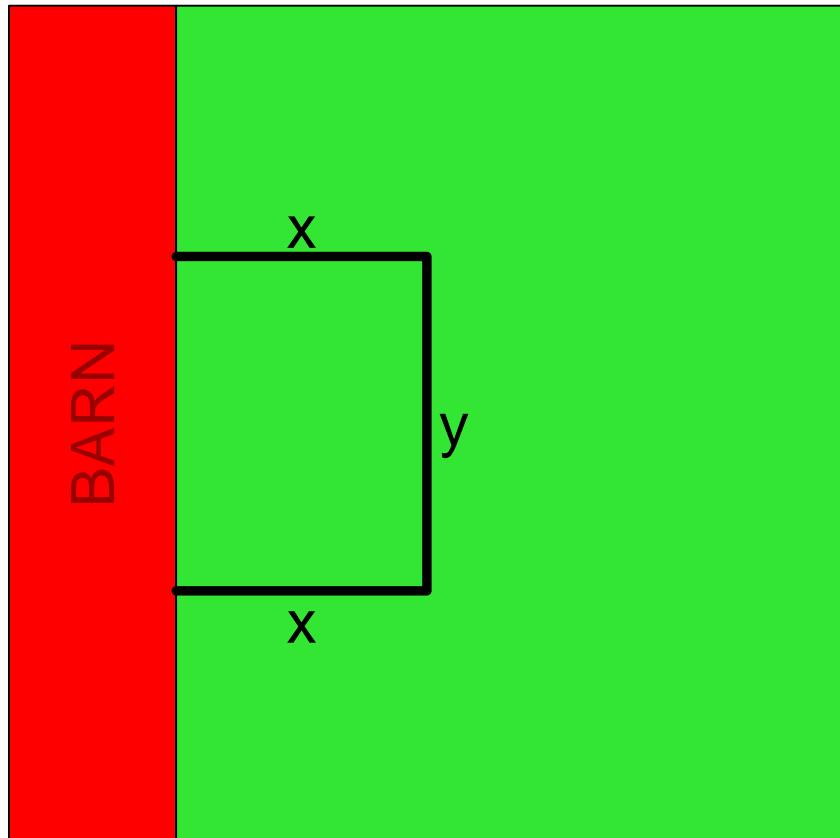


### 1. Problem

Amelia will use 96 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

Let  $x$  represent the length of fence perpendicular to the barn, and let  $y$  represent the length of fence parallel to the barn.



She wants to give her dog as much area as possible. Find the value of  $x$  that maximizes the area.

#### Solution

The total length of fence is 96 feet. There are 2 sides with lengths of  $x$  and one side with a length of  $y$ .

$$2x + y = 96$$

Solve for  $y$ .

$$y = 96 - 2x$$

Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

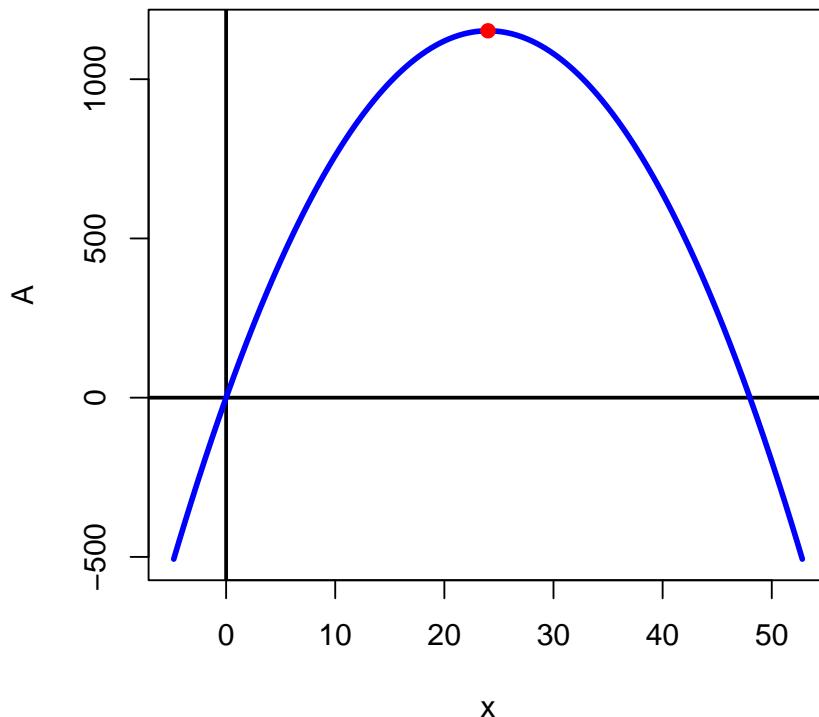
Substitute  $96 - 2x$  in place of  $y$  to get area in terms of only  $x$ .

$$A = x(96 - 2x)$$

From here, we can expand into standard form, and use the formula  $h = \frac{-b}{2a}$  to find the vertex.  
Expand into standard form.

$$A = -2x^2 + 192x$$

You can graph  $A$  versus  $x$  and get a downwards facing parabola.



Now, remember the vertex (in this case the maximum) occurs when  $x = \frac{-b}{2a}$ .

$$x_{\text{optimal}} = \frac{-(192)}{2(-2)} = 24$$

So, the area is maximized when  $x = 24$  feet and  $y = 48$  feet.