Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 9)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= 6x^5 + 8x^4 - 3x^3 - 4x + 7 \\ q(x) &= 3x^5 - 6x^3 + x^2 + 7x + 5 \end{aligned}$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (6)x^5 + (8)x^4 + (-3)x^3 + (0)x^2 + (-4)x^1 + (7)x^0 \\ q(x) &= (3)x^5 + (0)x^4 + (-6)x^3 + (1)x^2 + (7)x^1 + (5)x^0 \\ p(x) - q(x) &= (3)x^5 + (8)x^4 + (3)x^3 + (-1)x^2 + (-11)x^1 + (2)x^0 \\ p(x) - q(x) &= 3x^5 + 8x^4 + 3x^3 - x^2 - 11x + 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 4x^2 - 2x - 8 \\ b(x) &= -2x - 5 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$4x^2$	$-2x$	-8
$-2x$	$-8x^3$	$4x^2$	$16x$
-5	$-20x^2$	$10x$	40

$$a(x) \cdot b(x) = -8x^3 + 4x^2 - 20x^2 + 16x + 10x + 40$$

Combine like terms.

$$a(x) \cdot b(x) = -8x^3 - 16x^2 + 26x + 40$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 6x^3 - 24x^2 - 28x - 17 \\ g(x) &= x - 5 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-5}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 6 & -24 & -28 & -17 \\ 5 & & 30 & 30 & 10 \\ \hline & 6 & 6 & 2 & -7 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 6x^2 + 6x + 2 + \frac{-7}{x-5}$$

In other words, $h(x) = 6x^2 + 6x + 2$ and the remainder is $R = -7$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 6x^3 - 24x^2 - 28x - 17$. Evaluate $f(5)$.

You could do this the hard way.

$$\begin{aligned} f(5) &= (6) \cdot (5)^3 \ + \ (-24) \cdot (5)^2 \ + \ (-28) \cdot (5) \ + \ (-17) \\ &= (6) \cdot (125) \ + \ (-24) \cdot (25) \ + \ (-28) \cdot (5) \ + \ (-17) \\ &= (750) \ + \ (-600) \ + \ (-140) \ + \ (-17) \\ &= -7 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(5)$ equals the remainder when $f(x)$ is divided by $x - 5$. Thus, $f(5) = -7$.