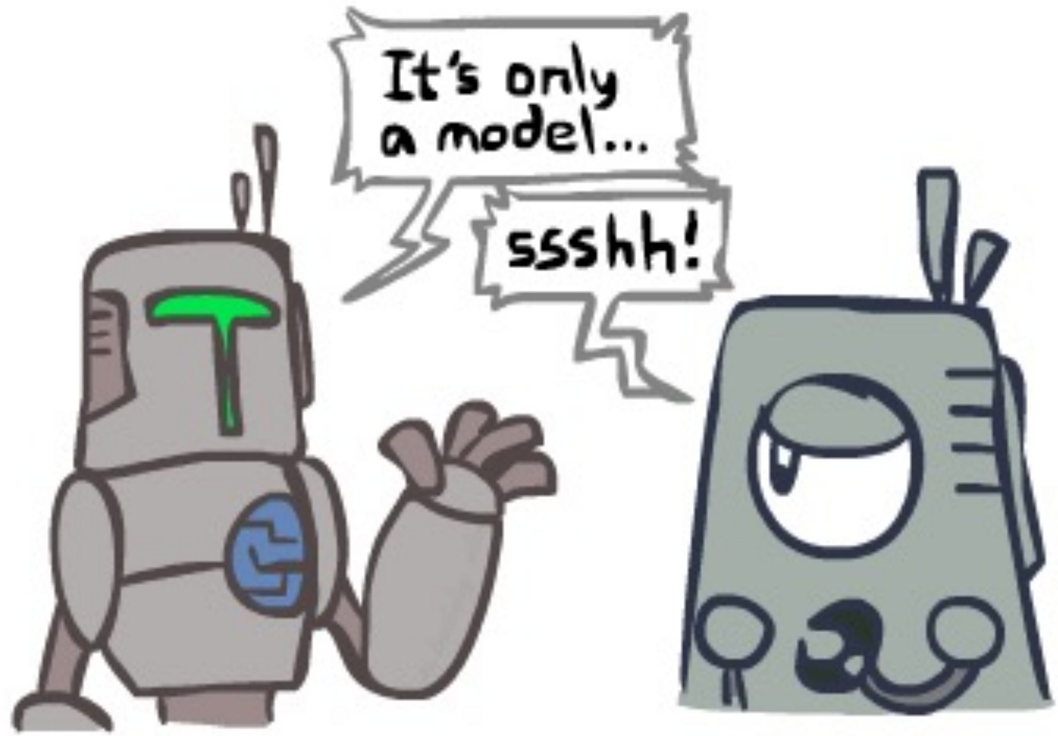


But…

- A* keeps the entire explored region in memory
- => will run out of space before you get bored waiting for the answer 🙁

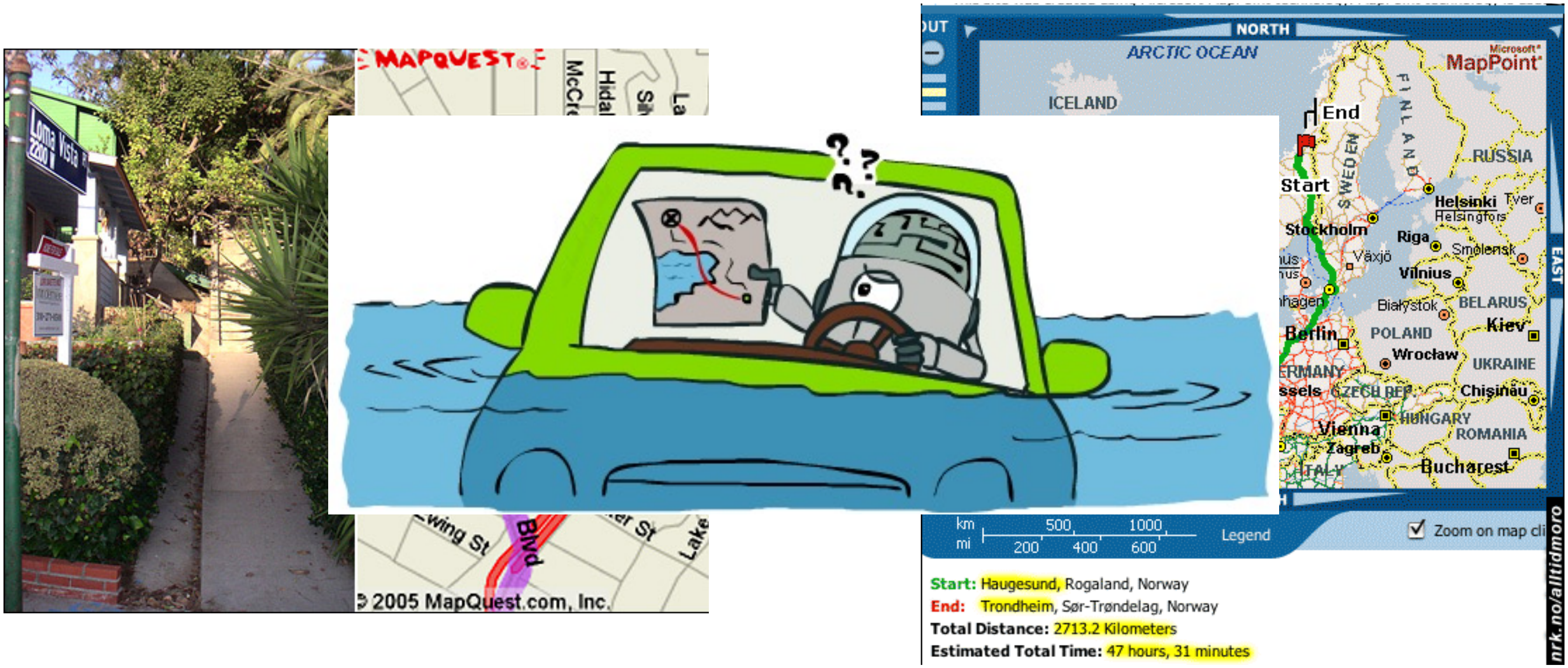
Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Search Gone Wrong?

Search Gone Wrong?



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

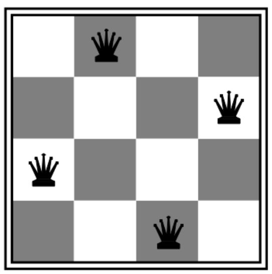
Graph Search Pseudo-Code

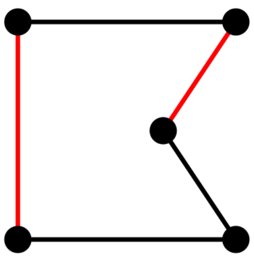
