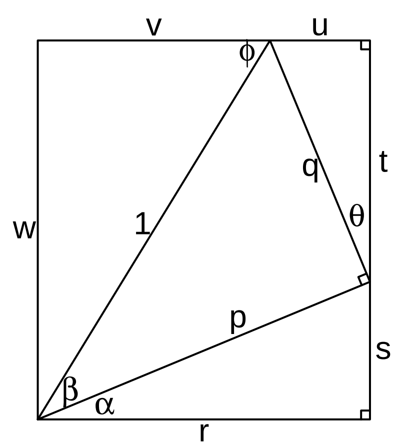
Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).



Variable	Algebraic expression
p =	
q =	
r =	
s =	
θ =	
t =	
u =	
ϕ =	
v =	
w =	

Question 2

The angle-sum and angle-difference identities are listed below:

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)\\ \cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)\end{aligned}$$

You know the following:

$$\sin(240^\circ) = \frac{-\sqrt{3}}{2} \qquad \sin(135^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(240^\circ) = \frac{-1}{2} \qquad \cos(135^\circ) = \frac{-\sqrt{2}}{2}$$

Determine $\sin(105^\circ)$ exactly.

Question 3

Prove the (sine) double-angle identity: $\sin(2x) = 2\sin(x)\cos(x)$

(Hint: start with an angle-sum formula from Question 2.)

Question 4

Prove the (cosine) double-angle identity: $\cos(2x) = 2\cos^2(x) - 1$

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$*)*

Question 5

Prove the (cosine) half-angle identity: $\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$.

(Hint: start with the double-angle identity from Question 4.)

Question 6

Given $\cos(108^\circ) \approx -0.31$, what is $\cos(54^\circ)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $108/2 = 54$.*)*