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## s17 Geometric Series Exam (Solution v5)

### Question 1

Consider the partial geometric series represented below with first term  $a = 847$ , common ratio  $r = \left(\frac{65}{77}\right)^{1/10}$ , and  $n = 10$  terms.

$$S = 847 + 832.77 + 818.78 + 805.03 + 791.5 + 778.21 + 765.13 + 752.28 + 739.64 + 727.22$$

We can multiply both sides by  $r$ .

$$rS = 832.77 + 818.78 + 805.03 + 791.5 + 778.21 + 765.13 + 752.28 + 739.64 + 727.22 + 715$$

What is the value of  $S - rS$ ?

Most terms cancel.

$$847 - 715 = 132$$

### Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 6 + 6(4) + 6(4)^2 + 6(4)^3 + \cdots + 6(4)^{60} + 6(4)^{61} + 6(4)^{62} + 6(4)^{63}$$

Identify the initial term, the common ratio, and the number of terms.

$$\text{first term} = a = 6$$

$$\text{common ratio} = r = 4$$

$$\text{number of terms} = n = 64$$

### Question 3

Write a proof for the partial geometric series formula.

- Define the variables.
- Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- Using annotated algebraic manipulation, produce the partial geometric series formula.

### Definitions

$a$  = first term

$r$  = common ratio

$n$  = number of terms

$S$  = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by  $r$ .

$$rS = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out  $S$  from left side.

$$S(1 - r) = a - ar^n$$

Divide both sides by  $(1 - r)$ . We technically need to enforce  $r \neq 1$  as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$