Name: ________________________ Date: ____________

Polynomial Operations SOLUTION (version 156)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 5x^5 + 4x^4 + 3x^3 + 10x^2 + 8$$

$$q(x) = 9x^5 - 8x^4 + 4x^3 - 10x - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (5)x^5 + (4)x^4 + (3)x^3 + (10)x^2 + (0)x^1 + (8)x^0 \\ q(x) &= (9)x^5 + (-8)x^4 + (4)x^3 + (0)x^2 + (-10)x^1 + (-5)x^0 \\ p(x) - q(x) &= (-4)x^5 + (12)x^4 + (-1)x^3 + (10)x^2 + (10)x^1 + (13)x^0 \\ p(x) - q(x) &= -4x^5 + 12x^4 - x^3 + 10x^2 + 10x + 13 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -4x^2 - 5x - 7$$

$$b(x) = 3x + 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-4x^2$	$-5x$	-7
$3x$	$-12x^3$	$-15x^2$	$-21x$
8	$-32x^2$	$-40x$	-56

$$a(x) \cdot b(x) = -12x^3 - 15x^2 - 32x^2 - 21x - 40x - 56$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 - 47x^2 - 61x - 56$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 156)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 5x^3 - 25x^2 - 4x + 22 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 5 & -25 & -4 & 22 \\ 5 & & 25 & 0 & -20 \\ \hline & 5 & 0 & -4 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 5x^2 - 4 + \frac{2}{x-5}$$

In other words, $h(x) = 5x^2 - 4$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 5x^3 - 25x^2 - 4x + 22$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (5)\cdot(5)^3 \ + \ (-25)\cdot(5)^2 \ + \ (-4)\cdot(5) \ + \ (22) \\ &= (5)\cdot(125) \ + \ (-25)\cdot(25) \ + \ (-4)\cdot(5) \ + \ (22) \\ &= (625) \ + \ (-625) \ + \ (-20) \ + \ (22) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x - 5$. Thus, $f(5) = 2$.