Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 123)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -3x^5 + 10x^4 + 2x^3 - 6x + 9 \\ q(x) &= -5x^5 - 3x^4 - 9x^3 + x^2 - 7 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-3)x^5 + (10)x^4 + (2)x^3 + (0)x^2 + (-6)x^1 + (9)x^0 \\ q(x) &= (-5)x^5 + (-3)x^4 + (-9)x^3 + (1)x^2 + (0)x^1 + (-7)x^0 \\ p(x) - q(x) &= (2)x^5 + (13)x^4 + (11)x^3 + (-1)x^2 + (-6)x^1 + (16)x^0 \\ p(x) - q(x) &= 2x^5 + 13x^4 + 11x^3 - x^2 - 6x + 16 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -8x^2 - 3x - 6 \\ b(x) &= -3x + 8 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-3x$	-6
$-3x$	$24x^3$	$9x^2$	$18x$
8	$-64x^2$	$-24x$	-48

$$a(x) \cdot b(x) = 24x^3 + 9x^2 - 64x^2 + 18x - 24x - 48$$

Combine like terms.

$$a(x) \cdot b(x) = 24x^3 - 55x^2 - 6x - 48$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 123)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 - 28x^2 - 29x - 18 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 4 & -28 & -29 & -18 \\ 8 & & 32 & 32 & 24 \\ \hline & 4 & 4 & 3 & 6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 4x^2 + 4x + 3 + \frac{6}{x-8}$$

In other words, $h(x) = 4x^2 + 4x + 3$ and the remainder is $R = 6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 - 28x^2 - 29x - 18$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (4) \cdot (8)^3 \ + \ (-28) \cdot (8)^2 \ + \ (-29) \cdot (8) \ + \ (-18) \\ &= (4) \cdot (512) \ + \ (-28) \cdot (64) \ + \ (-29) \cdot (8) \ + \ (-18) \\ &= (2048) \ + \ (-1792) \ + \ (-232) \ + \ (-18) \\ &= 6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 6$.