Name: ______________________________ Date: ______________

# Polynomial Operations SOLUTION (version 236)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) = 6x^5 - 3x^4 + 4x^3 - 7x + 5$$

$$q(x) = -x^5 + 5x^4 - 8x^2 + 9x + 6$$

**Express the sum of $p(x) + q(x)$ in standard form.**

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$p(x) = (6)x^5 + (-3)x^4 + (4)x^3 + (0)x^2 + (-7)x^1 + (5)x^0$$
$$q(x) = (-1)x^5 + (5)x^4 + (0)x^3 + (-8)x^2 + (9)x^1 + (6)x^0$$
$$p(x) + q(x) = (5)x^5 + (2)x^4 + (4)x^3 + (-8)x^2 + (2)x^1 + (11)x^0$$
$$p(x) + q(x) = 5x^5 + 2x^4 + 4x^3 - 8x^2 + 2x + 11$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) = -3x^2 + 4x + 9$$

$$b(x) = 7x - 3$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $-3x^2$ | $4x$ | $9$ |
|---|---|---|---|
| $7x$ | $-21x^3$ | $28x^2$ | $63x$ |
| $-3$ | $9x^2$ | $-12x$ | $-27$ |

$$a(x) \cdot b(x) = -21x^3 + 28x^2 + 9x^2 + 63x - 12x - 27$$

Combine like terms.

$$a(x) \cdot b(x) = -21x^3 + 37x^2 + 51x - 27$$

**3. Express $(x+1)^6$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

# Polynomial Operations SOLUTION (version 236)

**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= -2x^3 - 10x^2 + 28x - 3 \\ g(x) &= x + 7 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+7}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

| | | | | |
|---|---|---|---|---|
| | -2 | -10 | 28 | -3 |
| -7 | | 14 | -28 | 0 |
| | -2 | 4 | 0 | -3 |

So,

$$\frac{f(x)}{g(x)} = -2x^2 + 4x + \frac{-3}{x+7}$$

In other words, $h(x) = -2x^2 + 4x$ and the remainder is $R = -3$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 - 10x^2 + 28x - 3$. Evaluate $f(-7)$.**

You could do this the hard way.

$$\begin{aligned} f(-7) &= (-2)\cdot(-7)^3 \ + \ (-10)\cdot(-7)^2 \ + \ (28)\cdot(-7) \ + \ (-3) \\ &= (-2)\cdot(-343) \ + \ (-10)\cdot(49) \ + \ (28)\cdot(-7) \ + \ (-3) \\ &= (686) \ + \ (-490) \ + \ (-196) \ + \ (-3) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-7)$ equals the remainder when $f(x)$ is divided by $x + 7$. Thus, $f(-7) = -3$.