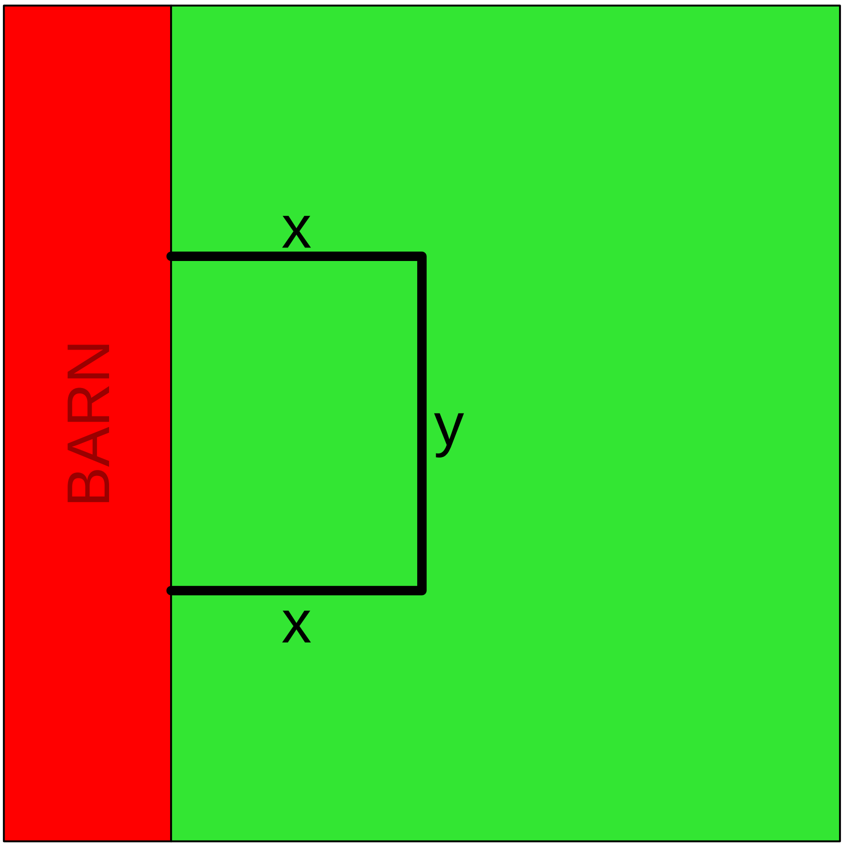
1. **Problem**
Amelia will use 96 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.



She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

Solution
The total length of fence is 96 feet. There are 2 sides with lengths of x and one side with a length of y.

$$2x + y = 96$$

Solve for y.

$$y = 96 - 2x$$

Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

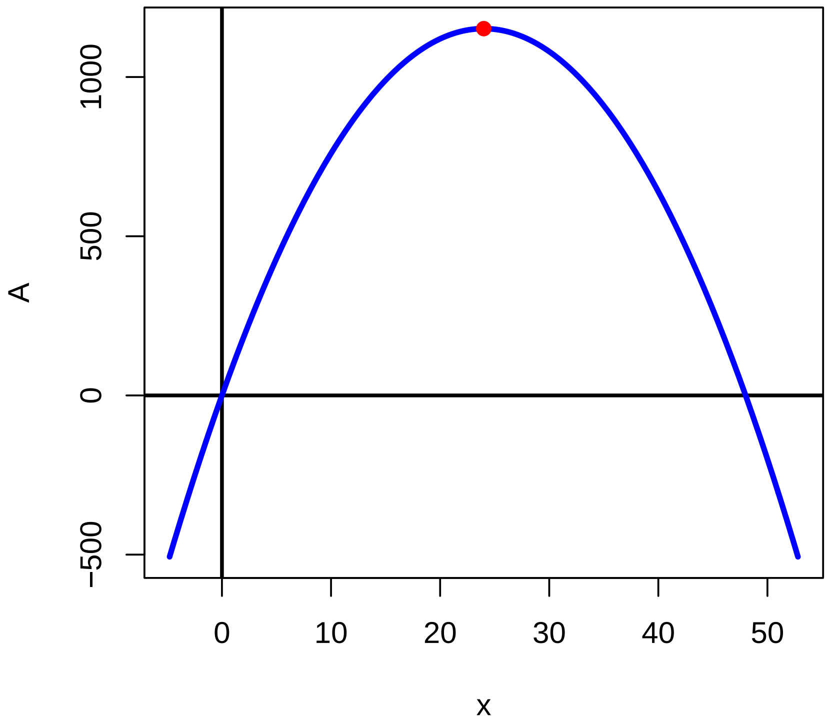
Substitute $96 - 2x$ in place of y to get area in terms of only x.

