

# Constellation Optimization for Coherent Optical Channels Distorted by Nonlinear Phase Noise

Christian Häger<sup>1</sup> Alexandre Graell i Amat<sup>1</sup> Alex Alvarado<sup>2</sup> Erik Agrell<sup>1</sup>

<sup>1</sup>Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden,  
Fiber Optic Communications Research Center



<sup>2</sup>Department of Engineering, University of Cambridge, UK

*{christian.haege, alexandre.graell, agrell}@chalmers.se, alex.alvarado@ieee.org}*

GLOBECOM, Anaheim, CA – December 4, 2012



# Motivation

## Motivation

- Higher order modulation formats for optical communications to **increase spectral efficiency**

## Motivation

- Higher order modulation formats for optical communications to **increase spectral efficiency**
- Optical channel suffers from distortions that are absent for example in wireless channels

## Motivation

- Higher order modulation formats for optical communications to **increase spectral efficiency**
- Optical channel suffers from distortions that are absent for example in wireless channels
- Focus here: **Nonlinear phase noise**

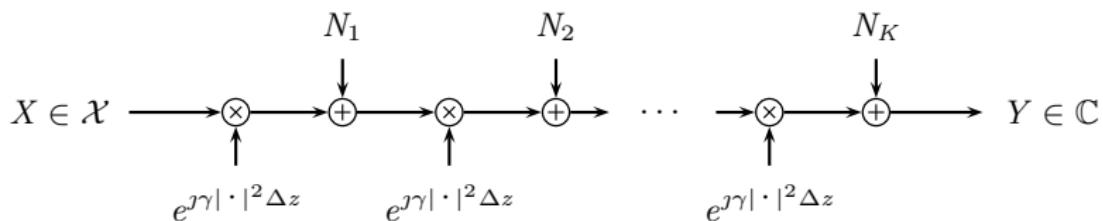
## Motivation

- Higher order modulation formats for optical communications to **increase spectral efficiency**
- Optical channel suffers from distortions that are absent for example in wireless channels
- Focus here: **Nonlinear phase noise**
- **Practical question:** How much can we gain by optimizing the constellation compared to standard QAM?

## Motivation

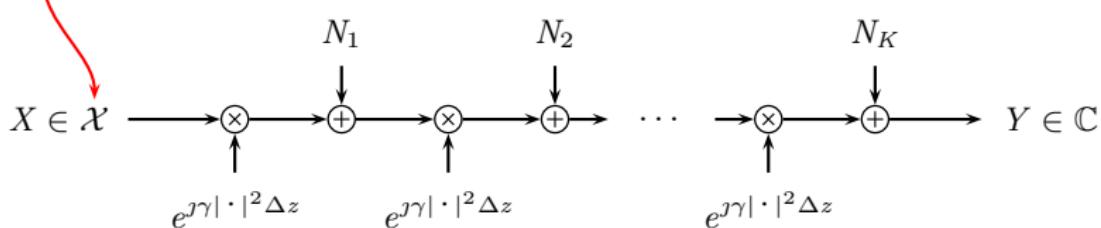
- Higher order modulation formats for optical communications to **increase spectral efficiency**
- Optical channel suffers from distortions that are absent for example in wireless channels
- Focus here: **Nonlinear phase noise**
- **Practical question:** How much can we gain by optimizing the constellation compared to standard QAM?
- **Theoretical question:** How do optimal constellations look like for very strong nonlinearities?

## Discrete-Time Channel Model

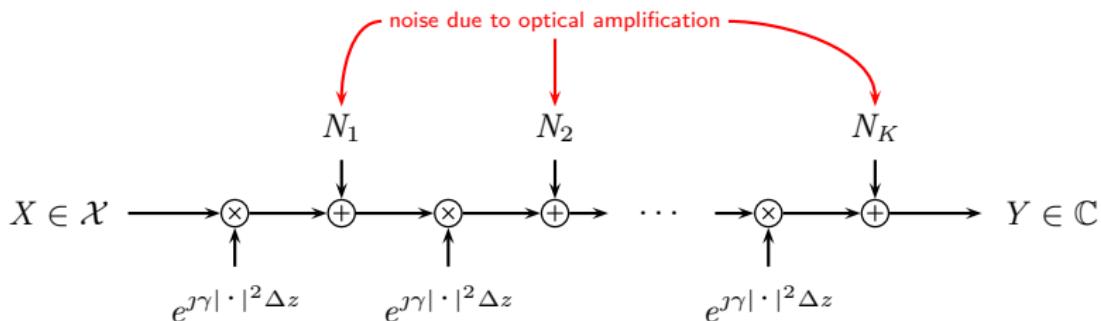


# Discrete-Time Channel Model

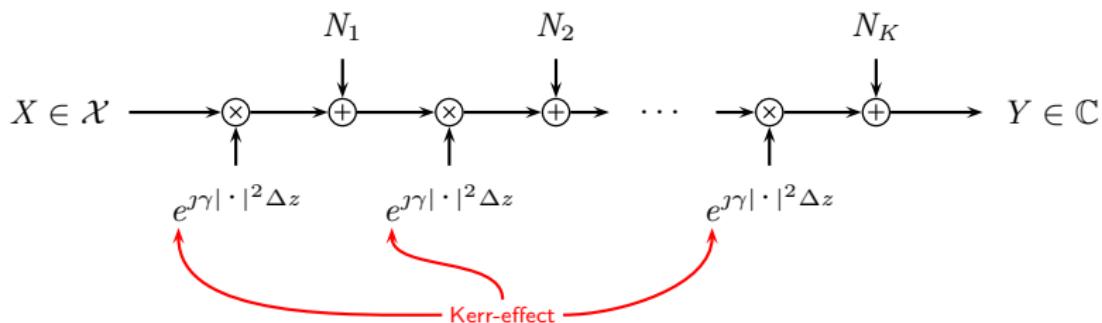
signal constellation, here  $|\mathcal{X}| = 16$



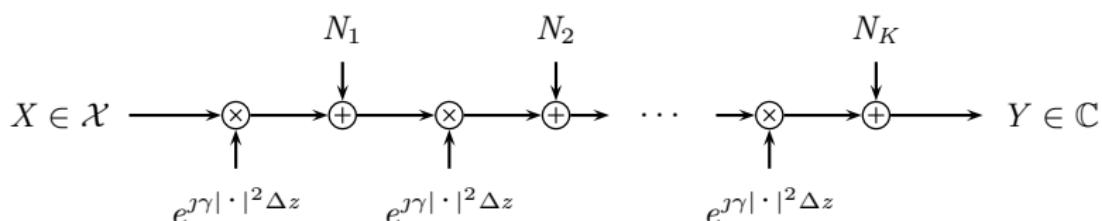
## Discrete-Time Channel Model



## Discrete-Time Channel Model

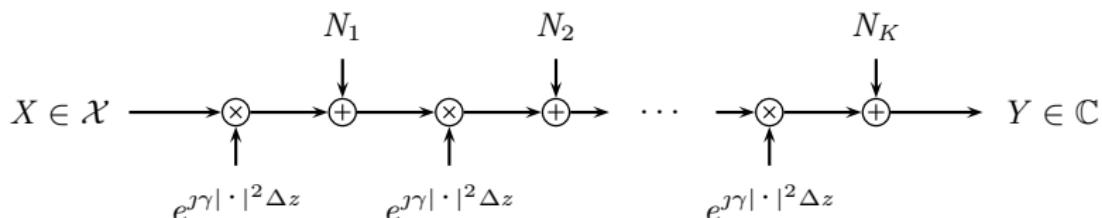


## Discrete-Time Channel Model



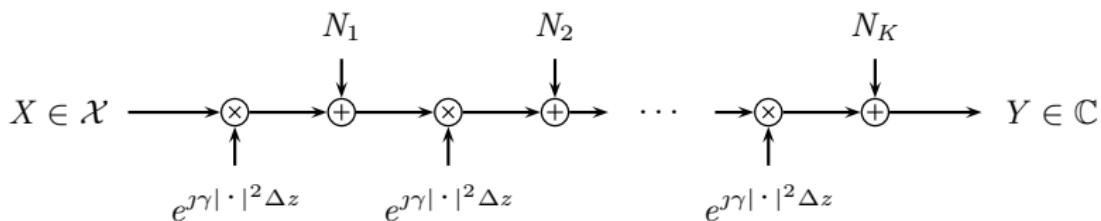
- **Additive noise** per segment  $N_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2/K)$

## Discrete-Time Channel Model



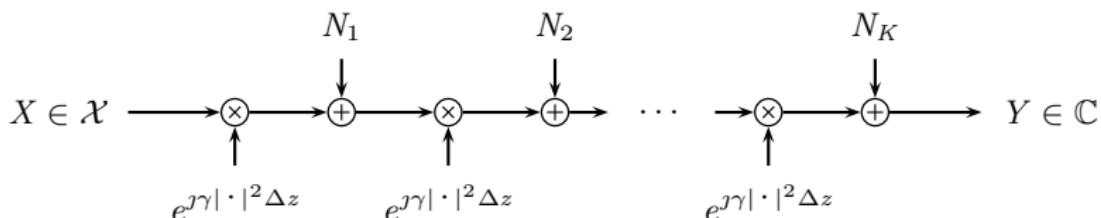
- **Additive noise per segment**  $N_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2 / K)$
- **Distributed Raman amplification**, i.e., the limit  $K \rightarrow \infty$

## Discrete-Time Channel Model



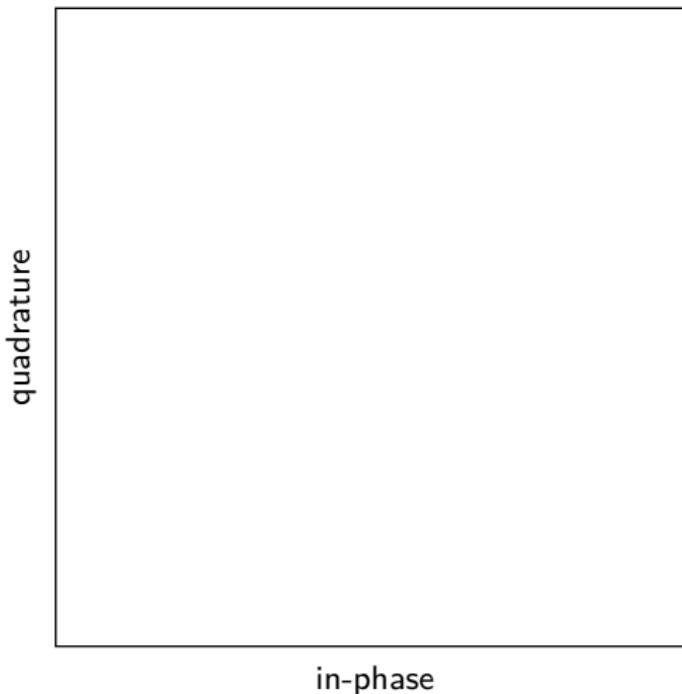
- **Additive noise per segment**  $N_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2/K)$
- **Distributed Raman amplification**, i.e., the limit  $K \rightarrow \infty$
- **Power**  $P = \mathbb{E} [|X|^2]$  and **signal-to(-additive)-noise ratio**  $\text{SNR} = P/\sigma^2$

## Discrete-Time Channel Model

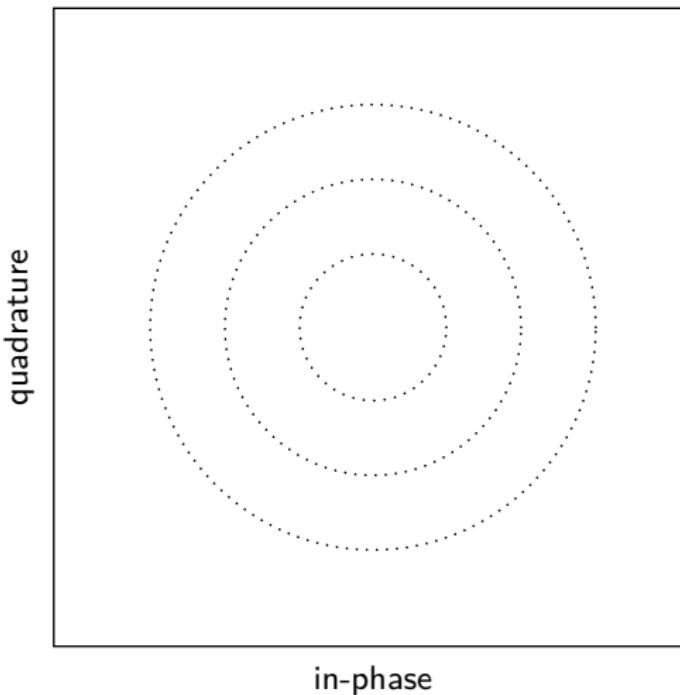


- Additive noise per segment  $N_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2/K)$
- Distributed Raman amplification, i.e., the limit  $K \rightarrow \infty$
- Power  $P = \mathbb{E} [|X|^2]$  and signal-to(-additive)-noise ratio  $\text{SNR} = P/\sigma^2$
- $L = 5500$  km, other parameters  $\gamma, \sigma^2$  taken from [Lau and Kahn, 2007]

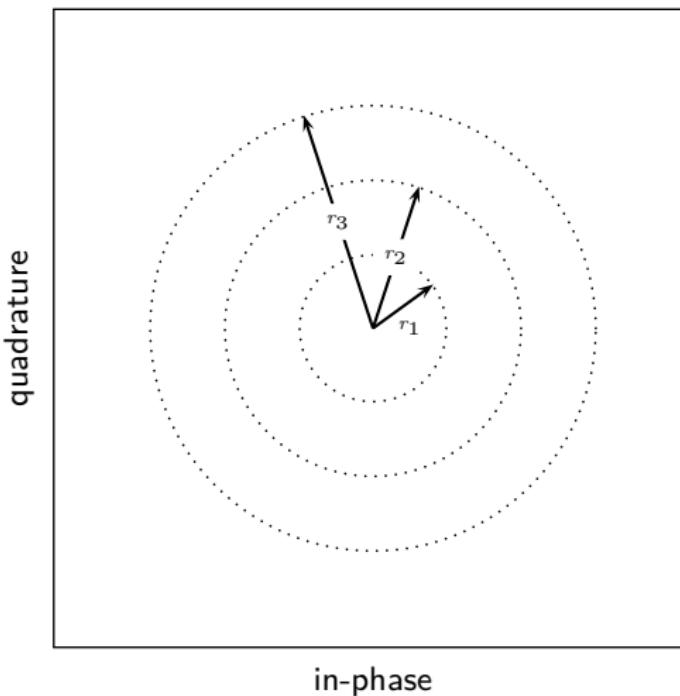
# Amplitude Phase-Shift Keying (APSK), Example



