

Name: \_\_\_\_\_

## at1204p\_vertex\_and\_roots... from standard-form quadratic functions (v111)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

### Example

$$f(x) = 6x^2 + 4x - 5$$

#### Example solution

1. Find the axis of symmetry. Use the formula  $h = \frac{-b}{2a}$ , where  $h$  is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)} = \frac{-1}{3}$$

$$\text{axis of symmetry: } x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula  $w = \frac{\sqrt{b^2 - 4ac}}{2a}$ .

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The  $x$ -intercepts can be found by adding  $w$  to or subtracting  $w$  from  $h$ .

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know  $h = \frac{-1}{3}$ , so we just need  $k$ . Use the formula  $k = \frac{4ac - b^2}{4a}$ .

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex: } \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 4x^2 - 3x - 9$$

1. Axis of symmetry

$$h = \frac{-(-3)}{2(4)} = \frac{3}{8}$$

$$\text{axis of symmetry: } x = \frac{3}{8}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-3)^2 - 4(4)(-9)}}{2(4)}$$

$$w = \frac{\sqrt{153}}{8} = \frac{\sqrt{3 \cdot 3 \cdot 17}}{8} = \frac{3\sqrt{17}}{8}$$

$$w = \frac{3\sqrt{17}}{8}$$

3. Roots

$$\left(\frac{3}{8} - \frac{3\sqrt{17}}{8}, 0\right) \quad \text{and} \quad \left(\frac{3}{8} + \frac{3\sqrt{17}}{8}, 0\right)$$

4. Vertex

$$k = \frac{4(4)(-9) - (-3)^2}{4(4)}$$

$$k = \frac{-153}{16}$$

$$\text{vertex: } \left(\frac{3}{8}, \frac{-153}{16}\right)$$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = x^2 + 10x + 8$$

1. Axis of symmetry

$$h = \frac{-(10)}{2(1)} = -5$$

axis of symmetry:  $x = -5$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(10)^2 - 4(1)(8)}}{2(1)}$$

$$w = \frac{\sqrt{68}}{2} = \frac{\sqrt{2 \cdot 2 \cdot 17}}{2} = \frac{2\sqrt{17}}{2}$$
$$w = \sqrt{17}$$

3. Roots

$$\left(-5 - \sqrt{17}, 0\right) \text{ and } \left(-5 + \sqrt{17}, 0\right)$$

4. Vertex

$$k = \frac{4(1)(8) - (10)^2}{4(1)}$$

$$k = \frac{-68}{4} = -17$$

vertex:  $(-5, -17)$

### Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 3x^2 + 10x - 10$$

1. Axis of symmetry

$$h = \frac{-(10)}{2(3)} = \frac{-5}{3}$$

$$\text{axis of symmetry: } x = \frac{-5}{3}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(10)^2 - 4(3)(-10)}}{2(3)}$$

$$w = \frac{\sqrt{220}}{6} = \frac{\sqrt{2 \cdot 2 \cdot 5 \cdot 11}}{6} = \frac{2\sqrt{55}}{6}$$

$$w = \frac{\sqrt{55}}{3}$$

3. Roots

$$\left(\frac{-5}{3} - \frac{\sqrt{55}}{3}, 0\right) \quad \text{and} \quad \left(\frac{-5}{3} + \frac{\sqrt{55}}{3}, 0\right)$$

4. Vertex

$$k = \frac{4(3)(-10) - (10)^2}{4(3)}$$

$$k = \frac{-220}{12} = \frac{-55}{3}$$

$$\text{vertex: } \left(\frac{-5}{3}, \frac{-55}{3}\right)$$