Name: ____ Date: ____________________

Polynomial Operations SOLUTIONS (version 7)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) = 7x^5 + 2x^4 - 9x^2 + 6x - 3$$

$$q(x) = -10x^5 + 3x^4 - 4x^3 + 6x - 9$$

Express the difference $p(x) - q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) - q(x)$ with addition/subtraction.

$$\begin{aligned} p(x) &= (7)x^5 + (2)x^4 + (0)x^3 + (-9)x^2 + (6)x^1 + (-3)x^0 \\ q(x) &= (-10)x^5 + (3)x^4 + (-4)x^3 + (0)x^2 + (6)x^1 + (-9)x^0 \\ p(x) - q(x) &= (17)x^5 + (-1)x^4 + (4)x^3 + (-9)x^2 + (0)x^1 + (6)x^0 \\ p(x) - q(x) &= 17x^5 - x^4 + 4x^3 - 9x^2 + 6 \end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) = 5x^2 + 7x - 4$$

$$b(x) = 4x - 7$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$5x^2$	$7x$	-4
$4x$	$20x^3$	$28x^2$	$-16x$
-7	$-35x^2$	$-49x$	28

$$a(x) \cdot b(x) = 20x^3 + 28x^2 - 35x^2 - 16x - 49x + 28$$

Combine like terms.

$$a(x) \cdot b(x) = 20x^3 - 7x^2 - 65x + 28$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

Polynomial Operations SOLUTIONS (version 7)

4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= x^3 - 6x^2 - 19x + 26 \\ g(x) &= x - 8 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x-8}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{c|cccc} & 1 & \text{-6} & \text{-19} & 26 \\ 8 & & 8 & 16 & \text{-24} \\ \hline & 1 & 2 & \text{-3} & 2 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = x^2 + 2x - 3 + \frac{2}{x-8}$$

In other words, $h(x) = x^2 + 2x - 3$ and the remainder is $R = 2$.

5. Let polynomial $f(x)$ still be defined as $f(x) = x^3 - 6x^2 - 19x + 26$. Evaluate $f(8)$.

You could do this the hard way.

$$\begin{aligned} f(8) &= (1)\cdot(8)^3 \;+\; (-6)\cdot(8)^2 \;+\; (-19)\cdot(8) \;+\; (26) \\ &= (1)\cdot(512) \;+\; (-6)\cdot(64) \;+\; (-19)\cdot(8) \;+\; (26) \\ &= (512) \;+\; (-384) \;+\; (-152) \;+\; (26) \\ &= 2 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(8)$ equals the remainder when $f(x)$ is divided by $x - 8$. Thus, $f(8) = 2$.