Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 209)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -3x^5 - 6x^4 - 2x^3 - 5x^2 + 10 \\ q(x) &= 4x^5 + 6x^4 - 10x^2 - 9x - 5 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-3)x^5 + (-6)x^4 + (-2)x^3 + (-5)x^2 + (0)x^1 + (10)x^0 \\ q(x) &= (4)x^5 + (6)x^4 + (0)x^3 + (-10)x^2 + (-9)x^1 + (-5)x^0 \\ q(x) - p(x) &= (7)x^5 + (12)x^4 + (2)x^3 + (-5)x^2 + (-9)x^1 + (-15)x^0 \\ q(x) - p(x) &= 7x^5 + 12x^4 + 2x^3 - 5x^2 - 9x - 15 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -3x^2 - 9x + 2 \\ b(x) &= 5x + 3 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-9x$	2
$5x$	$-15x^3$	$-45x^2$	$10x$
3	$-9x^2$	$-27x$	6

$$a(x) \cdot b(x) = -15x^3 - 45x^2 - 9x^2 + 10x - 27x + 6$$

Combine like terms.

$$a(x) \cdot b(x) = -15x^3 - 54x^2 - 17x + 6$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -4x^3 - 29x^2 + 24x + 6 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -4 & -29 & 24 & 6 \\ -8 & & 32 & -24 & 0 \\ \hline & -4 & 3 & 0 & 6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -4x^2 + 3x + \frac{6}{x+8}$$

In other words, $h(x) = -4x^2 + 3x$ and the remainder is $R = 6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -4x^3 - 29x^2 + 24x + 6$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-4)\cdot(-8)^3 \ + \ (-29)\cdot(-8)^2 \ + \ (24)\cdot(-8) \ + \ (6) \\ &= (-4)\cdot(-512) \ + \ (-29)\cdot(64) \ + \ (24)\cdot(-8) \ + \ (6) \\ &= (2048) \ + \ (-1856) \ + \ (-192) \ + \ (6) \\ &= 6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = 6$.