

## Polynomial Operations SOLUTION (version 147)

1. Let polynomials  $p(x)$  and  $q(x)$  be defined below.

$$p(x) = -2x^5 + 9x^3 - 3x^2 - 6x + 7$$

$$q(x) = 7x^5 - x^4 - 2x^3 + 6x - 4$$

Express the difference  $p(x) - q(x)$  in standard form.

Get “unsimplified” forms. Then find  $p(x) - q(x)$  with addition/subtraction.

$$p(x) = (-2)x^5 + (0)x^4 + (9)x^3 + (-3)x^2 + (-6)x^1 + (7)x^0$$

$$q(x) = (7)x^5 + (-1)x^4 + (-2)x^3 + (0)x^2 + (6)x^1 + (-4)x^0$$

$$p(x) - q(x) = (-9)x^5 + (1)x^4 + (11)x^3 + (-3)x^2 + (-12)x^1 + (11)x^0$$

$$p(x) - q(x) = -9x^5 + x^4 + 11x^3 - 3x^2 - 12x + 11$$

2. Let polynomials  $a(x)$  and  $b(x)$  be defined below.

$$a(x) = -6x^2 - 5x - 7$$

$$b(x) = -2x - 9$$

Express the product  $a(x) \cdot b(x)$  in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-5x$	$-7$
$-2x$	$12x^3$	$10x^2$	$14x$
$-9$	$54x^2$	$45x$	$63$

$$a(x) \cdot b(x) = 12x^3 + 10x^2 + 54x^2 + 14x + 45x + 63$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 + 64x^2 + 59x + 63$$

3. Express  $(x + 1)^6$  in standard (expanded) form.

Remember [the binomial theorem](#). It tells us to use Pascal’s triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials  $f(x)$  and  $g(x)$  be defined below.

$$\begin{aligned}f(x) &= -3x^3 - 23x^2 + 6x - 10 \\g(x) &= x + 8\end{aligned}$$

The quotient of  $\frac{f(x)}{g(x)}$  can be expressed as a polynomial,  $h(x)$ , and a remainder,  $R$  (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express  $h(x)$  in standard form, and find the remainder  $R$ .

I prefer using synthetic division.

$$\begin{array}{r|rrrr}-8 & -3 & -23 & 6 & -10 \\ & & 24 & -8 & 16 \\ \hline & -3 & 1 & -2 & 6\end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + x - 2 + \frac{6}{x+8}$$

In other words,  $h(x) = -3x^2 + x - 2$  and the remainder is  $R = 6$ .

5. Let polynomial  $f(x)$  still be defined as  $f(x) = -3x^3 - 23x^2 + 6x - 10$ . Evaluate  $f(-8)$ .

You could do this the hard way.

$$\begin{aligned}f(-8) &= (-3) \cdot (-8)^3 + (-23) \cdot (-8)^2 + (6) \cdot (-8) + (-10) \\ &= (-3) \cdot (-512) + (-23) \cdot (64) + (6) \cdot (-8) + (-10) \\ &= (1536) + (-1472) + (-48) + (-10) \\ &= 6\end{aligned}$$

Or, if you reference the [polynomial remainder theorem](#), you can state that you know  $f(-8)$  equals the remainder when  $f(x)$  is divided by  $x + 8$ . Thus,  $f(-8) = 6$ .