Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 112)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 - 6x^4 + 4x^2 - 5x + 3$$

$$q(x) = -2x^5 - 7x^4 - x^3 - 9x^2 - 6$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (-6)x^4 + (0)x^3 + (4)x^2 + (-5)x^1 + (3)x^0 \\ q(x) &= (-2)x^5 + (-7)x^4 + (-1)x^3 + (-9)x^2 + (0)x^1 + (-6)x^0 \\ q(x) - p(x) &= (7)x^5 + (-1)x^4 + (-1)x^3 + (-13)x^2 + (5)x^1 + (-9)x^0 \\ q(x) - p(x) &= 7x^5 - x^4 - x^3 - 13x^2 + 5x - 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 - 7x + 3$$

$$b(x) = -3x + 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$-7x$	3
$-3x$	$-15x^3$	$21x^2$	$-9x$
2	$10x^2$	$-14x$	6

$$a(x) \cdot b(x) = -15x^3 + 21x^2 + 10x^2 - 9x - 14x + 6$$

Combine like terms.

$$a(x) \cdot b(x) = -15x^3 + 31x^2 - 23x + 6$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 112)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 + 6x^2 + 27x + 1 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & 6 & 27 & 1 \\ 9 & & -9 & -27 & 0 \\ \hline & -1 & -3 & 0 & 1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 3x + \frac{1}{x-9}$$

In other words, $h(x) = -x^2 - 3x$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 6x^2 + 27x + 1$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-1)\cdot(9)^3 + (6)\cdot(9)^2 + (27)\cdot(9) + (1) \\ &= (-1)\cdot(729) + (6)\cdot(81) + (27)\cdot(9) + (1) \\ &= (-729) + (486) + (243) + (1) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 1$.