Name: __________ Date: __________

Polynomial Operations SOLUTION (version 103)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 - 8x^4 + 6x^3 + 10x^2 + 1$$

$$q(x) = -6x^5 - 2x^4 + 7x^3 + 3x - 8$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (-8)x^4 + (6)x^3 + (10)x^2 + (0)x^1 + (1)x^0 \\ q(x) &= (-6)x^5 + (-2)x^4 + (7)x^3 + (0)x^2 + (3)x^1 + (-8)x^0 \\ p(x) + q(x) &= (-15)x^5 + (-10)x^4 + (13)x^3 + (10)x^2 + (3)x^1 + (-7)x^0 \\ p(x) + q(x) &= -15x^5 - 10x^4 + 13x^3 + 10x^2 + 3x - 7 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 + 8x + 7$$

$$b(x) = 6x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$8x$	7
$6x$	$30x^3$	$48x^2$	$42x$
-5	$-25x^2$	$-40x$	-35

$$a(x) \cdot b(x) = 30x^3 + 48x^2 - 25x^2 + 42x - 40x - 35$$

Combine like terms.

$$a(x) \cdot b(x) = 30x^3 + 23x^2 + 2x - 35$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 103)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 15x^2 - 27x + 4 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 15 & -27 & 4 \\ -9 & & -18 & 27 & 0 \\ \hline & 2 & -3 & 0 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 - 3x + \frac{4}{x+9}$$

In other words, $h(x) = 2x^2 - 3x$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 15x^2 - 27x + 4$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (2)\cdot(-9)^3 \ + \ (15)\cdot(-9)^2 \ + \ (-27)\cdot(-9) \ + \ (4) \\ &= (2)\cdot(-729) \ + \ (15)\cdot(81) \ + \ (-27)\cdot(-9) \ + \ (4) \\ &= (-1458) \ + \ (1215) \ + \ (243) \ + \ (4) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x+9$. Thus, $f(-9) = 4$.