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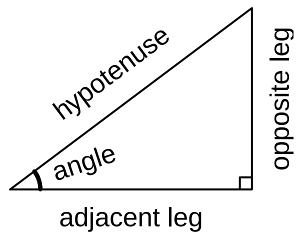
Trig cheat sheet

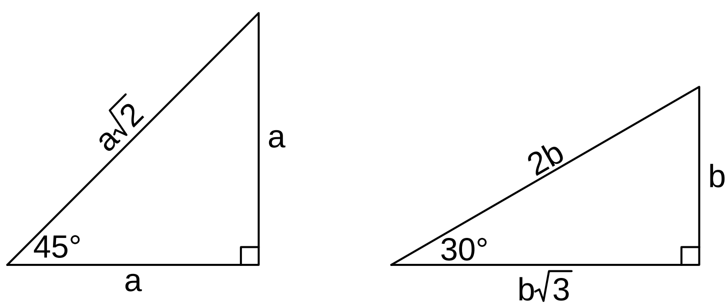
Below are the right-triangle definitions of the 6 trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. The hypotenuse is the longest side of a right triangle, always across from the right angle. The opposite leg is across from the indicated angle, and the adjacent leg is the other side.



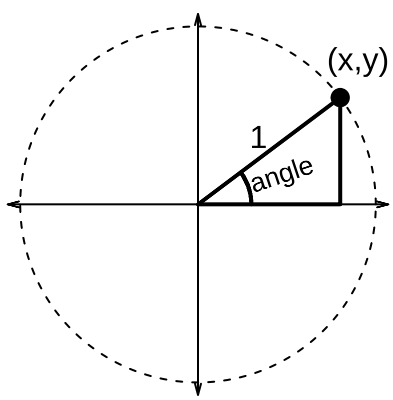
$$\sin(\text{angle}) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\text{angle}) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\text{angle}) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc(\text{angle}) = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec(\text{angle}) = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot(\text{angle}) = \frac{\text{adjacent}}{\text{opposite}}$$

Some special right triangles whose angles and ratios can be found exactly with a tad of algebra are the 45-45-90 and 30-60-90 triangles.

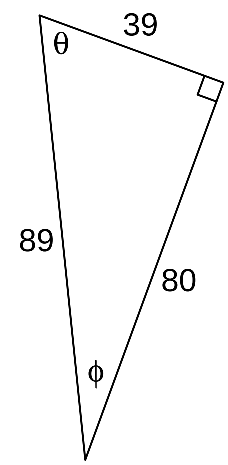


To extend beyond acute angles, we use a unit circle: replace the hypotenuse length with 1, the opposite-leg length with y, and the adjacent-leg length with x.



Question 1

Consider the right triangle below, with side lengths 39, 80, and 89 and acute angle measures θ and ϕ.

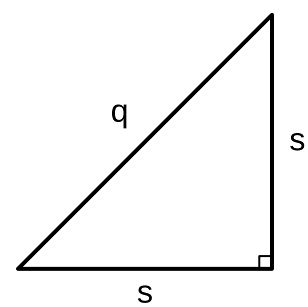


Express the 6 trigonometric ratios of angle θ. Write each ratio as a fraction. When relevant, use an improper fraction (like $\frac{5}{3}$), not a mixed number (not like $1 + \frac{2}{3}$).

Trig function	Ratio (function's output)
$\sin(\theta)$ =	80/89
$\cos(\theta)$ =	39/89
$\tan(\theta)$ =	80/39
$\csc(\theta)$ =	89/80
$\sec(\theta)$ =	89/39
$\cot(\theta)$ =	39/80

Question 2

Consider the isosceles right triangle below.



Prove that $q = s\sqrt{2}$.

(Remember Pythagorean Theorem: a triangle with lengths a, b, and c, where $a \leq b < c$, is a right triangle if and only if $a^2 + b^2 = c^2$.)

Substitute into Pythagorean equation:

$$s^2 + s^2 = q^2$$

Combine similar terms.

$$2s^2 = q^2$$

Take the square root of both sides.

$$\sqrt{2s^2} = \sqrt{q^2}$$

Distribute the radical over the product.

$$\sqrt{2} \cdot \sqrt{s^2} = \sqrt{q^2}$$

Both s and q are positive, so the square root is the inverse of the squaring. Simplify.

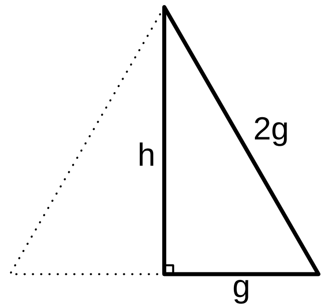
$$\sqrt{2} \cdot s = q$$

Rearrange.

$$q = s\sqrt{2}$$

Question 3

Consider the triangle below, generated by bisecting an equilateral triangle.



Prove that $h = g\sqrt{3}$.

(Remember Pythagorean Theorem: a triangle with lengths a, b, and c, where $a \leq b < c$, is a right triangle if and only if $a^2 + b^2 = c^2$.)

