Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 211)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$\begin{aligned} p(x) &= -3x^5 + 4x^4 - 2x^3 - 5x^2 - 7 \\ q(x) &= -4x^5 + 7x^4 + 10x^3 + 5x + 1 \end{aligned}$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-3)x^5 + (4)x^4 + (-2)x^3 + (-5)x^2 + (0)x^1 + (-7)x^0 \\ q(x) &= (-4)x^5 + (7)x^4 + (10)x^3 + (0)x^2 + (5)x^1 + (1)x^0 \\ q(x) - p(x) &= (-1)x^5 + (3)x^4 + (12)x^3 + (5)x^2 + (5)x^1 + (8)x^0 \\ q(x) - p(x) &= -x^5 + 3x^4 + 12x^3 + 5x^2 + 5x + 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$\begin{aligned} a(x) &= 5x^2 - 9x + 8 \\ b(x) &= 7x + 4 \end{aligned}$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$-9x$	8
$7x$	$35x^3$	$-63x^2$	$56x$
4	$20x^2$	$-36x$	32

$$a(x) \cdot b(x) = 35x^3 - 63x^2 + 20x^2 + 56x - 36x + 32$$

Combine like terms.

$$a(x) \cdot b(x) = 35x^3 - 43x^2 + 20x + 32$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 211)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -x^3 - 10x^2 - 16x + 1 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -1 & -10 & -16 & 1 \\ -8 & & 8 & 16 & 0 \\ \hline & -1 & -2 & 0 & 1 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -x^2 - 2x + \frac{1}{x+8}$$

In other words, $h(x) = -x^2 - 2x$ and the remainder is $R = 1$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -x^3 - 10x^2 - 16x + 1$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-1)\cdot(-8)^3 + (-10)\cdot(-8)^2 + (-16)\cdot(-8) + (1) \\ &= (-1)\cdot(-512) + (-10)\cdot(64) + (-16)\cdot(-8) + (1) \\ &= (512) + (-640) + (128) + (1) \\ &= 1 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x+8$. Thus, $f(-8) = 1$.