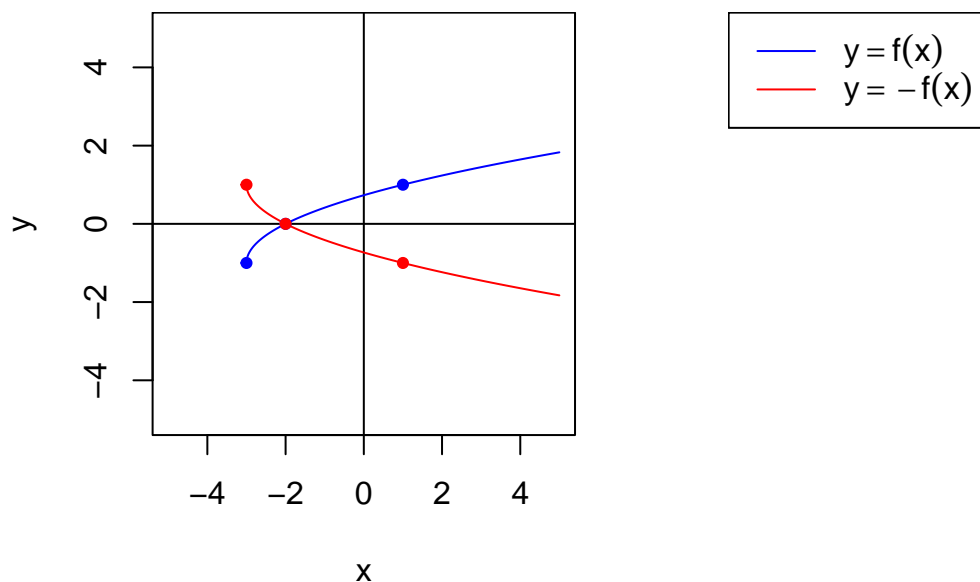


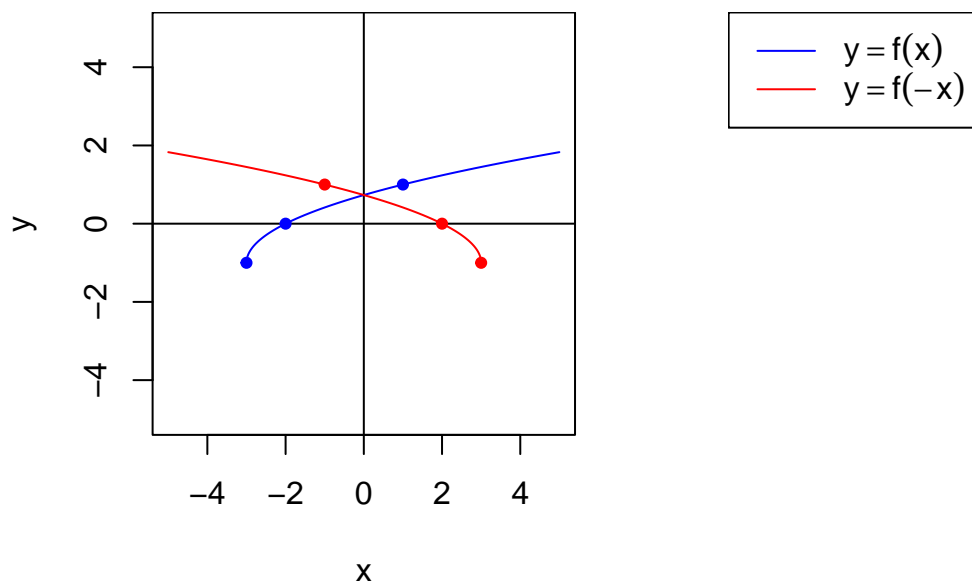
## Vertical reflection: $y = -f(x)$

- Vertical reflection (reflection over the  $x$  axis)
- Let corresponding points  $(x_1, y_1)$  and  $(x_2, y_2)$  exist such that  $y_1 = f(x_1)$  if and only if  $y_2 = -f(x_2)$ .
  - Then we know  $x_2 = x_1$  and  $y_2 = -y_1$ . (Negate the  $y$  values.)
  - In other words:  $(a, b) \rightarrow (a, -b)$  for any point  $(a, b)$  on curve  $y = f(x)$ .
  - For example,  $(3, 1)$  is on  $y = f(x)$  if and only if  $(3, -1)$  is on  $y = -f(x)$



