Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 126)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 5x^5 - 7x^4 - 2x^2 + 10x + 4$$

$$q(x) = 10x^5 - 7x^4 - 9x^3 - 8x^2 + 6$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (5)x^5 + (-7)x^4 + (0)x^3 + (-2)x^2 + (10)x^1 + (4)x^0 \\ q(x) &= (10)x^5 + (-7)x^4 + (-9)x^3 + (-8)x^2 + (0)x^1 + (6)x^0 \\ p(x) - q(x) &= (-5)x^5 + (0)x^4 + (9)x^3 + (6)x^2 + (10)x^1 + (-2)x^0 \\ p(x) - q(x) &= -5x^5 + 9x^3 + 6x^2 + 10x - 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 - 8x + 2$$

$$b(x) = 4x + 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$-8x$	2
$4x$	$20x^3$	$-32x^2$	$8x$
8	$40x^2$	$-64x$	16

$$a(x) \cdot b(x) = 20x^3 - 32x^2 + 40x^2 + 8x - 64x + 16$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 + 8x^2 - 56x + 16$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 126)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 17x^2 + 27x - 9 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	2	17	27	-9
-6		-12	-30	18
	2	5	-3	9

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 5x - 3 + \frac{9}{x+6}$$

In other words, $h(x) = 2x^2 + 5x - 3$ and the remainder is $R = 9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 17x^2 + 27x - 9$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (2) \cdot (-6)^3 \;+\; (17) \cdot (-6)^2 \;+\; (27) \cdot (-6) \;+\; (-9) \\ &= (2) \cdot (-216) \;+\; (17) \cdot (36) \;+\; (27) \cdot (-6) \;+\; (-9) \\ &= (-432) \;+\; (612) \;+\; (-162) \;+\; (-9) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x + 6$. Thus, $f(-6) = 9$.