Name: ________________________________ Date: ________________

Polynomial Operations SOLUTION (version 245)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 10x^5 + 6x^4 + 8x^2 + 7x + 3$$

$$q(x) = 10x^5 - 4x^3 + 7x^2 - 5x + 6$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (10)x^5 + (6)x^4 + (0)x^3 + (8)x^2 + (7)x^1 + (3)x^0 \\ q(x) &= (10)x^5 + (0)x^4 + (-4)x^3 + (7)x^2 + (-5)x^1 + (6)x^0 \\ p(x) - q(x) &= (0)x^5 + (6)x^4 + (4)x^3 + (1)x^2 + (12)x^1 + (-3)x^0 \\ p(x) - q(x) &= 6x^4 + 4x^3 + x^2 + 12x - 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 - 9x + 7$$

$$b(x) = -5x + 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-9x$	7
$-5x$	$40x^3$	$45x^2$	$-35x$
2	$-16x^2$	$-18x$	14

$$a(x) \cdot b(x) = 40x^3 + 45x^2 - 16x^2 - 35x - 18x + 14$$

Combine like terms.

$$a(x) \cdot b(x) = 40x^3 + 29x^2 - 53x + 14$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 - 21x^2 - 23x + 1 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & -21 & -23 & 1 \\ 8 & & 24 & 24 & 8 \\ \hline & 3 & 3 & 1 & 9 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 3x + 1 + \frac{9}{x-8}$$

In other words, $h(x) = 3x^2 + 3x + 1$ and the remainder is $R = 9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 - 21x^2 - 23x + 1$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (3)\cdot(8)^3 \;+\; (-21)\cdot(8)^2 \;+\; (-23)\cdot(8) \;+\; (1) \\ &= (3)\cdot(512) \;+\; (-21)\cdot(64) \;+\; (-23)\cdot(8) \;+\; (1) \\ &= (1536) \;+\; (-1344) \;+\; (-184) \;+\; (1) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 9$.