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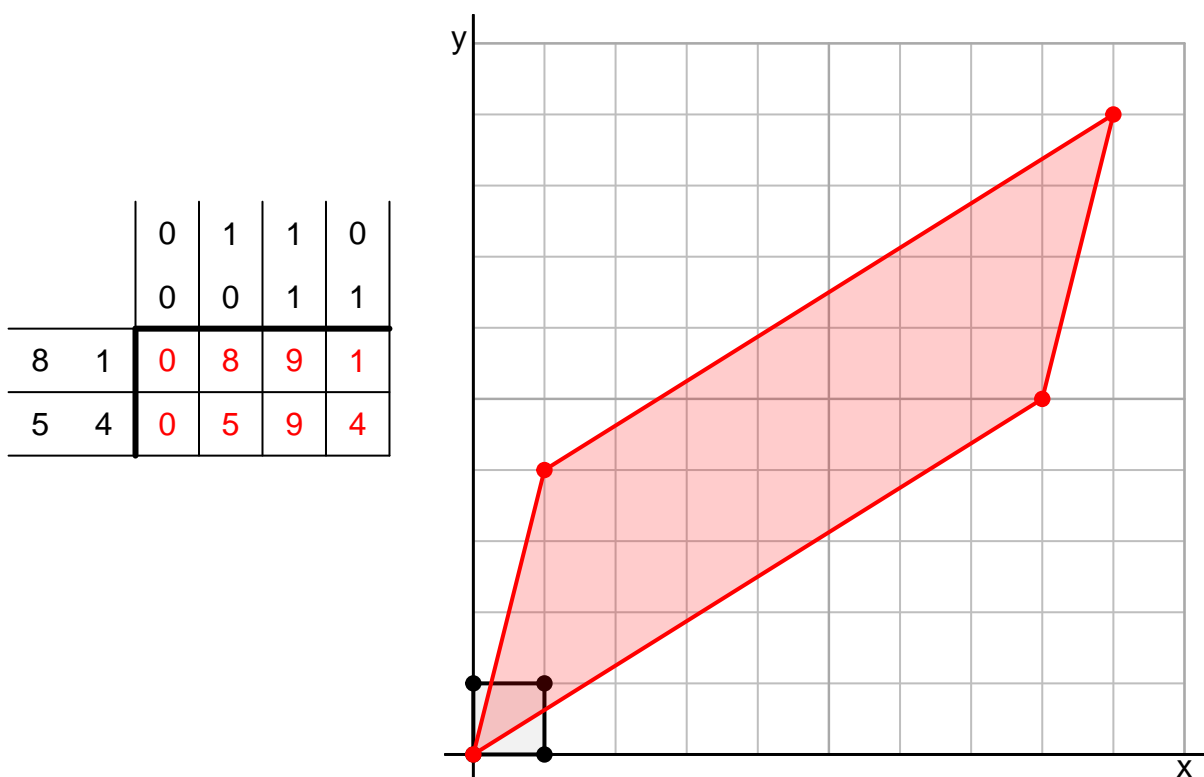
**s19 Matrix Exam (solution v147)**

Let the  $2 \times 4$  matrix  $U$  represent four points in the  $xy$ -plane (so each column represents a point). When those four points are connected as a convex polygon, matrix  $U$  represents a unit square. Also, let the  $2 \times 2$  matrix  $L$  represent a linear transformation.

$$U = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix}$$

Let matrix  $P = L \cdot U$ , so  $P$  is found by matrix multiplication of  $L$  times  $U$ . Matrix  $P$  also represents 4 points of a polygon. Use the diagram below to calculate the elements of  $P$ . Then, draw the polygon represented by matrix  $P$  on the  $xy$ -plane below. Notice I have already drawn the unit square represented by matrix  $U$ .

1. Multiply  $L \cdot U$  and draw resulting polygon.



2. What is the area of the convex polygon represented by matrix  $P$ ? *Hint: the area equals the absolute value of the determinant of matrix  $L$ .*

$$\text{area} = \det(L) = (8 \cdot 4) - (1 \cdot 5)$$

$$\text{area} = 27$$

The triangle shown below is composed of the three points represented by matrix  $A = \begin{bmatrix} 0 & 5 & 10 \\ 10 & 10 & 5 \end{bmatrix}$ . In order to reflect over the  $x$  axis, reflect over the  $y$  axis, and then rotate by  $53.13^\circ$  counterclockwise we can multiply by the transformation matrix  $R = \begin{bmatrix} -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix}$ .

3. Calculate the matrix  $R \cdot A$ .

$$R \cdot A = \begin{bmatrix} 8 & 5 & -2 \\ -6 & -10 & -11 \end{bmatrix}$$

4. Draw the triangle represented by  $R \cdot A$ .

