Name:____ Date:____________________

Polynomial Operations SOLUTIONS (version 14)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -4x^5 - 2x^3 - 7x^2 - x + 6$$

$$q(x) = 7x^5 + x^4 - 6x^3 + 3x^2 - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get “unsimplified” forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-4)x^5 + (0)x^4 + (-2)x^3 + (-7)x^2 + (-1)x^1 + (6)x^0 \\ q(x) &= (7)x^5 + (1)x^4 + (-6)x^3 + (3)x^2 + (0)x^1 + (-5)x^0 \\ p(x) - q(x) &= (-11)x^5 + (-1)x^4 + (4)x^3 + (-10)x^2 + (-1)x^1 + (11)x^0 \\ p(x) - q(x) &= -11x^5 - x^4 + 4x^3 - 10x^2 - x + 11 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 - 5x + 3$$

$$b(x) = 3x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$-5x$	3
$3x$	$-21x^3$	$-15x^2$	$9x$
-5	$35x^2$	$25x$	-15

$$a(x) \cdot b(x) = -21x^3 - 15x^2 + 35x^2 + 9x + 25x - 15$$

Combine like terms.

$$a(x) \cdot b(x) = -21x^3 + 20x^2 + 34x - 15$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal’s triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 + 9x^2 + 4x - 29 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & 9 & 4 & -29 \\ 9 & & -9 & 0 & 36 \\ \hline & -1 & 0 & 4 & 7 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 + 4 + \frac{7}{x-9}$$

In other words, $h(x) = -x^2 + 4$ and the remainder is $R = 7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 9x^2 + 4x - 29$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-1)\cdot(9)^3 \ + \ (9)\cdot(9)^2 \ + \ (4)\cdot(9) \ + \ (-29) \\ &= (-1)\cdot(729) \ + \ (9)\cdot(81) \ + \ (4)\cdot(9) \ + \ (-29) \\ &= (-729) \ + \ (729) \ + \ (36) \ + \ (-29) \\ &= 7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 7$.