$$A = x(96 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex. Expand into standard form.

$$A = -2x^2 + 192x$$

You can graph A versus x and get a downwards facing parabola.



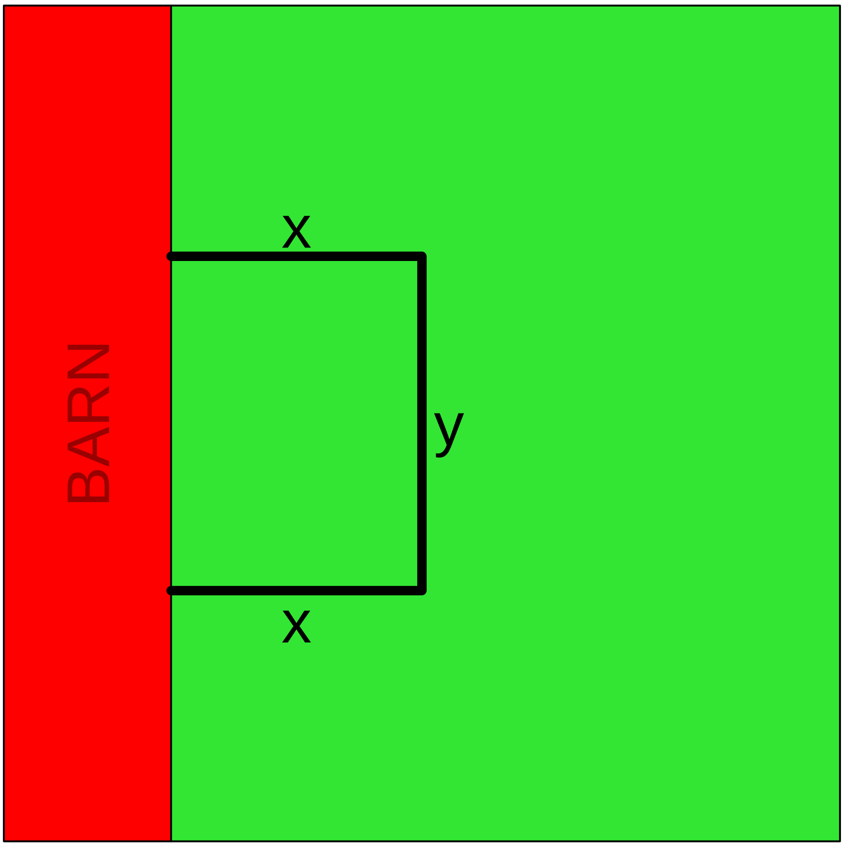
Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(192)}{2(-2)} = 24$$

So, the area is maximized when $x = 24$ feet and $y = 48$ feet.

1. **Problem**
Amelia will use 64 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.



She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

Solution
The total length of fence is 64 feet. There are 2 sides with lengths of x and one side with a length of y.

$$2x + y = 64$$

Solve for y.

