Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 210)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= x^5 - 10x^4 + 6x^3 + 7x^2 + 9 \\ q(x) &= -2x^5 + 4x^4 - 9x^3 + 6x - 8 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (1)x^5 + (-10)x^4 + (6)x^3 + (7)x^2 + (0)x^1 + (9)x^0 \\ q(x) &= (-2)x^5 + (4)x^4 + (-9)x^3 + (0)x^2 + (6)x^1 + (-8)x^0 \\ q(x) - p(x) &= (-3)x^5 + (14)x^4 + (-15)x^3 + (-7)x^2 + (6)x^1 + (-17)x^0 \\ q(x) - p(x) &= -3x^5 + 14x^4 - 15x^3 - 7x^2 + 6x - 17 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 5x^2 - 4x + 2 \\ b(x) &= -8x + 6 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$-4x$	2
$-8x$	$-40x^3$	$32x^2$	$-16x$
6	$30x^2$	$-24x$	12

$$a(x) \cdot b(x) = -40x^3 + 32x^2 + 30x^2 - 16x - 24x + 12$$

Combine like terms.

$$a(x) \cdot b(x) = -40x^3 + 62x^2 - 40x + 12$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 210)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 16x^2 + 18x + 26 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & -16 & 18 & 26 \\ -6 & & 18 & -12 & -36 \\ \hline & -3 & 2 & 6 & -10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 2x + 6 + \frac{-10}{x+6}$$

In other words, $h(x) = -3x^2 + 2x + 6$ and the remainder is $R = -10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 16x^2 + 18x + 26$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-3)\cdot(-6)^3 \;+\; (-16)\cdot(-6)^2 \;+\; (18)\cdot(-6) \;+\; (26) \\ &= (-3)\cdot(-216) \;+\; (-16)\cdot(36) \;+\; (18)\cdot(-6) \;+\; (26) \\ &= (648) \;+\; (-576) \;+\; (-108) \;+\; (26) \\ &= -10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x+6$. Thus, $f(-6) = -10$.