

# Model-Based Machine Learning for Physical-Layer Communication over Optical Fiber

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Summer School on “AI for Optical Networks &  
Neuromorphic Photonics for AI Acceleration”  
September 6, 2021



**CHALMERS**

**FORCE**  
FIBER-OPTIC COMMUNICATIONS  
RESEARCH CENTER

# Thank You!



**Henry D. Pfister**  
Duke



**Christoffer Fougstedt**  
Chalmers (now: Ericsson)



**Lars Svensson**  
Chalmers



**Per Larsson-Edefors**  
Chalmers



**Rick M. Bütler**  
TU/e (now: TU Delft)



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**Menno van den Hout**  
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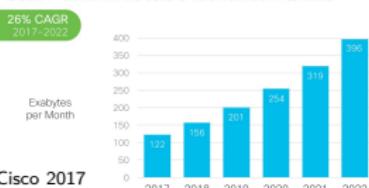
**Sjoerd van der Heide**  
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**Chigo Okonkwo**  
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## Motivation and Challenges

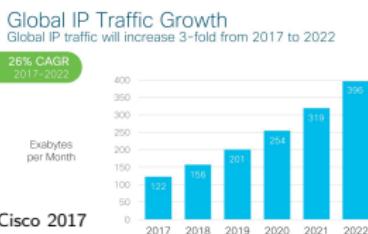
Global IP Traffic Growth  
Global IP traffic will increase 3-fold from 2017 to 2022



Source: Cisco 2017

- The COVID-19 pandemic has highlighted the importance of our global communication infrastructure
- Data traffic has been and will continue to grow exponentially
- Simply scaling current technology is not sustainable: fiber infrastructure would consume all world-wide electricity within less than 10 years

## Motivation and Challenges



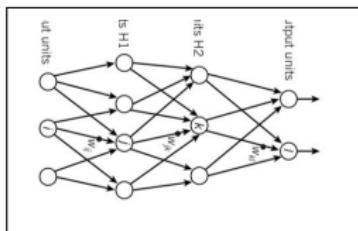
- Higher data rates?
- More energy efficiency?
- New functionalities?

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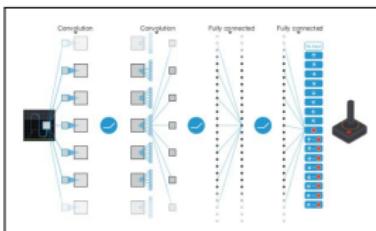
How can machine learning (ML) be used productively in communications to improve future systems?

# This work started with a simple observation ...

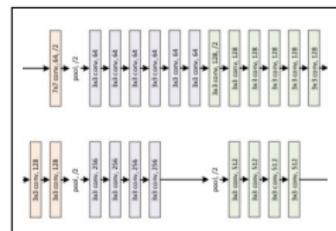
Deep Learning [LeCun et al., 2015]



Deep Q-Learning [Mnih et al., 2015]



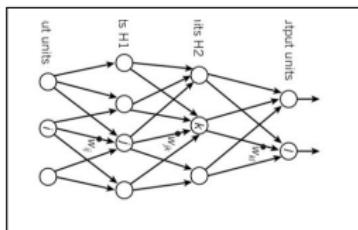
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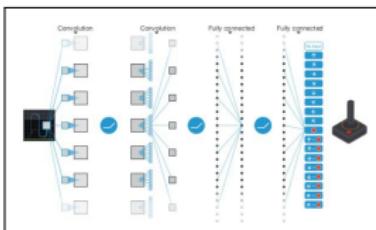
Multi-layer neural networks: impressive performance, countless applications

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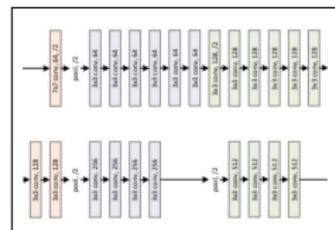
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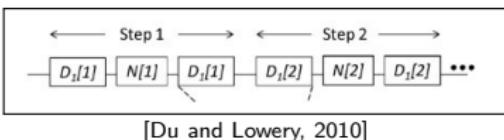
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Multi-layer neural networks: impressive performance, countless applications



[Du and Lowery, 2010]



[Nakashima et al., 2017]

Split-step methods for solving the propagation equation in fiber-optics

# Agenda

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In this talk, we ...

1. show that multi-layer neural networks and the split-step method have the same functional form: both alternate linear and pointwise nonlinear steps
2. propose a physics-based machine-learning approach based on parameterizing the split-step method (no black-box neural networks)
3. revisit hardware-efficient nonlinear equalization via learned digital backpropagation

# Outline

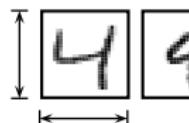
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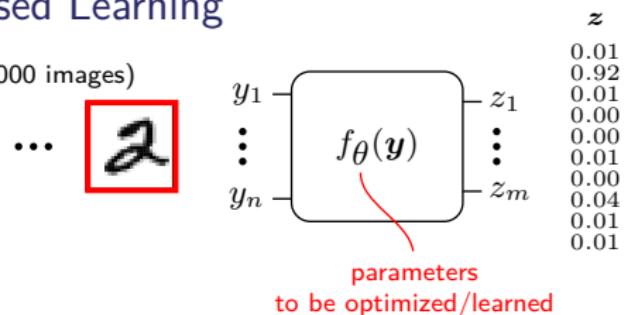
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## Supervised Learning

## handwritten digit recognition (MNIST: 70,000 images)

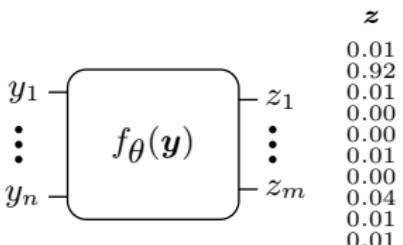


$28 \times 28$  pixels  $\implies n = 784$

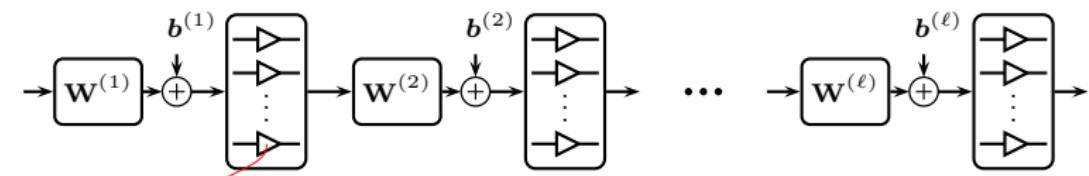


## Supervised Learning

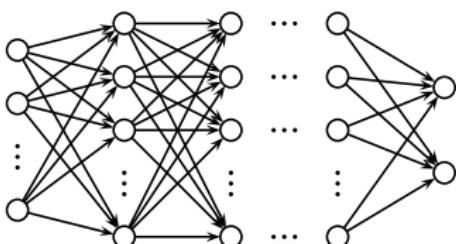
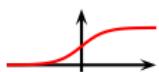
handwritten digit recognition (MNIST: 70,000 images)



