Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 20)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -8x^5 - 2x^4 - 9x^2 - 5x - 6$$

$$q(x) = -8x^5 - 2x^4 + 7x^3 + x + 5$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-8)x^5 + (-2)x^4 + (0)x^3 + (-9)x^2 + (-5)x^1 + (-6)x^0 \\ q(x) &= (-8)x^5 + (-2)x^4 + (7)x^3 + (0)x^2 + (1)x^1 + (5)x^0 \\ p(x) + q(x) &= (-16)x^5 + (-4)x^4 + (7)x^3 + (-9)x^2 + (-4)x^1 + (-1)x^0 \\ p(x) + q(x) &= -16x^5 - 4x^4 + 7x^3 - 9x^2 - 4x - 1 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -2x^2 + 6x + 3$$

$$b(x) = -6x + 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$6x$	3
$-6x$	$12x^3$	$-36x^2$	$-18x$
3	$-6x^2$	$18x$	9

$$a(x) \cdot b(x) = 12x^3 - 36x^2 - 6x^2 - 18x + 18x + 9$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 - 42x^2 + 9$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 + 10x^2 + 9x + 14 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -2 & 10 & 9 & 14 \\ 6 & & -12 & -12 & -18 \\ \hline & -2 & -2 & -3 & -4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 - 2x - 3 + \frac{-4}{x-6}$$

In other words, $h(x) = -2x^2 - 2x - 3$ and the remainder is $R = -4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 + 10x^2 + 9x + 14$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (-2)\cdot(6)^3 \ + \ (10)\cdot(6)^2 \ + \ (9)\cdot(6) \ + \ (14) \\ &= (-2)\cdot(216) \ + \ (10)\cdot(36) \ + \ (9)\cdot(6) \ + \ (14) \\ &= (-432) \ + \ (360) \ + \ (54) \ + \ (14) \\ &= -4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = -4$.