Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 19)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -6x^5 - 5x^3 - 2x^2 - x + 9$$

$$q(x) = -4x^5 - 10x^4 - 7x^2 - 5x + 9$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-6)x^5 + (0)x^4 + (-5)x^3 + (-2)x^2 + (-1)x^1 + (9)x^0 \\ q(x) &= (-4)x^5 + (-10)x^4 + (0)x^3 + (-7)x^2 + (-5)x^1 + (9)x^0 \\ q(x) - p(x) &= (2)x^5 + (-10)x^4 + (5)x^3 + (-5)x^2 + (-4)x^1 + (0)x^0 \\ q(x) - p(x) &= 2x^5 - 10x^4 + 5x^3 - 5x^2 - 4x \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 - 5x + 8$$

$$b(x) = 3x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-5x$	8
$3x$	$-18x^3$	$-15x^2$	$24x$
7	$-42x^2$	$-35x$	56

$$a(x) \cdot b(x) = -18x^3 - 15x^2 - 42x^2 + 24x - 35x + 56$$

Combine like terms.

$$a(x) \cdot b(x) = -18x^3 - 57x^2 - 11x + 56$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 10x^2 + 26x - 4 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 1 & -10 & 26 & -4 \\ 6 & & 6 & -24 & 12 \\ \hline & 1 & -4 & 2 & 8 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 - 4x + 2 + \frac{8}{x-6}$$

In other words, $h(x) = x^2 - 4x + 2$ and the remainder is $R = 8$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 10x^2 + 26x - 4$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (1)\cdot(6)^3 \ + \ (-10)\cdot(6)^2 \ + \ (26)\cdot(6) \ + \ (-4) \\ &= (1)\cdot(216) \ + \ (-10)\cdot(36) \ + \ (26)\cdot(6) \ + \ (-4) \\ &= (216) \ + \ (-360) \ + \ (156) \ + \ (-4) \\ &= 8 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = 8$.