Name: ________________________________ Date: ________________

Polynomial Operations SOLUTION (version 129)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = -2x^5 - 10x^3 + 8x^2 - 3x - 7$$

$$q(x) = -3x^5 - 2x^4 - 9x^2 - x + 5$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (-2)x^5 + (0)x^4 + (-10)x^3 + (8)x^2 + (-3)x^1 + (-7)x^0 \\ q(x) &= (-3)x^5 + (-2)x^4 + (0)x^3 + (-9)x^2 + (-1)x^1 + (5)x^0 \\ p(x) + q(x) &= (-5)x^5 + (-2)x^4 + (-10)x^3 + (-1)x^2 + (-4)x^1 + (-2)x^0 \\ p(x) + q(x) &= -5x^5 - 2x^4 - 10x^3 - x^2 - 4x - 2 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = -7x^2 + 5x + 2$$

$$b(x) = 4x - 2$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-7x^2$	$5x$	2
$4x$	$-28x^3$	$20x^2$	$8x$
-2	$14x^2$	$-10x$	-4

$$a(x) \cdot b(x) = -28x^3 + 20x^2 + 14x^2 + 8x - 10x - 4$$

Combine like terms.

$$a(x) \cdot b(x) = -28x^3 + 34x^2 - 2x - 4$$

3. Express $(x + 1)^5$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Polynomial Operations SOLUTION (version 129)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 8x^3 + 28x^2 - 16x + 9 \\ g(x) &= x + 4 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+4}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

	8	28	-16	9
-4		-32	16	0
	8	-4	0	9

So,

$$\frac{f(x)}{g(x)} = 8x^2 - 4x + \frac{9}{x+4}$$

In other words, $h(x) = 8x^2 - 4x$ and the remainder is $R = 9$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 8x^3 + 28x^2 - 16x + 9$. Evaluate $f(-4)$.

You could do this the hard way.

$$\begin{aligned} f(-4) &= (8)\cdot(-4)^3 \ + \ (28)\cdot(-4)^2 \ + \ (-16)\cdot(-4) \ + \ (9) \\ &= (8)\cdot(-64) \ + \ (28)\cdot(16) \ + \ (-16)\cdot(-4) \ + \ (9) \\ &= (-512) \ + \ (448) \ + \ (64) \ + \ (9) \\ &= 9 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-4)$ equals the remainder when $f(x)$ is divided by $x + 4$. Thus, $f(-4) = 9$.