Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 122)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -10x^5 - 7x^4 + x^3 - 9x^2 - 4$$
$$q(x) = 10x^5 - 4x^3 + 6x^2 + 3x - 2$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-10)x^5 + (-7)x^4 + (1)x^3 + (-9)x^2 + (0)x^1 + (-4)x^0 \\
q(x) &= (10)x^5 + (0)x^4 + (-4)x^3 + (6)x^2 + (3)x^1 + (-2)x^0 \\
p(x) + q(x) &= (0)x^5 + (-7)x^4 + (-3)x^3 + (-3)x^2 + (3)x^1 + (-6)x^0 \\
p(x) + q(x) &= -7x^4 - 3x^3 - 3x^2 + 3x - 6
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 - 2x - 3$$
$$b(x) = -5x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$-2x$	-3
$-5x$	$35x^3$	$10x^2$	$15x$
4	$-28x^2$	$-8x$	-12

$$a(x) \cdot b(x) = 35x^3 + 10x^2 - 28x^2 + 15x - 8x - 12$$

Combine like terms.

$$a(x) \cdot b(x) = 35x^3 - 18x^2 + 7x - 12$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 122)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -6x^3 + 29x^2 + 4x + 3 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -6 & 29 & 4 & 3 \\ 5 & & -30 & -5 & -5 \\ \hline & -6 & -1 & -1 & -2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -6x^2 - x - 1 + \frac{-2}{x-5}$$

In other words, $h(x) = -6x^2 - x - 1$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -6x^3 + 29x^2 + 4x + 3$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (-6)\cdot(5)^3 \;+\; (29)\cdot(5)^2 \;+\; (4)\cdot(5) \;+\; (3) \\ &= (-6)\cdot(125) \;+\; (29)\cdot(25) \;+\; (4)\cdot(5) \;+\; (3) \\ &= (-750) \;+\; (725) \;+\; (20) \;+\; (3) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x-5$. Thus, $f(5) = -2$.