Name: ______________________ Date: ______________

Polynomial Operations SOLUTIONS (version 4)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 3x^5 - 10x^4 + 7x^3 + 4x^2 + 5 \\ q(x) &= -2x^5 - 8x^4 + x^3 - 3x - 6 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (-10)x^4 + (7)x^3 + (4)x^2 + (0)x^1 + (5)x^0 \\ q(x) &= (-2)x^5 + (-8)x^4 + (1)x^3 + (0)x^2 + (-3)x^1 + (-6)x^0 \\ p(x) - q(x) &= (5)x^5 + (-2)x^4 + (6)x^3 + (4)x^2 + (3)x^1 + (11)x^0 \\ p(x) - q(x) &= 5x^5 - 2x^4 + 6x^3 + 4x^2 + 3x + 11 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 2x^2 + 7x - 4 \\ b(x) &= 5x + 2 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$7x$	-4
$5x$	$10x^3$	$35x^2$	$-20x$
2	$4x^2$	$14x$	-8

$$a(x) \cdot b(x) = 10x^3 + 35x^2 + 4x^2 - 20x + 14x - 8$$

Combine like terms.

$$a(x) \cdot b(x) = 10x^3 + 39x^2 - 6x - 8$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTIONS (version 4)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 16x^2 + 7x - 26 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & -16 & 7 & -26 \\ -6 & & 18 & -12 & 30 \\ \hline & -3 & 2 & -5 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 2x - 5 + \frac{4}{x+6}$$

In other words, $h(x) = -3x^2 + 2x - 5$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 16x^2 + 7x - 26$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-3)\cdot(-6)^3 \ + \ (-16)\cdot(-6)^2 \ + \ (7)\cdot(-6) \ + \ (-26) \\ &= (-3)\cdot(-216) \ + \ (-16)\cdot(36) \ + \ (7)\cdot(-6) \ + \ (-26) \\ &= (648) \ + \ (-576) \ + \ (-42) \ + \ (-26) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 4$.