Derivation of Quadratic Formula

Begin with a generalized quadratic equation in standard form.

$$ax^2 + bx + c = 0$$

Subtract c from both sides.

$$ax^2 + bx + c - c = -c$$

Simplify left side.

$$ax^2 + bx = -c$$

Divide both sides by a.

$$\frac{ax^2 + bx}{a} = \frac{-c}{a}$$

On the left side, distribute the divisor to both terms in the numerator.

$$\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$$

Simplify the quadratic term.

$$x^2 + \frac{bx}{a} = \frac{-c}{a}$$

Complete the square by adding the square of half the linear coefficient to both sides of the equation.

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Factor the perfect square.

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

On the right side, distribute the exponent to all three factors of the quotient.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

On the right side, made a common denominator by multiplying second term by $\frac{4a}{4a}$.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

On right side, combine fractions.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Undo the squaring.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Subtract $\frac{b}{2a}$ from both sides.

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Distribute the radical over the division.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

Simplify the denominator.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Combine the fractions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$