

## Polynomial Operations SOLUTION (version 159)

1. Let polynomials  $p(x)$  and  $q(x)$  be defined below.

$$p(x) = -8x^5 - 10x^4 - 9x^3 + 7x + 1$$

$$q(x) = 10x^5 + 2x^4 - 3x^3 + 4x^2 - 7$$

Express the difference  $p(x) - q(x)$  in standard form.

Get “unsimplified” forms. Then find  $p(x) - q(x)$  with addition/subtraction.

$$p(x) = (-8)x^5 + (-10)x^4 + (-9)x^3 + (0)x^2 + (7)x^1 + (1)x^0$$

$$q(x) = (10)x^5 + (2)x^4 + (-3)x^3 + (4)x^2 + (0)x^1 + (-7)x^0$$

$$p(x) - q(x) = (-18)x^5 + (-12)x^4 + (-6)x^3 + (-4)x^2 + (7)x^1 + (8)x^0$$

$$p(x) - q(x) = -18x^5 - 12x^4 - 6x^3 - 4x^2 + 7x + 8$$

2. Let polynomials  $a(x)$  and  $b(x)$  be defined below.

$$a(x) = 7x^2 - 6x - 3$$

$$b(x) = -7x + 2$$

Express the product  $a(x) \cdot b(x)$  in standard form.

You can use a table for multiplication.

*	$7x^2$	$-6x$	$-3$
$-7x$	$-49x^3$	$42x^2$	$21x$
$2$	$14x^2$	$-12x$	$-6$

$$a(x) \cdot b(x) = -49x^3 + 42x^2 + 14x^2 + 21x - 12x - 6$$

Combine like terms.

$$a(x) \cdot b(x) = -49x^3 + 56x^2 + 9x - 6$$

3. Express  $(x + 1)^6$  in standard (expanded) form.

Remember [the binomial theorem](#). It tells us to use Pascal’s triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials  $f(x)$  and  $g(x)$  be defined below.

$$\begin{aligned}f(x) &= 2x^3 + 17x^2 + 10x + 14 \\g(x) &= x + 8\end{aligned}$$

The quotient of  $\frac{f(x)}{g(x)}$  can be expressed as a polynomial,  $h(x)$ , and a remainder,  $R$  (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express  $h(x)$  in standard form, and find the remainder  $R$ .

I prefer using synthetic division.

$$\begin{array}{r|rrrr}-8 & 2 & 17 & 10 & 14 \\ & & -16 & -8 & -16 \\ \hline & 2 & 1 & 2 & -2\end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + x + 2 + \frac{-2}{x+8}$$

In other words,  $h(x) = 2x^2 + x + 2$  and the remainder is  $R = -2$ .

5. Let polynomial  $f(x)$  still be defined as  $f(x) = 2x^3 + 17x^2 + 10x + 14$ . Evaluate  $f(-8)$ .

You could do this the hard way.

$$\begin{aligned}f(-8) &= (2) \cdot (-8)^3 + (17) \cdot (-8)^2 + (10) \cdot (-8) + (14) \\ &= (2) \cdot (-512) + (17) \cdot (64) + (10) \cdot (-8) + (14) \\ &= (-1024) + (1088) + (-80) + (14) \\ &= -2\end{aligned}$$

Or, if you reference the [polynomial remainder theorem](#), you can state that you know  $f(-8)$  equals the remainder when  $f(x)$  is divided by  $x + 8$ . Thus,  $f(-8) = -2$ .