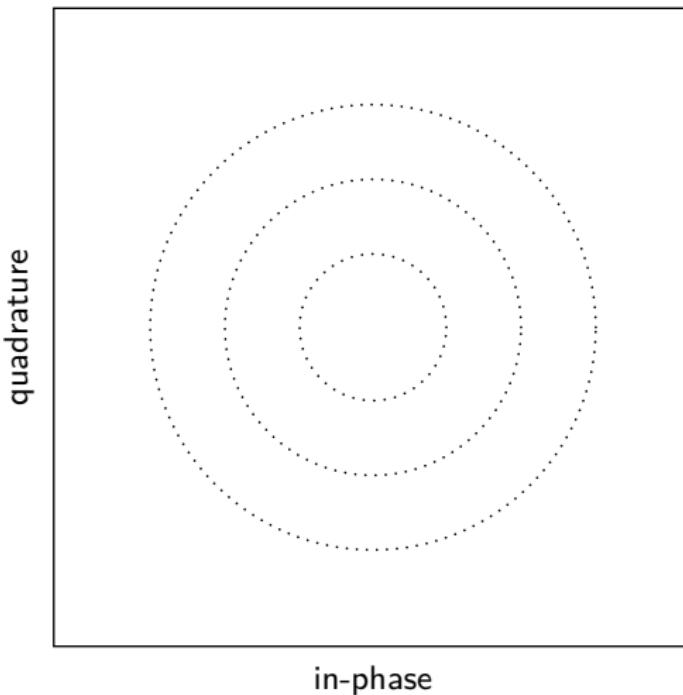
# Amplitude Phase-Shift Keying (APSK), Example



# Amplitude Phase-Shift Keying (APSK), Example

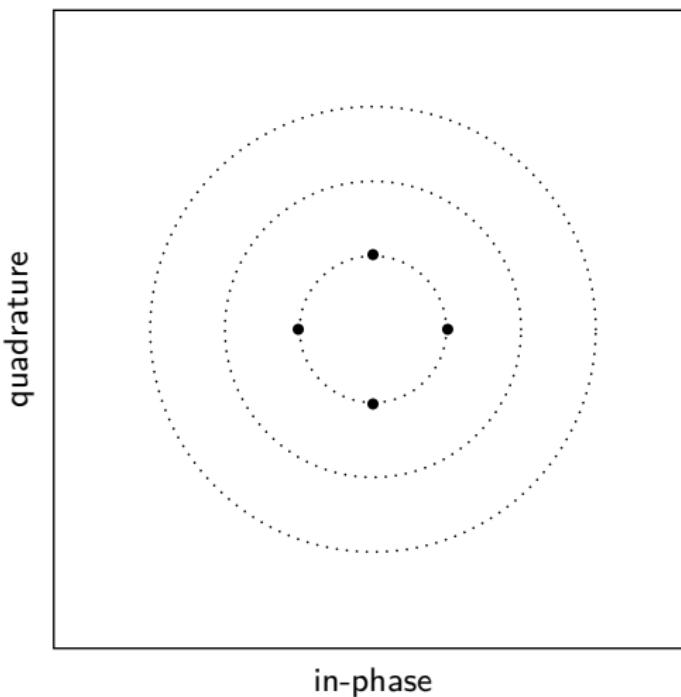


# Amplitude Phase-Shift Keying (APSK), Example

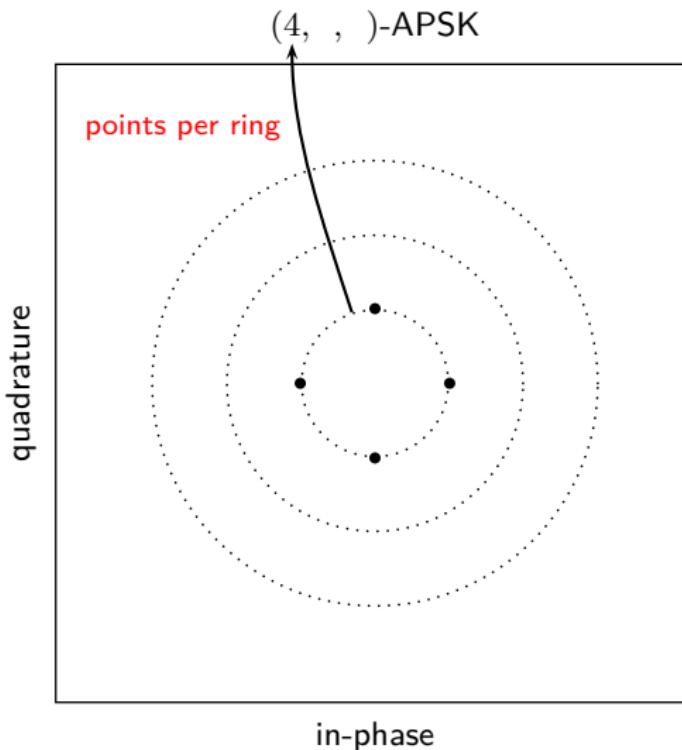


# Amplitude Phase-Shift Keying (APSK), Example

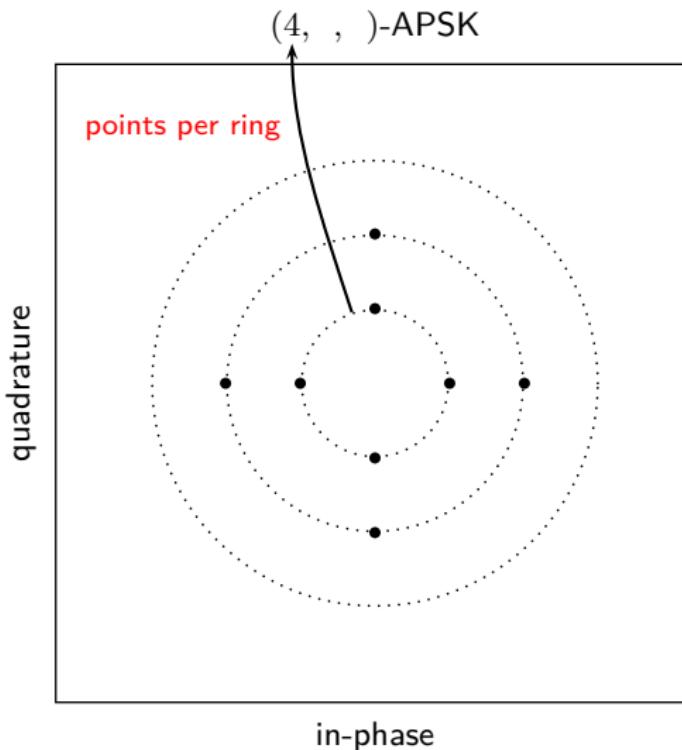
( , , )-APSK



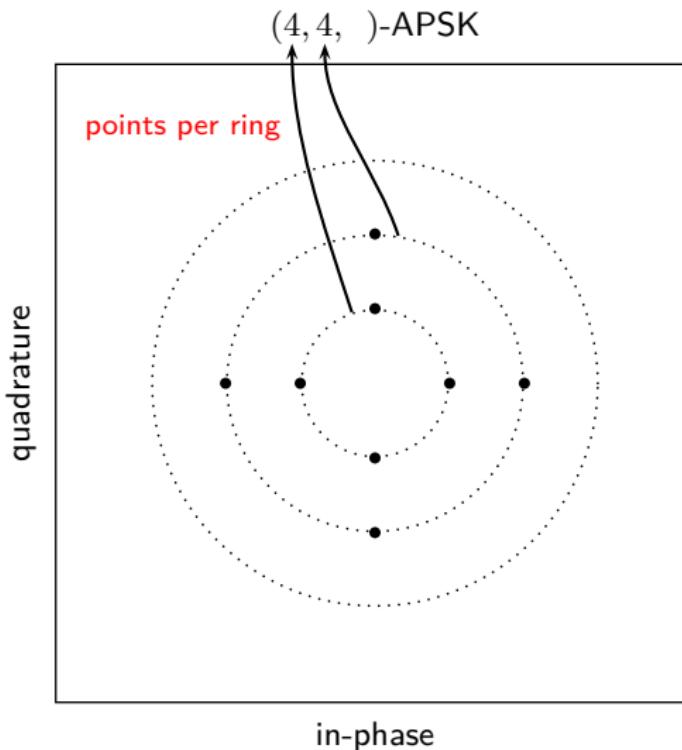
# Amplitude Phase-Shift Keying (APSK), Example



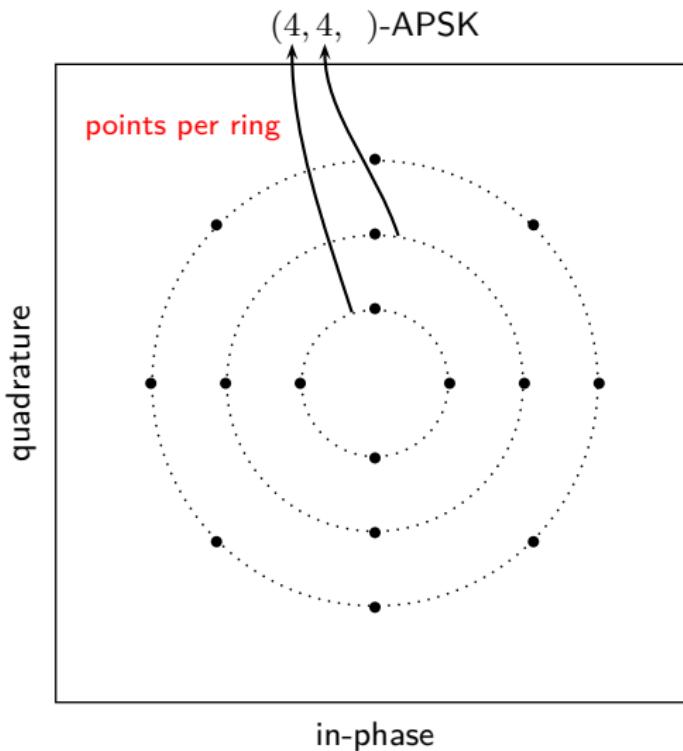
# Amplitude Phase-Shift Keying (APSK), Example



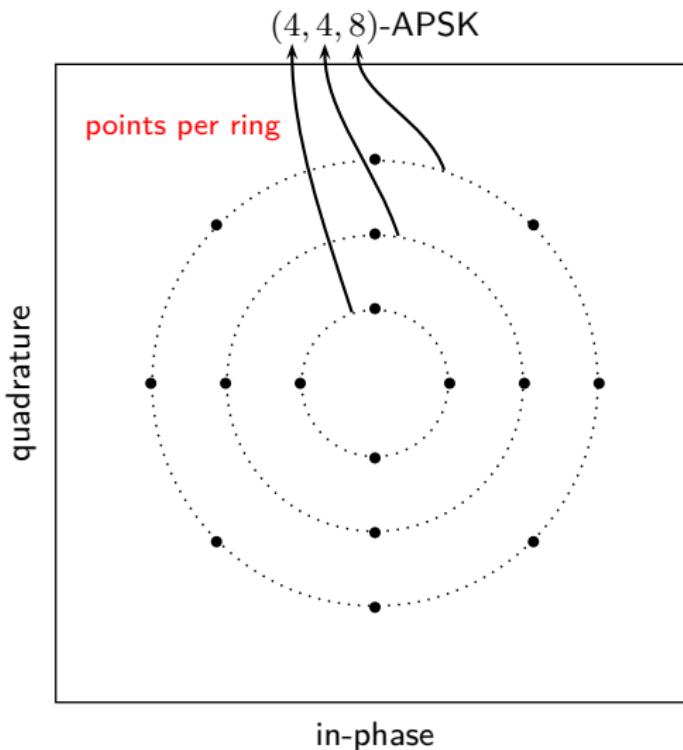
# Amplitude Phase-Shift Keying (APSK), Example



# Amplitude Phase-Shift Keying (APSK), Example

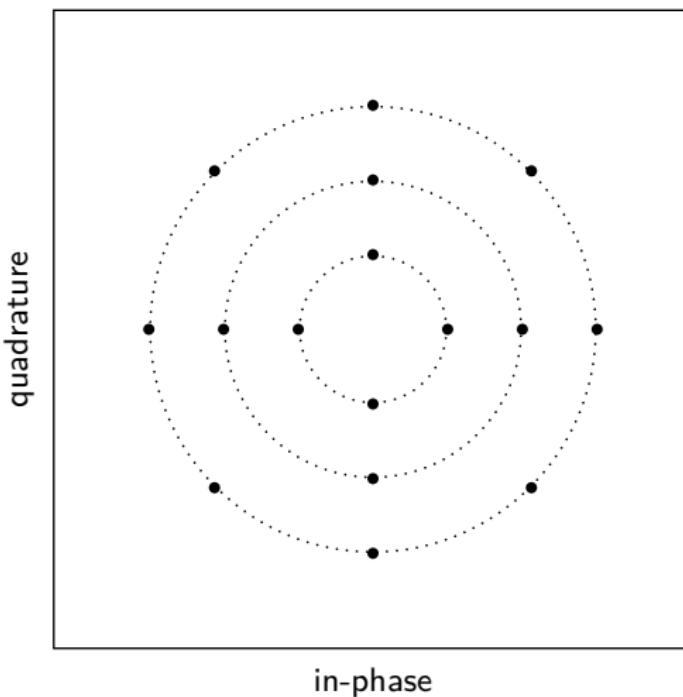


# Amplitude Phase-Shift Keying (APSK), Example



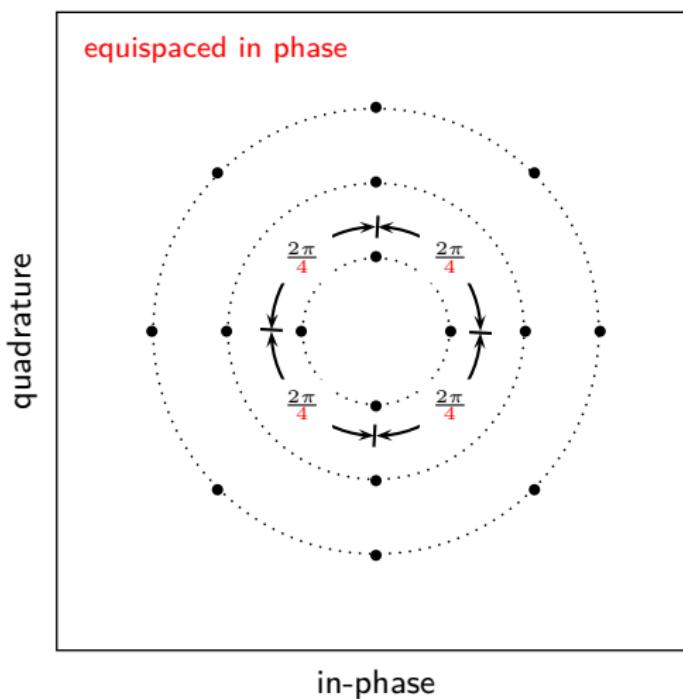
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK



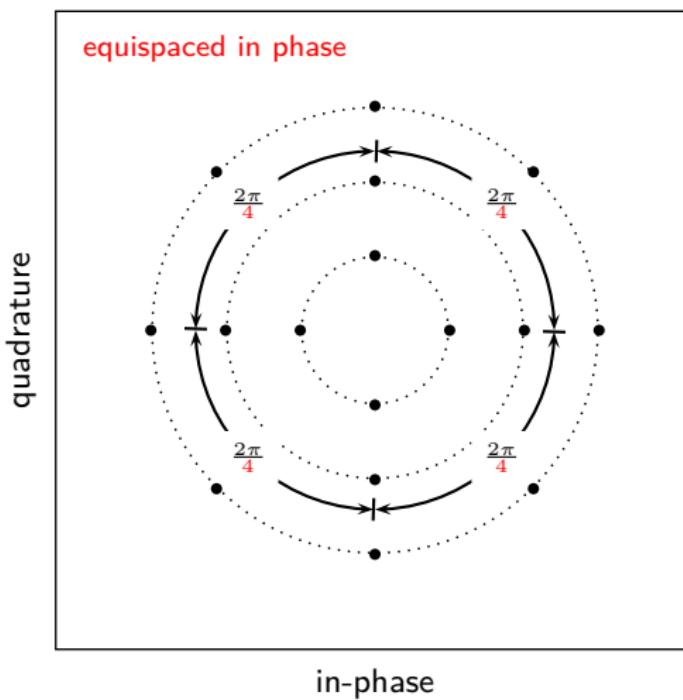
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK



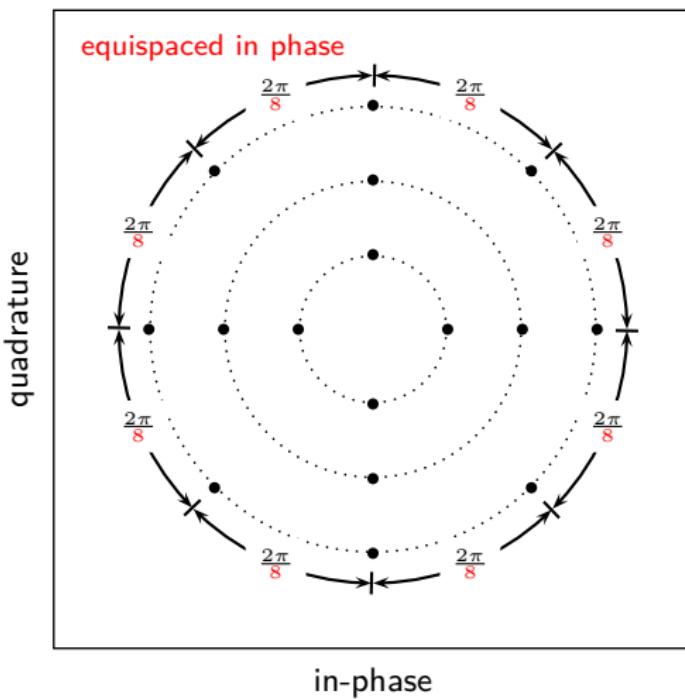
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK



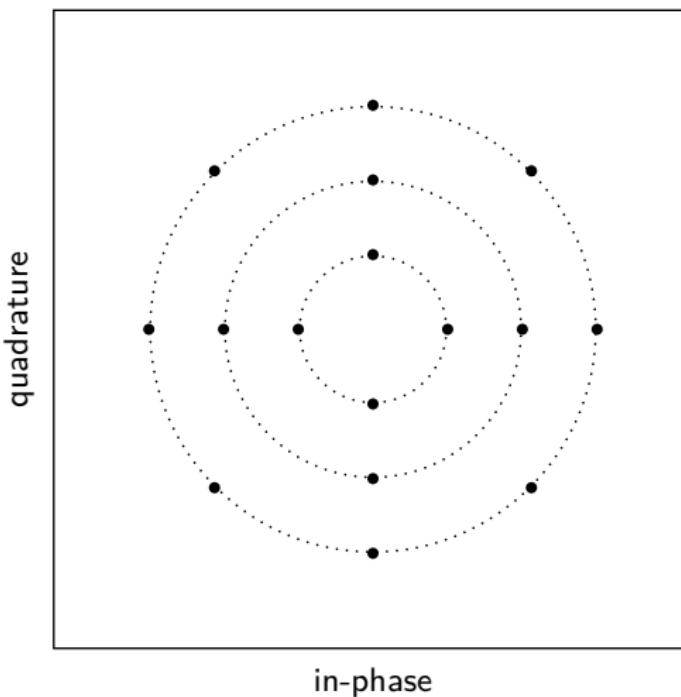
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK



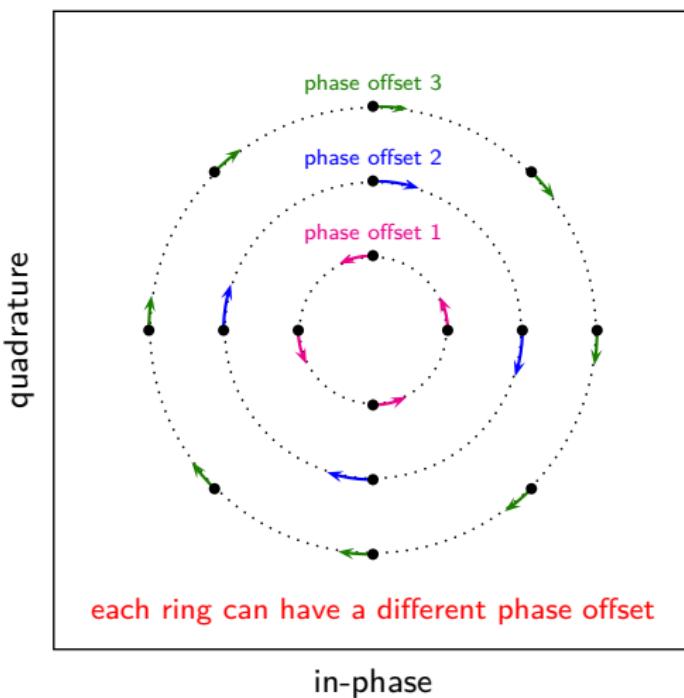
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK



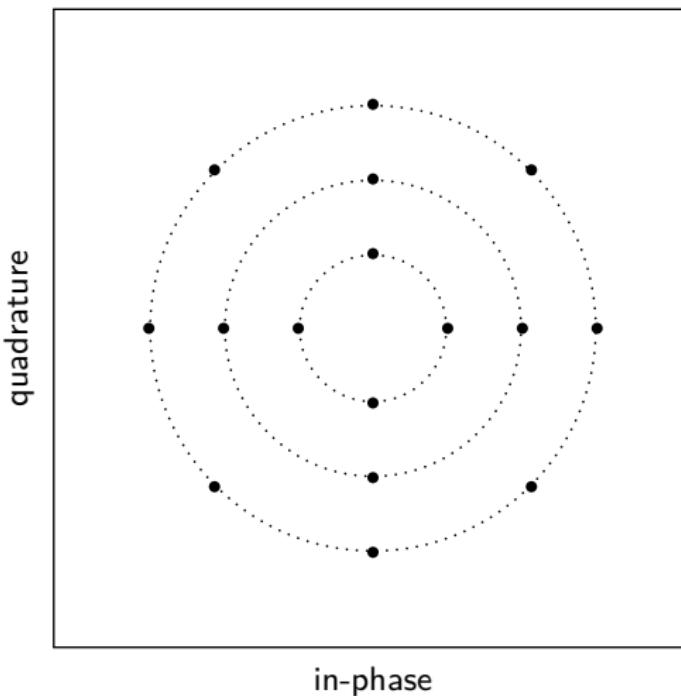
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 8)-APSK

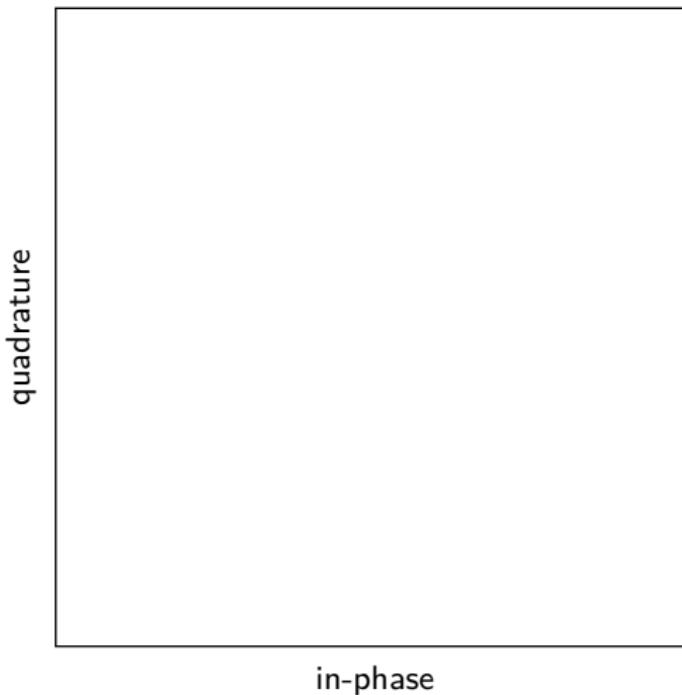


# Amplitude Phase-Shift Keying (APSK), Example

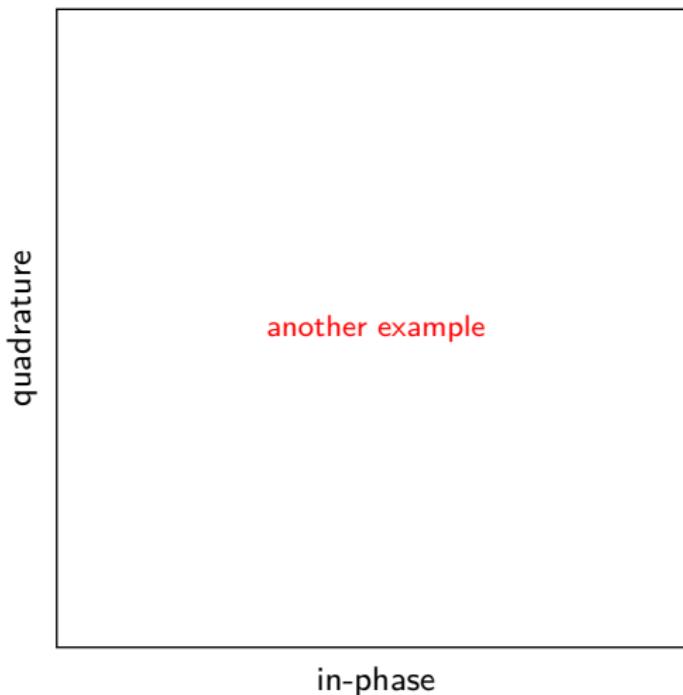
(4, 4, 8)-APSK



# Amplitude Phase-Shift Keying (APSK), Example

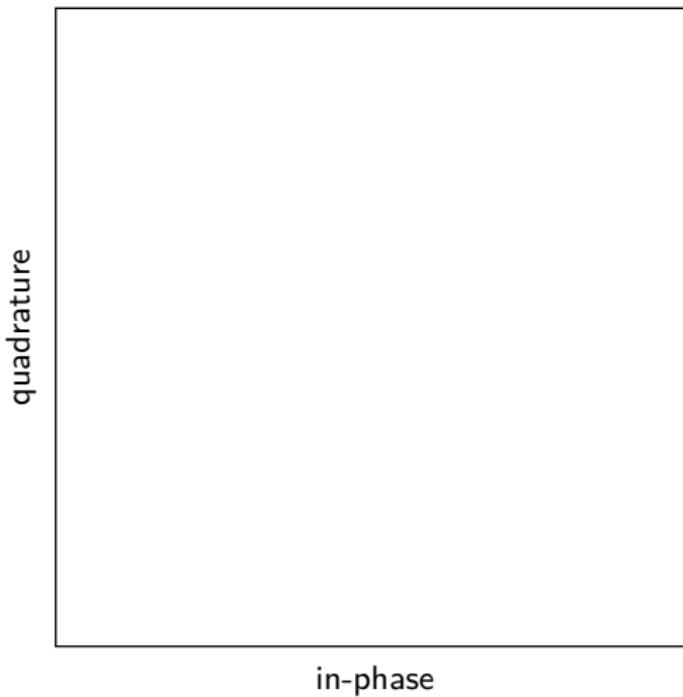


# Amplitude Phase-Shift Keying (APSK), Example



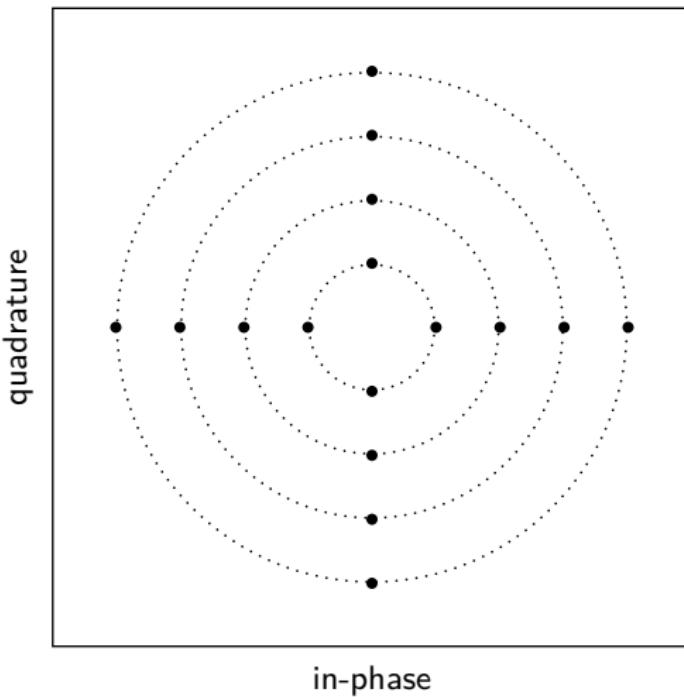
# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 4, 4)-APSK



