Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 131)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 4x^5 + 10x^4 - 2x^2 - 5x - 6$$
$$q(x) = -7x^5 - 5x^3 + 4x^2 - x - 2$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (4)x^5 + (10)x^4 + (0)x^3 + (-2)x^2 + (-5)x^1 + (-6)x^0 \\ q(x) &= (-7)x^5 + (0)x^4 + (-5)x^3 + (4)x^2 + (-1)x^1 + (-2)x^0 \\ q(x) - p(x) &= (-11)x^5 + (-10)x^4 + (-5)x^3 + (6)x^2 + (4)x^1 + (4)x^0 \\ q(x) - p(x) &= -11x^5 - 10x^4 - 5x^3 + 6x^2 + 4x + 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 - 2x + 7$$
$$b(x) = -5x - 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-2x$	7
$-5x$	$40x^3$	$10x^2$	$-35x$
-3	$24x^2$	$6x$	-21

$$a(x) \cdot b(x) = 40x^3 + 10x^2 + 24x^2 - 35x + 6x - 21$$

Combine like terms.

$$a(x) \cdot b(x) = 40x^3 + 34x^2 - 29x - 21$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 131)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 - 19x^2 - 13x - 5 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 3 & -19 & -13 & -5 \\ 7 & & 21 & 14 & 7 \\ \hline & 3 & 2 & 1 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 2x + 1 + \frac{2}{x-7}$$

In other words, $h(x) = 3x^2 + 2x + 1$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 - 19x^2 - 13x - 5$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (3)\cdot(7)^3 \ + \ (-19)\cdot(7)^2 \ + \ (-13)\cdot(7) \ + \ (-5) \\ &= (3)\cdot(343) \ + \ (-19)\cdot(49) \ + \ (-13)\cdot(7) \ + \ (-5) \\ &= (1029) \ + \ (-931) \ + \ (-91) \ + \ (-5) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x-7$. Thus, $f(7) = 2$.