at1118paper: Complete the Square (v407)

Example

By completing the square, find both solutions to the given equation:

$$x^2 - 54x = -440$$

Add $\left(\frac{-54}{2}\right)^2$, which equals 729, to both sides of the equation.

$$x^2 - 54x + 729 = 289$$

Factor the left side.

$$(x - 27)^2 = 289$$

Undo the squaring. We need to consider both $\pm\sqrt{289}$.

$$x - 27 = -17 \qquad\qquad \text{or} \qquad\qquad x - 27 = 17$$
$$x = 10 \qquad\qquad \text{or} \qquad\qquad x = 44$$

Question 1

By completing the square, find both solutions to the given equation:

$$x^2 + 56x = -159$$

$$x^2 + 56x + 784 = 625$$
$$(x + 28)^2 = 625$$
$$x + 28 = \pm 25$$
$$x = -53 \qquad \text{or} \qquad x = -3$$

Question 2

By completing the square, find both solutions to the given equation:

$$x^2 - 6x = 616$$

$$x^2 - 6x + 9 = 625$$
$$(x - 3)^2 = 625$$
$$x - 3 = \pm 25$$
$$x = -22 \qquad \text{or} \qquad x = 28$$

Question 3

By completing the square, find both solutions to the given equation:

$$x^2 - 18x \;=\; 1008$$

$$x^2 - 18x + 81 \;=\; 1089$$
$$(x - 9)^2 \;=\; 1089$$
$$x - 9 \;=\; \pm 33$$
$$x = -24 \qquad \text{or} \qquad x = 42$$

Question 4

By completing the square, find both solutions to the given equation:

$$x^2 + 28x \;=\; 380$$

$$x^2 + 28x + 196 \;=\; 576$$
$$(x + 14)^2 \;=\; 576$$
$$x + 14 \;=\; \pm 24$$
$$x = -38 \qquad \text{or} \qquad x = 10$$

Question 5

By completing the square, find both solutions to the given equation:

$$x^2 - 48x \;=\; 265$$

$$x^2 - 48x + 576 \;=\; 841$$
$$(x - 24)^2 \;=\; 841$$
$$x - 24 \;=\; \pm 29$$
$$x = -5 \qquad \text{or} \qquad x = 53$$

Question 6

By completing the square, find both solutions to the given equation:

$$x^2 + 42x \;=\; 184$$

$$x^2 + 42x + 441 \;=\; 625$$
$$(x + 21)^2 \;=\; 625$$
$$x + 21 \;=\; \pm 25$$
$$x = -46 \qquad \text{or} \qquad x = 4$$