at1117paper: Complete the Square (v328)

Example

A square's edge length is x feet. A rectangle has a height of x feet and a width of 56 feet. Their combined area, found by adding the square's area and the rectangle's area, is 897 square feet. What is the value of x?

Example's Solution

$$x^2 + 56x = 897$$

To complete the square, add $\left(\frac{56}{2}\right)^2 = 784$ to both sides.

$$x^2 + 56x + 784 = 1681$$

Recognize the left side is now a perfect-square trinomial. Factor the left side.

$$(x + 28)^2 = 1681$$

Undo the squaring.

$$x + 28 = \pm\sqrt{1681}$$

$$x + 28 = \pm 41$$

Subtract 28 from both sides.

$$x = -28 \pm 41$$

In this geometric example, we are only concerned about the positive solution. So,

$$x = 13$$

Question 1

A square's edge length is x feet. A rectangle has a height of x feet and a width of 58 feet. The total area, of the square and rectangle, is 840 square feet. What is the value of x?

$$x^2 + 58x = 840$$

$$x^2 + 58x + 841 = 1681$$

$$(x + 29)^2 = 1681$$

$$x + 29 = \pm 41$$

$$x = 12$$

Question 2

A square's edge length is x feet. A rectangle has a height of x feet and a width of 42 feet. The total area, of the square and rectangle, is 855 square feet. What is the value of x?

$$x^2 + 42x = 855$$
$$x^2 + 42x + 441 = 1296$$
$$(x + 21)^2 = 1296$$
$$x + 21 = \pm 36$$
$$x = 15$$

Question 3

A square's edge length is x feet. A rectangle has a height of x feet and a width of 56 feet. The total area, of the square and rectangle, is 372 square feet. What is the value of x?

$$x^2 + 56x = 372$$
$$x^2 + 56x + 784 = 1156$$
$$(x + 28)^2 = 1156$$
$$x + 28 = \pm 34$$
$$x = 6$$