Name:

Question 1

In terms of α and β, express all the lengths and other angle measures (θ and ϕ).

Variable	Algebraic expression
$p \ =$	
$q \ =$	
$r \ =$	
$s \ =$	
$\theta \ =$	
$t \ =$	
$u \ =$	
$\phi \ =$	
$v \ =$	
$w \ =$	

Question 2

The angle-sum and angle-difference identities are true for any α and β:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

You know the following:

$$\sin(60°) = \frac{\sqrt{3}}{2} \qquad\qquad \sin(45°) = \frac{\sqrt{2}}{2}$$

$$\cos(60°) = \frac{1}{2} \qquad\qquad \cos(45°) = \frac{\sqrt{2}}{2}$$

Determine $\cos(105°)$ exactly.

Question 3

Prove that $\sin(2x) = 2\sin(x)\cos(x)$ for any x.

(Hint: start with an angle-sum formula from Question 2.)

Question 4

Prove that $\cos(2x) = 2\cos^2(x) - 1$ for any x.

(Hint: start with an angle-sum formula from Question 2. Also, you will need to use the Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$)

Question 5

Prove that $\cos\left(\frac{y}{2}\right) = \sqrt{\frac{1+\cos(y)}{2}}$. (Technically this assumes $\cos(y/2) > 0$, but let's not worry about that here.)

(Hint: use the identity proved in Question 4.)

Question 6

If you knew that $\cos(100°) \approx -0.17$, then what is $\cos(50°)$? Please set up an expression, but do **not** try to simplify or evaluate a decimal approximation.

(Hint: $100/2 = 50$.)