Name:__________________________________ Date:____________________

Polynomial Operations SOLUTION (version 138)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 10x^5 - 6x^4 - x^2 - 7x + 5$$

$$q(x) = 4x^5 + 10x^4 - 8x^3 - 3x^2 + 5$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (10)x^5 + (-6)x^4 + (0)x^3 + (-1)x^2 + (-7)x^1 + (5)x^0 \\ q(x) &= (4)x^5 + (10)x^4 + (-8)x^3 + (-3)x^2 + (0)x^1 + (5)x^0 \\ q(x) - p(x) &= (-6)x^5 + (16)x^4 + (-8)x^3 + (-2)x^2 + (7)x^1 + (0)x^0 \\ q(x) - p(x) &= -6x^5 + 16x^4 - 8x^3 - 2x^2 + 7x \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 + 2x - 8$$

$$b(x) = -8x - 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$2x$	-8
$-8x$	$24x^3$	$-16x^2$	$64x$
-4	$12x^2$	$-8x$	32

$$a(x) \cdot b(x) = 24x^3 - 16x^2 + 12x^2 + 64x - 8x + 32$$

Combine like terms.

$$a(x) \cdot b(x) = 24x^3 - 4x^2 + 56x + 32$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 138)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 + 20x^2 + 16x + 28 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & 20 & 16 & 28 \\ -6 & & -18 & -12 & -24 \\ \hline & 3 & 2 & 4 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 2x + 4 + \frac{4}{x+6}$$

In other words, $h(x) = 3x^2 + 2x + 4$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 + 20x^2 + 16x + 28$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (3)\cdot(-6)^3 \ + \ (20)\cdot(-6)^2 \ + \ (16)\cdot(-6) \ + \ (28) \\ &= (3)\cdot(-216) \ + \ (20)\cdot(36) \ + \ (16)\cdot(-6) \ + \ (28) \\ &= (-648) \ + \ (720) \ + \ (-96) \ + \ (28) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 4$.