Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 106)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) \;=\; -8x^5 - 4x^3 + x^2 - 2x + 5$$

$$q(x) \;=\; 6x^5 + 3x^4 + x^2 - 9x - 8$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-8)x^5 + (0)x^4 + (-4)x^3 + (1)x^2 + (-2)x^1 + (5)x^0 \\
q(x) &= (6)x^5 + (3)x^4 + (0)x^3 + (1)x^2 + (-9)x^1 + (-8)x^0 \\
p(x) + q(x) &= (-2)x^5 + (3)x^4 + (-4)x^3 + (2)x^2 + (-11)x^1 + (-3)x^0 \\
p(x) + q(x) &= -2x^5 + 3x^4 - 4x^3 + 2x^2 - 11x - 3
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) \;=\; -5x^2 + 6x + 7$$

$$b(x) \;=\; 2x + 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-5x^2$	$6x$	7
$2x$	$-10x^3$	$12x^2$	$14x$
8	$-40x^2$	$48x$	56

$$a(x) \cdot b(x) \;=\; -10x^3 + 12x^2 - 40x^2 + 14x + 48x + 56$$

Combine like terms.

$$a(x) \cdot b(x) \;=\; -10x^3 - 28x^2 + 62x + 56$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 106)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 7x^2 - 16x - 19 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & -7 & -16 & -19 \\ 9 & & 9 & 18 & 18 \\ \hline & 1 & 2 & 2 & -1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 2x + 2 + \frac{-1}{x-9}$$

In other words, $h(x) = x^2 + 2x + 2$ and the remainder is $R = -1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 7x^2 - 16x - 19$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (1)\cdot(9)^3 \;+\; (-7)\cdot(9)^2 \;+\; (-16)\cdot(9) \;+\; (-19) \\ &= (1)\cdot(729) \;+\; (-7)\cdot(81) \;+\; (-16)\cdot(9) \;+\; (-19) \\ &= (729) \;+\; (-567) \;+\; (-144) \;+\; (-19) \\ &= -1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = -1$.