Name:

Date:

s17 Geometric Series Exam (SLTN v398)

Question 1

Consider the partial geometric series represented below with first term a = 779, common ratio $r = \left(\frac{13}{19}\right)^{1/10}$, and n = 10 terms.

$$S \ = \ 779 + 749.99 + 722.06 + 695.18 + 669.29 + 644.37 + 620.37 + 597.27 + 575.03 + 553.62$$

We can multiply both sides by r.

$$rS \ = \ 749.99 + 722.06 + 695.18 + 669.29 + 644.37 + 620.37 + 597.27 + 575.03 + 553.62 + 533$$

What is the value of S - rS?

Most terms cancel.

$$779 - 533 = 246$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 2 + 2(3) + 2(3)^{2} + 2(3)^{3} + \cdots + 2(3)^{51} + 2(3)^{52} + 2(3)^{53} + 2(3)^{54}$$

Identify the initial term, the common ratio, and the number of terms.

first term
$$= a = 2$$

common ratio =
$$r = 3$$

number of terms =
$$n = 55$$

Question 3

Write a proof for the partial geometric series formula.

- a. Define the variables.
- b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a =first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^{2} + ar^{3} + \dots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1-r) = a - ar^n$$

Divide both sides by (1-r). We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$