Name:

Date:

s17 Geometric Series Exam (SLTN v350)

Question 1

Consider the partial geometric series represented below with first term a = 703, common ratio $r = \left(\frac{30}{37}\right)^{1/10}$, and n = 10 terms.

$$S \ = \ 703 + 688.41 + 674.12 + 660.13 + 646.43 + 633.02 + 619.88 + 607.01 + 594.42 + 582.08$$

We can multiply both sides by r.

$$rS \; = \; 688.41 + 674.12 + 660.13 + 646.43 + 633.02 + 619.88 + 607.01 + 594.42 + 582.08 + 570$$

What is the value of S - rS?

Most terms cancel.

$$703 - 570 = 133$$

Question 2

Consider the geometric series shown below, using ellipsis notation to indicate a continuation of the pattern without writing every term.

$$S = 8 + 8(2) + 8(2)^{2} + 8(2)^{3} + \cdots + 8(2)^{80} + 8(2)^{81} + 8(2)^{82} + 8(2)^{83}$$

Identify the initial term, the common ratio, and the number of terms.

first term
$$= a = 8$$

common ratio =
$$r = 2$$

number of terms =
$$n = 84$$

Question 3

Write a proof for the partial geometric series formula.

- a. Define the variables.
- b. Write the sum using variables and ellipsis notation. You can implicitly assume the number of terms is more than the number of terms you choose to write.
- c. Using annotated algebraic manipulation, produce the partial geometric series formula.

Definitions

a =first term

r = common ratio

n = number of terms

S = sum of partial geometric series

The partial geometric series is expressed using ellipsis notation.

$$S = a + ar + ar^{2} + ar^{3} + \dots + ar^{n-4} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

Multiply both sides by r.

$$rS = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract the second equation from the first equation.

$$S - rS = a - ar^n$$

Factor out S from left side.

$$S(1-r) = a - ar^n$$

Divide both sides by (1-r). We technically need to enforce $r \neq 1$ as a condition of the formula because otherwise we'd be dividing by 0 in this step, and division by 0 is not defined.

$$S = \frac{a - ar^n}{1 - r}$$