at1118paper: Complete the Square (v415)

Example

By completing the square, find both solutions to the given equation:

$$x^2 - 26x = -153$$

Add $\left(\frac{-26}{2}\right)^2$, which equals 169, to both sides of the equation.

$$x^2 - 26x + 169 = 16$$

Factor the left side.

$$(x - 13)^2 = 16$$

Undo the squaring. We need to consider both $\pm\sqrt{16}$.

$$x - 13 = -4 \qquad \text{or} \qquad x - 13 = 4$$
$$x = -17 \qquad \text{or} \qquad x = -9$$

Question 1

By completing the square, find both solutions to the given equation:

$$x^2 - 40x = -396$$

$$x^2 - 40x + 400 = 4$$
$$(x - 20)^2 = 4$$
$$x - 20 = \pm 2$$
$$x = 18 \qquad \text{or} \qquad x = 22$$

Question 2

By completing the square, find both solutions to the given equation:

$$x^2 - 44x = 2117$$

$$x^2 - 44x + 484 = 2601$$
$$(x - 22)^2 = 2601$$
$$x - 22 = \pm 51$$
$$x = -29 \qquad \text{or} \qquad x = 73$$

Question 3

By completing the square, find both solutions to the given equation:

$$x^2 + 52x = -235$$

$$x^2 + 52x + 676 = 441$$
$$(x + 26)^2 = 441$$
$$x + 26 = \pm 21$$
$$x = -47 \qquad \text{or} \qquad x = -5$$

Question 4

By completing the square, find both solutions to the given equation:

$$x^2 + 16x = -63$$

$$x^2 + 16x + 64 = 1$$
$$(x + 8)^2 = 1$$
$$x + 8 = \pm 1$$
$$x = -9 \qquad \text{or} \qquad x = -7$$

Question 5

By completing the square, find both solutions to the given equation:

$$x^2 + 42x = -360$$

$$x^2 + 42x + 441 = 81$$
$$(x + 21)^2 = 81$$
$$x + 21 = \pm 9$$
$$x = -30 \qquad \text{or} \qquad x = -12$$

Question 6

By completing the square, find both solutions to the given equation:

$$x^2 - 22x = -96$$

$$x^2 - 22x + 121 = 25$$
$$(x - 11)^2 = 25$$
$$x - 11 = \pm 5$$
$$x = 6 \qquad \text{or} \qquad x = 16$$