Name: ____ Date: ____________________

Polynomial Operations SOLUTIONS (version 25)

1. Let polynomials $p(x)$ and $q(x)$ be defined below.

$$p(x) \;=\; 8x^5 + 9x^4 + x^3 + 2x - 3$$

$$q(x) \;=\; 9x^5 - 2x^3 - 3x^2 + 5x + 1$$

Express the sum of $p(x) + q(x)$ in standard form.

Get "unsimplified" forms. Then find $p(x) + q(x)$ with addition/subtraction.

$$\begin{aligned}
p(x) &= (8)x^5 + (9)x^4 + (1)x^3 + (0)x^2 + (2)x^1 + (-3)x^0 \\
q(x) &= (9)x^5 + (0)x^4 + (-2)x^3 + (-3)x^2 + (5)x^1 + (1)x^0 \\
p(x) + q(x) &= (17)x^5 + (9)x^4 + (-1)x^3 + (-3)x^2 + (7)x^1 + (-2)x^0 \\
p(x) + q(x) &= 17x^5 + 9x^4 - x^3 - 3x^2 + 7x - 2
\end{aligned}$$

2. Let polynomials $a(x)$ and $b(x)$ be defined below.

$$a(x) \;=\; -2x^2 - 6x + 8$$

$$b(x) \;=\; -6x - 5$$

Express the product $a(x) \cdot b(x)$ in standard form.

You can use a table for multiplication.

*	$-2x^2$	$-6x$	8
$-6x$	$12x^3$	$36x^2$	$-48x$
-5	$10x^2$	$30x$	-40

$$a(x) \cdot b(x) \;=\; 12x^3 + 36x^2 + 10x^2 - 48x + 30x - 40$$

Combine like terms.

$$a(x) \cdot b(x) \;=\; 12x^3 + 46x^2 - 18x - 40$$

3. Express $(x + 1)^6$ in standard (expanded) form.

Remember the binomial theorem. It tells us to use Pascal's triangle.

$$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

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4. Let polynomials $f(x)$ and $g(x)$ be defined below.

$$\begin{aligned} f(x) &= 2x^3 + 16x^2 + 13x - 4 \\ g(x) &= x + 7 \end{aligned}$$

The quotient of $\frac{f(x)}{g(x)}$ can be expressed as a polynomial, $h(x)$, and a remainder, R (a real number).

$$\frac{f(x)}{g(x)} = h(x) + \frac{R}{x+7}$$

By using synthetic division or long division, express $h(x)$ in standard form, and find the remainder R.

I prefer using synthetic division.

$$\begin{array}{r|rrrr} & 2 & 16 & 13 & -4 \\ -7 & & -14 & -14 & 7 \\ \hline & 2 & 2 & -1 & 3 \end{array}$$

So,

$$\frac{f(x)}{g(x)} = 2x^2 + 2x - 1 + \frac{3}{x+7}$$

In other words, $h(x) = 2x^2 + 2x - 1$ and the remainder is $R = 3$.

5. Let polynomial $f(x)$ still be defined as $f(x) = 2x^3 + 16x^2 + 13x - 4$. Evaluate $f(-7)$.

You could do this the hard way.

$$\begin{aligned} f(-7) &= (2)\cdot(-7)^3 \ + \ (16)\cdot(-7)^2 \ + \ (13)\cdot(-7) \ + \ (-4) \\ &= (2)\cdot(-343) \ + \ (16)\cdot(49) \ + \ (13)\cdot(-7) \ + \ (-4) \\ &= (-686) \ + \ (784) \ + \ (-91) \ + \ (-4) \\ &= 3 \end{aligned}$$

Or, if you reference the polynomial remainder theorem, you can state that you know $f(-7)$ equals the remainder when $f(x)$ is divided by $x + 7$. Thus, $f(-7) = 3$.