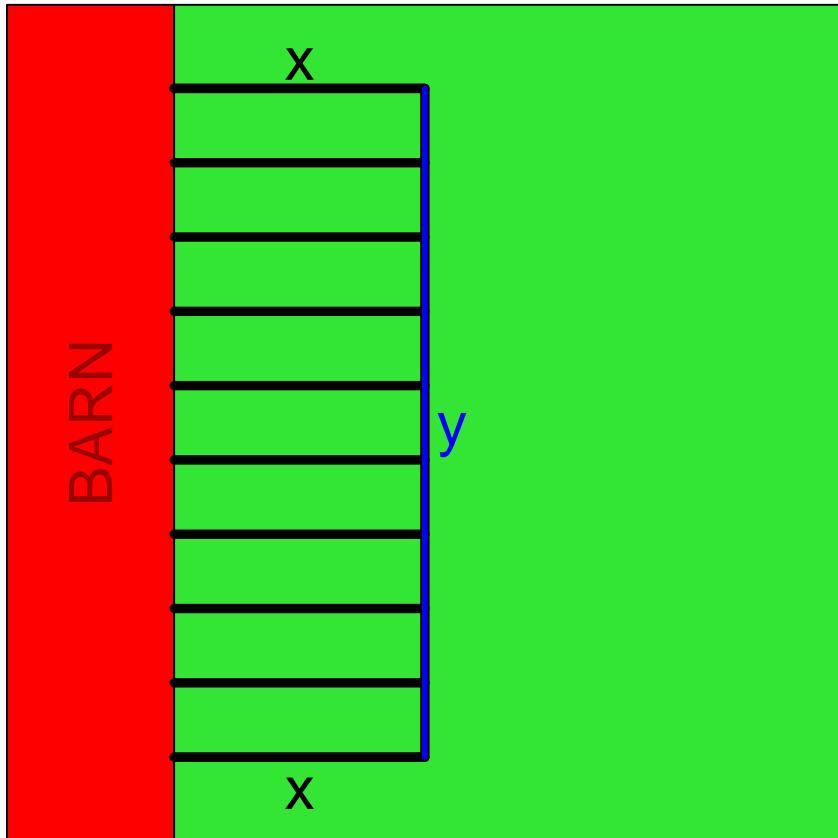


1. Problem

Amelia will use 780 feet of fence to build 9 rectangular enclosures. As shown in the figure below, the 9 enclosures will be built so each one is against a barn. Neighboring enclosures will use a single fence to separate them. Let x represent the length of fence perpendicular to the barn, and let y represent the length of fence parallel to the barn.



Amelia wants to maximize the total area of the enclosures. Find the value of x that maximizes the area.

Solution

The total area is simply the product of x and y .

$$A = xy$$

The total length of fence is 780 feet. Notice for 9 enclosures, we need 10 lengths of x .

$$780 = 10x + y$$

Solve this equation for y by subtracting $10x$ from both sides.

$$780 - 10x = y$$

Substitute $780 - 10x$ for y in the area equation.

$$A = x \cdot (780 - 10x)$$

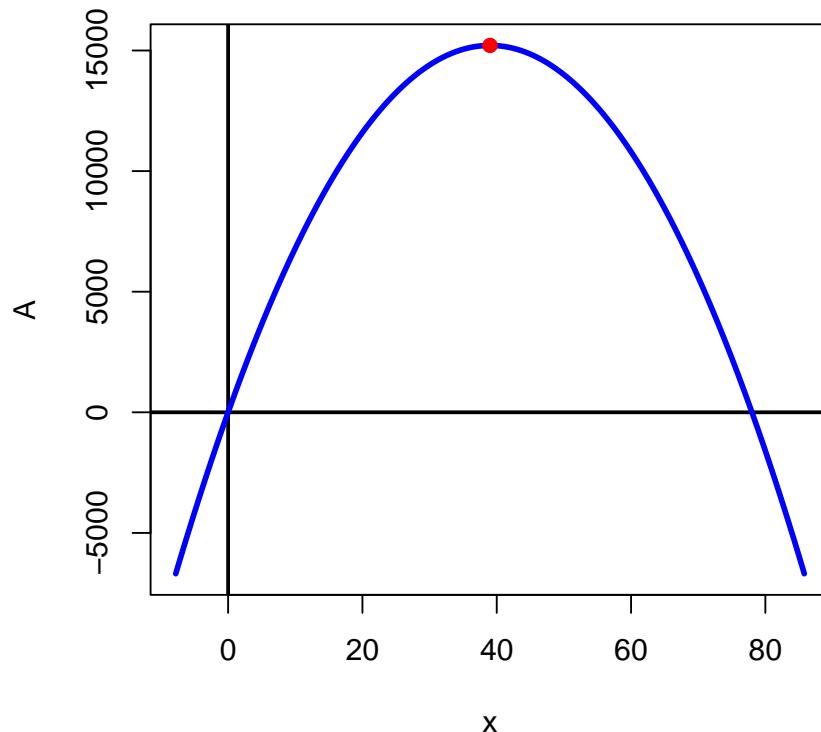
Distribute.

$$A = 780x - 10x^2$$

Put quadratic expression in standard order.

$$A = -10x^2 + 780x$$

If you draw a graph of A versus x , you'll get a parabola.



Notice, the maximum area occurs at the parabola's vertex. So, we can use $h = \frac{-b}{2a}$ to find the optimal x value.

$$x_{\text{optimal}} = \frac{-b}{2a}$$

$$x_{\text{optimal}} = \frac{-(780)}{2(-10)}$$

$$x_{\text{optimal}} = 39$$