Name: ____ Date: ____

Polynomial Operations SOLUTION (version 228)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 6x^5 + 4x^4 - 8x^3 + 10x^2 + 7$$

$$q(x) = x^5 - 4x^3 - 5x^2 - 3x + 2$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (6)x^5 + (4)x^4 + (-8)x^3 + (10)x^2 + (0)x^1 + (7)x^0 \\ q(x) &= (1)x^5 + (0)x^4 + (-4)x^3 + (-5)x^2 + (-3)x^1 + (2)x^0 \\ q(x) - p(x) &= (-5)x^5 + (-4)x^4 + (4)x^3 + (-15)x^2 + (-3)x^1 + (-5)x^0 \\ q(x) - p(x) &= -5x^5 - 4x^4 + 4x^3 - 15x^2 - 3x - 5 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 - 6x + 3$$

$$b(x) = 7x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$-6x$	3
$7x$	$-56x^3$	$-42x^2$	$21x$
-5	$40x^2$	$30x$	-15

$$a(x) \cdot b(x) = -56x^3 - 42x^2 + 40x^2 + 21x + 30x - 15$$

Combine like terms.

$$a(x) \cdot b(x) = -56x^3 - 2x^2 + 51x - 15$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 228)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 20x^2 + 21x + 28 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	2	20	21	28
-9		-18	-18	-27
	2	2	3	1

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 2x + 3 + \frac{1}{x+9}$$

In other words, $h(x) = 2x^2 + 2x + 3$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 20x^2 + 21x + 28$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (2)\cdot(-9)^3 \ + \ (20)\cdot(-9)^2 \ + \ (21)\cdot(-9) \ + \ (28) \\ &= (2)\cdot(-729) \ + \ (20)\cdot(81) \ + \ (21)\cdot(-9) \ + \ (28) \\ &= (-1458) \ + \ (1620) \ + \ (-189) \ + \ (28) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 1$.