Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 139)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -4x^5 - 10x^4 - 2x^2 - 9x - 5$$

$$q(x) = -6x^5 + 9x^4 + 4x^3 + 10x - 3$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-4)x^5 + (-10)x^4 + (0)x^3 + (-2)x^2 + (-9)x^1 + (-5)x^0 \\ q(x) &= (-6)x^5 + (9)x^4 + (4)x^3 + (0)x^2 + (10)x^1 + (-3)x^0 \\ p(x) + q(x) &= (-10)x^5 + (-1)x^4 + (4)x^3 + (-2)x^2 + (1)x^1 + (-8)x^0 \\ p(x) + q(x) &= -10x^5 - x^4 + 4x^3 - 2x^2 + x - 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 - 5x - 3$$

$$b(x) = -4x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$-5x$	-3
$-4x$	$-8x^3$	$20x^2$	$12x$
-5	$-10x^2$	$25x$	15

$$a(x) \cdot b(x) = -8x^3 + 20x^2 - 10x^2 + 12x + 25x + 15$$

Combine like terms.

$$a(x) \cdot b(x) = -8x^3 + 10x^2 + 37x + 15$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 139)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 + 27x^2 + x - 12 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & 27 & 1 & -12 \\ 9 & & -27 & 0 & 9 \\ \hline & -3 & 0 & 1 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 1 + \frac{-3}{x-9}$$

In other words, $h(x) = -3x^2 + 1$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 + 27x^2 + x - 12$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-3)\cdot(9)^3 \; + \; (27)\cdot(9)^2 \; + \; (1)\cdot(9) \; + \; (-12) \\ &= (-3)\cdot(729) \; + \; (27)\cdot(81) \; + \; (1)\cdot(9) \; + \; (-12) \\ &= (-2187) \; + \; (2187) \; + \; (9) \; + \; (-12) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = -3$.