Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 225)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = x^5 - 8x^4 - 10x^3 - 6x^2 - 5$$

$$q(x) = 7x^5 + 2x^3 - x^2 - 8x + 4$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (1)x^5 + (-8)x^4 + (-10)x^3 + (-6)x^2 + (0)x^1 + (-5)x^0 \\ q(x) &= (7)x^5 + (0)x^4 + (2)x^3 + (-1)x^2 + (-8)x^1 + (4)x^0 \\ q(x) - p(x) &= (6)x^5 + (8)x^4 + (12)x^3 + (5)x^2 + (-8)x^1 + (9)x^0 \\ q(x) - p(x) &= 6x^5 + 8x^4 + 12x^3 + 5x^2 - 8x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -9x^2 + 5x + 6$$

$$b(x) = -6x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-9x^2$	$5x$	6
$-6x$	$54x^3$	$-30x^2$	$-36x$
-5	$45x^2$	$-25x$	-30

$$a(x) \cdot b(x) = 54x^3 - 30x^2 + 45x^2 - 36x - 25x - 30$$

Combine like terms.

$$a(x) \cdot b(x) = 54x^3 + 15x^2 - 61x - 30$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 225)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 + 27x^2 - 25x - 21 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -5 & 27 & -25 & -21 \\ 4 & & -20 & 28 & 12 \\ \hline & -5 & 7 & 3 & -9 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -5x^2 + 7x + 3 + \frac{-9}{x-4}$$

In other words, $h(x) = -5x^2 + 7x + 3$ and the remainder is $R = -9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 + 27x^2 - 25x - 21$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (-5)\cdot(4)^3 + (27)\cdot(4)^2 + (-25)\cdot(4) + (-21) \\ &= (-5)\cdot(64) + (27)\cdot(16) + (-25)\cdot(4) + (-21) \\ &= (-320) + (432) + (-100) + (-21) \\ &= -9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -9$.