Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 203)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 9x^5 + 3x^4 + 2x^2 - 4x + 6$$
$$q(x) = 8x^5 - 3x^3 - 9x^2 - 5x - 1$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (9)x^5 + (3)x^4 + (0)x^3 + (2)x^2 + (-4)x^1 + (6)x^0 \\ q(x) &= (8)x^5 + (0)x^4 + (-3)x^3 + (-9)x^2 + (-5)x^1 + (-1)x^0 \\ p(x) - q(x) &= (1)x^5 + (3)x^4 + (3)x^3 + (11)x^2 + (1)x^1 + (7)x^0 \\ p(x) - q(x) &= x^5 + 3x^4 + 3x^3 + 11x^2 + x + 7 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -8x^2 + 7x + 2$$
$$b(x) = 2x - 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-8x^2$	$7x$	2
$2x$	$-16x^3$	$14x^2$	$4x$
-3	$24x^2$	$-21x$	-6

$$a(x) \cdot b(x) = -16x^3 + 14x^2 + 24x^2 + 4x - 21x - 6$$

Combine like terms.

$$a(x) \cdot b(x) = -16x^3 + 38x^2 - 17x - 6$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 3x^2 - 27x - 2 \\ g(x) &= x + 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 3 & -27 & -2 \\ -7 & & -7 & 28 & -7 \\ \hline & 1 & -4 & 1 & -9 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 - 4x + 1 + \frac{-9}{x+7}$$

In other words, $h(x) = x^2 - 4x + 1$ and the remainder is $R = -9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 3x^2 - 27x - 2$. Evaluate $f(-7)$.

You could do this the hard way.

$$\begin{aligned} f(-7) &= (1)\cdot(-7)^3 \ + \ (3)\cdot(-7)^2 \ + \ (-27)\cdot(-7) \ + \ (-2) \\ &= (1)\cdot(-343) \ + \ (3)\cdot(49) \ + \ (-27)\cdot(-7) \ + \ (-2) \\ &= (-343) \ + \ (147) \ + \ (189) \ + \ (-2) \\ &= -9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-7)$ equals the remainder when $f(x)$ is divided by $x + 7$. Thus, $f(-7) = -9$.