```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

Local Search

Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem





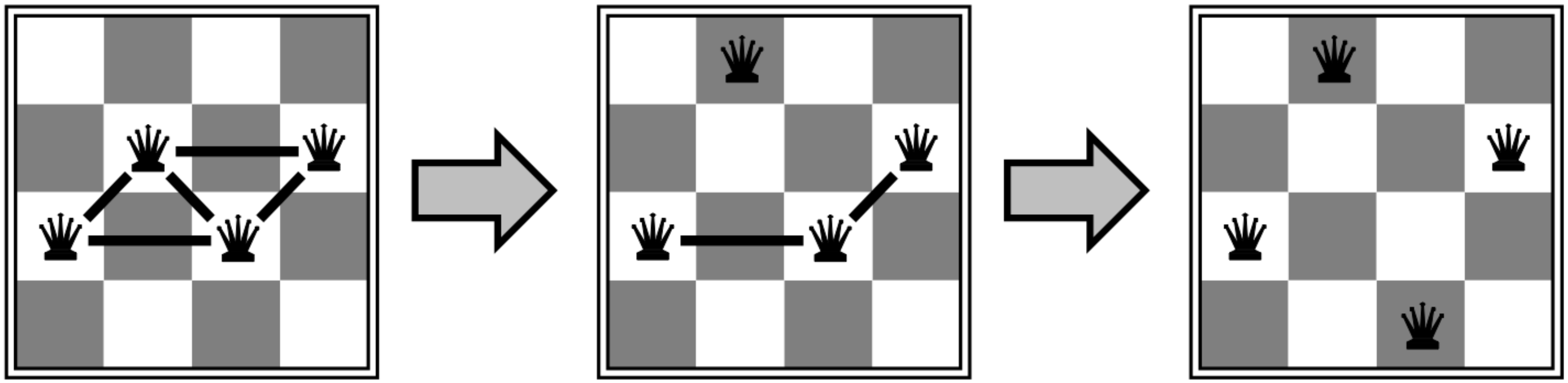
- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Actions: move a queen in its column
- Heuristic value function: number of conflicts



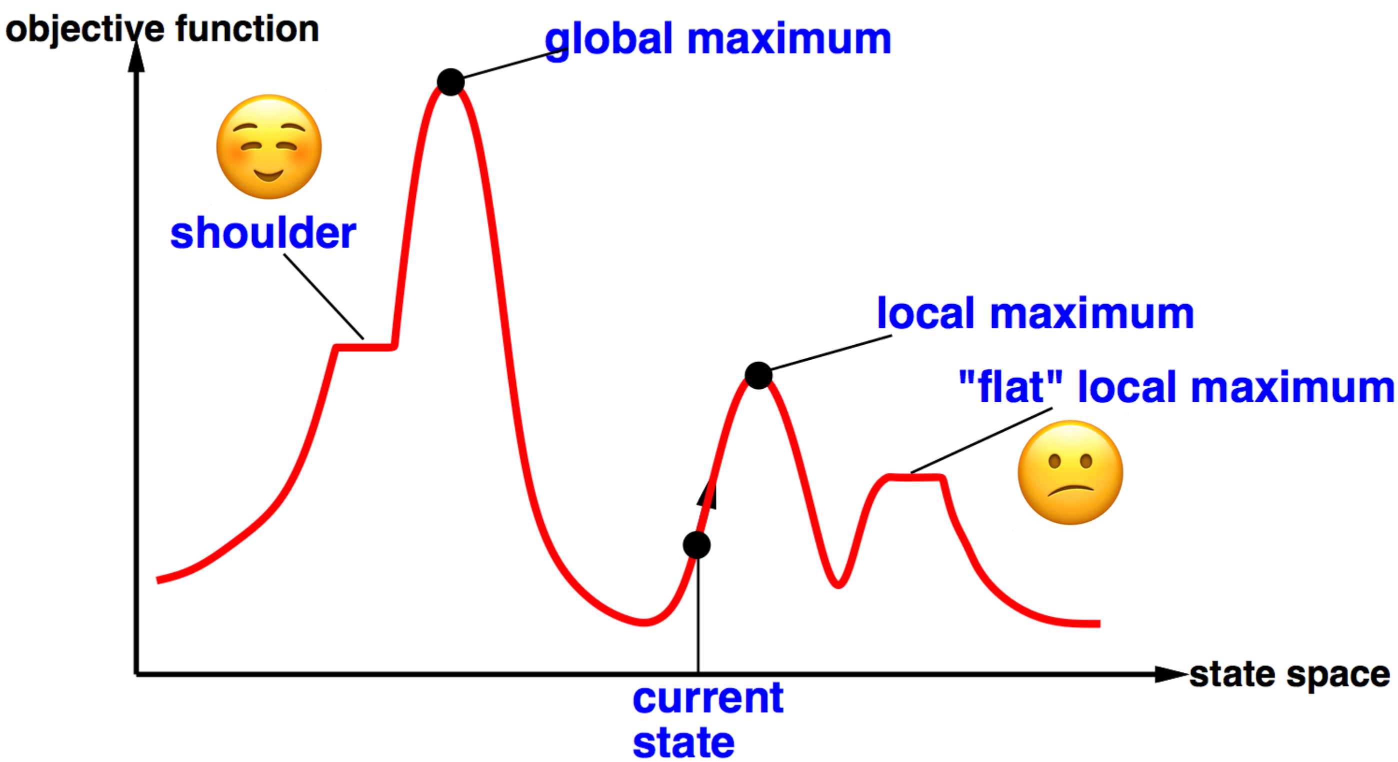
h = 5 **h = 2** **h = 0**

Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
  current ← make-node(problem.initial-state)
  loop do
