Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 110)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -3x^5 + 9x^4 - 8x^3 - 7x^2 + 1$$

$$q(x) = 10x^5 - x^4 - 9x^2 - 5x + 4$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (-3)x^5 + (9)x^4 + (-8)x^3 + (-7)x^2 + (0)x^1 + (1)x^0 \\
q(x) &= (10)x^5 + (-1)x^4 + (0)x^3 + (-9)x^2 + (-5)x^1 + (4)x^0 \\
p(x) - q(x) &= (-13)x^5 + (10)x^4 + (-8)x^3 + (2)x^2 + (5)x^1 + (-3)x^0 \\
p(x) - q(x) &= -13x^5 + 10x^4 - 8x^3 + 2x^2 + 5x - 3
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 - 3x + 5$$

$$b(x) = 3x - 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$-3x$	5
$3x$	$-21x^3$	$-9x^2$	$15x$
-4	$28x^2$	$12x$	-20

$$a(x) \cdot b(x) = -21x^3 - 9x^2 + 28x^2 + 15x + 12x - 20$$

Combine like terms.

$$a(x) \cdot b(x) = -21x^3 + 19x^2 + 27x - 20$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 - 19x^2 + 23x + 9 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & -19 & 23 & 9 \\ 8 & & 16 & -24 & -8 \\ \hline & 2 & -3 & -1 & 1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 - 3x - 1 + \frac{1}{x-8}$$

In other words, $h(x) = 2x^2 - 3x - 1$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 - 19x^2 + 23x + 9$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (2)\cdot(8)^3 \ + \ (-19)\cdot(8)^2 \ + \ (23)\cdot(8) \ + \ (9) \\ &= (2)\cdot(512) \ + \ (-19)\cdot(64) \ + \ (23)\cdot(8) \ + \ (9) \\ &= (1024) \ + \ (-1216) \ + \ (184) \ + \ (9) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 1$.