

Derivation of Quadratic Formula

Begin with a generalized quadratic equation in standard form.

$$ax^2 + bx + c = 0$$

Subtract c from both sides, and divide both sides by a .

$$x^2 + \frac{bx}{a} = \frac{-c}{a}$$

Complete the square by adding the square of half the linear coefficient to both sides of the equation.

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Factor the perfect square.

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Simplify the right side by distributing the exponent, making common denominator, and combining fractions.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Undo the squaring.

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Subtract $\frac{b}{2a}$ from both sides. Also distribute the radical over the division.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Combine the fractions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$