Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 18)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -2x^5 + 6x^4 - 4x^2 + 3x + 7 \\ q(x) &= -x^5 - 5x^3 - 8x^2 + 7x + 3 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (6)x^4 + (0)x^3 + (-4)x^2 + (3)x^1 + (7)x^0 \\ q(x) &= (-1)x^5 + (0)x^4 + (-5)x^3 + (-8)x^2 + (7)x^1 + (3)x^0 \\ q(x) - p(x) &= (1)x^5 + (-6)x^4 + (-5)x^3 + (-4)x^2 + (4)x^1 + (-4)x^0 \\ q(x) - p(x) &= x^5 - 6x^4 - 5x^3 - 4x^2 + 4x - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -2x^2 - 5x - 8 \\ b(x) &= 8x - 5 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$-5x$	-8
$8x$	$-16x^3$	$-40x^2$	$-64x$
-5	$10x^2$	$25x$	40

$$a(x) \cdot b(x) = -16x^3 - 40x^2 + 10x^2 - 64x + 25x + 40$$

Combine like terms.

$$a(x) \cdot b(x) = -16x^3 - 30x^2 - 39x + 40$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -6x^3 + 27x^2 + 18x - 13 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-6	27	18	-13
5		-30	-15	15
	-6	-3	3	2

So,

$$\frac{f(x)}{g(x)} = -6x^2 - 3x + 3 + \frac{2}{x-5}$$

In other words, $h(x) = -6x^2 - 3x + 3$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -6x^3 + 27x^2 + 18x - 13$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (-6)\cdot(5)^3 \ + \ (27)\cdot(5)^2 \ + \ (18)\cdot(5) \ + \ (-13) \\ &= (-6)\cdot(125) \ + \ (27)\cdot(25) \ + \ (18)\cdot(5) \ + \ (-13) \\ &= (-750) \ + \ (675) \ + \ (90) \ + \ (-13) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x-5$. Thus, $f(5) = 2$.