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## s17 Geometric Series Exam (Solution v21)

### Question 1

Consider the partial geometric series represented below with first term  $a = 702$ , common ratio  $r = \left(\frac{20}{27}\right)^{1/10}$ , and  $n = 10$  terms.

$$S = 702 + 681.25 + 661.1 + 641.56 + 622.59 + 604.19 + 586.32 + 568.99 + 552.17 + 535.84$$

We can multiply both sides by  $r$ .

$$rS = 681.25 + 661.1 + 641.56 + 622.59 + 604.19 + 586.32 + 568.99 + 552.17 + 535.84 + 520$$

What is the value of  $S - rS$ ?

Most terms cancel.

$$702 - 520 = 182$$

### Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 2 + 2(8) + 2(8)^2 + 2(8)^3 + \cdots + 2(8)^{47} + 2(8)^{48} + 2(8)^{49} + 2(8)^{50}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 2$$

$$\text{common ratio} = r = 8$$

$$\text{number of terms} = n = 51$$

### Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

### Definitions

$a$  = first term

$r$  = common ratio

$n$  = number of terms

$S$  = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by  $r$ .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out  $S$  from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by  $(1 - r)$ . We technically need to enforce  $r \neq 1$  as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$