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## s17 Geometric Series Exam (Solution v108)

### Question 1

Consider the partial geometric series represented below with first term  $a = 924$ , common ratio  $r = \left(\frac{29}{66}\right)^{1/10}$ , and  $n = 10$  terms.

$$S = 924 + 851.05 + 783.87 + 721.99 + 664.99 + 612.49 + 564.14 + 519.6 + 478.58 + 440.8$$

We can multiply both sides by  $r$ .

$$rS = 851.05 + 783.87 + 721.99 + 664.99 + 612.49 + 564.14 + 519.6 + 478.58 + 440.8 + 406$$

What is the value of  $S - rS$ ?

Most terms cancel.

$$924 - 406 = 518$$

### Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 5 + 5(6) + 5(6)^2 + 5(6)^3 + \cdots + 5(6)^{48} + 5(6)^{49} + 5(6)^{50} + 5(6)^{51}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 5$$

$$\text{common ratio} = r = 6$$

$$\text{number of terms} = n = 52$$

### Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

### Definitions

$a$  = first term

$r$  = common ratio

$n$  = number of terms

$S$  = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by  $r$ .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out  $S$  from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by  $(1 - r)$ . We technically need to enforce  $r \neq 1$  as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$