Name: ____________________ Date: __________

Polynomial Operations SOLUTION (version 221)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -3x^5 - 9x^4 + 4x^2 - 10x + 1$$

$$q(x) = 8x^5 + 3x^4 + 6x^3 - 2x - 5$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-3)x^5 + (-9)x^4 + (0)x^3 + (4)x^2 + (-10)x^1 + (1)x^0 \\ q(x) &= (8)x^5 + (3)x^4 + (6)x^3 + (0)x^2 + (-2)x^1 + (-5)x^0 \\ q(x) - p(x) &= (11)x^5 + (12)x^4 + (6)x^3 + (-4)x^2 + (8)x^1 + (-6)x^0 \\ q(x) - p(x) &= 11x^5 + 12x^4 + 6x^3 - 4x^2 + 8x - 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 + 4x + 7$$

$$b(x) = 6x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$4x$	7
$6x$	$12x^3$	$24x^2$	$42x$
4	$8x^2$	$16x$	28

$$a(x) \cdot b(x) = 12x^3 + 24x^2 + 8x^2 + 42x + 16x + 28$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 + 32x^2 + 58x + 28$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 221)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 + 17x^2 + 12x - 29 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -2 & 17 & 12 & -29 \\ 9 & & -18 & -9 & 27 \\ \hline & -2 & -1 & 3 & -2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 - x + 3 + \frac{-2}{x-9}$$

In other words, $h(x) = -2x^2 - x + 3$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 + 17x^2 + 12x - 29$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-2)\cdot(9)^3 \;+\; (17)\cdot(9)^2 \;+\; (12)\cdot(9) \;+\; (-29) \\ &= (-2)\cdot(729) \;+\; (17)\cdot(81) \;+\; (12)\cdot(9) \;+\; (-29) \\ &= (-1458) \;+\; (1377) \;+\; (108) \;+\; (-29) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = -2$.