Instantaneous Slope Writeup

Due 2023 Oct 11

Slope of a secant line

A secant line connects two points along a function. The slope is called the average rate of change. A function f over an interval $[x_a, x_b]$ has an average rate of change equal to the quotient of rise over run.

$$\text{averate rate of change} = \frac{f(x_b) - f(x_a)}{x_b - x_a}$$

Let's say a ball's height h in meters is a function of time t in seconds. It is dropped from a tower 100 meters tall, so it's position is approximately quadratic in time during the free fall.

$$h(t) = 100 - 10t^2$$

1. Find the average rate of change between $t_a = 1$ and $t_b = 2$. *Please give appropriate units and explain its meaning in context.*

"Instantaneous" rate of change

Another way to write the average-rate-of-change formula uses $\Delta x = x_b - x_a$.

$$\text{AROC} = \frac{f(x_a + \Delta x) - f(x_a)}{\Delta x}$$

2. Going back to the falling ball, with $h(t) = 100 - 10t^2$, find the average rate of change when $t_a = 2$ and $\Delta t = 0.01$. *Please give appropriate units and explain its meaning in context.*

Desmos

On Desmos graphing calculator, enter the following expressions.

$$h(t) = 100 - 10t^2$$

$$t_a = 2$$

$$D = 0.01$$

$$m = \frac{h(t_a + D) - h(t_a)}{D}$$

$$y = m \cdot (t - t_a) + h(t_a)$$

Fill out the table:

t_a	m (an instantaneous rate of change)
0.0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	

3. Summarize the results. *Please give appropriate units and explain the results in context.*