Name: _______________________________________________ Date: _______________________

# Polynomial Operations SOLUTION (version 240)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) \;=\; -6x^5 + 2x^4 - 5x^2 - x + 4$$

$$q(x) \;=\; 5x^5 + 9x^3 - 10x^2 - 6x - 8$$

**Express the difference $p(x) - q(x)$ in standard form.**

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-6)x^5 + (2)x^4 + (0)x^3 + (-5)x^2 + (-1)x^1 + (4)x^0 \\ q(x) &= (5)x^5 + (0)x^4 + (9)x^3 + (-10)x^2 + (-6)x^1 + (-8)x^0 \\ p(x) - q(x) &= (-11)x^5 + (2)x^4 + (-9)x^3 + (5)x^2 + (5)x^1 + (12)x^0 \\ p(x) - q(x) &= -11x^5 + 2x^4 - 9x^3 + 5x^2 + 5x + 12 \end{aligned}$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) \;=\; -4x^2 - 8x + 5$$

$$b(x) \;=\; -3x - 7$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $-4x^2$ | $-8x$ | $5$ |
|---|---|---|---|
| $-3x$ | $12x^3$ | $24x^2$ | $-15x$ |
| $-7$ | $28x^2$ | $56x$ | $-35$ |

$$a(x) \cdot b(x) \;=\; 12x^3 + 24x^2 + 28x^2 - 15x + 56x - 35$$

Combine like terms.

$$a(x) \cdot b(x) \;=\; 12x^3 + 52x^2 + 41x - 35$$

**3. Express $(x+1)^4$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

# Polynomial Operations SOLUTION (version 240)

**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= 2x^3 - 14x^2 + x - 3 \\ g(x) &= x - 7 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 2 & -14 & 1 & -3 \\ 7 & & 14 & 0 & 7 \\ \hline & 2 & 0 & 1 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 1 + \frac{4}{x-7}$$

In other words, $h(x) = 2x^2 + 1$ and the remainder is $R = 4$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 - 14x^2 + x - 3$. Evaluate $f(7)$.**

You could do this the hard way.

$$\begin{aligned} f(7) &= (2) \cdot (7)^3 \ + \ (-14) \cdot (7)^2 \ + \ (1) \cdot (7) \ + \ (-3) \\ &= (2) \cdot (343) \ + \ (-14) \cdot (49) \ + \ (1) \cdot (7) \ + \ (-3) \\ &= (686) \ + \ (-686) \ + \ (7) \ + \ (-3) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = 4$.