Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 141)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -2x^5 - 10x^4 + 4x^2 + 5x + 7 \\ q(x) &= -6x^5 - 2x^3 + 3x^2 + x + 9 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (-10)x^4 + (0)x^3 + (4)x^2 + (5)x^1 + (7)x^0 \\ q(x) &= (-6)x^5 + (0)x^4 + (-2)x^3 + (3)x^2 + (1)x^1 + (9)x^0 \\ q(x) - p(x) &= (-4)x^5 + (10)x^4 + (-2)x^3 + (-1)x^2 + (-4)x^1 + (2)x^0 \\ q(x) - p(x) &= -4x^5 + 10x^4 - 2x^3 - x^2 - 4x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 5x^2 + 6x - 4 \\ b(x) &= 2x - 8 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$6x$	-4
$2x$	$10x^3$	$12x^2$	$-8x$
-8	$-40x^2$	$-48x$	32

$$a(x) \cdot b(x) = 10x^3 + 12x^2 - 40x^2 - 8x - 48x + 32$$

Combine like terms.

$$a(x) \cdot b(x) = 10x^3 - 28x^2 - 56x + 32$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 141)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 - 15x^2 + 27x + 7 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -2 & -15 & 27 & 7 \\ -9 & & 18 & -27 & 0 \\ \hline & -2 & 3 & 0 & 7 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 + 3x + \frac{7}{x+9}$$

In other words, $h(x) = -2x^2 + 3x$ and the remainder is $R = 7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 - 15x^2 + 27x + 7$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (-2)\cdot(-9)^3 \ + \ (-15)\cdot(-9)^2 \ + \ (27)\cdot(-9) \ + \ (7) \\ &= (-2)\cdot(-729) \ + \ (-15)\cdot(81) \ + \ (27)\cdot(-9) \ + \ (7) \\ &= (1458) \ + \ (-1215) \ + \ (-243) \ + \ (7) \\ &= 7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 7$.