Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 22)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 + x^4 + 3x^2 - 7x + 4$$

$$q(x) = x^5 - 2x^3 + 10x^2 + 9x - 5$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (1)x^4 + (0)x^3 + (3)x^2 + (-7)x^1 + (4)x^0 \\ q(x) &= (1)x^5 + (0)x^4 + (-2)x^3 + (10)x^2 + (9)x^1 + (-5)x^0 \\ p(x) - q(x) &= (7)x^5 + (1)x^4 + (2)x^3 + (-7)x^2 + (-16)x^1 + (9)x^0 \\ p(x) - q(x) &= 7x^5 + x^4 + 2x^3 - 7x^2 - 16x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 + 3x - 9$$

$$b(x) = -5x + 4$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$3x$	-9
$-5x$	$30x^3$	$-15x^2$	$45x$
4	$-24x^2$	$12x$	-36

$$a(x) \cdot b(x) = 30x^3 - 15x^2 - 24x^2 + 45x + 12x - 36$$

Combine like terms.

$$a(x) \cdot b(x) = 30x^3 - 39x^2 + 57x - 36$$

3. Express $(x+1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -4x^3 + 27x^2 - 17x + 3 \\ g(x) &= x - 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & -4 & 27 & -17 & 3 \\ 6 & & -24 & 18 & 6 \\ \hline & -4 & 3 & 1 & 9 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -4x^2 + 3x + 1 + \frac{9}{x-6}$$

In other words, $h(x) = -4x^2 + 3x + 1$ and the remainder is $R = 9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -4x^3 + 27x^2 - 17x + 3$. Evaluate $f(6)$.

You could do this the hard way.

$$\begin{aligned} f(6) &= (-4)\cdot(6)^3 + (27)\cdot(6)^2 + (-17)\cdot(6) + (3) \\ &= (-4)\cdot(216) + (27)\cdot(36) + (-17)\cdot(6) + (3) \\ &= (-864) + (972) + (-102) + (3) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(6)$ equals the remainder when $f(x)$ is divided by $x - 6$. Thus, $f(6) = 9$.