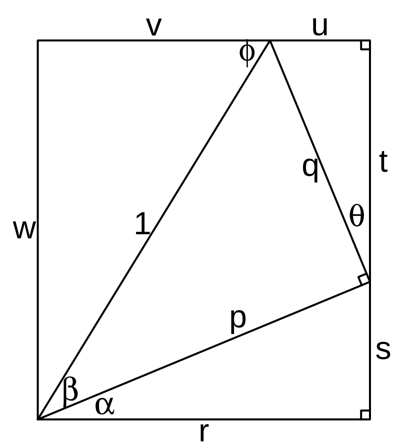
Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).



Variable	Algebraic expression
$p\ =$	
$q\ =$	
$r\ =$	
$s\ =$	
$\theta\ =$	
$t\ =$	
$u\ =$	
$\phi\ =$	
$v\ =$	
$w\ =$	

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)\\ \cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)\end{aligned}$$

You know the following:

$$\sin(135^\circ) = \frac{\sqrt{2}}{2} \qquad \sin(120^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(135^\circ) = \frac{-\sqrt{2}}{2} \qquad \cos(120^\circ) = \frac{-1}{2}$$

Determine $\sin(255^\circ)$ exactly.

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

Question 4

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$*)*

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

Question 6

If you knew that $\cos(40°) \approx 0.77$, then what is $\cos(20°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $40/2 = 20$.*)*