Name: ______________________ Date: ______________

Polynomial Operations SOLUTION (version 204)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -10x^5 + 3x^4 + 2x^3 - 9x + 6$$

$$q(x) = -5x^5 + x^4 + 4x^3 - 7x^2 + 9$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-10)x^5 + (3)x^4 + (2)x^3 + (0)x^2 + (-9)x^1 + (6)x^0 \\ q(x) &= (-5)x^5 + (1)x^4 + (4)x^3 + (-7)x^2 + (0)x^1 + (9)x^0 \\ p(x) - q(x) &= (-5)x^5 + (2)x^4 + (-2)x^3 + (7)x^2 + (-9)x^1 + (-3)x^0 \\ p(x) - q(x) &= -5x^5 + 2x^4 - 2x^3 + 7x^2 - 9x - 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 3x^2 + 2x - 7$$

$$b(x) = -7x - 8$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$3x^2$	$2x$	-7
$-7x$	$-21x^3$	$-14x^2$	$49x$
-8	$-24x^2$	$-16x$	56

$$a(x) \cdot b(x) = -21x^3 - 14x^2 - 24x^2 + 49x - 16x + 56$$

Combine like terms.

$$a(x) \cdot b(x) = -21x^3 - 38x^2 + 33x + 56$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 204)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 + 23x^2 - 9x - 20 \\ g(x) &= x + 6 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+6}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 4 & 23 & -9 & -20 \\ -6 & & -24 & 6 & 18 \\ \hline & 4 & -1 & -3 & -2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 4x^2 - x - 3 + \frac{-2}{x+6}$$

In other words, $h(x) = 4x^2 - x - 3$ and the remainder is $R = -2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 + 23x^2 - 9x - 20$. Evaluate $f(-6)$.

You could do this the hard way.

$$\begin{aligned} f(-6) &= (4)\cdot(-6)^3 \ + \ (23)\cdot(-6)^2 \ + \ (-9)\cdot(-6) \ + \ (-20) \\ &= (4)\cdot(-216) \ + \ (23)\cdot(36) \ + \ (-9)\cdot(-6) \ + \ (-20) \\ &= (-864) \ + \ (828) \ + \ (54) \ + \ (-20) \\ &= -2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-6)$ equals the remainder when $f(x)$ is divided by $x+6$. Thus, $f(-6) = -2$.