Name: ____________________ Date: ____________

Polynomial Operations SOLUTION (version 157)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -8x^5 - 10x^3 - 7x^2 + x + 2$$

$$q(x) = 3x^5 + 5x^4 - 10x^2 + 2x - 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-8)x^5 + (0)x^4 + (-10)x^3 + (-7)x^2 + (1)x^1 + (2)x^0 \\ q(x) &= (3)x^5 + (5)x^4 + (0)x^3 + (-10)x^2 + (2)x^1 + (-7)x^0 \\ p(x) - q(x) &= (-11)x^5 + (-5)x^4 + (-10)x^3 + (3)x^2 + (-1)x^1 + (9)x^0 \\ p(x) - q(x) &= -11x^5 - 5x^4 - 10x^3 + 3x^2 - x + 9 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 9x^2 - 4x - 5$$

$$b(x) = 2x - 3$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$9x^2$	$-4x$	-5
$2x$	$18x^3$	$-8x^2$	$-10x$
-3	$-27x^2$	$12x$	15

$$a(x) \cdot b(x) = 18x^3 - 8x^2 - 27x^2 - 10x + 12x + 15$$

Combine like terms.

$$a(x) \cdot b(x) = 18x^3 - 35x^2 + 2x + 15$$

3. Express $(x+1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 157)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -5x^3 + 18x^2 + 13x - 25 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-5	18	13	-25
4		-20	-8	20
	-5	-2	5	-5

So,

$$\frac{f(x)}{g(x)} = -5x^2 - 2x + 5 + \frac{-5}{x-4}$$

In other words, $h(x) = -5x^2 - 2x + 5$ and the remainder is $R = -5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -5x^3 + 18x^2 + 13x - 25$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (-5)\cdot(4)^3 \;+\; (18)\cdot(4)^2 \;+\; (13)\cdot(4) \;+\; (-25) \\ &= (-5)\cdot(64) \;+\; (18)\cdot(16) \;+\; (13)\cdot(4) \;+\; (-25) \\ &= (-320) \;+\; (288) \;+\; (52) \;+\; (-25) \\ &= -5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -5$.