at1118paper: Complete the Square (v410)

Example

By completing the square, find both solutions to the given equation:

$$x^2 - 58x = -517$$

Add $\left(\frac{-58}{2}\right)^2$, which equals 841, to both sides of the equation.

$$x^2 - 58x + 841 = 324$$

Factor the left side.

$$(x - 29)^2 = 324$$

Undo the squaring. We need to consider both $\pm\sqrt{324}$.

$x - 29 = -18$	or	$x - 29 = 18$
$x = 11$	or	$x = 47$

Question 1

By completing the square, find both solutions to the given equation:

$$x^2 + 40x = -399$$

$$x^2 + 40x + 400 = 1$$
$$(x + 20)^2 = 1$$
$$x + 20 = \pm 1$$
$$x = -21 \quad \text{or} \quad x = -19$$

Question 2

By completing the square, find both solutions to the given equation:

$$x^2 + 50x = -609$$

$$x^2 + 50x + 625 = 16$$
$$(x + 25)^2 = 16$$
$$x + 25 = \pm 4$$
$$x = -29 \quad \text{or} \quad x = -21$$

Question 3

By completing the square, find both solutions to the given equation:

$$x^2 - 20x \;=\; -84$$

$$x^2 - 20x + 100 \;=\; 16$$
$$(x - 10)^2 \;=\; 16$$
$$x - 10 \;=\; \pm 4$$
$$x = 6 \qquad \text{or} \qquad x = 14$$

Question 4

By completing the square, find both solutions to the given equation:

$$x^2 + 46x \;=\; 560$$

$$x^2 + 46x + 529 \;=\; 1089$$
$$(x + 23)^2 \;=\; 1089$$
$$x + 23 \;=\; \pm 33$$
$$x = -56 \qquad \text{or} \qquad x = 10$$

Question 5

By completing the square, find both solutions to the given equation:

$$x^2 - 40x \;=\; 624$$

$$x^2 - 40x + 400 \;=\; 1024$$
$$(x - 20)^2 \;=\; 1024$$
$$x - 20 \;=\; \pm 32$$
$$x = -12 \qquad \text{or} \qquad x = 52$$

Question 6

By completing the square, find both solutions to the given equation:

$$x^2 - 24x \;=\; 1300$$

$$x^2 - 24x + 144 \;=\; 1444$$
$$(x - 12)^2 \;=\; 1444$$
$$x - 12 \;=\; \pm 38$$
$$x = -26 \qquad \text{or} \qquad x = 50$$