Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 146)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 8x^5 + 4x^4 + 10x^3 - 3x^2 + 5$$

$$q(x) = 10x^5 - 4x^4 + x^2 - 9x - 8$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (8)x^5 + (4)x^4 + (10)x^3 + (-3)x^2 + (0)x^1 + (5)x^0 \\ q(x) &= (10)x^5 + (-4)x^4 + (0)x^3 + (1)x^2 + (-9)x^1 + (-8)x^0 \\ p(x) + q(x) &= (18)x^5 + (0)x^4 + (10)x^3 + (-2)x^2 + (-9)x^1 + (-3)x^0 \\ p(x) + q(x) &= 18x^5 + 10x^3 - 2x^2 - 9x - 3 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 4x^2 - 6x + 5$$

$$b(x) = -7x - 9$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$4x^2$	$-6x$	5
$-7x$	$-28x^3$	$42x^2$	$-35x$
-9	$-36x^2$	$54x$	-45

$$a(x) \cdot b(x) = -28x^3 + 42x^2 - 36x^2 - 35x + 54x - 45$$

Combine like terms.

$$a(x) \cdot b(x) = -28x^3 + 6x^2 + 19x - 45$$

3. Express $(x+1)^4$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Polynomial Operations SOLUTION (version 146)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 6x^2 - 26x - 6 \\ g(x) &= x - 9 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-9}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 1 & -6 & -26 & -6 \\ 9 & & 9 & 27 & 9 \\ \hline & 1 & 3 & 1 & 3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 3x + 1 + \frac{3}{x-9}$$

In other words, $h(x) = x^2 + 3x + 1$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 6x^2 - 26x - 6$. Evaluate $f(9)$.

You could do this the hard way.

$$\begin{aligned} f(9) &= (1)\cdot(9)^3 \ + \ (-6)\cdot(9)^2 \ + \ (-26)\cdot(9) \ + \ (-6) \\ &= (1)\cdot(729) \ + \ (-6)\cdot(81) \ + \ (-26)\cdot(9) \ + \ (-6) \\ &= (729) \ + \ (-486) \ + \ (-234) \ + \ (-6) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(9)$ equals the remainder when $f(x)$ is divided by $x - 9$. Thus, $f(9) = 3$.