Name: __________ Date: __________

Polynomial Operations SOLUTION (version 136)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) \;=\; -3x^5 - 5x^4 - 9x^3 - 2x^2 - 4$$

$$q(x) \;=\; 2x^5 - 9x^3 + 4x^2 + x - 8$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-3)x^5 + (-5)x^4 + (-9)x^3 + (-2)x^2 + (0)x^1 + (-4)x^0 \\ q(x) &= (2)x^5 + (0)x^4 + (-9)x^3 + (4)x^2 + (1)x^1 + (-8)x^0 \\ p(x) - q(x) &= (-5)x^5 + (-5)x^4 + (0)x^3 + (-6)x^2 + (-1)x^1 + (4)x^0 \\ p(x) - q(x) &= -5x^5 - 5x^4 - 6x^2 - x + 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) \;=\; -6x^2 - 8x - 5$$

$$b(x) \;=\; 4x - 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-8x$	-5
$4x$	$-24x^3$	$-32x^2$	$-20x$
-6	$36x^2$	$48x$	30

$$a(x) \cdot b(x) \;=\; -24x^3 - 32x^2 + 36x^2 - 20x + 48x + 30$$

Combine like terms.

$$a(x) \cdot b(x) \;=\; -24x^3 + 4x^2 + 28x + 30$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 136)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 25x^2 - 9x - 1 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & -25 & -9 & -1 \\ -8 & & 24 & 8 & 8 \\ \hline & -3 & -1 & -1 & 7 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 - x - 1 + \frac{7}{x+8}$$

In other words, $h(x) = -3x^2 - x - 1$ and the remainder is $R = 7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 25x^2 - 9x - 1$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-3)\cdot(-8)^3 + (-25)\cdot(-8)^2 + (-9)\cdot(-8) + (-1) \\ &= (-3)\cdot(-512) + (-25)\cdot(64) + (-9)\cdot(-8) + (-1) \\ &= (1536) + (-1600) + (72) + (-1) \\ &= 7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = 7$.