Practice Exam 1

Date: September 21, 2023

DIRECTIONS:

- Write your answers on the exam paper.
- If you need extra space, please use the back of a page.
- You have 80 minutes to complete the exam.
- Please do not turn the exam over until you are instructed to do so.
- Good Luck!

1	/25
2	/15
3	/25
4	/10
5	/25
Total	/100

1. (25 points, 5 each) For each of the following problems, answer **True** or **False** and BRIEFLY JUSTIFY your answer.

   (a) If $T(n) = n^2$, $T(n) = O(n \lg n)$.

   (b) Here is a pseudocode to calculate a factorial of n, where $n \in \mathbb{Z}^+$ (positive integers).

   FACTORIAL(n)

   ```
   1  x = 1
   2  for i = 1 to n
   3        x = x * i
   4  return x
   ```

   A loop invariant for this algorithm is:

   Before i-th iteration, $k = i - 1$, $x = 1 * 2 * .. * k = k!$, and $0! = 1$.

(c) Given a n-element heap, $\lfloor n/2 \rfloor$ is a leaf in the binary heap.

(d) Here is a pseudocode for bubblesort:

BUBBLESORT(A)

```
1  for i = 1 to A.length − 1
2      for j = A.length downto i + 1
3          if A[j] < A[j − 1]
4              exchange A[j] with A[j − 1]
```

In the worst-case, the runtime of this algorithm is $T(n) = \Theta(n^2)$.

(e) Given $f(n) = n$ and $g(n) = 2n^2 + 3n + 5$, where $n \in \mathbb{Z}$ (integers), $f(n) \geq g(n)$ for all $n \geq 4$.

2. (15 points)PALINDROME procedure iterates over a string to check if a string is a palindrome or not. A string is considered palindrome if the string reads the same backward as forward. For instance, "madam" is a palindrome.

   Input: S is a string with indices $S[1, 2, ...n]$
   Output: True if S is a palindrome, otherwise False.

   PALINDROME(S)

```
1  i = ⌊(S.length + 1)/2⌋
2  j = ⌈(S.length + 1)/2⌉
3  while i ≥ 1 and j ≤ S.length
4      if S[i] ≠ S[j]
5          return False
6      else
7          i = i − 1
8          j = j + 1
9  return True
```

   Here is a loop invariant for PALINDROME procedure:

   $S[i + 1 \,..\, j - 1]$ is a palindrome.

   Use the loop invariant to prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

   [Hint: The size of a string will either be even or odd. Therefore, you should argue for each case, especially for initialization/base case and maintenance/inductive step.]

(Please use this page to write your answer for question (2) if the previous page is not enough.)

3. (25 points) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n/2) + n^2$, $T(1) = 1$

   (a) Draw a recursion tree for $T(n) = 2T(n/2)+n^2$ to determine an asymptotic **upper bound**.

(b) Use the substitution method to verify your upper bound.

4. (10 points) Using the definition, show that $T(n) = \frac{1}{2}n^2 - 3n$ is $T(n) = \Theta(n^2)$.

5. (25 points) Heaps

   (a) (10 points) Write pseudocode for BUILD-MAX-HEAP(A) procedure. You may assume that MAX-HEAPIFY(A, i) has already been implemented, where i is an index of any node in the binary heap.

   (b) (5 points) State an upper bound for the runtime of BUILD-MAX-HEAP(A) procedure. Justify your answer.

(c) (10 points) Illustrate BUILD-MAX-HEAP(A) on array $A = \langle 4, 1, 3, 2, 16, 9, 10, 14, 8, 7 \rangle$.