Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 150)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 3x^5 - x^4 - 10x^3 - 5x + 6 \\ q(x) &= -2x^5 + 7x^4 + 9x^2 + x + 5 \end{aligned}$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (-1)x^4 + (-10)x^3 + (0)x^2 + (-5)x^1 + (6)x^0 \\ q(x) &= (-2)x^5 + (7)x^4 + (0)x^3 + (9)x^2 + (1)x^1 + (5)x^0 \\ p(x) + q(x) &= (1)x^5 + (6)x^4 + (-10)x^3 + (9)x^2 + (-4)x^1 + (11)x^0 \\ p(x) + q(x) &= x^5 + 6x^4 - 10x^3 + 9x^2 - 4x + 11 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -6x^2 + 7x + 5 \\ b(x) &= 3x + 7 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

$*$	$-6x^2$	$7x$	5
$3x$	$-18x^3$	$21x^2$	$15x$
7	$-42x^2$	$49x$	35

$$a(x) \cdot b(x) = -18x^3 + 21x^2 - 42x^2 + 15x + 49x + 35$$

Combine like terms.

$$a(x) \cdot b(x) = -18x^3 - 21x^2 + 64x + 35$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 150)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 2x^2 - 26x + 22 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & -2 & -26 & 22 \\ 6 & & 6 & 24 & -12 \\ \hline & 1 & 4 & -2 & 10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 4x - 2 + \frac{10}{x-6}$$

In other words, $h(x) = x^2 + 4x - 2$ and the remainder is $R = 10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 2x^2 - 26x + 22$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (1)\cdot(6)^3 \ + \ (-2)\cdot(6)^2 \ + \ (-26)\cdot(6) \ + \ (22) \\ &= (1)\cdot(216) \ + \ (-2)\cdot(36) \ + \ (-26)\cdot(6) \ + \ (22) \\ &= (216) \ + \ (-72) \ + \ (-156) \ + \ (22) \\ &= 10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = 10$.