Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 227)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -7x^5 + 9x^4 - 8x^3 + 3x + 4 \\ q(x) &= 10x^5 - 2x^3 - 5x^2 + 9x - 4 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-7)x^5 + (9)x^4 + (-8)x^3 + (0)x^2 + (3)x^1 + (4)x^0 \\ q(x) &= (10)x^5 + (0)x^4 + (-2)x^3 + (-5)x^2 + (9)x^1 + (-4)x^0 \\ q(x) - p(x) &= (17)x^5 + (-9)x^4 + (6)x^3 + (-5)x^2 + (6)x^1 + (-8)x^0 \\ q(x) - p(x) &= 17x^5 - 9x^4 + 6x^3 - 5x^2 + 6x - 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 7x^2 - 6x - 2 \\ b(x) &= 6x + 8 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$7x^2$	$-6x$	-2
$6x$	$42x^3$	$-36x^2$	$-12x$
8	$56x^2$	$-48x$	-16

$$a(x) \cdot b(x) = 42x^3 - 36x^2 + 56x^2 - 12x - 48x - 16$$

Combine like terms.

$$a(x) \cdot b(x) = 42x^3 + 20x^2 - 60x - 16$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 227)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 11x^2 - 21x - 24 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-1	-11	-21	-24
-9		9	18	27
	-1	-2	-3	3

So,

$$\frac{f(x)}{g(x)} = -x^2 - 2x - 3 + \frac{3}{x+9}$$

In other words, $h(x) = -x^2 - 2x - 3$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 11x^2 - 21x - 24$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (-1)\cdot(-9)^3 \ + \ (-11)\cdot(-9)^2 \ + \ (-21)\cdot(-9) \ + \ (-24) \\ &= (-1)\cdot(-729) \ + \ (-11)\cdot(81) \ + \ (-21)\cdot(-9) \ + \ (-24) \\ &= (729) \ + \ (-891) \ + \ (189) \ + \ (-24) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 3$.