Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 6)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 + 9x^3 + 3x^2 - x - 2$$

$$q(x) = -5x^5 + 8x^4 + 6x^2 + 9x + 2$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (0)x^4 + (9)x^3 + (3)x^2 + (-1)x^1 + (-2)x^0 \\ q(x) &= (-5)x^5 + (8)x^4 + (0)x^3 + (6)x^2 + (9)x^1 + (2)x^0 \\ q(x) - p(x) &= (-13)x^5 + (8)x^4 + (-9)x^3 + (3)x^2 + (10)x^1 + (4)x^0 \\ q(x) - p(x) &= -13x^5 + 8x^4 - 9x^3 + 3x^2 + 10x + 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 8x + 5$$

$$b(x) = -3x - 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-8x$	5
$-3x$	$-18x^3$	$24x^2$	$-15x$
-6	$-36x^2$	$48x$	-30

$$a(x) \cdot b(x) = -18x^3 + 24x^2 - 36x^2 - 15x + 48x - 30$$

Combine like terms.

$$a(x) \cdot b(x) = -18x^3 - 12x^2 + 33x - 30$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 + 19x^2 - 11x + 23 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -2 & 19 & -11 & 23 \\ 9 & & -18 & 9 & -18 \\ \hline & -2 & 1 & -2 & 5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 + x - 2 + \frac{5}{x-9}$$

In other words, $h(x) = -2x^2 + x - 2$ and the remainder is $R = 5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 + 19x^2 - 11x + 23$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-2)\cdot(9)^3 \;+\; (19)\cdot(9)^2 \;+\; (-11)\cdot(9) \;+\; (23) \\ &= (-2)\cdot(729) \;+\; (19)\cdot(81) \;+\; (-11)\cdot(9) \;+\; (23) \\ &= (-1458) \;+\; (1539) \;+\; (-99) \;+\; (23) \\ &= 5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 5$.