Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 206)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 + 5x^4 + 10x^3 + 4x + 8$$

$$q(x) = -5x^5 - 6x^3 - 10x^2 + 2x - 1$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (5)x^4 + (10)x^3 + (0)x^2 + (4)x^1 + (8)x^0 \\ q(x) &= (-5)x^5 + (0)x^4 + (-6)x^3 + (-10)x^2 + (2)x^1 + (-1)x^0 \\ p(x) - q(x) &= (3)x^5 + (5)x^4 + (16)x^3 + (10)x^2 + (2)x^1 + (9)x^0 \\ p(x) - q(x) &= 3x^5 + 5x^4 + 16x^3 + 10x^2 + 2x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 + 5x + 4$$

$$b(x) = 8x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$5x$	4
$8x$	$-24x^3$	$40x^2$	$32x$
4	$-12x^2$	$20x$	16

$$a(x) \cdot b(x) = -24x^3 + 40x^2 - 12x^2 + 32x + 20x + 16$$

Combine like terms.

$$a(x) \cdot b(x) = -24x^3 + 28x^2 + 52x + 16$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 3x - 26 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 6 & 3 & -26 \\ -4 & & -4 & -8 & 20 \\ \hline & 1 & 2 & -5 & -6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 2x - 5 + \frac{-6}{x+4}$$

In other words, $h(x) = x^2 + 2x - 5$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 6x^2 + 3x - 26$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (1)\cdot(-4)^3 \ + \ (6)\cdot(-4)^2 \ + \ (3)\cdot(-4) \ + \ (-26) \\ &= (1)\cdot(-64) \ + \ (6)\cdot(16) \ + \ (3)\cdot(-4) \ + \ (-26) \\ &= (-64) \ + \ (96) \ + \ (-12) \ + \ (-26) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = -6$.