      neighbor ← a highest-valued successor of current
      if neighbor.value ≤ current.value then
          return current.state
      current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

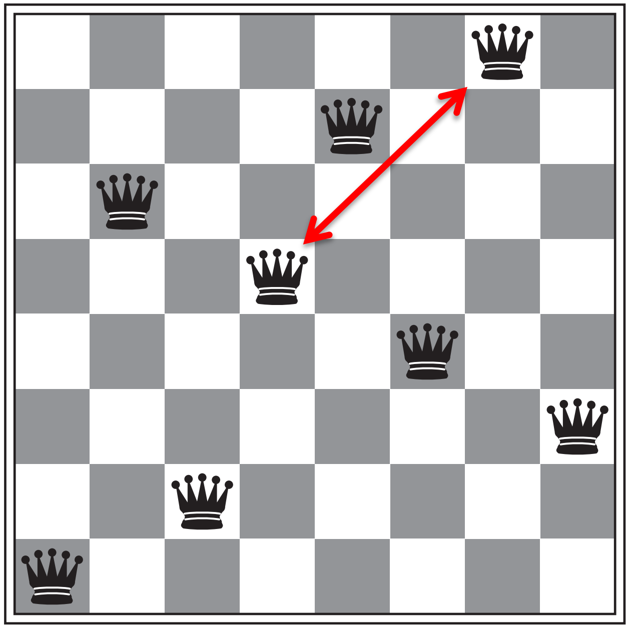
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



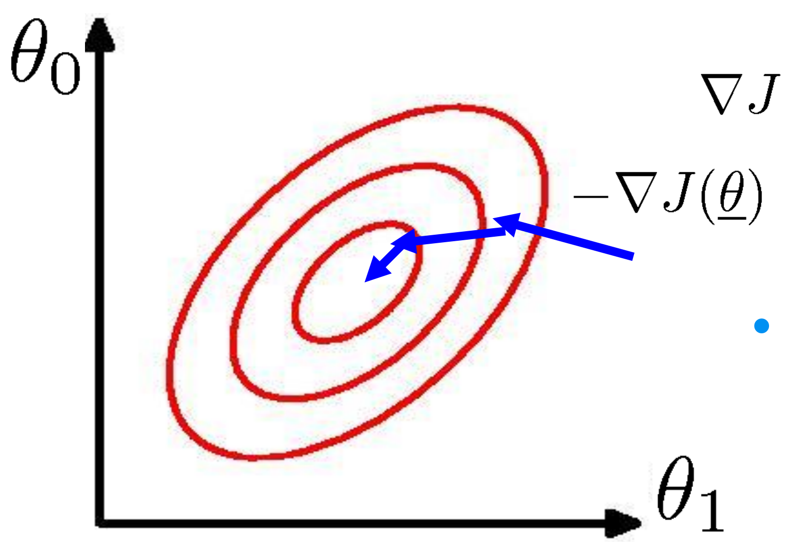
Moral: algorithms with knobs to twiddle are irritating

Variants of Hill Climbing

- **Stochastic Hill Climbing** selects at random from the uphill moves. The probability of selection varies with the steepness of the uphill move. In fact it selects a random state from the available better states. This usually converges slower than steepest ascent, but in some state landscapes it finds better landscapes
- **First-Choice Climbing** implements the above one by generating successors randomly until a better one (i.e. the first found better state) is found.
- **Random-restart hill climbing** searches from randomly generated initial moves until the goal state is reached.