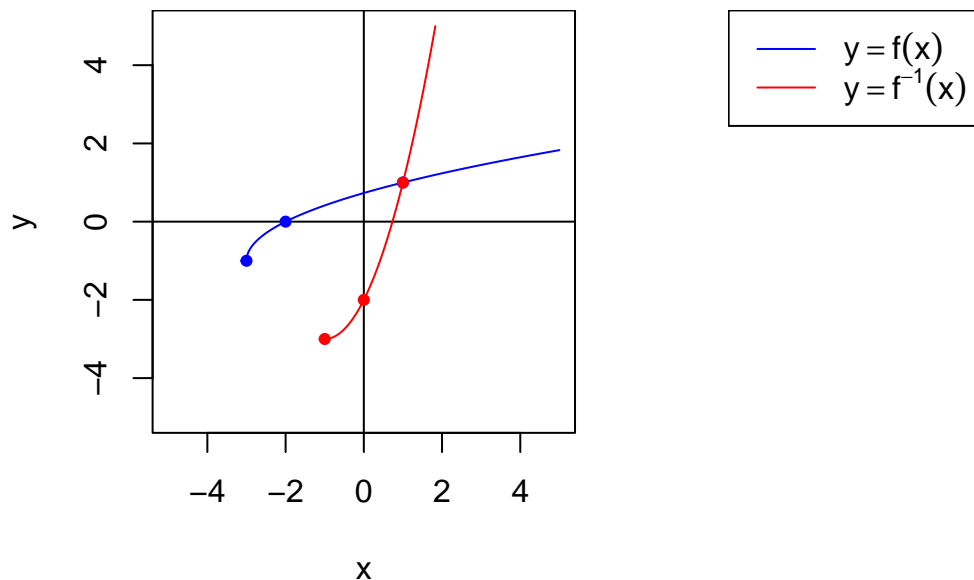
## Horizontal reflection: $y = f(-x)$

- Horizontal reflection (reflection over the  $y$  axis)
- Let corresponding points  $(x_1, y_1)$  and  $(x_2, y_2)$  exist such that  $y_1 = f(x_1)$  if and only if  $y_2 = f(-x_2)$ .
  - Then we know  $x_2 = -x_1$  and  $y_2 = y_1$ . (Negate the  $x$  values.)
  - In other words:  $(a, b) \rightarrow (-a, b)$  for any point  $(a, b)$  on curve  $y = f(x)$ .
  - For example,  $(3, 1)$  is on  $y = f(x)$  if and only if  $(-3, 1)$  is on  $y = f(-x)$



## Inverse function: $y = f^{-1}(x)$

- Reflection over the  $y = x$  line.
- Let  $f$  represent a one-to-one function.
- Let corresponding points  $(x_1, y_1)$  and  $(x_2, y_2)$  exist such that  $y_1 = f(x_1)$  if and only if  $y_2 = f^{-1}(x_2)$ .
  - Then we know  $x_2 = y_1$  and  $y_2 = x_1$ . (Swap the  $x$  and  $y$ .)
  - In other words:  $(a, b) \rightarrow (b, a)$  for any point  $(a, b)$  on curve  $y = f(x)$ .
  - For example,  $(3, 1)$  is on  $y = f(x)$  if and only if  $(1, 3)$  is on  $y = f^{-1}(x)$



## Double reflection: $y = -f(-x)$

- Double reflection over  $x$  axis and then  $y$  axis (or  $180^\circ$  rotation around origin).
- Let corresponding points  $(x_1, y_1)$  and  $(x_2, y_2)$  exist such that  $y_1 = f(x_1)$  if and only if  $y_2 = -f(-x_2)$ .
  - Then we know  $x_2 = -x_1$  and  $y_2 = -y_1$ . (Negate the  $x$  and  $y$  values.)
  - In other words:  $(a, b) \rightarrow (-a, -b)$  for any point  $(a, b)$  on curve  $y = f(x)$ .
  - For example,  $(3, 1)$  is on  $y = f(x)$  if and only if  $(-3, -1)$  is on  $y = -f(-x)$