How to choose  $f_\theta(y)$ ? Deep feed-forward neural networks



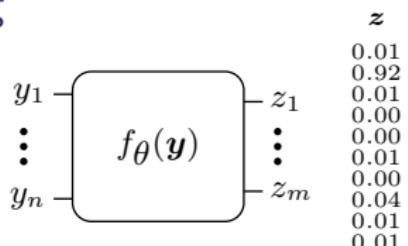
activation function



**equivalent graph representation**

## Supervised Learning

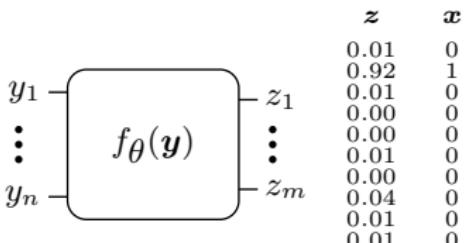
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How to optimize  $\theta = \{W^{(1)}, \dots, W^{(\ell)}, b^{(1)}, \dots, b^{(\ell)}\}$ ?

## Supervised Learning

handwritten digit recognition (MNIST: 70,000 images)



How to optimize  $\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(\ell)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(\ell)}\}$ ?

Given a **data set**  $\mathcal{D} = \{(\mathbf{y}^{(i)}, \mathbf{x}^{(i)})\}_{i=1}^N$ , where  $\mathbf{y}^{(i)}$  are **model inputs** and  $\mathbf{x}^{(i)}$  are **labels**, we iteratively minimize

$$\frac{1}{|\mathcal{B}_k|} \sum_{(\mathbf{y}, \mathbf{x}) \in \mathcal{B}_k} \mathcal{L}(f_\theta(\mathbf{y}), \mathbf{x}) \triangleq g(\theta) \quad \text{using } \theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k)$$

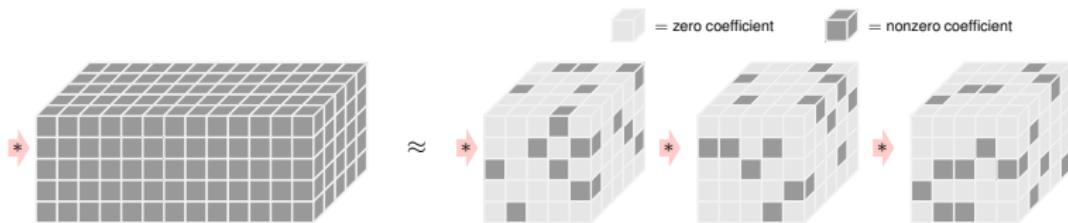
stochastic gradient descent

- $\mathcal{B}_k \subset \mathcal{D}$  and  $|\mathcal{B}_k|$  is called the **batch (or minibatch) size**
  - Typical **loss function**: mean squared error  $\mathcal{L}(a, b) = \|a - b\|^2$  (regression)
  - $\lambda$  is called the **step size or learning rate**

# Why Deep Models?

Many possible answers

One advantage is complexity: **deep** computation graphs tend to be **more parameter efficient than shallow** graphs [Lin et al., 2017]



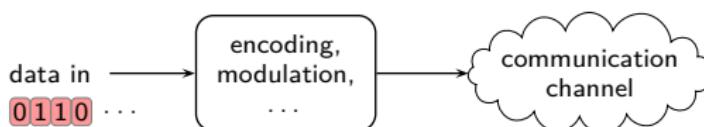
- **Sparsity** can emerge due to **(approximate) factorization** (even for linear models, e.g., FFT)
- Deep computation graphs allow for **very simple elementary steps**
- Deep models typically have **many “good” parameter configurations** that are close to each other  $\implies$  **robustness** to, e.g., quantization noise

## Physical-Layer Design: Conventional vs. Machine Learning



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[Shen and Lau, 2011], Fiber nonlinearity compensation using extreme learning machine for DSP-based ..., (*OECC*)  
[Giacoumidis et al., 2015], Fiber nonlinearity-induced penalty reduction in CO-OFDM by ANN-based ..., (*Opt. Lett.*)

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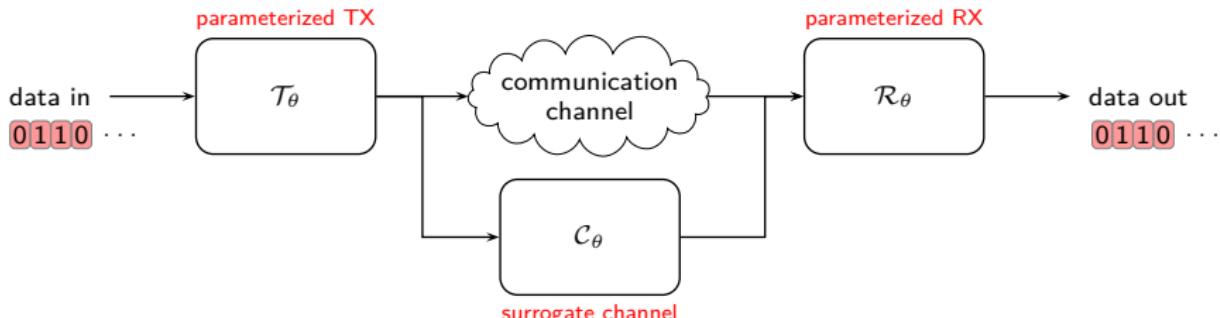
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- **Joint transmitter–receiver learning** via autoencoder [O'Shea and Hoydis, 2017]

[Karanov et al., 2018], End-to-end deep learning of optical fiber communications (*J. Lightw. Technol.*)

[Li et al., 2018], Achievable information rates for nonlinear fiber communication via end-to-end autoencoder learning, (ECOC)

