Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 148)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 3x^5 - x^4 + 2x^3 - 6x - 5$$

$$q(x) = 8x^5 - 3x^3 - 5x^2 - 2x - 9$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (3)x^5 + (-1)x^4 + (2)x^3 + (0)x^2 + (-6)x^1 + (-5)x^0 \\ q(x) &= (8)x^5 + (0)x^4 + (-3)x^3 + (-5)x^2 + (-2)x^1 + (-9)x^0 \\ q(x) - p(x) &= (5)x^5 + (1)x^4 + (-5)x^3 + (-5)x^2 + (4)x^1 + (-4)x^0 \\ q(x) - p(x) &= 5x^5 + x^4 - 5x^3 - 5x^2 + 4x - 4 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 + 7x + 6$$

$$b(x) = 7x - 9$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$7x$	6
$7x$	$14x^3$	$49x^2$	$42x$
-9	$-18x^2$	$-63x$	-54

$$a(x) \cdot b(x) = 14x^3 + 49x^2 - 18x^2 + 42x - 63x - 54$$

Combine like terms.

$$a(x) \cdot b(x) = 14x^3 + 31x^2 - 21x - 54$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 148)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 4x^3 - 29x^2 + 4x + 26 \\ g(x) &= x - 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	4	-29	4	26
7		28	-7	-21
	4	-1	-3	5

So,

$$\frac{f(x)}{g(x)} = 4x^2 - x - 3 + \frac{5}{x-7}$$

In other words, $h(x) = 4x^2 - x - 3$ and the remainder is $R = 5$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 4x^3 - 29x^2 + 4x + 26$. Evaluate $f(7)$.

You could do this the hard way.

$$\begin{aligned} f(7) &= (4)\cdot(7)^3 + (-29)\cdot(7)^2 + (4)\cdot(7) + (26) \\ &= (4)\cdot(343) + (-29)\cdot(49) + (4)\cdot(7) + (26) \\ &= (1372) + (-1421) + (28) + (26) \\ &= 5 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(7)$ equals the remainder when $f(x)$ is divided by $x - 7$. Thus, $f(7) = 5$.