Name:____ Date:____________________

Polynomial Operations SOLUTION (version 219)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 7x^5 + 2x^3 - 10x^2 - 6x + 3$$

$$q(x) = -8x^5 + 2x^4 + 4x^2 - x + 7$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (7)x^5 + (0)x^4 + (2)x^3 + (-10)x^2 + (-6)x^1 + (3)x^0 \\
q(x) &= (-8)x^5 + (2)x^4 + (0)x^3 + (4)x^2 + (-1)x^1 + (7)x^0 \\
q(x) - p(x) &= (-15)x^5 + (2)x^4 + (-2)x^3 + (14)x^2 + (5)x^1 + (4)x^0 \\
q(x) - p(x) &= -15x^5 + 2x^4 - 2x^3 + 14x^2 + 5x + 4
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -2x^2 + 4x - 9$$

$$b(x) = 5x - 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$4x$	-9
$5x$	$-10x^3$	$20x^2$	$-45x$
-4	$8x^2$	$-16x$	36

$$a(x) \cdot b(x) = -10x^3 + 20x^2 + 8x^2 - 45x - 16x + 36$$

Combine like terms.

$$a(x) \cdot b(x) = -10x^3 + 28x^2 - 61x + 36$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 219)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 19x^2 + 21x - 20 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 19 & 21 & -20 \\ -8 & & -16 & -24 & 24 \\ \hline & 2 & 3 & -3 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 3x - 3 + \frac{4}{x+8}$$

In other words, $h(x) = 2x^2 + 3x - 3$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 19x^2 + 21x - 20$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (2)\cdot(-8)^3 \ + \ (19)\cdot(-8)^2 \ + \ (21)\cdot(-8) \ + \ (-20) \\ &= (2)\cdot(-512) \ + \ (19)\cdot(64) \ + \ (21)\cdot(-8) \ + \ (-20) \\ &= (-1024) \ + \ (1216) \ + \ (-168) \ + \ (-20) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x + 8$. Thus, $f(-8) = 4$.