Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 107)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 - 3x^4 + 6x^3 + 10x^2 + 5$$

$$q(x) = -3x^5 - 10x^4 - 9x^3 + 7x - 6$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (-3)x^4 + (6)x^3 + (10)x^2 + (0)x^1 + (5)x^0 \\ q(x) &= (-3)x^5 + (-10)x^4 + (-9)x^3 + (0)x^2 + (7)x^1 + (-6)x^0 \\ p(x) + q(x) &= (-5)x^5 + (-13)x^4 + (-3)x^3 + (10)x^2 + (7)x^1 + (-1)x^0 \\ p(x) + q(x) &= -5x^5 - 13x^4 - 3x^3 + 10x^2 + 7x - 1 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 7x + 4$$

$$b(x) = -5x - 9$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-7x$	4
$-5x$	$-30x^3$	$35x^2$	$-20x$
-9	$-54x^2$	$63x$	-36

$$a(x) \cdot b(x) = -30x^3 + 35x^2 - 54x^2 - 20x + 63x - 36$$

Combine like terms.

$$a(x) \cdot b(x) = -30x^3 - 19x^2 + 43x - 36$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 107)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 - 26x^2 + 13x + 4 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 4 & -26 & 13 & 4 \\ 6 & & 24 & -12 & 6 \\ \hline & 4 & -2 & 1 & 10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 4x^2 - 2x + 1 + \frac{10}{x-6}$$

In other words, $h(x) = 4x^2 - 2x + 1$ and the remainder is $R = 10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 - 26x^2 + 13x + 4$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (4) \cdot (6)^3 \ + \ (-26) \cdot (6)^2 \ + \ (13) \cdot (6) \ + \ (4) \\ &= (4) \cdot (216) \ + \ (-26) \cdot (36) \ + \ (13) \cdot (6) \ + \ (4) \\ &= (864) \ + \ (-936) \ + \ (78) \ + \ (4) \\ &= 10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = 10$.