s19 Matrix Exam (solution v126)

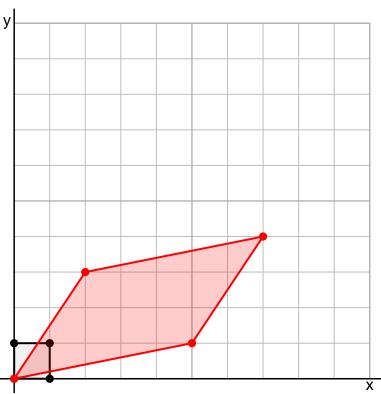
Let the 2×4 matrix U represent four points in the xy-plane (so each column represents a point). When those four points are connected as a convex polygon, matrix U represents a unit square. Also, let the 2×2 matrix L represent a linear transformation.

$$U = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$$

Let matrix $P = L \cdot U$, so P is found by matrix multiplication of L times U. Matrix P also represents 4 points of a polygon. Use the diagram below to calculate the elements of P. Then, draw the polygon represented by matrix P on the xy-plane below. Notice I have already drawn the unit square represented by matrix U.

1. Multiply $L \cdot U$ and draw resulting polygon.

		0	1	1	0
		0	0	1	1
5	2	0	5	7	2
1	3	0	1	4	3



2. What is the area of the convex polygon represented by matrix P? Hint: the area equals the absolute value of the determinant of matrix L.

area =
$$det(L)$$
 = $(5 \cdot 3) - (2 \cdot 1)$

The triangle shown below is composed of the three points represented by matrix $A = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \end{bmatrix}$. In order to reflect over the x axis, reflect over the y axis, and then rotate by 36.87° counterclockwise we can multiply by the transformation matrix $R = \begin{bmatrix} -0.8 & 0.6 \\ -0.6 & -0.8 \end{bmatrix}$.

3. Calculate the matrix $R \cdot A$.

$$R \cdot A = \begin{bmatrix} -4 & 3 & -2 \\ -3 & -4 & -14 \end{bmatrix}$$

4. Draw the triangle represented by $R \cdot A$.

