Name:____ Date:____

Polynomial Operations SOLUTION (version 220)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 6x^5 + 5x^3 + 8x^2 - 9x + 4 \\ q(x) &= 8x^5 - 6x^4 + 5x^3 + 10x + 2 \end{aligned}$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (6)x^5 + (0)x^4 + (5)x^3 + (8)x^2 + (-9)x^1 + (4)x^0 \\ q(x) &= (8)x^5 + (-6)x^4 + (5)x^3 + (0)x^2 + (10)x^1 + (2)x^0 \\ p(x) + q(x) &= (14)x^5 + (-6)x^4 + (10)x^3 + (8)x^2 + (1)x^1 + (6)x^0 \\ p(x) + q(x) &= 14x^5 - 6x^4 + 10x^3 + 8x^2 + x + 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= -4x^2 + 3x + 2 \\ b(x) &= -5x + 9 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-4x^2$	$3x$	2
$-5x$	$20x^3$	$-15x^2$	$-10x$
9	$-36x^2$	$27x$	18

$$a(x) \cdot b(x) = 20x^3 - 15x^2 - 36x^2 - 10x + 27x + 18$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 - 51x^2 + 17x + 18$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 29x^2 - 20x - 25 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-3	-29	-20	-25
-9		27	18	18
	-3	-2	-2	-7

So,

$$\frac{f(x)}{g(x)} = -3x^2 - 2x - 2 + \frac{-7}{x+9}$$

In other words, $h(x) = -3x^2 - 2x - 2$ and the remainder is $R = -7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 29x^2 - 20x - 25$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (-3)\cdot(-9)^3 \ + \ (-29)\cdot(-9)^2 \ + \ (-20)\cdot(-9) \ + \ (-25) \\ &= (-3)\cdot(-729) \ + \ (-29)\cdot(81) \ + \ (-20)\cdot(-9) \ + \ (-25) \\ &= (2187) \ + \ (-2349) \ + \ (180) \ + \ (-25) \\ &= -7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x+9$. Thus, $f(-9) = -7$.