

Name: \_\_\_\_\_

### at1203p\_qf: Finding roots and vertex of quadratic functions (v103)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

#### Example

$$f(x) = 6x^2 + 4x - 5$$

#### Example solution

1. Find the axis of symmetry. Use the formula  $h = \frac{-b}{2a}$ , where  $h$  is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)}$$

$$\text{axis of symmetry: } x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula  $w = \frac{\sqrt{b^2 - 4ac}}{2a}$ .

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The  $x$ -intercepts can be found by adding  $w$  to or subtracting  $w$  from  $h$ .

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know  $h = \frac{-1}{3}$ , so we just need  $k$ . Use the formula  $k = \frac{4ac - b^2}{4a}$ .

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex: } \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 6x^2 - 2x - 5$$

1. Axis of symmetry

$$h = \frac{-(2)}{2(6)}$$

$$\text{axis of symmetry: } x = \frac{1}{6}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-2)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{124}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 31}}{12} = \frac{2\sqrt{31}}{12}$$

$$w = \frac{\sqrt{31}}{6}$$

3. Roots

$$\left(\frac{1}{6} - \frac{\sqrt{31}}{6}, 0\right) \quad \text{and} \quad \left(\frac{1}{6} + \frac{\sqrt{31}}{6}, 0\right)$$

4. Vertex

$$k = \frac{4(6)(-5) - (-2)^2}{4(6)}$$

$$k = \frac{-124}{24} = \frac{-31}{6}$$

$$\text{vertex: } \left(\frac{1}{6}, \frac{-31}{6}\right)$$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 2x^2 - 2x - 5$$

1. Axis of symmetry

$$h = \frac{-(2)}{2(2)}$$

$$\text{axis of symmetry: } x = \frac{1}{2}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-2)^2 - 4(2)(-5)}}{2(2)}$$

$$w = \frac{\sqrt{44}}{4} = \frac{\sqrt{2 \cdot 2 \cdot 11}}{4} = \frac{2\sqrt{11}}{4}$$

$$w = \frac{\sqrt{11}}{2}$$

3. Roots

$$\left(\frac{1}{2} - \frac{\sqrt{11}}{2}, 0\right) \quad \text{and} \quad \left(\frac{1}{2} + \frac{\sqrt{11}}{2}, 0\right)$$

4. Vertex

$$k = \frac{4(2)(-5) - (-2)^2}{4(2)}$$

$$k = \frac{-44}{8} = \frac{-11}{2}$$

$$\text{vertex: } \left(\frac{1}{2}, \frac{-11}{2}\right)$$

### Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 4x^2 + 2x - 7$$

1. Axis of symmetry

$$h = \frac{-(-2)}{2(4)}$$

$$\text{axis of symmetry: } x = \frac{-1}{4}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(2)^2 - 4(4)(-7)}}{2(4)}$$

$$w = \frac{\sqrt{116}}{8} = \frac{\sqrt{2 \cdot 2 \cdot 29}}{8} = \frac{2\sqrt{29}}{8}$$

$$w = \frac{\sqrt{29}}{4}$$

3. Roots

$$\left(\frac{-1}{4} - \frac{\sqrt{29}}{4}, 0\right) \quad \text{and} \quad \left(\frac{-1}{4} + \frac{\sqrt{29}}{4}, 0\right)$$

4. Vertex

$$k = \frac{4(4)(-7) - (2)^2}{4(4)}$$

$$k = \frac{-116}{16} = \frac{-29}{4}$$

$$\text{vertex: } \left(\frac{-1}{4}, \frac{-29}{4}\right)$$