at1118paper: Complete the Square (v411)

Example

By completing the square, find both solutions to the given equation:

$$x^2 - 38x = -325$$

Add $\left(\frac{-38}{2}\right)^2$, which equals 361, to both sides of the equation.

$$x^2 - 38x + 361 = 36$$

Factor the left side.

$$(x - 19)^2 = 36$$

Undo the squaring. We need to consider both $\pm\sqrt{36}$.

$$x - 19 = -6 \qquad\qquad \text{or} \qquad\qquad x - 19 = 6$$
$$x = 13 \qquad\qquad \text{or} \qquad\qquad x = 25$$

Question 1

By completing the square, find both solutions to the given equation:

$$x^2 - 34x = -240$$

$$x^2 - 34x + 289 = 49$$
$$(x - 17)^2 = 49$$
$$x - 17 = \pm 7$$
$$x = 10 \qquad \text{or} \qquad x = 24$$

Question 2

By completing the square, find both solutions to the given equation:

$$x^2 + 22x = -120$$

$$x^2 + 22x + 121 = 1$$
$$(x + 11)^2 = 1$$
$$x + 11 = \pm 1$$
$$x = -12 \qquad \text{or} \qquad x = -10$$

Question 3

By completing the square, find both solutions to the given equation:

$$x^2 - 40x \;=\; 624$$

$$
\begin{aligned}
x^2 - 40x + 400 &= 1024 \\
(x - 20)^2 &= 1024 \\
x - 20 &= \pm 32 \\
x = -12 \qquad \text{or} &\qquad x = 52
\end{aligned}
$$

Question 4

By completing the square, find both solutions to the given equation:

$$x^2 + 40x \;=\; 384$$

$$
\begin{aligned}
x^2 + 40x + 400 &= 784 \\
(x + 20)^2 &= 784 \\
x + 20 &= \pm 28 \\
x = -48 \qquad \text{or} &\qquad x = 8
\end{aligned}
$$

Question 5

By completing the square, find both solutions to the given equation:

$$x^2 - 10x \;=\; 200$$

$$
\begin{aligned}
x^2 - 10x + 25 &= 225 \\
(x - 5)^2 &= 225 \\
x - 5 &= \pm 15 \\
x = -10 \qquad \text{or} &\qquad x = 20
\end{aligned}
$$

Question 6

By completing the square, find both solutions to the given equation:

$$x^2 - 36x \;=\; -275$$

$$
\begin{aligned}
x^2 - 36x + 324 &= 49 \\
(x - 18)^2 &= 49 \\
x - 18 &= \pm 7 \\
x = 11 \qquad \text{or} &\qquad x = 25
\end{aligned}
$$