Gradient descent

Hill-climbing in continuous spaces



- Gradient vector

$$\nabla J(\underline{\theta}) = \left[\frac{\partial J(\underline{\theta})}{\partial \theta_0} \quad \frac{\partial J(\underline{\theta})}{\partial \theta_1} \quad \ldots \right]$$

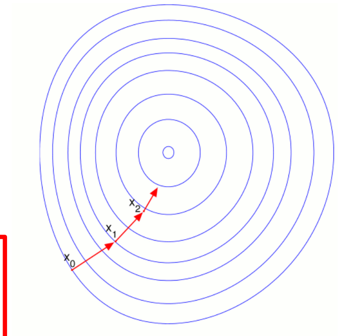
- Indicates direction of steepest ascent

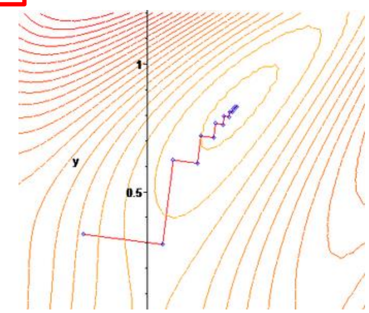
  (negative = steepest descent)

Gradient descent

Hill-climbing in continuous spaces

Gradient = the most direct direction up-hill in the objective (cost) function, so its negative minimizes the cost function.





* Assume we have some cost-function: $J(x_1, x_2, \ldots, x_n)$
and we want minimize over continuous variables $x_1, x_2, .., x_n$

1. Compute the *gradient :* $\dfrac{\partial}{\partial x_i} J(x_1, \ldots, x_n) \qquad \forall i$

2. Take a small step downhill in the direction of the gradient:

$$x_i' = x_i - \lambda \frac{\partial}{\partial x_i} J(x_1, \ldots, x_n)$$

3. Check if $J(x_1', \ldots, x_n') < J(x_1, \ldots, x_n)$

(or, Armijo rule, etc.)

4. If true then accept move, if not “reject”.

(decrease step size, etc.)

