Name:

Date:

s17 Geometric Series Exam (Solution v35)

Question 1

Consider the partial geometric series represented below with first term a=310, common ratio $r=\left(\frac{17}{31}\right)^{1/10}$, and n=10 terms.

$$S = 310 + 291.92 + 274.9 + 258.87 + 243.78 + 229.56 + 216.18 + 203.57 + 191.7 + 180.53$$

We can multiply both sides by r.

$$rS = 291.92 + 274.9 + 258.87 + 243.78 + 229.56 + 216.18 + 203.57 + 191.7 + 180.53 + 170$$

What is the value of S - rS?

Most terms cancel.

$$310 - 170 = 140$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 6 + 6(4) + 6(4)^{2} + 6(4)^{3} + \cdots + 6(4)^{61} + 6(4)^{62} + 6(4)^{63} + 6(4)^{64}$$

Identify the initial term, the common ratio, and the number of terms.

first term
$$= a = 6$$

common ratio =
$$r = 4$$

number of terms =
$$n = 65$$

Question 3

Write a proof for the partial geometric series formula.

- a. Define the variables.
- b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a =first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^{2} + ar^{3} + \dots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1-r) = a - ar^n$$

Divide both sides by (1-r). We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$