Name:

## at1204p_vertex_and_roots... from standard-form quadratic functions (v102)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ($w$)
3. Both $x$-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex $(h, k)$ shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

## Example

$$f(x) = 6x^2 + 4x - 5$$

### Example solution

1. Find the axis of symmetry. Use the formula $h = \frac{-b}{2a}$, where $h$ is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)} = \frac{-1}{3}$$

$$\text{axis of symmetry:} \quad x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula $w = \frac{\sqrt{b^2-4ac}}{2a}$.

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The $x$-intercepts can be found by adding $w$ to or subtracting $w$ from $h$.

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know $h = \frac{-1}{3}$, so we just need $k$. Use the formula $k = \frac{4ac-b^2}{4a}$.

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex:} \quad \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ($w$)
3. Both $x$-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex $(h, k)$ shown as cartesian coordinates

Box your answers.

$$f(x) \;=\; x^2 + 10x - 3$$

1. Axis of symmetry

$$h = \frac{-(10)}{2(1)} \;=\; -5$$

$$\text{axis of symmetry:} \quad x = -5$$

2. Distance from axis of symmetry to root

$$w \;=\; \frac{\sqrt{(10)^2 - 4(1)(-3)}}{2(1)}$$

$$w \;=\; \frac{\sqrt{112}}{2} \;=\; \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 7}}{2} \;=\; \frac{4\sqrt{7}}{2}$$

$$w \;=\; 2\sqrt{7}$$

3. Roots

$$\left(-5 - 2\sqrt{7},\, 0\right) \quad \text{and} \quad \left(-5 + 2\sqrt{7},\, 0\right)$$

4. Vertex

$$k = \frac{4(1)(-3) - (10)^2}{4(1)}$$

$$k \;=\; \frac{-112}{4} \;=\; -28$$

$$\text{vertex:} \quad (-5,\, -28)$$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ($w$)
3. Both $x$-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex $(h, k)$ shown as cartesian coordinates

Box your answers.

$$f(x) \;=\; 4x^2 - 8x + 1$$

1. Axis of symmetry

$$h = \frac{-(-8)}{2(4)} \;=\; 1$$

axis of symmetry: $\quad x = 1$

2. Distance from axis of symmetry to root

$$w \;=\; \frac{\sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$w \;=\; \frac{\sqrt{48}}{8} \;=\; \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}}{8} \;=\; \frac{4\sqrt{3}}{8}$$

$$w \;=\; \frac{\sqrt{3}}{2}$$

3. Roots

$$\left(1 - \frac{\sqrt{3}}{2},\, 0\right) \quad \text{and} \quad \left(1 + \frac{\sqrt{3}}{2},\, 0\right)$$

4. Vertex

$$k = \frac{4(4)(1) - (-8)^2}{4(4)}$$

$$k \;=\; \frac{-48}{16} \;=\; -3$$

vertex: $\quad (1,\, -3)$

## Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ($w$)
3. Both $x$-intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex $(h, k)$ shown as cartesian coordinates

Box your answers.

$$f(x) = 5x^2 - 2x - 10$$

1. Axis of symmetry

$$h = \frac{-(-2)}{2(5)} = \frac{1}{5}$$

$$\text{axis of symmetry:} \quad x = \frac{1}{5}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-2)^2 - 4(5)(-10)}}{2(5)}$$

$$w = \frac{\sqrt{204}}{10} = \frac{\sqrt{2 \cdot 2 \cdot 3 \cdot 17}}{10} = \frac{2\sqrt{51}}{10}$$

$$w = \frac{\sqrt{51}}{5}$$

3. Roots

$$\left(\frac{1}{5} - \frac{\sqrt{51}}{5}, 0\right) \quad \text{and} \quad \left(\frac{1}{5} + \frac{\sqrt{51}}{5}, 0\right)$$

4. Vertex

$$k = \frac{4(5)(-10) - (-2)^2}{4(5)}$$

$$k = \frac{-204}{20} = \frac{-51}{5}$$

$$\text{vertex:} \quad \left(\frac{1}{5}, \frac{-51}{5}\right)$$