Name: ______________________________ Date: ______________

Polynomial Operations SOLUTIONS (version 31)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -8x^5 + 3x^3 + 6x^2 + x + 10$$
$$q(x) = 2x^5 + x^4 - 6x^3 - 8x^2 + 9$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-8)x^5 + (0)x^4 + (3)x^3 + (6)x^2 + (1)x^1 + (10)x^0 \\ q(x) &= (2)x^5 + (1)x^4 + (-6)x^3 + (-8)x^2 + (0)x^1 + (9)x^0 \\ p(x) - q(x) &= (-10)x^5 + (-1)x^4 + (9)x^3 + (14)x^2 + (1)x^1 + (1)x^0 \\ p(x) - q(x) &= -10x^5 - x^4 + 9x^3 + 14x^2 + x + 1 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -9x^2 - 7x + 3$$
$$b(x) = 3x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-9x^2$	$-7x$	3
$3x$	$-27x^3$	$-21x^2$	$9x$
-7	$63x^2$	$49x$	-21

$$a(x) \cdot b(x) = -27x^3 - 21x^2 + 63x^2 + 9x + 49x - 21$$

Combine like terms.

$$a(x) \cdot b(x) = -27x^3 + 42x^2 + 58x - 21$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 3x^3 - 12x^2 + x - 5 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 3 & -12 & 1 & -5 \\ 4 & & 12 & 0 & 4 \\ \hline & 3 & 0 & 1 & -1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 3x^2 + 1 + \frac{-1}{x-4}$$

In other words, $h(x) = 3x^2 + 1$ and the remainder is $R = -1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 3x^3 - 12x^2 + x - 5$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (3) \cdot (4)^3 \ + \ (-12) \cdot (4)^2 \ + \ (1) \cdot (4) \ + \ (-5) \\ &= (3) \cdot (64) \ + \ (-12) \cdot (16) \ + \ (1) \cdot (4) \ + \ (-5) \\ &= (192) \ + \ (-192) \ + \ (4) \ + \ (-5) \\ &= -1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -1$.