Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 215)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -7x^5 - 5x^4 + x^3 - 6x^2 + 10$$

$$q(x) = 6x^5 + 10x^4 + 7x^3 - 2x - 9$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-7)x^5 + (-5)x^4 + (1)x^3 + (-6)x^2 + (0)x^1 + (10)x^0 \\
q(x) &= (6)x^5 + (10)x^4 + (7)x^3 + (0)x^2 + (-2)x^1 + (-9)x^0 \\
p(x) - q(x) &= (-13)x^5 + (-15)x^4 + (-6)x^3 + (-6)x^2 + (2)x^1 + (19)x^0 \\
p(x) - q(x) &= -13x^5 - 15x^4 - 6x^3 - 6x^2 + 2x + 19
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 4x^2 + 6x - 9$$

$$b(x) = 6x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$4x^2$	$6x$	-9
$6x$	$24x^3$	$36x^2$	$-54x$
7	$28x^2$	$42x$	-63

$$a(x) \cdot b(x) = 24x^3 + 36x^2 + 28x^2 - 54x + 42x - 63$$

Combine like terms.

$$a(x) \cdot b(x) = 24x^3 + 64x^2 - 12x - 63$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 215)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 + 19x^2 - 23x - 7 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -2 & 19 & -23 & -7 \\ 8 & & -16 & 24 & 8 \\ \hline & -2 & 3 & 1 & 1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 + 3x + 1 + \frac{1}{x-8}$$

In other words, $h(x) = -2x^2 + 3x + 1$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 + 19x^2 - 23x - 7$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (-2)\cdot(8)^3 \ + \ (19)\cdot(8)^2 \ + \ (-23)\cdot(8) \ + \ (-7) \\ &= (-2)\cdot(512) \ + \ (19)\cdot(64) \ + \ (-23)\cdot(8) \ + \ (-7) \\ &= (-1024) \ + \ (1216) \ + \ (-184) \ + \ (-7) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 1$.