Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 140)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -x^5 + 10x^4 + 3x^3 - 4x - 5$$

$$q(x) = -2x^5 + 3x^3 - 5x^2 - 9x - 8$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-1)x^5 + (10)x^4 + (3)x^3 + (0)x^2 + (-4)x^1 + (-5)x^0 \\ q(x) &= (-2)x^5 + (0)x^4 + (3)x^3 + (-5)x^2 + (-9)x^1 + (-8)x^0 \\ p(x) - q(x) &= (1)x^5 + (10)x^4 + (0)x^3 + (5)x^2 + (5)x^1 + (3)x^0 \\ p(x) - q(x) &= x^5 + 10x^4 + 5x^2 + 5x + 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 + 8x - 9$$

$$b(x) = 5x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$8x$	-9
$5x$	$25x^3$	$40x^2$	$-45x$
3	$15x^2$	$24x$	-27

$$a(x) \cdot b(x) = 25x^3 + 40x^2 + 15x^2 - 45x + 24x - 27$$

Combine like terms.

$$a(x) \cdot b(x) = 25x^3 + 55x^2 - 21x - 27$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 140)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -6x^3 - 25x^2 - 4x + 1 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-6	-25	-4	1
-4		24	4	0
	-6	-1	0	1

So,

$$\frac{f(x)}{g(x)} = -6x^2 - x + \frac{1}{x+4}$$

In other words, $h(x) = -6x^2 - x$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -6x^3 - 25x^2 - 4x + 1$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (-6)\cdot(-4)^3 \ + \ (-25)\cdot(-4)^2 \ + \ (-4)\cdot(-4) \ + \ (1) \\ &= (-6)\cdot(-64) \ + \ (-25)\cdot(16) \ + \ (-4)\cdot(-4) \ + \ (1) \\ &= (384) \ + \ (-400) \ + \ (16) \ + \ (1) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = 1$.