Name: ______________________________ Date: ______________

# Polynomial Operations SOLUTIONS (version 29)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) = -10x^5 + 5x^3 + 3x^2 + 4x + 9$$

$$q(x) = -4x^5 - 6x^4 - 8x^3 + 2x^2 - 3$$

**Express the sum of $p(x) + q(x)$ in standard form.**

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-10)x^5 + (0)x^4 + (5)x^3 + (3)x^2 + (4)x^1 + (9)x^0 \\ q(x) &= (-4)x^5 + (-6)x^4 + (-8)x^3 + (2)x^2 + (0)x^1 + (-3)x^0 \\ p(x) + q(x) &= (-14)x^5 + (-6)x^4 + (-3)x^3 + (5)x^2 + (4)x^1 + (6)x^0 \\ p(x) + q(x) &= -14x^5 - 6x^4 - 3x^3 + 5x^2 + 4x + 6 \end{aligned}$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) = 6x^2 - 5x - 4$$

$$b(x) = -2x - 8$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $6x^2$ | $-5x$ | $-4$ |
|---|---|---|---|
| $-2x$ | $-12x^3$ | $10x^2$ | $8x$ |
| $-8$ | $-48x^2$ | $40x$ | $32$ |

$$a(x) \cdot b(x) = -12x^3 + 10x^2 - 48x^2 + 8x + 40x + 32$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 - 38x^2 + 48x + 32$$

**3. Express $(x + 1)^4$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= -4x^3 + 25x^2 + 23x - 13 \\ g(x) &= x - 7 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

| | | | | |
|---|---|---|---|---|
| | -4 | 25 | 23 | -13 |
| 7 | | -28 | -21 | 14 |
| | -4 | -3 | 2 | 1 |

So,

$$\frac{f(x)}{g(x)} = -4x^2 - 3x + 2 + \frac{1}{x-7}$$

In other words, $h(x) = -4x^2 - 3x + 2$ and the remainder is $R = 1$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = -4x^3 + 25x^2 + 23x - 13$. Evaluate $f(7)$.**

You could do this the hard way.

$$\begin{aligned} f(7) &= (-4)\cdot(7)^3 \;+\; (25)\cdot(7)^2 \;+\; (23)\cdot(7) \;+\; (-13) \\ &= (-4)\cdot(343) \;+\; (25)\cdot(49) \;+\; (23)\cdot(7) \;+\; (-13) \\ &= (-1372) \;+\; (1225) \;+\; (161) \;+\; (-13) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = 1$.