Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 217)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = x^5 + 10x^4 + 4x^2 - 6x + 8$$

$$q(x) = -x^5 + 10x^3 - 9x^2 + 8x - 6$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (1)x^5 + (10)x^4 + (0)x^3 + (4)x^2 + (-6)x^1 + (8)x^0 \\ q(x) &= (-1)x^5 + (0)x^4 + (10)x^3 + (-9)x^2 + (8)x^1 + (-6)x^0 \\ p(x) + q(x) &= (0)x^5 + (10)x^4 + (10)x^3 + (-5)x^2 + (2)x^1 + (2)x^0 \\ p(x) + q(x) &= 10x^4 + 10x^3 - 5x^2 + 2x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 7x^2 + 6x + 8$$

$$b(x) = -3x + 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$7x^2$	$6x$	8
$-3x$	$-21x^3$	$-18x^2$	$-24x$
8	$56x^2$	$48x$	64

$$a(x) \cdot b(x) = -21x^3 - 18x^2 + 56x^2 - 24x + 48x + 64$$

Combine like terms.

$$a(x) \cdot b(x) = -21x^3 + 38x^2 + 24x + 64$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 217)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 7x^2 - 4x + 23 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 1 & -7 & -4 & 23 \\ 7 & & 7 & 0 & -28 \\ \hline & 1 & 0 & -4 & -5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 - 4 + \frac{-5}{x-7}$$

In other words, $h(x) = x^2 - 4$ and the remainder is $R = -5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 7x^2 - 4x + 23$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (1)\cdot(7)^3 \ + \ (-7)\cdot(7)^2 \ + \ (-4)\cdot(7) \ + \ (23) \\ &= (1)\cdot(343) \ + \ (-7)\cdot(49) \ + \ (-4)\cdot(7) \ + \ (23) \\ &= (343) \ + \ (-343) \ + \ (-28) \ + \ (23) \\ &= -5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = -5$.