Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 205)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 4x^5 - x^4 + 9x^2 + 5x - 8$$

$$q(x) = -3x^5 - 10x^4 - 9x^3 + 2x + 6$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (4)x^5 + (-1)x^4 + (0)x^3 + (9)x^2 + (5)x^1 + (-8)x^0 \\ q(x) &= (-3)x^5 + (-10)x^4 + (-9)x^3 + (0)x^2 + (2)x^1 + (6)x^0 \\ p(x) + q(x) &= (1)x^5 + (-11)x^4 + (-9)x^3 + (9)x^2 + (7)x^1 + (-2)x^0 \\ p(x) + q(x) &= x^5 - 11x^4 - 9x^3 + 9x^2 + 7x - 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 + 8x + 3$$

$$b(x) = -2x + 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$8x$	3
$-2x$	$-10x^3$	$-16x^2$	$-6x$
8	$40x^2$	$64x$	24

$$a(x) \cdot b(x) = -10x^3 - 16x^2 + 40x^2 - 6x + 64x + 24$$

Combine like terms.

$$a(x) \cdot b(x) = -10x^3 + 24x^2 + 58x + 24$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 205)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 - 28x^2 + 10x - 15 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -5 & -28 & 10 & -15 \\ -6 & & 30 & -12 & 12 \\ \hline & -5 & 2 & -2 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -5x^2 + 2x - 2 + \frac{-3}{x+6}$$

In other words, $h(x) = -5x^2 + 2x - 2$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 - 28x^2 + 10x - 15$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-5)\cdot(-6)^3 \ + \ (-28)\cdot(-6)^2 \ + \ (10)\cdot(-6) \ + \ (-15) \\ &= (-5)\cdot(-216) \ + \ (-28)\cdot(36) \ + \ (10)\cdot(-6) \ + \ (-15) \\ &= (1080) \ + \ (-1008) \ + \ (-60) \ + \ (-15) \\ &= \ -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x+6$. Thus, $f(-6) = -3$.