Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 132)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 2x^5 + 9x^4 - 6x^3 + 7x - 8$$

$$q(x) = 7x^5 - 2x^4 + 3x^3 - 8x^2 - 1$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (2)x^5 + (9)x^4 + (-6)x^3 + (0)x^2 + (7)x^1 + (-8)x^0 \\
q(x) &= (7)x^5 + (-2)x^4 + (3)x^3 + (-8)x^2 + (0)x^1 + (-1)x^0 \\
p(x) - q(x) &= (-5)x^5 + (11)x^4 + (-9)x^3 + (8)x^2 + (7)x^1 + (-7)x^0 \\
p(x) - q(x) &= -5x^5 + 11x^4 - 9x^3 + 8x^2 + 7x - 7
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 - 5x + 4$$

$$b(x) = 6x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-5x$	4
$6x$	$-18x^3$	$-30x^2$	$24x$
7	$-21x^2$	$-35x$	28

$$a(x) \cdot b(x) = -18x^3 - 30x^2 - 21x^2 + 24x - 35x + 28$$

Combine like terms.

$$a(x) \cdot b(x) = -18x^3 - 51x^2 - 11x + 28$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 132)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -2x^3 - 14x^2 - 21x - 15 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -2 & -14 & -21 & -15 \\ -5 & & 10 & 20 & 5 \\ \hline & -2 & -4 & -1 & -10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -2x^2 - 4x - 1 + \frac{-10}{x+5}$$

In other words, $h(x) = -2x^2 - 4x - 1$ and the remainder is $R = -10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -2x^3 - 14x^2 - 21x - 15$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (-2)\cdot(-5)^3 \; + \; (-14)\cdot(-5)^2 \; + \; (-21)\cdot(-5) \; + \; (-15) \\ &= (-2)\cdot(-125) \; + \; (-14)\cdot(25) \; + \; (-21)\cdot(-5) \; + \; (-15) \\ &= (250) \; + \; (-350) \; + \; (105) \; + \; (-15) \\ &= -10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x+5$. Thus, $f(-5) = -10$.