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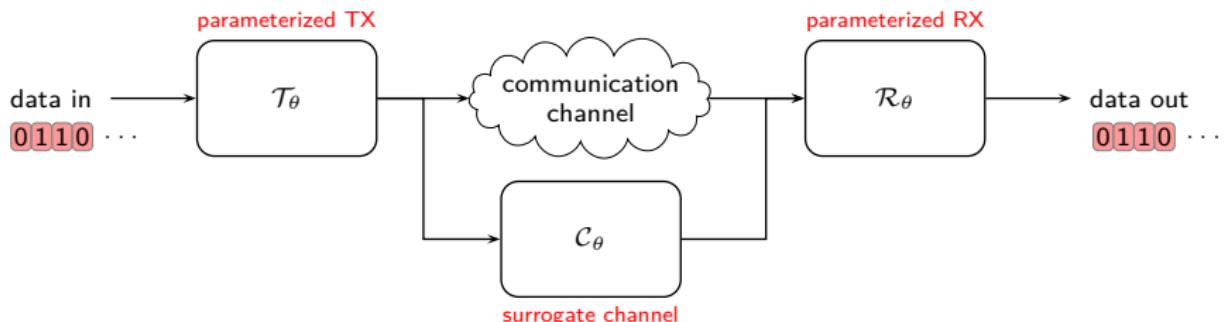


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- Use function approximators and learn parameter configurations  $\theta$  from data
- Joint transmitter–receiver learning via autoencoder [O'Shea and Hoydis, 2017]
- Surrogate channel models for gradient-based TX training

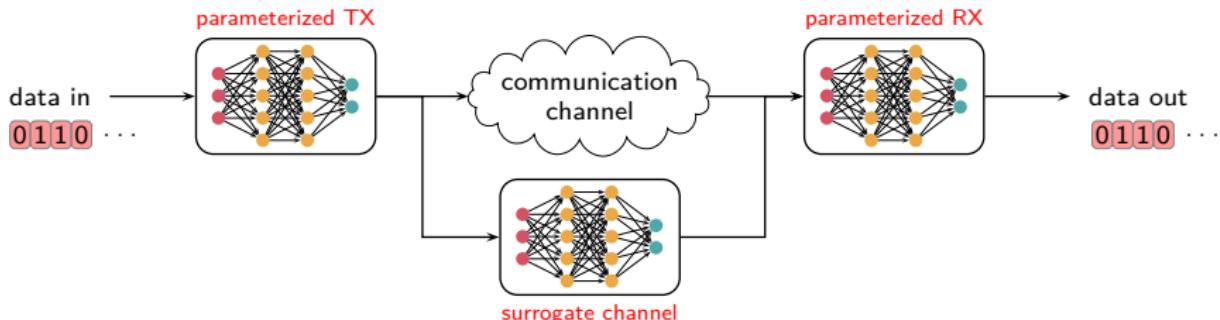
[O'Shea et al., 2018], Approximating the void: Learning stochastic channel models from observation with variational GANs, (*arXiv*)  
[Ye et al., 2018], Channel agnostic end-to-end learning based communication systems with conditional GAN, (*arXiv*)

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# Physical-Layer Design: Conventional vs. Machine Learning



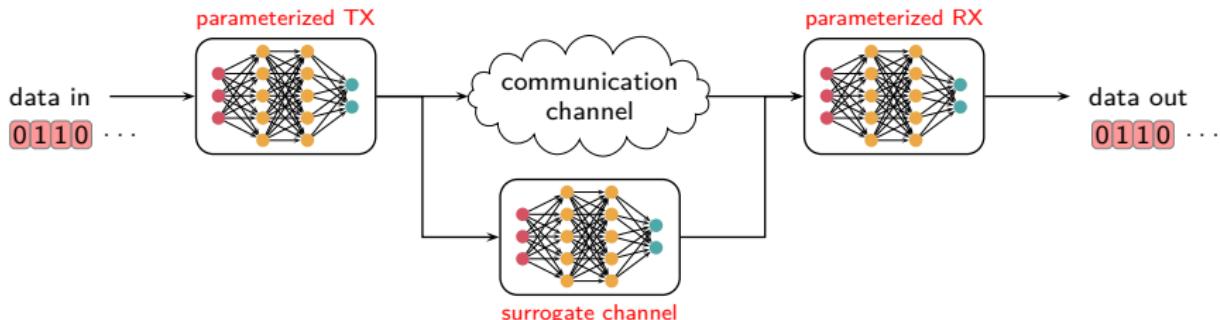
# Physical-Layer Design: Conventional vs. Machine Learning



Using (deep) neural networks for  $\mathcal{T}_\theta, \mathcal{R}_\theta, \mathcal{C}_\theta$ ? Possible, but ...

- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?
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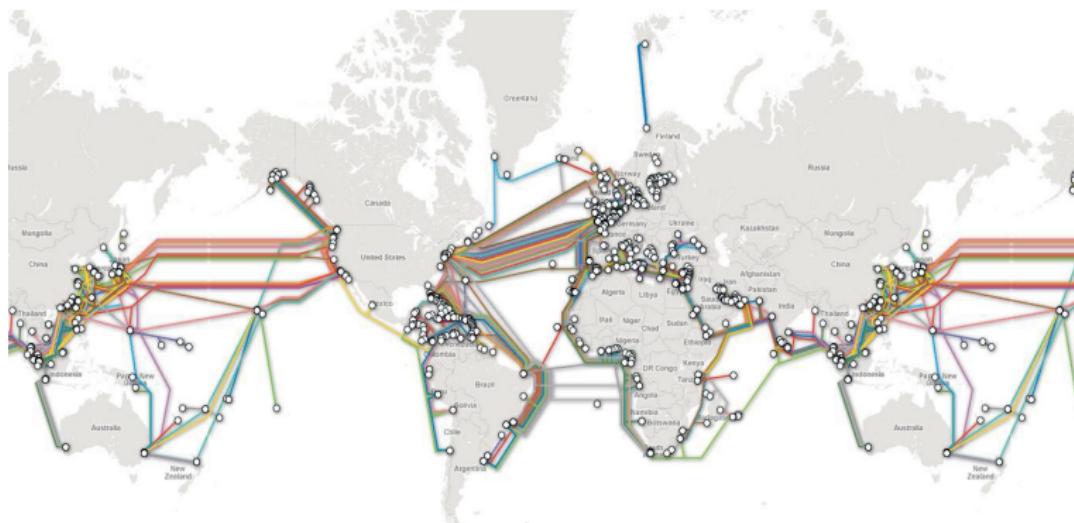
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Our contribution: designing “neural-network-like” machine-learning models by exploiting the underlying physics of the propagation.