$$y = 64 - 2x$$

Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

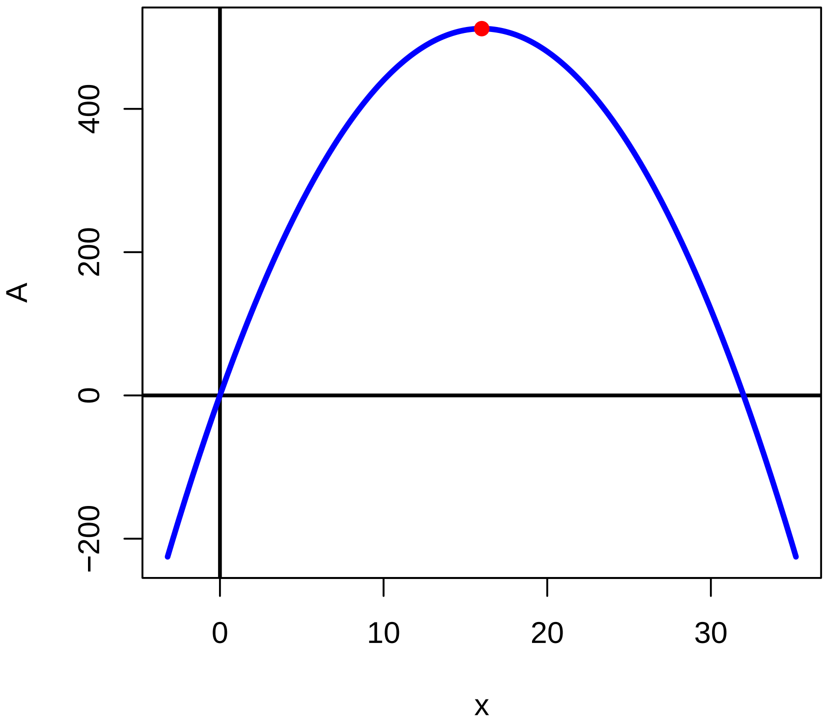
Substitute $64 - 2x$ in place of y to get area in terms of only x.

$$A = x(64 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex. Expand into standard form.

$$A = -2x^2 + 128x$$

You can graph A versus x and get a downwards facing parabola.



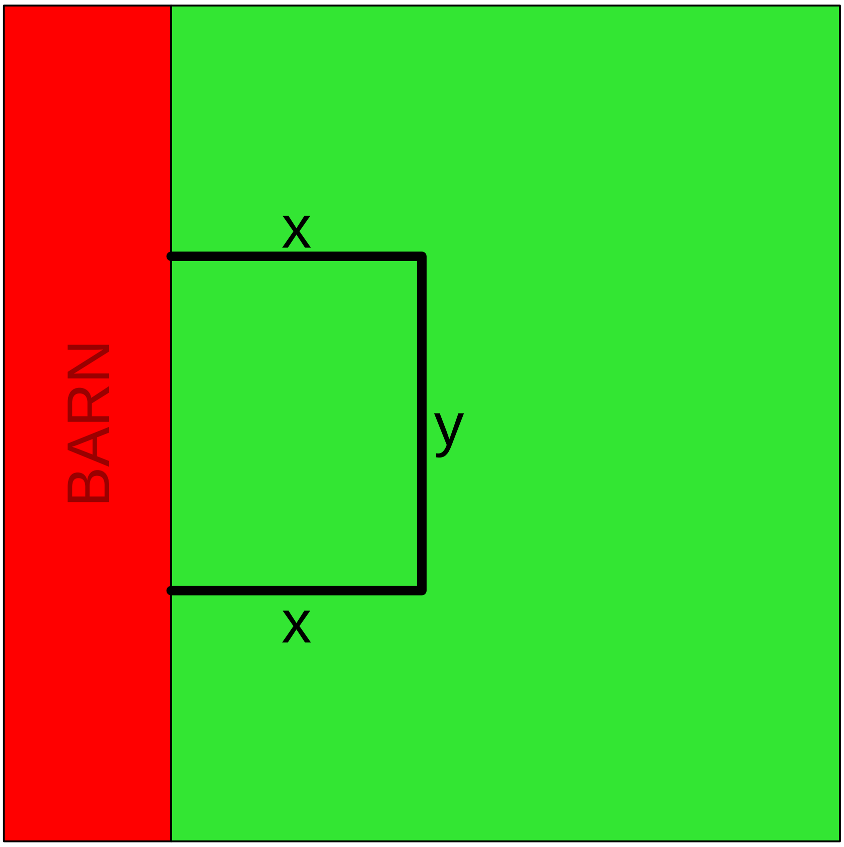
Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(128)}{2(-2)} = 16$$

So, the area is maximized when $x = 16$ feet and $y = 32$ feet.

1. **Problem**
   Amelia will use 160 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

   Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.

