Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 155)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) \;=\; 10x^5 + 9x^4 - x^2 + 4x + 6$$

$$q(x) \;=\; -8x^5 - 7x^4 - 3x^3 + 5x^2 + 2$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (10)x^5 + (9)x^4 + (0)x^3 + (-1)x^2 + (4)x^1 + (6)x^0 \\ q(x) &= (-8)x^5 + (-7)x^4 + (-3)x^3 + (5)x^2 + (0)x^1 + (2)x^0 \\ q(x) - p(x) &= (-18)x^5 + (-16)x^4 + (-3)x^3 + (6)x^2 + (-4)x^1 + (-4)x^0 \\ q(x) - p(x) &= -18x^5 - 16x^4 - 3x^3 + 6x^2 - 4x - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) \;=\; -6x^2 + 3x - 7$$

$$b(x) \;=\; 8x - 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$3x$	-7
$8x$	$-48x^3$	$24x^2$	$-56x$
-4	$24x^2$	$-12x$	28

$$a(x) \cdot b(x) \;=\; -48x^3 + 24x^2 + 24x^2 - 56x - 12x + 28$$

Combine like terms.

$$a(x) \cdot b(x) \;=\; -48x^3 + 48x^2 - 68x + 28$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 14x^2 + 24x + 24 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 14 & 24 & 24 \\ \text{-5} & & \text{-10} & \text{-20} & \text{-20} \\ \hline & 2 & 4 & 4 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 4x + 4 + \frac{4}{x+5}$$

In other words, $h(x) = 2x^2 + 4x + 4$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 14x^2 + 24x + 24$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (2)\cdot(-5)^3 \ + \ (14)\cdot(-5)^2 \ + \ (24)\cdot(-5) \ + \ (24) \\ &= (2)\cdot(-125) \ + \ (14)\cdot(25) \ + \ (24)\cdot(-5) \ + \ (24) \\ &= (-250) \ + \ (350) \ + \ (-120) \ + \ (24) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x + 5$. Thus, $f(-5) = 4$.