Name: ____ Date: ____________________

Polynomial Operations SOLUTION (version 249)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 - 9x^3 + x^2 - 8x - 6$$

$$q(x) = 3x^5 - 5x^4 + 10x^3 + 6x + 7$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (0)x^4 + (-9)x^3 + (1)x^2 + (-8)x^1 + (-6)x^0 \\ q(x) &= (3)x^5 + (-5)x^4 + (10)x^3 + (0)x^2 + (6)x^1 + (7)x^0 \\ p(x) - q(x) &= (-5)x^5 + (5)x^4 + (-19)x^3 + (1)x^2 + (-14)x^1 + (-13)x^0 \\ p(x) - q(x) &= -5x^5 + 5x^4 - 19x^3 + x^2 - 14x - 13 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 7x^2 - 4x - 9$$

$$b(x) = 2x - 6$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$7x^2$	$-4x$	-9
$2x$	$14x^3$	$-8x^2$	$-18x$
-6	$-42x^2$	$24x$	54

$$a(x) \cdot b(x) = 14x^3 - 8x^2 - 42x^2 - 18x + 24x + 54$$

Combine like terms.

$$a(x) \cdot b(x) = 14x^3 - 50x^2 + 6x + 54$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 249)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 + 19x^2 - 27x - 9 \\ g(x) &= x - 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & -3 & 19 & -27 & -9 \\ 4 & & -12 & 28 & 4 \\ \hline & -3 & 7 & 1 & -5 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = -3x^2 + 7x + 1 + \frac{-5}{x-4}$$

In other words, $h(x) = -3x^2 + 7x + 1$ and the remainder is $R = -5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 + 19x^2 - 27x - 9$. Evaluate $f(4)$.

You could do this the hard way.

$$\begin{aligned} f(4) &= (-3)\cdot(4)^3 \ + \ (19)\cdot(4)^2 \ + \ (-27)\cdot(4) \ + \ (-9) \\ &= (-3)\cdot(64) \ + \ (19)\cdot(16) \ + \ (-27)\cdot(4) \ + \ (-9) \\ &= (-192) \ + \ (304) \ + \ (-108) \ + \ (-9) \\ &= \ -5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(4)$ equals the remainder when $f(x)$ is divided by $x - 4$. Thus, $f(4) = -5$.