

Name: \_\_\_\_\_

### at1204p\_vertex\_and\_roots... from standard-form quadratic functions (v128)

For each quadratic function, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Your answers should be in simplified exact form, no decimal approximations. Improper fractions are preferred to mixed numbers.

#### Example

$$f(x) = 6x^2 + 4x - 5$$

#### Example solution

1. Find the axis of symmetry. Use the formula  $h = \frac{-b}{2a}$ , where  $h$  is the horizontal coordinate of the vertex. Remember that the vertical axis of symmetry intersects the vertex.

$$h = \frac{-(4)}{2(6)} = \frac{-1}{3}$$

$$\text{axis of symmetry: } x = \frac{-1}{3}$$

2. Find the distance of each root from the axis of symmetry. Use the formula  $w = \frac{\sqrt{b^2 - 4ac}}{2a}$ .

$$w = \frac{\sqrt{(4)^2 - 4(6)(-5)}}{2(6)}$$

$$w = \frac{\sqrt{136}}{12} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 17}}{12} = \frac{2\sqrt{34}}{12}$$

$$w = \frac{\sqrt{34}}{6}$$

3. The  $x$ -intercepts can be found by adding  $w$  to or subtracting  $w$  from  $h$ .

$$\left(\frac{-1}{3} - \frac{\sqrt{34}}{6}, 0\right) \quad \text{and} \quad \left(\frac{-1}{3} + \frac{\sqrt{34}}{6}, 0\right)$$

4. Find the vertex. We already know  $h = \frac{-1}{3}$ , so we just need  $k$ . Use the formula  $k = \frac{4ac - b^2}{4a}$ .

$$k = \frac{4(6)(-5) - (4)^2}{4(6)}$$

$$k = \frac{-136}{24} = \frac{-17}{3}$$

$$\text{vertex: } \left(\frac{-1}{3}, \frac{-17}{3}\right)$$

## Question 1

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Box your answers.

$$f(x) = 9x^2 - 6x - 5$$

1. Axis of symmetry

$$h = \frac{-(-6)}{2(9)} = \frac{1}{3}$$

$$\text{axis of symmetry: } x = \frac{1}{3}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-6)^2 - 4(9)(-5)}}{2(9)}$$

$$w = \frac{\sqrt{216}}{18} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}}{18} = \frac{6\sqrt{6}}{18}$$

$$w = \frac{\sqrt{6}}{3}$$

3. Roots

$$\left(\frac{1}{3} - \frac{\sqrt{6}}{3}, 0\right) \quad \text{and} \quad \left(\frac{1}{3} + \frac{\sqrt{6}}{3}, 0\right)$$

4. Vertex

$$k = \frac{4(9)(-5) - (-6)^2}{4(9)}$$

$$k = \frac{-216}{36} = -6$$

$$\text{vertex: } \left(\frac{1}{3}, -6\right)$$

## Question 2

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Box your answers.

$$f(x) = 3x^2 + 8x - 8$$

1. Axis of symmetry

$$h = \frac{-(8)}{2(3)} = \frac{-4}{3}$$

$$\text{axis of symmetry: } x = \frac{-4}{3}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(8)^2 - 4(3)(-8)}}{2(3)}$$

$$w = \frac{\sqrt{160}}{6} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}}{6} = \frac{4\sqrt{10}}{6}$$

$$w = \frac{2\sqrt{10}}{3}$$

3. Roots

$$\left(\frac{-4}{3} - \frac{2\sqrt{10}}{3}, 0\right) \quad \text{and} \quad \left(\frac{-4}{3} + \frac{2\sqrt{10}}{3}, 0\right)$$

4. Vertex

$$k = \frac{4(3)(-8) - (8)^2}{4(3)}$$

$$k = \frac{-160}{12} = \frac{-40}{3}$$

$$\text{vertex: } \left(\frac{-4}{3}, \frac{-40}{3}\right)$$

### Question 3

For the quadratic function listed below, find:

1. The equation of the axis of symmetry
2. The distance of each root to the axis of symmetry ( $w$ )
3. Both  $x$ -intercepts (also called the roots or the zeros), each shown as cartesian coordinates
4. The location of the vertex ( $h, k$ ) shown as cartesian coordinates

Box your answers.

$$f(x) = 7x^2 - 6x - 2$$

1. Axis of symmetry

$$h = \frac{-(-6)}{2(7)} = \frac{3}{7}$$

$$\text{axis of symmetry: } x = \frac{3}{7}$$

2. Distance from axis of symmetry to root

$$w = \frac{\sqrt{(-6)^2 - 4(7)(-2)}}{2(7)}$$

$$w = \frac{\sqrt{92}}{14} = \frac{\sqrt{2 \cdot 2 \cdot 23}}{14} = \frac{2\sqrt{23}}{14}$$

$$w = \frac{\sqrt{23}}{7}$$

3. Roots

$$\left(\frac{3}{7} - \frac{\sqrt{23}}{7}, 0\right) \quad \text{and} \quad \left(\frac{3}{7} + \frac{\sqrt{23}}{7}, 0\right)$$

4. Vertex

$$k = \frac{4(7)(-2) - (-6)^2}{4(7)}$$

$$k = \frac{-92}{28} = \frac{-23}{7}$$

$$\text{vertex: } \left(\frac{3}{7}, \frac{-23}{7}\right)$$