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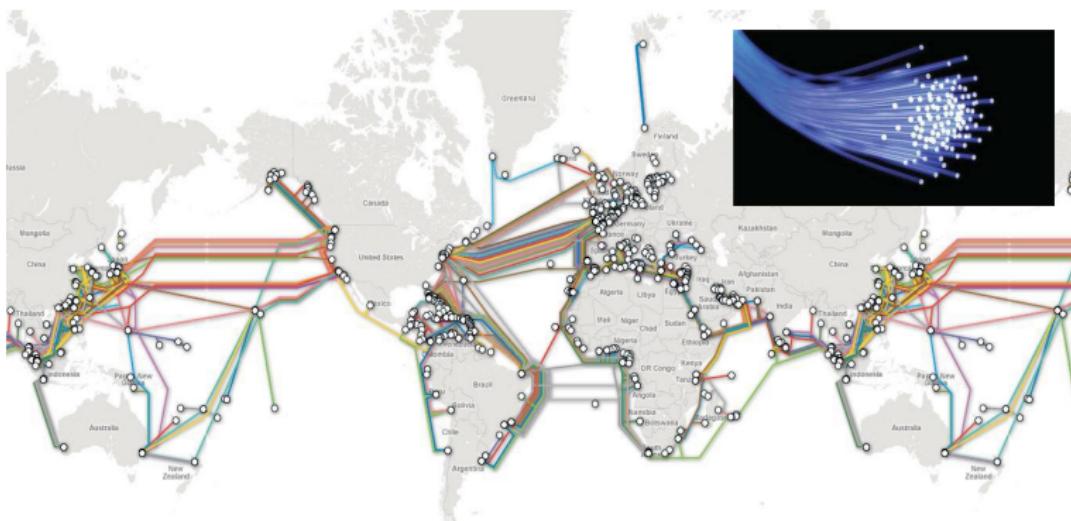
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# Fiber-Optic Communications



Fiber-optic systems enable **data traffic over very long distances** connecting cities, countries, and continents.

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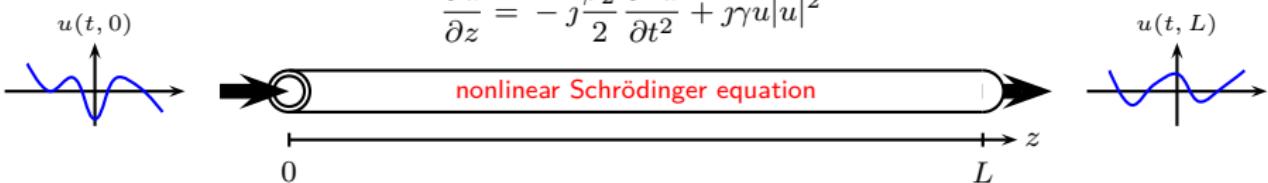


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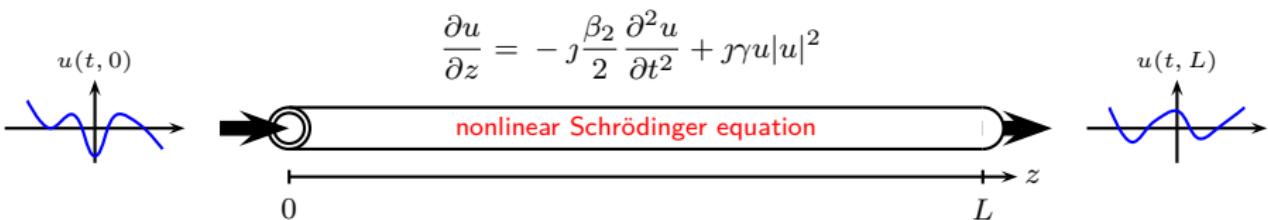
- **Dispersion:** different wavelengths travel at different speeds (linear)
- **Kerr effect:** refractive index changes with signal intensity (nonlinear)

# Nonlinear Fiber Channel Modeling

$$\frac{\partial u}{\partial z} = -j \frac{\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \gamma u |u|^2$$

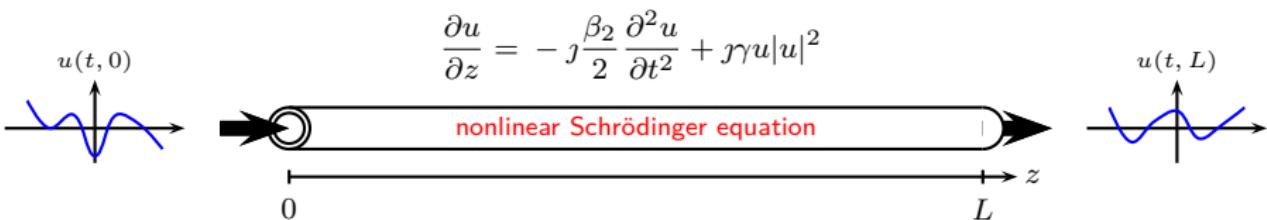


## Nonlinear Fiber Channel Modeling



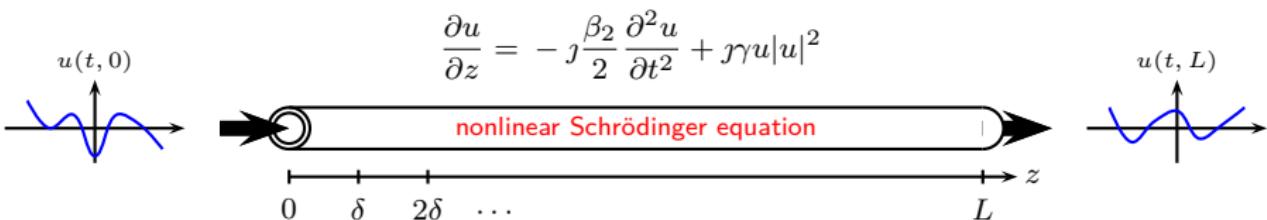
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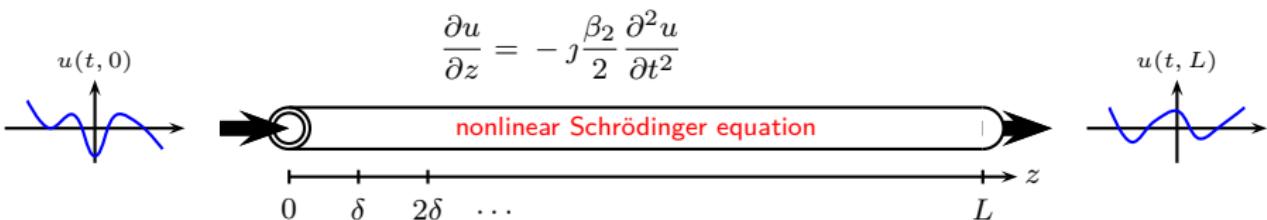
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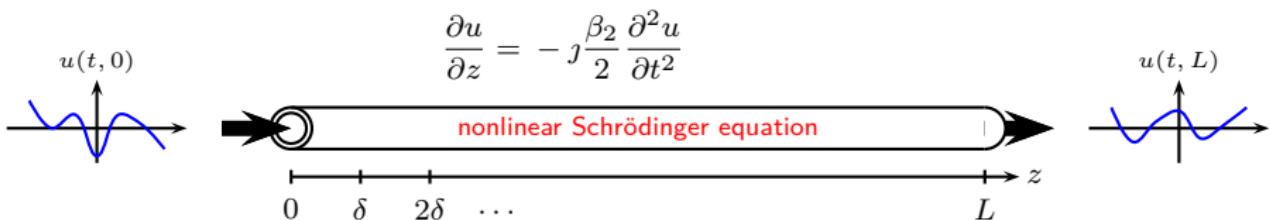
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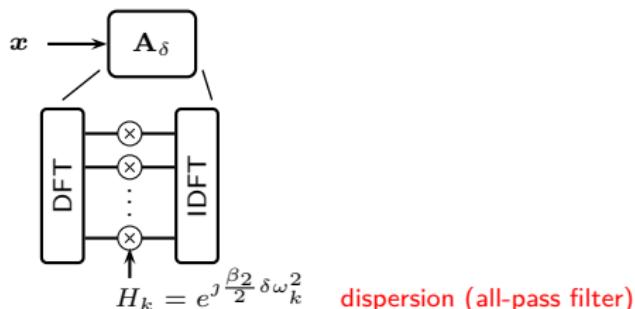


