Name:__________________________________ Date:__________________

Polynomial Operations SOLUTION (version 243)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 2x^5 - 6x^4 - 3x^3 - 9x^2 + 7$$

$$q(x) = 9x^5 - 5x^4 - 10x^3 + 8x - 4$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (2)x^5 + (-6)x^4 + (-3)x^3 + (-9)x^2 + (0)x^1 + (7)x^0 \\ q(x) &= (9)x^5 + (-5)x^4 + (-10)x^3 + (0)x^2 + (8)x^1 + (-4)x^0 \\ p(x) - q(x) &= (-7)x^5 + (-1)x^4 + (7)x^3 + (-9)x^2 + (-8)x^1 + (11)x^0 \\ p(x) - q(x) &= -7x^5 - x^4 + 7x^3 - 9x^2 - 8x + 11 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 + 2x - 6$$

$$b(x) = -6x - 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$2x$	-6
$-6x$	$18x^3$	$-12x^2$	$36x$
-3	$9x^2$	$-6x$	18

$$a(x) \cdot b(x) = 18x^3 - 12x^2 + 9x^2 + 36x - 6x + 18$$

Combine like terms.

$$a(x) \cdot b(x) = 18x^3 - 3x^2 + 30x + 18$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 243)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 + 10x^2 - 17x - 25 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & 10 & -17 & -25 \\ 7 & & -7 & 21 & 28 \\ \hline & -1 & 3 & 4 & 3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 + 3x + 4 + \frac{3}{x-7}$$

In other words, $h(x) = -x^2 + 3x + 4$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 10x^2 - 17x - 25$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (-1)\cdot(7)^3 \;+\; (10)\cdot(7)^2 \;+\; (-17)\cdot(7) \;+\; (-25) \\ &= (-1)\cdot(343) \;+\; (10)\cdot(49) \;+\; (-17)\cdot(7) \;+\; (-25) \\ &= (-343) \;+\; (490) \;+\; (-119) \;+\; (-25) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x-7$. Thus, $f(7) = 3$.