Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 231)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 10x^5 - 9x^4 + 5x^3 - 2x - 3$$

$$q(x) = -6x^5 - x^4 - 2x^3 + 3x^2 - 9$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (10)x^5 + (-9)x^4 + (5)x^3 + (0)x^2 + (-2)x^1 + (-3)x^0 \\ q(x) &= (-6)x^5 + (-1)x^4 + (-2)x^3 + (3)x^2 + (0)x^1 + (-9)x^0 \\ p(x) + q(x) &= (4)x^5 + (-10)x^4 + (3)x^3 + (3)x^2 + (-2)x^1 + (-12)x^0 \\ p(x) + q(x) &= 4x^5 - 10x^4 + 3x^3 + 3x^2 - 2x - 12 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 7x^2 + 8x - 4$$

$$b(x) = 7x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$7x^2$	$8x$	-4
$7x$	$49x^3$	$56x^2$	$-28x$
-5	$-35x^2$	$-40x$	20

$$a(x) \cdot b(x) = 49x^3 + 56x^2 - 35x^2 - 28x - 40x + 20$$

Combine like terms.

$$a(x) \cdot b(x) = 49x^3 + 21x^2 - 68x + 20$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 231)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 22x^2 + 19x + 18 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & -22 & 19 & 18 \\ -8 & & 24 & -16 & -24 \\ \hline & -3 & 2 & 3 & -6 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 2x + 3 + \frac{-6}{x+8}$$

In other words, $h(x) = -3x^2 + 2x + 3$ and the remainder is $R = -6$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 22x^2 + 19x + 18$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-3)\cdot(-8)^3 \ + \ (-22)\cdot(-8)^2 \ + \ (19)\cdot(-8) \ + \ (18) \\ &= (-3)\cdot(-512) \ + \ (-22)\cdot(64) \ + \ (19)\cdot(-8) \ + \ (18) \\ &= (1536) \ + \ (-1408) \ + \ (-152) \ + \ (18) \\ &= -6 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = -6$.