Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 135)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 5x^5 - 2x^3 - 10x^2 - 7x + 6$$

$$q(x) = -4x^5 - 5x^4 + 9x^2 + 2x + 1$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (5)x^5 + (0)x^4 + (-2)x^3 + (-10)x^2 + (-7)x^1 + (6)x^0 \\ q(x) &= (-4)x^5 + (-5)x^4 + (0)x^3 + (9)x^2 + (2)x^1 + (1)x^0 \\ q(x) - p(x) &= (-9)x^5 + (-5)x^4 + (2)x^3 + (19)x^2 + (9)x^1 + (-5)x^0 \\ q(x) - p(x) &= -9x^5 - 5x^4 + 2x^3 + 19x^2 + 9x - 5 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -4x^2 + 7x + 6$$

$$b(x) = -5x + 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-4x^2$	$7x$	6
$-5x$	$20x^3$	$-35x^2$	$-30x$
2	$-8x^2$	$14x$	12

$$a(x) \cdot b(x) = 20x^3 - 35x^2 - 8x^2 - 30x + 14x + 12$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 - 43x^2 - 16x + 12$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 135)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 + 14x^2 + 25x - 4 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & 14 & 25 & -4 \\ 6 & & -18 & -24 & 6 \\ \hline & -3 & -4 & 1 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 - 4x + 1 + \frac{2}{x-6}$$

In other words, $h(x) = -3x^2 - 4x + 1$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 + 14x^2 + 25x - 4$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (-3)\cdot(6)^3 \; + \; (14)\cdot(6)^2 \; + \; (25)\cdot(6) \; + \; (-4) \\ &= (-3)\cdot(216) \; + \; (14)\cdot(36) \; + \; (25)\cdot(6) \; + \; (-4) \\ &= (-648) \; + \; (504) \; + \; (150) \; + \; (-4) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = 2$.