Notes: Coordinate transformations

It is helpful to rewrite function transformations as coordinate transformations. For example:

$$g[x] \;=\; \frac{f[2x+3]}{5}+7$$

Can be rewritten as:

f		g
(a,b)	$\rightarrow$	$\left(\dfrac{a-3}{2}\ ,\ \dfrac{b}{5}+7\right)$

This tells us that if $(25,30)$ is on curve f we can find the corresponding point on curve g:

$$\left(\frac{25-3}{2}\ ,\ \frac{30}{5}+7\right)$$

$$(11\ ,\ 13)$$

In other words, since $f[25]=30$ we know $g[11]=13$.

Detailed logic

Notice, this could be worked out as follows. Since (a,b) is a point on the f curve, we know $f[a]=b$. To use that information in the main equation, we need $2x+3$ to equal a.

$$2x+3=a$$

Solve for x. (This is a two-step equation.)

$$x=\frac{a-3}{2}$$

Going back to the main equation, replace $2x+3$ with a and replace x with $\frac{a-3}{2}$.

$$g\left[\frac{a-3}{2}\right] \;=\; \frac{f[a]}{5}+7$$

Replace $f[a]$ with b.

$$g\left[\frac{a-3}{2}\right] \;=\; \frac{b}{5}+7$$

And so we know point $\left(\frac{a-3}{2}\ ,\ \frac{b}{5}+7\right)$ is on curve g.

Shortcut

Use the INVERSE of the ARGUMENT for x and the OUTER operations for y.

Some examples:

Example 1

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; \frac{f\left[\frac{x}{7}+3\right]}{9}+2$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \rightarrow \left(7\,(a-3)\;\;,\;\;\frac{b}{9}+2\right)$$

Example 2

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; 6\cdot f[8\,(x+9)]+2$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \rightarrow \left(\frac{a}{8}-9\;,\;\;6b+2\right)$$

Example 3

Consider the two functions f and g, where g is defined as a transformation of f:

$$g[x] \;=\; 7\cdot(f[2x-6]-8)$$

For point (a, b) on curve f there is a corresponding point on the curve g. Write the coordinate transformation.

$$(a, b) \rightarrow \left(\frac{a+6}{2}\;,\;\;7\,(b-8)\right)$$