In terms of  $\alpha$  and  $\beta$ , express all the lengths and other angle measures ( $\theta$  and  $\phi$ ).



| Variable   | Algebraic expression |
|------------|----------------------|
| p =        |                      |
| q =        |                      |
| r =        |                      |
| s =        |                      |
| $\theta =$ |                      |
| t =        |                      |
| u =        |                      |
| $\phi =$   |                      |
| v =        |                      |
| w =        |                      |

The angle-sum and angle-difference identities are listed below:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

You know the following:

$$\sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\sin(120^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(120^\circ) = \frac{-1}{2}$$

Determine  $\cos(165^{\circ})$  exactly.

Prove the (sine) double-angle identity:  $\sin(2x) = 2\sin(x)\cos(x)$ 

(Hint: start with an angle-sum formula from Question 2.)

## Question 4

Prove the (cosine) double-angle identity:  $cos(2x) = 2cos^2(x) - 1$ 

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity:  $\sin^2(x) + \cos^2(x) = 1$ )

Prove the (cosine) half-angle identity:  $\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1+\cos(y)}{2}}$ .

 $({\it Hint: start with the double-angle identity from \ Question \ 4.})$ 

## Question 6

Given  $\cos(80^\circ) \approx 0.17$ , what is  $\cos(40^\circ)$ ? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: 80/2 = 40.)