Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 233)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 - 5x^4 - 3x^2 + 7x - 2$$

$$q(x) = -7x^5 - 3x^4 + 2x^3 - 10x^2 + 5$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (-5)x^4 + (0)x^3 + (-3)x^2 + (7)x^1 + (-2)x^0 \\ q(x) &= (-7)x^5 + (-3)x^4 + (2)x^3 + (-10)x^2 + (0)x^1 + (5)x^0 \\ p(x) + q(x) &= (1)x^5 + (-8)x^4 + (2)x^3 + (-13)x^2 + (7)x^1 + (3)x^0 \\ p(x) + q(x) &= x^5 - 8x^4 + 2x^3 - 13x^2 + 7x + 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 8x^2 - 7x + 9$$

$$b(x) = 5x - 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$8x^2$	$-7x$	9
$5x$	$40x^3$	$-35x^2$	$45x$
-2	$-16x^2$	$14x$	-18

$$a(x) \cdot b(x) = 40x^3 - 35x^2 - 16x^2 + 45x + 14x - 18$$

Combine like terms.

$$a(x) \cdot b(x) = 40x^3 - 51x^2 + 59x - 18$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 233)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 7x^2 + 18x - 10 \\ g(x) &= x + 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & -7 & 18 & -10 \\ -9 & & 9 & -18 & 0 \\ \hline & -1 & 2 & 0 & -10 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 + 2x + \frac{-10}{x+9}$$

In other words, $h(x) = -x^2 + 2x$ and the remainder is $R = -10$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 7x^2 + 18x - 10$. Evaluate $f(-9)$.

You could do this the hard way.

$$\begin{aligned} f(-9) &= (-1)\cdot(-9)^3 \ + \ (-7)\cdot(-9)^2 \ + \ (18)\cdot(-9) \ + \ (-10) \\ &= (-1)\cdot(-729) \ + \ (-7)\cdot(81) \ + \ (18)\cdot(-9) \ + \ (-10) \\ &= (729) \ + \ (-567) \ + \ (-162) \ + \ (-10) \\ &= -10 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-9)$ equals the remainder when $f(x)$ is divided by $x+9$. Thus, $f(-9) = -10$.