

# Constellation Optimization for Coherent Optical Channels Distorted by Nonlinear Phase Noise

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Fiber Optic Communications Research Center



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Cambridge, April 17, 2013

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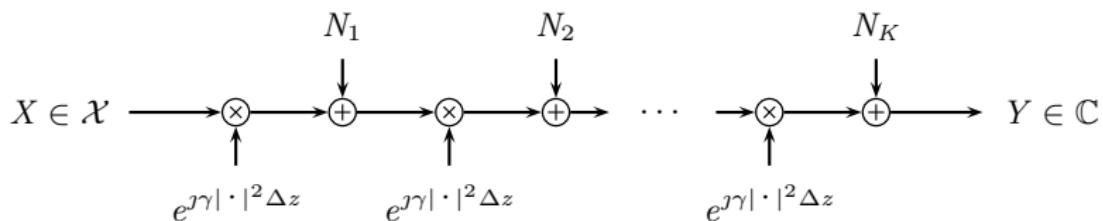
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- **Theoretical question:** How do optimal constellations look like for very strong nonlinearities?

# Outline

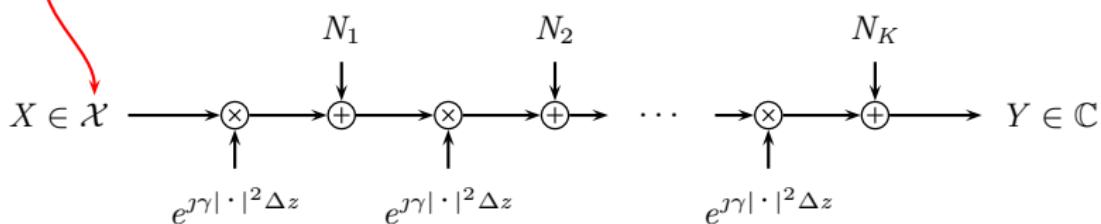
1. Introduction: Channel Model and APSK
2. Detection Methods
3. APSK Optimization
4. Conclusions

## Discrete-Time Channel Model

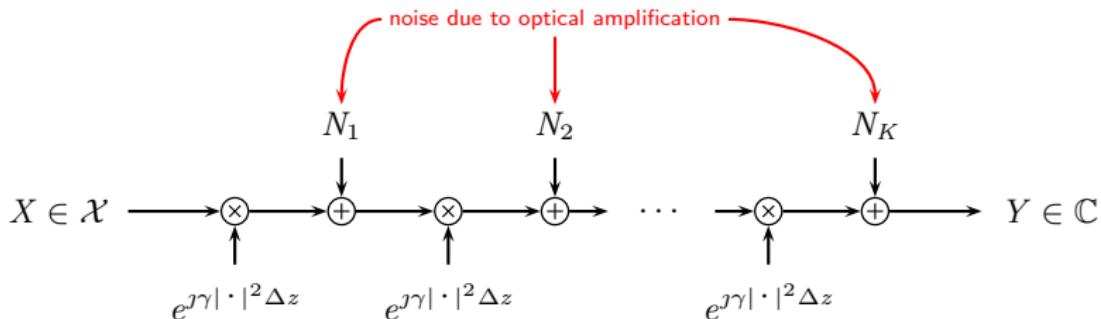


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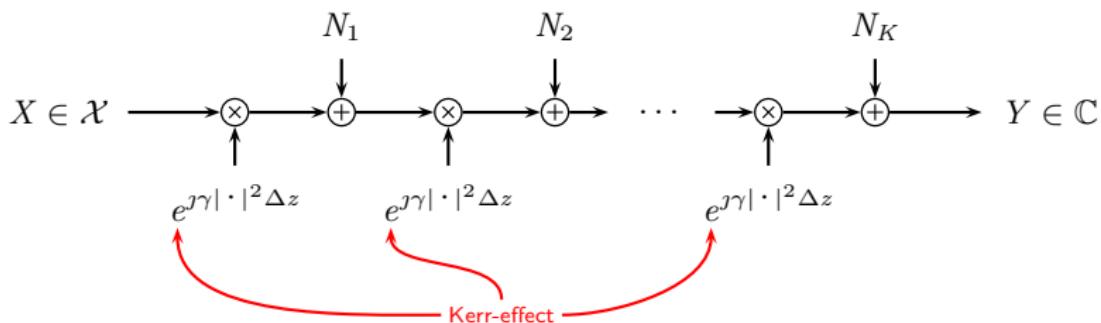
signal constellation, here  $|\mathcal{X}| = 16$



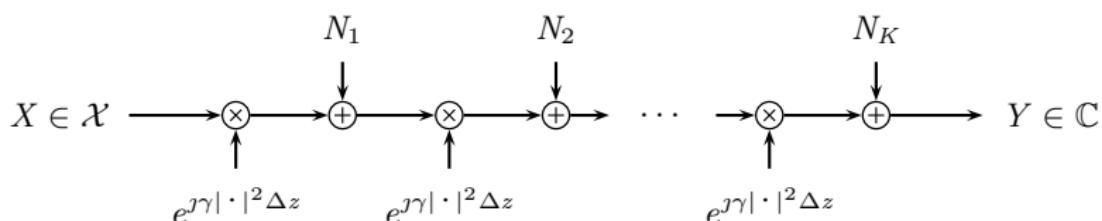
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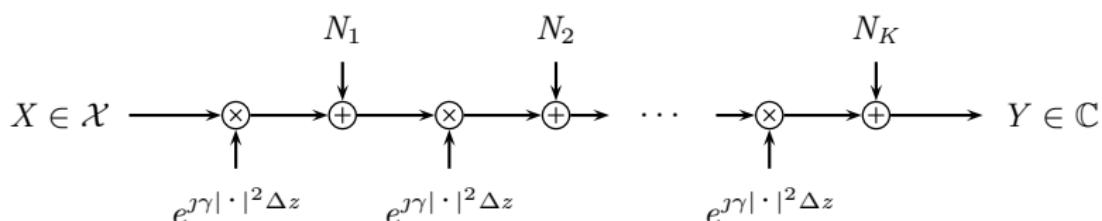


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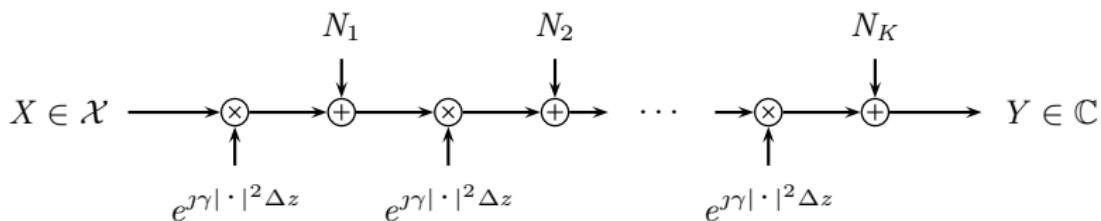
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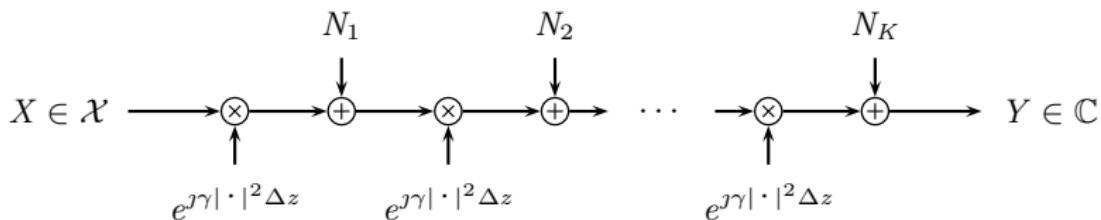
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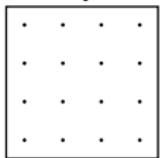


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- $L = 5500$  km, other parameters  $\gamma, \sigma^2$  taken from [Lau and Kahn, 2007]

## Power Dependent Phase Noise

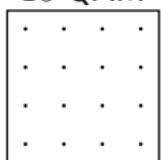
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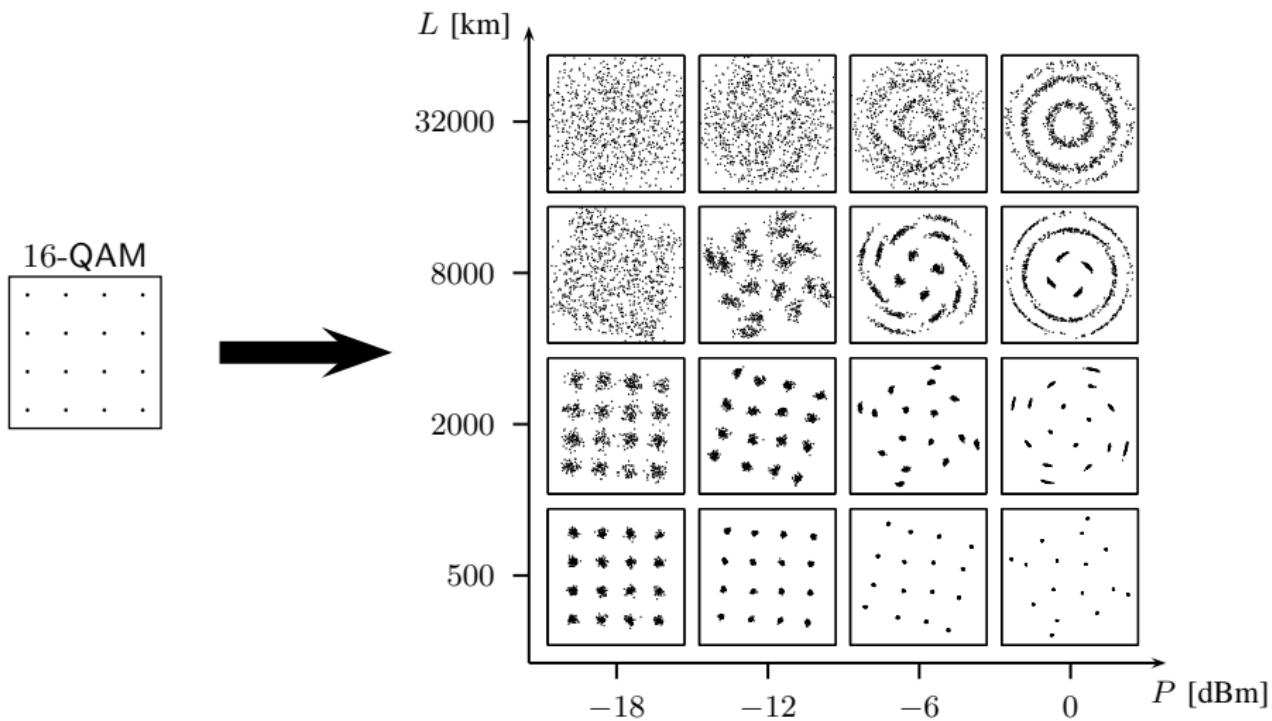