   She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

   Solution
   The total length of fence is 160 feet. There are 2 sides with lengths of x and one side with a length of y.

$$2x + y = 160$$

   Solve for y.

$$y = 160 - 2x$$

   Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

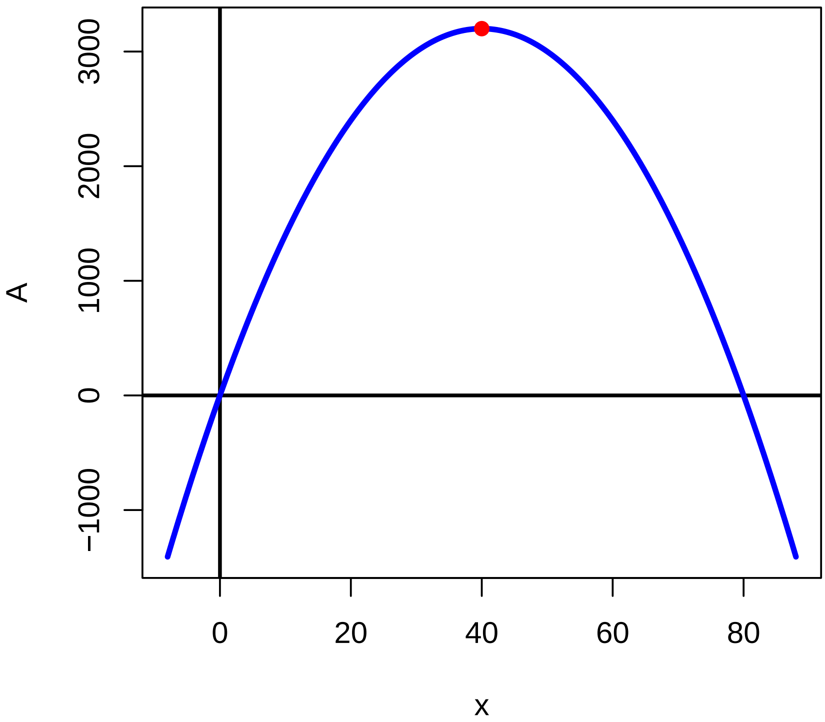
   Substitute $160 - 2x$ in place of y to get area in terms of only x.

$$A = x(160 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex. Expand into standard form.

$$A = -2x^2 + 320x$$

You can graph A versus x and get a downwards facing parabola.



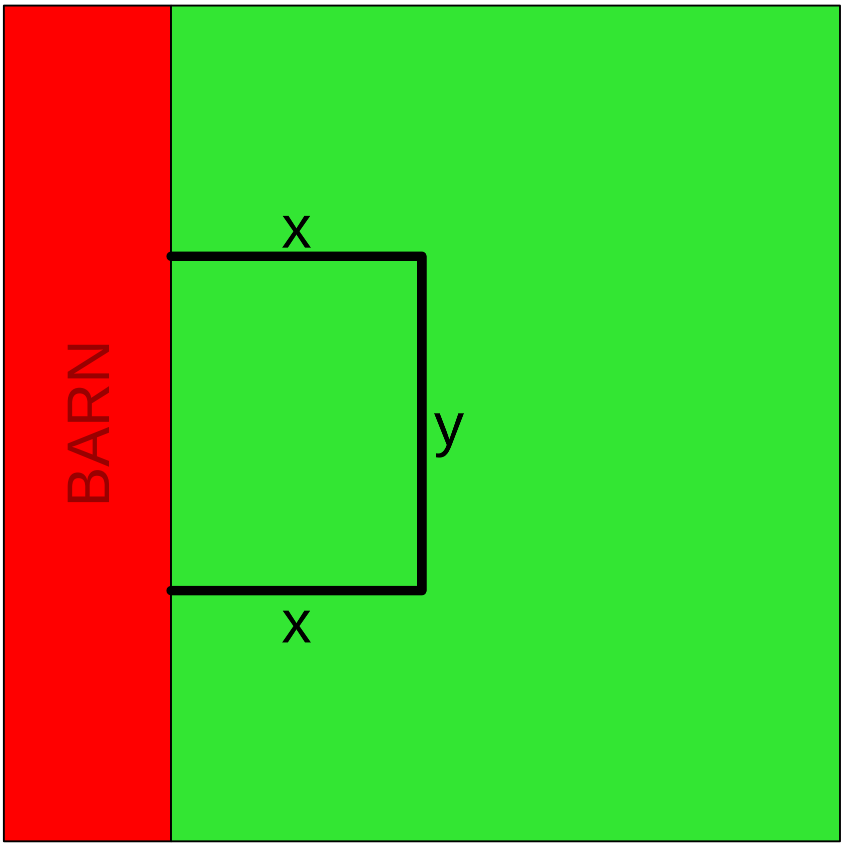
Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(320)}{2(-2)} = 40$$

