Name: ____________________ Date: ____________________

Polynomial Operations SOLUTION (version 200)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 + 7x^4 - 10x^2 + 5x - 2$$

$$q(x) = 8x^5 + 4x^4 - 6x^3 + 3x^2 - 5$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (7)x^4 + (0)x^3 + (-10)x^2 + (5)x^1 + (-2)x^0 \\ q(x) &= (8)x^5 + (4)x^4 + (-6)x^3 + (3)x^2 + (0)x^1 + (-5)x^0 \\ q(x) - p(x) &= (0)x^5 + (-3)x^4 + (-6)x^3 + (13)x^2 + (-5)x^1 + (-3)x^0 \\ q(x) - p(x) &= -3x^4 - 6x^3 + 13x^2 - 5x - 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 4x^2 - 6x + 2$$

$$b(x) = 8x + 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$4x^2$	$-6x$	2
$8x$	$32x^3$	$-48x^2$	$16x$
6	$24x^2$	$-36x$	12

$$a(x) \cdot b(x) = 32x^3 - 48x^2 + 24x^2 + 16x - 36x + 12$$

Combine like terms.

$$a(x) \cdot b(x) = 32x^3 - 24x^2 - 20x + 12$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 200)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 17x^2 - 9x + 2 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 17 & -9 & 2 \\ -9 & & -18 & 9 & 0 \\ \hline & 2 & -1 & 0 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 - x + \frac{2}{x+9}$$

In other words, $h(x) = 2x^2 - x$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 17x^2 - 9x + 2$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (2)\cdot(-9)^3 \ + \ (17)\cdot(-9)^2 \ + \ (-9)\cdot(-9) \ + \ (2) \\ &= (2)\cdot(-729) \ + \ (17)\cdot(81) \ + \ (-9)\cdot(-9) \ + \ (2) \\ &= (-1458) \ + \ (1377) \ + \ (81) \ + \ (2) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 2$.