Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 208)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 - x^3 - 7x^2 - 5x + 3$$

$$q(x) = -2x^5 + 8x^4 + 7x^3 - 5x + 9$$

Express the difference $p(x) - q(x)$ in standard form.

Get “unsimplified” forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (0)x^4 + (-1)x^3 + (-7)x^2 + (-5)x^1 + (3)x^0 \\ q(x) &= (-2)x^5 + (8)x^4 + (7)x^3 + (0)x^2 + (-5)x^1 + (9)x^0 \\ p(x) - q(x) &= (0)x^5 + (-8)x^4 + (-8)x^3 + (-7)x^2 + (0)x^1 + (-6)x^0 \\ p(x) - q(x) &= -8x^4 - 8x^3 - 7x^2 - 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 9x - 5$$

$$b(x) = 5x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-9x$	-5
$5x$	$30x^3$	$-45x^2$	$-25x$
4	$24x^2$	$-36x$	-20

$$a(x) \cdot b(x) = 30x^3 - 45x^2 + 24x^2 - 25x - 36x - 20$$

Combine like terms.

$$a(x) \cdot b(x) = 30x^3 - 21x^2 - 61x - 20$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal’s triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 208)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 + 16x^2 + 8x + 21 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & 16 & 8 & 21 \\ \text{-5} & & \text{-15} & \text{-5} & \text{-15} \\ \hline & 3 & 1 & 3 & 6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + x + 3 + \frac{6}{x+5}$$

In other words, $h(x) = 3x^2 + x + 3$ and the remainder is $R = 6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 + 16x^2 + 8x + 21$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (3)\cdot(-5)^3 \;+\; (16)\cdot(-5)^2 \;+\; (8)\cdot(-5) \;+\; (21) \\ &= (3)\cdot(-125) \;+\; (16)\cdot(25) \;+\; (8)\cdot(-5) \;+\; (21) \\ &= (-375) \;+\; (400) \;+\; (-40) \;+\; (21) \\ &= 6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x + 5$. Thus, $f(-5) = 6$.