Name: ______________________________ Date: ______________

# Polynomial Operations SOLUTIONS (version 24)

**1. Let polynomials $p(x)$ and $q(x)$ be defined below.**

$$p(x) = 8x^5 + 7x^4 + 2x^3 + 5x - 10$$

$$q(x) = -8x^5 - 7x^4 + 10x^3 - x^2 - 6$$

**Express the difference $p(x) - q(x)$ in standard form.**

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (7)x^4 + (2)x^3 + (0)x^2 + (5)x^1 + (-10)x^0 \\ q(x) &= (-8)x^5 + (-7)x^4 + (10)x^3 + (-1)x^2 + (0)x^1 + (-6)x^0 \\ p(x) - q(x) &= (16)x^5 + (14)x^4 + (-8)x^3 + (1)x^2 + (5)x^1 + (-4)x^0 \\ p(x) - q(x) &= 16x^5 + 14x^4 - 8x^3 + x^2 + 5x - 4 \end{aligned}$$

**2. Let polynomials $a(x)$ and $b(x)$ be defined below.**

$$a(x) = -2x^2 + 5x - 4$$

$$b(x) = 6x + 5$$

**Express the product $a(x) \cdot b(x)$ in standard form.**

You can use a table for multiplication.

| * | $-2x^2$ | $5x$ | $-4$ |
|---|---|---|---|
| $6x$ | $-12x^3$ | $30x^2$ | $-24x$ |
| $5$ | $-10x^2$ | $25x$ | $-20$ |

$$a(x) \cdot b(x) = -12x^3 + 30x^2 - 10x^2 - 24x + 25x - 20$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 + 20x^2 + x - 20$$

**3. Express $(x+1)^6$ in standard (expanded) form.**

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

# Polynomial Operations SOLUTIONS (version 24)

**4. Let polynomials $f(x)$ and $g(x)$ be defined below.**

$$\begin{aligned} f(x) &= x^3 + 12x^2 + 28x + 11 \\ g(x) &= x + 9 \end{aligned}$$

**The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, $R$ (a real number).**

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

**By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder $R$.**

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 12 & 28 & 11 \\ \text{-9} & & \text{-9} & \text{-27} & \text{-9} \\ \hline & 1 & 3 & 1 & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 3x + 1 + \frac{2}{x+9}$$

In other words, $h(x) = x^2 + 3x + 1$ and the remainder is $R = 2$.

**5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 12x^2 + 28x + 11$. Evaluate $f(-9)$.**

You could do this the hard way.

$$\begin{aligned} f(-9) &= (1) \cdot (-9)^3 \ + \ (12) \cdot (-9)^2 \ + \ (28) \cdot (-9) \ + \ (11) \\ &= (1) \cdot (-729) \ + \ (12) \cdot (81) \ + \ (28) \cdot (-9) \ + \ (11) \\ &= (-729) \ + \ (972) \ + \ (-252) \ + \ (11) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 2$.