

Question 1

In terms of  $\alpha$  and  $\beta$ , express all the lengths and other angle measures ( $\theta$  and  $\phi$ ).



Variable	Algebraic expression
$p =$	
$q =$	
$r =$	
$s =$	
$\theta =$	
$t =$	
$u =$	
$\phi =$	
$v =$	
$w =$	

**Question 2**

The angle-sum and angle-difference identities are true for any  $\alpha$  and  $\beta$ :

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

You know the following:

$$\sin(135^\circ) = \frac{\sqrt{2}}{2}$$

$$\sin(120^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(135^\circ) = \frac{-\sqrt{2}}{2}$$

$$\cos(120^\circ) = \frac{-1}{2}$$

Determine  $\sin(15^\circ)$  exactly.

**Question 3**

Prove that  $\sin(2x) = 2 \sin(x) \cos(x)$  for any  $x$ .

*(Hint: start with an angle-sum formula from Question 2.)*

**Question 4**

Prove that  $\cos(2x) = 2 \cos^2(x) - 1$  for any  $x$ .

*(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity:  $\sin^2(x) + \cos^2(x) = 1$ )*

**Question 5**

Prove that  $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$ . (Technically this assumes  $\cos(y/2) > 0$ , but let's not worry about that here.)

*(Hint: use the identity proved in Question 4.)*

**Question 6**

If you knew that  $\cos(160^\circ) \approx -0.94$ , then what is  $\cos(80^\circ)$ ? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

*(Hint:  $160/2 = 80$ .)*