Name: ____________________ Date: ____________

Polynomial Operations SOLUTIONS (version 5)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 + 10x^4 - 5x^2 - 6x + 3$$
$$q(x) = -10x^5 + 5x^4 - 8x^3 + x - 4$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-9)x^5 + (10)x^4 + (0)x^3 + (-5)x^2 + (-6)x^1 + (3)x^0 \\
q(x) &= (-10)x^5 + (5)x^4 + (-8)x^3 + (0)x^2 + (1)x^1 + (-4)x^0 \\
p(x) + q(x) &= (-19)x^5 + (15)x^4 + (-8)x^3 + (-5)x^2 + (-5)x^1 + (-1)x^0 \\
p(x) + q(x) &= -19x^5 + 15x^4 - 8x^3 - 5x^2 - 5x - 1
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 - 7x - 5$$
$$b(x) = 4x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-7x$	-5
$4x$	$-12x^3$	$-28x^2$	$-20x$
7	$-21x^2$	$-49x$	-35

$$a(x) \cdot b(x) = -12x^3 - 28x^2 - 21x^2 - 20x - 49x - 35$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 - 49x^2 - 69x - 35$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTIONS (version 5)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 - 7x^2 + 14x - 11 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -2 & -7 & 14 & -11 \\ -5 & & 10 & -15 & 5 \\ \hline & -2 & 3 & -1 & -6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 + 3x - 1 + \frac{-6}{x+5}$$

In other words, $h(x) = -2x^2 + 3x - 1$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 - 7x^2 + 14x - 11$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (-2)\cdot(-5)^3 + (-7)\cdot(-5)^2 + (14)\cdot(-5) + (-11) \\ &= (-2)\cdot(-125) + (-7)\cdot(25) + (14)\cdot(-5) + (-11) \\ &= (250) + (-175) + (-70) + (-11) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x + 5$. Thus, $f(-5) = -6$.