Name: ____________________ Date: ____________________

Polynomial Operations SOLUTIONS (version 32)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 4x^5 - 9x^4 + x^3 + 3x - 6$$
$$q(x) = -3x^5 + 9x^4 - 6x^2 + 4x - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get “unsimplified” forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (4)x^5 + (-9)x^4 + (1)x^3 + (0)x^2 + (3)x^1 + (-6)x^0 \\
q(x) &= (-3)x^5 + (9)x^4 + (0)x^3 + (-6)x^2 + (4)x^1 + (-5)x^0 \\
p(x) - q(x) &= (7)x^5 + (-18)x^4 + (1)x^3 + (6)x^2 + (-1)x^1 + (-1)x^0 \\
p(x) - q(x) &= 7x^5 - 18x^4 + x^3 + 6x^2 - x - 1
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 - 3x + 9$$
$$b(x) = -5x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-3x$	9
$-5x$	$40x^3$	$15x^2$	$-45x$
7	$-56x^2$	$-21x$	63

$$a(x) \cdot b(x) = 40x^3 + 15x^2 - 56x^2 - 45x - 21x + 63$$

Combine like terms.

$$a(x) \cdot b(x) = 40x^3 - 41x^2 - 66x + 63$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal’s triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 9x^2 + 3x + 20 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & \text{-1} & \text{-9} & 3 & 20 \\ \text{-9} & & 9 & 0 & \text{-27} \\ \hline & \text{-1} & 0 & 3 & \text{-7} \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 + 3 + \frac{-7}{x+9}$$

In other words, $h(x) = -x^2 + 3$ and the remainder is $R = -7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 9x^2 + 3x + 20$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (-1)\cdot(-9)^3 \ + \ (-9)\cdot(-9)^2 \ + \ (3)\cdot(-9) \ + \ (20) \\ &= (-1)\cdot(-729) \ + \ (-9)\cdot(81) \ + \ (3)\cdot(-9) \ + \ (20) \\ &= (729) \ + \ (-729) \ + \ (-27) \ + \ (20) \\ &= -7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = -7$.