Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 226)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -6x^5 + 5x^4 + 9x^3 + 2x + 10$$
$$q(x) = -3x^5 + 10x^4 - 9x^3 + 4x^2 + 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-6)x^5 + (5)x^4 + (9)x^3 + (0)x^2 + (2)x^1 + (10)x^0 \\
q(x) &= (-3)x^5 + (10)x^4 + (-9)x^3 + (4)x^2 + (0)x^1 + (5)x^0 \\
p(x) - q(x) &= (-3)x^5 + (-5)x^4 + (18)x^3 + (-4)x^2 + (2)x^1 + (5)x^0 \\
p(x) - q(x) &= -3x^5 - 5x^4 + 18x^3 - 4x^2 + 2x + 5
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 - 4x - 2$$
$$b(x) = -3x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-4x$	-2
$-3x$	$9x^3$	$12x^2$	$6x$
-7	$21x^2$	$28x$	14

$$a(x) \cdot b(x) = 9x^3 + 12x^2 + 21x^2 + 6x + 28x + 14$$

Combine like terms.

$$a(x) \cdot b(x) = 9x^3 + 33x^2 + 34x + 14$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 226)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 12x^2 + 29x + 18 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	1	-12	29	18
8		8	-32	-24
	1	-4	-3	-6

So,

$$\frac{f(x)}{g(x)} = x^2 - 4x - 3 + \frac{-6}{x-8}$$

In other words, $h(x) = x^2 - 4x - 3$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 12x^2 + 29x + 18$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (1)\cdot(8)^3 \ + \ (-12)\cdot(8)^2 \ + \ (29)\cdot(8) \ + \ (18) \\ &= (1)\cdot(512) \ + \ (-12)\cdot(64) \ + \ (29)\cdot(8) \ + \ (18) \\ &= (512) \ + \ (-768) \ + \ (232) \ + \ (18) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = -6$.