Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 229)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = x^5 + 10x^3 - 4x^2 + 9x - 7$$

$$q(x) = 2x^5 - 9x^4 + x^3 + 10x - 3$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (1)x^5 + (0)x^4 + (10)x^3 + (-4)x^2 + (9)x^1 + (-7)x^0 \\ q(x) &= (2)x^5 + (-9)x^4 + (1)x^3 + (0)x^2 + (10)x^1 + (-3)x^0 \\ p(x) - q(x) &= (-1)x^5 + (9)x^4 + (9)x^3 + (-4)x^2 + (-1)x^1 + (-4)x^0 \\ p(x) - q(x) &= -x^5 + 9x^4 + 9x^3 - 4x^2 - x - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 2x - 9$$

$$b(x) = -2x + 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-2x$	-9
$-2x$	$-12x^3$	$4x^2$	$18x$
6	$36x^2$	$-12x$	-54

$$a(x) \cdot b(x) = -12x^3 + 4x^2 + 36x^2 + 18x - 12x - 54$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 + 40x^2 + 6x - 54$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 + 8x^2 - 25x + 25 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 4 & 8 & -25 & 25 \\ -4 & & -16 & 32 & -28 \\ \hline & 4 & -8 & 7 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 4x^2 - 8x + 7 + \frac{-3}{x+4}$$

In other words, $h(x) = 4x^2 - 8x + 7$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 + 8x^2 - 25x + 25$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (4) \cdot (-4)^3 \ + \ (8) \cdot (-4)^2 \ + \ (-25) \cdot (-4) \ + \ (25) \\ &= (4) \cdot (-64) \ + \ (8) \cdot (16) \ + \ (-25) \cdot (-4) \ + \ (25) \\ &= (-256) \ + \ (128) \ + \ (100) \ + \ (25) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = -3$.