Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 246)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 3x^5 - 8x^4 - 9x^3 + 2x + 6 \\ q(x) &= x^5 + 7x^4 + 8x^3 + 9x^2 - 2 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (-8)x^4 + (-9)x^3 + (0)x^2 + (2)x^1 + (6)x^0 \\ q(x) &= (1)x^5 + (7)x^4 + (8)x^3 + (9)x^2 + (0)x^1 + (-2)x^0 \\ q(x) - p(x) &= (-2)x^5 + (15)x^4 + (17)x^3 + (9)x^2 + (-2)x^1 + (-8)x^0 \\ q(x) - p(x) &= -2x^5 + 15x^4 + 17x^3 + 9x^2 - 2x - 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -6x^2 - 3x - 5 \\ b(x) &= -2x - 7 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-3x$	-5
$-2x$	$12x^3$	$6x^2$	$10x$
-7	$42x^2$	$21x$	35

$$a(x) \cdot b(x) = 12x^3 + 6x^2 + 42x^2 + 10x + 21x + 35$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 + 48x^2 + 31x + 35$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 11x^2 + 24x - 1 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 11 & 24 & -1 \\ -8 & & -8 & -24 & 0 \\ \hline & 1 & 3 & 0 & -1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 3x + \frac{-1}{x+8}$$

In other words, $h(x) = x^2 + 3x$ and the remainder is $R = -1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 11x^2 + 24x - 1$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (1)\cdot(-8)^3 \;+\; (11)\cdot(-8)^2 \;+\; (24)\cdot(-8) \;+\; (-1) \\ &= (1)\cdot(-512) \;+\; (11)\cdot(64) \;+\; (24)\cdot(-8) \;+\; (-1) \\ &= (-512) \;+\; (704) \;+\; (-192) \;+\; (-1) \\ &= -1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = -1$.