Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 242)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 3x^5 + 8x^4 - 6x^3 - 4x^2 - 9$$

$$q(x) = -8x^5 - 9x^3 + 6x^2 + x + 2$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (8)x^4 + (-6)x^3 + (-4)x^2 + (0)x^1 + (-9)x^0 \\ q(x) &= (-8)x^5 + (0)x^4 + (-9)x^3 + (6)x^2 + (1)x^1 + (2)x^0 \\ p(x) + q(x) &= (-5)x^5 + (8)x^4 + (-15)x^3 + (2)x^2 + (1)x^1 + (-7)x^0 \\ p(x) + q(x) &= -5x^5 + 8x^4 - 15x^3 + 2x^2 + x - 7 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 - 5x + 7$$

$$b(x) = 3x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-5x$	7
$3x$	$-18x^3$	$-15x^2$	$21x$
4	$-24x^2$	$-20x$	28

$$a(x) \cdot b(x) = -18x^3 - 15x^2 - 24x^2 + 21x - 20x + 28$$

Combine like terms.

$$a(x) \cdot b(x) = -18x^3 - 39x^2 + x + 28$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 + 23x^2 - 10x - 15 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	4	23	-10	-15
-6		-24	6	24
	4	-1	-4	9

So,

$$\frac{f(x)}{g(x)} = 4x^2 - x - 4 + \frac{9}{x+6}$$

In other words, $h(x) = 4x^2 - x - 4$ and the remainder is $R = 9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 + 23x^2 - 10x - 15$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (4) \cdot (-6)^3 \;+\; (23) \cdot (-6)^2 \;+\; (-10) \cdot (-6) \;+\; (-15) \\ &= (4) \cdot (-216) \;+\; (23) \cdot (36) \;+\; (-10) \cdot (-6) \;+\; (-15) \\ &= (-864) \;+\; (828) \;+\; (60) \;+\; (-15) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 9$.