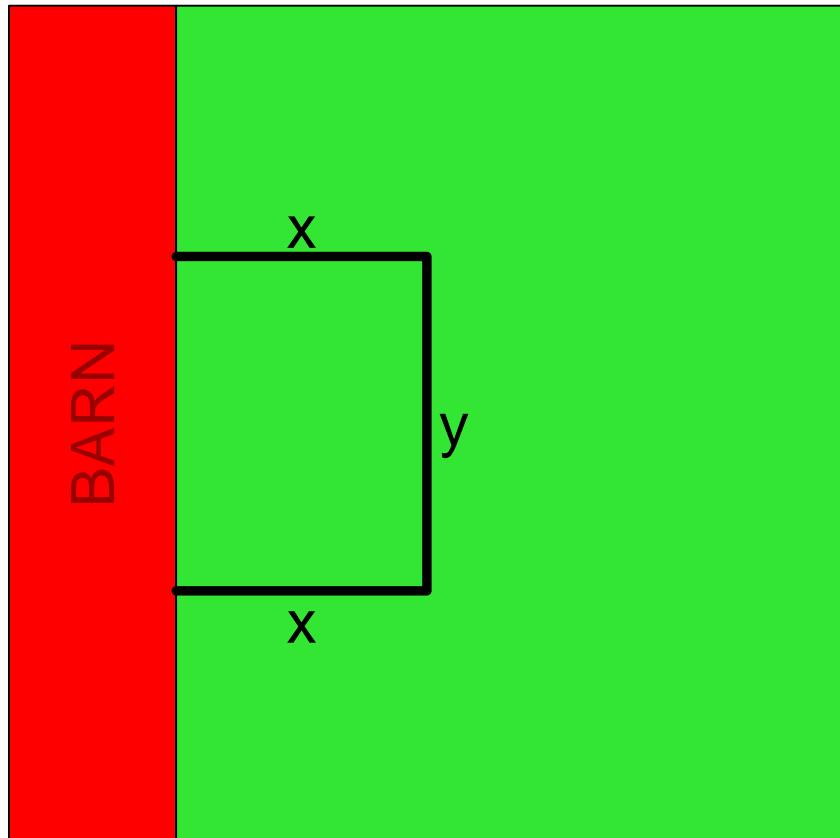


1. Problem

Amelia will use 104 feet of fence to build a rectangular enclosure for her dog. Amelia will build the enclosure next to a very long barn, so she'll only use the fencing for 3 of the sides of the rectangle.

Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.



She wants to give her dog as much area as possible. Find the value of x that maximizes the area.

Solution

The total length of fence is 104 feet. There are 2 sides with lengths of x and one side with a length of y .

$$2x + y = 104$$

Solve for y .

$$y = 104 - 2x$$

Write an equation for the area. A rectangular area equals the length times the width.

$$A = xy$$

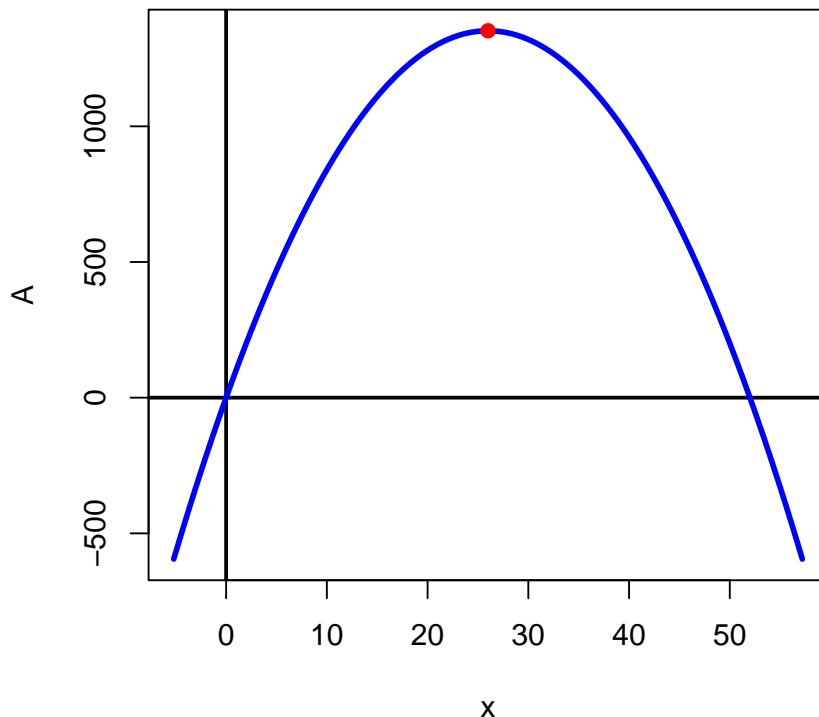
Substitute $104 - 2x$ in place of y to get area in terms of only x .

$$A = x(104 - 2x)$$

From here, we can expand into standard form, and use the formula $h = \frac{-b}{2a}$ to find the vertex.
Expand into standard form.

$$A = -2x^2 + 208x$$

You can graph A versus x and get a downwards facing parabola.



Now, remember the vertex (in this case the maximum) occurs when $x = \frac{-b}{2a}$.

$$x_{\text{optimal}} = \frac{-(208)}{2(-2)} = 26$$

So, the area is maximized when $x = 26$ feet and $y = 52$ feet.