

Partial Geometric Series Summation Formula Derivation

Let a geometric series have an initial term a and common ratio r . The sum, S , of the partial series ending at the n th term can be expressed using ellipsis notation.

$$S = a + ar + ar^2 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r , and distribute to all the terms on the right side.

$$rS = ar + ar^2 + ar^3 + \cdots + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation. Notice many terms “cancel out”.

$$S - rS = a - ar^n$$

Factor the left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by $(1 - r)$.

$$S = \frac{a - ar^n}{1 - r}$$