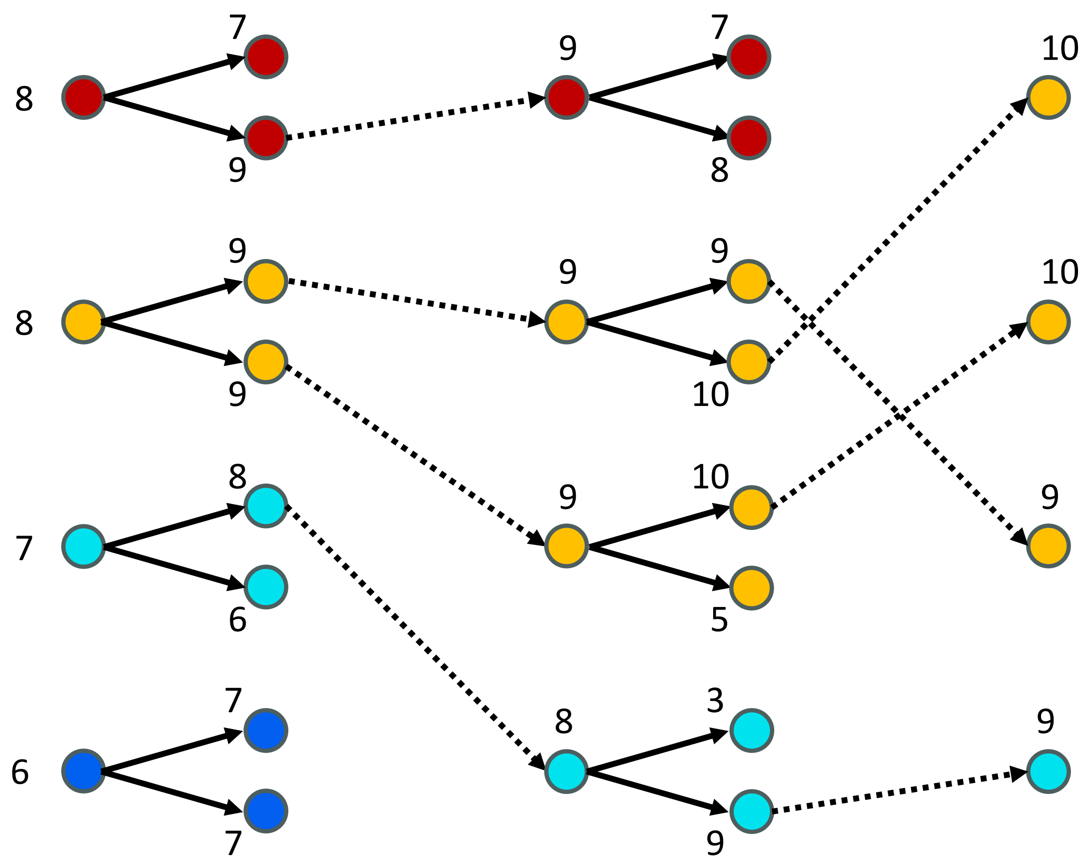
5. Repeat.

Local beam search

- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states

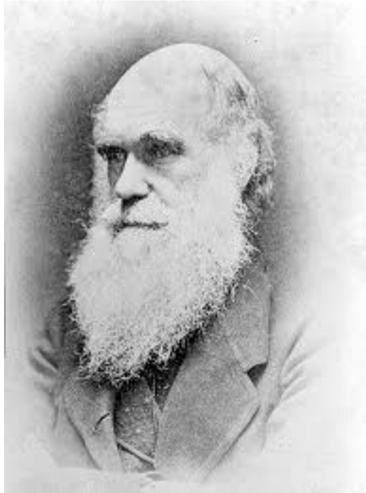
Or, K chosen randomly with a bias towards good ones

Beam search example (*K*=4)

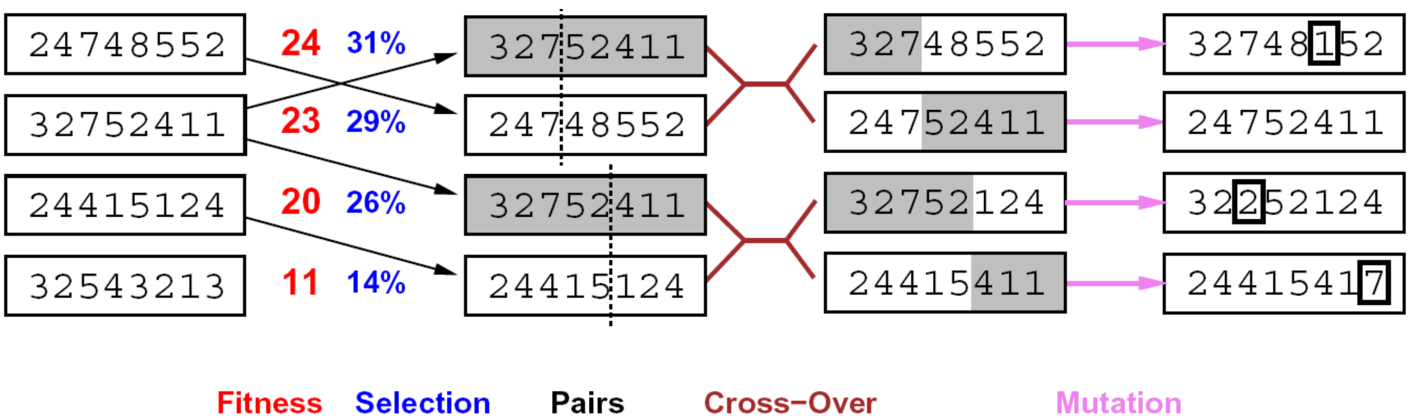


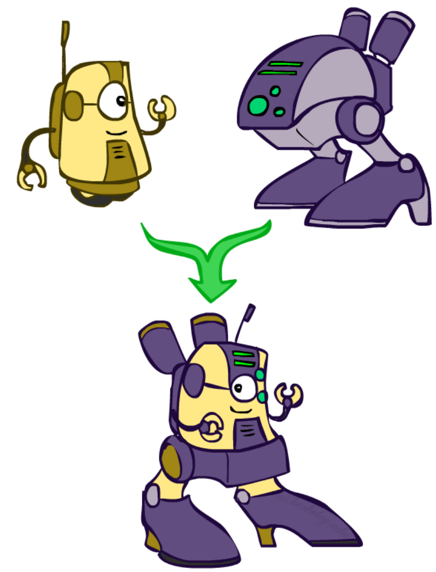
Local beam search

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! “Come over here, the grass is greener!”
- What other well-known algorithm does this remind you of?
  - Evolution!



Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety