In terms of α and β , express all the lengths and other angle measures (θ and ϕ).



| Variable | Algebraic expression |
|------------|----------------------|
| p = | |
| q = | |
| r = | |
| s = | |
| $\theta =$ | |
| t = | |
| u = | |
| $\phi =$ | |
| v = | |
| w = | |

The angle-sum and angle-difference identities are listed below:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

You know the following:

$$\sin(225^\circ) = \frac{-\sqrt{2}}{2}$$

$$\sin(300^\circ) = \frac{-\sqrt{3}}{2}$$

$$\cos(225^\circ) = \frac{-\sqrt{2}}{2}$$

$$\cos(300^\circ) = \frac{1}{2}$$

Determine $\cos(-75^{\circ})$ exactly.

Prove the (sine) double-angle identity: $\sin(2x) = 2\sin(x)\cos(x)$

(Hint: start with an angle-sum formula from Question 2.)

Question 4

Prove the (cosine) double-angle identity: $cos(2x) = 2cos^2(x) - 1$

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$)

Prove the (cosine) half-angle identity: $\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$.

 $({\it Hint: start with the double-angle identity from \ Question \ 4.})$

Question 6

Given $\cos(122^\circ) \approx -0.53$, what is $\cos(61^\circ)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: 122/2 = 61.)