Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 127)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -6x^5 + x^3 - 10x^2 - 8x - 7$$

$$q(x) = -2x^5 - 9x^4 - 7x^3 + 10x + 4$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-6)x^5 + (0)x^4 + (1)x^3 + (-10)x^2 + (-8)x^1 + (-7)x^0 \\ q(x) &= (-2)x^5 + (-9)x^4 + (-7)x^3 + (0)x^2 + (10)x^1 + (4)x^0 \\ q(x) - p(x) &= (4)x^5 + (-9)x^4 + (-8)x^3 + (10)x^2 + (18)x^1 + (11)x^0 \\ q(x) - p(x) &= 4x^5 - 9x^4 - 8x^3 + 10x^2 + 18x + 11 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 + 7x - 9$$

$$b(x) = 2x + 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$7x$	-9
$2x$	$12x^3$	$14x^2$	$-18x$
5	$30x^2$	$35x$	-45

$$a(x) \cdot b(x) = 12x^3 + 14x^2 + 30x^2 - 18x + 35x - 45$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 + 44x^2 + 17x - 45$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 127)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -7x^3 + 28x^2 - 7x + 25 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -7 & 28 & -7 & 25 \\ 4 & & -28 & 0 & -28 \\ \hline & -7 & 0 & -7 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -7x^2 - 7 + \frac{-3}{x-4}$$

In other words, $h(x) = -7x^2 - 7$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -7x^3 + 28x^2 - 7x + 25$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (-7)\cdot(4)^3 \; + \; (28)\cdot(4)^2 \; + \; (-7)\cdot(4) \; + \; (25) \\ &= (-7)\cdot(64) \; + \; (28)\cdot(16) \; + \; (-7)\cdot(4) \; + \; (25) \\ &= (-448) \; + \; (448) \; + \; (-28) \; + \; (25) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -3$.