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### s17 Geometric Series Exam (SLTN v376)

#### Question 1

Consider the partial geometric series represented below with first term  $a = 390$ , common ratio  $r = \left(\frac{4}{15}\right)^{1/10}$ , and  $n = 10$  terms.

$$S = 390 + 341.71 + 299.4 + 262.33 + 229.85 + 201.4 + 176.46 + 154.61 + 135.47 + 118.7$$

We can multiply both sides by  $r$ .

$$rS = 341.71 + 299.4 + 262.33 + 229.85 + 201.4 + 176.46 + 154.61 + 135.47 + 118.7 + 104$$

What is the value of  $S - rS$ ?

Most terms cancel.

$$390 - 104 = 286$$

#### Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 7 + 7(2) + 7(2)^2 + 7(2)^3 + \cdots + 7(2)^{66} + 7(2)^{67} + 7(2)^{68} + 7(2)^{69}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 7$$

$$\text{common ratio} = r = 2$$

$$\text{number of terms} = n = 70$$

### Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

### Definitions

$a$  = first term

$r$  = common ratio

$n$  = number of terms

$S$  = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by  $r$ .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out  $S$  from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by  $(1 - r)$ . We technically need to enforce  $r \neq 1$  as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$