Name:

at1204p_vertex_and_roots... from standard-form quadratic functions (v119)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry (w)
3. Both x-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex (h, k) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

Example

$$f(x) = 6x^2 + 4x - 5$$

Example solution

1. Find the axis of symmetry. Use the formula $h = \frac{-b}{2a}$, where h is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)} = \frac{-1}{3}$$

$$\text{axis of symmetry:} \quad x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula $w = \frac{\sqrt{b^2-4ac}}{2a}$.

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The x-intercepts can be found by adding w to or subtracting w from h.

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know $h = \frac{-1}{3}$, so we just need k. Use the formula $k = \frac{4ac-b^2}{4a}$.

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex:} \quad \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry (w)
3. Both x-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex (h, k) shown as cartesian coordinates

Box your answers.

$$f(x) = 7x^2 - 10x - 6$$

1. Axis of symmetry

$$h = \frac{-(-10)}{2(7)} = \frac{5}{7}$$

$$\text{axis of symmetry:} \quad x = \frac{5}{7}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-10)^2 - 4(7)(-6)}}{2(7)}$$

$$w = \frac{\sqrt{268}}{14} = \frac{\sqrt{2 \cdot 2 \cdot 67}}{14} = \frac{2\sqrt{67}}{14}$$

$$w = \frac{\sqrt{67}}{7}$$

3. Roots

$$\left(\frac{5}{7} - \frac{\sqrt{67}}{7}, 0\right) \quad \text{and} \quad \left(\frac{5}{7} + \frac{\sqrt{67}}{7}, 0\right)$$

4. Vertex

$$k = \frac{4(7)(-6) - (-10)^2}{4(7)}$$

$$k = \frac{-268}{28} = \frac{-67}{7}$$

$$\text{vertex:} \quad \left(\frac{5}{7}, \frac{-67}{7}\right)$$

Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry (w)
3. Both x-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex (h, k) shown as cartesian coordinates

Box your answers.

$$f(x) \;=\; x^2 - 4x + 2$$

1. Axis of symmetry

$$h = \frac{-(-4)}{2(1)} \;=\; 2$$

$$\text{axis of symmetry:} \quad x = 2$$

2. Distance from axis of symmetry to root

$$w \;=\; \frac{\sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$w \;=\; \frac{\sqrt{8}}{2} \;=\; \frac{\sqrt{2 \cdot 2 \cdot 2}}{2} \;=\; \frac{2\sqrt{2}}{2}$$

$$w \;=\; \sqrt{2}$$

3. Roots

$$\left(2 - \sqrt{2},\, 0\right) \quad \text{and} \quad \left(2 + \sqrt{2},\, 0\right)$$

4. Vertex

$$k = \frac{4(1)(2) - (-4)^2}{4(1)}$$

$$k \;=\; \frac{-8}{4} \;=\; -2$$

$$\text{vertex:} \quad (2,\, -2)$$

Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry (w)
3. Both x-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex (h, k) shown as cartesian coordinates

Box your answers.

$$f(x) = 4x^2 + 6x - 3$$

1. Axis of symmetry

$$h = \frac{-(6)}{2(4)} = \frac{-3}{4}$$

$$\text{axis of symmetry: } x = \frac{-3}{4}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(6)^2 - 4(4)(-3)}}{2(4)}$$

$$w = \frac{\sqrt{84}}{8} = \frac{\sqrt{2 \cdot 2 \cdot 3 \cdot 7}}{8} = \frac{2\sqrt{21}}{8}$$

$$w = \frac{\sqrt{21}}{4}$$

3. Roots

$$\left(\frac{-3}{4} - \frac{\sqrt{21}}{4}, 0\right) \quad \text{and} \quad \left(\frac{-3}{4} + \frac{\sqrt{21}}{4}, 0\right)$$

4. Vertex

$$k = \frac{4(4)(-3) - (6)^2}{4(4)}$$

$$k = \frac{-84}{16} = \frac{-21}{4}$$

$$\text{vertex: } \left(\frac{-3}{4}, \frac{-21}{4}\right)$$