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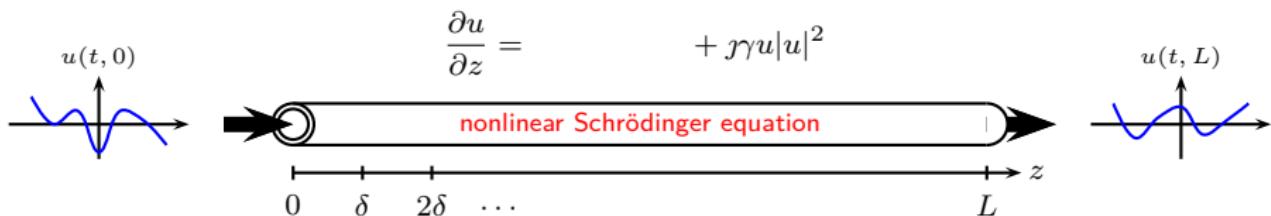
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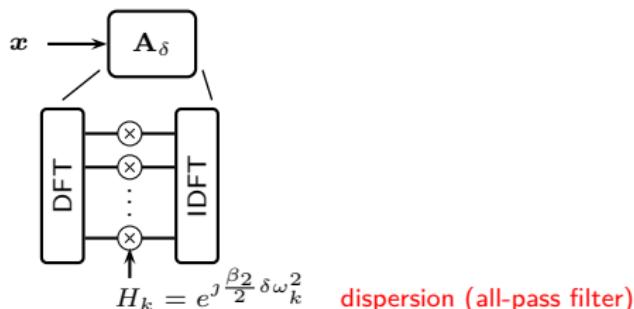
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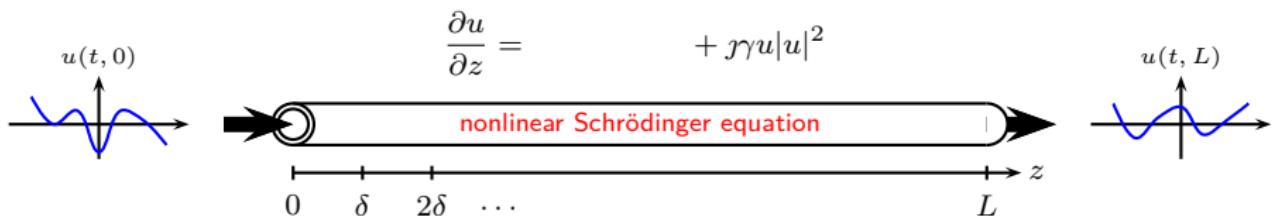
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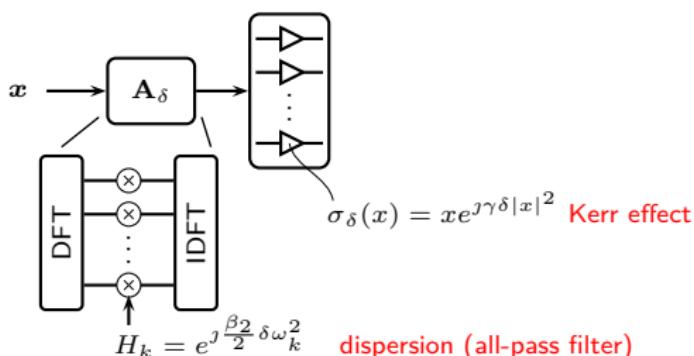
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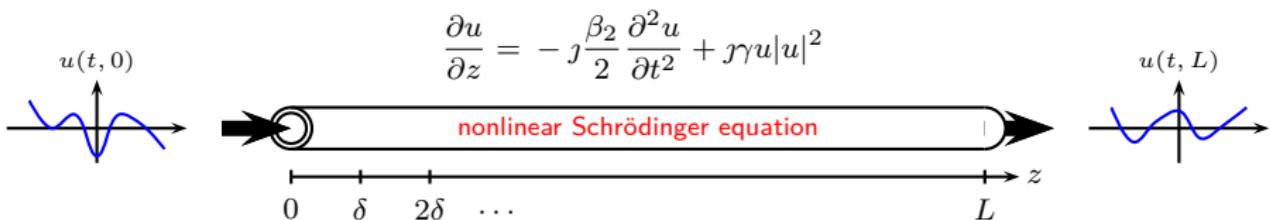
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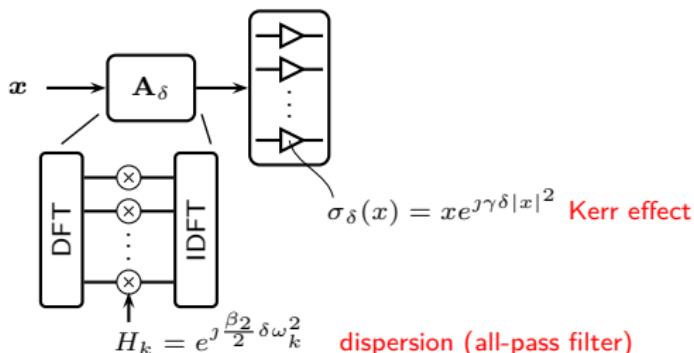
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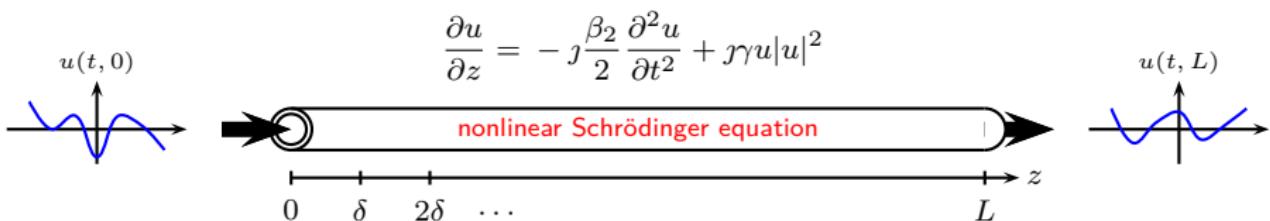
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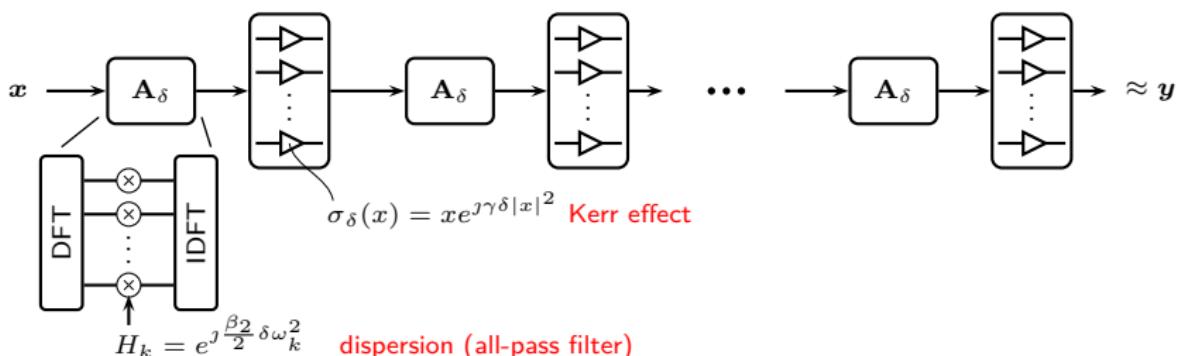
