Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 115)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 9x^5 + 2x^4 - 5x^3 + 7x + 8$$

$$q(x) = -10x^5 + 5x^4 - 8x^2 + 4x + 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (9)x^5 + (2)x^4 + (-5)x^3 + (0)x^2 + (7)x^1 + (8)x^0 \\ q(x) &= (-10)x^5 + (5)x^4 + (0)x^3 + (-8)x^2 + (4)x^1 + (7)x^0 \\ p(x) - q(x) &= (19)x^5 + (-3)x^4 + (-5)x^3 + (8)x^2 + (3)x^1 + (1)x^0 \\ p(x) - q(x) &= 19x^5 - 3x^4 - 5x^3 + 8x^2 + 3x + 1 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 + 9x - 5$$

$$b(x) = -5x - 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$9x$	-5
$-5x$	$-10x^3$	$-45x^2$	$25x$
-2	$-4x^2$	$-18x$	10

$$a(x) \cdot b(x) = -10x^3 - 45x^2 - 4x^2 + 25x - 18x + 10$$

Combine like terms.

$$a(x) \cdot b(x) = -10x^3 - 49x^2 + 7x + 10$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 + 12x^2 + 27x + 3 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & 12 & 27 & 3 \\ -9 & & -9 & -27 & 0 \\ \hline & 1 & 3 & 0 & 3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 3x + \frac{3}{x+9}$$

In other words, $h(x) = x^2 + 3x$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 + 12x^2 + 27x + 3$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (1) \cdot (-9)^3 \ + \ (12) \cdot (-9)^2 \ + \ (27) \cdot (-9) \ + \ (3) \\ &= (1) \cdot (-729) \ + \ (12) \cdot (81) \ + \ (27) \cdot (-9) \ + \ (3) \\ &= (-729) \ + \ (972) \ + \ (-243) \ + \ (3) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x + 9$. Thus, $f(-9) = 3$.