Name: ____ Date: ____________________

Polynomial Operations SOLUTIONS (version 13)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 5x^5 + 6x^4 - 8x^3 - 9x^2 + 10 \\ q(x) &= -7x^5 - 10x^4 + 5x^3 - x + 8 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (5)x^5 + (6)x^4 + (-8)x^3 + (-9)x^2 + (0)x^1 + (10)x^0 \\ q(x) &= (-7)x^5 + (-10)x^4 + (5)x^3 + (0)x^2 + (-1)x^1 + (8)x^0 \\ q(x) - p(x) &= (-12)x^5 + (-16)x^4 + (13)x^3 + (9)x^2 + (-1)x^1 + (-2)x^0 \\ q(x) - p(x) &= -12x^5 - 16x^4 + 13x^3 + 9x^2 - x - 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -3x^2 + 2x - 7 \\ b(x) &= 4x - 7 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$2x$	-7
$4x$	$-12x^3$	$8x^2$	$-28x$
-7	$21x^2$	$-14x$	49

$$a(x) \cdot b(x) = -12x^3 + 8x^2 + 21x^2 - 28x - 14x + 49$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 + 29x^2 - 42x + 49$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 + 22x^2 + 14x + 19 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 4 & 22 & 14 & 19 \\ -5 & & -20 & -10 & -20 \\ \hline & 4 & 2 & 4 & -1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 4x^2 + 2x + 4 + \frac{-1}{x+5}$$

In other words, $h(x) = 4x^2 + 2x + 4$ and the remainder is $R = -1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 + 22x^2 + 14x + 19$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (4)\cdot(-5)^3 + (22)\cdot(-5)^2 + (14)\cdot(-5) + (19) \\ &= (4)\cdot(-125) + (22)\cdot(25) + (14)\cdot(-5) + (19) \\ &= (-500) + (550) + (-70) + (19) \\ &= -1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x+5$. Thus, $f(-5) = -1$.