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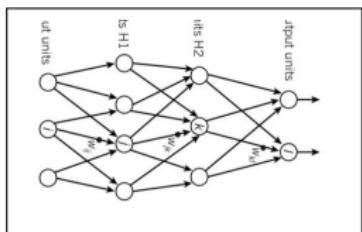
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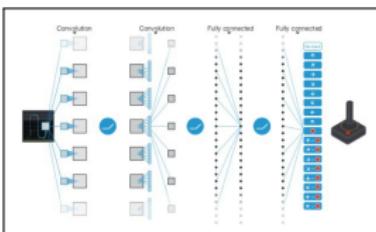
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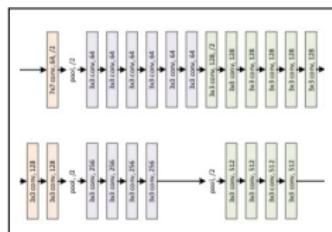
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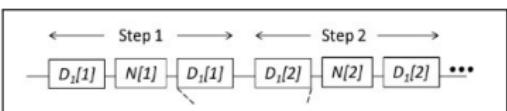
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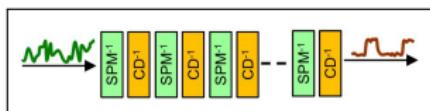
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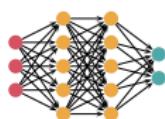
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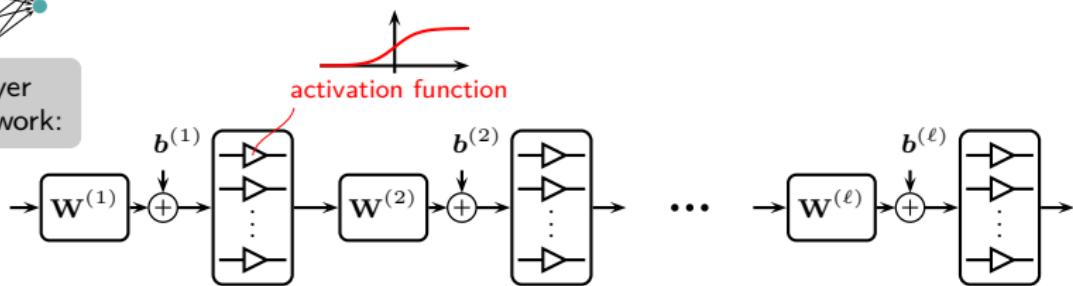
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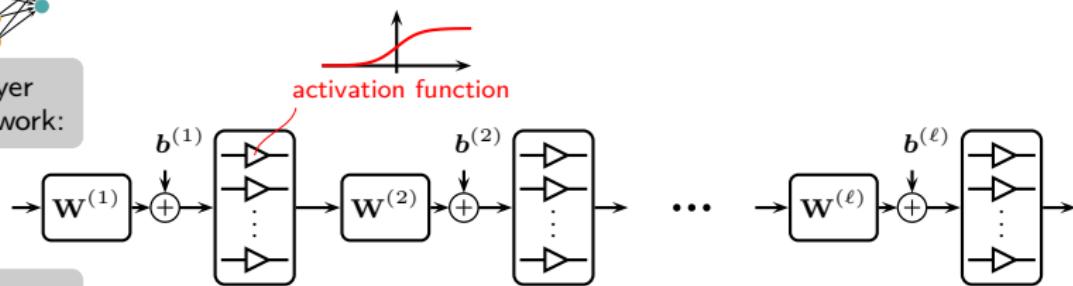
multi-layer  
neural network:



