Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 11)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 + 9x^4 + x^2 + 5x + 4$$

$$q(x) = 7x^5 - 5x^4 - 3x^3 + 9x^2 - 10$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (8)x^5 + (9)x^4 + (0)x^3 + (1)x^2 + (5)x^1 + (4)x^0 \\
q(x) &= (7)x^5 + (-5)x^4 + (-3)x^3 + (9)x^2 + (0)x^1 + (-10)x^0 \\
p(x) - q(x) &= (1)x^5 + (14)x^4 + (3)x^3 + (-8)x^2 + (5)x^1 + (14)x^0 \\
p(x) - q(x) &= x^5 + 14x^4 + 3x^3 - 8x^2 + 5x + 14
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 + 4x + 8$$

$$b(x) = -6x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$4x$	8
$-6x$	$42x^3$	$-24x^2$	$-48x$
3	$-21x^2$	$12x$	24

$$a(x) \cdot b(x) = 42x^3 - 24x^2 - 21x^2 - 48x + 12x + 24$$

Combine like terms.

$$a(x) \cdot b(x) = 42x^3 - 45x^2 - 36x + 24$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 8x^2 - 9x + 25 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & -8 & -9 & 25 \\ -6 & & 6 & 12 & -18 \\ \hline & -1 & -2 & 3 & 7 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 2x + 3 + \frac{7}{x+6}$$

In other words, $h(x) = -x^2 - 2x + 3$ and the remainder is $R = 7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 8x^2 - 9x + 25$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-1)\cdot(-6)^3 \ + \ (-8)\cdot(-6)^2 \ + \ (-9)\cdot(-6) \ + \ (25) \\ &= (-1)\cdot(-216) \ + \ (-8)\cdot(36) \ + \ (-9)\cdot(-6) \ + \ (25) \\ &= (216) \ + \ (-288) \ + \ (54) \ + \ (25) \\ &= 7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 7$.