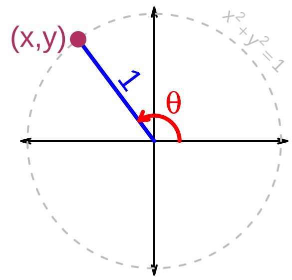
Unit-Circle Trigonometry Cheat Sheet



Definitions

$$\sin(\theta) = y \qquad \cos(\theta) = x \qquad \tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} = \text{slope}$$

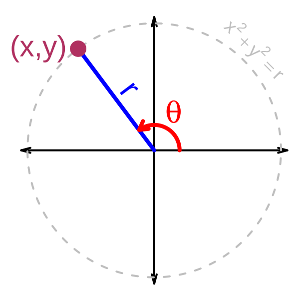
Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad |\sin(\theta)| = \sqrt{1 - \cos^2(\theta)} \qquad |\cos(\theta)| = \sqrt{1 - \sin^2(\theta)}$$

$$\tan^2(\theta) + 1 = \frac{1}{\cos^2(\theta)} \qquad |\tan(\theta)| = \sqrt{\frac{1 - \cos^2(\theta)}{\cos^2(\theta)}} \qquad |\cos(\theta)| = \sqrt{\frac{1}{\tan^2(\theta) + 1}}$$

$$\tan^2(\theta) + 1 = \frac{1}{1 - \sin^2(\theta)} \qquad |\tan(\theta)| = \sqrt{\frac{\sin^2(\theta)}{1 - \sin^2(\theta)}} \qquad |\sin(\theta)| = \sqrt{\frac{\tan^2(\theta)}{\tan^2(\theta) + 1}}$$

Polar Coordinates

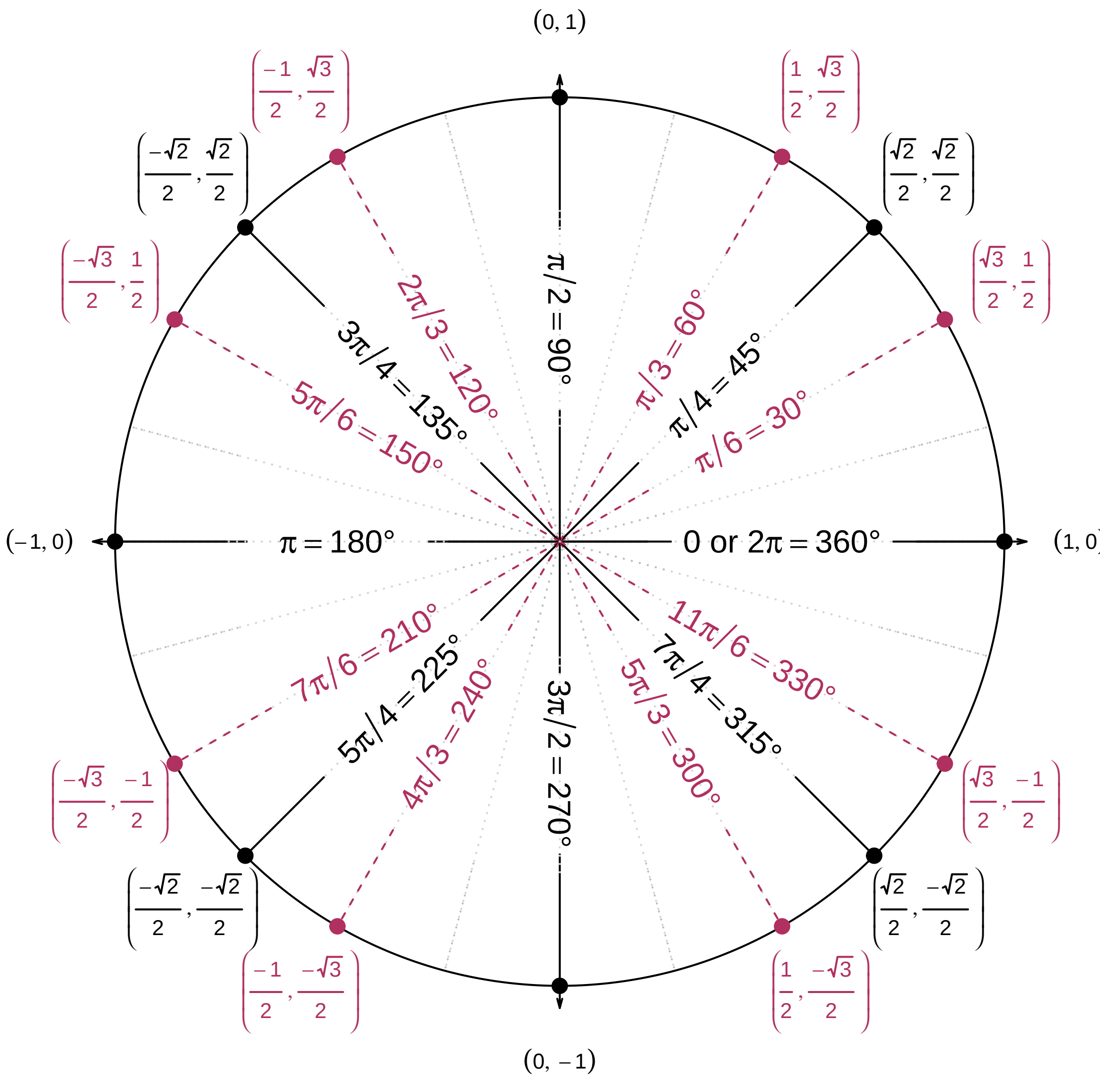


$$x = r \cdot \cos(\theta) \qquad y = r \cdot \sin(\theta)$$

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan(y, x)$$

Special angles

- Draw an isosceles right triangle with a hypotenuse of length 1 and leg length of x. Solve $x^2 + x^2 = 1^2$ to prove length ratios of $\frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1$ for the $45° - 45° - 90°$ triangle.
- Draw an equilateral triangle, and cut it in half to produce a right triangle with a hypotenuse of length 1, a leg of length 1/2, and another leg of length x. Solve $x^2 + \left(\frac{1}{2}\right)^2 = 1^2$ to prove length ratios of $\frac{1}{2} : \frac{\sqrt{3}}{2} : 1$ for the $30° - 60° - 90°$ triangle..
- See the right-triangle cheat sheet for diagrams.
- Use symmetry of the unit circle to determine all coordinates shown below.



So, for example:

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2} \qquad \tan\left(\frac{2\pi}{3}\right) = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{-1}{2}\right)} = -\sqrt{3}$$