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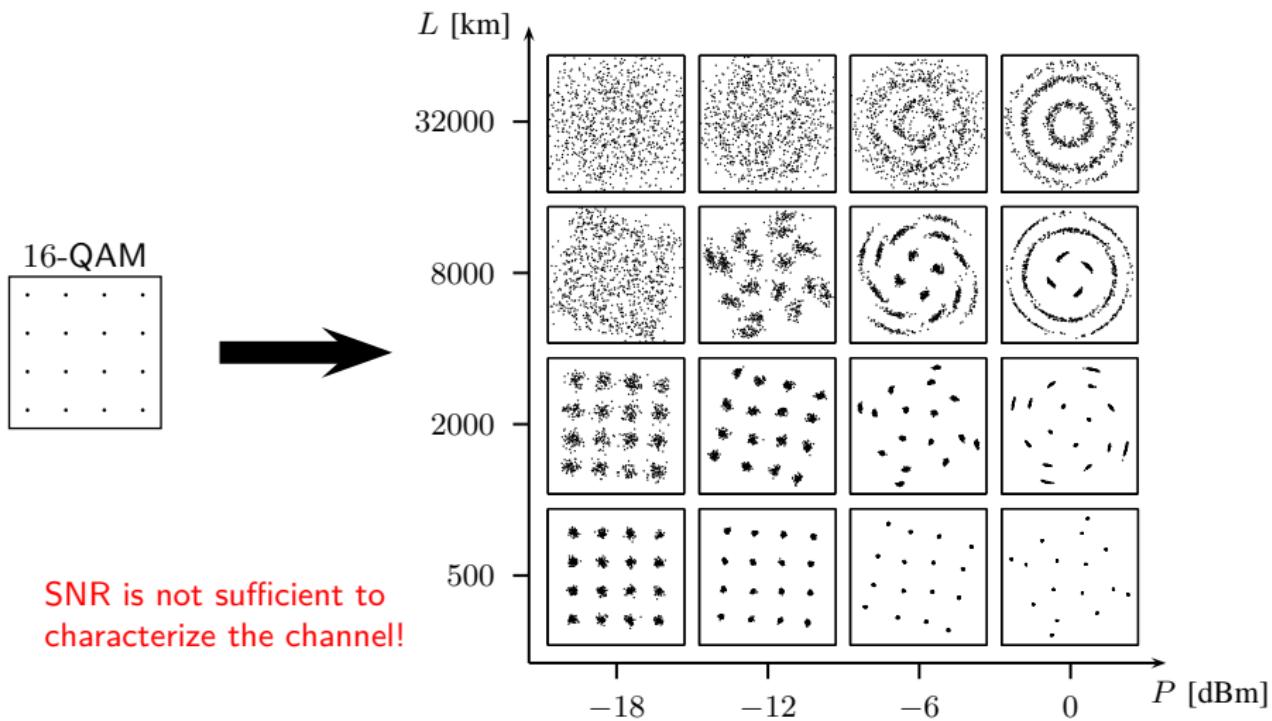
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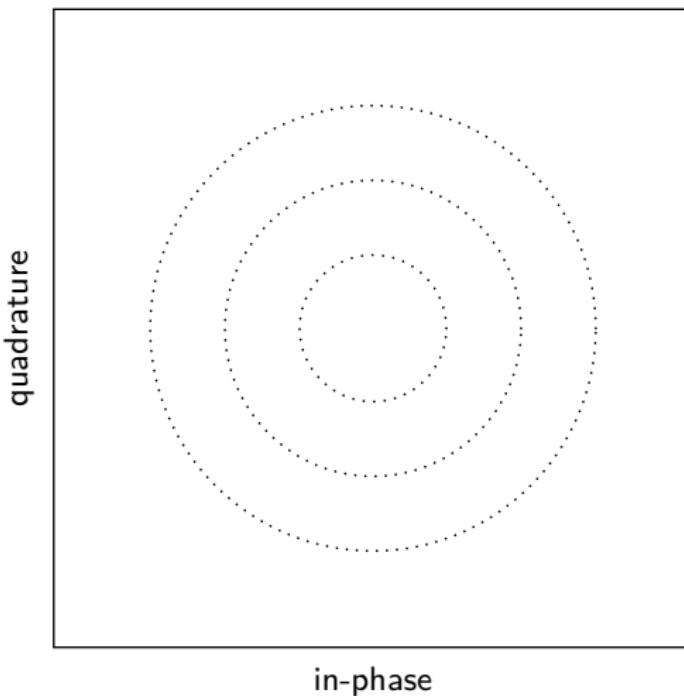
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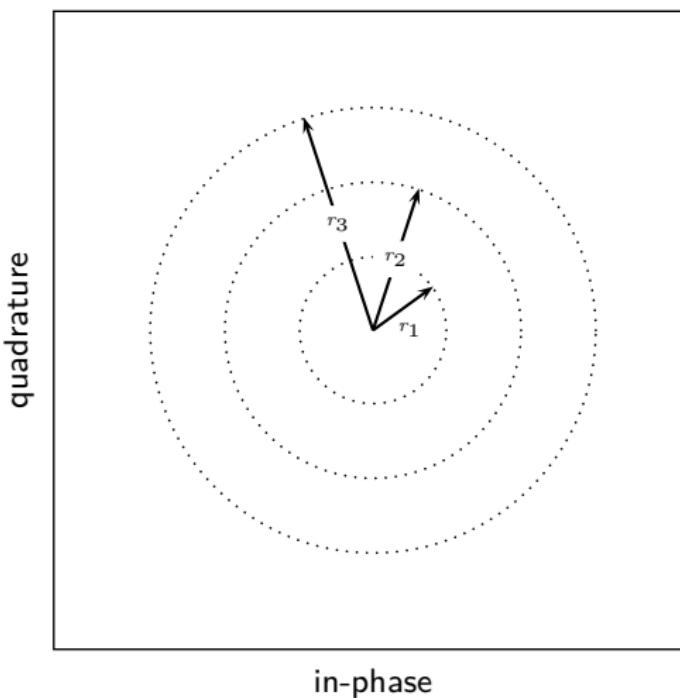
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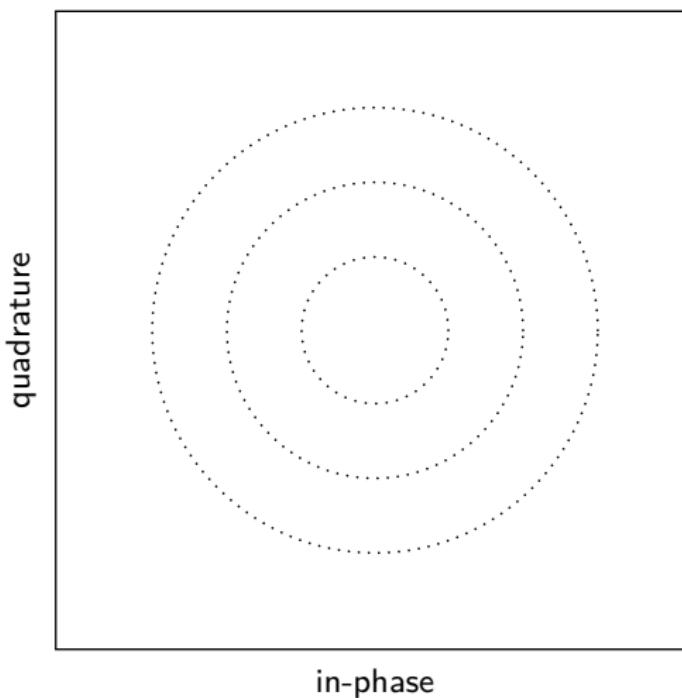
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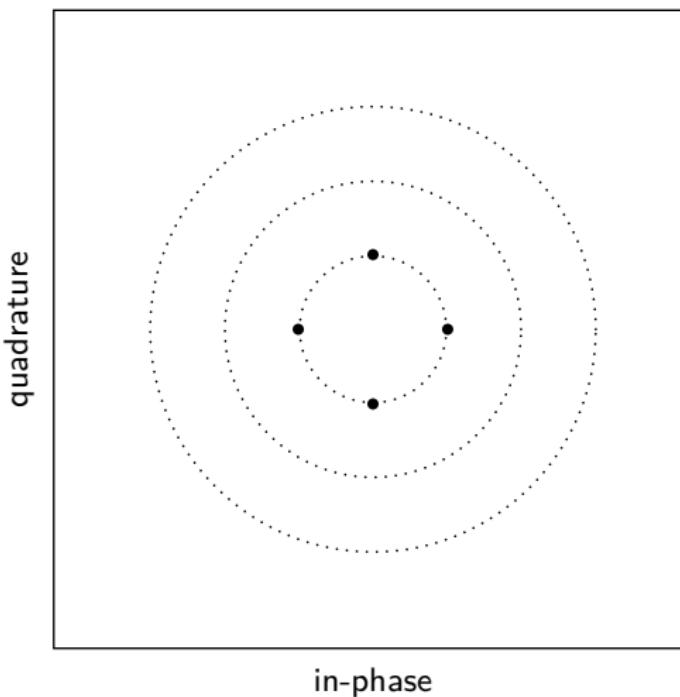


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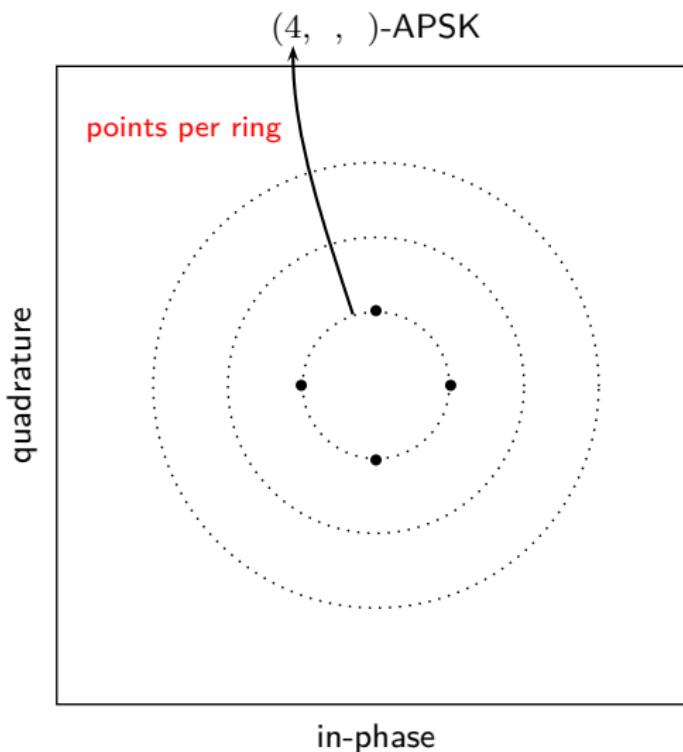


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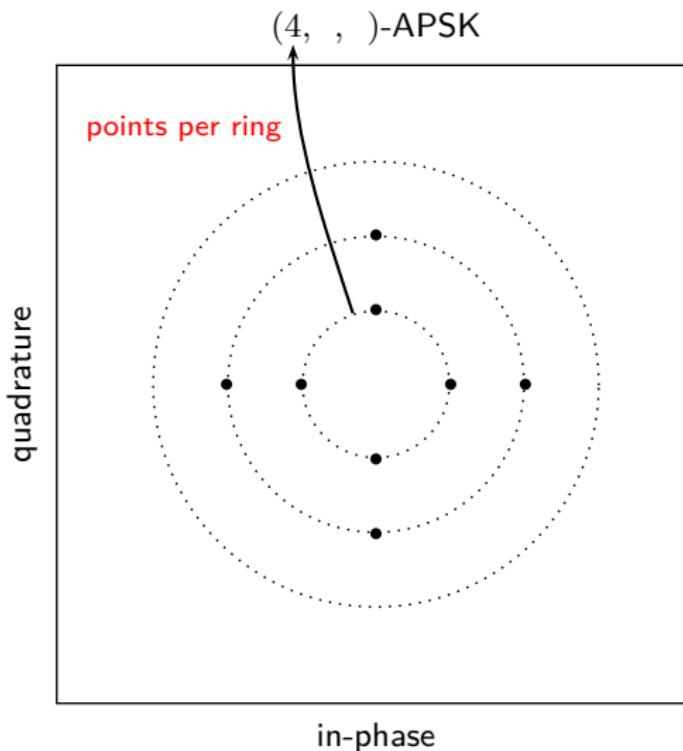
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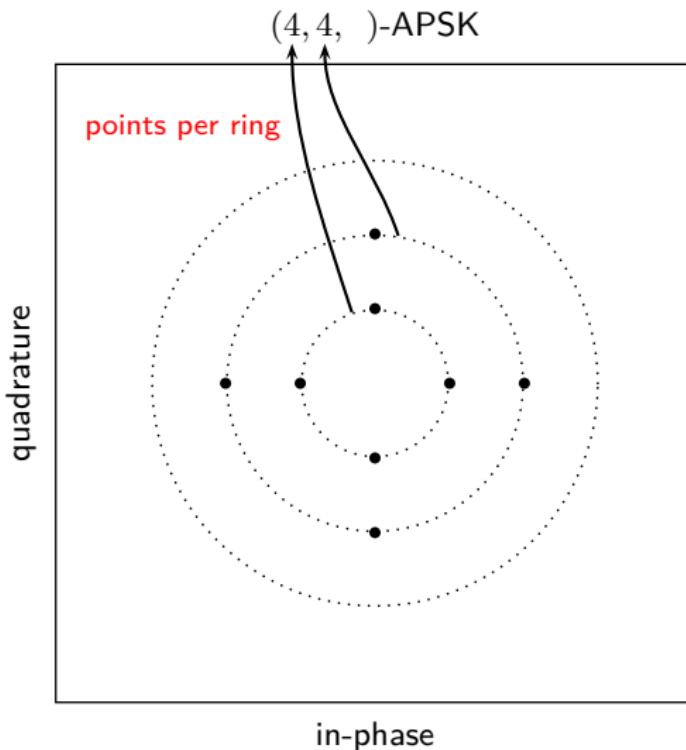
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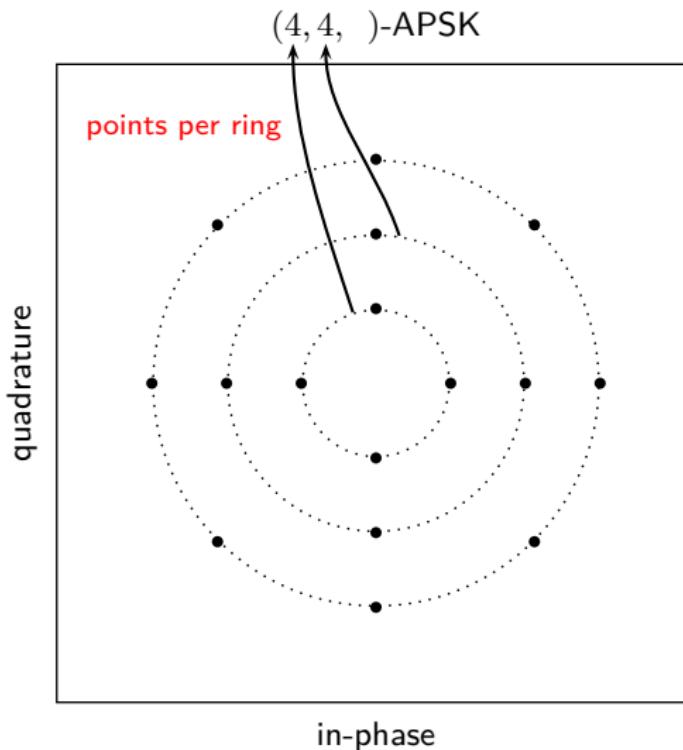
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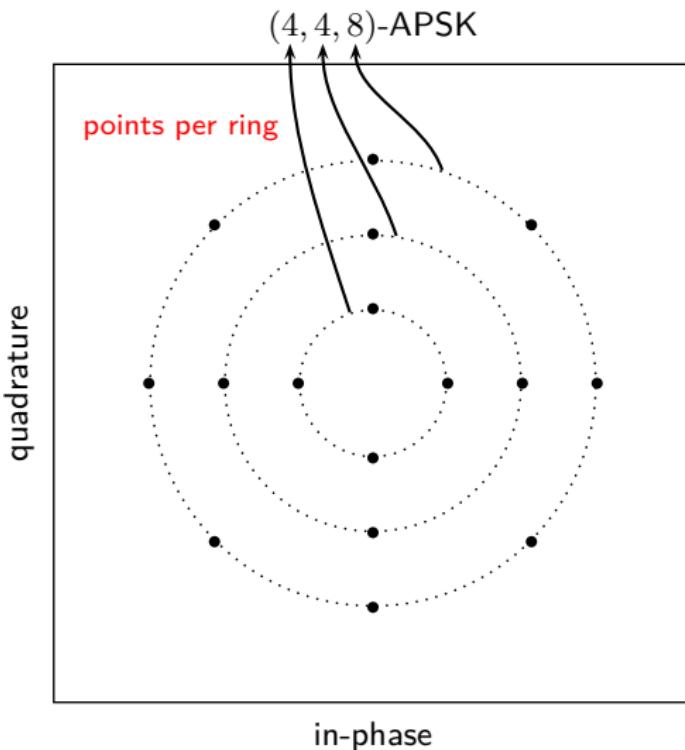
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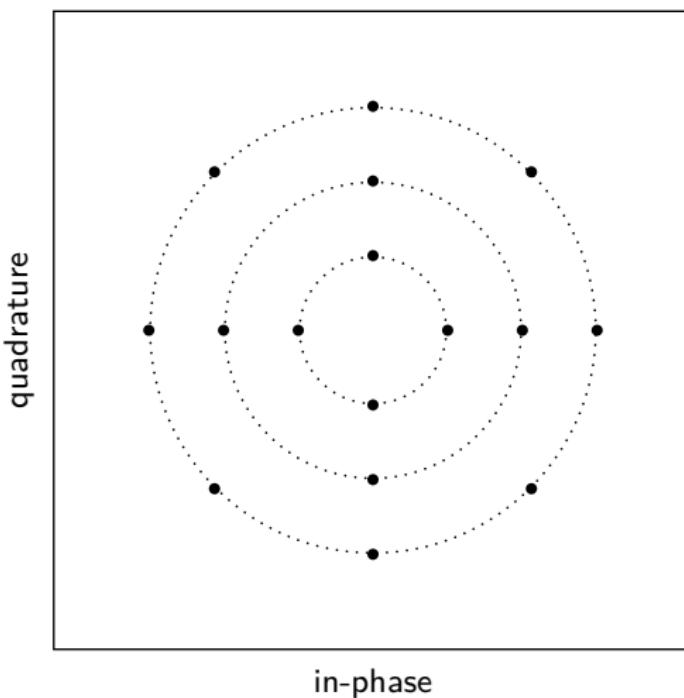


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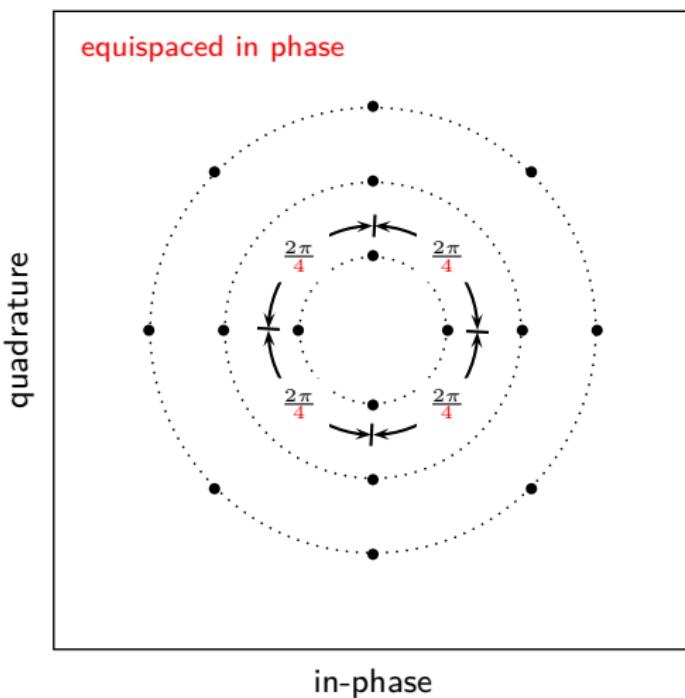
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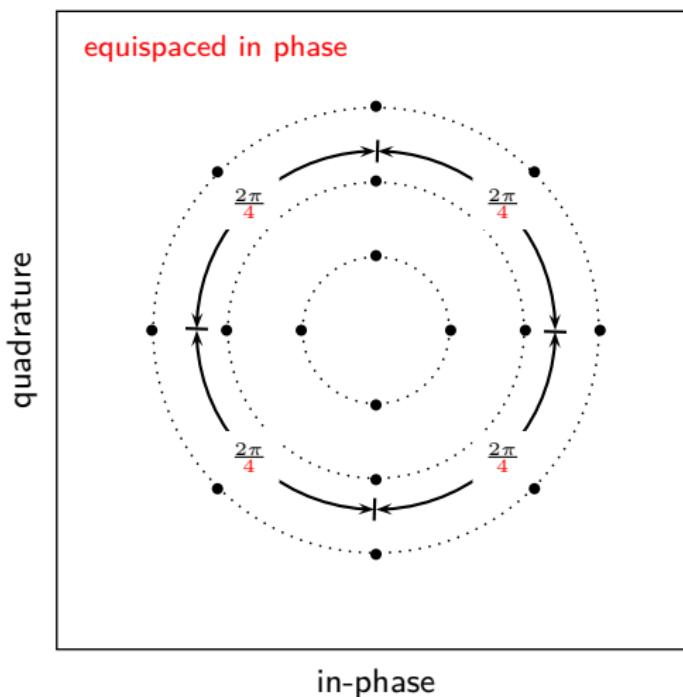
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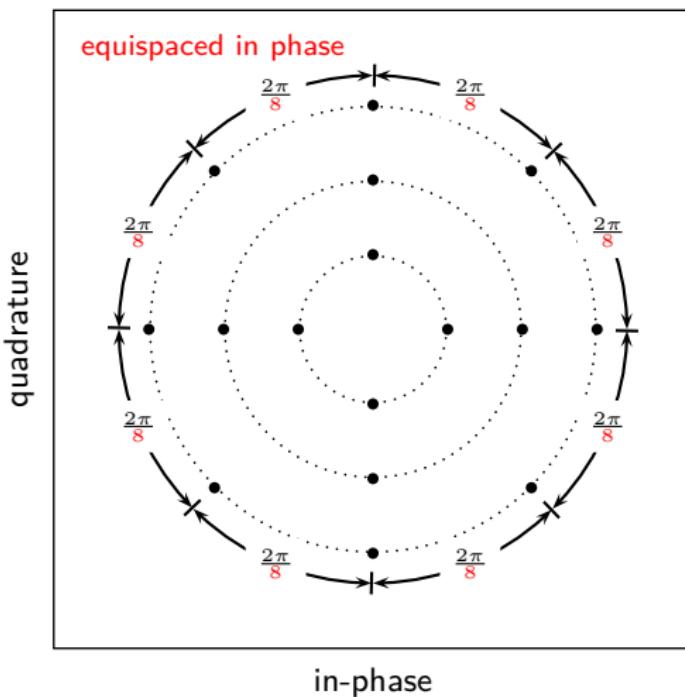
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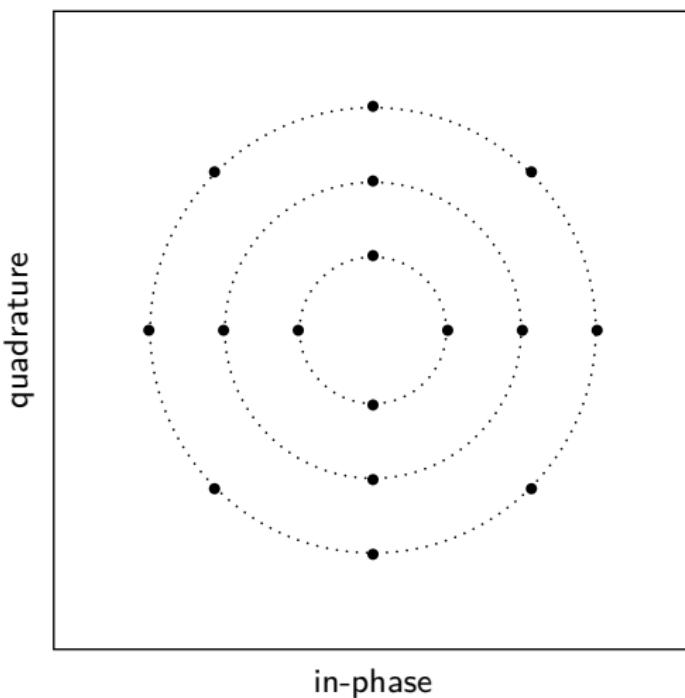
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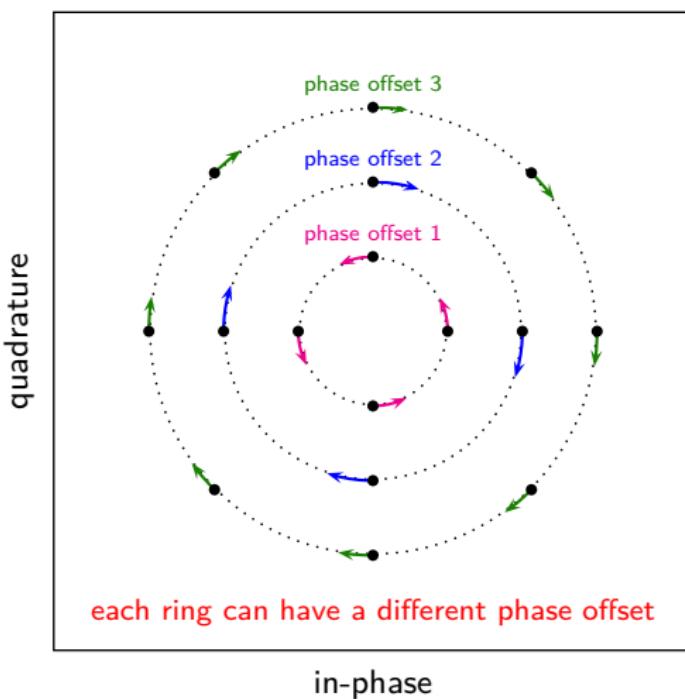
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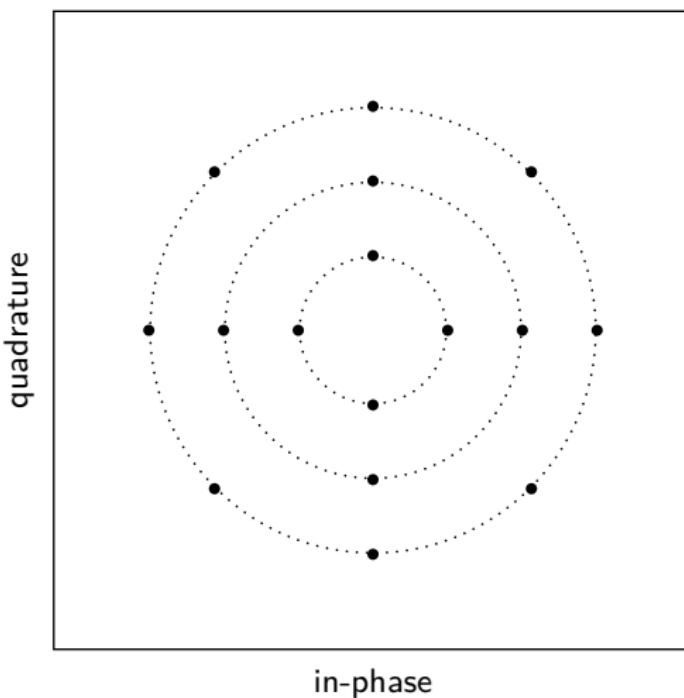
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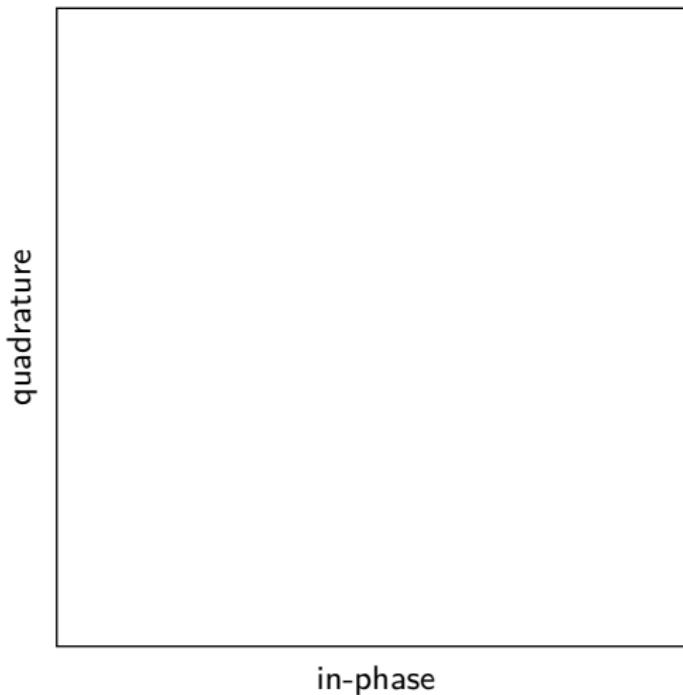


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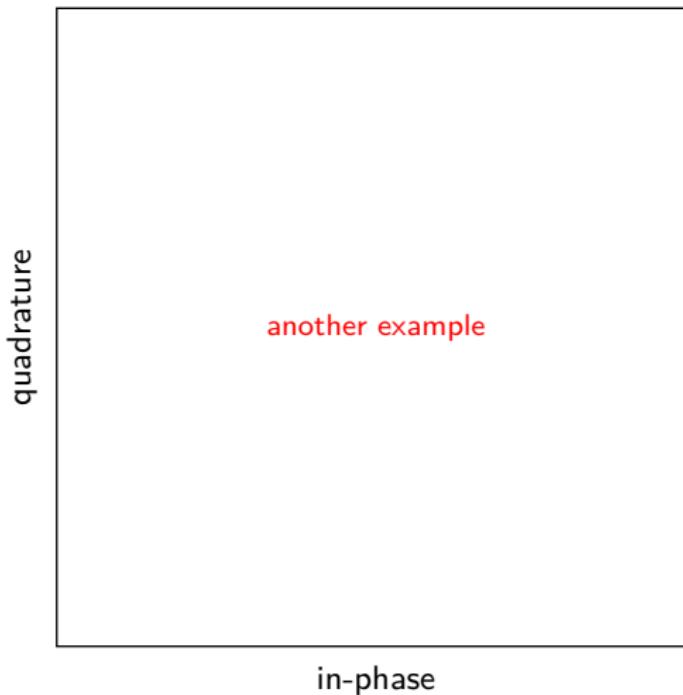
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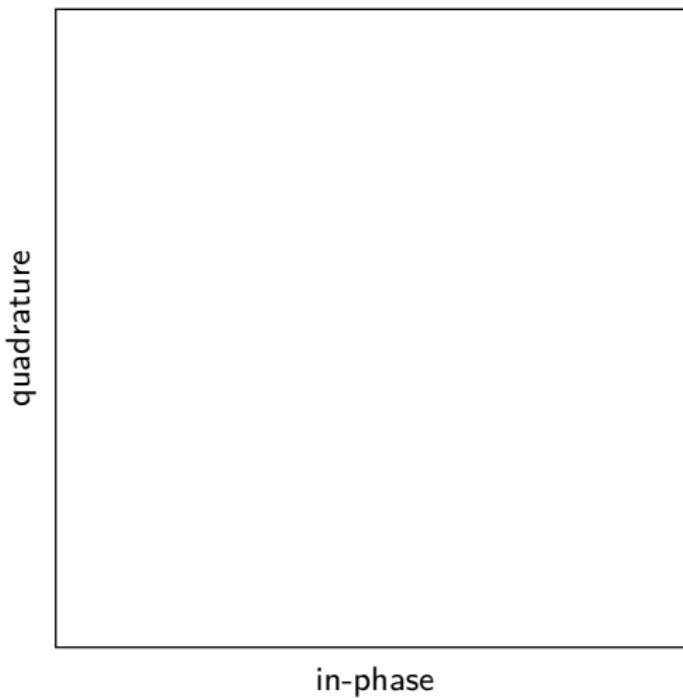


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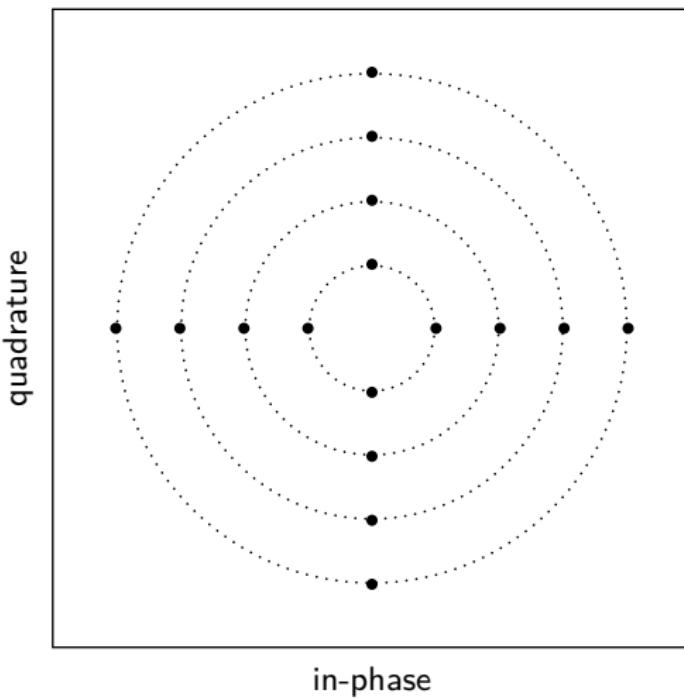
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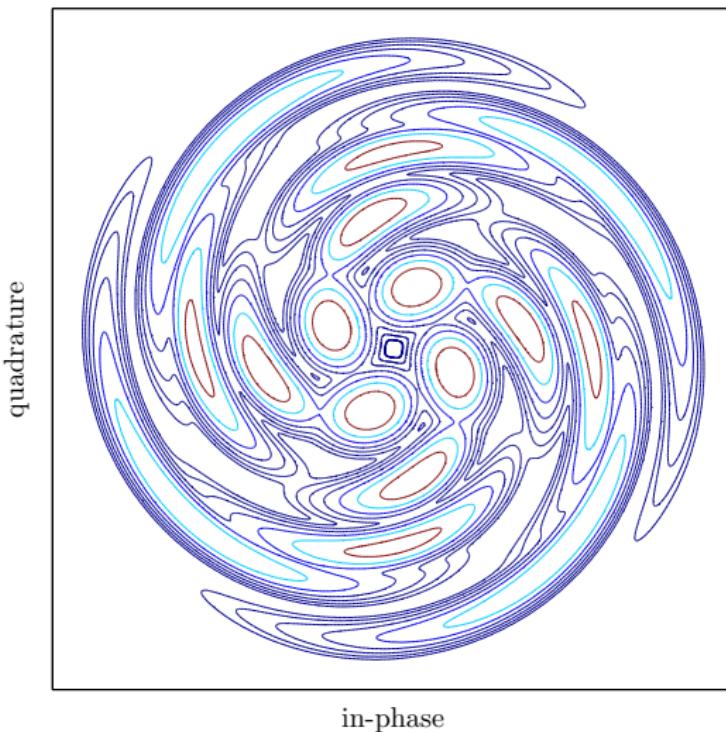
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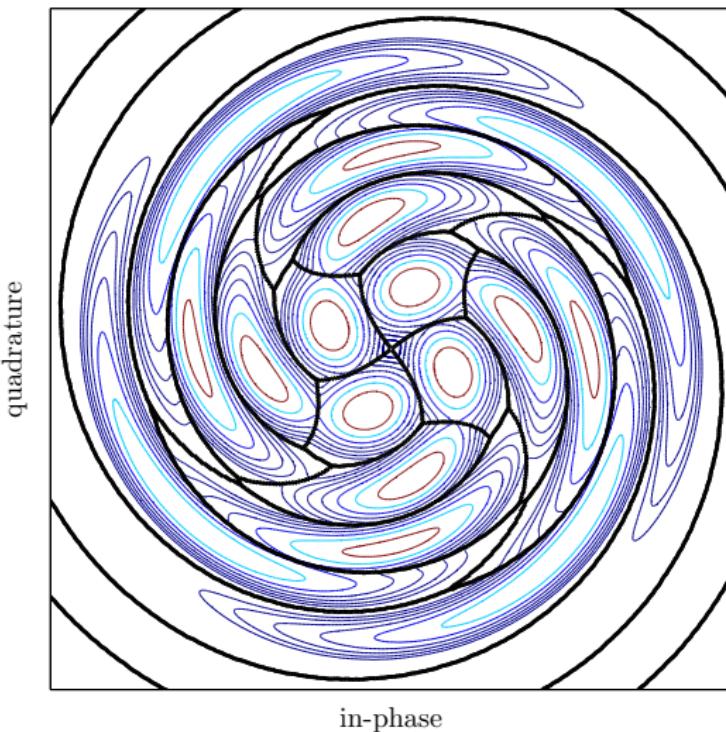
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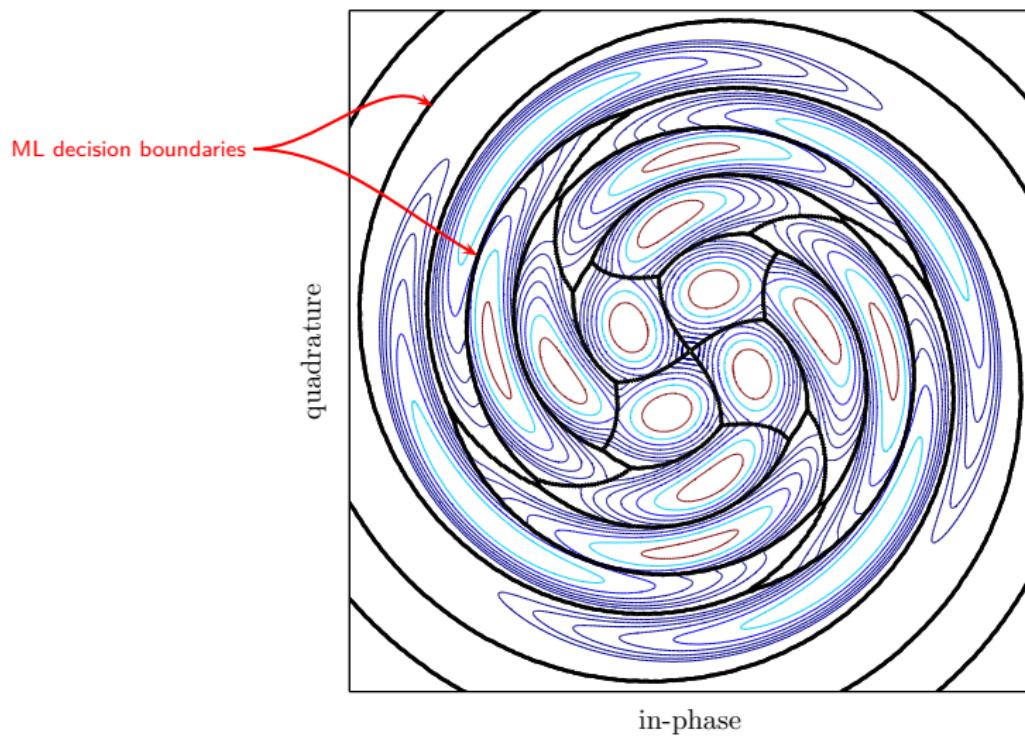
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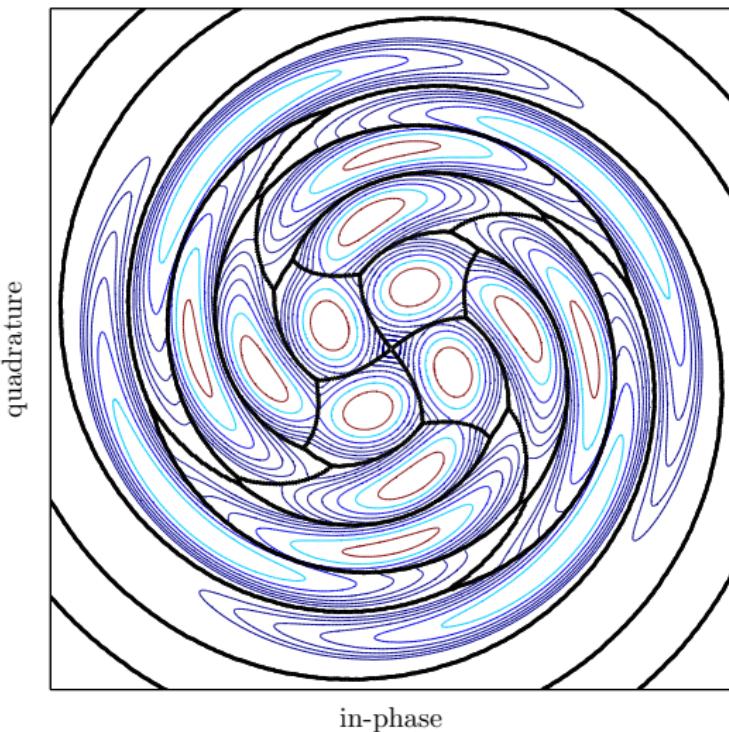
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- Probability density function (PDF)  $f_{Y|X=x_i}(y)$  **known**

PDF for (4,4,4,4)-APSK at  $P = -4$  dBm

PDF for  $(4,4,4,4)$ -APSK at  $P = -4$  dBm

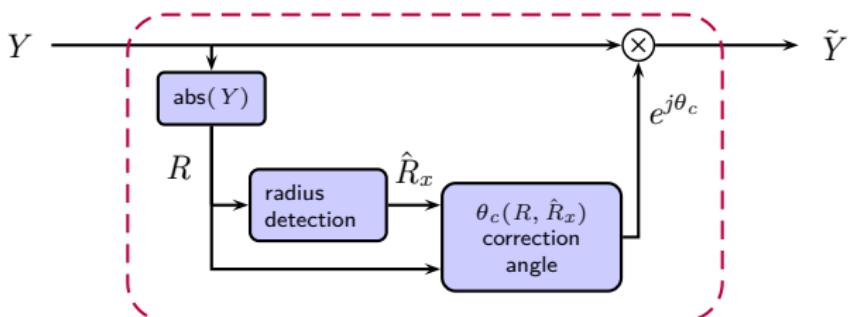
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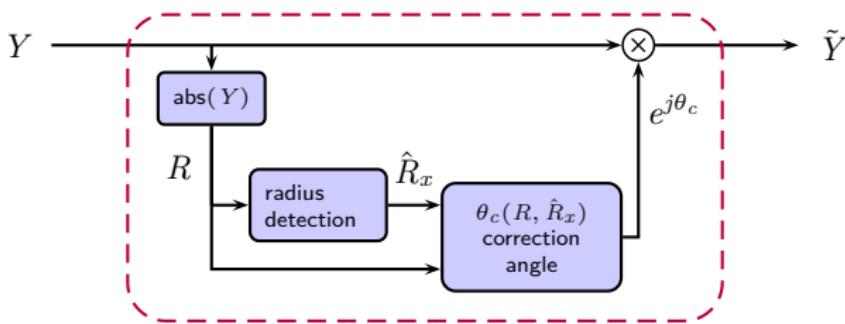
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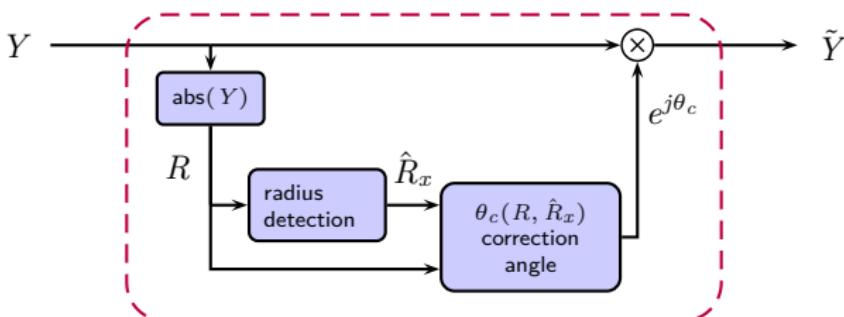


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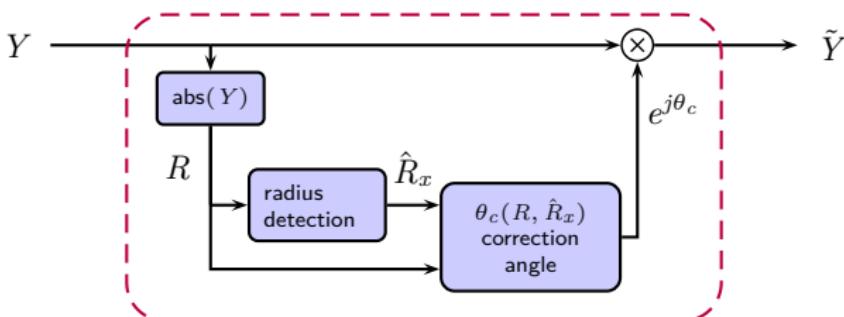
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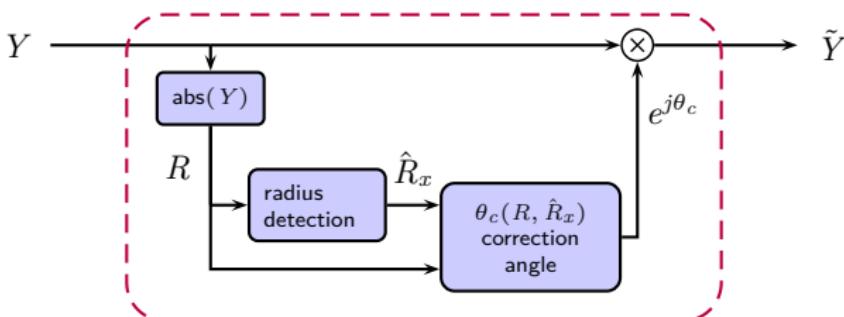
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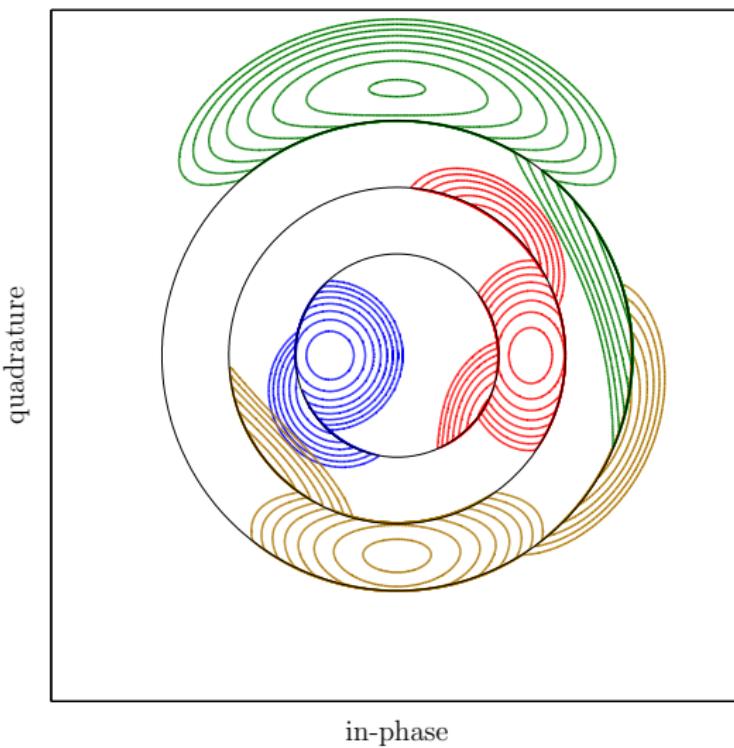
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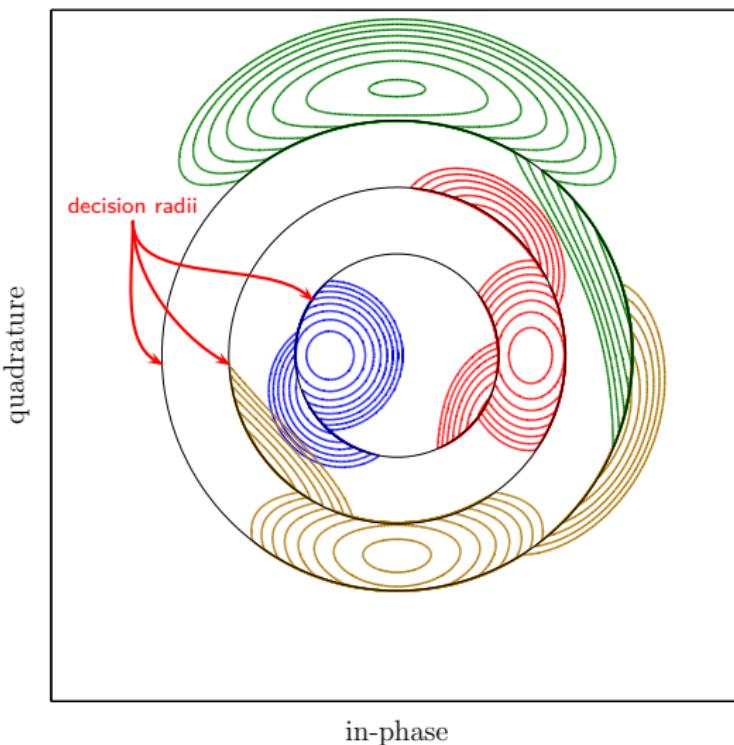
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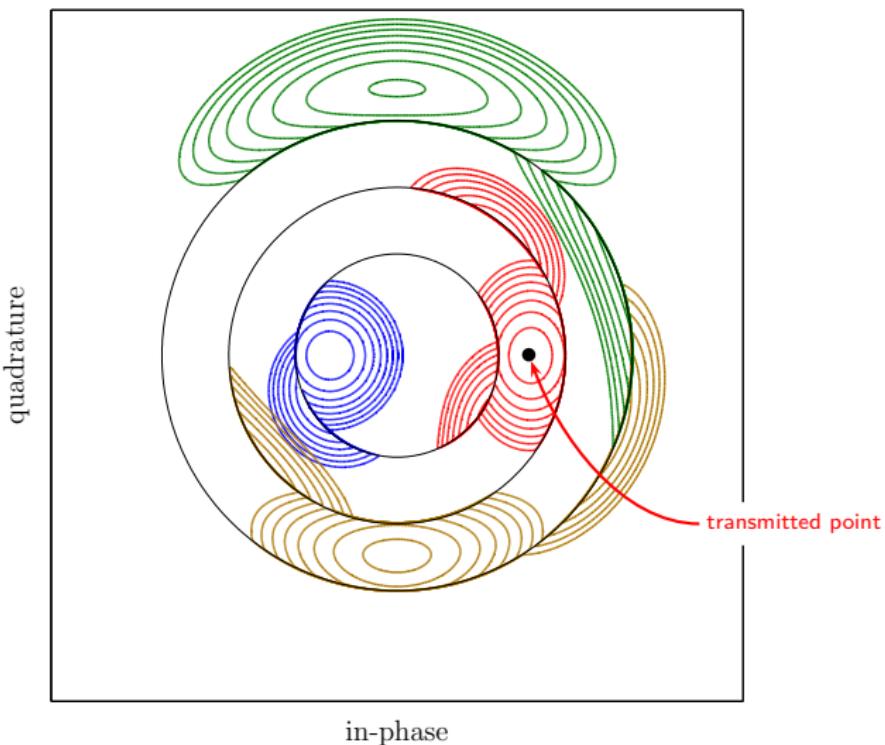


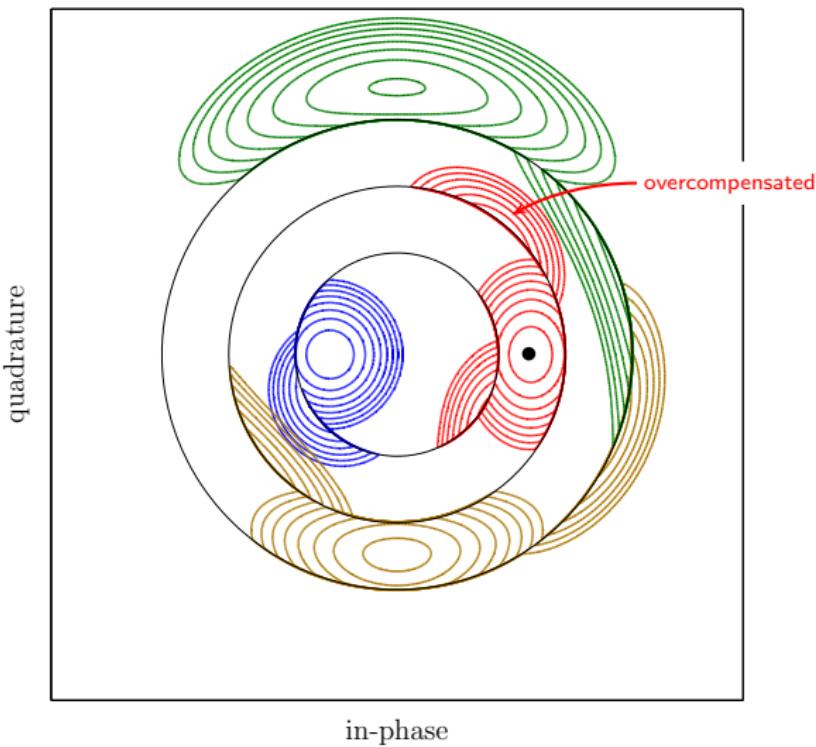
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- Note: the PDF of  $\tilde{Y}$  is defined piecewise

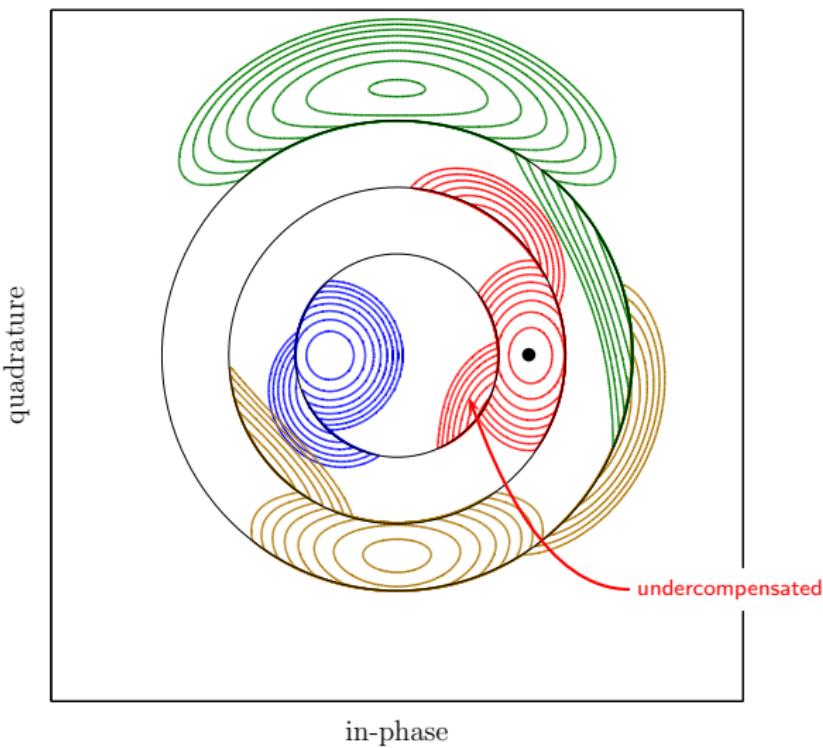
PDF of  $\tilde{Y}$  for (4,4,4,4)-APSK at  $P = -4$  dBm