So, the area is maximized when $x = 40$ feet and $y = 80$ feet.

1. **Problem**
   Amelia will use 104 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

   Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.

   She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

   Solution
   The total length of fence is 104 feet. There are 2 sides with lengths of x and one side with a length of y.

   $$2x + y = 104$$

   Solve for y.

   $$y = 104 - 2x$$

   Write an equation for the area. A rectangular area equals the length times the width.

   $$A = xy$$

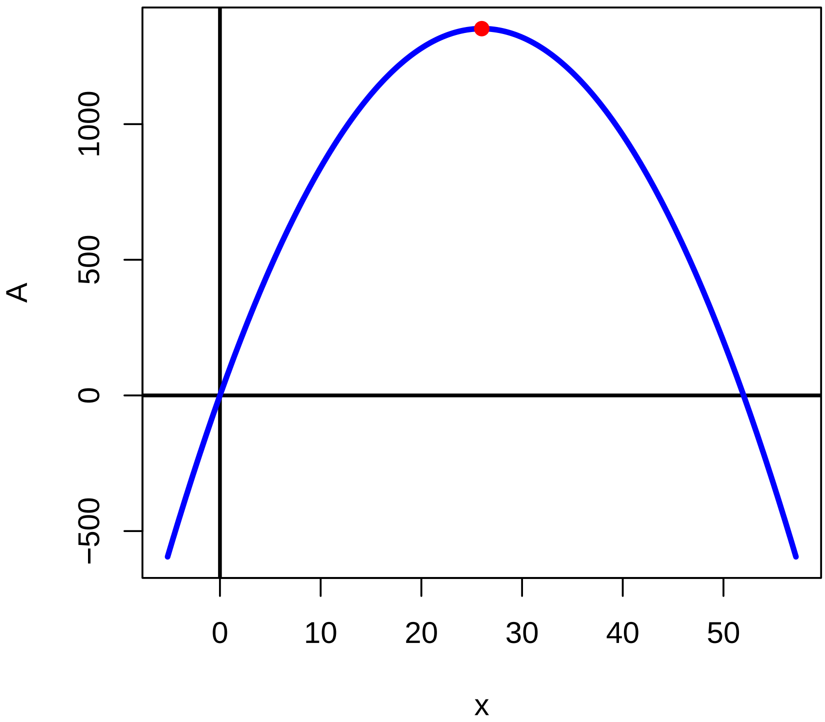
   Substitute $104 - 2x$ in place of y to get area in terms of only x.

$$A = x(104 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex. Expand into standard form.

$$A = -2x^2 + 208x$$

You can graph A versus x and get a downwards facing parabola.



