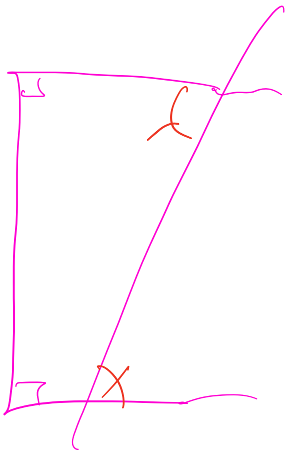
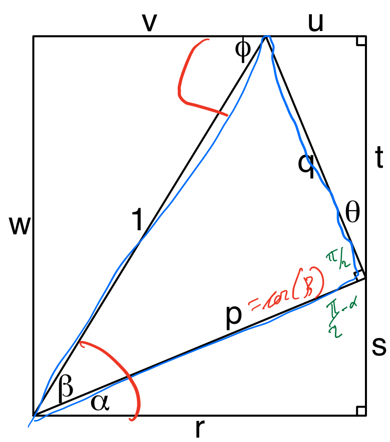
 Name: KEY

Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).





hyp = 1

$\cos(\beta) = \frac{p}{1}$

$\sin(\beta) = \frac{q}{1}$

$\cos(\alpha) = \frac{r}{\cos(\beta)}$

$r = \cos(\alpha) \cdot \cos(\beta)$

$\theta = \pi - \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right) = \alpha$

Variable	Algebraic expression
$p =$	$\cos(\beta)$
$q =$	$\sin(\beta)$
$r =$	$\cos(\alpha) \cdot \cos(\beta)$
$s =$	$\sin(\alpha) \cdot \cos(\beta)$
$\theta =$	α
$t =$	$\cos(\alpha) \cdot \sin(\beta)$
$u =$	$\sin(\alpha) \cdot \sin(\beta)$
$\phi =$	$\alpha + \beta$
$v =$	$\cos(\alpha+\beta)$
$w =$	$\sin(\alpha+\beta)$

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

You know the following:

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \cos(45^\circ) = \frac{\sqrt{2}}{2}$$

Determine $\cos(105^\circ)$ exactly.

$$\cos(60^\circ + 45^\circ) = \cos(60^\circ)\cos(45^\circ) - \sin(60^\circ)\sin(45^\circ)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

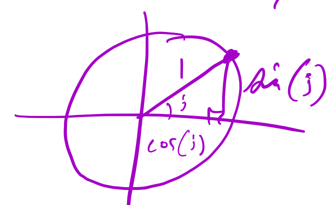
$\sin(\alpha+\beta) = \sin(\alpha)\cdot\cos(\beta) + \cos(\alpha)\cdot\sin(\beta)$

Let $\alpha = x$ and $\beta = x$.

$\sin(x+x) = \sin(x)\cdot\cos(x) + \cos(x)\cdot\sin(x)$

$\sin(2x) = 2\sin(x)\cdot\cos(x)$

NOT NECESSARY



$\sin^2(\theta) + \cos^2(\theta) = 1^2$

$p\cdot q + q\cdot p$

$2pq$

Question 4

$l + m = 1$

$l = 1 - m$

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$)

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

Let $\alpha = x$ and $\beta = x$.

$\cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x) = 1 - \cos^2(x)$

$\cos(2x) = \cos^2(x) - (1 - \cos^2(x))$

$\cos(2x) = \cos^2(x) - 1 + \cos^2(x)$

$\boxed{\cos(2x) = 2\cos^2(x) - 1}$

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

$\cos(2x) = 2\cos^2(x) - 1$

Let $2x = y$, so $x = \frac{y}{2}$.

$\cos(y) = 2\cos^2\left(\frac{y}{2}\right) - 1$

$\cos(y) + 1 = 2\cos^2\left(\frac{y}{2}\right)$

$\frac{\cos(y)+1}{2} = \cos^2\left(\frac{y}{2}\right)$

$\boxed{\cos\left(\frac{y}{2}\right) = \sqrt{\frac{\cos(y)+1}{2}}}$

Question 6

If you knew that $\cos(100°) \approx -0.17$, then what is $\cos(50°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: 100/2 = 50.)

$y = 100°$

$\cos\left(\frac{100°}{2}\right) = \sqrt{\frac{\cos(100°)+1}{2}}$

$\boxed{\cos(50°) = \sqrt{\frac{-0.17+1}{2}}}$