Name: ____________________________ Date: ________________

Polynomial Operations SOLUTIONS (version 1)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -x^5 - 5x^3 - 7x^2 - 3x + 9$$

$$q(x) = -6x^5 - 7x^4 + x^3 - 5x + 10$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-1)x^5 + (0)x^4 + (-5)x^3 + (-7)x^2 + (-3)x^1 + (9)x^0 \\ q(x) &= (-6)x^5 + (-7)x^4 + (1)x^3 + (0)x^2 + (-5)x^1 + (10)x^0 \\ p(x) - q(x) &= (5)x^5 + (7)x^4 + (-6)x^3 + (-7)x^2 + (2)x^1 + (-1)x^0 \\ p(x) - q(x) &= 5x^5 + 7x^4 - 6x^3 - 7x^2 + 2x - 1 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 - 2x + 5$$

$$b(x) = 4x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$-2x$	5
$4x$	$-28x^3$	$-8x^2$	$20x$
7	$-49x^2$	$-14x$	35

$$a(x) \cdot b(x) = -28x^3 - 8x^2 - 49x^2 + 20x - 14x + 35$$

Combine like terms.

$$a(x) \cdot b(x) = -28x^3 - 57x^2 + 6x + 35$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTIONS (version 1)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 14x^2 + 3x + 23 \\ g(x) &= x + 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 14 & 3 & 23 \\ \text{-7} & & \text{-14} & 0 & \text{-21} \\ \hline & 2 & 0 & 3 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 3 + \frac{2}{x+7}$$

In other words, $h(x) = 2x^2 + 3$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 14x^2 + 3x + 23$. Evaluate $f(-7)$.

You could do this the hard way.

$$\begin{aligned} f(-7) &= (2)\cdot(-7)^3 \ + \ (14)\cdot(-7)^2 \ + \ (3)\cdot(-7) \ + \ (23) \\ &= (2)\cdot(-343) \ + \ (14)\cdot(49) \ + \ (3)\cdot(-7) \ + \ (23) \\ &= (-686) \ + \ (686) \ + \ (-21) \ + \ (23) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-7)$ equals the remainder when $f(x)$ is divided by $x + 7$. Thus, $f(-7) = 2$.