Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 160)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -9x^5 + 3x^4 + 7x^2 - x + 2 \\ q(x) &= 4x^5 + 7x^3 - 8x^2 - x + 10 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (3)x^4 + (0)x^3 + (7)x^2 + (-1)x^1 + (2)x^0 \\ q(x) &= (4)x^5 + (0)x^4 + (7)x^3 + (-8)x^2 + (-1)x^1 + (10)x^0 \\ p(x) - q(x) &= (-13)x^5 + (3)x^4 + (-7)x^3 + (15)x^2 + (0)x^1 + (-8)x^0 \\ p(x) - q(x) &= -13x^5 + 3x^4 - 7x^3 + 15x^2 - 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -9x^2 - 6x - 4 \\ b(x) &= -7x - 2 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-9x^2$	$-6x$	-4
$-7x$	$63x^3$	$42x^2$	$28x$
-2	$18x^2$	$12x$	8

$$a(x) \cdot b(x) = 63x^3 + 42x^2 + 18x^2 + 28x + 12x + 8$$

Combine like terms.

$$a(x) \cdot b(x) = 63x^3 + 60x^2 + 40x + 8$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 160)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 + 20x^2 - 28x - 28 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -2 & 20 & -28 & -28 \\ 8 & & -16 & 32 & 32 \\ \hline & -2 & 4 & 4 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 + 4x + 4 + \frac{4}{x-8}$$

In other words, $h(x) = -2x^2 + 4x + 4$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 + 20x^2 - 28x - 28$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (-2) \cdot (8)^3 \;+\; (20) \cdot (8)^2 \;+\; (-28) \cdot (8) \;+\; (-28) \\ &= (-2) \cdot (512) \;+\; (20) \cdot (64) \;+\; (-28) \cdot (8) \;+\; (-28) \\ &= (-1024) \;+\; (1280) \;+\; (-224) \;+\; (-28) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 4$.