Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 218)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -4x^5 + 5x^4 - 9x^3 - 7x - 3$$

$$q(x) = 3x^5 - 6x^4 + 8x^2 + 2x + 1$$

Express the sum of $p(x) + q(x)$ in standard form.

Get “unsimplified” forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$p(x) = (-4)x^5 + (5)x^4 + (-9)x^3 + (0)x^2 + (-7)x^1 + (-3)x^0$$
$$q(x) = (3)x^5 + (-6)x^4 + (0)x^3 + (8)x^2 + (2)x^1 + (1)x^0$$
$$p(x) + q(x) = (-1)x^5 + (-1)x^4 + (-9)x^3 + (8)x^2 + (-5)x^1 + (-2)x^0$$
$$p(x) + q(x) = -x^5 - x^4 - 9x^3 + 8x^2 - 5x - 2$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 + 9x - 3$$

$$b(x) = 2x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$9x$	-3
$2x$	$-12x^3$	$18x^2$	$-6x$
-7	$42x^2$	$-63x$	21

$$a(x) \cdot b(x) = -12x^3 + 18x^2 + 42x^2 - 6x - 63x + 21$$

Combine like terms.

$$a(x) \cdot b(x) = -12x^3 + 60x^2 - 69x + 21$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal’s triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 218)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 + 16x^2 + 17x - 6 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -5 & 16 & 17 & -6 \\ 4 & & -20 & -16 & 4 \\ \hline & -5 & -4 & 1 & -2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -5x^2 - 4x + 1 + \frac{-2}{x-4}$$

In other words, $h(x) = -5x^2 - 4x + 1$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 + 16x^2 + 17x - 6$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (-5)\cdot(4)^3 \;+\; (16)\cdot(4)^2 \;+\; (17)\cdot(4) \;+\; (-6) \\ &= (-5)\cdot(64) \;+\; (16)\cdot(16) \;+\; (17)\cdot(4) \;+\; (-6) \\ &= (-320) \;+\; (256) \;+\; (68) \;+\; (-6) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -2$.