Name: ____________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 8)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 + 7x^4 - 10x^3 - 9x^2 + 1$$

$$q(x) = 4x^5 + 8x^4 - 10x^3 - x - 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (7)x^4 + (-10)x^3 + (-9)x^2 + (0)x^1 + (1)x^0 \\ q(x) &= (4)x^5 + (8)x^4 + (-10)x^3 + (0)x^2 + (-1)x^1 + (-7)x^0 \\ p(x) - q(x) &= (-6)x^5 + (-1)x^4 + (0)x^3 + (-9)x^2 + (1)x^1 + (8)x^0 \\ p(x) - q(x) &= -6x^5 - x^4 - 9x^2 + x + 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 8x^2 - 3x - 6$$

$$b(x) = 3x - 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$8x^2$	$-3x$	-6
$3x$	$24x^3$	$-9x^2$	$-18x$
-6	$-48x^2$	$18x$	36

$$a(x) \cdot b(x) = 24x^3 - 9x^2 - 48x^2 - 18x + 18x + 36$$

Combine like terms.

$$a(x) \cdot b(x) = 24x^3 - 57x^2 + 36$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 + 29x^2 - 19x + 15 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -3 & 29 & -19 & 15 \\ 9 & & -27 & 18 & -9 \\ \hline & -3 & 2 & -1 & 6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 2x - 1 + \frac{6}{x-9}$$

In other words, $h(x) = -3x^2 + 2x - 1$ and the remainder is $R = 6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 + 29x^2 - 19x + 15$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (-3)\cdot(9)^3 \ + \ (29)\cdot(9)^2 \ + \ (-19)\cdot(9) \ + \ (15) \\ &= (-3)\cdot(729) \ + \ (29)\cdot(81) \ + \ (-19)\cdot(9) \ + \ (15) \\ &= (-2187) \ + \ (2349) \ + \ (-171) \ + \ (15) \\ &= 6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 6$.