Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 239)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -5x^5 - 6x^4 - 10x^3 + x^2 + 8$$

$$q(x) = 8x^5 - 3x^3 - 10x^2 - 2x + 6$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-5)x^5 + (-6)x^4 + (-10)x^3 + (1)x^2 + (0)x^1 + (8)x^0 \\ q(x) &= (8)x^5 + (0)x^4 + (-3)x^3 + (-10)x^2 + (-2)x^1 + (6)x^0 \\ p(x) - q(x) &= (-13)x^5 + (-6)x^4 + (-7)x^3 + (11)x^2 + (2)x^1 + (2)x^0 \\ p(x) - q(x) &= -13x^5 - 6x^4 - 7x^3 + 11x^2 + 2x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 4x - 3$$

$$b(x) = 3x - 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-4x$	-3
$3x$	$18x^3$	$-12x^2$	$-9x$
-8	$-48x^2$	$32x$	24

$$a(x) \cdot b(x) = 18x^3 - 12x^2 - 48x^2 - 9x + 32x + 24$$

Combine like terms.

$$a(x) \cdot b(x) = 18x^3 - 60x^2 + 23x + 24$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 239)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 + 5x^2 + 22x + 20 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & 5 & 22 & 20 \\ 8 & & -8 & -24 & -16 \\ \hline & -1 & -3 & -2 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 3x - 2 + \frac{4}{x-8}$$

In other words, $h(x) = -x^2 - 3x - 2$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 5x^2 + 22x + 20$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (-1)\cdot(8)^3 \ + \ (5)\cdot(8)^2 \ + \ (22)\cdot(8) \ + \ (20) \\ &= (-1)\cdot(512) \ + \ (5)\cdot(64) \ + \ (22)\cdot(8) \ + \ (20) \\ &= (-512) \ + \ (320) \ + \ (176) \ + \ (20) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 4$.