Name: ______________________________ Date: ______________

# Polynomial Operations SOLUTION (version 104)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) = 2x^5 + 10x^3 - 9x^2 - 5x - 7$$

$$q(x) = -6x^5 - 3x^4 + 10x^3 + 7x + 1$$

**Express the sum of $p(x) + q(x)$ in standard form.**

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (2)x^5 + (0)x^4 + (10)x^3 + (-9)x^2 + (-5)x^1 + (-7)x^0 \\ q(x) &= (-6)x^5 + (-3)x^4 + (10)x^3 + (0)x^2 + (7)x^1 + (1)x^0 \\ p(x) + q(x) &= (-4)x^5 + (-3)x^4 + (20)x^3 + (-9)x^2 + (2)x^1 + (-6)x^0 \\ p(x) + q(x) &= -4x^5 - 3x^4 + 20x^3 - 9x^2 + 2x - 6 \end{aligned}$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) = 9x^2 - 3x + 6$$

$$b(x) = 7x - 3$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $9x^2$ | $-3x$ | $6$ |
|---|---|---|---|
| $7x$ | $63x^3$ | $-21x^2$ | $42x$ |
| $-3$ | $-27x^2$ | $9x$ | $-18$ |

$$a(x) \cdot b(x) = 63x^3 - 21x^2 - 27x^2 + 42x + 9x - 18$$

Combine like terms.

$$a(x) \cdot b(x) = 63x^3 - 48x^2 + 51x - 18$$

**3. Express $(x+1)^4$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

# Polynomial Operations SOLUTION (version 104)

**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= 3x^3 - 25x^2 - 18x - 8 \\ g(x) &= x - 9 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

| | | | | |
|---|---|---|---|---|
| | 3 | -25 | -18 | -8 |
| 9 | | 27 | 18 | 0 |
| | 3 | 2 | 0 | -8 |

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 2x + \frac{-8}{x-9}$$

In other words, $h(x) = 3x^2 + 2x$ and the remainder is $R = -8$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 - 25x^2 - 18x - 8$. Evaluate $f(9)$.**

You could do this the hard way.

$$\begin{aligned} f(9) &= (3)\cdot(9)^3 \ + \ (-25)\cdot(9)^2 \ + \ (-18)\cdot(9) \ + \ (-8) \\ &= (3)\cdot(729) \ + \ (-25)\cdot(81) \ + \ (-18)\cdot(9) \ + \ (-8) \\ &= (2187) \ + \ (-2025) \ + \ (-162) \ + \ (-8) \\ &= -8 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = -8$.