Notes: Coordinate transformations

It is helpful to rewrite function transformations as coordinate transformations. For example:

$$g[x] = \frac{f[2x+3]}{5} + 7$$

Can be rewritten as:

f	g
(a, b)	$\rightarrow$ $\left(\dfrac{a-3}{2} , \dfrac{b}{5} + 7 \right)$

This tells us that if $(25, 30)$ is on curve f we can find the corresponding point on curve g:

$$\left(\frac{25-3}{2} , \frac{30}{5} + 7 \right)$$

$$(11 , 13)$$

In other words, since $f[25] = 30$ we know $g[11] = 13$.

Detailed logic

Notice, this could be worked out as follows. Since (a, b) is a point on the f curve, we know $f[a] = b$. To use that information in the main equation, we need $2x + 3$ to equal a.

$$2x + 3 = a$$

Solve for x. (This is a two-step equation.)

$$x = \frac{a-3}{2}$$

Going back to the main equation, replace $2x + 3$ with a and replace x with $\frac{a-3}{2}$.

$$g\left[\frac{a-3}{2} \right] = \frac{f[a]}{5} + 7$$

Replace $f[a]$ with b.

$$g\left[\frac{a-3}{2} \right] = \frac{b}{5} + 7$$

And so we know point $\left(\frac{a-3}{2} , \frac{b}{5} + 7 \right)$ is on curve g.

Shortcut

Use the INVERSE of the ARGUMENT for x and the OUTER operations for y.

Some examples:

Example 1

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; \frac{f\left[\frac{x}{7} + 3\right]}{9} + 2$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \to \left(7\,(a - 3) \;,\; \frac{b}{9} + 2\right)$$

Example 2

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; 6 \cdot f[8\,(x + 9)] + 2$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \to \left(\frac{a}{8} - 9 \;,\; 6b + 2\right)$$

Example 3

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; 7 \cdot (f[2x - 6] - 8)$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \to \left(\frac{a + 6}{2} \;,\; 7\,(b - 8)\right)$$