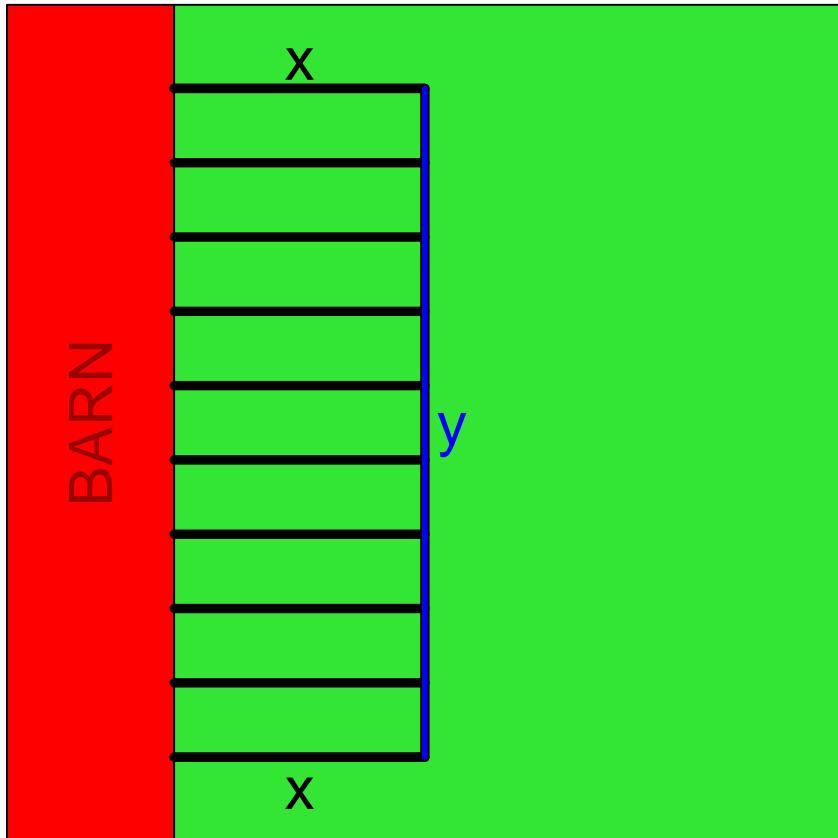


### 1. Problem

Amelia will use 460 feet of fence to build 9 rectangular enclosures. As shown in the figure below, the 9 enclosures will be built so each one is against a barn. Neighboring enclosures will use a single fence to separate them. Let  $x$  represent the length of fence perpendicular to the barn, and let  $y$  represent the length of fence parallel to the barn.



Amelia wants to maximize the total area of the enclosures. Find the value of  $x$  that maximizes the area.

#### Solution

The total area is simply the product of  $x$  and  $y$ .

$$A = xy$$

The total length of fence is 460 feet. Notice for 9 enclosures, we need 10 lengths of  $x$ .

$$460 = 10x + y$$

Solve this equation for  $y$  by subtracting  $10x$  from both sides.

$$460 - 10x = y$$

Substitute  $460 - 10x$  for  $y$  in the area equation.

$$A = x \cdot (460 - 10x)$$

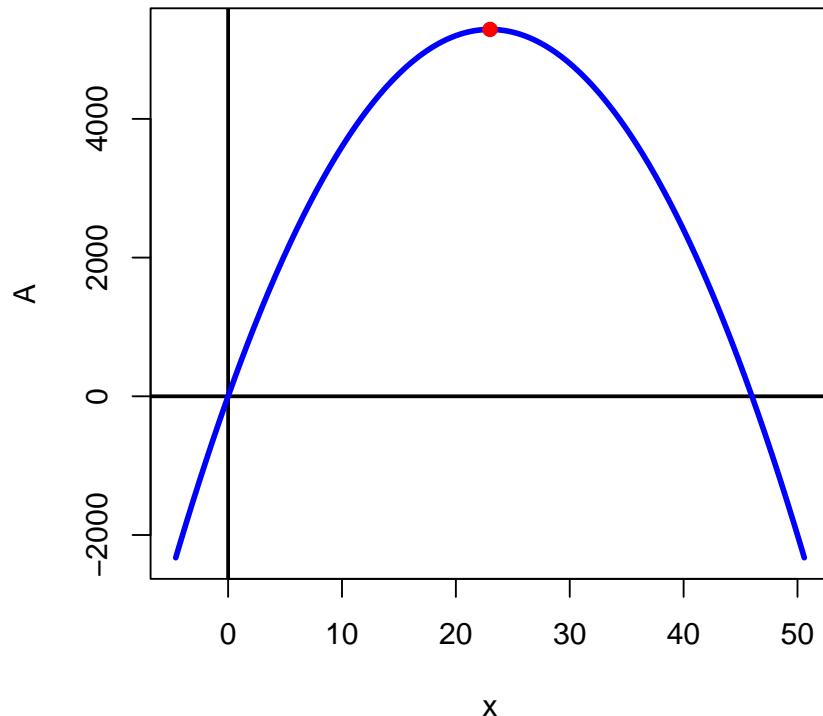
Distribute.

$$A = 460x - 10x^2$$

Put quadratic expression in standard order.

$$A = -10x^2 + 460x$$

If you draw a graph of  $A$  versus  $x$ , you'll get a parabola.



Notice, the maximum area occurs at the parabola's vertex. So, we can use  $h = \frac{-b}{2a}$  to find the optimal  $x$  value.

$$x_{\text{optimal}} = \frac{-b}{2a}$$

$$x_{\text{optimal}} = \frac{-(460)}{2(-10)}$$

$$x_{\text{optimal}} = 23$$