Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, and radical functions in the Model Algebra II course. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. For the high school Model Algebra II course, instructional time should focus on four critical areas: (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) expand understandings of functions and graphing to include trigonometric functions; (3) synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions; and (4) relate data display and summary statistics to probability and explore a variety of data collection methods.

1. A central theme of this Model Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.

2. Building on their previous work with functions and on their work with trigonometric ratios and circles in the Model Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

3. Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this Model Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

4. Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

- N-CN.A.1: Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with $x - a$ and b real.
- N-CN.A.2: Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.
- N-CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.C.8: (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$
- N-CN.C.9: (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- N-VM.A.1: (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

- N-VM.A.3: (+) Solve problems involving velocity and other quantities that can be represented by vectors.
- N-VM.C.6: (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- N-VM.C.8: (+) Add, subtract, and multiply matrices of appropriate dimensions.
- N-VM.C.12: (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.
- A-SSE.A.1.a: Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1.b: Interpret complicated expressions by viewing one or more of their parts as a single entity.
- A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus... $(x - y)(x + y)(x - yi)(x + yi)$.
- A-SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
- A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
- A-APR.A.1.a: Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.
- A-APR.B.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- A-APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.C.4: Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
- A-APR.C.5: (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- A-APR.D.6: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-APR.D.7: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple root and rational functions and exponential functions.
- A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent equations describing satellites orbiting Earth and constraints on Earth's size and atmosphere.
- A-REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- A-REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are polynomial, rational, and logarithmic functions.
- F-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior;

and periodicity.

- F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.C.7: Graph functions expressed symbolically and show key features of the graph; by hand in simple cases and using technology for more complicated cases.
- F-IF.C.7.b: Graph square root and cube root functions.
- F-IF.C.7.c: Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-IF.C.7.e: Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
- F-IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-IF.C.8.a: Use the process of factoring in a polynomial function to show zeros, extreme values, and symmetry of the graph and interpret these in terms of a context.
- F-IF.C.9: Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. For example, given a graph of one polynomial function and an algebraic expression for another, say which has the larger relative maximum and/or smaller relative minimum.
- F-IF.C.10: Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
- F-BF.A.1: Write a function (simple rational, radical, logarithmic, and trigonometric functions) that describes a relationship between two quantities. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F-BF.A.1.b: Combine standard function types using arithmetic operations.
- F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F-BF.B.4: Find inverse functions algebraically and graphically. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.
- F-BF.B.4.a: Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- F-LE.A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F-TF.C.8: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.
- S-ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- S-IC.A.1: Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.

- S-IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of five tails in a row cause you to question the model?
- S-IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- S-IC.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S-IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S-IC.B.6: Evaluate reports based on data.
- S-MD.B.6: (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.B.7: (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater).