1.

Y_d = daughter's height.
X_i = midparent value.

$$Y_d = \beta_0 + \beta_1 x_i$$

least squares.

$$SS = \sum_{i=1}^{n} (Y_{di} - \beta_0 - \beta_1 X_i)^2$$

$$\frac{\partial SS}{\partial \beta_0} = \sum_{i=1}^{n} (Y_{di} - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$$

$$\underline{\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}}$$

$\hat{\beta}_0, \hat{\beta}_1$ is same for Y_s, Which is the height of son.

$$\frac{\partial SS}{\partial \beta_1} = \sum_{i=1}^{n} X_i (Y_{di} - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$$

$$= \sum_{i=1}^{n} X_i (Y_{di} - \bar{y} + \hat{\beta}_1 \bar{x} - \hat{\beta}_1 X_i) = 0$$

$$\underline{\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(y_{di} - \bar{y})}{\sum (X_i - \bar{x})^2}}$$

for daughters. $\hat{\beta}_0 = 15.96711$ $\hat{\beta}_1 = 0.72140$

$$\hat{y} = 15.96711 + 0.72140x$$

for sons. $\hat{\beta}_0 = 44.48953$ $\hat{\beta}_1 = .31919$

$$\hat{y} = 44.48953 + 0.31919x$$

2 (R file is attached)

$$\hat{y} = 15.34476 + 0.3214951 X_1 + 0.4059780 X_2 + 5.2259513 X_3$$

X_1 = mother's height
X_2 = father's height
X_3 = gender (0 if female, 1 if male)

3.

MSE for daughter, simple

$$MSE = \frac{\sum (y_{di} - \hat{y})^2}{\#\text{ of daughters} - 2} = 3.98$$

MSE for sons, simple

$$MSE = \frac{\sum (y_{si} - \hat{y})^2}{\#\text{ of sons} - 2} = 2.27$$

MSE for general linear model

$$MSE = \frac{\sum (y_i - \hat{y})^2}{n - \underbrace{3}_{\#\text{ of predictors}} - 1} = 4.64$$

For both sons & daughters, Simple regression is a better method.