Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 241)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -9x^5 + 8x^4 - 5x^3 - 3x - 2 \\ q(x) &= -7x^5 - 9x^4 - 4x^3 - x^2 + 10 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (8)x^4 + (-5)x^3 + (0)x^2 + (-3)x^1 + (-2)x^0 \\ q(x) &= (-7)x^5 + (-9)x^4 + (-4)x^3 + (-1)x^2 + (0)x^1 + (10)x^0 \\ q(x) - p(x) &= (2)x^5 + (-17)x^4 + (1)x^3 + (-1)x^2 + (3)x^1 + (12)x^0 \\ q(x) - p(x) &= 2x^5 - 17x^4 + x^3 - x^2 + 3x + 12 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 5x^2 + 3x - 4 \\ b(x) &= 5x + 4 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$3x$	-4
$5x$	$25x^3$	$15x^2$	$-20x$
4	$20x^2$	$12x$	-16

$$a(x) \cdot b(x) = 25x^3 + 15x^2 + 20x^2 - 20x + 12x - 16$$

Combine like terms.

$$a(x) \cdot b(x) = 25x^3 + 35x^2 - 8x - 16$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 241)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 - 27x^2 + 12x - 26 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -5 & -27 & 12 & -26 \\ -6 & & 30 & -18 & 36 \\ \hline & -5 & 3 & -6 & 10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -5x^2 + 3x - 6 + \frac{10}{x+6}$$

In other words, $h(x) = -5x^2 + 3x - 6$ and the remainder is $R = 10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 - 27x^2 + 12x - 26$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-5)\cdot(-6)^3 \ + \ (-27)\cdot(-6)^2 \ + \ (12)\cdot(-6) \ + \ (-26) \\ &= (-5)\cdot(-216) \ + \ (-27)\cdot(36) \ + \ (12)\cdot(-6) \ + \ (-26) \\ &= (1080) \ + \ (-972) \ + \ (-72) \ + \ (-26) \\ &= 10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x+6$. Thus, $f(-6) = 10$.