Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 247)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -9x^5 - 4x^4 - 8x^3 + 5x + 6$$

$$q(x) = -x^5 + 3x^4 + 6x^3 + 9x^2 - 10$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-9)x^5 + (-4)x^4 + (-8)x^3 + (0)x^2 + (5)x^1 + (6)x^0 \\ q(x) &= (-1)x^5 + (3)x^4 + (6)x^3 + (9)x^2 + (0)x^1 + (-10)x^0 \\ p(x) - q(x) &= (-8)x^5 + (-7)x^4 + (-14)x^3 + (-9)x^2 + (5)x^1 + (16)x^0 \\ p(x) - q(x) &= -8x^5 - 7x^4 - 14x^3 - 9x^2 + 5x + 16 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -4x^2 - 3x + 7$$

$$b(x) = -9x - 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-4x^2$	$-3x$	7
$-9x$	$36x^3$	$27x^2$	$-63x$
-8	$32x^2$	$24x$	-56

$$a(x) \cdot b(x) = 36x^3 + 27x^2 + 32x^2 - 63x + 24x - 56$$

Combine like terms.

$$a(x) \cdot b(x) = 36x^3 + 59x^2 - 39x - 56$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 247)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 9x^2 - 5x + 28 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-1	-9	-5	28
-8		8	8	-24
	-1	-1	3	4

So,

$$\frac{f(x)}{g(x)} = -x^2 - x + 3 + \frac{4}{x+8}$$

In other words, $h(x) = -x^2 - x + 3$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 9x^2 - 5x + 28$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-1)\cdot(-8)^3 \ + \ (-9)\cdot(-8)^2 \ + \ (-5)\cdot(-8) \ + \ (28) \\ &= (-1)\cdot(-512) \ + \ (-9)\cdot(64) \ + \ (-5)\cdot(-8) \ + \ (28) \\ &= (512) \ + \ (-576) \ + \ (40) \ + \ (28) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = 4$.