

Name: _____

Date: _____

s17 Geometric Series Exam (Solution v49)

Question 1

Consider the partial geometric series represented below with first term $a = 735$, common ratio $r = \left(\frac{4}{15}\right)^{1/10}$, and $n = 10$ terms.

$$S = 735 + 644 + 564.26 + 494.4 + 433.19 + 379.55 + 332.56 + 291.38 + 255.31 + 223.7$$

We can multiply both sides by r .

$$rS = 644 + 564.26 + 494.4 + 433.19 + 379.55 + 332.56 + 291.38 + 255.31 + 223.7 + 196$$

What is the value of $S - rS$?

Most terms cancel.

$$735 - 196 = 539$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 5 + 5(4) + 5(4)^2 + 5(4)^3 + \cdots + 5(4)^{47} + 5(4)^{48} + 5(4)^{49} + 5(4)^{50}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 5$$

$$\text{common ratio} = r = 4$$

$$\text{number of terms} = n = 51$$

Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a = first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by $(1 - r)$. We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$