

# APSK Constellation Design for Coherent Optical Channels Distorted by Nonlinear Phase Noise

## FORCE Meeting

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<sup>1</sup> Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden

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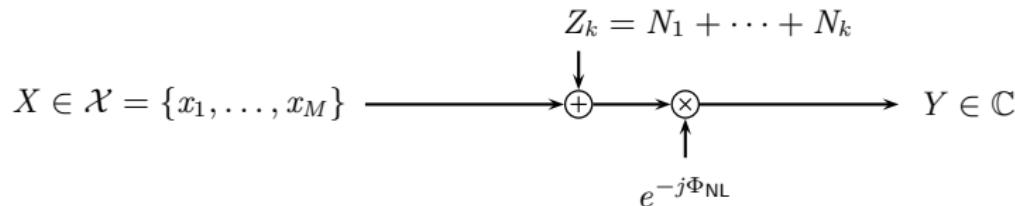
*{christian.haeger, alexandre.graell, agrell}@chalmers.se, alex.alvarado@ieee.org}*

January 25, 2012

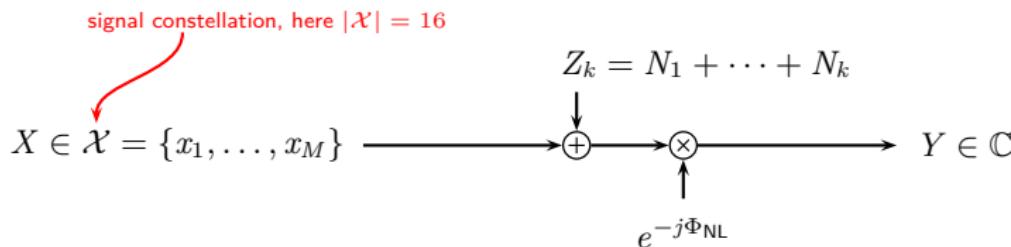
# Outline

- 1** Introduction: Channel Model and APSK
- 2** Detection Methods
- 3** APSK Optimization
- 4** Conclusions

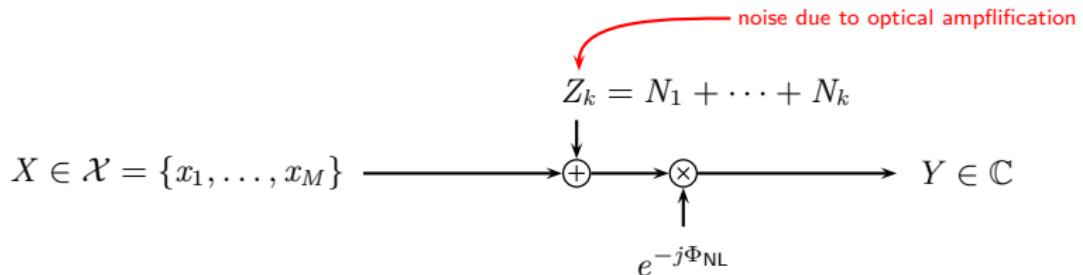
## Discrete-Time Channel Model



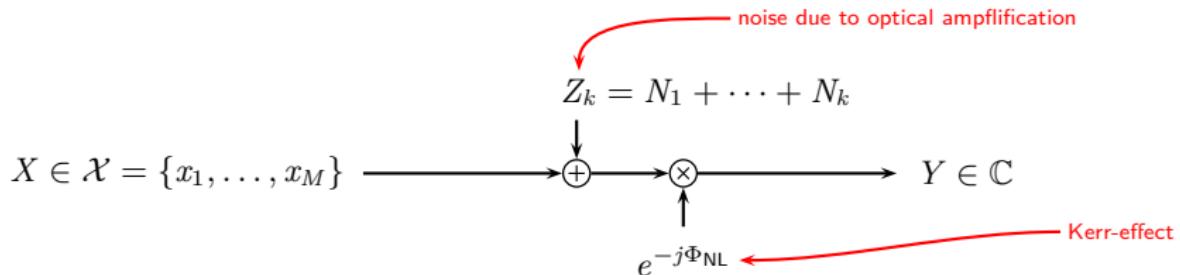
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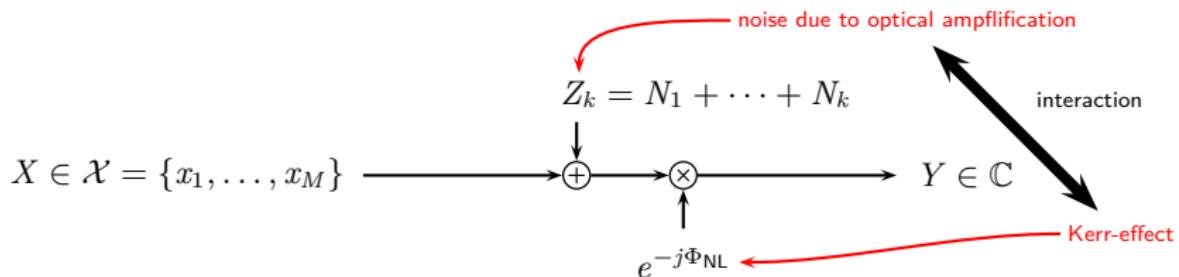
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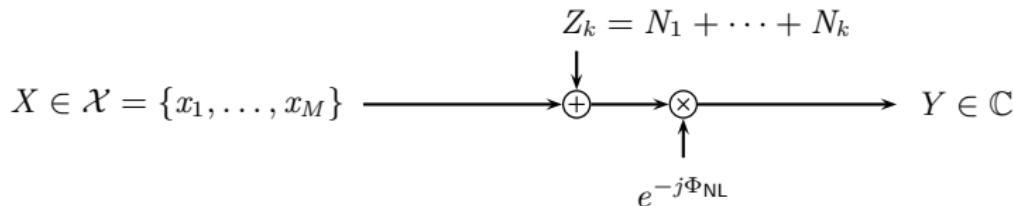
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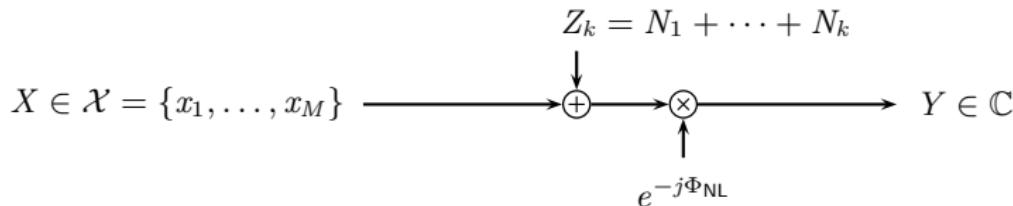
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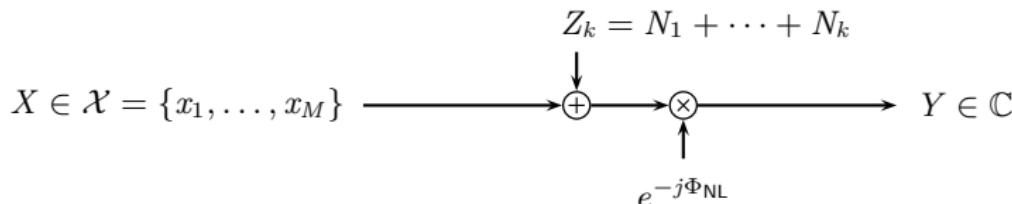


The nonlinear phase noise is defined as

$$\Phi_{NL} = \frac{\gamma L}{k} \sum_{i=1}^k |X + Z_i|^2$$

and the  $N_i$  are i.i.d. complex, zero-mean Gaussian RVs with variance  $\sigma_0^2 = \frac{\sigma^2}{k}$  per dimension.

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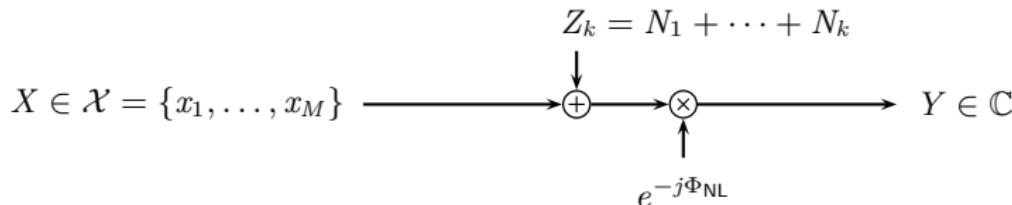
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Introduction

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Detection Methods

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APSK Optimization

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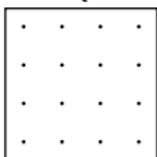
Conclusions

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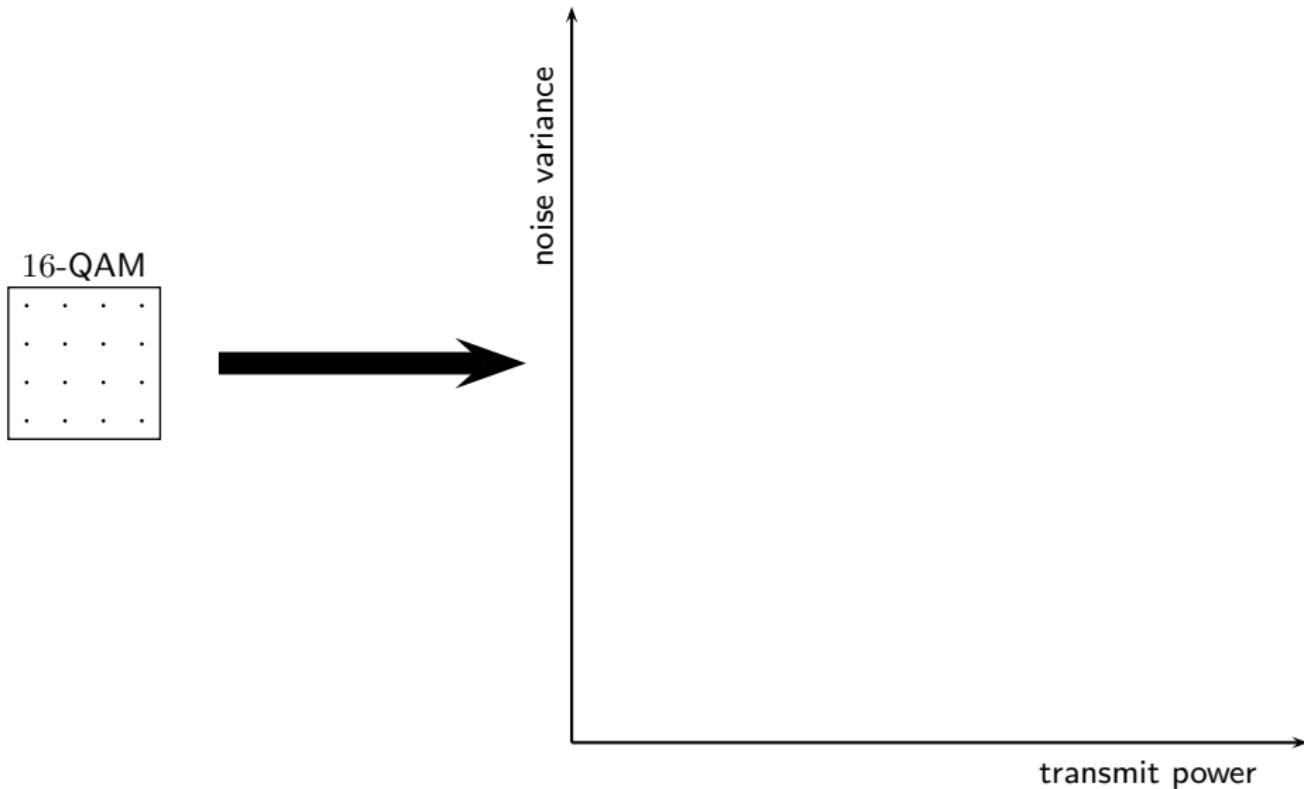
## Power Dependent Noise

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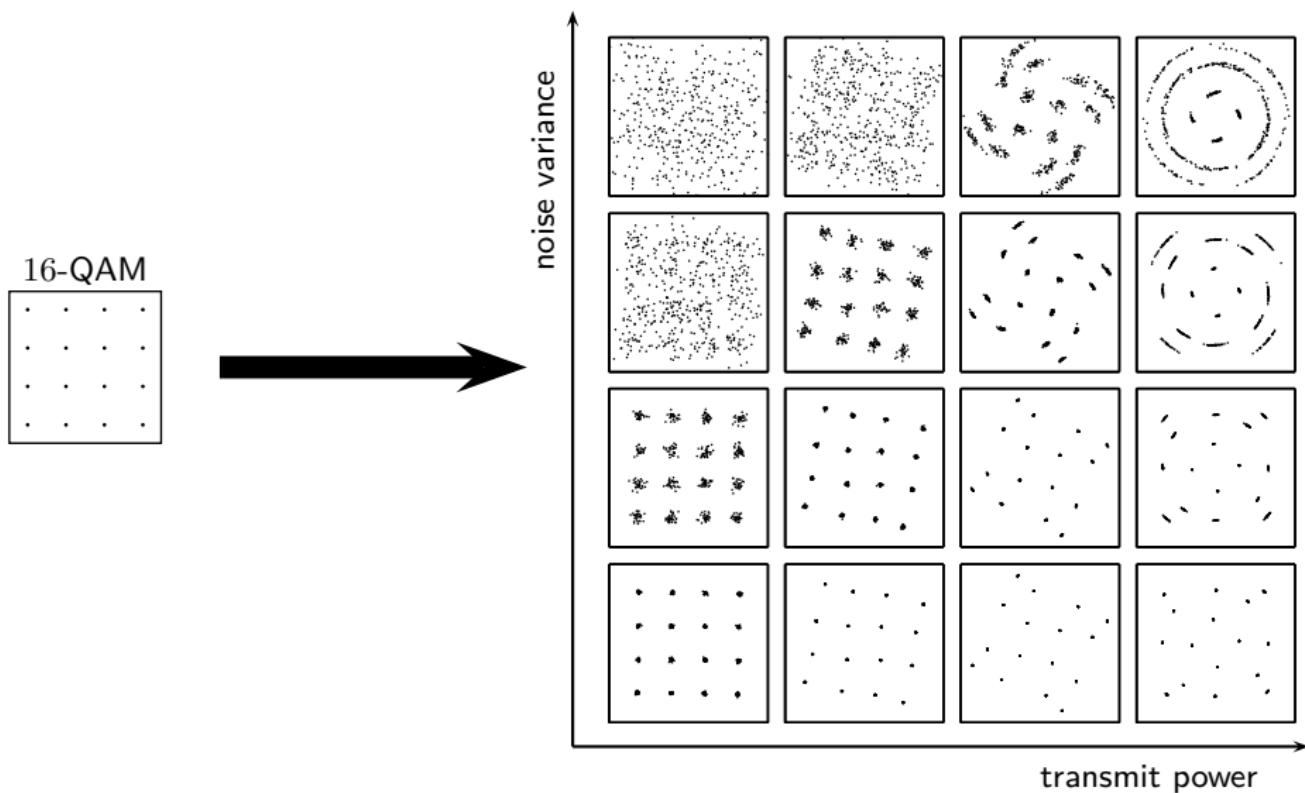
16-QAM



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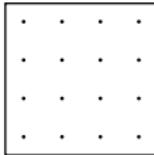


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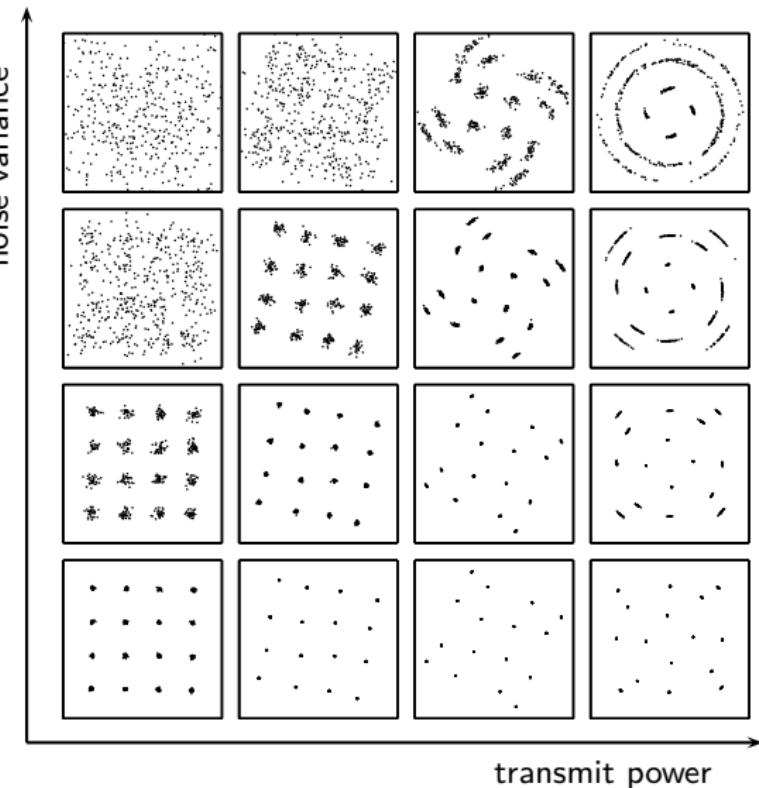


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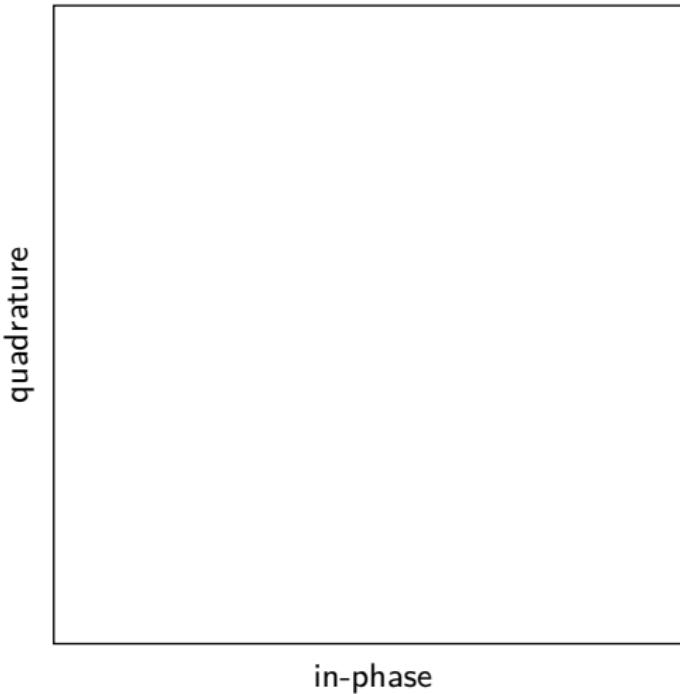
16-QAM



SNR is not sufficient to characterize the channel!



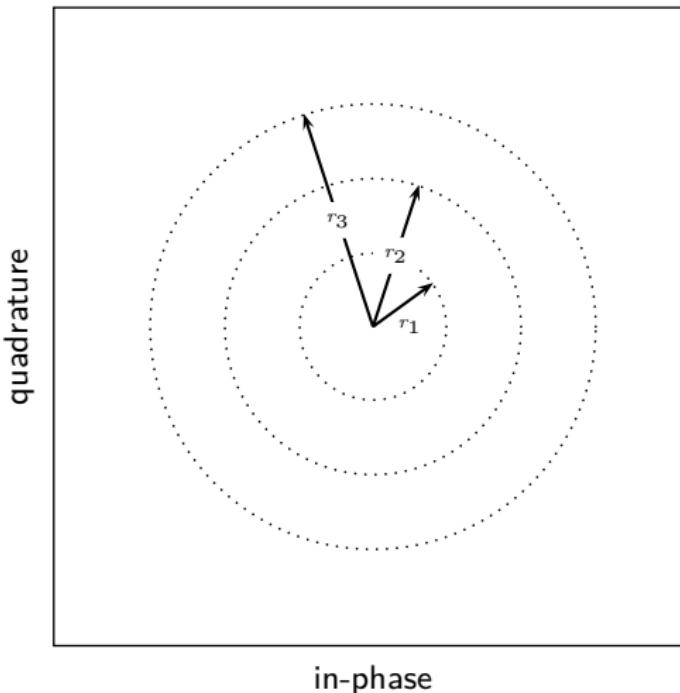
## Amplitude Phase-Shift Keying (APSK), Example



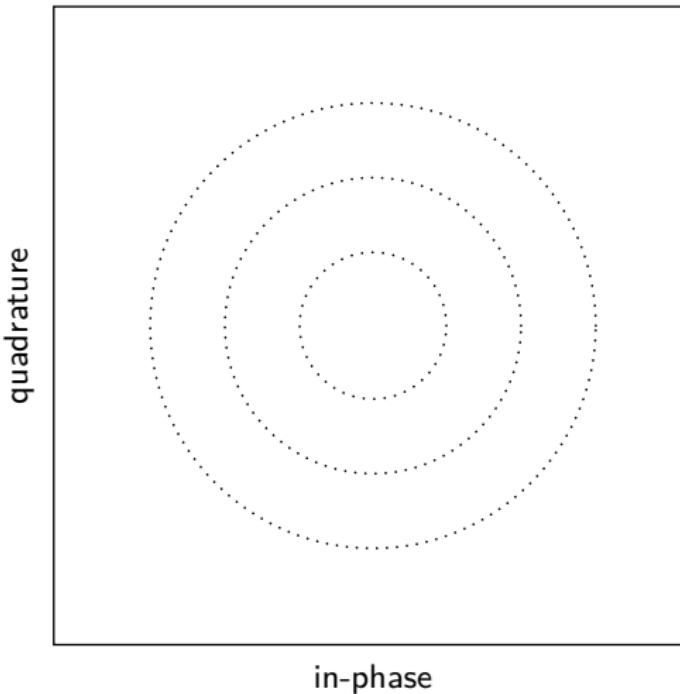
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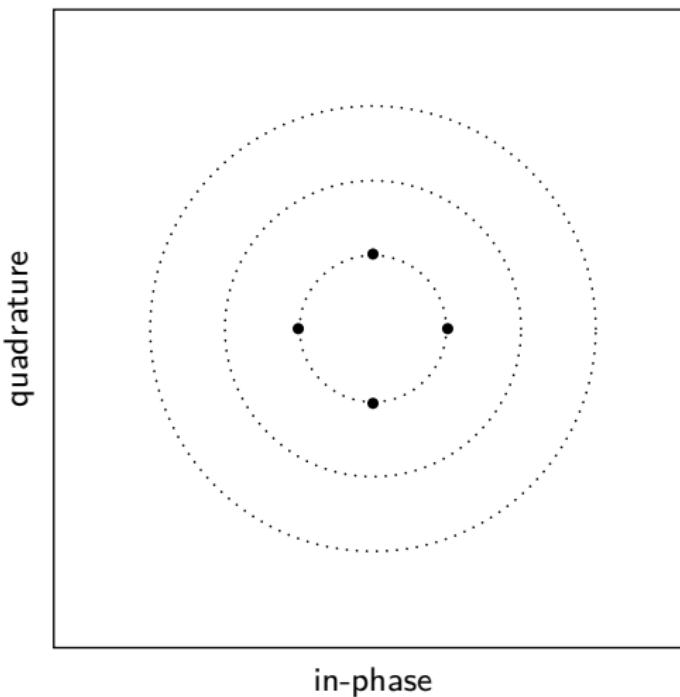


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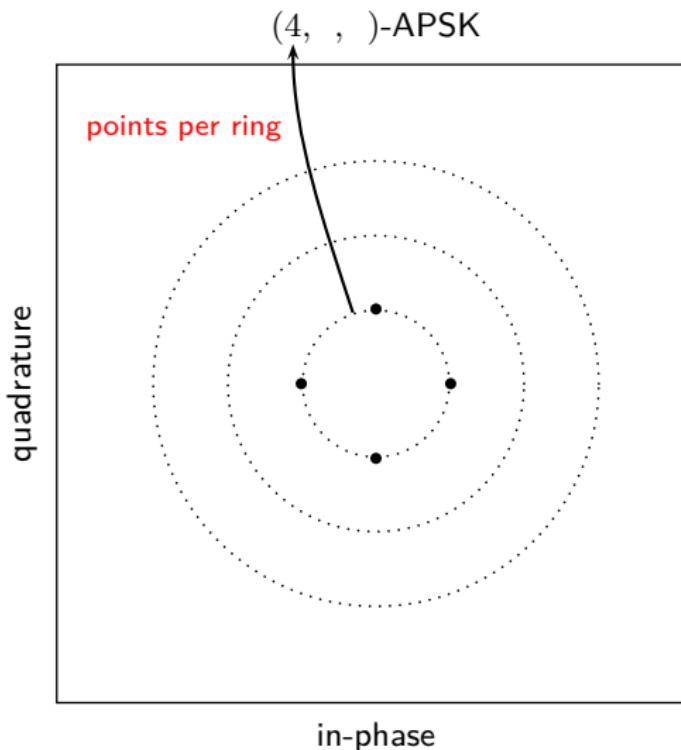


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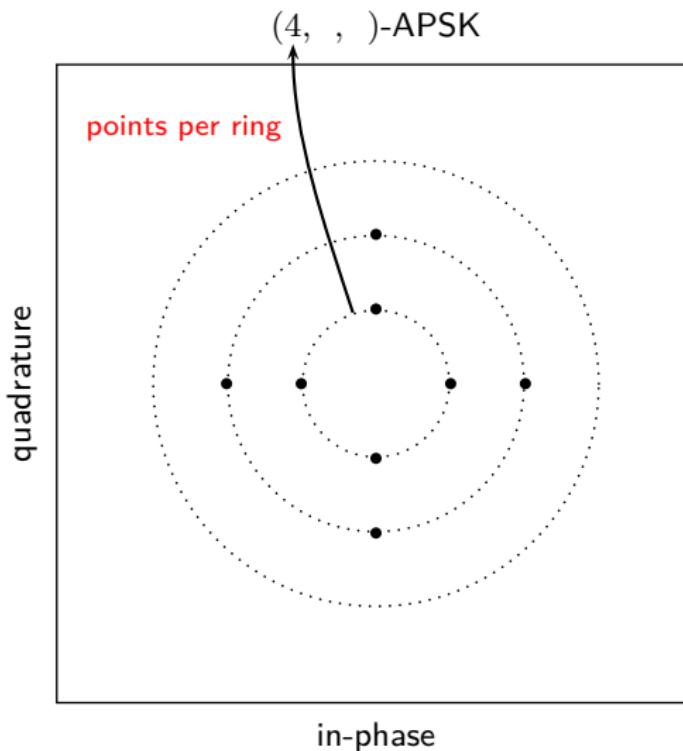
( , , )-APSK



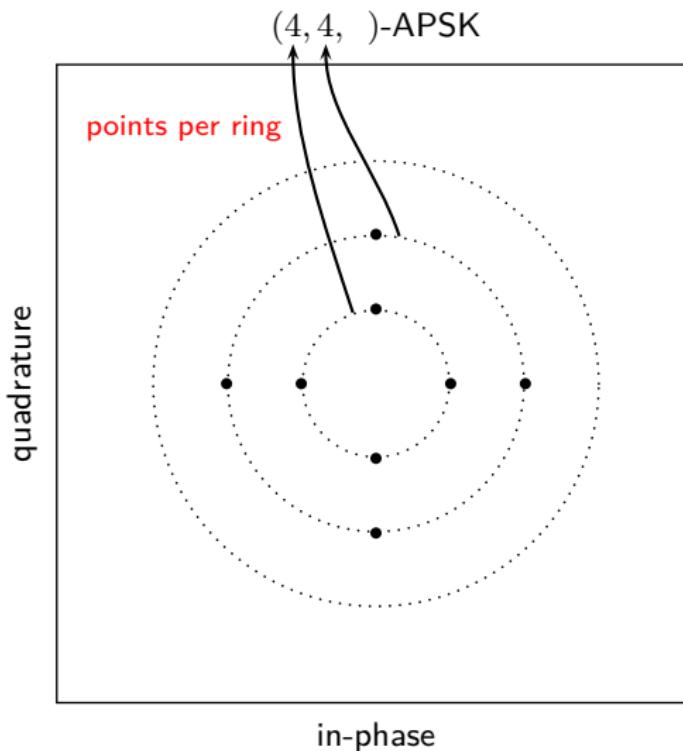
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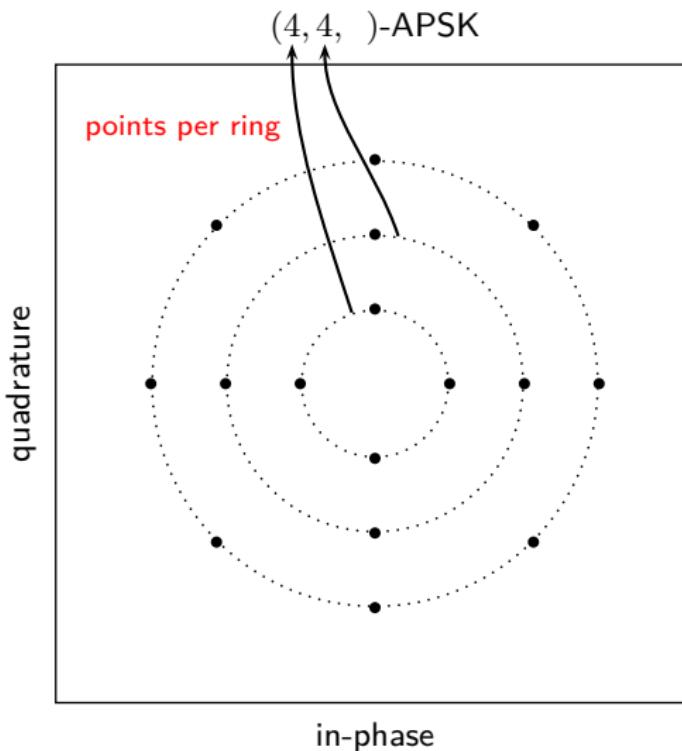
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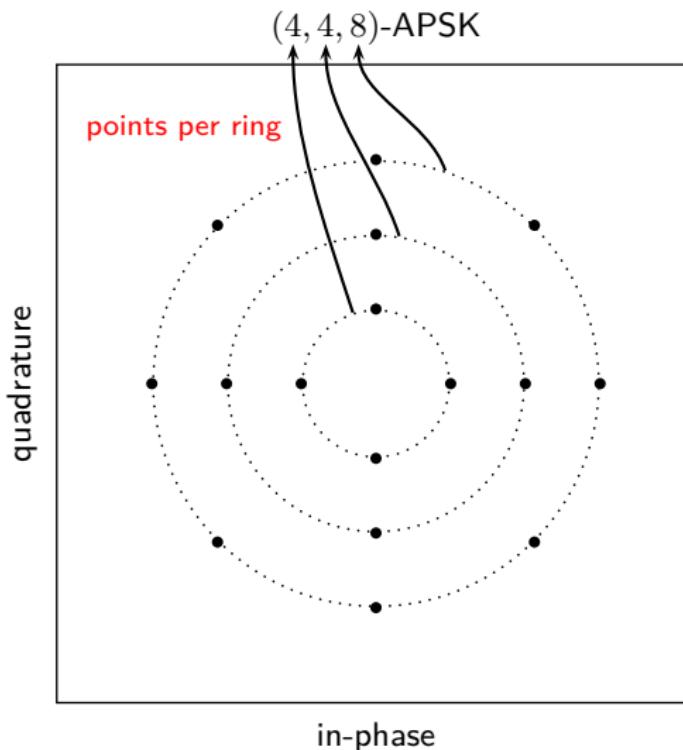
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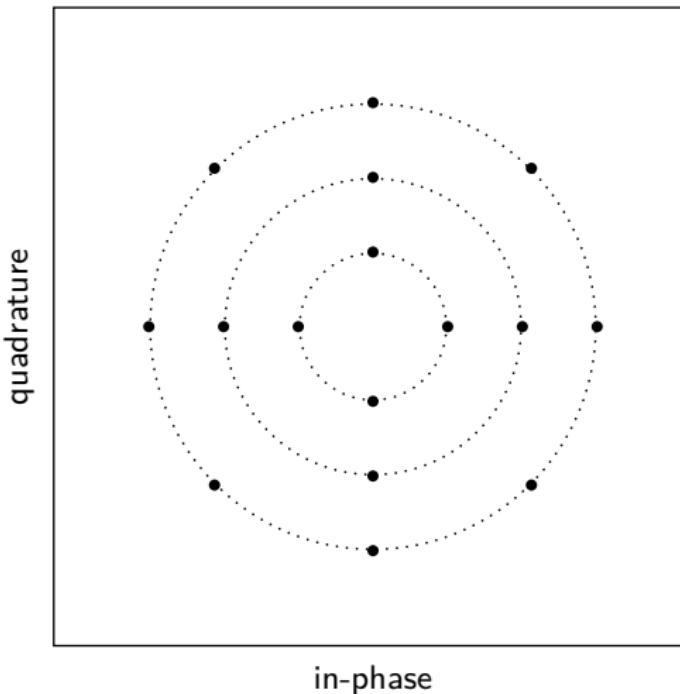


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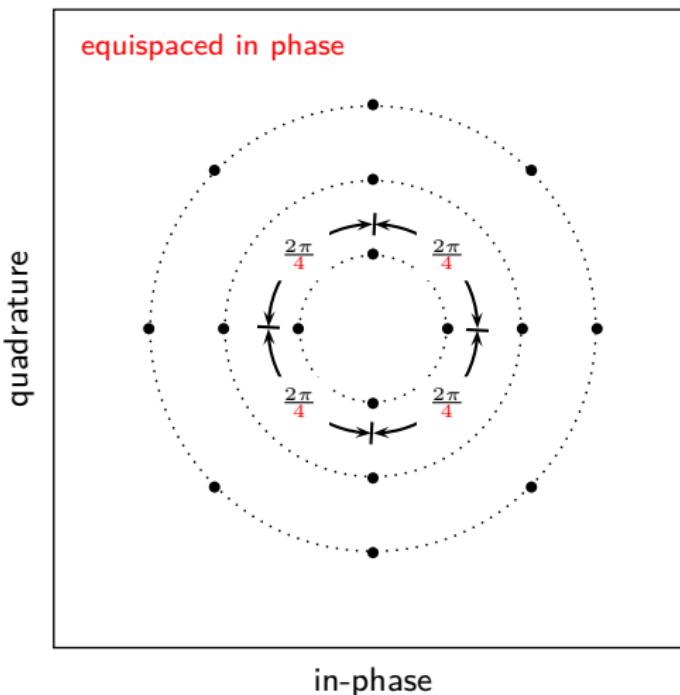
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(4, 4, 8)-APSK



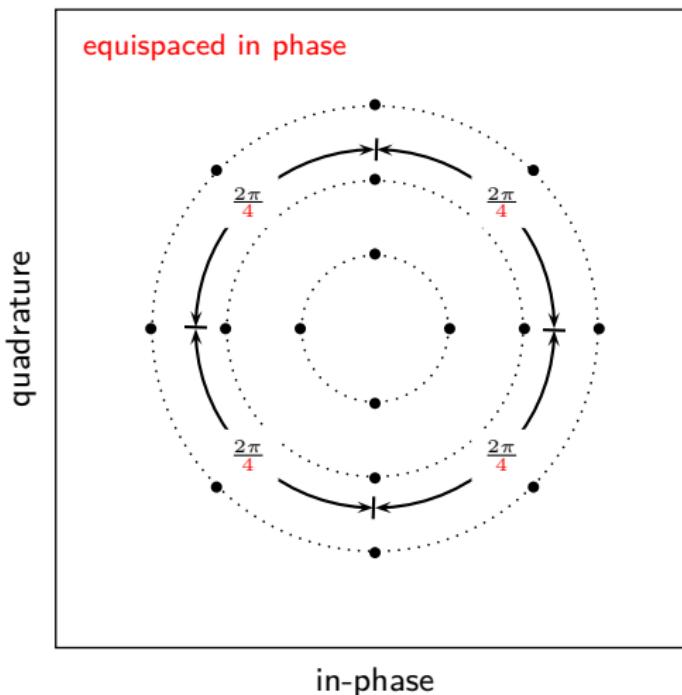
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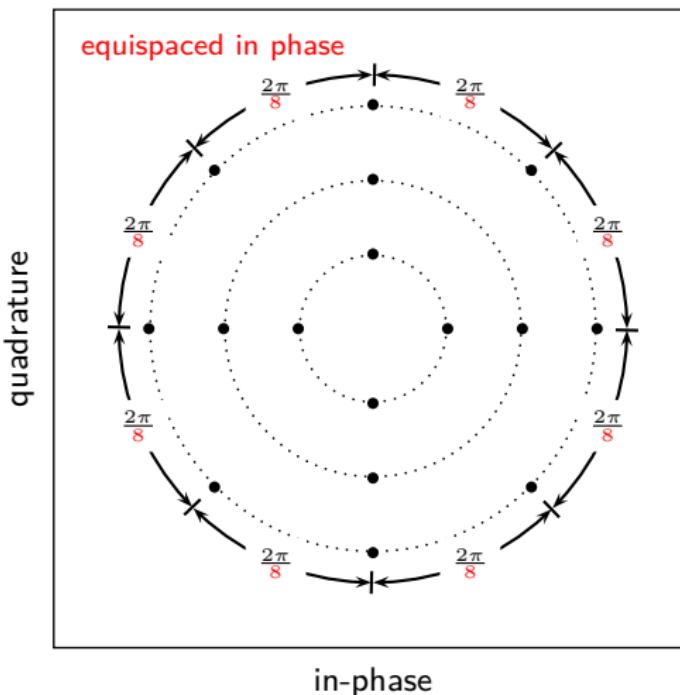
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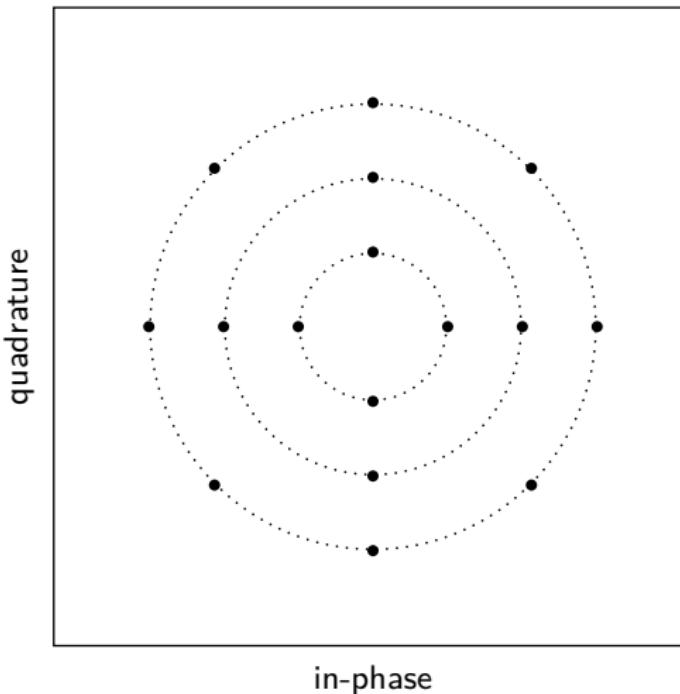
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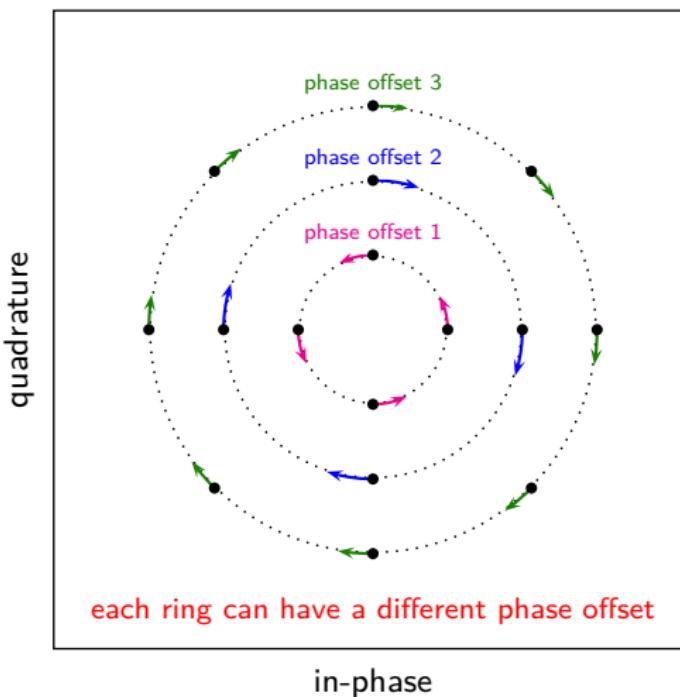
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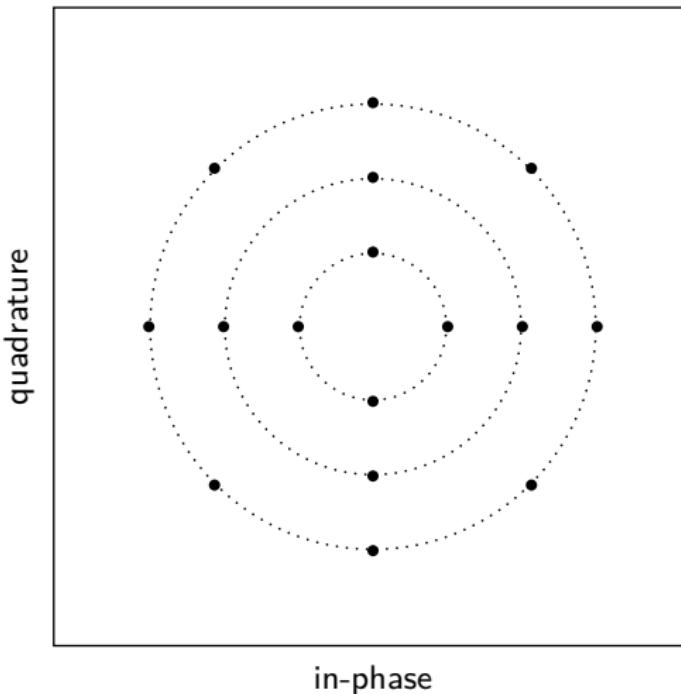
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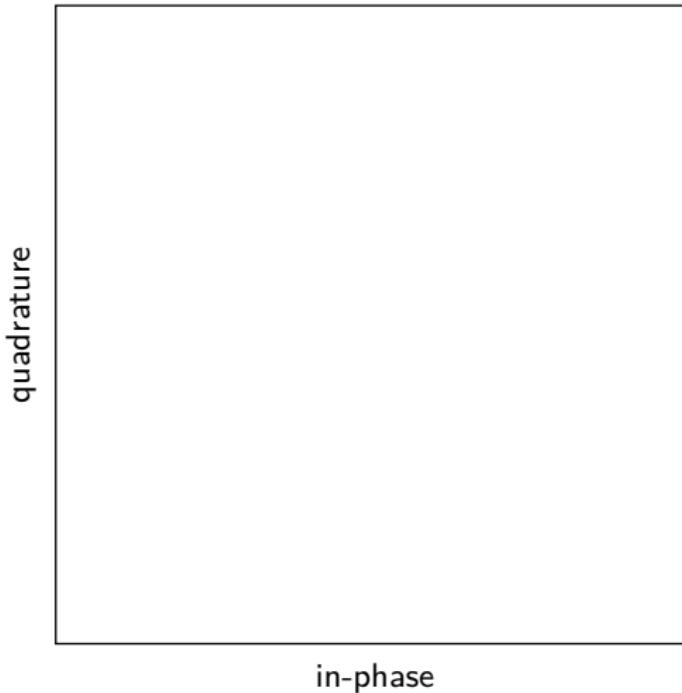


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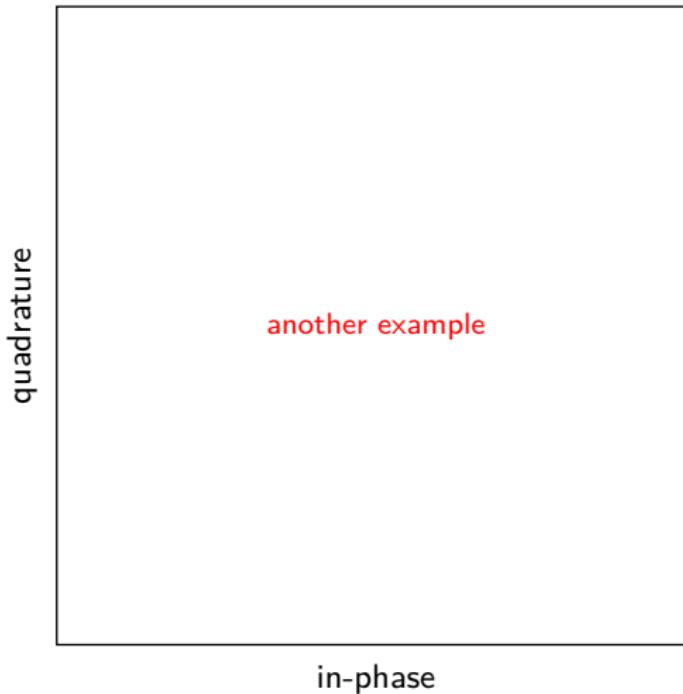
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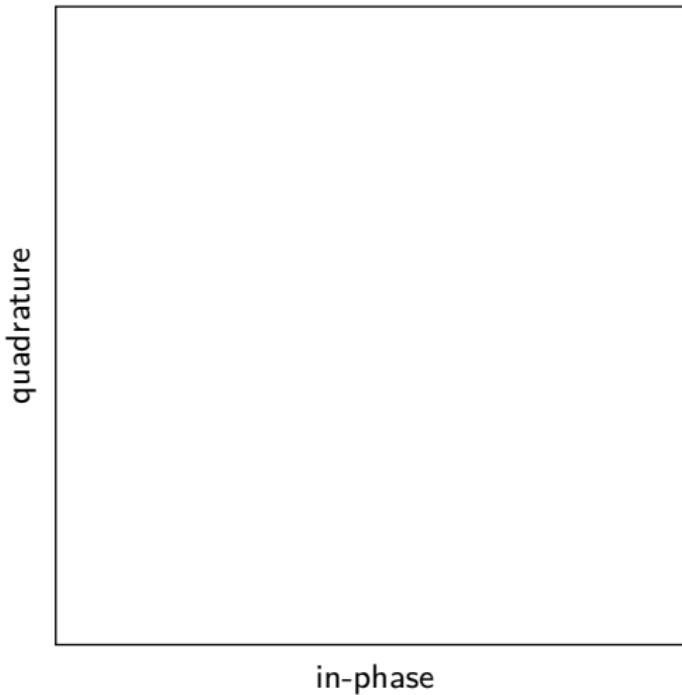


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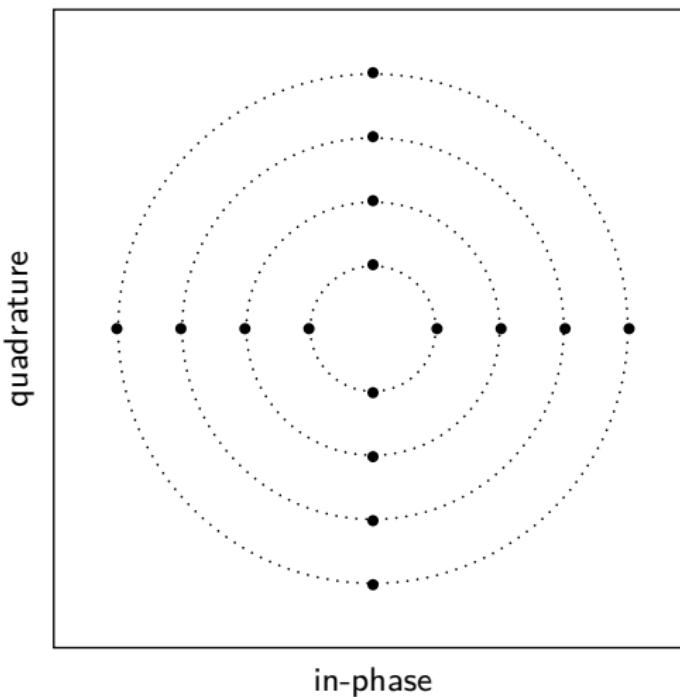
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Introduction  
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Detection Methods  
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APSK Optimization  
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Conclusions  
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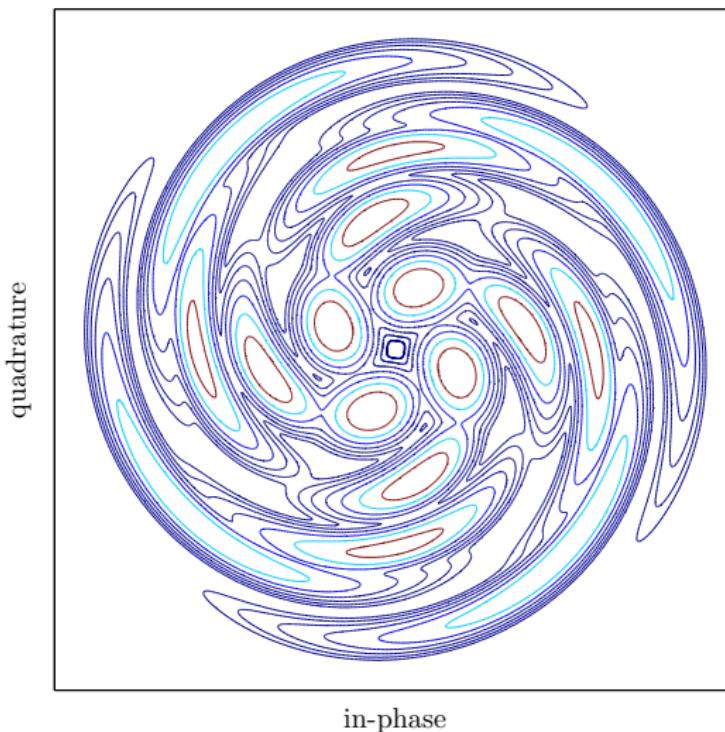
Introduction  
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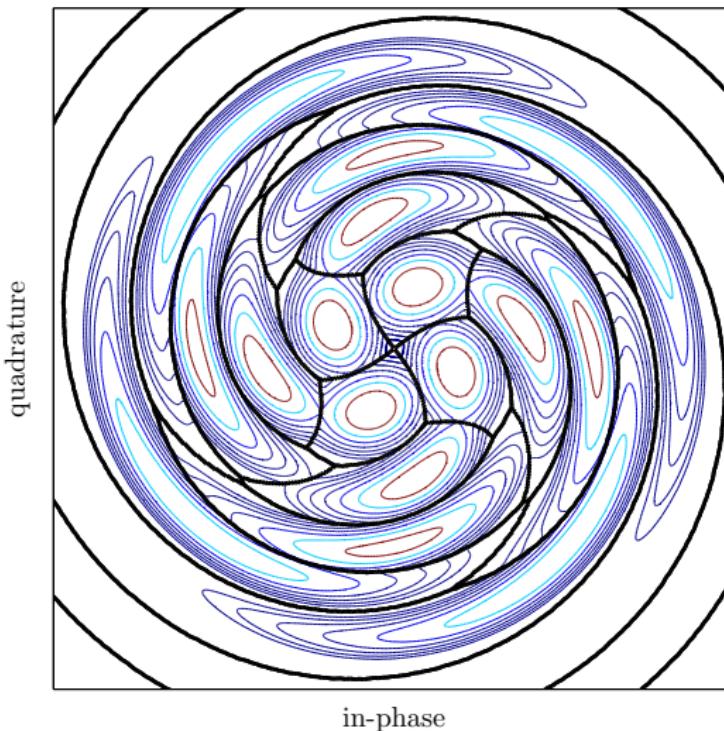
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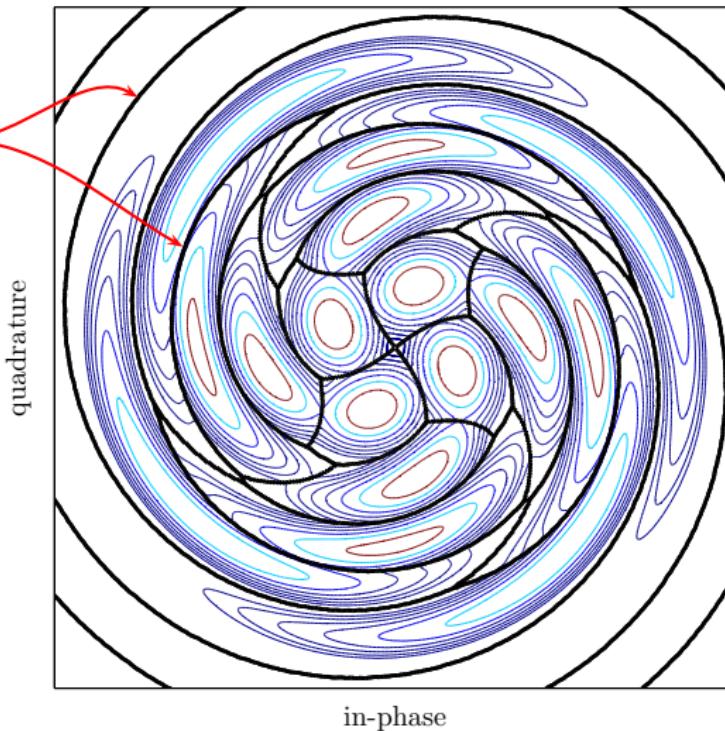
## PDF for (4,4,4,4)-APSK at $P = -4$ dBm

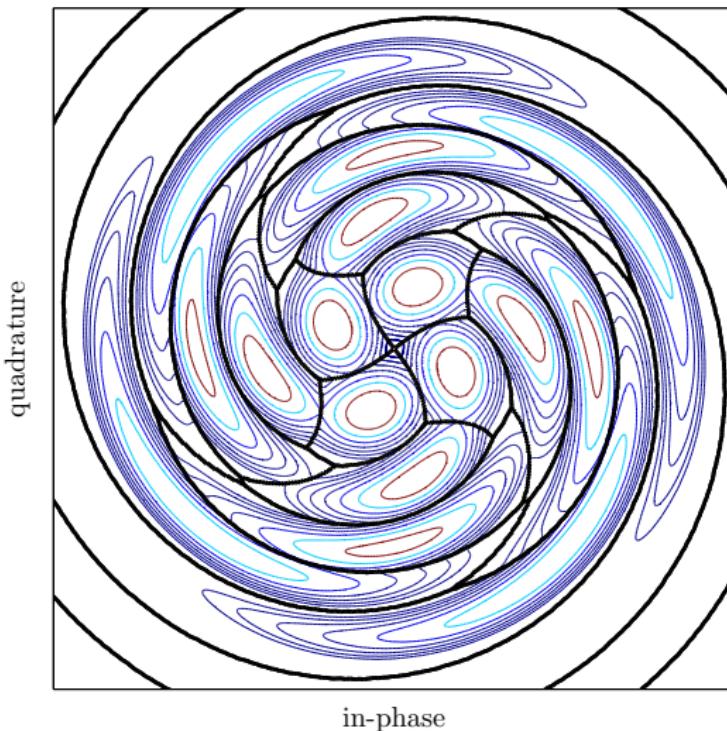
PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

ML decision boundaries



PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

Introduction  
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Detection Methods  
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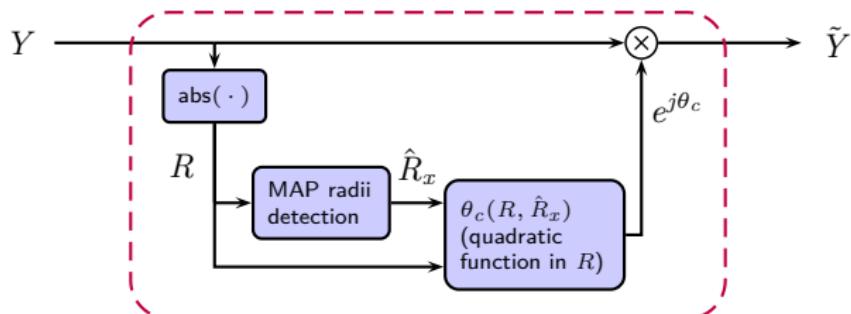
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## Nonlinear Phase Postcompensation

See [Ho, 2005],[Lau and Kahn, 2007].

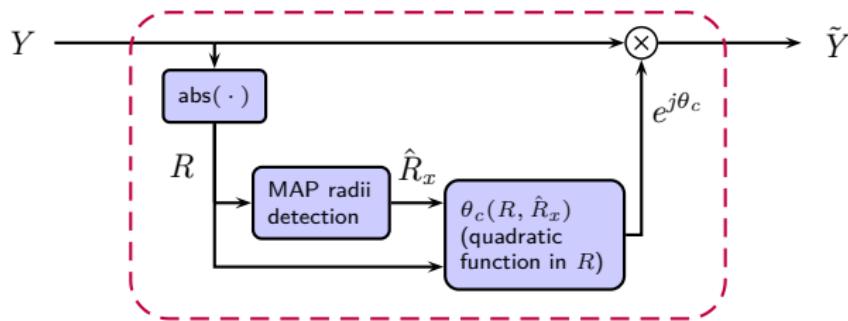
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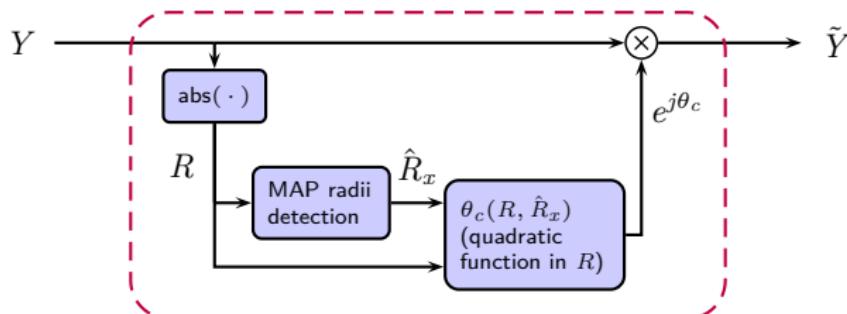
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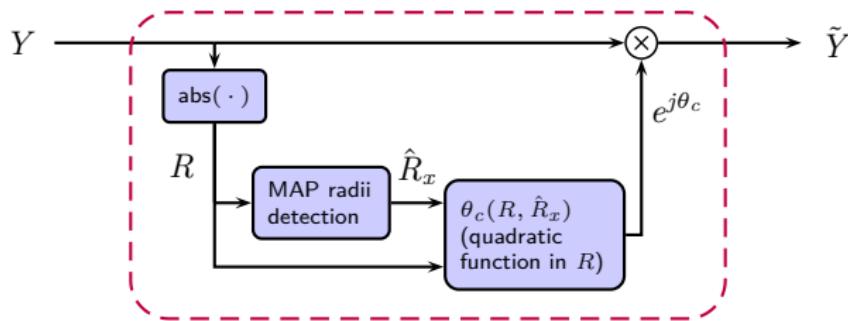
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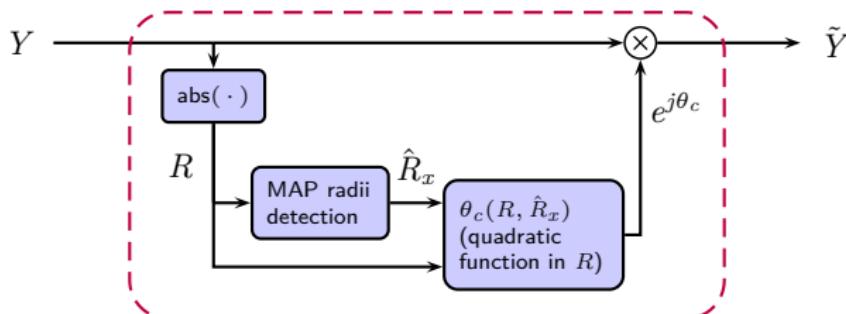
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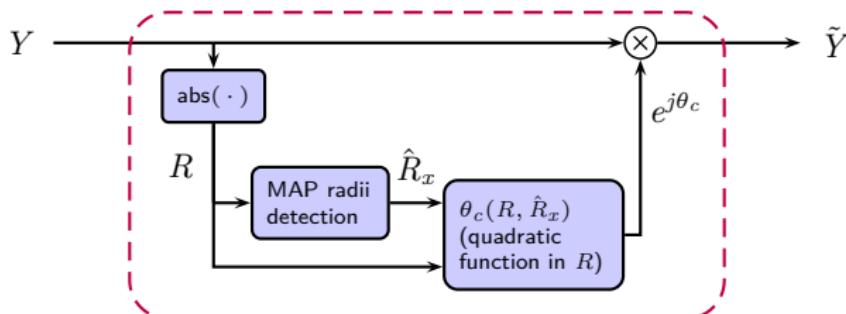
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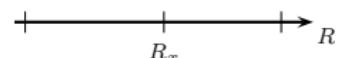
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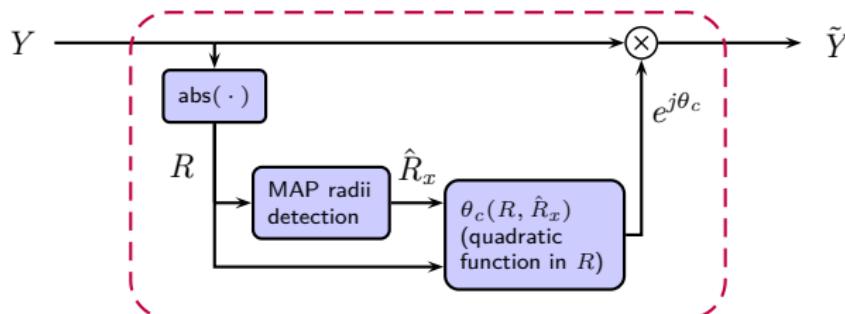


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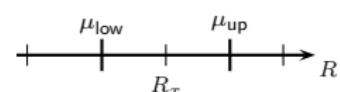


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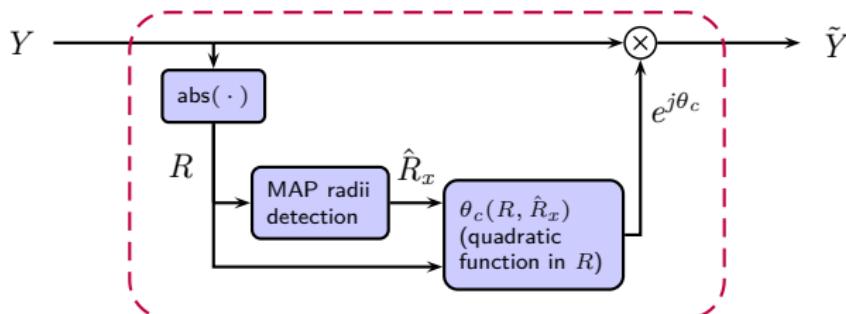


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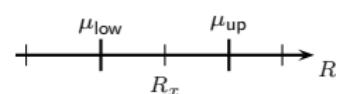


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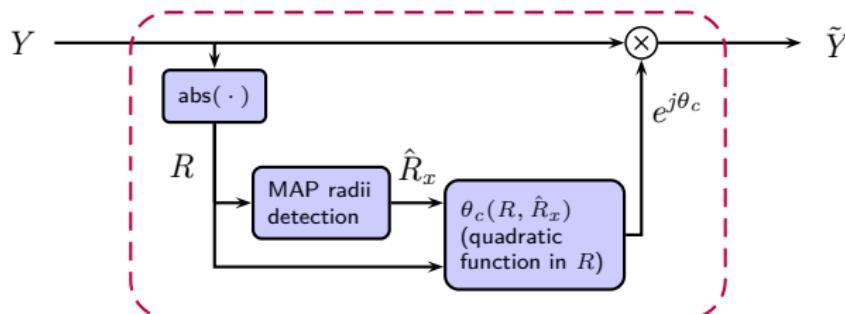
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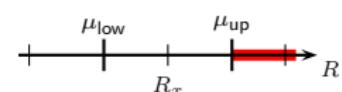
$$f_{\tilde{Y}|X=x_i}(y) = f_{Y|X=x_i}(y) \text{ rotated by } \left\{ \begin{array}{l} \end{array} \right.$$

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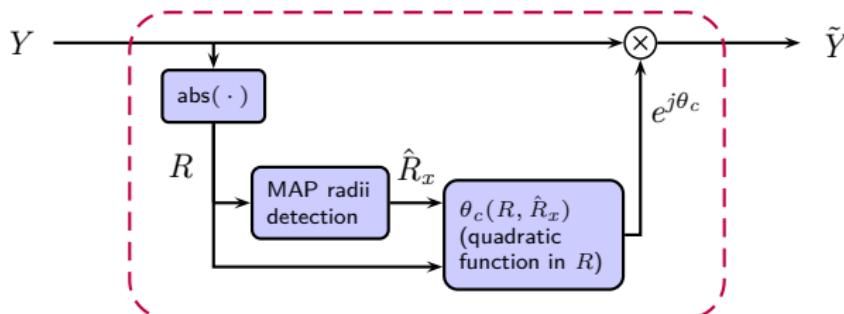
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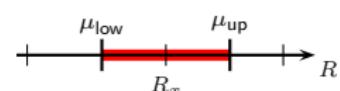
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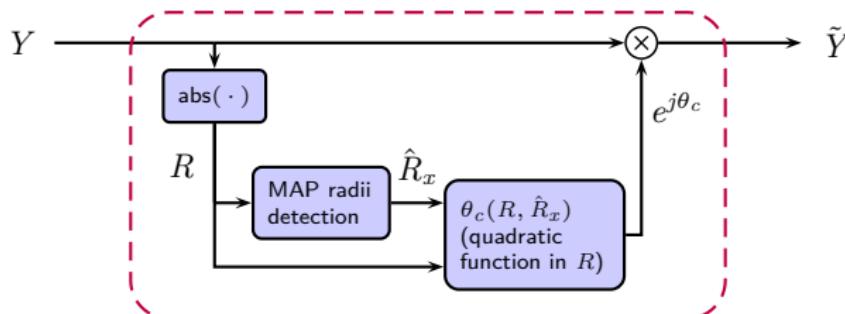
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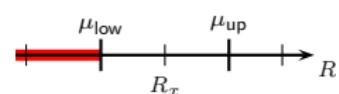
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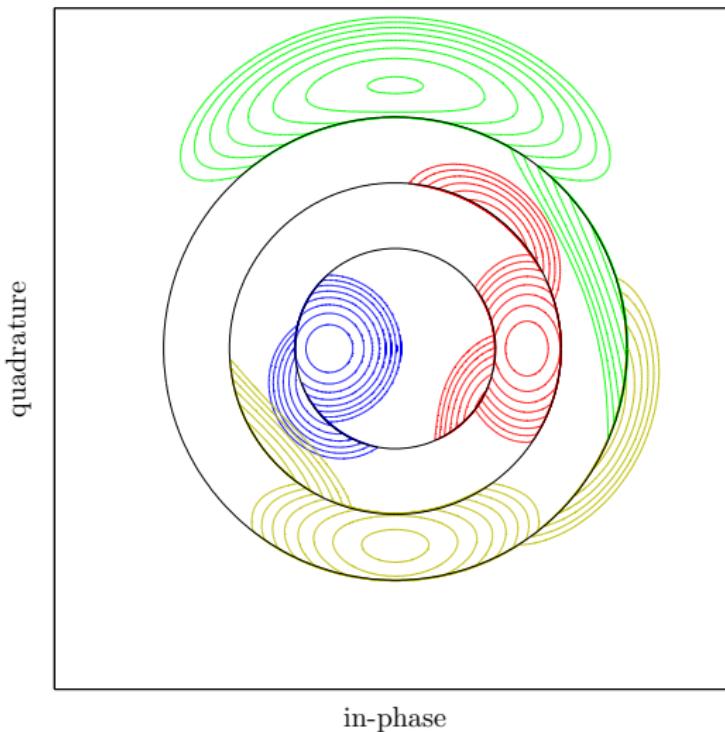


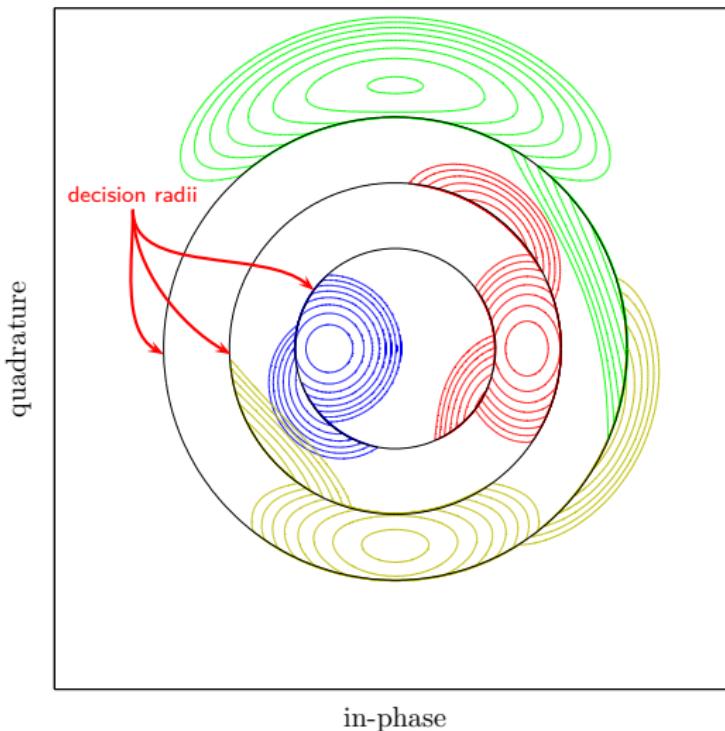
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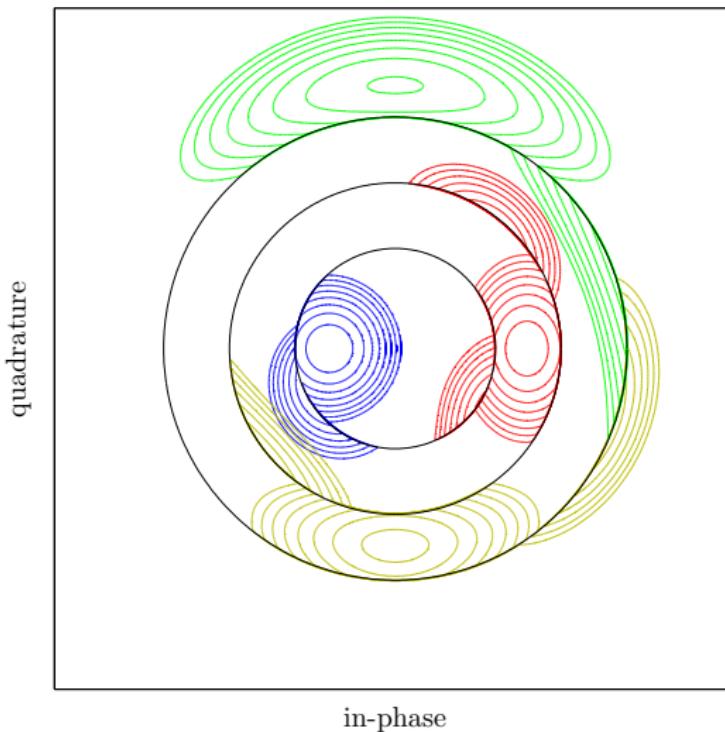


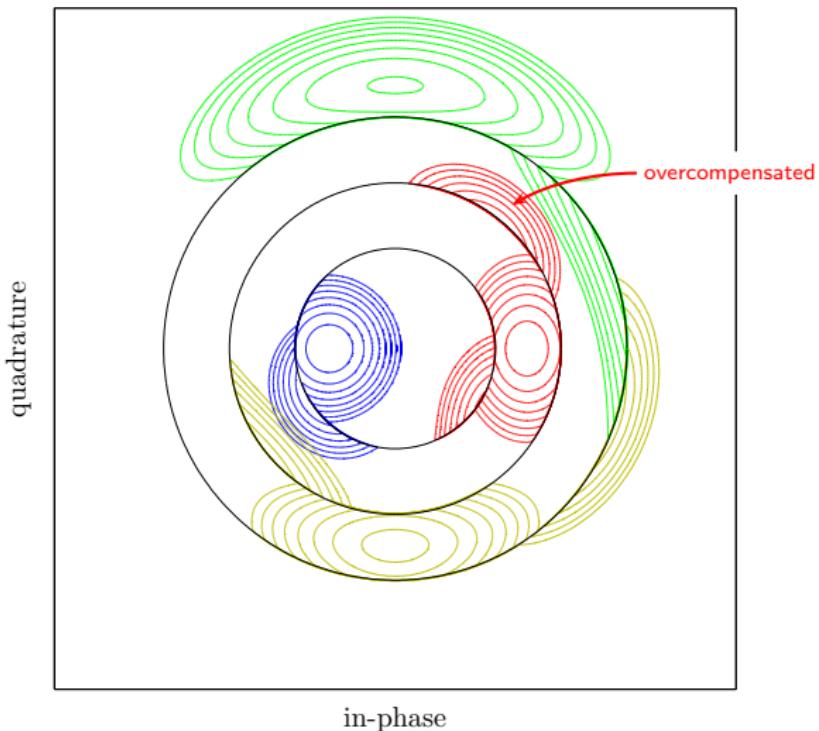
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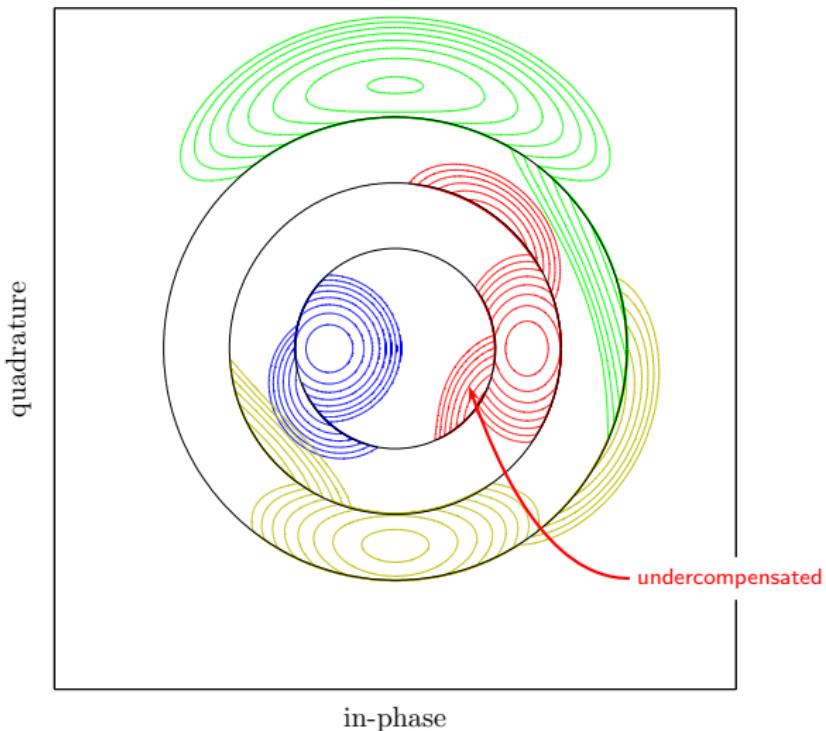
## Sliced PDF for (4,4,4,4)-APSK at $P = -4$ dBm

Sliced PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

Sliced PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

Sliced PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

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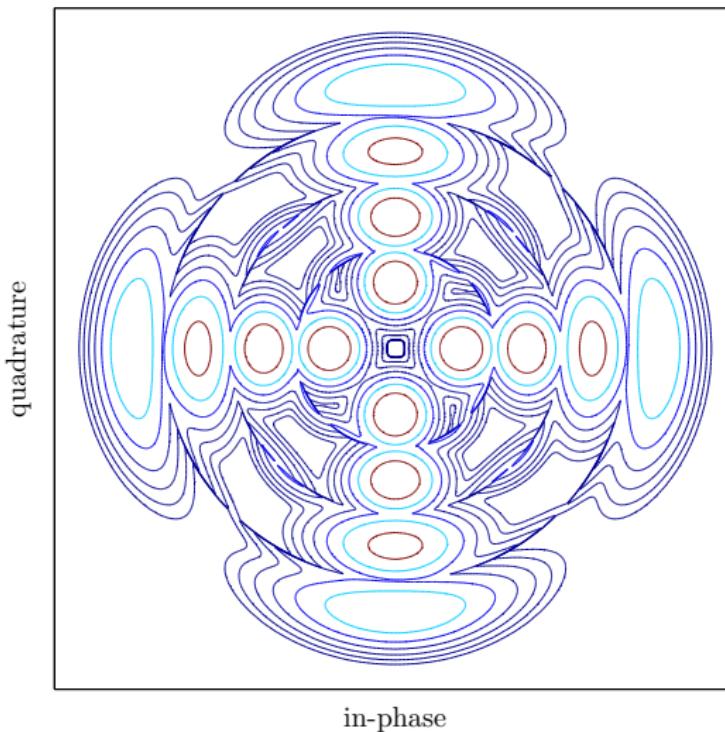
Introduction  
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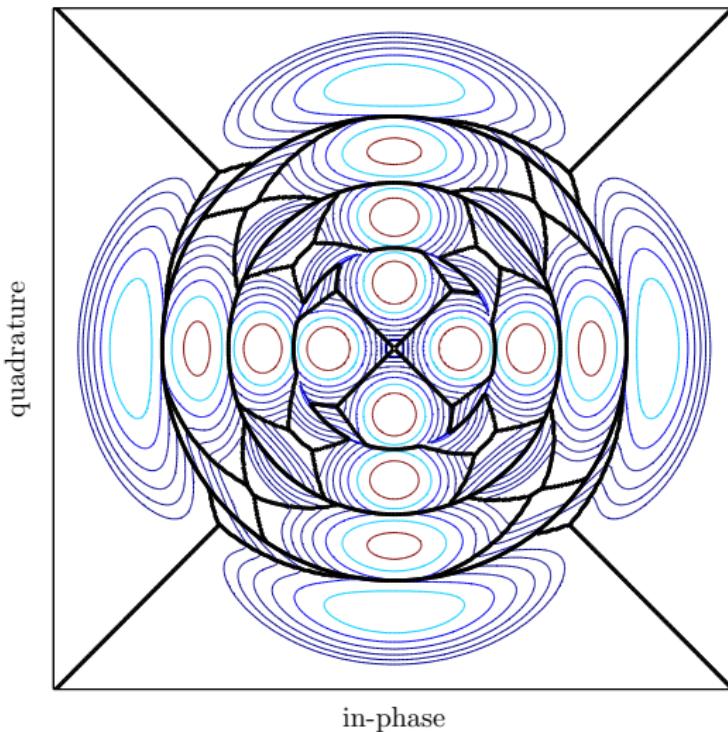
Detection Methods  
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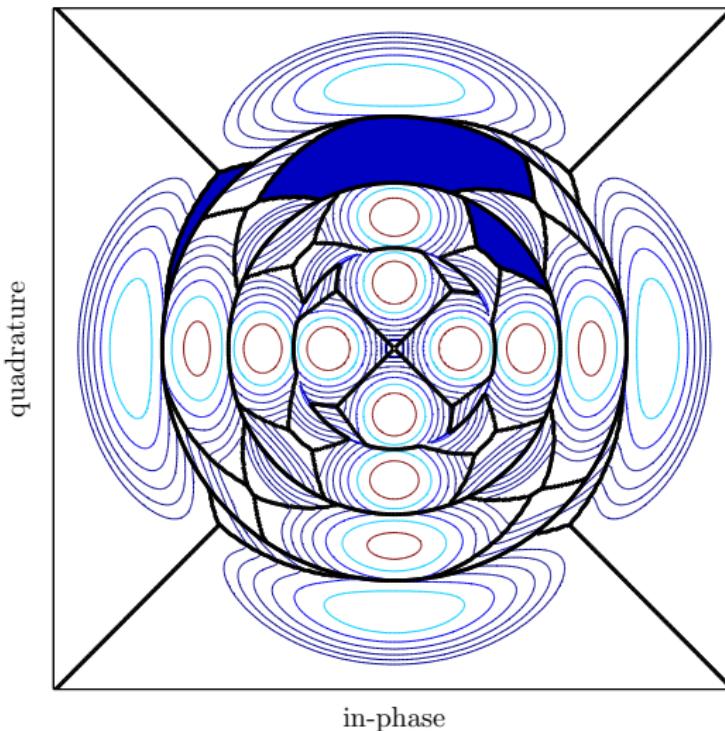
APSK Optimization  
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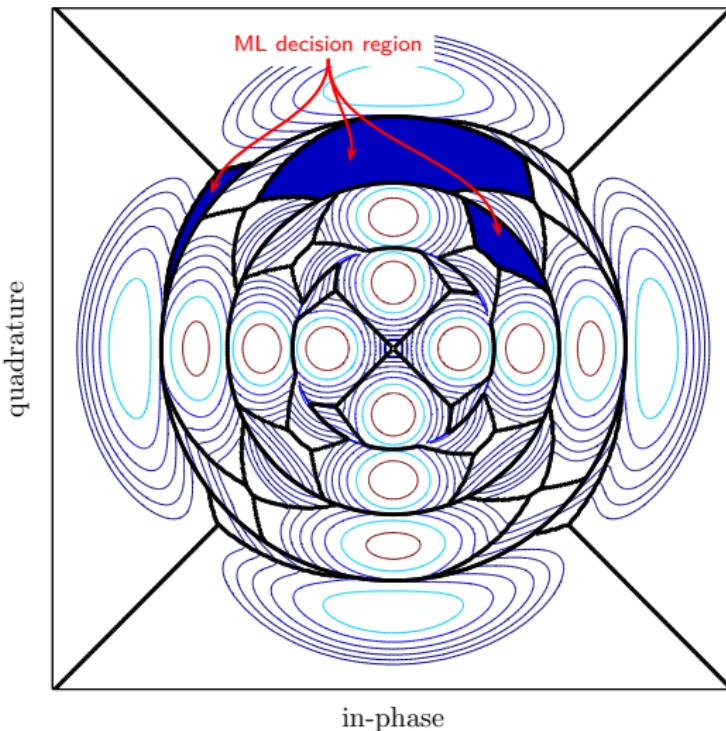
Conclusions  
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## ML Detection based on $\tilde{Y}$

