Name: ____________________ Date: ____________

Polynomial Operations SOLUTIONS (version 28)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 - 2x^3 + 6x^2 - 5x - 3$$

$$q(x) = 8x^5 + 10x^4 + 9x^3 - 5x^2 - 1$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (0)x^4 + (-2)x^3 + (6)x^2 + (-5)x^1 + (-3)x^0 \\ q(x) &= (8)x^5 + (10)x^4 + (9)x^3 + (-5)x^2 + (0)x^1 + (-1)x^0 \\ q(x) - p(x) &= (17)x^5 + (10)x^4 + (11)x^3 + (-11)x^2 + (5)x^1 + (2)x^0 \\ q(x) - p(x) &= 17x^5 + 10x^4 + 11x^3 - 11x^2 + 5x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 9x^2 + 3x + 8$$

$$b(x) = 4x + 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$9x^2$	$3x$	8
$4x$	$36x^3$	$12x^2$	$32x$
6	$54x^2$	$18x$	48

$$a(x) \cdot b(x) = 36x^3 + 12x^2 + 54x^2 + 32x + 18x + 48$$

Combine like terms.

$$a(x) \cdot b(x) = 36x^3 + 66x^2 + 50x + 48$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 8x^2 + x - 13 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 1 & \text{-8} & 1 & \text{-13} \\ 8 & & 8 & 0 & 8 \\ \hline & 1 & 0 & 1 & \text{-5} \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 1 + \frac{-5}{x-8}$$

In other words, $h(x) = x^2 + 1$ and the remainder is $R = -5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 8x^2 + x - 13$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (1)\cdot(8)^3 \ + \ (-8)\cdot(8)^2 \ + \ (1)\cdot(8) \ + \ (-13) \\ &= (1)\cdot(512) \ + \ (-8)\cdot(64) \ + \ (1)\cdot(8) \ + \ (-13) \\ &= (512) \ + \ (-512) \ + \ (8) \ + \ (-13) \\ &= -5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = -5$.