Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 15)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -10x^5 + x^3 - 6x^2 + 8x + 5$$

$$q(x) = x^5 + 5x^4 - 9x^3 - 2x - 10$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-10)x^5 + (0)x^4 + (1)x^3 + (-6)x^2 + (8)x^1 + (5)x^0 \\
q(x) &= (1)x^5 + (5)x^4 + (-9)x^3 + (0)x^2 + (-2)x^1 + (-10)x^0 \\
p(x) - q(x) &= (-11)x^5 + (-5)x^4 + (10)x^3 + (-6)x^2 + (10)x^1 + (15)x^0 \\
p(x) - q(x) &= -11x^5 - 5x^4 + 10x^3 - 6x^2 + 10x + 15
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -5x^2 - 3x + 8$$

$$b(x) = -4x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-5x^2$	$-3x$	8
$-4x$	$20x^3$	$12x^2$	$-32x$
-7	$35x^2$	$21x$	-56

$$a(x) \cdot b(x) = 20x^3 + 12x^2 + 35x^2 - 32x + 21x - 56$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 + 47x^2 - 11x - 56$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 - 29x^2 + 8x + 9 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -5 & -29 & 8 & 9 \\ -6 & & 30 & -6 & -12 \\ \hline & -5 & 1 & 2 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -5x^2 + x + 2 + \frac{-3}{x+6}$$

In other words, $h(x) = -5x^2 + x + 2$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 - 29x^2 + 8x + 9$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-5)\cdot(-6)^3 \ + \ (-29)\cdot(-6)^2 \ + \ (8)\cdot(-6) \ + \ (9) \\ &= (-5)\cdot(-216) \ + \ (-29)\cdot(36) \ + \ (8)\cdot(-6) \ + \ (9) \\ &= (1080) \ + \ (-1044) \ + \ (-48) \ + \ (9) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = -3$.