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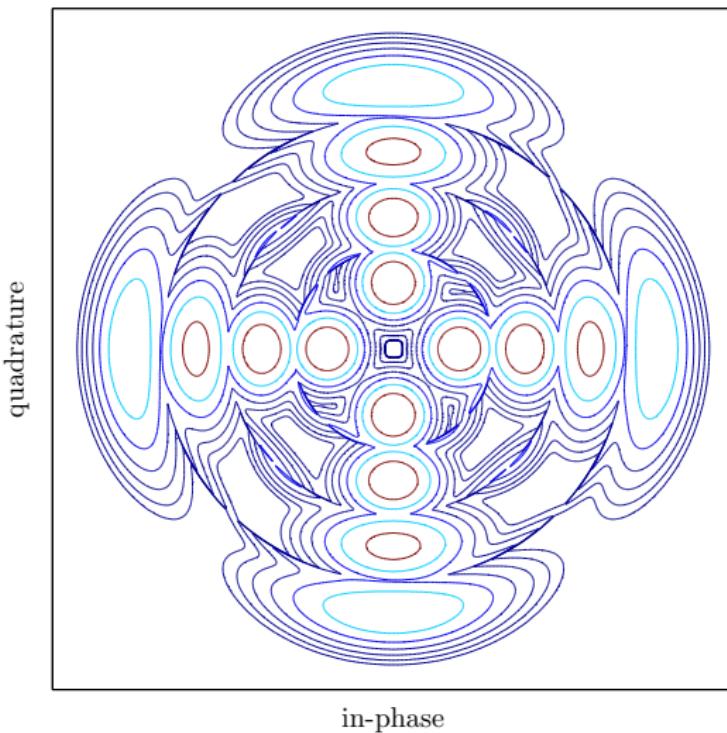
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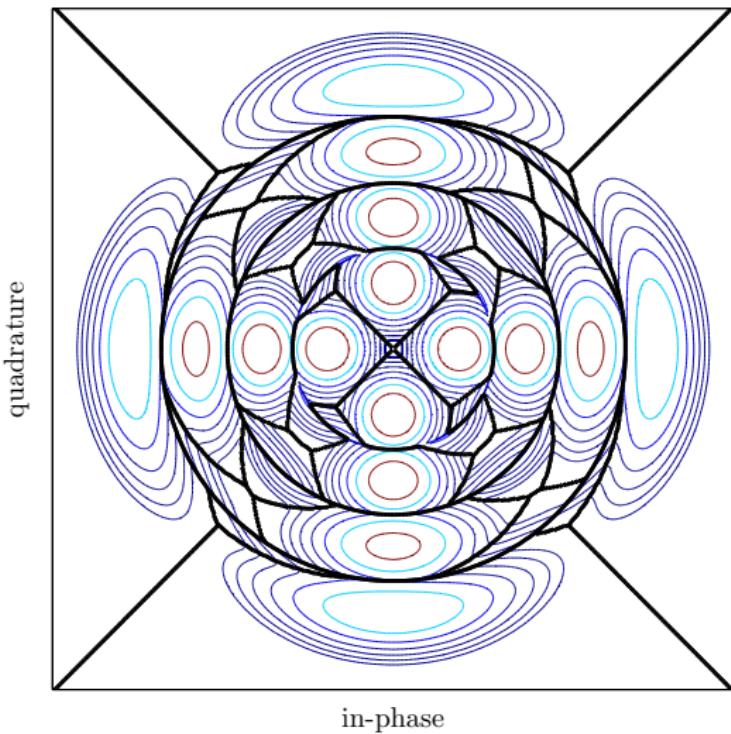
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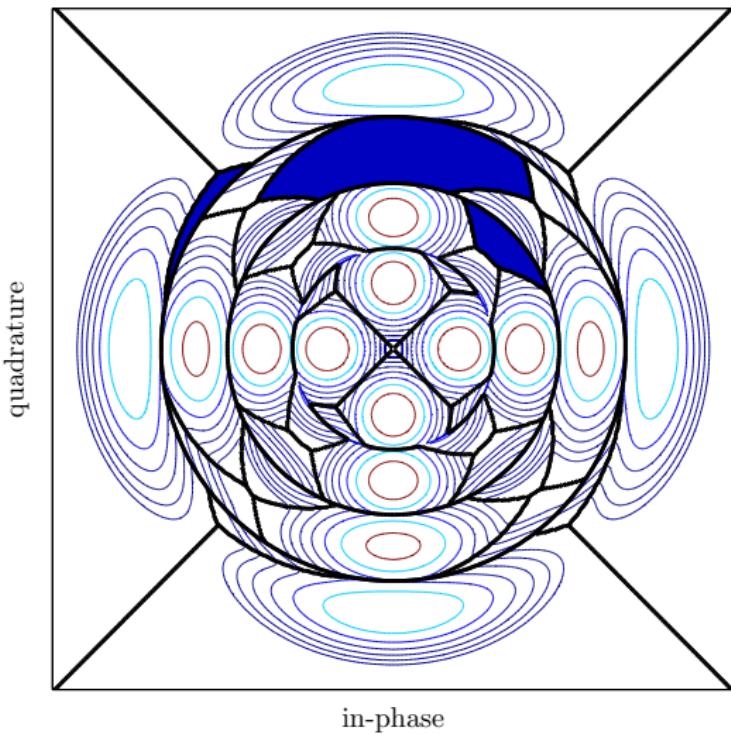
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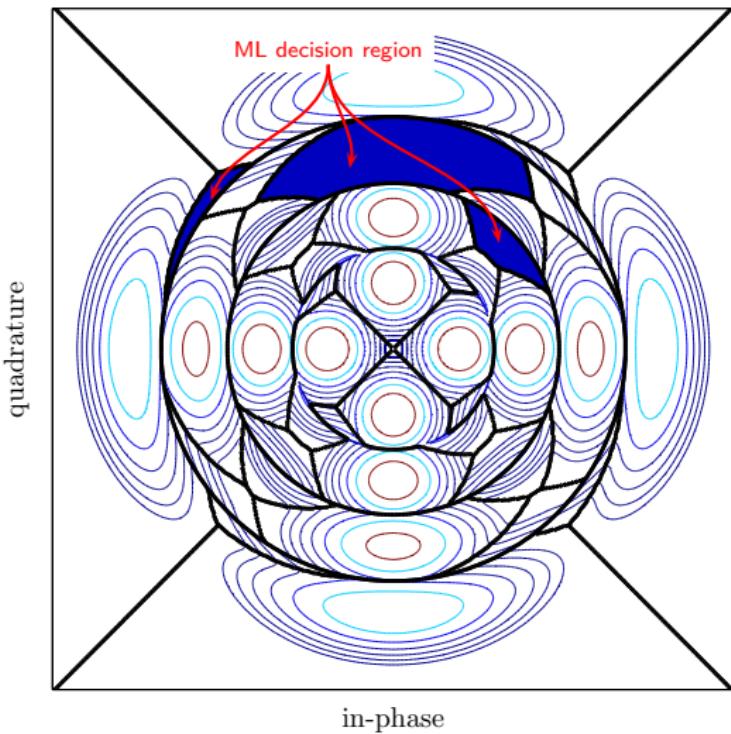
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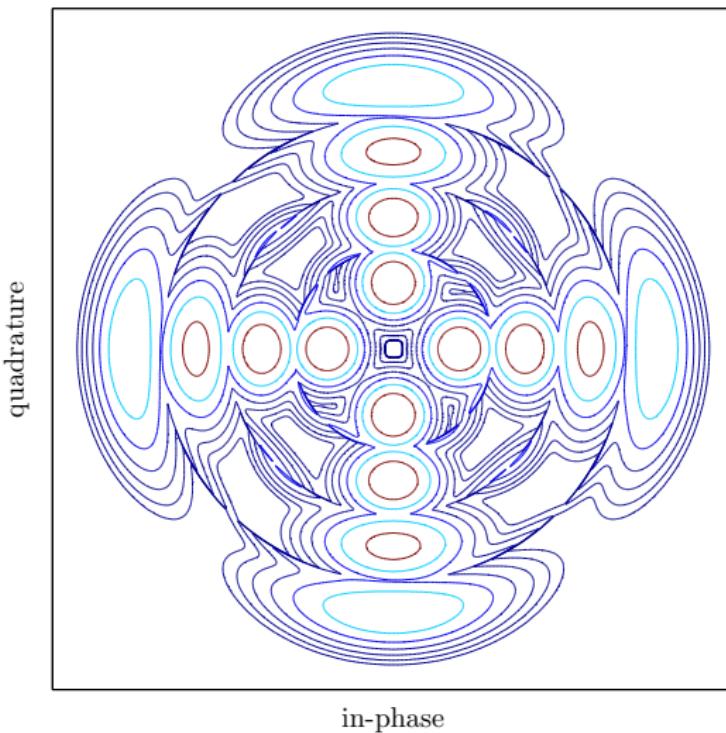
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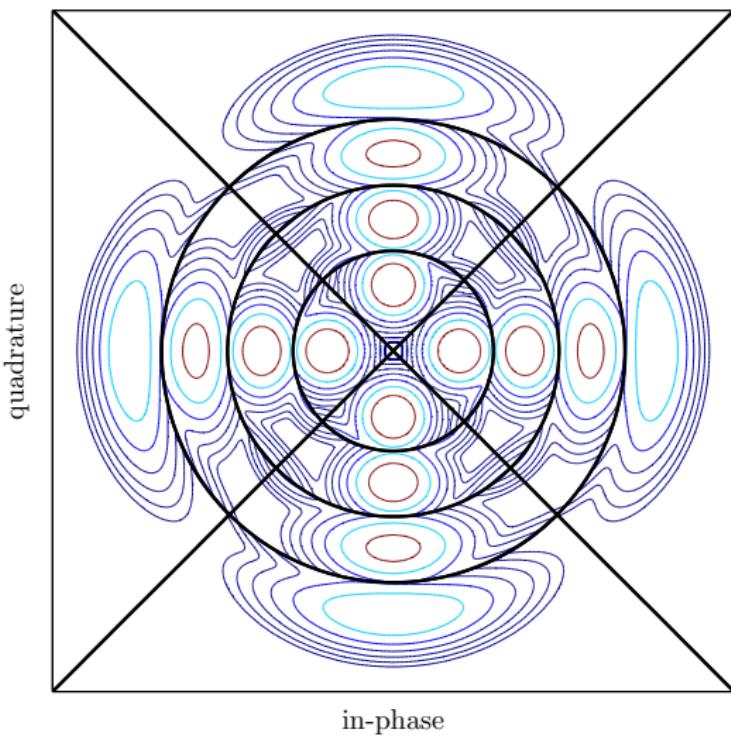
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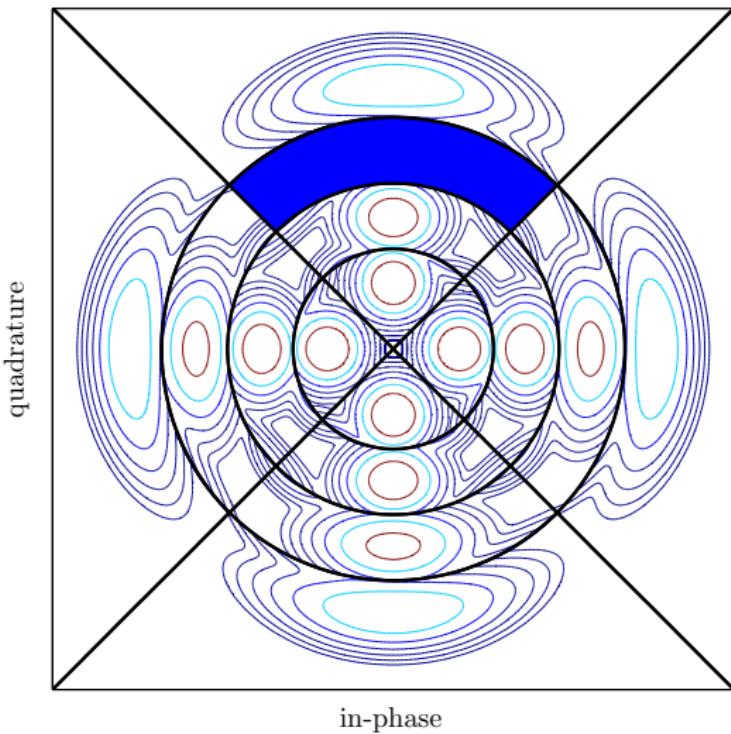
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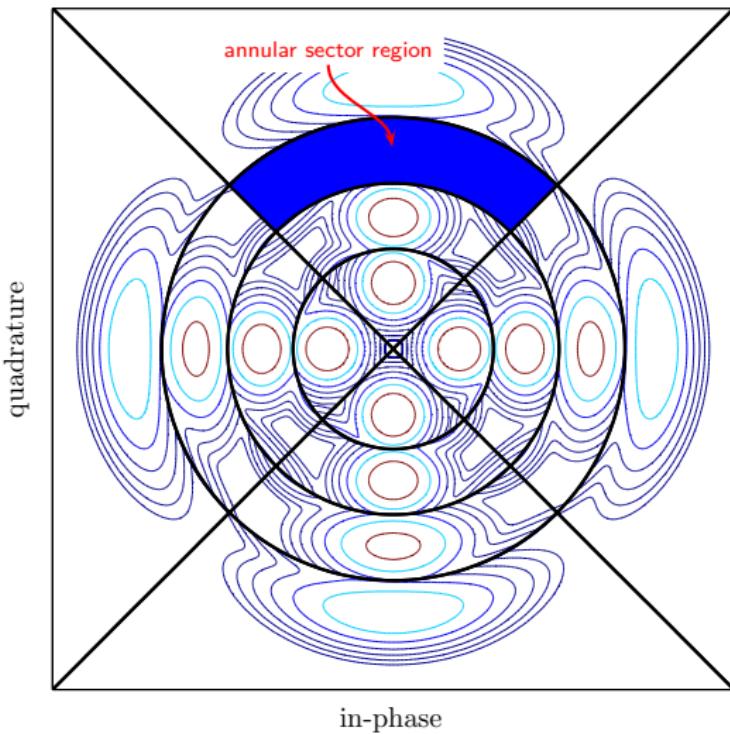
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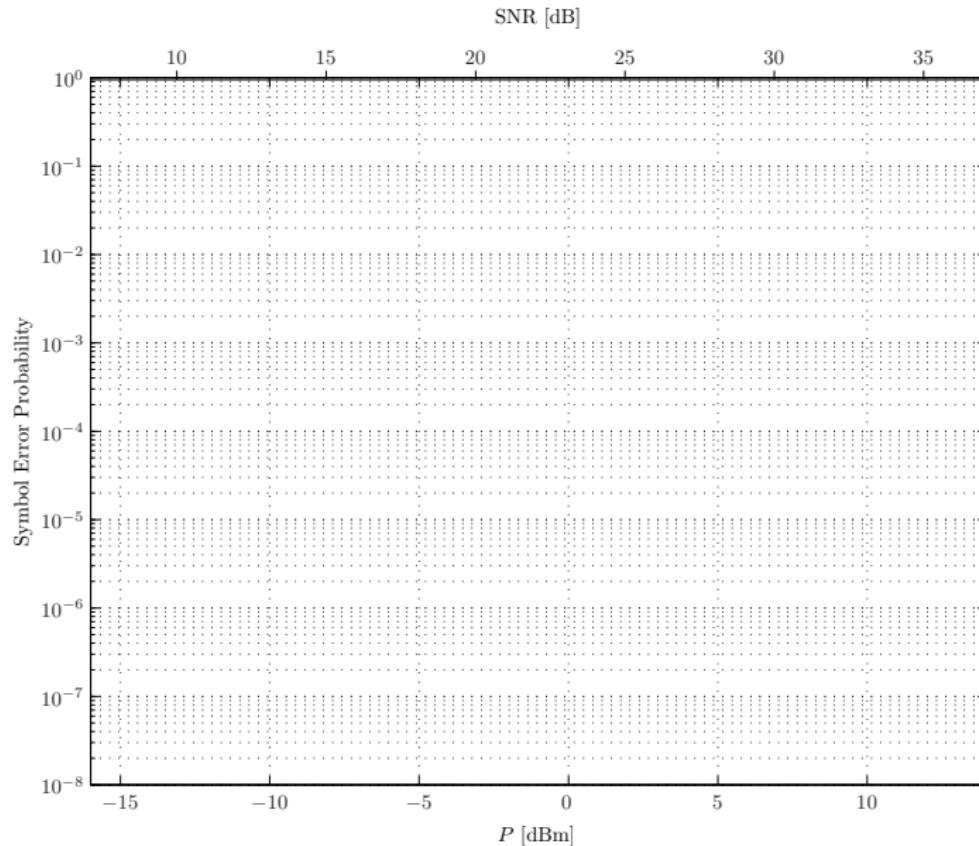
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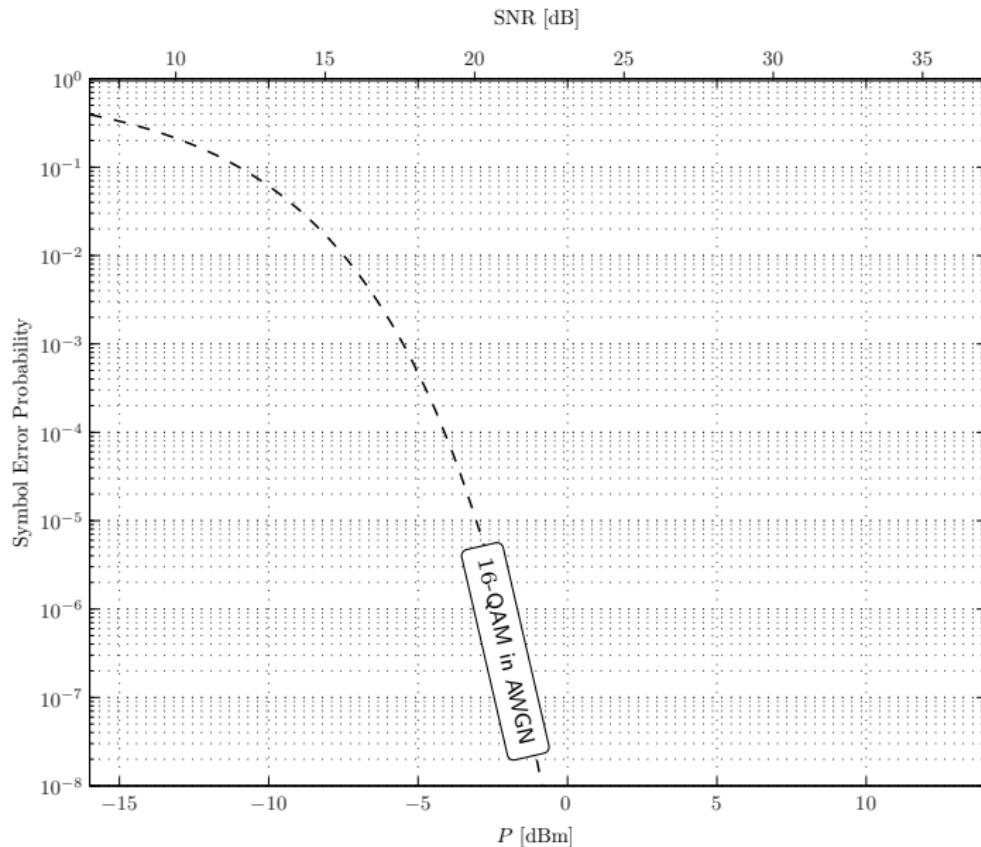
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- What phase offset? **Two-stage detection is insensitive to a phase offset.**

