Name:

Date:

s17 Geometric Series Exam (SLTN v399)

Question 1

Consider the partial geometric series represented below with first term a=297, common ratio $r=\left(\frac{16}{27}\right)^{1/10}$, and n=10 terms.

$$S = 297 + 281.86 + 267.49 + 253.85 + 240.91 + 228.63 + 216.98 + 205.91 + 195.42 + 185.45$$

We can multiply both sides by r.

$$rS \ = \ 281.86 + 267.49 + 253.85 + 240.91 + 228.63 + 216.98 + 205.91 + 195.42 + 185.45 + 176$$

What is the value of S - rS?

Most terms cancel.

$$297 - 176 = 121$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 4 + 4(3) + 4(3)^{2} + 4(3)^{3} + \cdots + 4(3)^{67} + 4(3)^{68} + 4(3)^{69} + 4(3)^{70}$$

Identify the initial term, the common ratio, and the number of terms.

first term =
$$a = 4$$

common ratio =
$$r = 3$$

number of terms =
$$n = 71$$

Question 3

Write a proof for the partial geometric series formula.

- a. Define the variables.
- b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a =first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^{2} + ar^{3} + \dots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1-r) = a - ar^n$$

Divide both sides by (1-r). We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$