

Name: _____

Date: _____

s17 Geometric Series Exam (Solution v12)

Question 1

Consider the partial geometric series represented below with first term $a = 552$, common ratio $r = \left(\frac{7}{23}\right)^{1/10}$, and $n = 10$ terms.

$$S = 552 + 490.09 + 435.12 + 386.32 + 342.99 + 304.53 + 270.37 + 240.05 + 213.13 + 189.22$$

We can multiply both sides by r .

$$rS = 490.09 + 435.12 + 386.32 + 342.99 + 304.53 + 270.37 + 240.05 + 213.13 + 189.22 + 168$$

What is the value of $S - rS$?

Most terms cancel.

$$552 - 168 = 384$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 5 + 5(3) + 5(3)^2 + 5(3)^3 + \cdots + 5(3)^{53} + 5(3)^{54} + 5(3)^{55} + 5(3)^{56}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 5$$

$$\text{common ratio} = r = 3$$

$$\text{number of terms} = n = 57$$

Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a = first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by $(1 - r)$. We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$