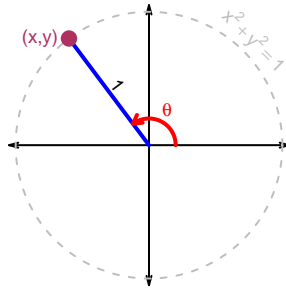


Unit-Circle Trigonometry Cheat Sheet



Definitions

$$\sin(\theta) = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} = \text{slope}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$|\sin(\theta)| = \sqrt{1 - \cos^2(\theta)}$$

$$|\cos(\theta)| = \sqrt{1 - \sin^2(\theta)}$$

$$\tan^2(\theta) + 1 = \frac{1}{\cos^2(\theta)}$$

$$|\tan(\theta)| = \sqrt{\frac{1 - \cos^2(\theta)}{\cos^2(\theta)}}$$

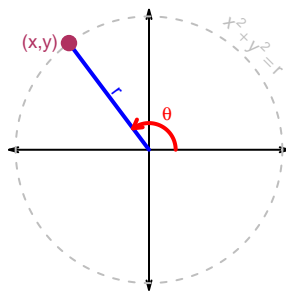
$$|\cos(\theta)| = \sqrt{\frac{1}{\tan^2(\theta) + 1}}$$

$$\tan^2(\theta) + 1 = \frac{1}{1 - \sin^2(\theta)}$$

$$|\tan(\theta)| = \sqrt{\frac{\sin^2(\theta)}{1 - \sin^2(\theta)}}$$

$$|\sin(\theta)| = \sqrt{\frac{\tan^2(\theta)}{\tan^2(\theta) + 1}}$$

Polar Coordinates

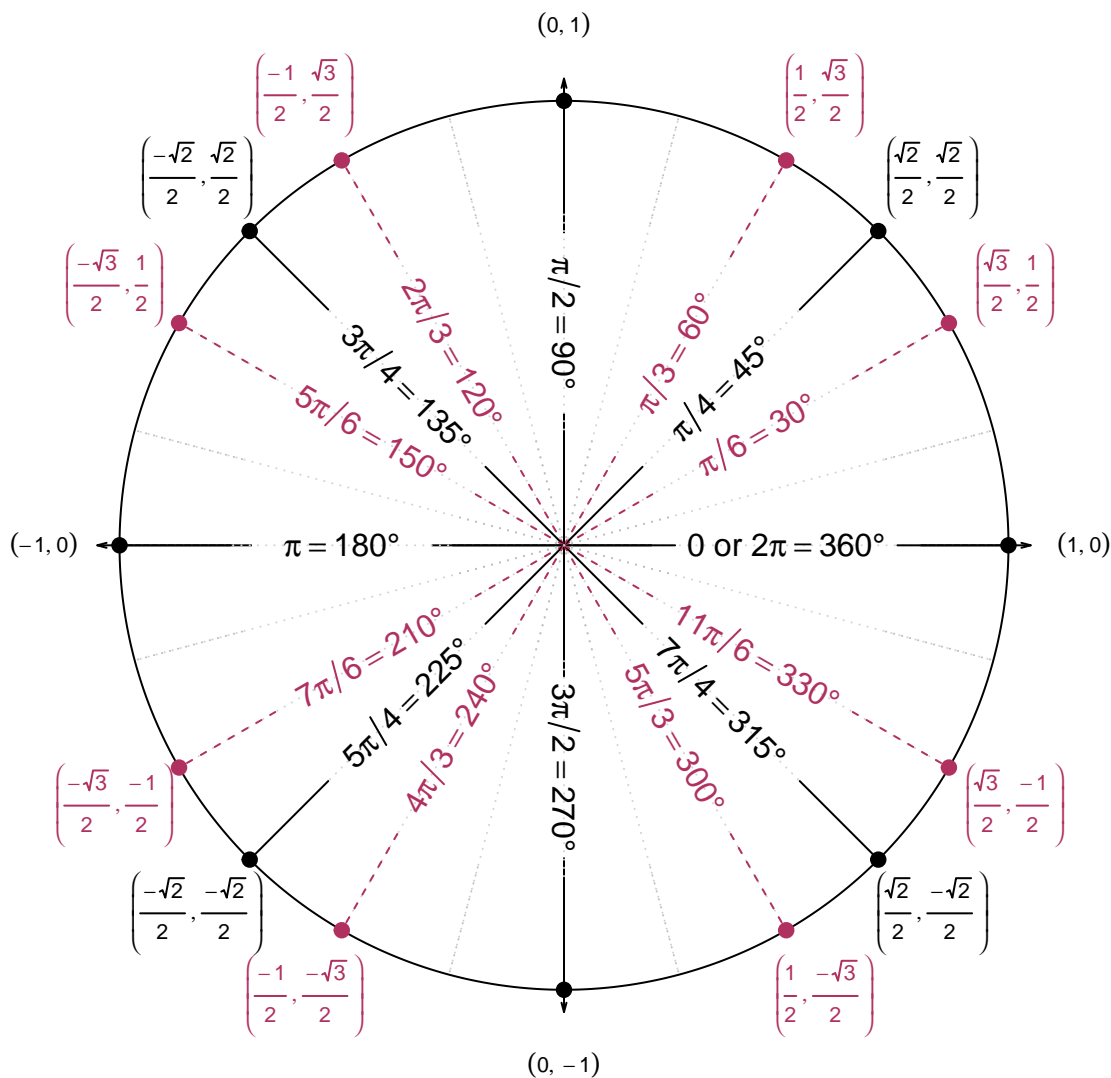


$$x = r \cdot \cos(\theta)$$

$$y = r \cdot \sin(\theta)$$

Special angles

- Draw an isosceles right triangle with a hypotenuse of length 1 and leg length of x . Solve $x^2 + x^2 = 1^2$ to prove length ratios of $\frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1$ for the $45^\circ - 45^\circ - 90^\circ$ triangle.
- Draw an equilateral triangle, and cut it in half to produce a right triangle with a hypotenuse of length 1, a leg of length $1/2$, and another leg of length x . Solve $x^2 + (\frac{1}{2})^2 = 1^2$ to prove length ratios of $\frac{1}{2} : \frac{\sqrt{3}}{2} : 1$ for the $30^\circ - 60^\circ - 90^\circ$ triangle..
- See the [right-triangle cheat sheet](#) for diagrams.
- Use symmetry of the unit circle to determine all coordinates shown below.



So, for example:

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \qquad \tan\left(\frac{2\pi}{3}\right) = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = -\sqrt{3}$$