Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 111)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -4x^5 + 3x^4 + x^2 - 9x - 5$$

$$q(x) = 10x^5 - 4x^4 + 7x^3 - 2x - 1$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-4)x^5 + (3)x^4 + (0)x^3 + (1)x^2 + (-9)x^1 + (-5)x^0 \\ q(x) &= (10)x^5 + (-4)x^4 + (7)x^3 + (0)x^2 + (-2)x^1 + (-1)x^0 \\ p(x) + q(x) &= (6)x^5 + (-1)x^4 + (7)x^3 + (1)x^2 + (-11)x^1 + (-6)x^0 \\ p(x) + q(x) &= 6x^5 - x^4 + 7x^3 + x^2 - 11x - 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 + 9x - 7$$

$$b(x) = 5x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$9x$	-7
$5x$	$30x^3$	$45x^2$	$-35x$
3	$18x^2$	$27x$	-21

$$a(x) \cdot b(x) = 30x^3 + 45x^2 + 18x^2 - 35x + 27x - 21$$

Combine like terms.

$$a(x) \cdot b(x) = 30x^3 + 63x^2 - 8x - 21$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 111)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 + 27x^2 - 25x + 1 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-3	27	-25	1
8		-24	24	-8
	-3	3	-1	-7

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 3x - 1 + \frac{-7}{x-8}$$

In other words, $h(x) = -3x^2 + 3x - 1$ and the remainder is $R = -7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 + 27x^2 - 25x + 1$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (-3)\cdot(8)^3 \;+\; (27)\cdot(8)^2 \;+\; (-25)\cdot(8) \;+\; (1) \\ &= (-3)\cdot(512) \;+\; (27)\cdot(64) \;+\; (-25)\cdot(8) \;+\; (1) \\ &= (-1536) \;+\; (1728) \;+\; (-200) \;+\; (1) \\ &= -7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = -7$.