Name: ____________________ Date: __________

Polynomial Operations SOLUTION (version 153)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 3x^5 - 8x^4 - 9x^3 - 5x - 10$$

$$q(x) = 9x^5 - 10x^4 - 8x^2 - x - 2$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (-8)x^4 + (-9)x^3 + (0)x^2 + (-5)x^1 + (-10)x^0 \\ q(x) &= (9)x^5 + (-10)x^4 + (0)x^3 + (-8)x^2 + (-1)x^1 + (-2)x^0 \\ p(x) - q(x) &= (-6)x^5 + (2)x^4 + (-9)x^3 + (8)x^2 + (-4)x^1 + (-8)x^0 \\ p(x) - q(x) &= -6x^5 + 2x^4 - 9x^3 + 8x^2 - 4x - 8 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -6x^2 - 9x - 3$$

$$b(x) = -5x + 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-6x^2$	$-9x$	-3
$-5x$	$30x^3$	$45x^2$	$15x$
2	$-12x^2$	$-18x$	-6

$$a(x) \cdot b(x) = 30x^3 + 45x^2 - 12x^2 + 15x - 18x - 6$$

Combine like terms.

$$a(x) \cdot b(x) = 30x^3 + 33x^2 - 3x - 6$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 153)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 11x^2 + 28x + 4 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 1 & -11 & 28 & 4 \\ 7 & & 7 & -28 & 0 \\ \hline & 1 & -4 & 0 & 4 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 - 4x + \frac{4}{x-7}$$

In other words, $h(x) = x^2 - 4x$ and the remainder is $R = 4$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 11x^2 + 28x + 4$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (1)\cdot(7)^3 \ + \ (-11)\cdot(7)^2 \ + \ (28)\cdot(7) \ + \ (4) \\ &= (1)\cdot(343) \ + \ (-11)\cdot(49) \ + \ (28)\cdot(7) \ + \ (4) \\ &= (343) \ + \ (-539) \ + \ (196) \ + \ (4) \\ &= 4 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = 4$.