

Name: _____

Date: _____

s17 Geometric Series Exam (Solution v104)

Question 1

Consider the partial geometric series represented below with first term $a = 990$, common ratio $r = \left(\frac{31}{55}\right)^{1/10}$, and $n = 10$ terms.

$$S = 990 + 934.84 + 882.74 + 833.56 + 787.11 + 743.25 + 701.83 + 662.73 + 625.8 + 590.93$$

We can multiply both sides by r .

$$rS = 934.84 + 882.74 + 833.56 + 787.11 + 743.25 + 701.83 + 662.73 + 625.8 + 590.93 + 558$$

What is the value of $S - rS$?

Most terms cancel.

$$990 - 558 = 432$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 2 + 2(5) + 2(5)^2 + 2(5)^3 + \cdots + 2(5)^{74} + 2(5)^{75} + 2(5)^{76} + 2(5)^{77}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 2$$

$$\text{common ratio} = r = 5$$

$$\text{number of terms} = n = 78$$

Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a = first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by $(1 - r)$. We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$