Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 121)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -4x^5 - 9x^4 + 3x^3 - x^2 + 6 \\ q(x) &= -x^5 + 8x^4 + 5x^3 + 7x - 3 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-4)x^5 + (-9)x^4 + (3)x^3 + (-1)x^2 + (0)x^1 + (6)x^0 \\ q(x) &= (-1)x^5 + (8)x^4 + (5)x^3 + (0)x^2 + (7)x^1 + (-3)x^0 \\ p(x) - q(x) &= (-3)x^5 + (-17)x^4 + (-2)x^3 + (-1)x^2 + (-7)x^1 + (9)x^0 \\ p(x) - q(x) &= -3x^5 - 17x^4 - 2x^3 - x^2 - 7x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -3x^2 + 9x - 7 \\ b(x) &= -5x - 6 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$9x$	-7
$-5x$	$15x^3$	$-45x^2$	$35x$
-6	$18x^2$	$-54x$	42

$$a(x) \cdot b(x) = 15x^3 - 45x^2 + 18x^2 + 35x - 54x + 42$$

Combine like terms.

$$a(x) \cdot b(x) = 15x^3 - 27x^2 - 19x + 42$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 121)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$f(x) = -x^3 + 3x^2 + 28x - 6$$
$$g(x) = x - 7$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -1 & 3 & 28 & -6 \\ 7 & & -7 & -28 & 0 \\ \hline & -1 & -4 & 0 & -6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 4x + \frac{-6}{x-7}$$

In other words, $h(x) = -x^2 - 4x$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 3x^2 + 28x - 6$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (-1)\cdot(7)^3 + (3)\cdot(7)^2 + (28)\cdot(7) + (-6) \\ &= (-1)\cdot(343) + (3)\cdot(49) + (28)\cdot(7) + (-6) \\ &= (-343) + (147) + (196) + (-6) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = -6$.