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## s17 Geometric Series Exam (Solution v27)

### Question 1

Consider the partial geometric series represented below with first term  $a = 510$ , common ratio  $r = \left(\frac{33}{85}\right)^{1/10}$ , and  $n = 10$  terms.

$$S = 510 + 463.96 + 422.07 + 383.97 + 349.31 + 317.77 + 289.09 + 262.99 + 239.25 + 217.65$$

We can multiply both sides by  $r$ .

$$rS = 463.96 + 422.07 + 383.97 + 349.31 + 317.77 + 289.09 + 262.99 + 239.25 + 217.65 + 198$$

What is the value of  $S - rS$ ?

Most terms cancel.

$$510 - 198 = 312$$

### Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 4 + 4(3) + 4(3)^2 + 4(3)^3 + \cdots + 4(3)^{49} + 4(3)^{50} + 4(3)^{51} + 4(3)^{52}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 4$$

$$\text{common ratio} = r = 3$$

$$\text{number of terms} = n = 53$$

### Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

### Definitions

$a$  = first term

$r$  = common ratio

$n$  = number of terms

$S$  = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by  $r$ .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out  $S$  from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by  $(1 - r)$ . We technically need to enforce  $r \neq 1$  as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$