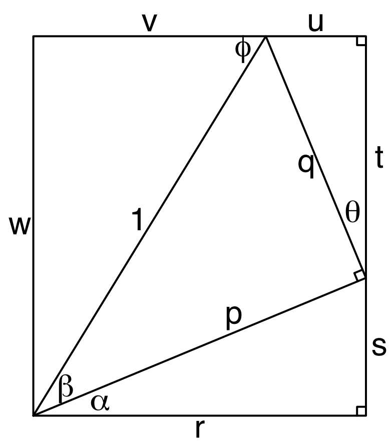
Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).



Variable	Algebraic expression
$p =$	$\cos(\beta)$
$q =$	$\sin(\beta)$
$r =$	$\cos(\alpha) \cdot \cos(\beta)$
$s =$	$\sin(\alpha) \cdot \cos(\beta)$
$\theta =$	α
$t =$	$\cos(\alpha) \cdot \sin(\beta)$
$u =$	$\sin(\alpha) \cdot \sin(\beta)$
$\phi =$	$\alpha + \beta$
$v =$	$\cos(\alpha + \beta)$
$w =$	$\sin(\alpha + \beta)$

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)\\ \cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)\end{aligned}$$

You know the following:

$$\sin(45°) = \frac{\sqrt{2}}{2} \qquad \sin(300°) = \frac{-\sqrt{3}}{2}$$

$$\cos(45°) = \frac{\sqrt{2}}{2} \qquad \cos(300°) = \frac{1}{2}$$

Determine $\cos(345°)$ exactly.

$$\cos(45° + 300°) = \cos(45°)\cdot\cos(300°) - \sin(45°)\sin(300°)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{-\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

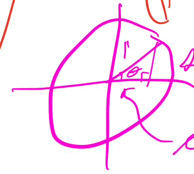
$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

Let $\alpha = x$ and $\beta = x$.

$$\sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Not needed (proof of Pythagorean identity)



$$a^2 + b^2 = c^2$$

$$\sin^2\theta + \cos^2\theta = 1^2$$

$$\sin^2\theta + \cos^2\theta = 1$$

Question 4

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$*)*

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Let $\alpha = x$ and $\beta = x$.

$$\cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = 1 - \cos^2(x)$$

$$\cos(2x) = \cos^2(x) - (1 - \cos^2(x))$$

$$\cos(2x) = \cos^2(x) - 1 + \cos^2(x)$$

$$\boxed{\cos(2x) = 2\cos^2(x) - 1}$$

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

$$\cos(2x) = 2\cos^2(x) - 1$$

Let $2x = y$, so $x = \frac{y}{2}$

$$\cos(y) = 2\cos^2\left(\frac{y}{2}\right) - 1$$

$$\cos(y) + 1 = 2\cos^2\left(\frac{y}{2}\right)$$

$$\frac{\cos(y)+1}{2} = \cos^2\left(\frac{y}{2}\right)$$

$$\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$$

Question 6

If you knew that $\cos(80°) \approx 0.17$, then what is $\cos(40°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $80/2 = 40$.*)* $y = 80°$

$$\cos\left(\frac{80°}{2}\right) = \sqrt{\frac{1+\cos(80°)}{2}}$$

$$\cos(40°) = \sqrt{\frac{1+0.17}{2}}$$