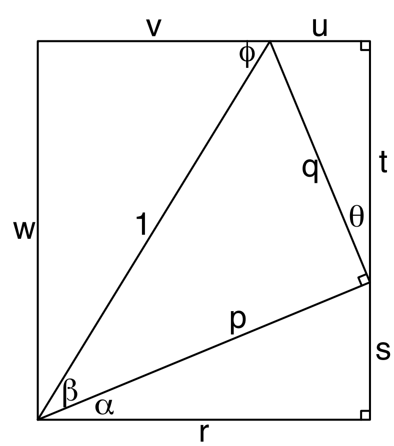
 Name: Answer Key

Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).



Variable	Algebraic expression
$p =$	$\cos(\beta)$
$q =$	$\sin(\beta)$
$r =$	$\cos(\alpha)\cdot\cos(\beta)$
$s =$	$\sin(\alpha)\cdot\cos(\beta)$
$\theta =$	α
$t =$	$\cos(\alpha)\cdot\sin(\beta)$
$u =$	$\sin(\alpha)\cdot\sin(\beta)$
$\phi =$	$\alpha+\beta$
$v =$	$\cos(\alpha+\beta)$
$w =$	$\sin(\alpha+\beta)$

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)\\ \cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)\end{aligned}$$

You know the following:

$$\sin(135°) = \frac{\sqrt{2}}{2} \qquad \sin(120°) = \frac{\sqrt{3}}{2}$$

$$\cos(135°) = \frac{-\sqrt{2}}{2} \qquad \cos(120°) = \frac{-1}{2}$$

Determine $\sin(15°)$ exactly.

$$\sin(15°) = \sin(135° - 120°)$$

$$= \sin(135°)\cos(120°) - \cos(135°)\sin(120°)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{-1}{2} - \frac{-\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\boxed{= \frac{\sqrt{6} - \sqrt{2}}{4}}$$

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

Let $x = \alpha = \beta$.

$$\sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Question 4

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$*)*

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Let $x = \alpha = \beta$.

$$\cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

from Pythagorean Identity: $\sin^2(x) = 1 - \cos^2(x)$

substitute

$$\cos(2x) = \cos^2(x) - (1 - \cos^2(x))$$

$$\cos(2x) = \cos^2(x) - 1 + \cos^2(x)$$

$$\cos(2x) = 2\cos(x) - 1$$

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

$\cos(2x) = 2\cos^2(x) - 1$

Let $y = 2x$ so $\frac{y}{2} = x$.

$\cos(y) = 2\cos^2\left(\frac{y}{2}\right) - 1$

$\cos(y) + 1 = 2\cos^2\left(\frac{y}{2}\right)$

$\frac{\cos(y)+1}{2} = \cos^2\left(\frac{y}{2}\right)$

$\pm\sqrt{\frac{1+\cos(y)}{2}} = \cos\left(\frac{y}{2}\right)$

$\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$

Question 6

If you knew that $\cos(160°) \approx -0.94$, then what is $\cos(80°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $160/2 = 80$.)

$\cos\left(\frac{160°}{2}\right) = \sqrt{\frac{1+\cos(160°)}{2}}$

$\cos(80°) = \sqrt{\frac{1-0.94}{2}}$