Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 109)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 + 5x^3 - 10x^2 - 7x + 4$$

$$q(x) = -5x^5 - 8x^4 + 6x^2 - x - 3$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (0)x^4 + (5)x^3 + (-10)x^2 + (-7)x^1 + (4)x^0 \\ q(x) &= (-5)x^5 + (-8)x^4 + (0)x^3 + (6)x^2 + (-1)x^1 + (-3)x^0 \\ q(x) - p(x) &= (-3)x^5 + (-8)x^4 + (-5)x^3 + (16)x^2 + (6)x^1 + (-7)x^0 \\ q(x) - p(x) &= -3x^5 - 8x^4 - 5x^3 + 16x^2 + 6x - 7 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 3x^2 - 2x + 7$$

$$b(x) = 8x + 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$3x^2$	$-2x$	7
$8x$	$24x^3$	$-16x^2$	$56x$
5	$15x^2$	$-10x$	35

$$a(x) \cdot b(x) = 24x^3 - 16x^2 + 15x^2 + 56x - 10x + 35$$

Combine like terms.

$$a(x) \cdot b(x) = 24x^3 - x^2 + 46x + 35$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 109)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 10x^2 + 19x + 25 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 10 & 19 & 25 \\ -8 & & -8 & -16 & -24 \\ \hline & 1 & 2 & 3 & 1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 2x + 3 + \frac{1}{x+8}$$

In other words, $h(x) = x^2 + 2x + 3$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 10x^2 + 19x + 25$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (1)\cdot(-8)^3 \ + \ (10)\cdot(-8)^2 \ + \ (19)\cdot(-8) \ + \ (25) \\ &= (1)\cdot(-512) \ + \ (10)\cdot(64) \ + \ (19)\cdot(-8) \ + \ (25) \\ &= (-512) \ + \ (640) \ + \ (-152) \ + \ (25) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = 1$.