Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 38)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -5x^5 + 2x^3 - 4x^2 + 8x + 7$$

$$q(x) = -8x^5 - 9x^4 - 6x^3 + 5x^2 - 10$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-5)x^5 + (0)x^4 + (2)x^3 + (-4)x^2 + (8)x^1 + (7)x^0 \\ q(x) &= (-8)x^5 + (-9)x^4 + (-6)x^3 + (5)x^2 + (0)x^1 + (-10)x^0 \\ p(x) - q(x) &= (3)x^5 + (9)x^4 + (8)x^3 + (-9)x^2 + (8)x^1 + (17)x^0 \\ p(x) - q(x) &= 3x^5 + 9x^4 + 8x^3 - 9x^2 + 8x + 17 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 8x^2 + 6x - 9$$

$$b(x) = -6x - 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$8x^2$	$6x$	-9
$-6x$	$-48x^3$	$-36x^2$	$54x$
-4	$-32x^2$	$-24x$	36

$$a(x) \cdot b(x) = -48x^3 - 36x^2 - 32x^2 + 54x - 24x + 36$$

Combine like terms.

$$a(x) \cdot b(x) = -48x^3 - 68x^2 + 30x + 36$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 - 25x^2 - 25x + 22 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	4	-25	-25	22
7		28	21	-28
	4	3	-4	-6

So,

$$\frac{f(x)}{g(x)} = 4x^2 + 3x - 4 + \frac{-6}{x-7}$$

In other words, $h(x) = 4x^2 + 3x - 4$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 - 25x^2 - 25x + 22$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (4)\cdot(7)^3 \ + \ (-25)\cdot(7)^2 \ + \ (-25)\cdot(7) \ + \ (22) \\ &= (4)\cdot(343) \ + \ (-25)\cdot(49) \ + \ (-25)\cdot(7) \ + \ (22) \\ &= (1372) \ + \ (-1225) \ + \ (-175) \ + \ (22) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = -6$.