Name: ______________________________ Date: ______________

Polynomial Operations SOLUTION (version 248)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 6x^5 - 8x^4 + 7x^3 + x^2 + 2$$

$$q(x) = 10x^5 + 3x^3 - 5x^2 - 6x + 8$$

Express the difference $q(x) - p(x)$ in standard form.

Get "unsimplified" forms. Then find $q(x) - p(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (6)x^5 + (-8)x^4 + (7)x^3 + (1)x^2 + (0)x^1 + (2)x^0 \\ q(x) &= (10)x^5 + (0)x^4 + (3)x^3 + (-5)x^2 + (-6)x^1 + (8)x^0 \\ q(x) - p(x) &= (4)x^5 + (8)x^4 + (-4)x^3 + (-6)x^2 + (-6)x^1 + (6)x^0 \\ q(x) - p(x) &= 4x^5 + 8x^4 - 4x^3 - 6x^2 - 6x + 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 2x^2 + 9x - 3$$

$$b(x) = 7x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$2x^2$	$9x$	-3
$7x$	$14x^3$	$63x^2$	$-21x$
-5	$-10x^2$	$-45x$	15

$$a(x) \cdot b(x) = 14x^3 + 63x^2 - 10x^2 - 21x - 45x + 15$$

Combine like terms.

$$a(x) \cdot b(x) = 14x^3 + 53x^2 - 66x + 15$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 248)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= -3x^3 - 26x^2 - 20x - 29 \\ g(x) &= x + 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	-3	-26	-20	-29
-8		24	16	32
	-3	-2	-4	3

So,

$$\frac{f(x)}{g(x)} = -3x^2 - 2x - 4 + \frac{3}{x+8}$$

In other words, $h(x) = -3x^2 - 2x - 4$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = -3x^3 - 26x^2 - 20x - 29$. Evaluate $f(-8)$.

You could do this the hard way.

$$\begin{aligned} f(-8) &= (-3)\cdot(-8)^3 \ + \ (-26)\cdot(-8)^2 \ + \ (-20)\cdot(-8) \ + \ (-29) \\ &= (-3)\cdot(-512) \ + \ (-26)\cdot(64) \ + \ (-20)\cdot(-8) \ + \ (-29) \\ &= (1536) \ + \ (-1664) \ + \ (160) \ + \ (-29) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-8)$ equals the remainder when $f(x)$ is divided by $x + 8$. Thus, $f(-8) = 3$.