Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 154)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -4x^5 - 8x^4 - x^3 + 3x^2 + 9$$

$$q(x) = x^5 + 4x^4 + 6x^2 + 2x + 3$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$p(x) = (-4)x^5 + (-8)x^4 + (-1)x^3 + (3)x^2 + (0)x^1 + (9)x^0$$
$$q(x) = (1)x^5 + (4)x^4 + (0)x^3 + (6)x^2 + (2)x^1 + (3)x^0$$
$$p(x) - q(x) = (-5)x^5 + (-12)x^4 + (-1)x^3 + (-3)x^2 + (-2)x^1 + (6)x^0$$
$$p(x) - q(x) = -5x^5 - 12x^4 - x^3 - 3x^2 - 2x + 6$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 - 7x + 5$$

$$b(x) = 2x - 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-7x$	5
$2x$	$-12x^3$	$-14x^2$	$10x$
-3	$18x^2$	$21x$	-15

$$a(x) \cdot b(x) = -12x^3 - 14x^2 + 18x^2 + 10x + 21x - 15$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 + 4x^2 + 31x - 15$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 154)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 + 24x^2 - 3x - 15 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	4	24	-3	-15
-6		-24	0	18
	4	0	-3	3

So,

$$\frac{f(x)}{g(x)} = 4x^2 - 3 + \frac{3}{x+6}$$

In other words, $h(x) = 4x^2 - 3$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 + 24x^2 - 3x - 15$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (4)\cdot(-6)^3 \;+\; (24)\cdot(-6)^2 \;+\; (-3)\cdot(-6) \;+\; (-15) \\ &= (4)\cdot(-216) \;+\; (24)\cdot(36) \;+\; (-3)\cdot(-6) \;+\; (-15) \\ &= (-864) \;+\; (864) \;+\; (18) \;+\; (-15) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x+6$. Thus, $f(-6) = 3$.