# Amplitude Phase-Shift Keying (APSK), Example

(4, 4, 4, 4)-APSK



# Probability Density Function of the Observation $Y$

# Probability Density Function of the Observation $Y$

- Extensive work on the statistics of  $\Phi_{NL}$

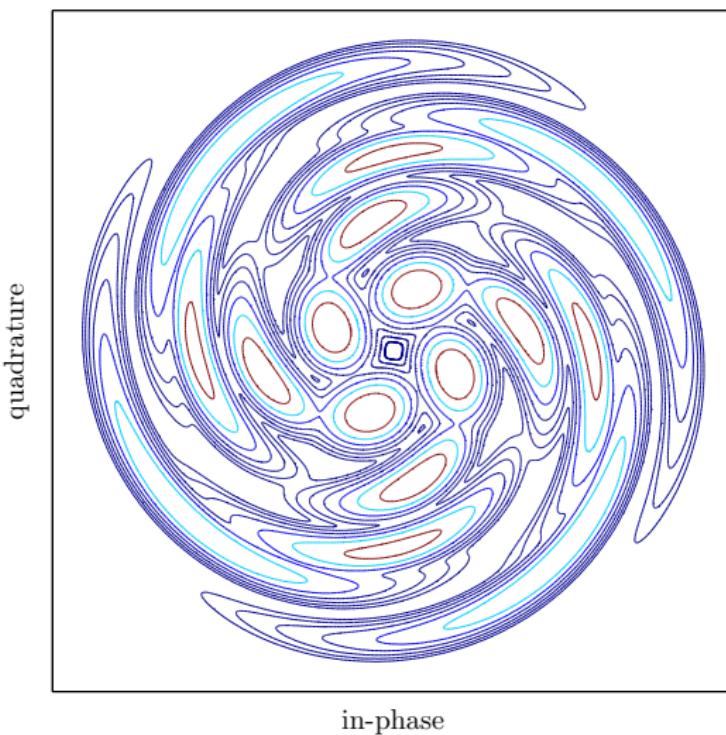
# Probability Density Function of the Observation $Y$

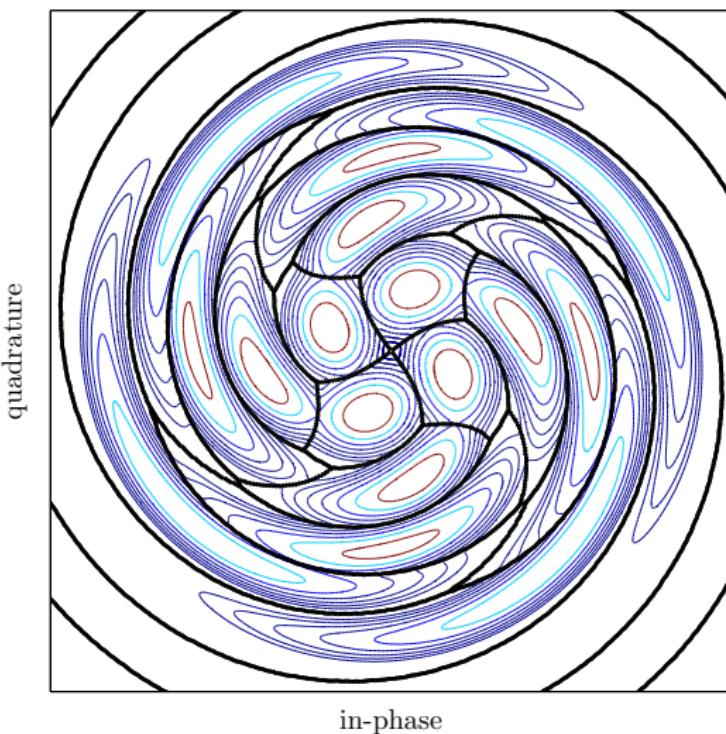
- Extensive work on the statistics of  $\Phi_{NL}$
- See, e.g., [Mecozzi, 1994], [Turitsyn et al., 2003], [Ho, 2005], [Yousefi and Kschischang, 2011]

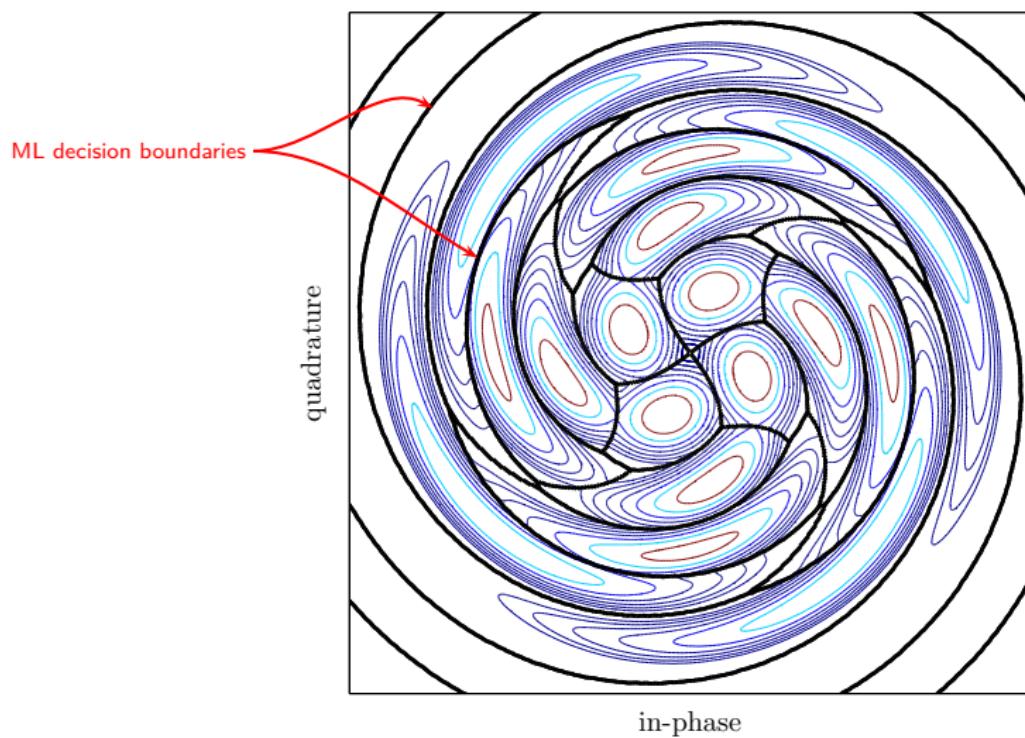
# Probability Density Function of the Observation $Y$

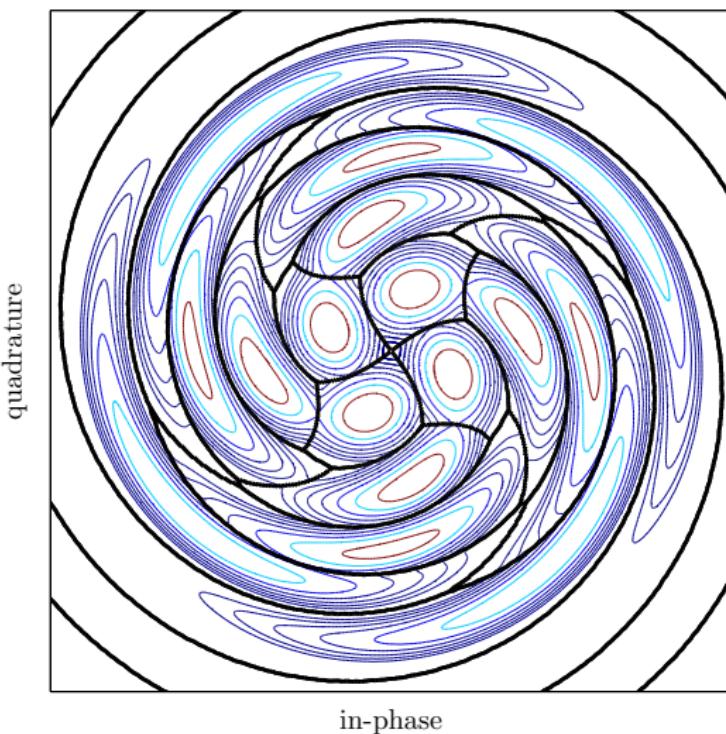
- Extensive work on the statistics of  $\Phi_{NL}$
- See, e.g., [Mecozzi, 1994], [Turitsyn et al., 2003], [Ho, 2005], [Yousefi and Kschischang, 2011]
- Probability density function (PDF)  $f_{Y|X=x_i}(y)$  **known**

PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

PDF for  $(4,4,4,4)$ 

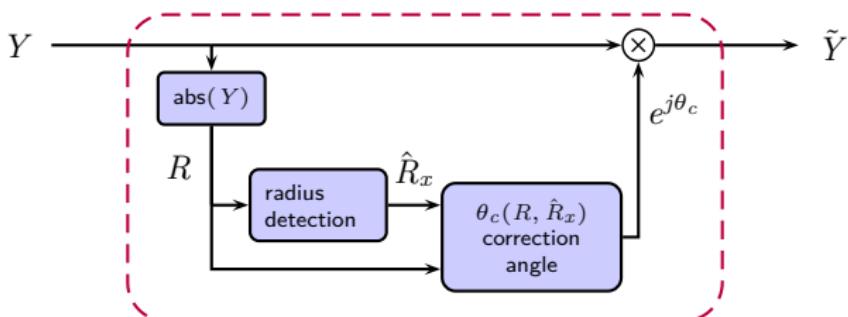
PDF for  $(4,4,4,4)$ 

PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

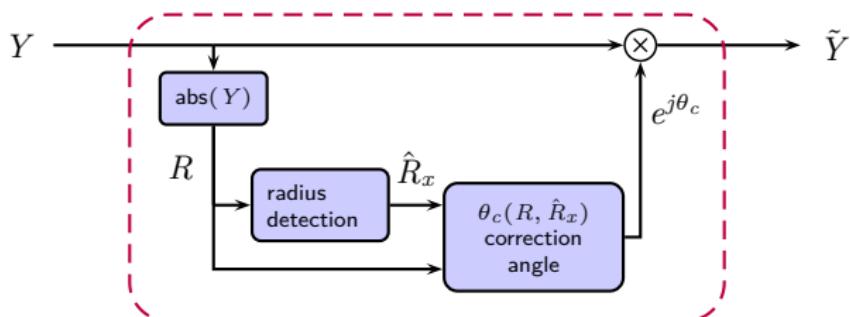
ML detection possible,  
but not practical.

# Nonlinear Phase Postcompensation

## Nonlinear Phase Postcompensation

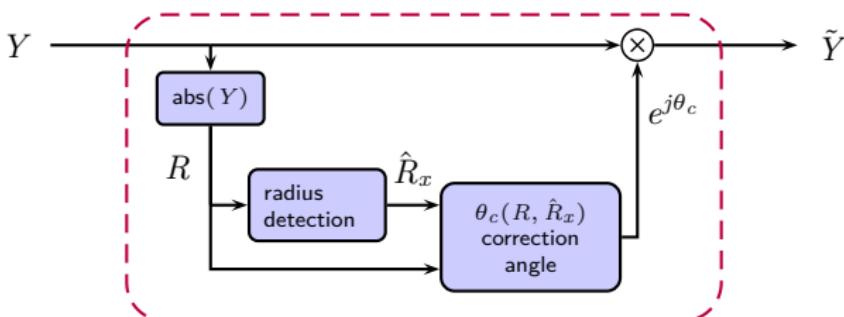


## Nonlinear Phase Postcompensation



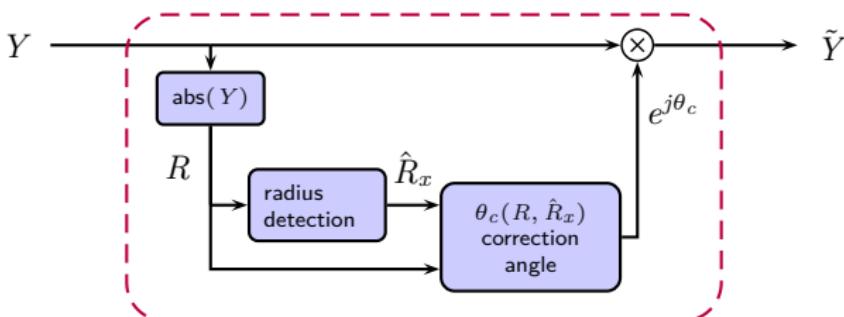
- Building block for suboptimal (but practical) **two-stage detector**

## Nonlinear Phase Postcompensation



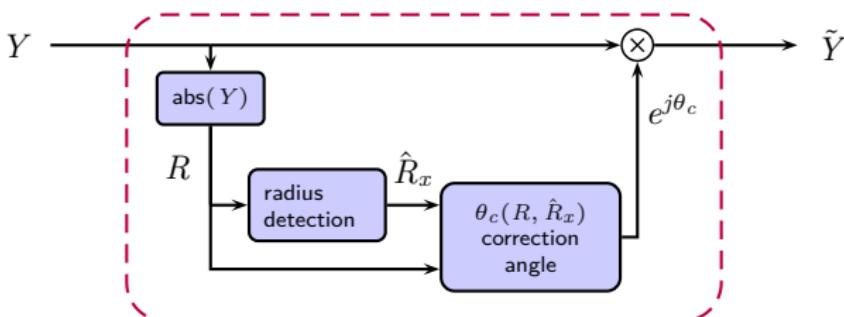
- Building block for suboptimal (but practical) **two-stage detector**
- However, in principle **no loss in optimality** due to postcompensation itself (ML detection based on  $\tilde{Y}$  still possible)

## Nonlinear Phase Postcompensation



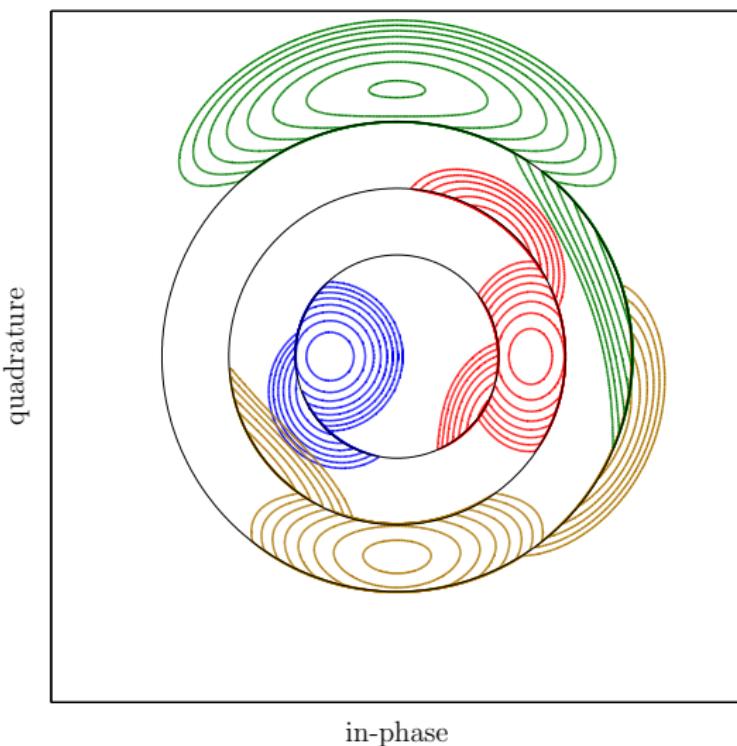
- Building block for suboptimal (but practical) **two-stage detector**
- However, in principle **no loss in optimality** due to postcompensation itself (ML detection based on  $\tilde{Y}$  still possible)
- Gives insight into why two-stage detection is suboptimal

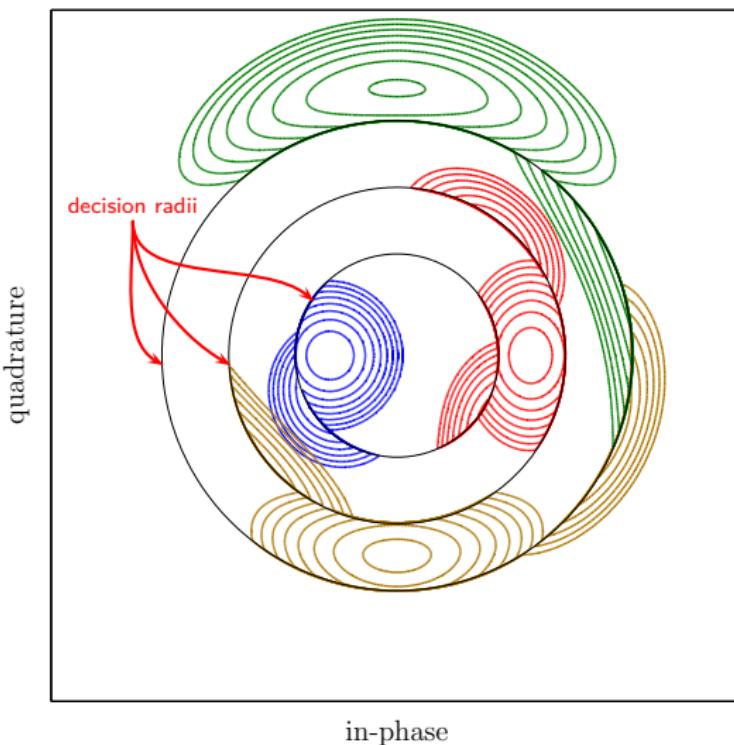
## Nonlinear Phase Postcompensation

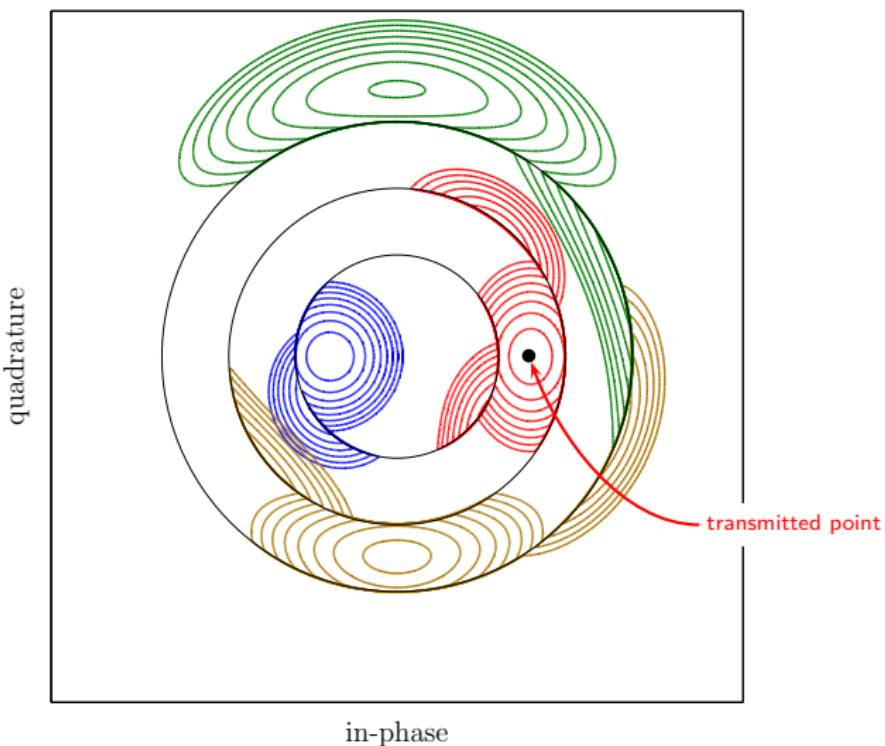


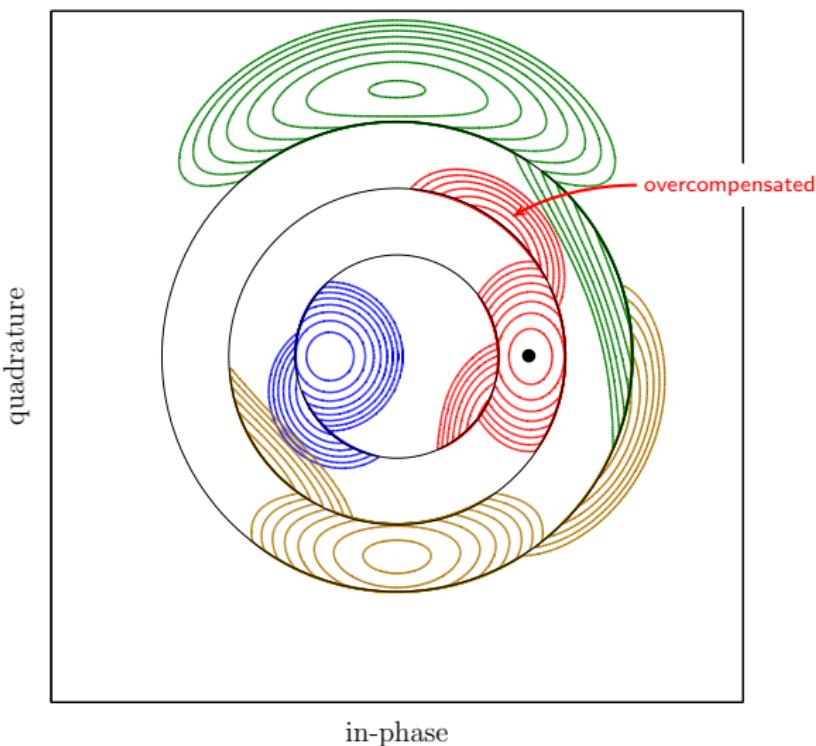
- Building block for suboptimal (but practical) **two-stage detector**
- However, in principle **no loss in optimality** due to postcompensation itself (ML detection based on  $\tilde{Y}$  still possible)
- Gives insight into why two-stage detection is suboptimal
- Note: the PDF of  $\tilde{Y}$  is defined piecewise

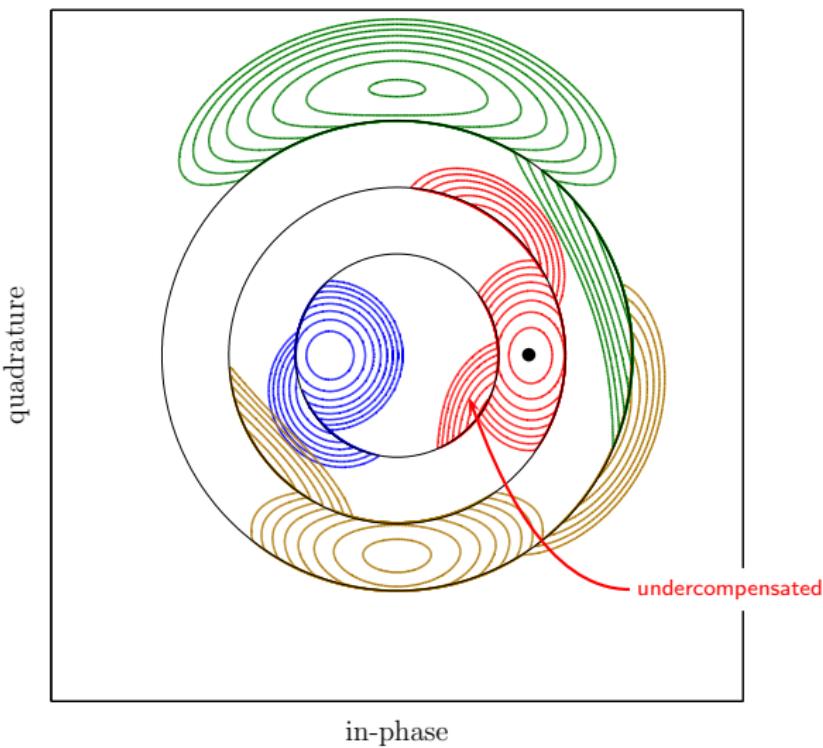
PDF of  $\tilde{Y}$  for (4,4,4,4)-APSK at  $P = -4$  dBm

PDF of  $\tilde{Y}$  for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

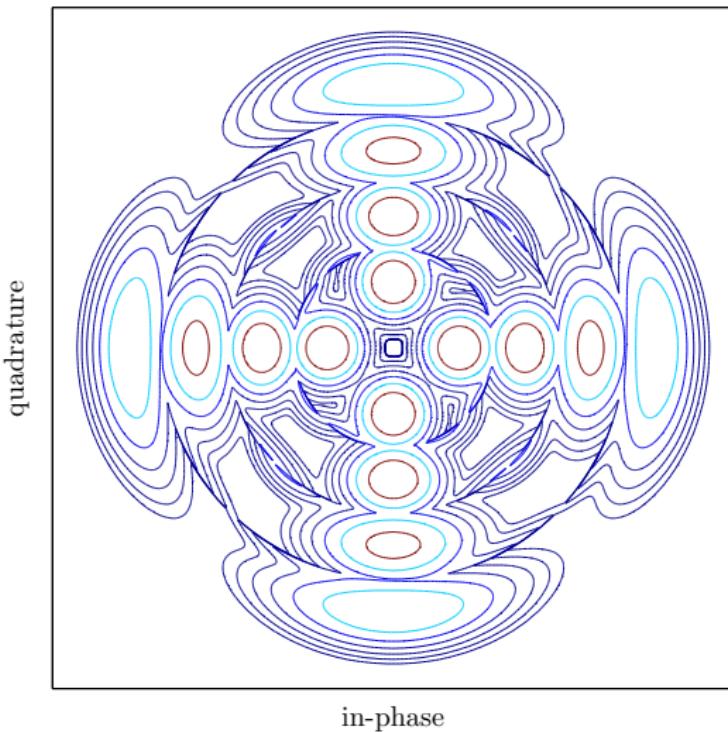
PDF of  $\tilde{Y}$  for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

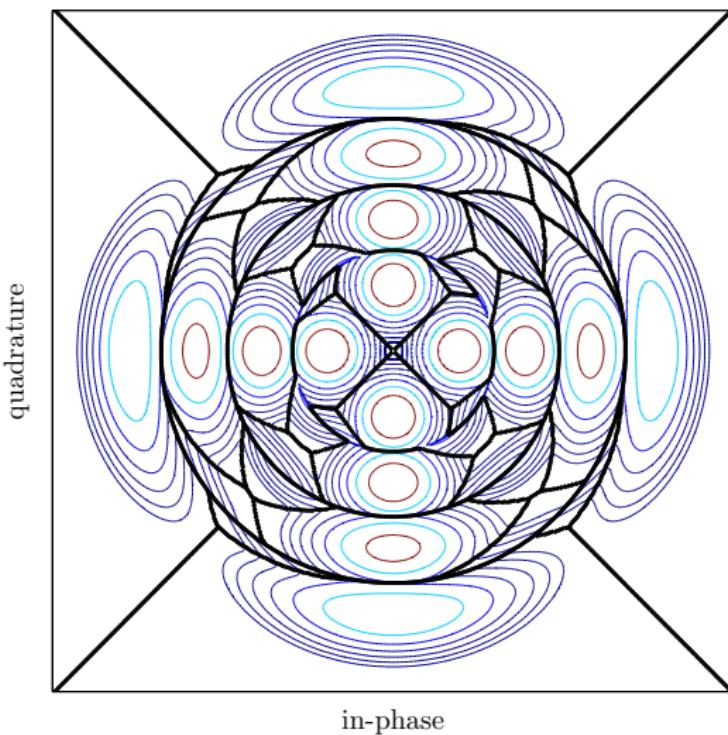
PDF of  $\tilde{Y}$  for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

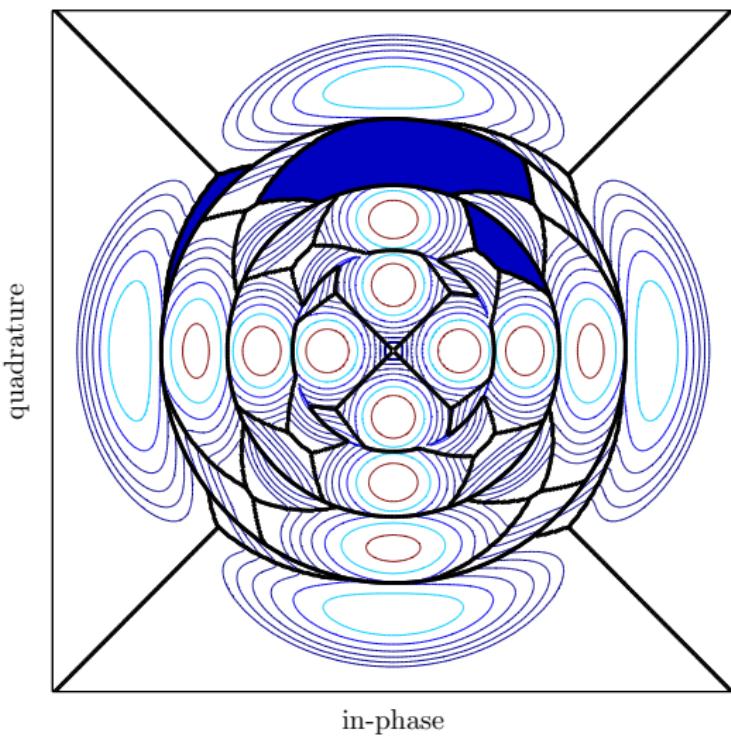
PDF of  $\tilde{Y}$  for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

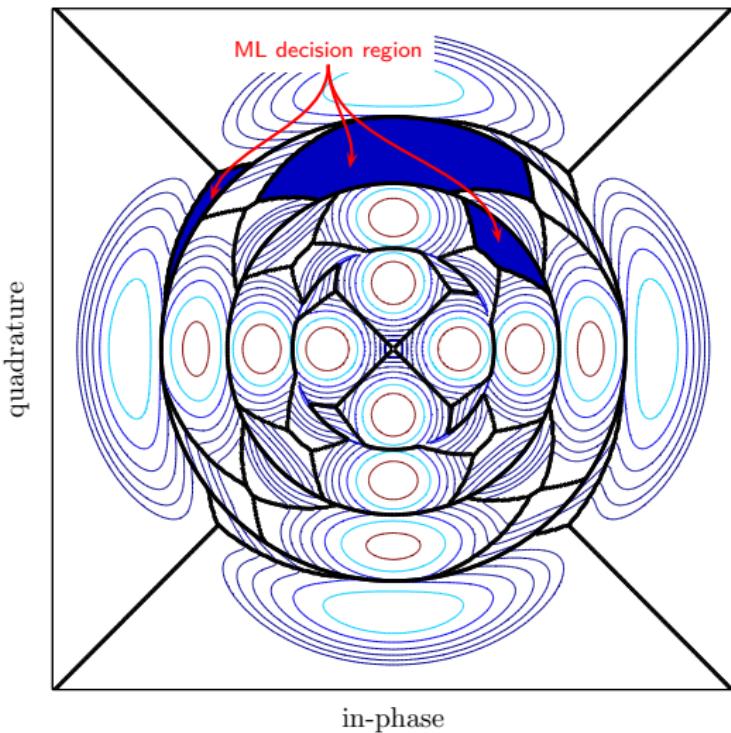
PDF of  $\tilde{Y}$  for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

ML Detection based on  $\tilde{Y}$

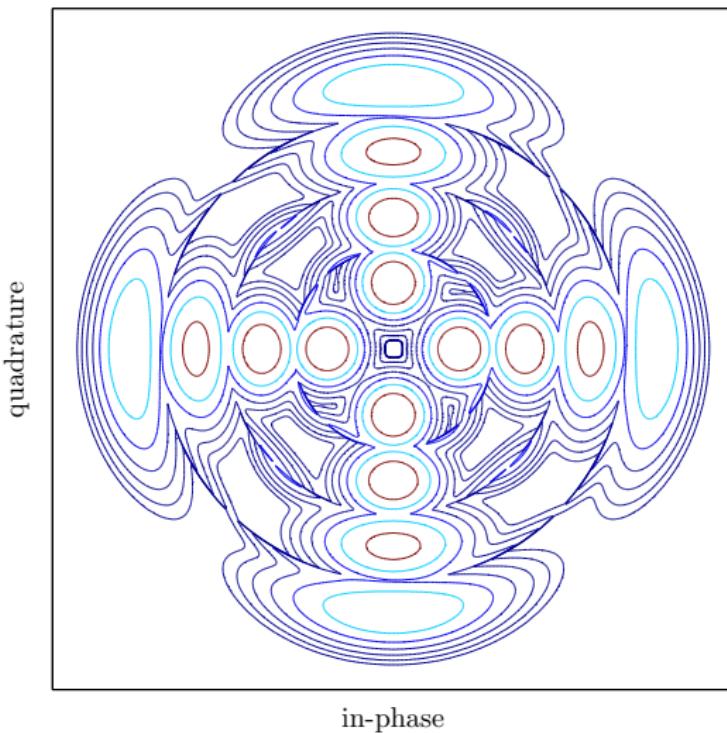
ML Detection based on  $\tilde{Y}$ 

ML Detection based on  $\tilde{Y}$ 

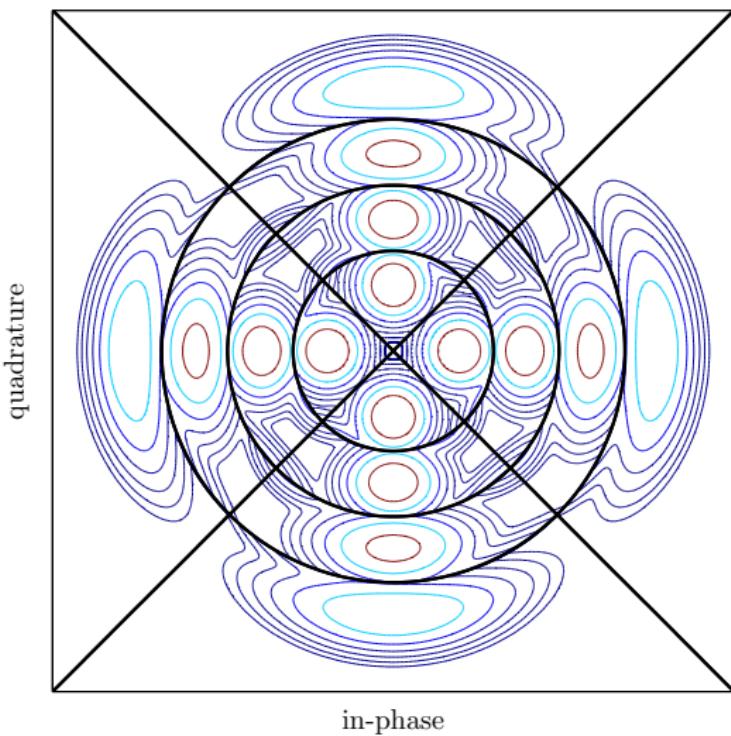
ML Detection based on  $\tilde{Y}$ 

ML Detection based on  $\tilde{Y}$ 

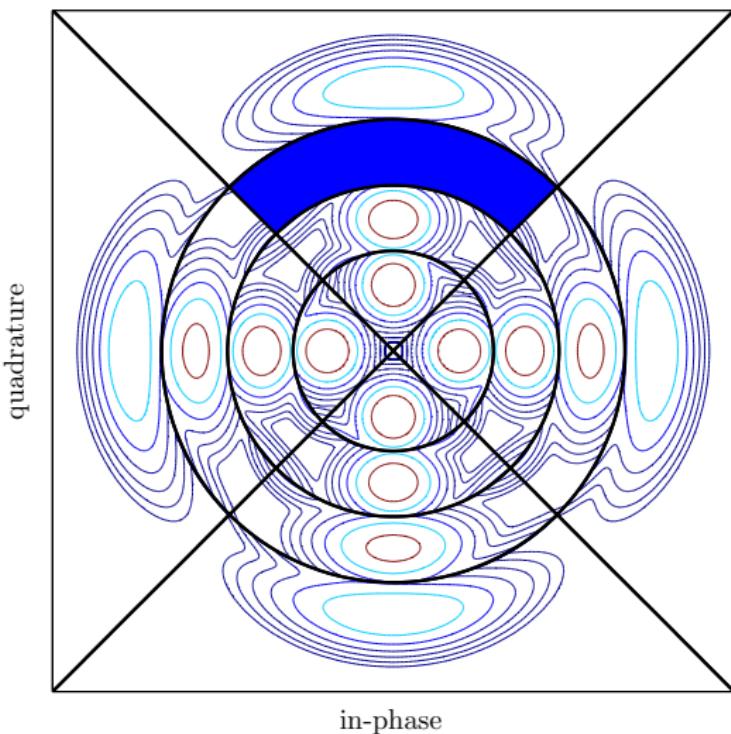
## Two-Stage Detection



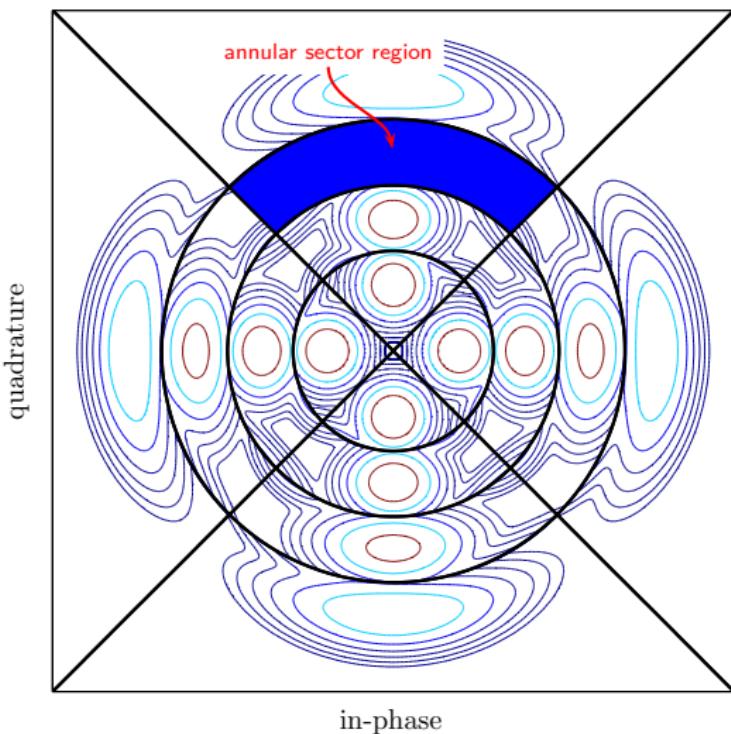
## Two-Stage Detection



## Two-Stage Detection



## Two-Stage Detection



# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

- How many rings?

# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

- How many rings? **More rings for increasing input power due to phase noise.**

# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

- How many rings? **More rings for increasing input power due to phase noise.**
- How many points per ring?

# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

- How many rings? **More rings for increasing input power due to phase noise.**
- How many points per ring?
- What radius distribution?

# Optimization Opportunities for APSK

## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

- How many rings? **More rings for increasing input power due to phase noise.**
- How many points per ring?
- What radius distribution?
- What phase offset?

# Optimization Opportunities for APSK

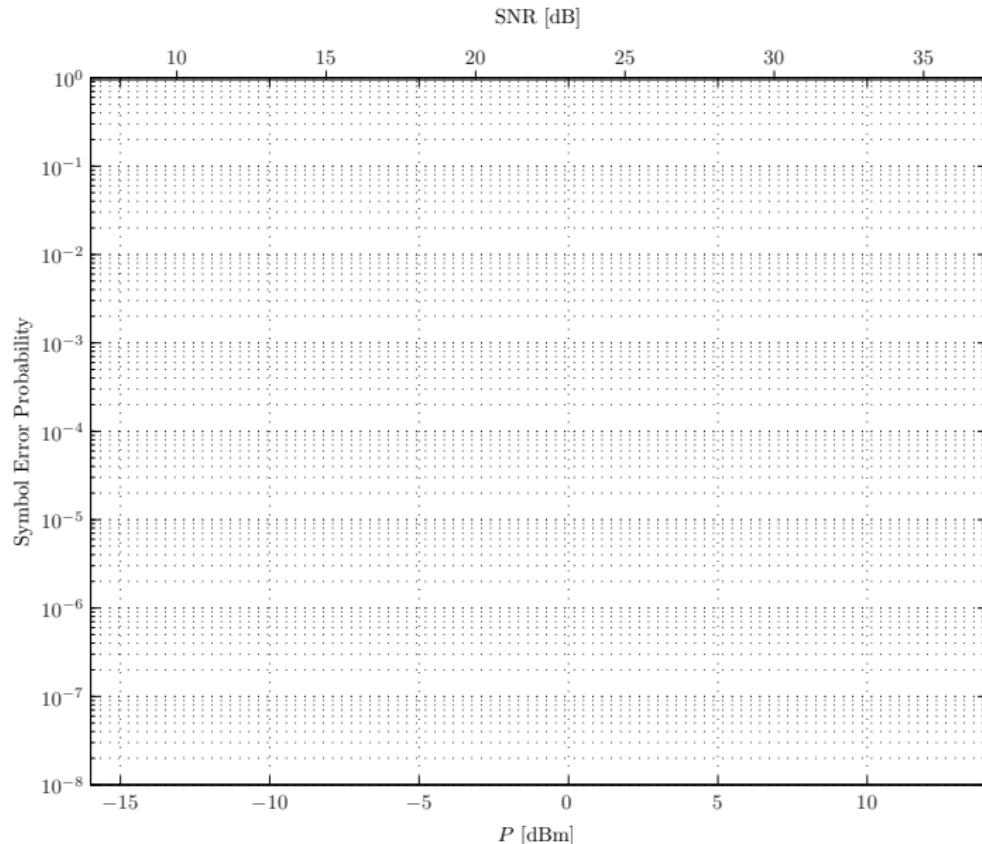
## Goal

Which APSK constellation **minimizes the symbol error probability under two-stage detection** for a given input power  $P$ ?

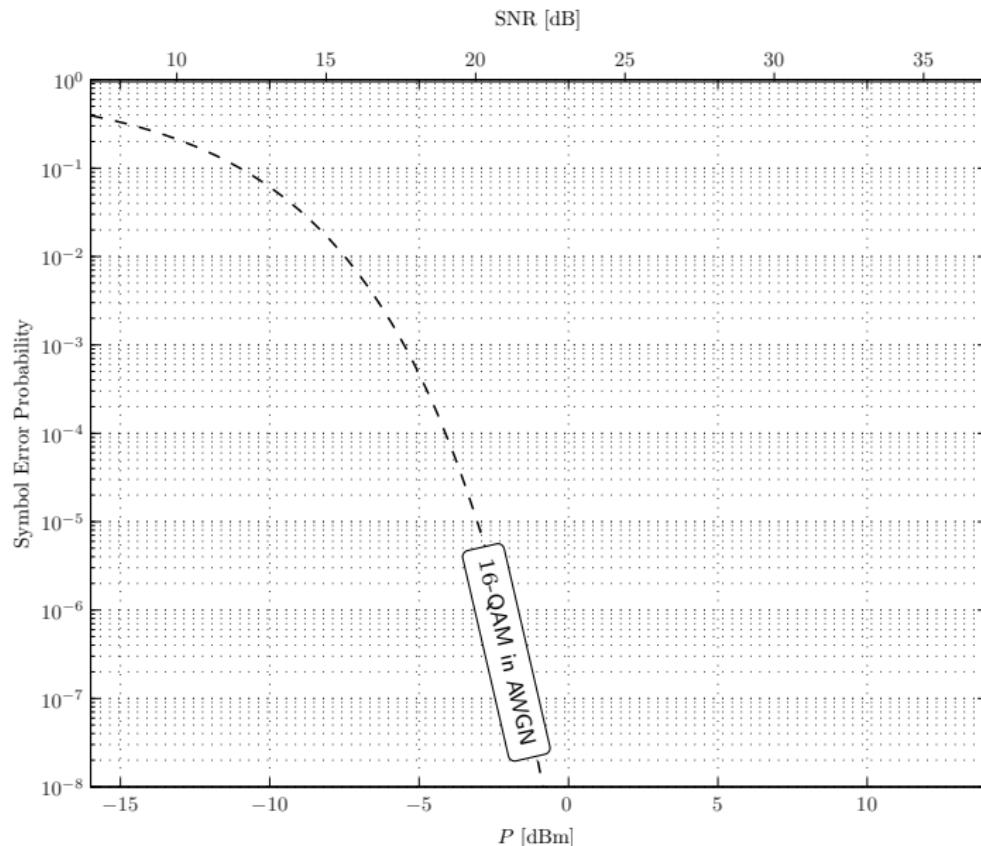
- How many rings? **More rings for increasing input power due to phase noise.**
- How many points per ring?
- What radius distribution?
- What phase offset? **Two-stage detection is insensitive to a phase offset.**

# Optimizing the Number of Rings and Points per Ring

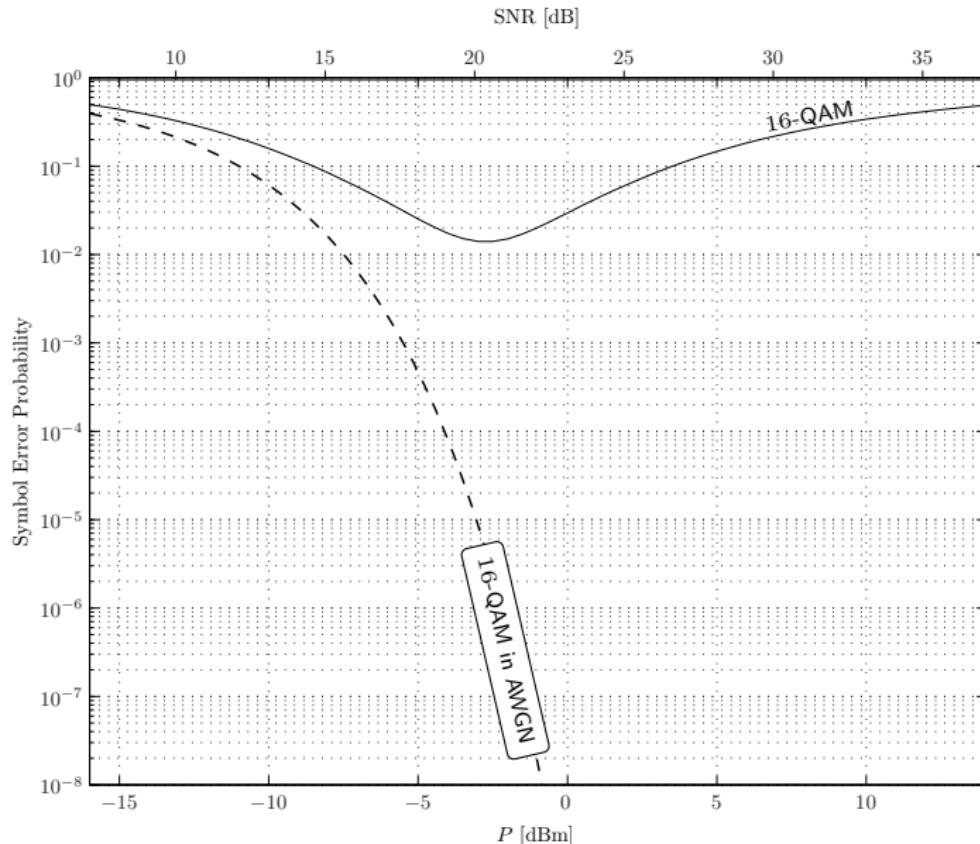
# Optimizing the Number of Rings and Points per Ring



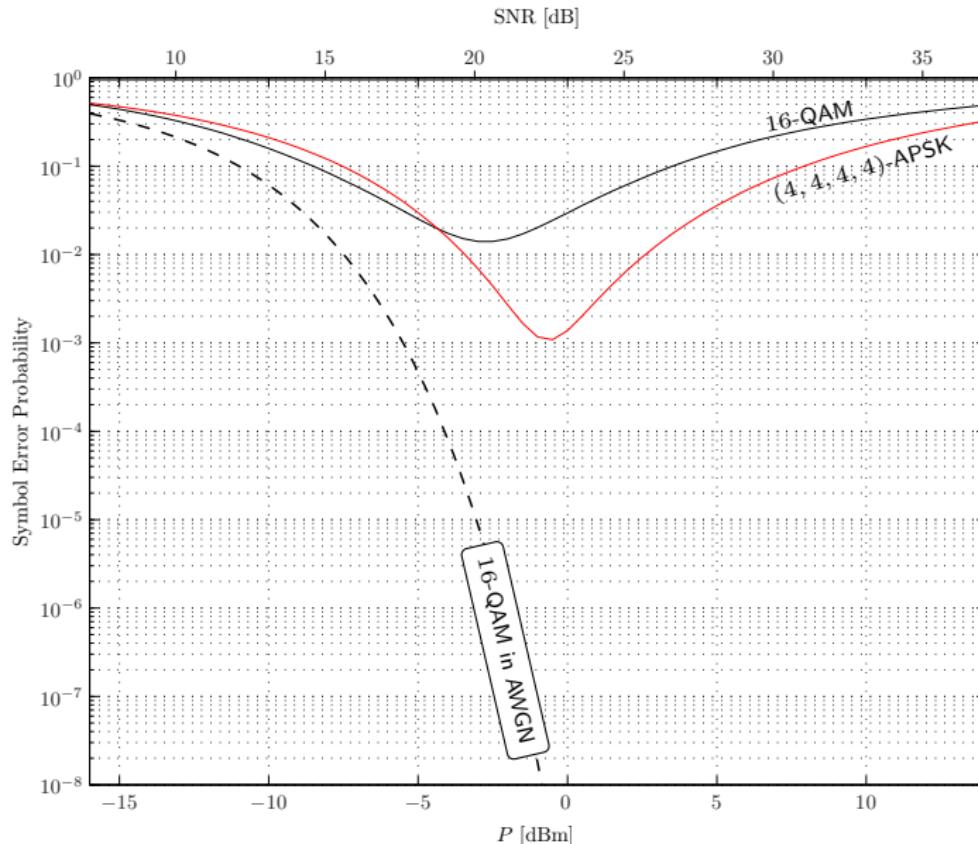
# Optimizing the Number of Rings and Points per Ring



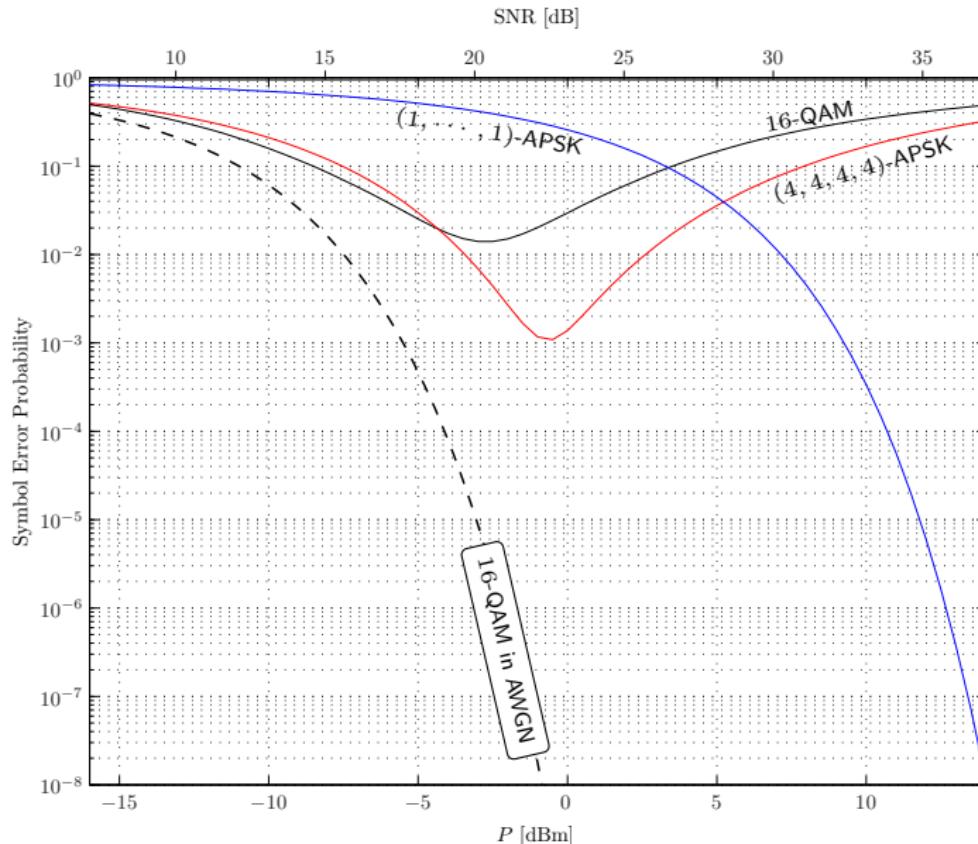
# Optimizing the Number of Rings and Points per Ring



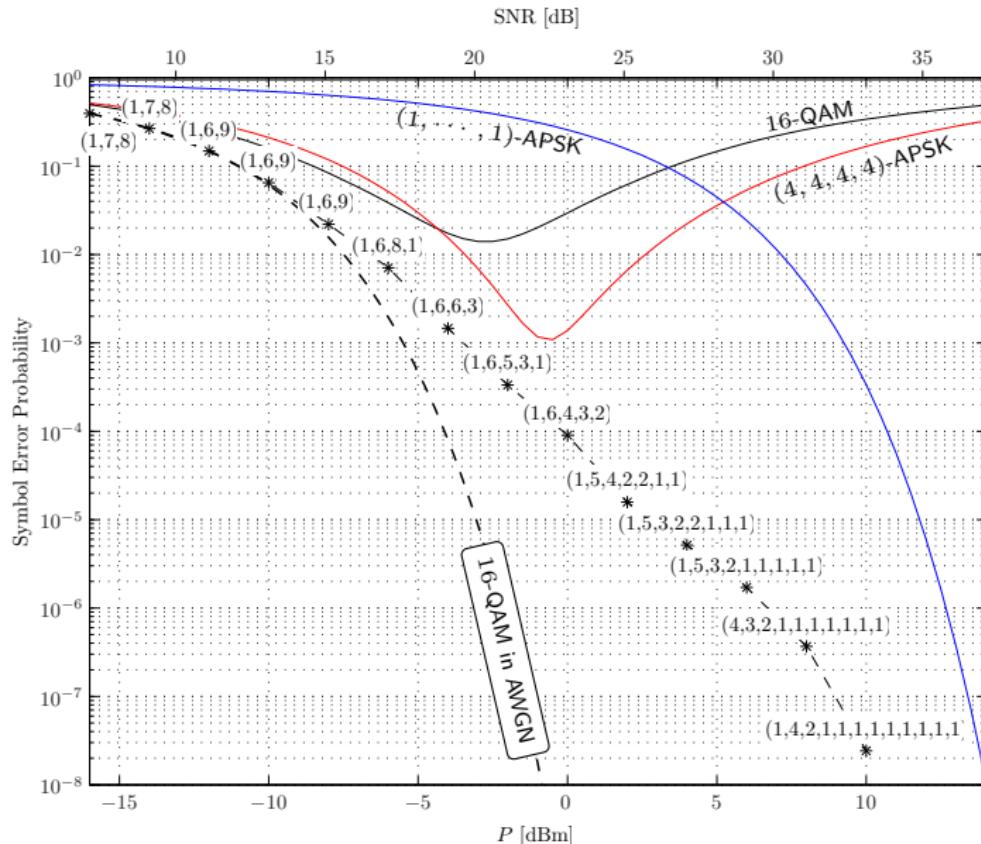
# Optimizing the Number of Rings and Points per Ring

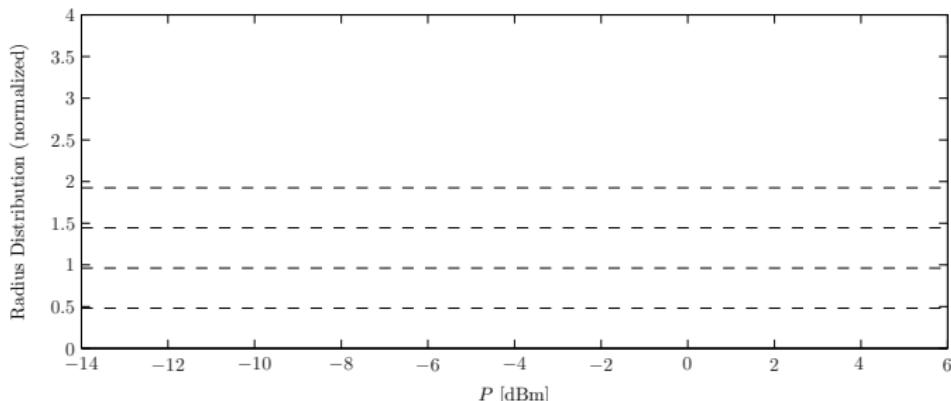
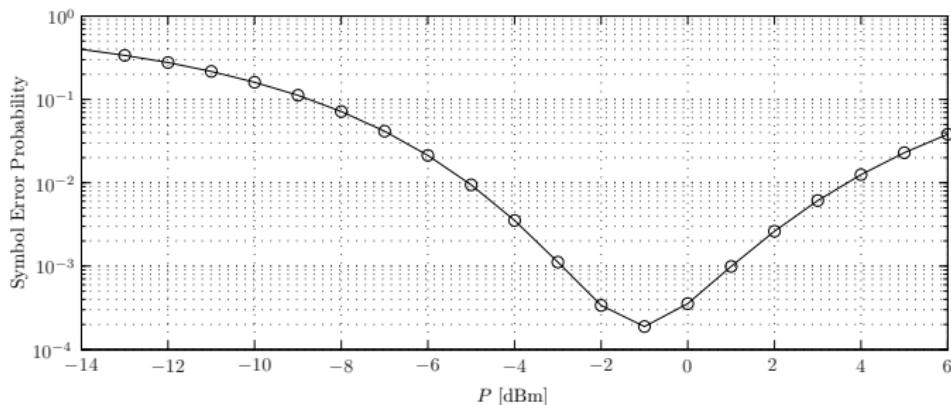


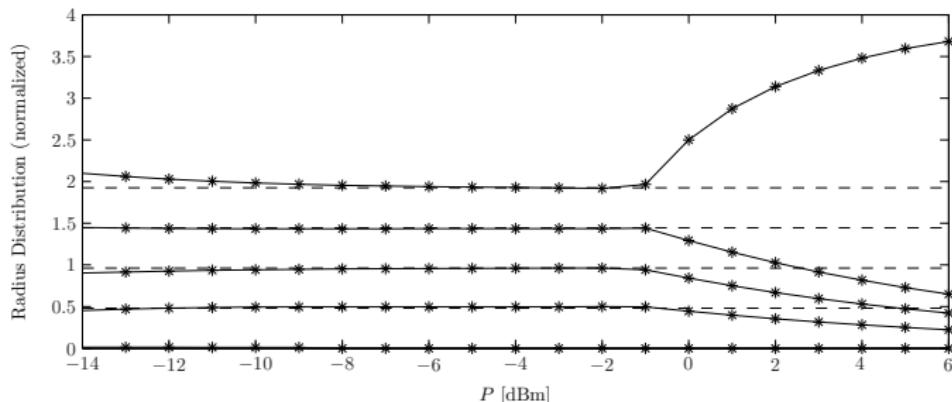
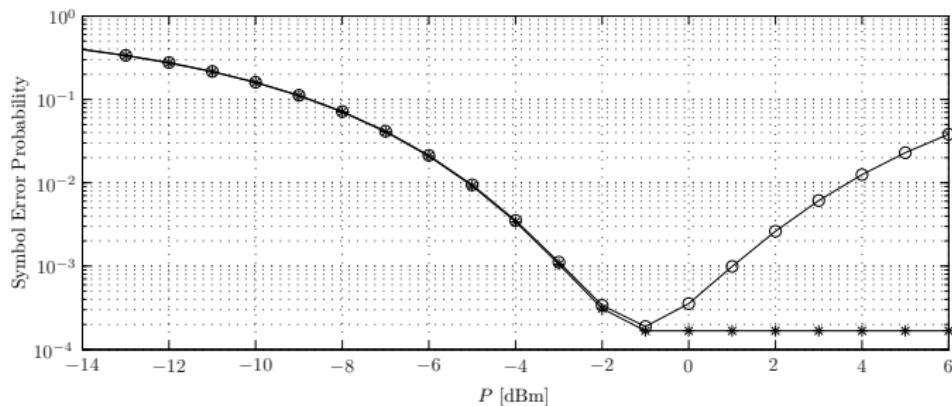
# Optimizing the Number of Rings and Points per Ring

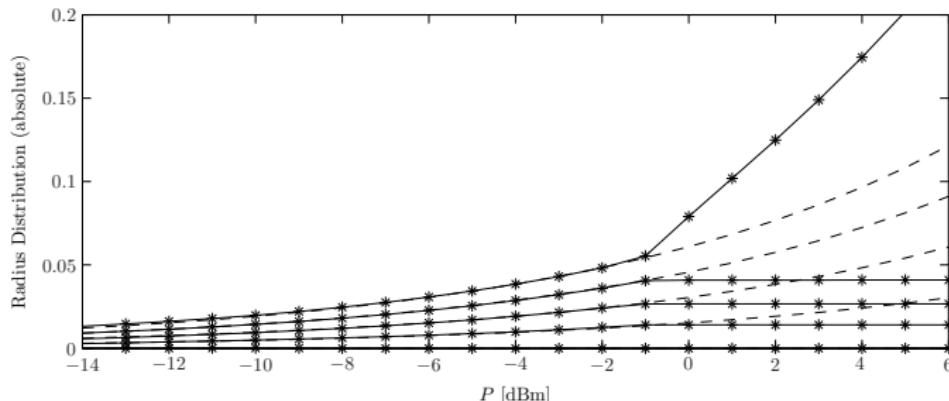
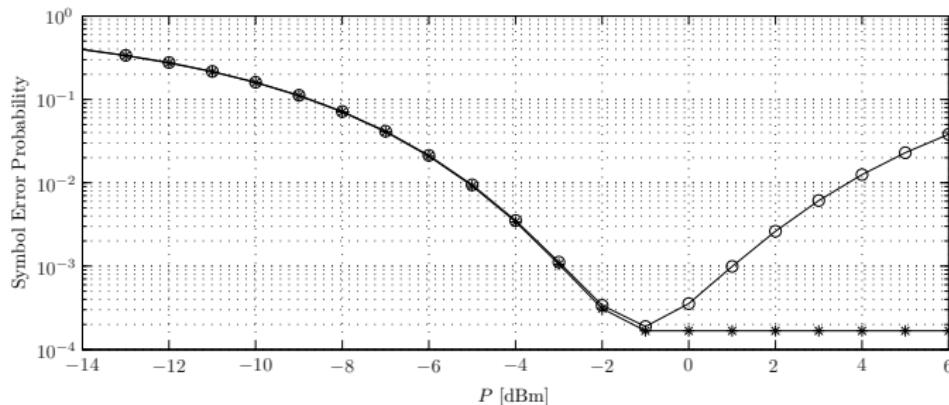


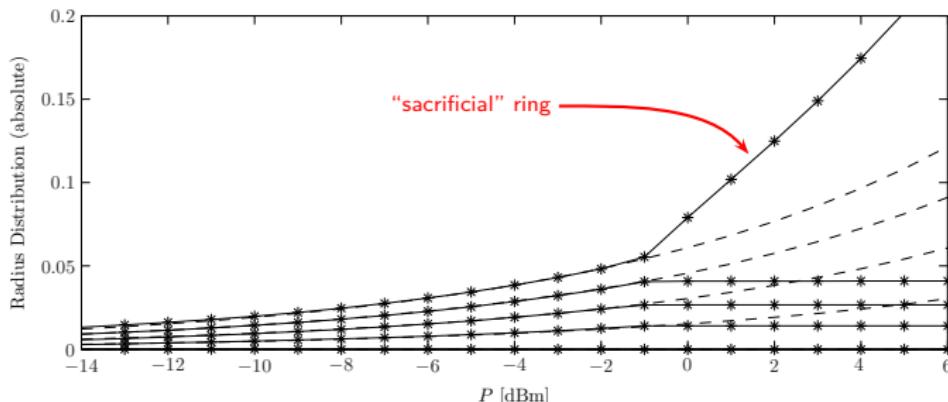
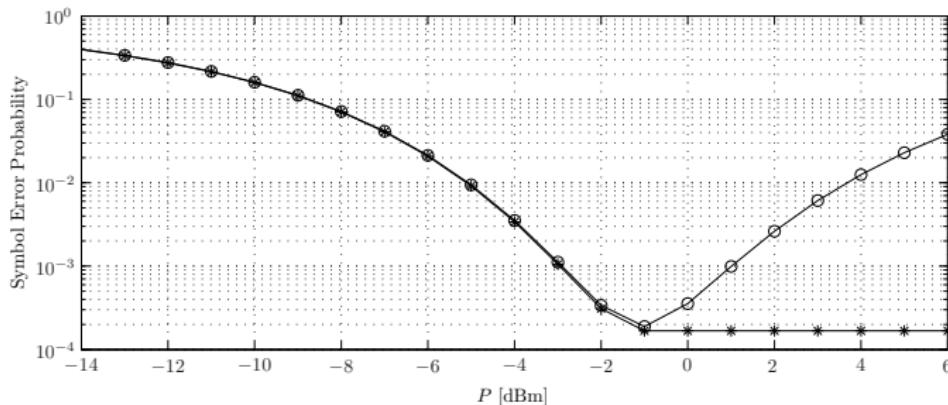
# Optimizing the Number of Rings and Points per Ring



Radius Optimization for  $(1,6,5,3,1)$ -APSK

Radius Optimization for  $(1,6,5,3,1)$ -APSK

Radius Optimization for  $(1,6,5,3,1)$ -APSK

Radius Optimization for  $(1,6,5,3,1)$ -APSK

## Conclusions and Future Work

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.
2. Significant gains can be obtained by choosing an optimized APSK constellations compared to regular 16-QAM for this channel model.

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.
2. Significant gains can be obtained by choosing an optimized APSK constellations compared to regular 16-QAM for this channel model.
3. High nonlinearities and an average power constraint may lead to counterintuitive optimization results.

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.
2. Significant gains can be obtained by choosing an optimized APSK constellations compared to regular 16-QAM for this channel model.
3. High nonlinearities and an average power constraint may lead to counterintuitive optimization results.

Future work include the effect of dual polarization and dispersion on the results.

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.
2. Significant gains can be obtained by choosing an optimized APSK constellations compared to regular 16-QAM for this channel model.
3. High nonlinearities and an average power constraint may lead to counterintuitive optimization results.

Future work include the effect of dual polarization and dispersion on the results.

Extended version available on Arxiv (submitted to IEEE Trans. Comm.): Häger, Graell i Amat, Alvarado, Agrell - Design of APSK Constellations for Coherent Optical Channels with Nonlinear Phase Noise, Sep. 2012

## Conclusions and Future Work

1. The performance loss of the two-stage detector originates from suboptimal detection regions, not from the postcompensation itself.
2. Significant gains can be obtained by choosing an optimized APSK constellations compared to regular 16-QAM for this channel model.
3. High nonlinearities and an average power constraint may lead to counterintuitive optimization results.

Future work include the effect of dual polarization and dispersion on the results.

Extended version available on Arxiv (submitted to IEEE Trans. Comm.): Häger, Graell i Amat, Alvarado, Agrell - Design of APSK Constellations for Coherent Optical Channels with Nonlinear Phase Noise, Sep. 2012

Thank you!



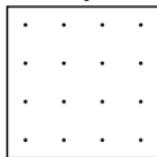
## References

-  Ho, K.-P. (2005).  
*Phase-modulated Optical Communication Systems.*  
Springer.
-  Lau, A. P. T. and Kahn, J. M. (2007).  
Signal Design and Detection in Presence of Nonlinear Phase Noise.  
*J. Lightw. Technol.*, 25(10):3008–3016.
-  Mecozzi, A. (1994).  
Limits to long-haul coherent transmission set by the Kerr nonlinearity and noise of the in-line amplifiers.  
*J. Lightw. Technol.*, 12(11):1993–2000.
-  Turitsyn, K. S., Derevyanko, S. A., Yurkevich, I. V., and Turitsyn, S. K. (2003).  
Information Capacity of Optical Fiber Channels with Zero Average Dispersion.  
*Phys. Rev. Lett.*, 91(20):203901.
-  Yousefi, M. I. and Kschischang, F. R. (2011).  
On the per-sample capacity of nondispersive optical fibers.  
*IEEE Trans. Inf. Theory*, 57(11):7522–7541.

# Power Dependent Phase Noise

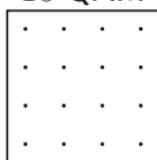
# Power Dependent Phase Noise

16-QAM

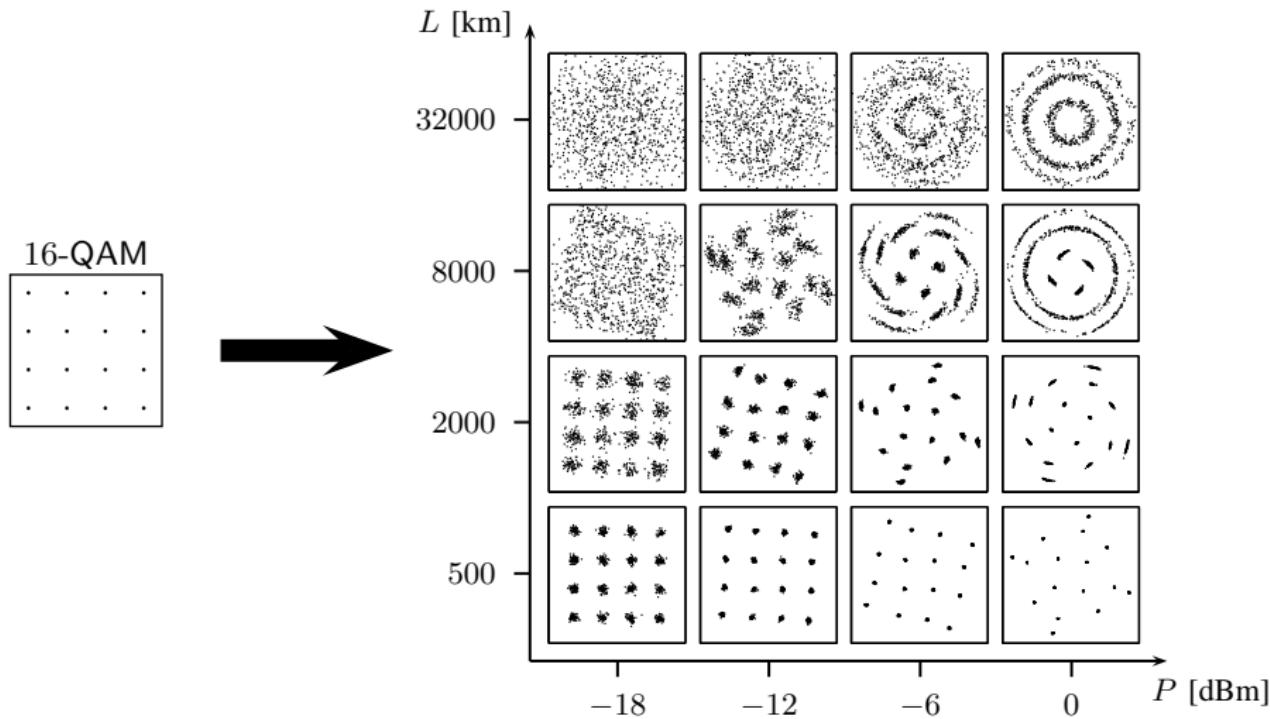


## Power Dependent Phase Noise

16-QAM



## Power Dependent Phase Noise



## Power Dependent Phase Noise

