

Name: \_\_\_\_\_

### at1203p\_qf: Finding roots and vertex of quadratic functions (v108)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

#### Example

$$f(x) = 6x^2 + 4x - 5$$

#### Example solution

1. Find the axis of symmetry. Use the formula  $h = \frac{-b}{2a}$ , where  $h$  is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)}$$

$$\text{axis of symmetry: } x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula  $w = \frac{\sqrt{b^2 - 4ac}}{2a}$ .

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The  $x$ -intercepts can be found by adding  $w$  to or subtracting  $w$  from  $h$ .

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know  $h = \frac{-1}{3}$ , so we just need  $k$ . Use the formula  $k = \frac{4ac - b^2}{4a}$ .

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex: } \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 7x^2 + 10x - 7$$

1. Axis of symmetry

$$h = \frac{-(-10)}{2(7)}$$

$$\text{axis of symmetry: } x = \frac{-5}{7}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(10)^2 - 4(7)(-7)}}{2(7)}$$

$$w = \frac{\sqrt{296}}{14} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 37}}{14} = \frac{2\sqrt{74}}{14}$$

$$w = \frac{\sqrt{74}}{7}$$

3. Roots

$$\left(\frac{-5}{7} - \frac{\sqrt{74}}{7}, 0\right) \quad \text{and} \quad \left(\frac{-5}{7} + \frac{\sqrt{74}}{7}, 0\right)$$

4. Vertex

$$k = \frac{4(7)(-7) - (10)^2}{4(7)}$$

$$k = \frac{-296}{28} = \frac{-74}{7}$$

$$\text{vertex: } \left(\frac{-5}{7}, \frac{-74}{7}\right)$$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 5x^2 - 3x - 9$$

1. Axis of symmetry

$$h = \frac{-(3)}{2(5)}$$

$$\text{axis of symmetry: } x = \frac{3}{10}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-3)^2 - 4(5)(-9)}}{2(5)}$$

$$w = \frac{\sqrt{189}}{10} = \frac{\sqrt{3 \cdot 3 \cdot 3 \cdot 7}}{10} = \frac{3\sqrt{21}}{10}$$

$$w = \frac{3\sqrt{21}}{10}$$

3. Roots

$$\left( \frac{3}{10} - \frac{3\sqrt{21}}{10}, 0 \right) \quad \text{and} \quad \left( \frac{3}{10} + \frac{3\sqrt{21}}{10}, 0 \right)$$

4. Vertex

$$k = \frac{4(5)(-9) - (-3)^2}{4(5)}$$

$$k = \frac{-189}{20}$$

$$\text{vertex: } \left( \frac{3}{10}, \frac{-189}{20} \right)$$

### Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 5x^2 + 6x - 1$$

1. Axis of symmetry

$$h = \frac{-(-6)}{2(5)}$$

$$\text{axis of symmetry: } x = \frac{-3}{5}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(6)^2 - 4(5)(-1)}}{2(5)}$$

$$w = \frac{\sqrt{56}}{10} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 7}}{10} = \frac{2\sqrt{14}}{10}$$

$$w = \frac{\sqrt{14}}{5}$$

3. Roots

$$\left(\frac{-3}{5} - \frac{\sqrt{14}}{5}, 0\right) \quad \text{and} \quad \left(\frac{-3}{5} + \frac{\sqrt{14}}{5}, 0\right)$$

4. Vertex

$$k = \frac{4(5)(-1) - (6)^2}{4(5)}$$

$$k = \frac{-56}{20} = \frac{-14}{5}$$

$$\text{vertex: } \left(\frac{-3}{5}, \frac{-14}{5}\right)$$