Substitute into Pythagorean equation:

$$g^2 + h^2 = (2g)^2$$

Distribute the exponent over the product.

$$g^2 + h^2 = 4g^2$$

Subtract g^2 from both sides.

$$h^2 = 3g^2$$

Take the square root of both sides.

$$\sqrt{h^2} = \sqrt{3g^2}$$

Distribute the radical across the product.

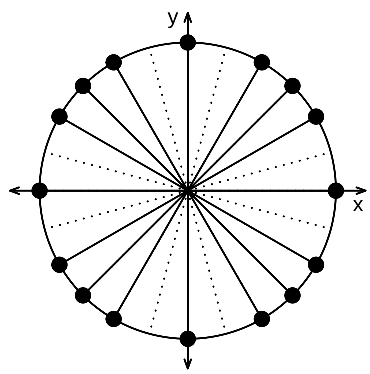
$$\sqrt{h^2} = \sqrt{3} \cdot \sqrt{g^2}$$

Both h and g are positive, so squaring and rooting undo each other. Also, multiplication is commutative.

$$h = g\sqrt{3}$$

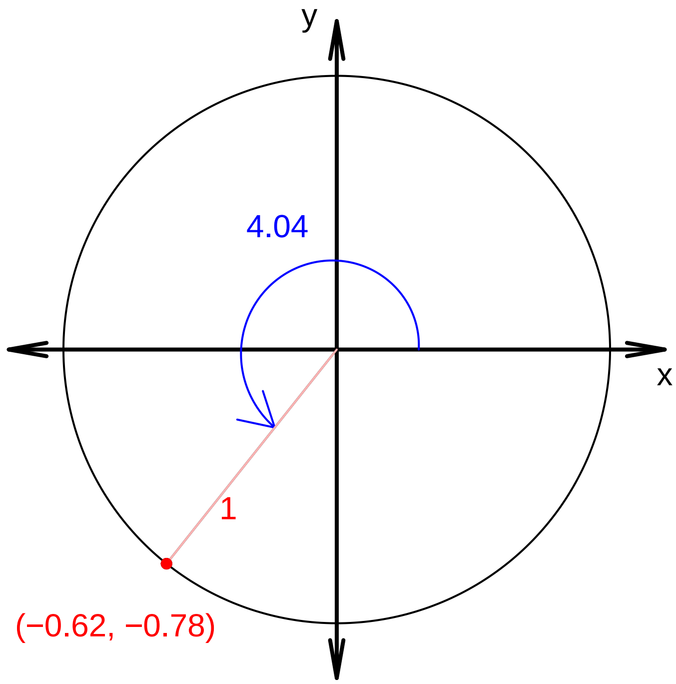
Question 4

A unit circle (with radius = 1) is drawn with its center at the origin. Based on the two proofs in Question 2 and Question 3, determine the angles and (exact) coordinates of the indicated 16 points, with the angle measure increasing, starting at 0, and all angles in standard position. All angles are multiples of 1/24 of a revolution. Remember, half a turn equals 180°, which equals π radians.



Angle measure (degrees)	Angle measure (radians)	x	y
0°	0	1	0
30°	$\frac{2\pi}{12} = \pi/6$	$\sqrt{3}/2$	$1/2$
45°	$\frac{3\pi}{12} = \pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
60°	$\frac{4\pi}{12} = \pi/3$	$1/2$	$\sqrt{3}/2$
90°	$\frac{6\pi}{12} = \pi/2$	0	1
120°	$\frac{8\pi}{12} = 2\pi/3$	$-1/2$	$\sqrt{3}/2$
135°	$\frac{9\pi}{12} = 3\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$
150°	$\frac{10\pi}{12} = 5\pi/6$	$-\sqrt{3}/2$	$1/2$
180°	$\frac{12\pi}{12} = \pi$	-1	0
210°	$\frac{14\pi}{12} = 7\pi/6$	$-\sqrt{3}/2$	$-1/2$
225°	$\frac{15\pi}{12} = 5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$
240°	$\frac{16\pi}{12} = 4\pi/3$	$-1/2$	$-\sqrt{3}/2$
270°	$\frac{18\pi}{12} = 3\pi/2$	0	-1
300°	$\frac{20\pi}{12} = 5\pi/3$	$1/2$	$-\sqrt{3}/2$
315°	$\frac{21\pi}{12} = 7\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$
330°	$\frac{22\pi}{12} = 11\pi/6$	$\sqrt{3}/2$	$-1/2$

Question 5



An angle of 4.04 radians intersects the unit circle at coordinates $(-0.62, -0.78)$. Fill the blanks in the two equations below.

$$\sin\left(\boxed{4.04}\right) = \boxed{-0.78}$$

$$\cos\left(\boxed{4.04}\right) = \boxed{-0.62}$$

$$\tan\left(\boxed{4.04}\right) = \frac{\boxed{-0.78}}{\boxed{-0.62}}$$