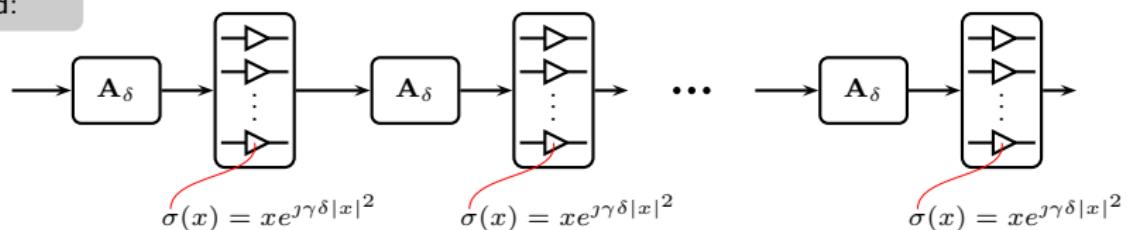
## The Main Idea



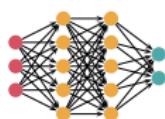
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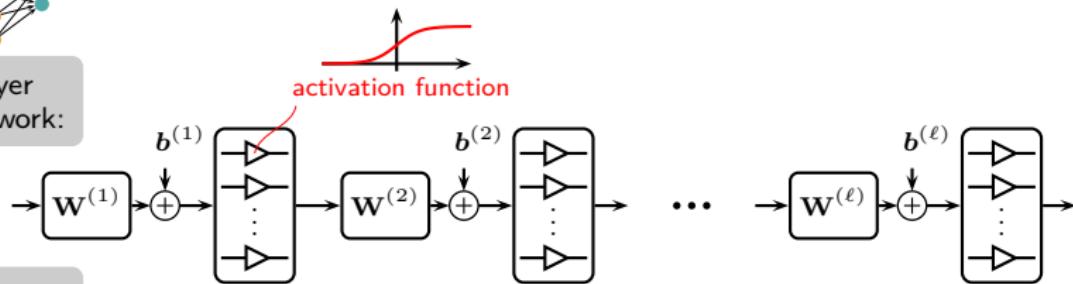
split-step  
method:



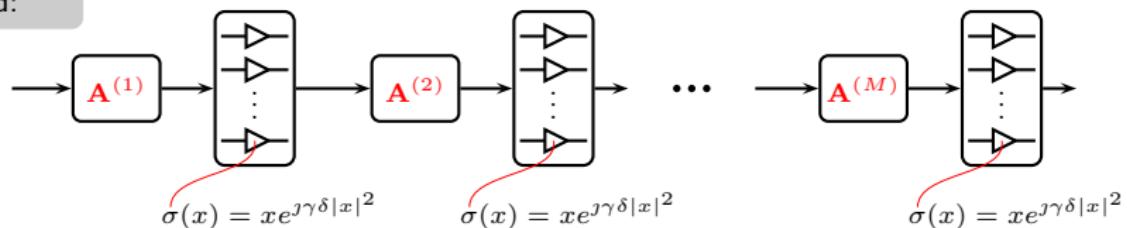
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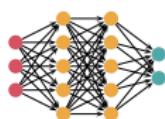
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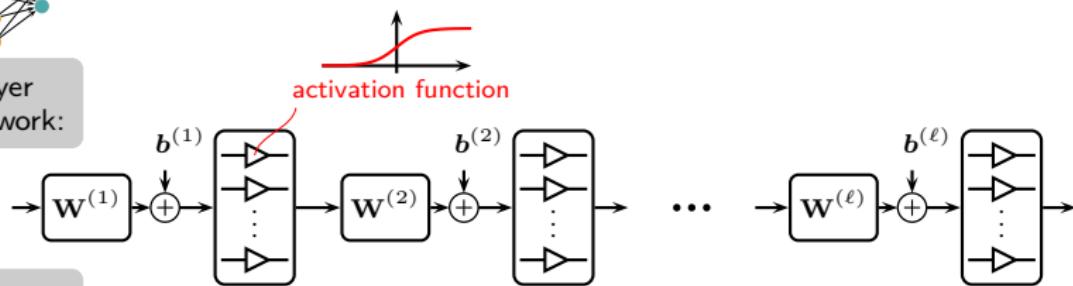
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- Parameterize all linear steps:  $f_\theta$  with  $\theta = \{A^{(1)}, \dots, A^{(M)}\}$

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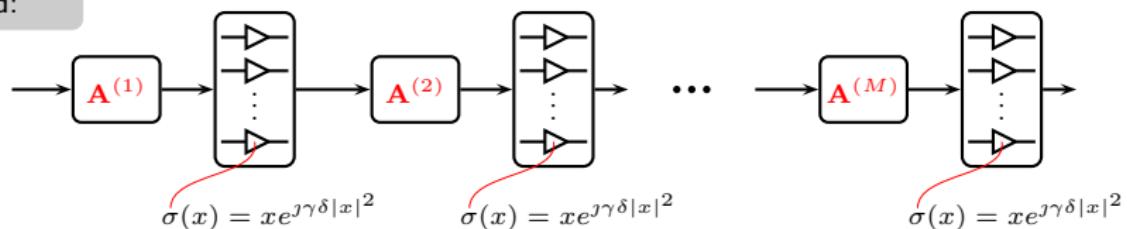
[Häger & Pfister, 2021], Physics-Based Deep Learning for Fiber-Optic Communication Systems, IEEE J. Sel. Areas Commun.



multi-layer  
neural network:



split-step  
method:

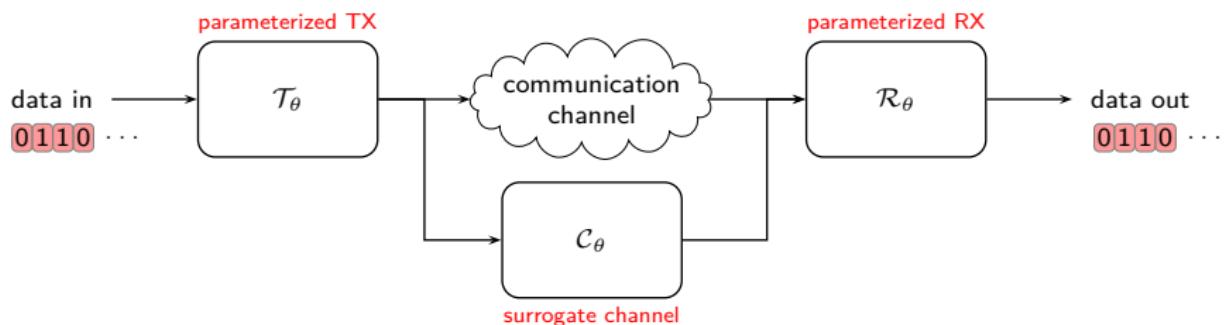


- This almost looks like a deep neural net!
- Parameterize all linear steps:  $f_\theta$  with  $\theta = \{A^{(1)}, \dots, A^{(M)}\}$
- Special cases: step-size optimization, nonlinear operator “placement”, ...

