Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 223)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 2x^5 + 6x^3 - 3x^2 - 8x - 5$$

$$q(x) = -6x^5 + 2x^4 - 4x^3 - 3x^2 - 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (2)x^5 + (0)x^4 + (6)x^3 + (-3)x^2 + (-8)x^1 + (-5)x^0 \\ q(x) &= (-6)x^5 + (2)x^4 + (-4)x^3 + (-3)x^2 + (0)x^1 + (-7)x^0 \\ p(x) - q(x) &= (8)x^5 + (-2)x^4 + (10)x^3 + (0)x^2 + (-8)x^1 + (2)x^0 \\ p(x) - q(x) &= 8x^5 - 2x^4 + 10x^3 - 8x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -2x^2 - 4x + 6$$

$$b(x) = 7x - 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$-4x$	6
$7x$	$-14x^3$	$-28x^2$	$42x$
-6	$12x^2$	$24x$	-36

$$a(x) \cdot b(x) = -14x^3 - 28x^2 + 12x^2 + 42x + 24x - 36$$

Combine like terms.

$$a(x) \cdot b(x) = -14x^3 - 16x^2 + 66x - 36$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 6x^3 + 20x^2 - 15x - 6 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 6 & 20 & -15 & -6 \\ -4 & & -24 & 16 & -4 \\ \hline & 6 & -4 & 1 & -10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 6x^2 - 4x + 1 + \frac{-10}{x+4}$$

In other words, $h(x) = 6x^2 - 4x + 1$ and the remainder is $R = -10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 6x^3 + 20x^2 - 15x - 6$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (6)\cdot(-4)^3 \ + \ (20)\cdot(-4)^2 \ + \ (-15)\cdot(-4) \ + \ (-6) \\ &= (6)\cdot(-64) \ + \ (20)\cdot(16) \ + \ (-15)\cdot(-4) \ + \ (-6) \\ &= (-384) \ + \ (320) \ + \ (60) \ + \ (-6) \\ &= -10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = -10$.