

Question 1

In terms of α and β , express all the lengths and other angle measures (θ and ϕ).



Variable	Algebraic expression
$p =$	
$q =$	
$r =$	
$s =$	
$\theta =$	
$t =$	
$u =$	
$\phi =$	
$v =$	
$w =$	

Question 2

The angle-sum and angle-difference identities are true for any α and β :

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

You know the following:

$$\sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\sin(150^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(150^\circ) = \frac{-\sqrt{3}}{2}$$

Determine $\cos(-105^\circ)$ exactly.

Question 3

Prove that $\sin(2x) = 2 \sin(x) \cos(x)$ for any x .

(Hint: start with an angle-sum formula from Question 2.)

Question 4

Prove that $\cos(2x) = 2 \cos^2(x) - 1$ for any x .

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$)

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

Question 6

If you knew that $\cos(110^\circ) \approx -0.34$, then what is $\cos(55^\circ)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $110/2 = 55$.)