Name: Date:

s17 Geometric Series Exam (SLTN v388)

Question 1

Consider the partial geometric series represented below with first term $a = 728$, common ratio $r = \left(\frac{76}{91}\right)^{1/10}$, and $n = 10$ terms.

$$S = 728 + 715 + 702.24 + 689.7 + 677.39 + 665.3 + 653.42 + 641.76 + 630.3 + 619.05$$

We can multiply both sides by r.

$$rS = 715 + 702.24 + 689.7 + 677.39 + 665.3 + 653.42 + 641.76 + 630.3 + 619.05 + 608$$

What is the value of $S - rS$?

Most terms cancel.

$$728 - 608 = 120$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 4 + 4(8) + 4(8)^2 + 4(8)^3 + \cdots + 4(8)^{85} + 4(8)^{86} + 4(8)^{87} + 4(8)^{88}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 4$$

$$\text{common ratio} = r = 8$$

$$\text{number of terms} = n = 89$$

Question 3

Write a proof for the partial geometric series formula.

a. Define the variables.
b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

$$\begin{aligned} a &= \text{first term} \\ r &= \text{common ratio} \\ n &= \text{number of terms} \\ S &= \text{sum of partial geometric series} \end{aligned}$$

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by $(1 - r)$. We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$