Name: __________ Date: __________

# Polynomial Operations SOLUTION (version 102)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) = x^5 - 4x^4 - 8x^3 - 10x - 2$$

$$q(x) = 5x^5 - 4x^4 + 8x^2 + x - 6$$

**Express the difference $q(x) - p(x)$ in standard form.**

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (1)x^5 + (-4)x^4 + (-8)x^3 + (0)x^2 + (-10)x^1 + (-2)x^0 \\ q(x) &= (5)x^5 + (-4)x^4 + (0)x^3 + (8)x^2 + (1)x^1 + (-6)x^0 \\ q(x) - p(x) &= (4)x^5 + (0)x^4 + (8)x^3 + (8)x^2 + (11)x^1 + (-4)x^0 \\ q(x) - p(x) &= 4x^5 + 8x^3 + 8x^2 + 11x - 4 \end{aligned}$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) = 4x^2 - 3x + 5$$

$$b(x) = 5x - 3$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $4x^2$ | $-3x$ | $5$ |
|---|---|---|---|
| $5x$ | $20x^3$ | $-15x^2$ | $25x$ |
| $-3$ | $-12x^2$ | $9x$ | $-15$ |

$$a(x) \cdot b(x) = 20x^3 - 15x^2 - 12x^2 + 25x + 9x - 15$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 - 27x^2 + 34x - 15$$

**3. Express $(x + 1)^6$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= 2x^3 + 13x^2 - 23x + 17 \\ g(x) &= x + 8 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 13 & -23 & 17 \\ -8 & & -16 & 24 & -8 \\ \hline & 2 & -3 & 1 & 9 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 - 3x + 1 + \frac{9}{x+8}$$

In other words, $h(x) = 2x^2 - 3x + 1$ and the remainder is $R = 9$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 13x^2 - 23x + 17$. Evaluate $f(-8)$.**

You could do this the hard way.

$$\begin{aligned} f(-8) &= (2)\cdot(-8)^3 \ + \ (13)\cdot(-8)^2 \ + \ (-23)\cdot(-8) \ + \ (17) \\ &= (2)\cdot(-512) \ + \ (13)\cdot(64) \ + \ (-23)\cdot(-8) \ + \ (17) \\ &= (-1024) \ + \ (832) \ + \ (184) \ + \ (17) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x + 8$. Thus, $f(-8) = 9$.