# Optimizing the Number of Rings and Points per Ring

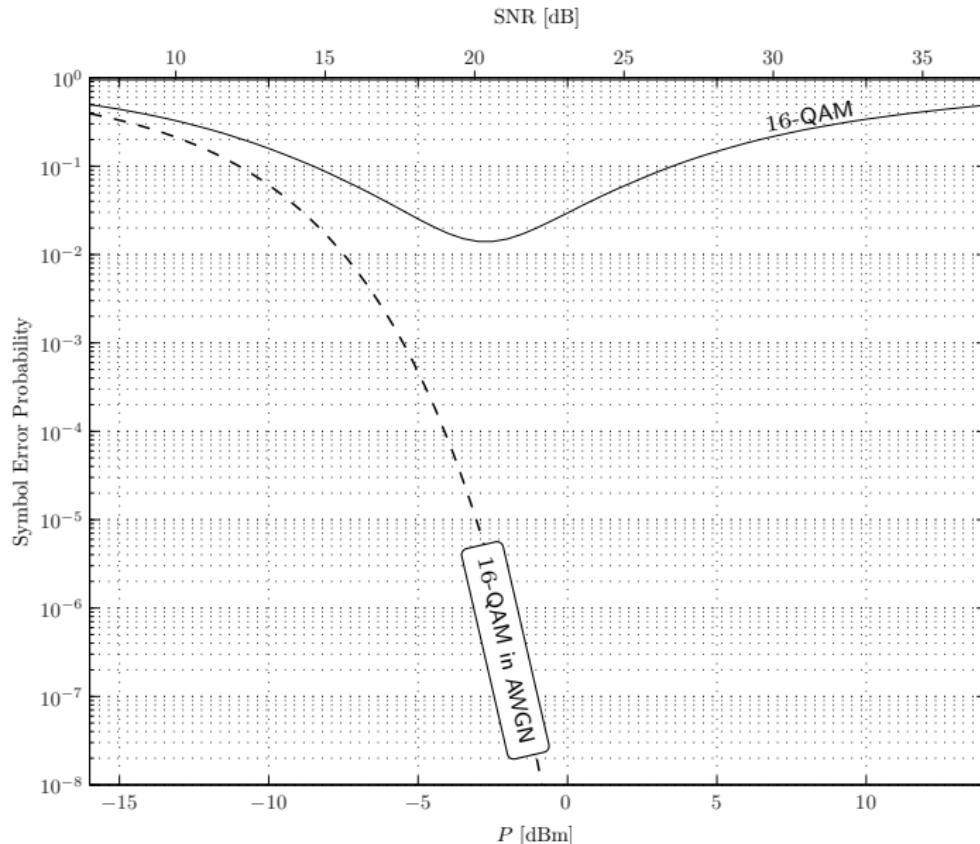
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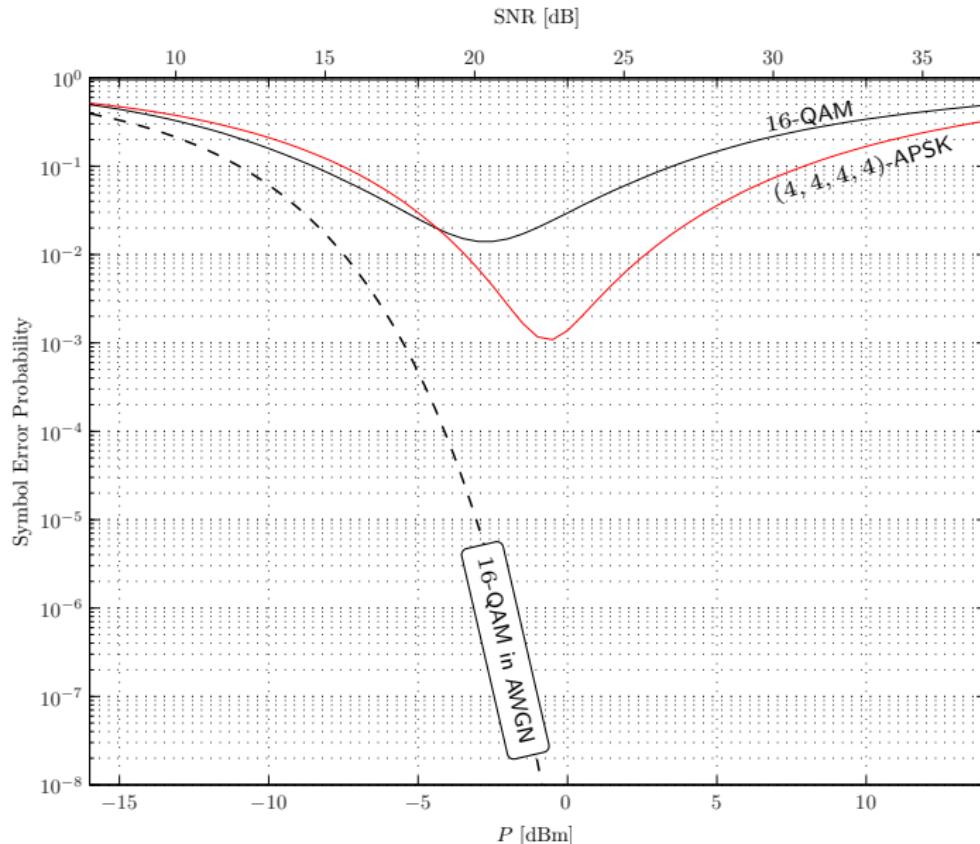
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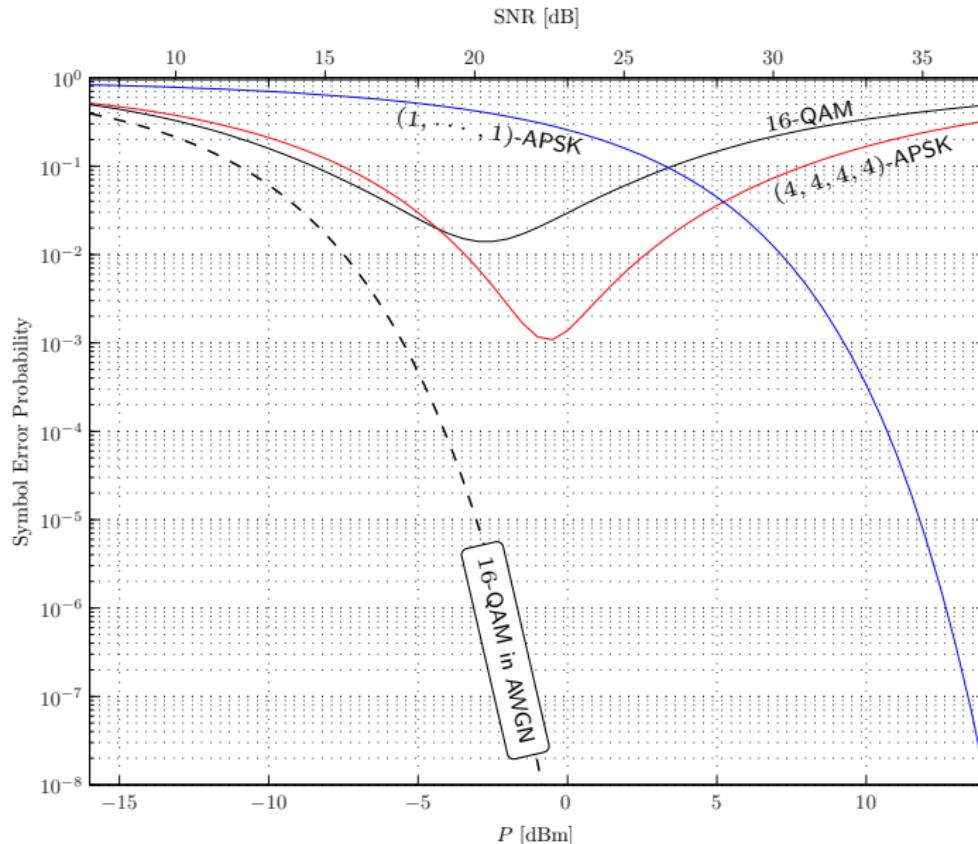
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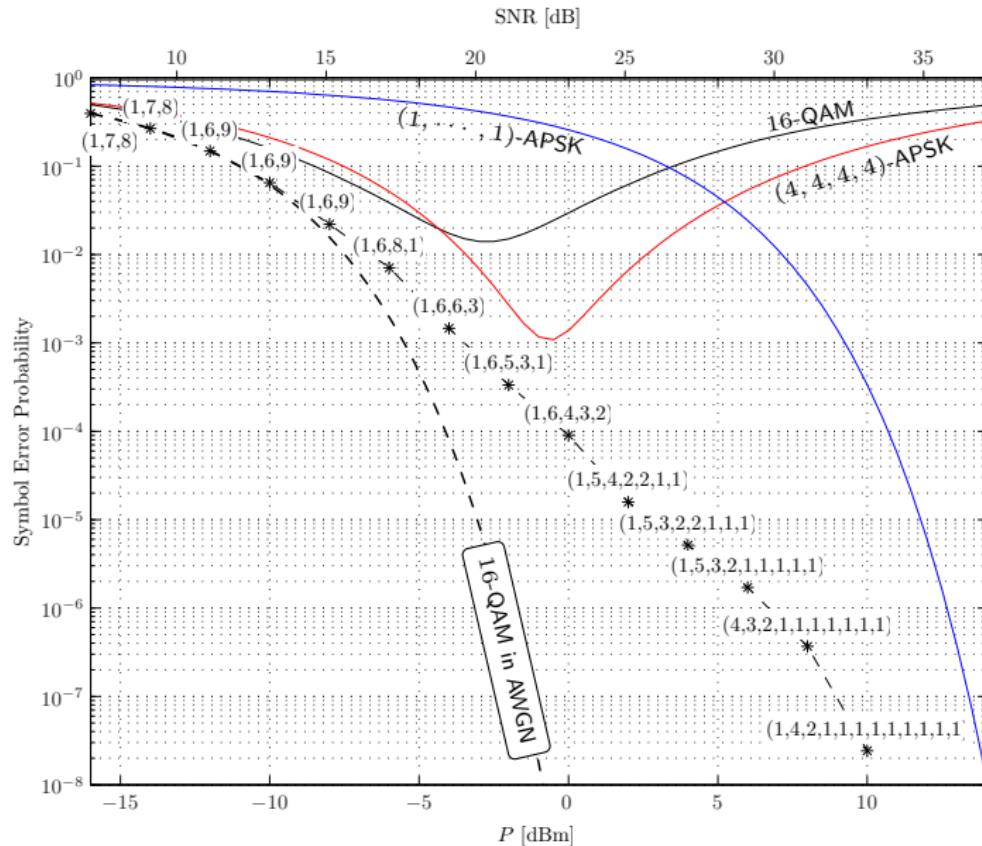
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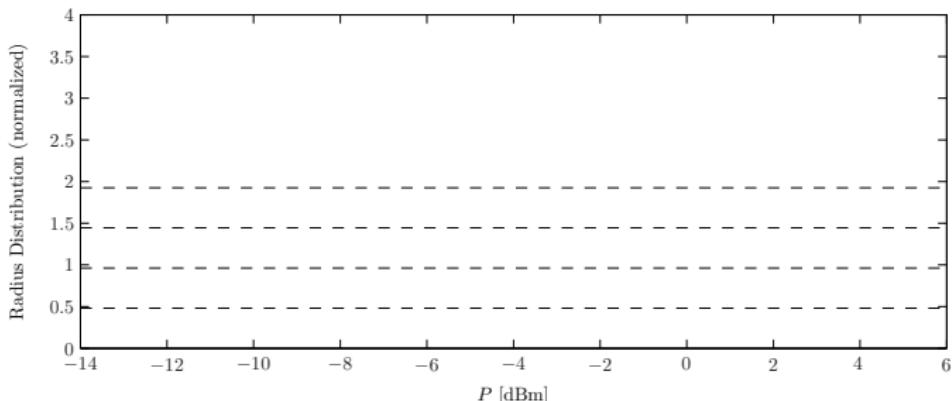
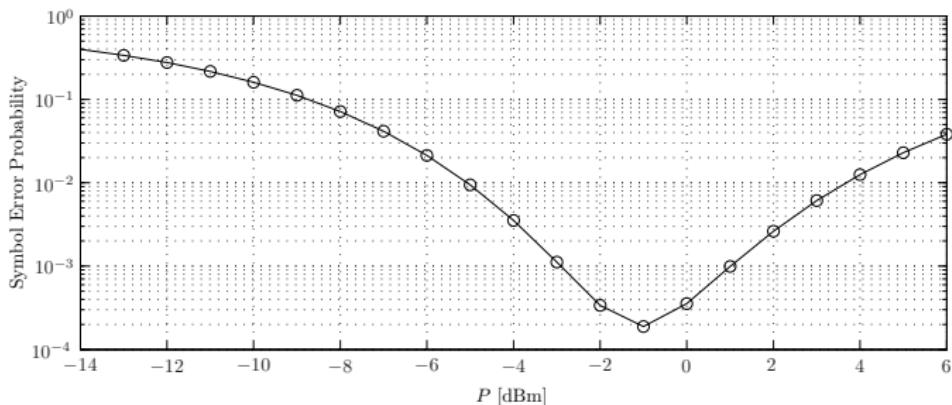


