Name: ______________________________________ Date: ________________

Polynomial Operations SOLUTION (version 134)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 10x^5 - 3x^4 + 6x^3 + 4x^2 - 9$$

$$q(x) = -5x^5 - 8x^4 + 7x^2 - 6x + 3$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (10)x^5 + (-3)x^4 + (6)x^3 + (4)x^2 + (0)x^1 + (-9)x^0 \\ q(x) &= (-5)x^5 + (-8)x^4 + (0)x^3 + (7)x^2 + (-6)x^1 + (3)x^0 \\ p(x) - q(x) &= (15)x^5 + (5)x^4 + (6)x^3 + (-3)x^2 + (6)x^1 + (-12)x^0 \\ p(x) - q(x) &= 15x^5 + 5x^4 + 6x^3 - 3x^2 + 6x - 12 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 - 7x + 4$$

$$b(x) = 6x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$-7x$	4
$6x$	$12x^3$	$-42x^2$	$24x$
3	$6x^2$	$-21x$	12

$$a(x) \cdot b(x) = 12x^3 - 42x^2 + 6x^2 + 24x - 21x + 12$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 - 36x^2 + 3x + 12$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 134)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 12x^2 + 27x - 6 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 12 & 27 & -6 \\ -9 & & -9 & -27 & 0 \\ \hline & 1 & 3 & 0 & -6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 3x + \frac{-6}{x+9}$$

In other words, $h(x) = x^2 + 3x$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 12x^2 + 27x - 6$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (1)\cdot(-9)^3 \ + \ (12)\cdot(-9)^2 \ + \ (27)\cdot(-9) \ + \ (-6) \\ &= (1)\cdot(-729) \ + \ (12)\cdot(81) \ + \ (27)\cdot(-9) \ + \ (-6) \\ &= (-729) \ + \ (972) \ + \ (-243) \ + \ (-6) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x+9$. Thus, $f(-9) = -6$.