Name:____________________________________ Date:__________________

Polynomial Operations SOLUTIONS (version 10)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -10x^5 + 7x^4 + 8x^3 - 3x - 2$$

$$q(x) = -8x^5 - 6x^3 - 9x^2 + 3x + 2$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-10)x^5 + (7)x^4 + (8)x^3 + (0)x^2 + (-3)x^1 + (-2)x^0 \\ q(x) &= (-8)x^5 + (0)x^4 + (-6)x^3 + (-9)x^2 + (3)x^1 + (2)x^0 \\ p(x) + q(x) &= (-18)x^5 + (7)x^4 + (2)x^3 + (-9)x^2 + (0)x^1 + (0)x^0 \\ p(x) + q(x) &= -18x^5 + 7x^4 + 2x^3 - 9x^2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -3x^2 - 6x + 9$$

$$b(x) = -3x + 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-3x^2$	$-6x$	9
$-3x$	$9x^3$	$18x^2$	$-27x$
7	$-21x^2$	$-42x$	63

$$a(x) \cdot b(x) = 9x^3 + 18x^2 - 21x^2 - 27x - 42x + 63$$

Combine like terms.

$$a(x) \cdot b(x) = 9x^3 - 3x^2 - 69x + 63$$

3. Express $(x + 1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 + 21x^2 + 25x - 28 \\ g(x) &= x + 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & 21 & 25 & -28 \\ -5 & & -15 & -30 & 25 \\ \hline & 3 & 6 & -5 & -3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 6x - 5 + \frac{-3}{x+5}$$

In other words, $h(x) = 3x^2 + 6x - 5$ and the remainder is $R = -3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 + 21x^2 + 25x - 28$. Evaluate $f(-5)$.

You could do this the hard way.

$$\begin{aligned} f(-5) &= (3)\cdot(-5)^3 \; + \; (21)\cdot(-5)^2 \; + \; (25)\cdot(-5) \; + \; (-28) \\ &= (3)\cdot(-125) \; + \; (21)\cdot(25) \; + \; (25)\cdot(-5) \; + \; (-28) \\ &= (-375) \; + \; (525) \; + \; (-125) \; + \; (-28) \\ &= -3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-5)$ equals the remainder when $f(x)$ is divided by $x + 5$. Thus, $f(-5) = -3$.