Name: ____________________ Date: ____________

Polynomial Operations SOLUTIONS (version 33)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 + 2x^4 - 4x^2 + x + 5$$
$$q(x) = 6x^5 + 2x^4 + 10x^3 - 4x^2 - 3$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-9)x^5 + (2)x^4 + (0)x^3 + (-4)x^2 + (1)x^1 + (5)x^0 \\
q(x) &= (6)x^5 + (2)x^4 + (10)x^3 + (-4)x^2 + (0)x^1 + (-3)x^0 \\
p(x) + q(x) &= (-3)x^5 + (4)x^4 + (10)x^3 + (-8)x^2 + (1)x^1 + (2)x^0 \\
p(x) + q(x) &= -3x^5 + 4x^4 + 10x^3 - 8x^2 + x + 2
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 - 5x + 2$$
$$b(x) = -8x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$-5x$	2
$-8x$	$56x^3$	$40x^2$	$-16x$
7	$-49x^2$	$-35x$	14

$$a(x) \cdot b(x) = 56x^3 + 40x^2 - 49x^2 - 16x - 35x + 14$$

Combine like terms.

$$a(x) \cdot b(x) = 56x^3 - 9x^2 - 51x + 14$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 5x^3 + 15x^2 - 19x + 12 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 5 & 15 & -19 & 12 \\ -4 & & -20 & 20 & -4 \\ \hline & 5 & -5 & 1 & 8 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 5x^2 - 5x + 1 + \frac{8}{x+4}$$

In other words, $h(x) = 5x^2 - 5x + 1$ and the remainder is $R = 8$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 5x^3 + 15x^2 - 19x + 12$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (5)\cdot(-4)^3 \ + \ (15)\cdot(-4)^2 \ + \ (-19)\cdot(-4) \ + \ (12) \\ &= (5)\cdot(-64) \ + \ (15)\cdot(16) \ + \ (-19)\cdot(-4) \ + \ (12) \\ &= (-320) \ + \ (240) \ + \ (76) \ + \ (12) \\ &= 8 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = 8$.