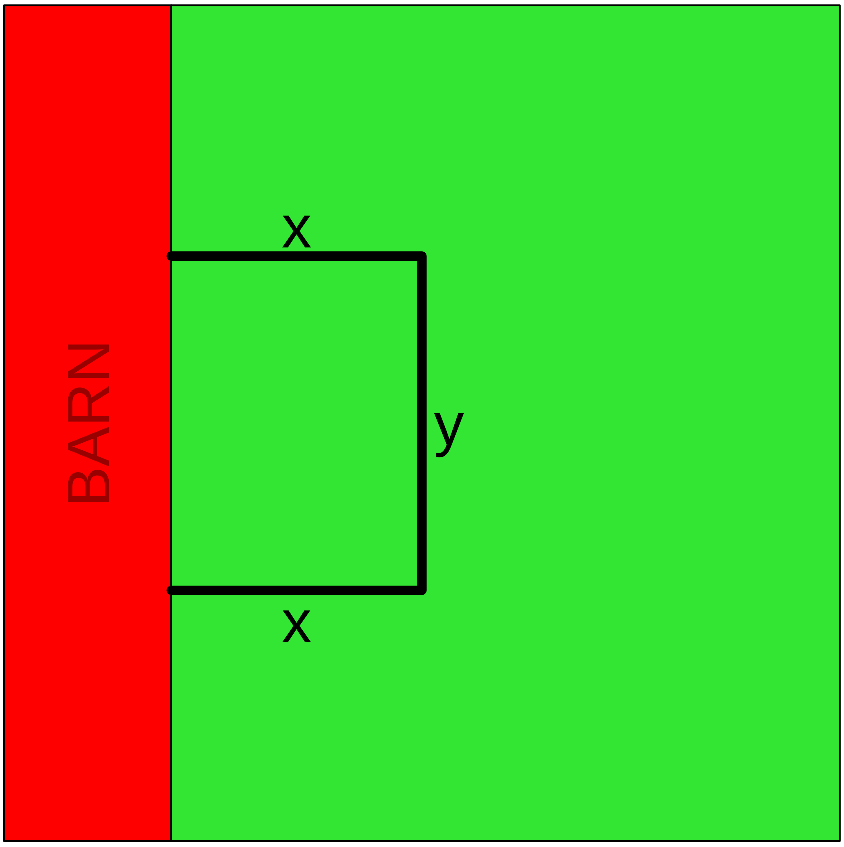
Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(208)}{2(-2)} = 26$$

So, the area is maximized when $x = 26$ feet and $y = 52$ feet.

1. **Problem**
Amelia will use 88 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.



She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

Solution
The total length of fence is 88 feet. There are 2 sides with lengths of x and one side with a length of y.

$$2x + y = 88$$

Solve for y.

$$y = 88 - 2x$$

Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

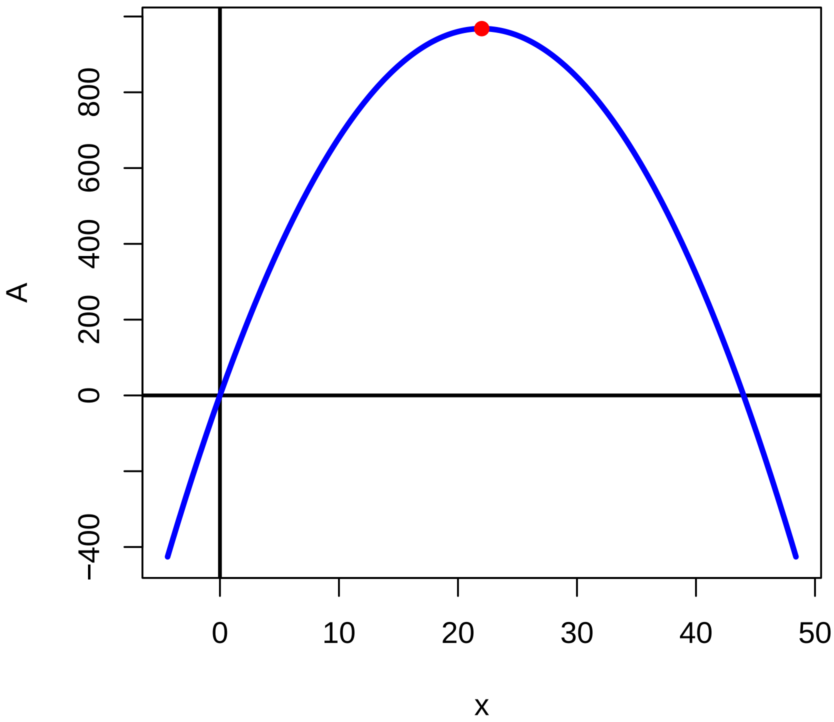
Substitute $88 - 2x$ in place of y to get area in terms of only x.

$$A = x(88 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex. Expand into standard form.

$$A = -2x^2 + 176x$$

You can graph A versus x and get a downwards facing parabola.



Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(176)}{2(-2)} = 22$$

So, the area is maximized when $x = 22$ feet and $y = 44$ feet.