

Name: \_\_\_\_\_

## at1204p\_vertex\_and\_roots... from standard-form quadratic functions (v113)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

### Example

$$f(x) = 6x^2 + 4x - 5$$

#### Example solution

1. Find the axis of symmetry. Use the formula  $h = \frac{-b}{2a}$ , where  $h$  is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)} = \frac{-1}{3}$$

$$\text{axis of symmetry: } x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula  $w = \frac{\sqrt{b^2 - 4ac}}{2a}$ .

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The  $x$ -intercepts can be found by adding  $w$  to or subtracting  $w$  from  $h$ .

$$\left( \frac{-1}{3} - \frac{\sqrt{34}}{6}, 0 \right) \quad \text{and} \quad \left( \frac{-1}{3} + \frac{\sqrt{34}}{6}, 0 \right)$$

4. Find the vertex. We already know  $h = \frac{-1}{3}$ , so we just need  $k$ . Use the formula  $k = \frac{4ac - b^2}{4a}$ .

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex: } \left( \frac{-1}{3}, \frac{-17}{3} \right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 5x^2 - 10x - 2$$

1. Axis of symmetry

$$h = \frac{-(-10)}{2(5)} = 1$$

axis of symmetry:  $x = 1$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-10)^2 - 4(5)(-2)}}{2(5)}$$

$$w = \frac{\sqrt{140}}{10} = \frac{\sqrt{2 \cdot 2 \cdot 5 \cdot 7}}{10} = \frac{2\sqrt{35}}{10}$$

$$w = \frac{\sqrt{35}}{5}$$

3. Roots

$$\left(1 - \frac{\sqrt{35}}{5}, 0\right) \text{ and } \left(1 + \frac{\sqrt{35}}{5}, 0\right)$$

4. Vertex

$$k = \frac{4(5)(-2) - (-10)^2}{4(5)}$$

$$k = \frac{-140}{20} = -7$$

vertex:  $(1, -7)$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 9x^2 - 4x - 3$$

1. Axis of symmetry

$$h = \frac{-(-4)}{2(9)} = \frac{2}{9}$$

$$\text{axis of symmetry: } x = \frac{2}{9}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-4)^2 - 4(9)(-3)}}{2(9)}$$

$$w = \frac{\sqrt{124}}{18} = \frac{\sqrt{2 \cdot 2 \cdot 31}}{18} = \frac{2\sqrt{31}}{18}$$

$$w = \frac{\sqrt{31}}{9}$$

3. Roots

$$\left(\frac{2}{9} - \frac{\sqrt{31}}{9}, 0\right) \quad \text{and} \quad \left(\frac{2}{9} + \frac{\sqrt{31}}{9}, 0\right)$$

4. Vertex

$$k = \frac{4(9)(-3) - (-4)^2}{4(9)}$$

$$k = \frac{-124}{36} = \frac{-31}{9}$$

$$\text{vertex: } \left(\frac{2}{9}, \frac{-31}{9}\right)$$

### Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex  $(h, k)$  shown as cartesian coordinates

Box your answers.

$$f(x) = 4x^2 - 10x - 7$$

1. Axis of symmetry

$$h = \frac{-(-10)}{2(4)} = \frac{5}{4}$$

$$\text{axis of symmetry: } x = \frac{5}{4}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-10)^2 - 4(4)(-7)}}{2(4)}$$

$$w = \frac{\sqrt{212}}{8} = \frac{\sqrt{2 \cdot 2 \cdot 53}}{8} = \frac{2\sqrt{53}}{8}$$

$$w = \frac{\sqrt{53}}{4}$$

3. Roots

$$\left(\frac{5}{4} - \frac{\sqrt{53}}{4}, 0\right) \quad \text{and} \quad \left(\frac{5}{4} + \frac{\sqrt{53}}{4}, 0\right)$$

4. Vertex

$$k = \frac{4(4)(-7) - (-10)^2}{4(4)}$$

$$k = \frac{-212}{16} = \frac{-53}{4}$$

$$\text{vertex: } \left(\frac{5}{4}, \frac{-53}{4}\right)$$