Name: ____________________ Date: ____________

Polynomial Operations SOLUTIONS (version 17)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 7x^5 - 10x^4 + 9x^2 + 2x + 1$$

$$q(x) = -4x^5 - 6x^3 + 3x^2 - 10x + 1$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (7)x^5 + (-10)x^4 + (0)x^3 + (9)x^2 + (2)x^1 + (1)x^0 \\ q(x) &= (-4)x^5 + (0)x^4 + (-6)x^3 + (3)x^2 + (-10)x^1 + (1)x^0 \\ q(x) - p(x) &= (-11)x^5 + (10)x^4 + (-6)x^3 + (-6)x^2 + (-12)x^1 + (0)x^0 \\ q(x) - p(x) &= -11x^5 + 10x^4 - 6x^3 - 6x^2 - 12x \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 - 8x - 6$$

$$b(x) = -7x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$-8x$	-6
$-7x$	$-14x^3$	$56x^2$	$42x$
3	$6x^2$	$-24x$	-18

$$a(x) \cdot b(x) = -14x^3 + 56x^2 + 6x^2 + 42x - 24x - 18$$

Combine like terms.

$$a(x) \cdot b(x) = -14x^3 + 62x^2 + 18x - 18$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 - 18x^2 + x - 6 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 2 & -18 & 1 & -6 \\ 9 & & 18 & 0 & 9 \\ \hline & 2 & 0 & 1 & 3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 1 + \frac{3}{x-9}$$

In other words, $h(x) = 2x^2 + 1$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 - 18x^2 + x - 6$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (2)\cdot(9)^3 \ + \ (-18)\cdot(9)^2 \ + \ (1)\cdot(9) \ + \ (-6) \\ &= (2)\cdot(729) \ + \ (-18)\cdot(81) \ + \ (1)\cdot(9) \ + \ (-6) \\ &= (1458) \ + \ (-1458) \ + \ (9) \ + \ (-6) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 3$.