Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 137)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 - 6x^4 - 7x^3 + 3x - 5$$

$$q(x) = -6x^5 - 5x^4 + 8x^2 - 10x - 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (-6)x^4 + (-7)x^3 + (0)x^2 + (3)x^1 + (-5)x^0 \\ q(x) &= (-6)x^5 + (-5)x^4 + (0)x^3 + (8)x^2 + (-10)x^1 + (-7)x^0 \\ p(x) - q(x) &= (14)x^5 + (-1)x^4 + (-7)x^3 + (-8)x^2 + (13)x^1 + (2)x^0 \\ p(x) - q(x) &= 14x^5 - x^4 - 7x^3 - 8x^2 + 13x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -4x^2 + 7x - 8$$

$$b(x) = 2x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-4x^2$	$7x$	-8
$2x$	$-8x^3$	$14x^2$	$-16x$
-7	$28x^2$	$-49x$	56

$$a(x) \cdot b(x) = -8x^3 + 14x^2 + 28x^2 - 16x - 49x + 56$$

Combine like terms.

$$a(x) \cdot b(x) = -8x^3 + 42x^2 - 65x + 56$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 137)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 18x^2 + 28x + 5 \\ g(x) &= x + 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 18 & 28 & 5 \\ \text{-7} & & \text{-14} & \text{-28} & 0 \\ \hline & 2 & 4 & 0 & 5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 4x + \frac{5}{x+7}$$

In other words, $h(x) = 2x^2 + 4x$ and the remainder is $R = 5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 18x^2 + 28x + 5$. Evaluate $f(-7)$.

You could do this the hard way.

$$\begin{aligned} f(-7) &= (2)\cdot(-7)^3 \;+\; (18)\cdot(-7)^2 \;+\; (28)\cdot(-7) \;+\; (5) \\ &= (2)\cdot(-343) \;+\; (18)\cdot(49) \;+\; (28)\cdot(-7) \;+\; (5) \\ &= (-686) \;+\; (882) \;+\; (-196) \;+\; (5) \\ &= 5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-7)$ equals the remainder when $f(x)$ is divided by $x + 7$. Thus, $f(-7) = 5$.