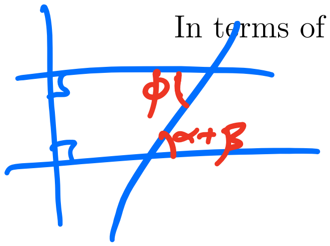
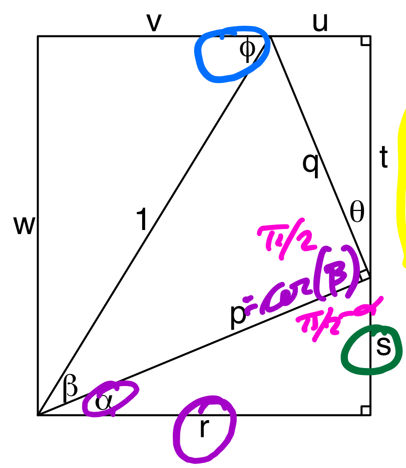
Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).





$\theta = \pi - \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right) = \alpha$

$\boxed{\cos(\alpha) = \frac{r}{\cos(\beta)}}$

$r = \cos(\alpha)\cdot\cos(\beta)$

$\sin(\alpha) = \frac{s}{\cos(\beta)}$

Variable	Algebraic expression
$p\ =$	$\cos(\beta)$
$q\ =$	$\sin(\beta)$
$r\ =$	$\cos(\alpha)\cos(\beta)$
$s\ =$	$\sin(\alpha)\cdot\cos(\beta)$
$\theta\ =$	α
$t\ =$	$\cos(\alpha)\cdot\sin(\beta)$
$u\ =$	$\sin(\alpha)\cdot\sin(\beta)$
$\phi\ =$	$\alpha+\beta$
$v\ =$	$\cos(\alpha+\beta)$
$w\ =$	$\sin(\alpha+\beta)$

$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)\\ \cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)\end{aligned}$$

You know the following:

$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)$

$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$

$$\sin(45^\circ) = \frac{\sqrt{2}}{2} \qquad \sin(150^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2} \qquad \cos(150^\circ) = \frac{-\sqrt{3}}{2}$$

Determine $\cos(-105^\circ)$ exactly.

$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$

$$\cos(45^\circ-150^\circ)=\cos(45^\circ)\cos(150^\circ)+\sin(45^\circ)\sin(150^\circ)$$

$$=\frac{\sqrt{2}}{2}\cdot\frac{-\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$=\frac{-\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$$

$$\boxed{=\frac{\sqrt{2}-\sqrt{6}}{4}}$$

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

Let $\alpha = x$.
Let $\beta = x$.

$\sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

Question 4

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$)

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

Let $\alpha = x$ and $\beta = x$.

$\cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x) = 1 - \cos^2(x)$

$\cos(2x) = \cos^2(x) - (1 - \cos^2(x))$

$\cos(2x) = \cos^2(x) - 1 + \cos^2(x)$

$\cos(2x) = 2\cos^2(x) - 1$

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

$$\cos(2x) = 2\cos^2(x) - 1$$

Let $2x = y$, so $x = \frac{y}{2}$.

$$\cos(y) = 2\cos^2(y/2) - 1$$

$$\cos(y) + 1 = 2\cos^2(y/2)$$

$$\frac{\cos(y)+1}{2} = \cos^2(y/2)$$

$$\pm\sqrt{\frac{\cos(y)+1}{2}} = \cos(y/2)$$

$$m = 2\ell^2 - 1$$

$$\ell = \pm\sqrt{\frac{m+1}{2}}$$

Question 6

If you knew that $\cos(110°) \approx \cancel{-1}$ -0.34, then what is $\cos(55°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $110/2 = 55$.)

$$\cos\left(\frac{110°}{2}\right) = \sqrt{\frac{1+\cos(110°)}{2}}$$

$$\cos(55°) = \sqrt{\frac{1+(-0.34)}{2}}$$

$$\sqrt{\frac{1-0.34}{2}}$$