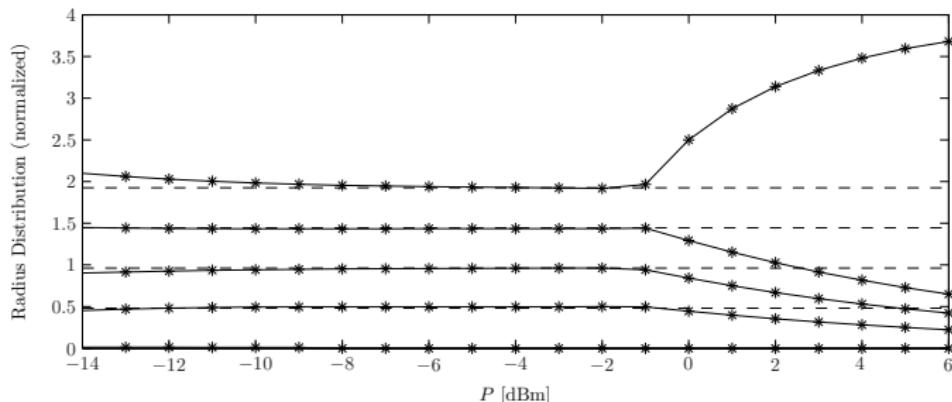
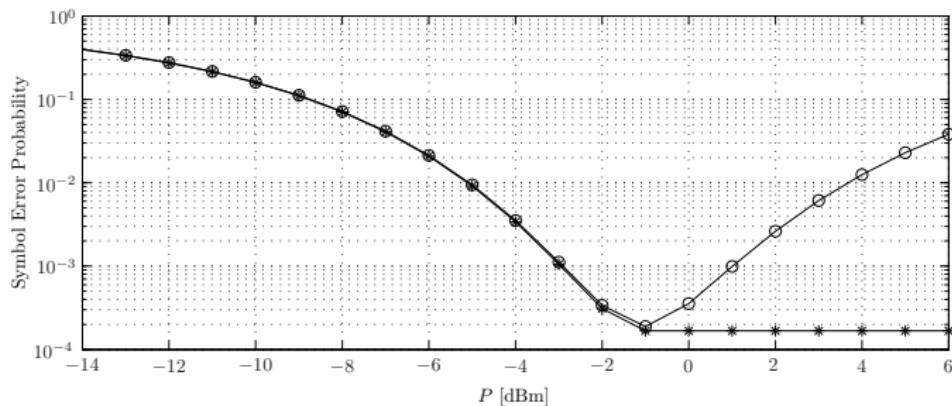
# Optimizing the Number of Rings and Points per Ring

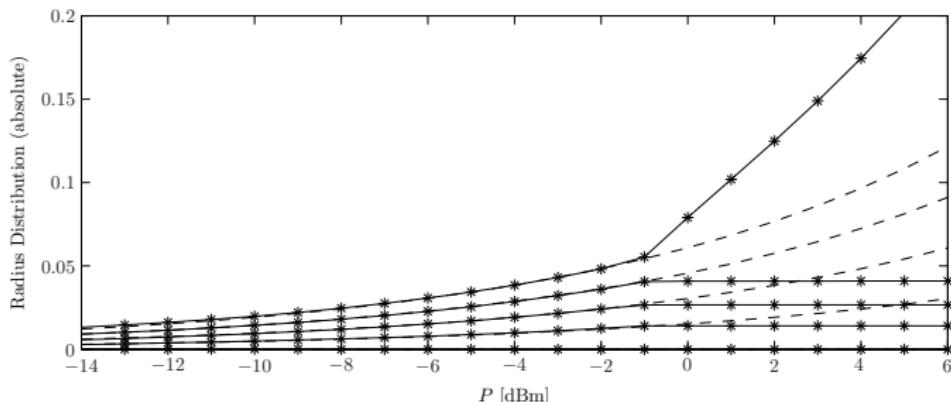
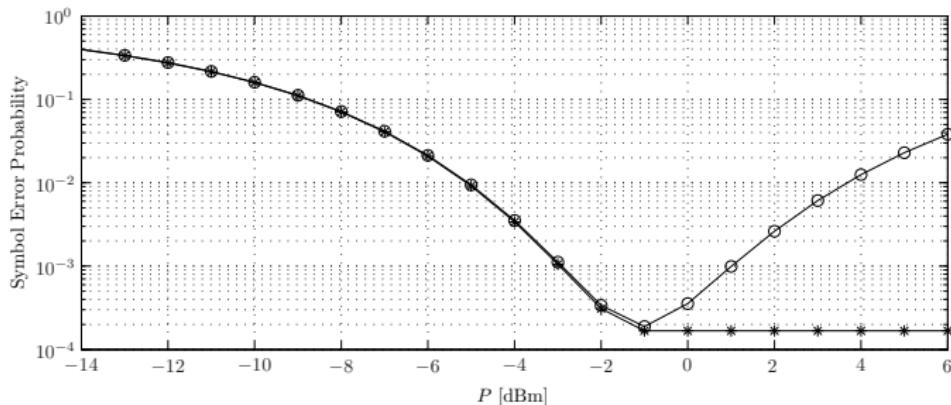


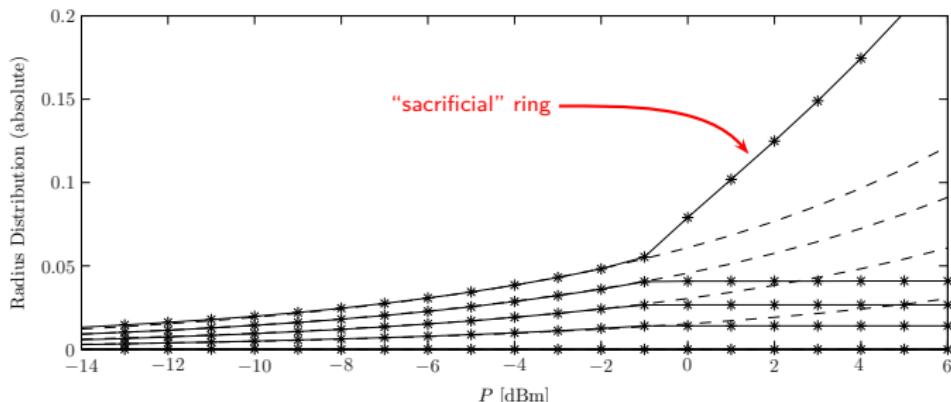
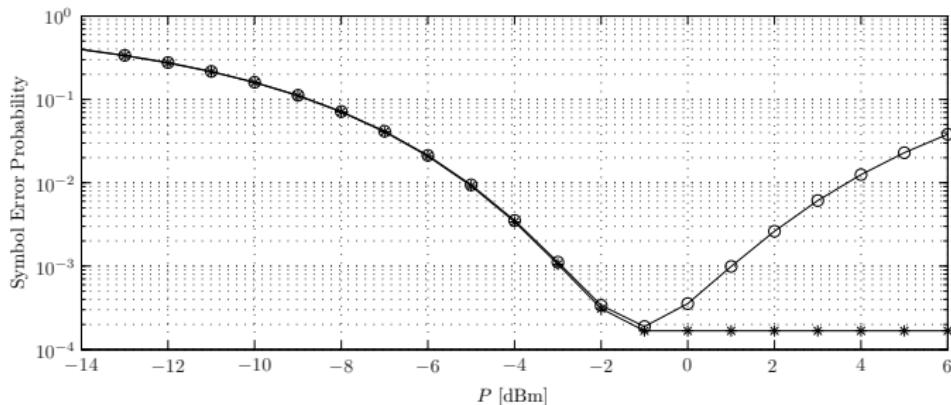
# Optimizing the Number of Rings and Points per Ring



Radius Optimization for  $(1,6,5,3,1)$ -APSK

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Thank you!



## References

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