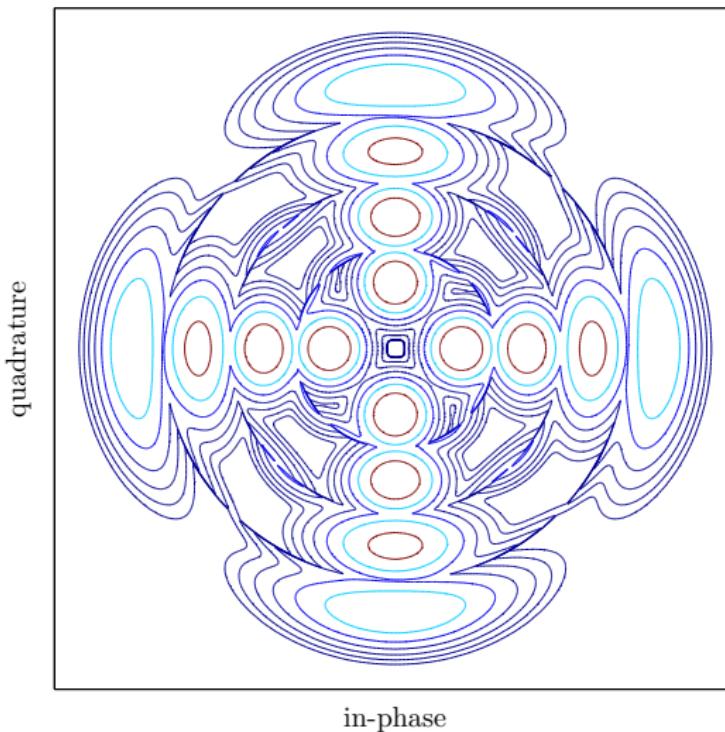
ML Detection based on  $\tilde{Y}$ 

ML Detection based on  $\tilde{Y}$ 

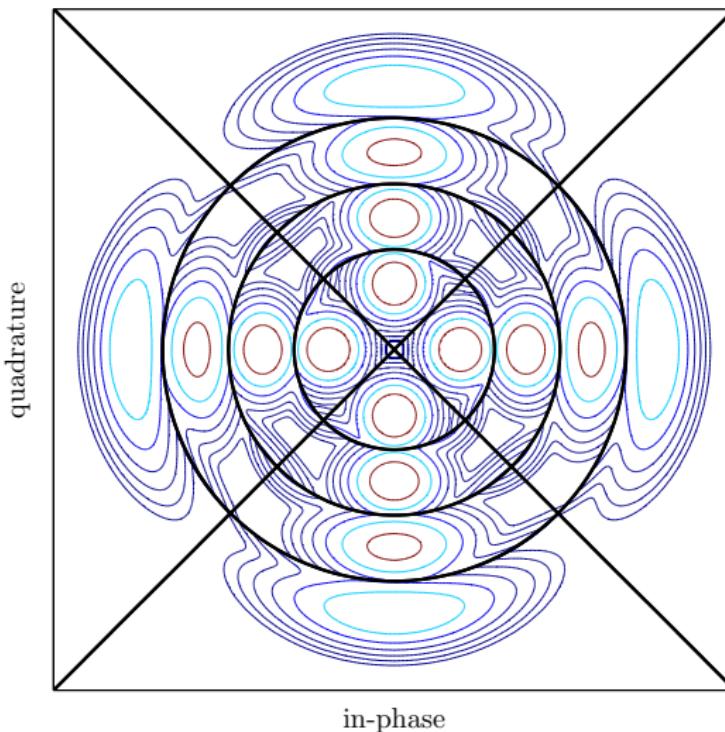
ML Detection based on  $\tilde{Y}$ 

ML Detection based on  $\tilde{Y}$ 

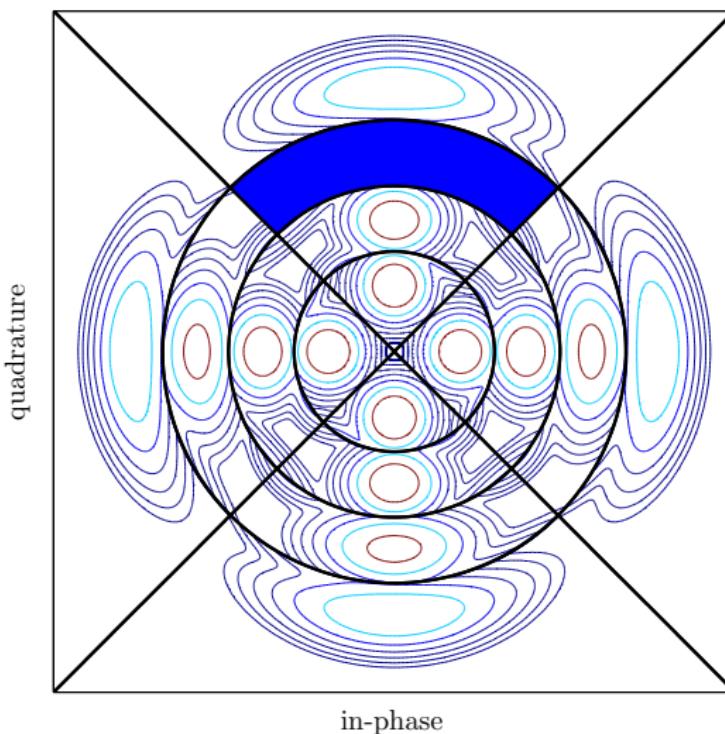
## Two-Stage Detection



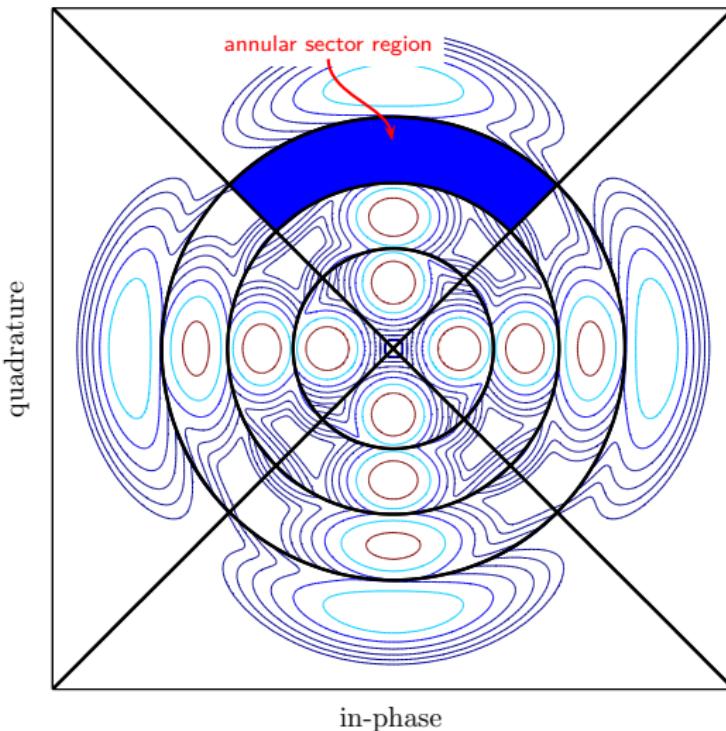
## Two-Stage Detection



## Two-Stage Detection



## Two-Stage Detection



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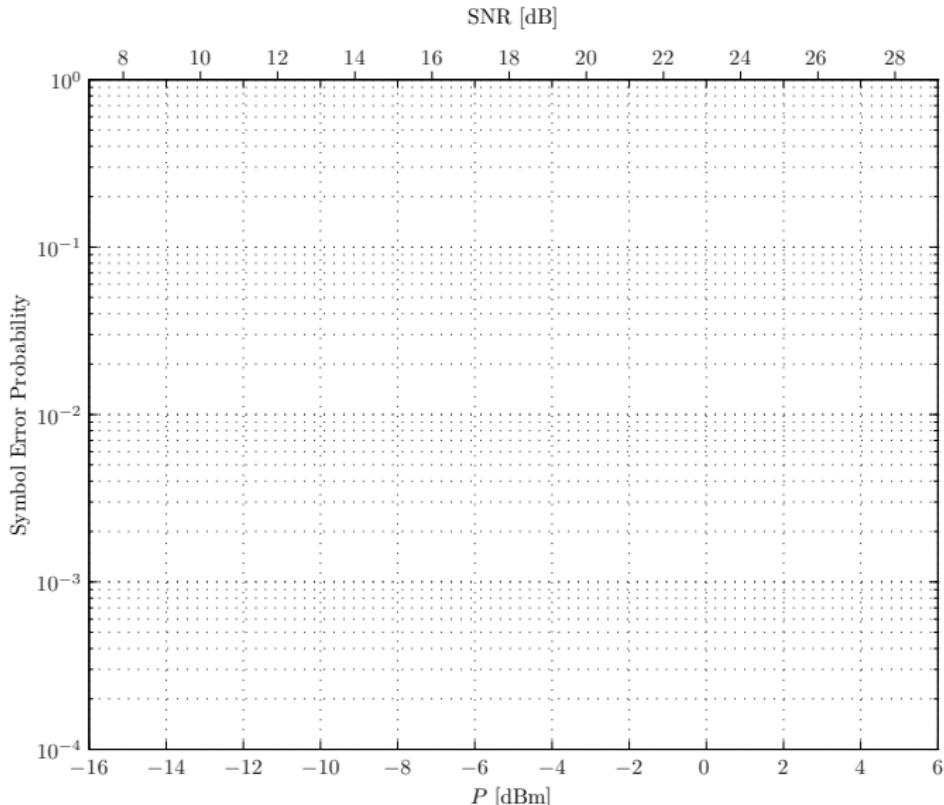
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## ML vs. Two-Stage Detection

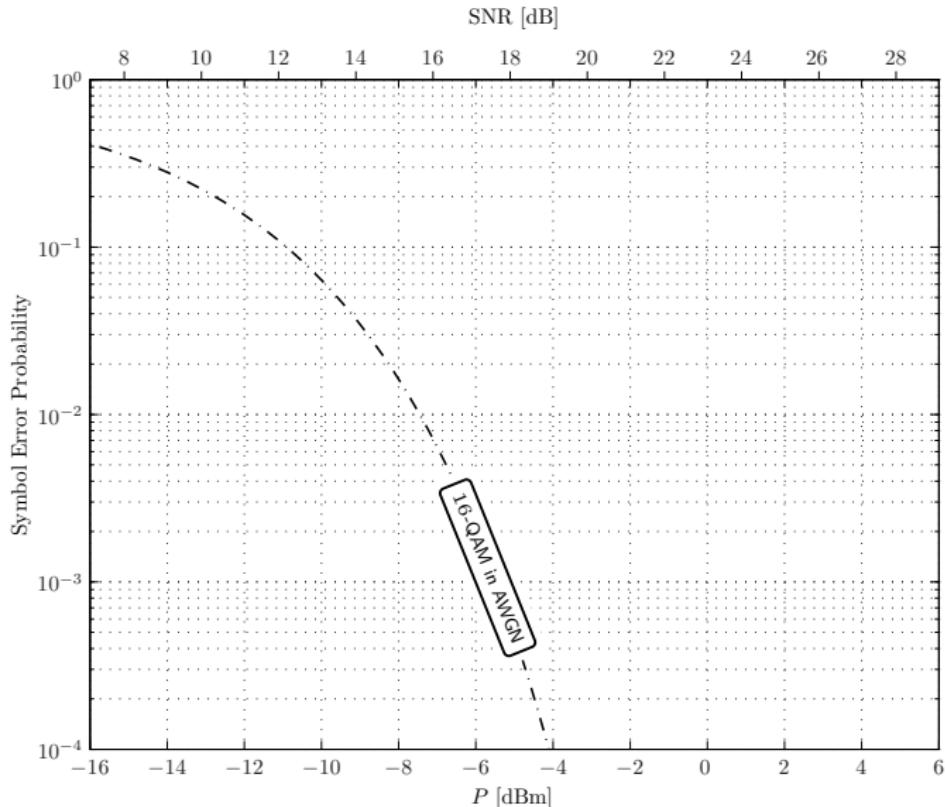
## ML vs. Two-Stage Detection

Question: How much do we lose in terms of symbol error probability?

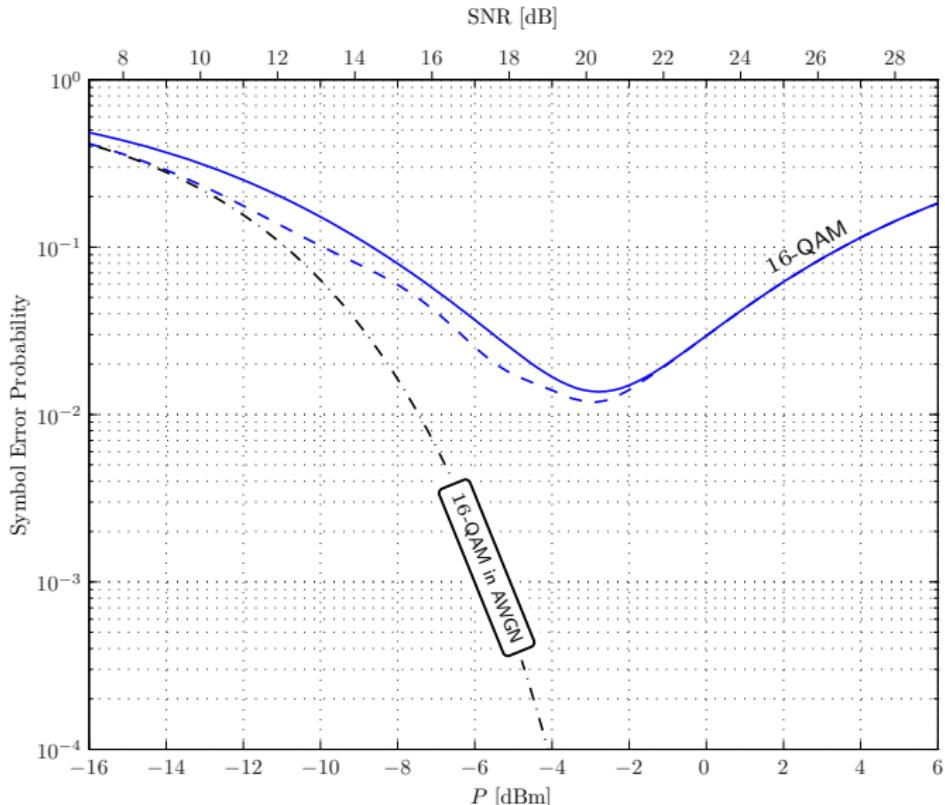
## ML vs. Two-Stage Detection



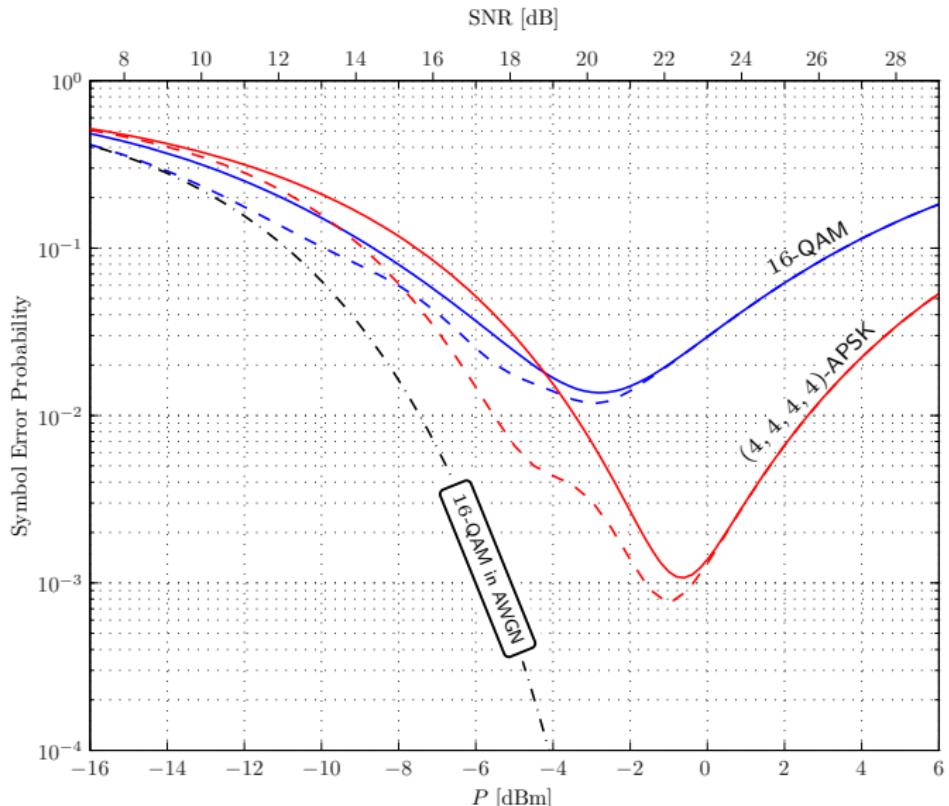
## ML vs. Two-Stage Detection



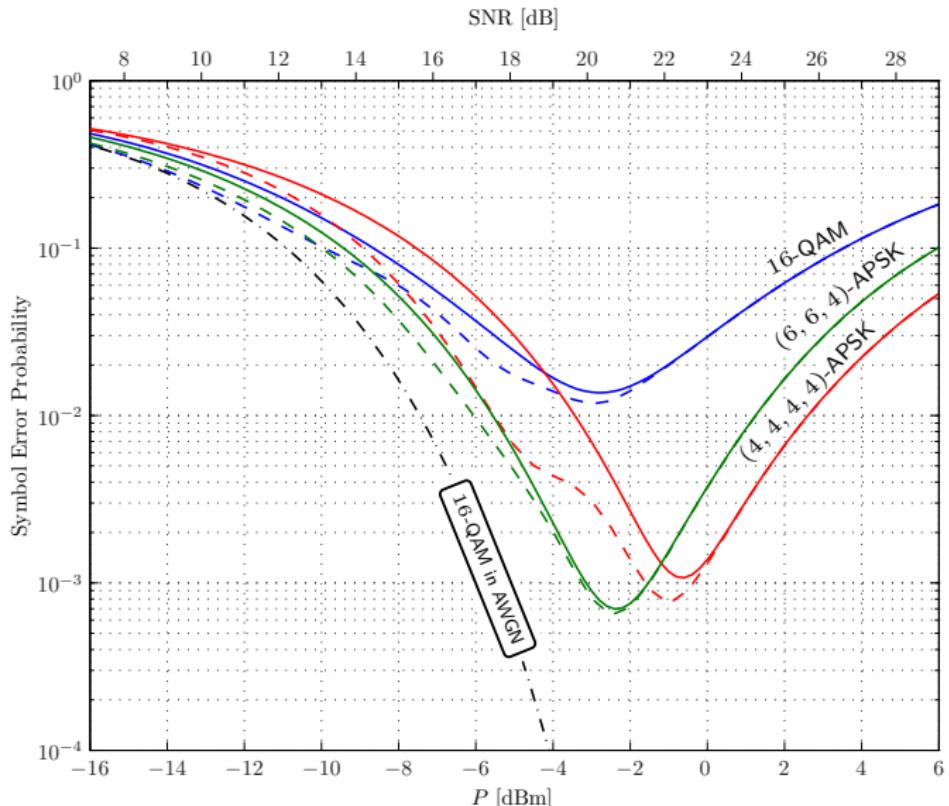
## ML vs. Two-Stage Detection



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## ML vs. Two-Stage Detection



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## Optimization Opportunities for APSK

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## Optimization Opportunities for APSK

- How many rings? More rings for increasing input power due to phase noise.
- How many points per ring?
- What radii distribution?
- What phase offset? Two-stage detection is insensitive to a phase offset.

