Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 250)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 10x^5 + 8x^4 - 5x^2 - 9x + 1$$

$$q(x) = x^5 + 8x^4 - 10x^3 - 7x + 4$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (10)x^5 + (8)x^4 + (0)x^3 + (-5)x^2 + (-9)x^1 + (1)x^0 \\
q(x) &= (1)x^5 + (8)x^4 + (-10)x^3 + (0)x^2 + (-7)x^1 + (4)x^0 \\
p(x) + q(x) &= (11)x^5 + (16)x^4 + (-10)x^3 + (-5)x^2 + (-16)x^1 + (5)x^0 \\
p(x) + q(x) &= 11x^5 + 16x^4 - 10x^3 - 5x^2 - 16x + 5
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -2x^2 + 6x + 7$$

$$b(x) = -8x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$6x$	7
$-8x$	$16x^3$	$-48x^2$	$-56x$
4	$-8x^2$	$24x$	28

$$a(x) \cdot b(x) = 16x^3 - 48x^2 - 8x^2 - 56x + 24x + 28$$

Combine like terms.

$$a(x) \cdot b(x) = 16x^3 - 56x^2 - 32x + 28$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 250)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 10x^2 + 23x - 12 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-3	-10	23	-12
-5		15	-25	10
	-3	5	-2	-2

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 5x - 2 + \frac{-2}{x+5}$$

In other words, $h(x) = -3x^2 + 5x - 2$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 10x^2 + 23x - 12$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (-3)\cdot(-5)^3 \;+\; (-10)\cdot(-5)^2 \;+\; (23)\cdot(-5) \;+\; (-12) \\ &= (-3)\cdot(-125) \;+\; (-10)\cdot(25) \;+\; (23)\cdot(-5) \;+\; (-12) \\ &= (375) \;+\; (-250) \;+\; (-115) \;+\; (-12) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x+5$. Thus, $f(-5) = -2$.