Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 234)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 2x^5 + 9x^3 - 5x^2 - x + 4$$
$$q(x) = -2x^5 - 8x^4 + x^3 + 3x + 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (2)x^5 + (0)x^4 + (9)x^3 + (-5)x^2 + (-1)x^1 + (4)x^0 \\
q(x) &= (-2)x^5 + (-8)x^4 + (1)x^3 + (0)x^2 + (3)x^1 + (7)x^0 \\
p(x) - q(x) &= (4)x^5 + (8)x^4 + (8)x^3 + (-5)x^2 + (-4)x^1 + (-3)x^0 \\
p(x) - q(x) &= 4x^5 + 8x^4 + 8x^3 - 5x^2 - 4x - 3
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 - 2x - 8$$
$$b(x) = -2x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-2x$	-8
$-2x$	$6x^3$	$4x^2$	$16x$
-7	$21x^2$	$14x$	56

$$a(x) \cdot b(x) = 6x^3 + 4x^2 + 21x^2 + 16x + 14x + 56$$

Combine like terms.

$$a(x) \cdot b(x) = 6x^3 + 25x^2 + 30x + 56$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 234)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 + 23x^2 + 6x + 18 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-5	23	6	18
5		-25	-10	-20
	-5	-2	-4	-2

So,

$$\frac{f(x)}{g(x)} = -5x^2 - 2x - 4 + \frac{-2}{x-5}$$

In other words, $h(x) = -5x^2 - 2x - 4$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 + 23x^2 + 6x + 18$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (-5)\cdot(5)^3 \ + \ (23)\cdot(5)^2 \ + \ (6)\cdot(5) \ + \ (18) \\ &= (-5)\cdot(125) \ + \ (23)\cdot(25) \ + \ (6)\cdot(5) \ + \ (18) \\ &= (-625) \ + \ (575) \ + \ (30) \ + \ (18) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x - 5$. Thus, $f(5) = -2$.