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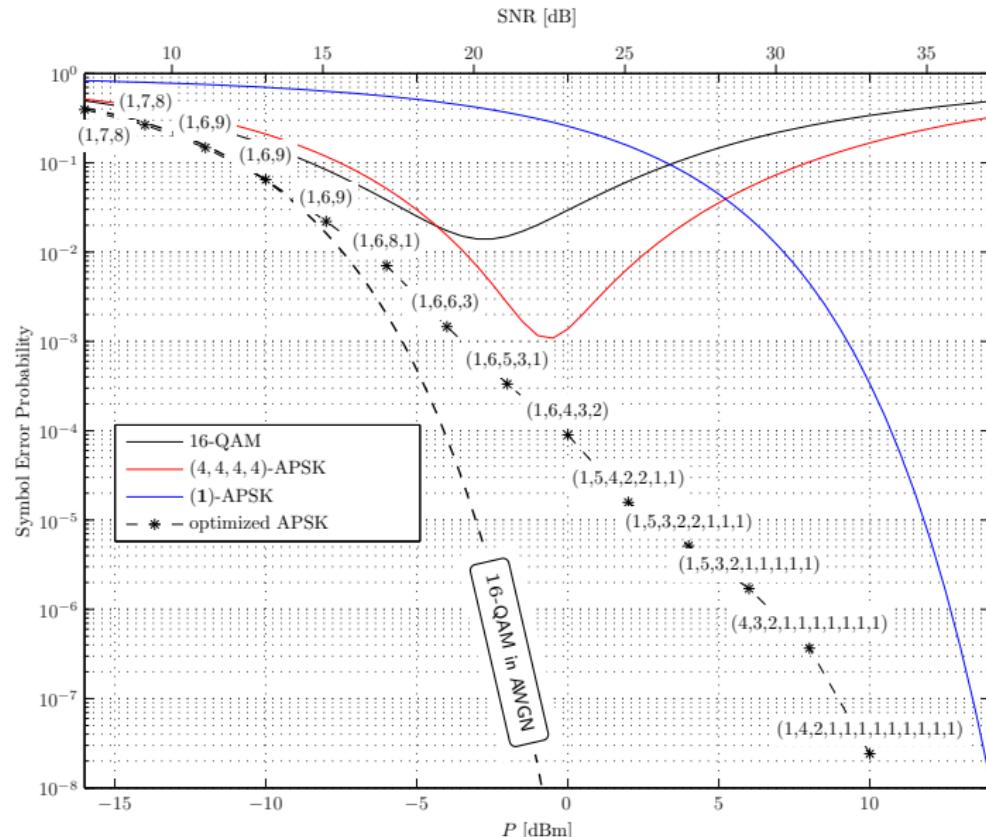
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APSK Optimization  
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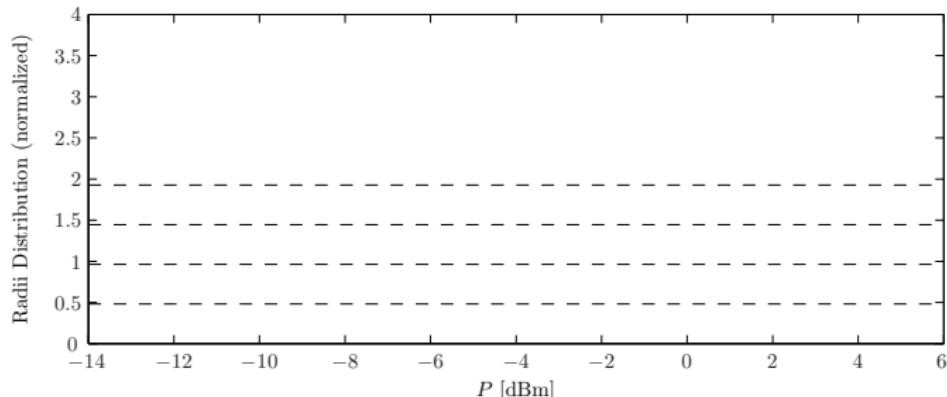
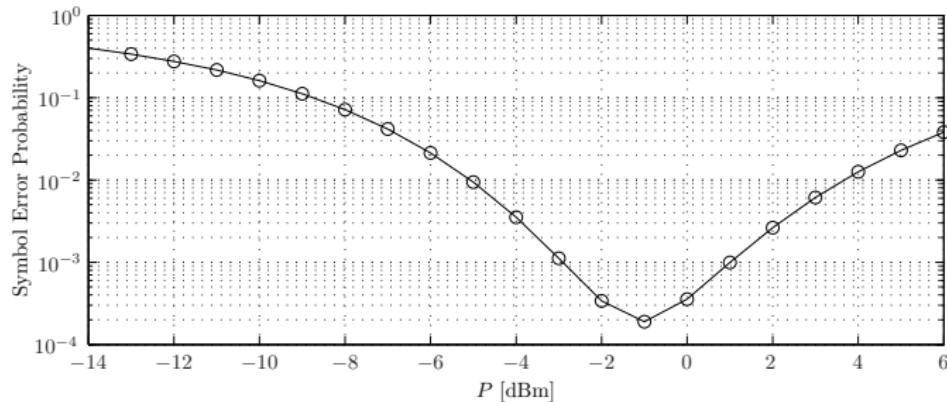
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## Optimizing the Number of Rings and Points per Ring

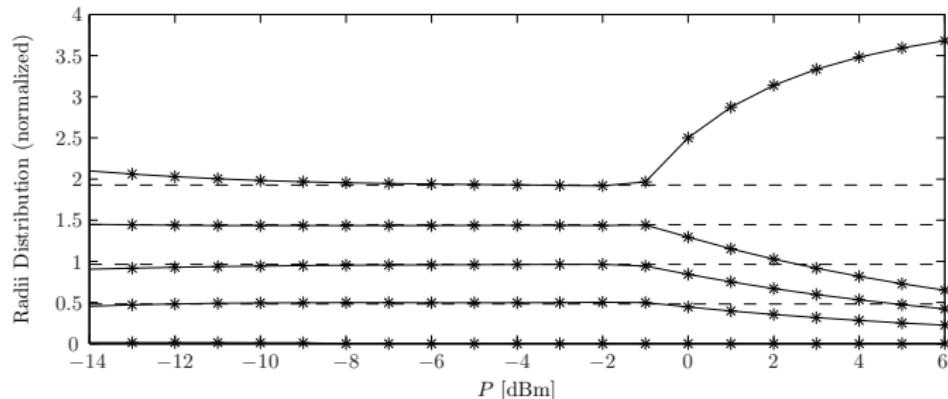
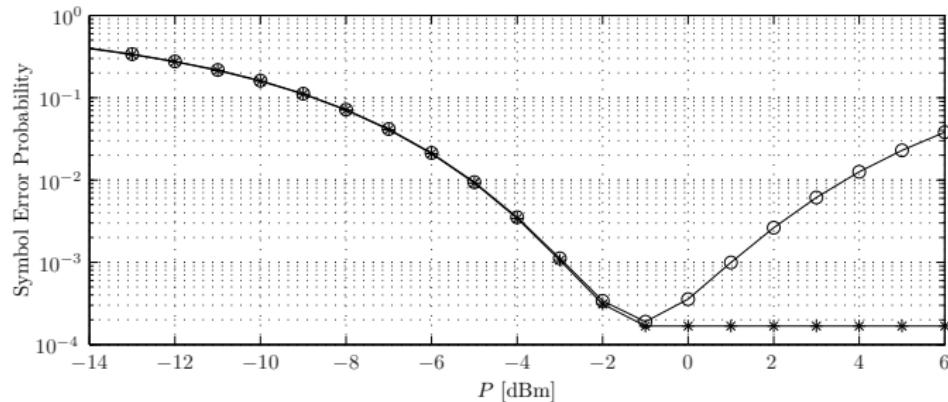
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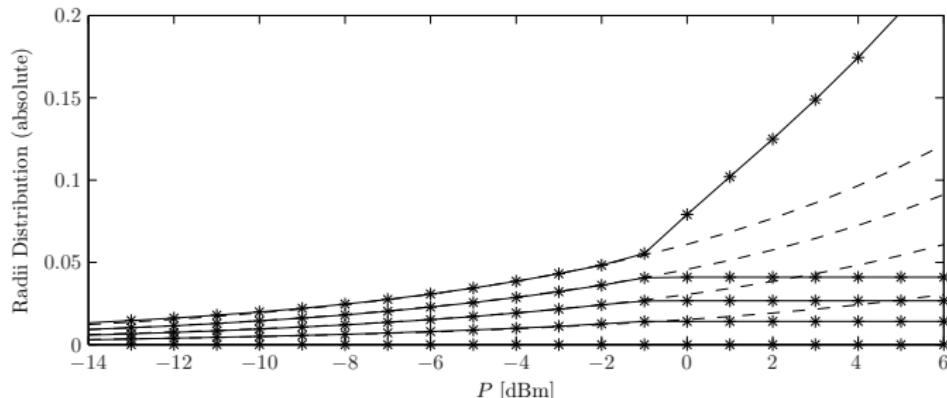
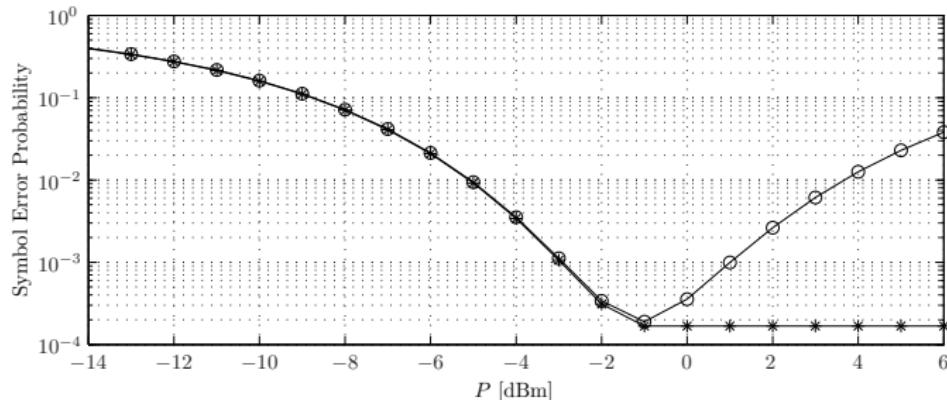
## Radii Optimization for (1,6,5,3,1)-APSK



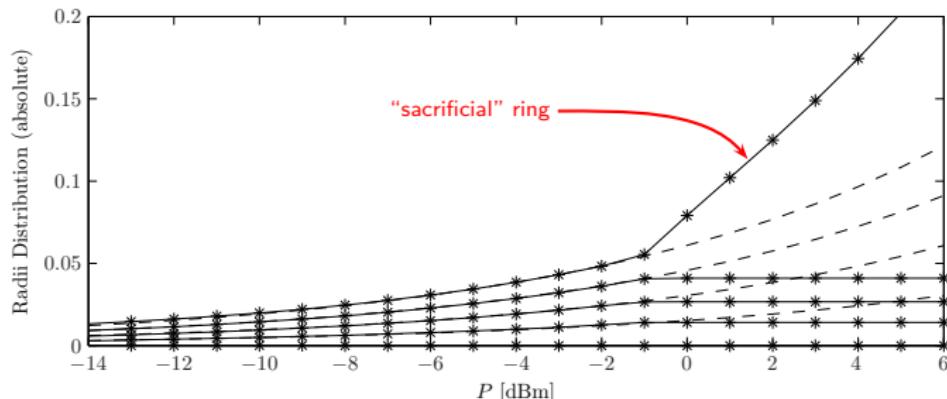
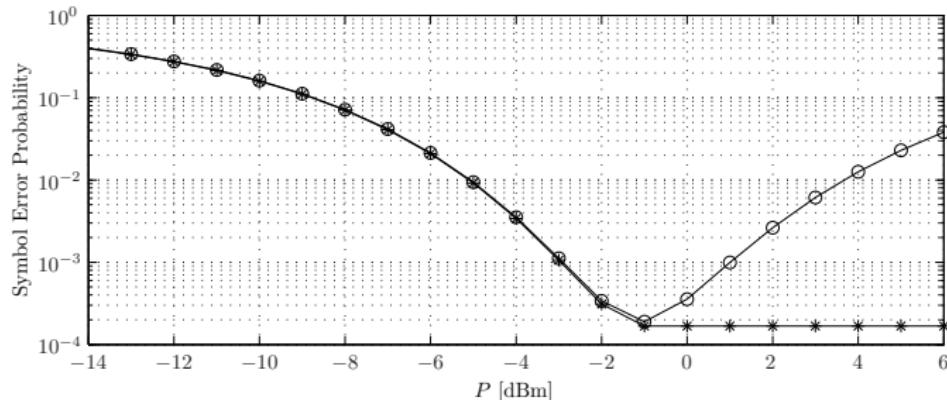
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## Joint Optimization

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- Computationally heavy to do for all 16-point APSK constellations
- Joint optimization for up to 6 rings shows almost no improvements over equidistant spacing of rings for low and moderate non-linearities.

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## Conclusions and Future Work

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- 2 Long term future work could incorporate other channel models including dispersion.

## References and Further Reading

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