Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 39)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -8x^5 + 2x^3 + 5x^2 - 6x + 4$$

$$q(x) = -8x^5 - 10x^4 - 7x^3 + x - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-8)x^5 + (0)x^4 + (2)x^3 + (5)x^2 + (-6)x^1 + (4)x^0 \\ q(x) &= (-8)x^5 + (-10)x^4 + (-7)x^3 + (0)x^2 + (1)x^1 + (-5)x^0 \\ p(x) - q(x) &= (0)x^5 + (10)x^4 + (9)x^3 + (5)x^2 + (-7)x^1 + (9)x^0 \\ p(x) - q(x) &= 10x^4 + 9x^3 + 5x^2 - 7x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 - 3x + 7$$

$$b(x) = -2x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-3x$	7
$-2x$	$16x^3$	$6x^2$	$-14x$
3	$-24x^2$	$-9x$	21

$$a(x) \cdot b(x) = 16x^3 + 6x^2 - 24x^2 - 14x - 9x + 21$$

Combine like terms.

$$a(x) \cdot b(x) = 16x^3 - 18x^2 - 23x + 21$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 7x^3 - 29x^2 - 29x - 13 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	7	-29	-29	-13
5		35	30	5
	7	6	1	-8

So,

$$\frac{f(x)}{g(x)} = 7x^2 + 6x + 1 + \frac{-8}{x-5}$$

In other words, $h(x) = 7x^2 + 6x + 1$ and the remainder is $R = -8$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 7x^3 - 29x^2 - 29x - 13$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (7)\cdot(5)^3 \;+\; (-29)\cdot(5)^2 \;+\; (-29)\cdot(5) \;+\; (-13) \\ &= (7)\cdot(125) \;+\; (-29)\cdot(25) \;+\; (-29)\cdot(5) \;+\; (-13) \\ &= (875) \;+\; (-725) \;+\; (-145) \;+\; (-13) \\ &= -8 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x - 5$. Thus, $f(5) = -8$.