Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 235)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -5x^5 + 6x^4 + x^2 + 10x + 7$$

$$q(x) = 3x^5 - 5x^3 - 6x^2 + 8x - 4$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-5)x^5 + (6)x^4 + (0)x^3 + (1)x^2 + (10)x^1 + (7)x^0 \\
q(x) &= (3)x^5 + (0)x^4 + (-5)x^3 + (-6)x^2 + (8)x^1 + (-4)x^0 \\
p(x) + q(x) &= (-2)x^5 + (6)x^4 + (-5)x^3 + (-5)x^2 + (18)x^1 + (3)x^0 \\
p(x) + q(x) &= -2x^5 + 6x^4 - 5x^3 - 5x^2 + 18x + 3
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 + 3x + 2$$

$$b(x) = 9x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$3x$	2
$9x$	$-54x^3$	$27x^2$	$18x$
-5	$30x^2$	$-15x$	-10

$$a(x) \cdot b(x) = -54x^3 + 27x^2 + 30x^2 + 18x - 15x - 10$$

Combine like terms.

$$a(x) \cdot b(x) = -54x^3 + 57x^2 + 3x - 10$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 235)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 + 15x^2 + 20x + 27 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & 15 & 20 & 27 \\ -4 & & -12 & -12 & -32 \\ \hline & 3 & 3 & 8 & -5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 3x + 8 + \frac{-5}{x+4}$$

In other words, $h(x) = 3x^2 + 3x + 8$ and the remainder is $R = -5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 + 15x^2 + 20x + 27$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (3)\cdot(-4)^3 \ + \ (15)\cdot(-4)^2 \ + \ (20)\cdot(-4) \ + \ (27) \\ &= (3)\cdot(-64) \ + \ (15)\cdot(16) \ + \ (20)\cdot(-4) \ + \ (27) \\ &= (-192) \ + \ (240) \ + \ (-80) \ + \ (27) \\ &= -5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x+4$. Thus, $f(-4) = -5$.