Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 12)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 7x^5 - 5x^4 - 4x^2 + 2x + 3$$

$$q(x) = 10x^5 - 7x^4 + 8x^3 - 5x^2 - 1$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (7)x^5 + (-5)x^4 + (0)x^3 + (-4)x^2 + (2)x^1 + (3)x^0 \\ q(x) &= (10)x^5 + (-7)x^4 + (8)x^3 + (-5)x^2 + (0)x^1 + (-1)x^0 \\ q(x) - p(x) &= (3)x^5 + (-2)x^4 + (8)x^3 + (-1)x^2 + (-2)x^1 + (-4)x^0 \\ q(x) - p(x) &= 3x^5 - 2x^4 + 8x^3 - x^2 - 2x - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 4x^2 - 6x + 5$$

$$b(x) = -8x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$4x^2$	$-6x$	5
$-8x$	$-32x^3$	$48x^2$	$-40x$
4	$16x^2$	$-24x$	20

$$a(x) \cdot b(x) = -32x^3 + 48x^2 + 16x^2 - 40x - 24x + 20$$

Combine like terms.

$$a(x) \cdot b(x) = -32x^3 + 64x^2 - 64x + 20$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTIONS (version 12)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 + 6x^2 + 16x - 2 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -1 & 6 & 16 & -2 \\ 8 & & -8 & -16 & 0 \\ \hline & -1 & -2 & 0 & -2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 2x + \frac{-2}{x-8}$$

In other words, $h(x) = -x^2 - 2x$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 + 6x^2 + 16x - 2$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (-1)\cdot(8)^3 \ + \ (6)\cdot(8)^2 \ + \ (16)\cdot(8) \ + \ (-2) \\ &= (-1)\cdot(512) \ + \ (6)\cdot(64) \ + \ (16)\cdot(8) \ + \ (-2) \\ &= (-512) \ + \ (384) \ + \ (128) \ + \ (-2) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = -2$.