Name:

Date:

s17 Geometric Series Exam (SLTN v303)

Question 1

Consider the partial geometric series represented below with first term a = 795, common ratio $r = \left(\frac{2}{5}\right)^{1/10}$, and n = 10 terms.

$$S \ = \ 795 + 725.39 + 661.88 + 603.93 + 551.05 + 502.8 + 458.78 + 418.61 + 381.96 + 348.51$$

We can multiply both sides by r.

$$rS = 725.39 + 661.88 + 603.93 + 551.05 + 502.8 + 458.78 + 418.61 + 381.96 + 348.51 + 318$$

What is the value of S - rS?

Most terms cancel.

$$795 - 318 = 477$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 7 + 7(8) + 7(8)^{2} + 7(8)^{3} + \dots + 7(8)^{67} + 7(8)^{68} + 7(8)^{69} + 7(8)^{70}$$

Identify the initial term, the common ratio, and the number of terms.

first term =
$$a = 7$$

common ratio =
$$r = 8$$

number of terms =
$$n = 71$$

Question 3

Write a proof for the partial geometric series formula.

- a. Define the variables.
- b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a =first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^{2} + ar^{3} + \dots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1-r) = a - ar^n$$

Divide both sides by (1-r). We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$