Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 230)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -4x^5 - 2x^3 - 6x^2 + 3x - 7 \\ q(x) &= 10x^5 - 2x^4 - 4x^3 + 7x + 5 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-4)x^5 + (0)x^4 + (-2)x^3 + (-6)x^2 + (3)x^1 + (-7)x^0 \\ q(x) &= (10)x^5 + (-2)x^4 + (-4)x^3 + (0)x^2 + (7)x^1 + (5)x^0 \\ p(x) - q(x) &= (-14)x^5 + (2)x^4 + (2)x^3 + (-6)x^2 + (-4)x^1 + (-12)x^0 \\ p(x) - q(x) &= -14x^5 + 2x^4 + 2x^3 - 6x^2 - 4x - 12 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -7x^2 + 5x + 3 \\ b(x) &= -7x + 4 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$5x$	3
$-7x$	$49x^3$	$-35x^2$	$-21x$
4	$-28x^2$	$20x$	12

$$a(x) \cdot b(x) = 49x^3 - 35x^2 - 28x^2 - 21x + 20x + 12$$

Combine like terms.

$$a(x) \cdot b(x) = 49x^3 - 63x^2 - x + 12$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 230)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 11x^2 + 26x + 21 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & -11 & 26 & 21 \\ 6 & & 6 & -30 & -24 \\ \hline & 1 & -5 & -4 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 - 5x - 4 + \frac{-3}{x-6}$$

In other words, $h(x) = x^2 - 5x - 4$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 11x^2 + 26x + 21$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (1)\cdot(6)^3 \ + \ (-11)\cdot(6)^2 \ + \ (26)\cdot(6) \ + \ (21) \\ &= (1)\cdot(216) \ + \ (-11)\cdot(36) \ + \ (26)\cdot(6) \ + \ (21) \\ &= (216) \ + \ (-396) \ + \ (156) \ + \ (21) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = -3$.