Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 108)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -7x^5 - 6x^4 + 5x^3 - 8x - 9$$

$$q(x) = 9x^5 - 6x^4 - 4x^2 - 8x - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-7)x^5 + (-6)x^4 + (5)x^3 + (0)x^2 + (-8)x^1 + (-9)x^0 \\ q(x) &= (9)x^5 + (-6)x^4 + (0)x^3 + (-4)x^2 + (-8)x^1 + (-5)x^0 \\ p(x) - q(x) &= (-16)x^5 + (0)x^4 + (5)x^3 + (4)x^2 + (0)x^1 + (-4)x^0 \\ p(x) - q(x) &= -16x^5 + 5x^3 + 4x^2 - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 6x^2 - 5x - 9$$

$$b(x) = 2x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$6x^2$	$-5x$	-9
$2x$	$12x^3$	$-10x^2$	$-18x$
4	$24x^2$	$-20x$	-36

$$a(x) \cdot b(x) = 12x^3 - 10x^2 + 24x^2 - 18x - 20x - 36$$

Combine like terms.

$$a(x) \cdot b(x) = 12x^3 + 14x^2 - 38x - 36$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTION (version 108)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 16x^2 + 7x - 25 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & -16 & 7 & -25 \\ -6 & & 18 & -12 & 30 \\ \hline & -3 & 2 & -5 & 5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 2x - 5 + \frac{5}{x+6}$$

In other words, $h(x) = -3x^2 + 2x - 5$ and the remainder is $R = 5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 16x^2 + 7x - 25$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (-3)\cdot(-6)^3 \ + \ (-16)\cdot(-6)^2 \ + \ (7)\cdot(-6) \ + \ (-25) \\ &= (-3)\cdot(-216) \ + \ (-16)\cdot(36) \ + \ (7)\cdot(-6) \ + \ (-25) \\ &= (648) \ + \ (-576) \ + \ (-42) \ + \ (-25) \\ &= 5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 5$.