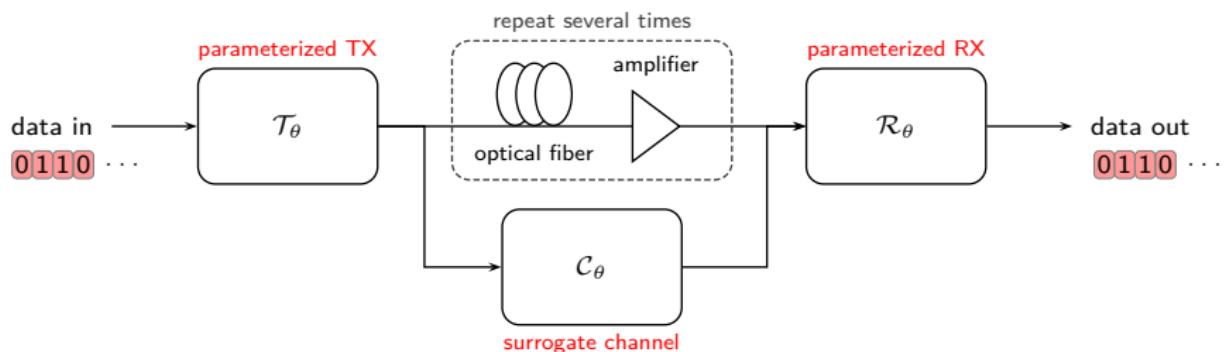
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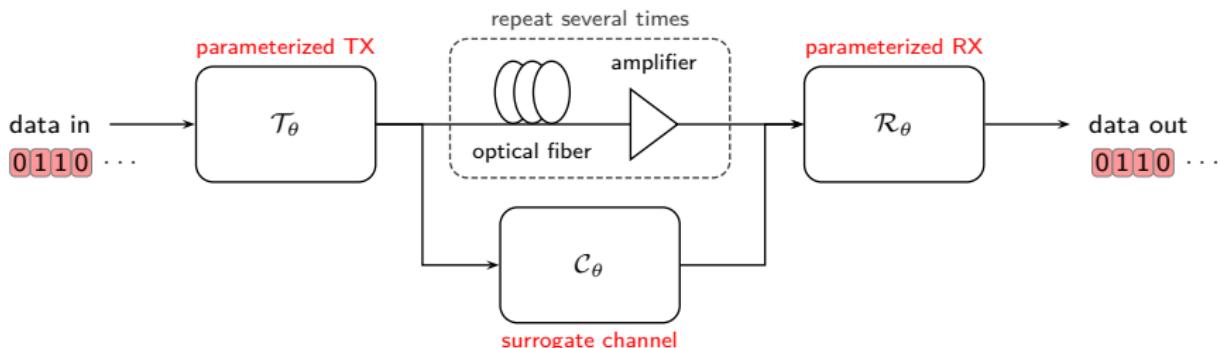
## Possible Applications



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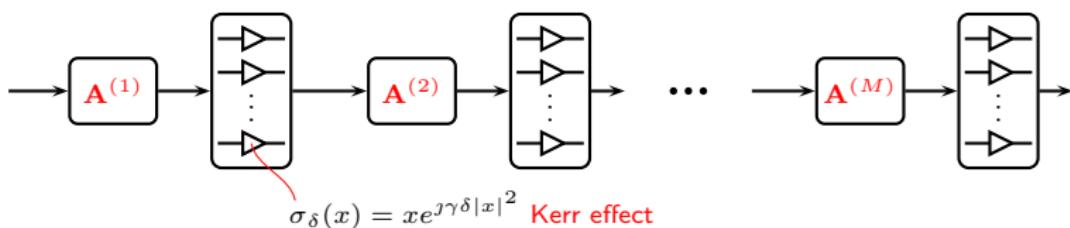


## Possible Applications



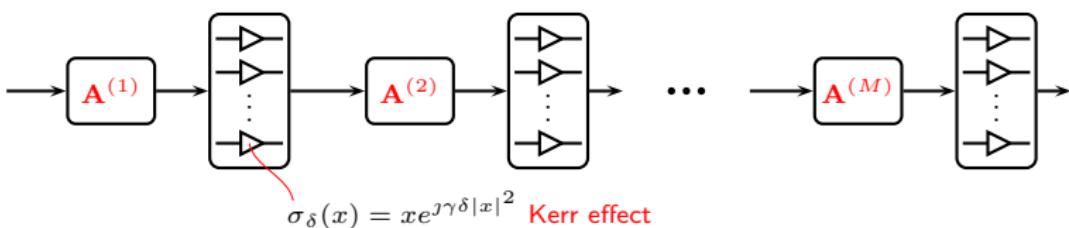
- **Channel  $C_\theta$ :** fine-tune model based on experimental data, reduce simulation time [Leibrich and Rosenkranz, 2003], [Li et al., 2005]
- **Receiver  $R_\theta$ :** nonlinear equalization (**focus in this talk**)
- **Transmitter  $T_\theta$ :** digital pre-distortion [Essiambre and Winzer, 2005], [Roberts et al., 2006], “split” nonlinearity compensation [Lavery et al., 2016]

## Potential Benefits



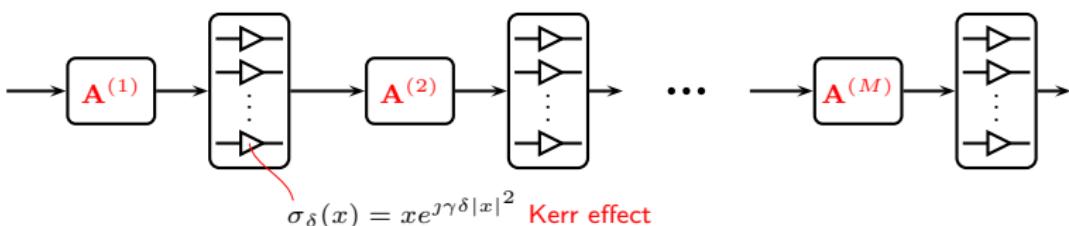
- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?

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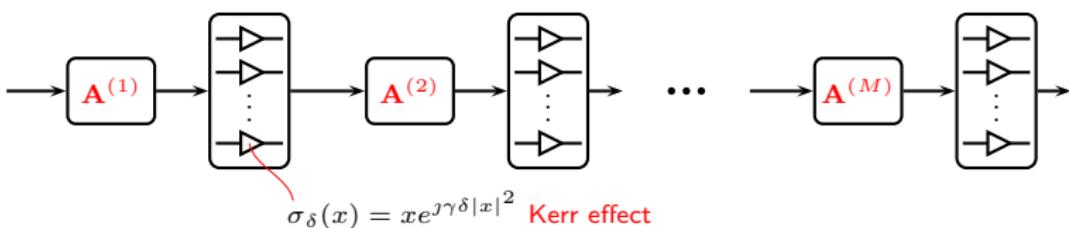
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  - Activation function is fixed; number of layers = number of steps
  - Hidden feature representations  $\approx$  signal at intermediate fiber locations
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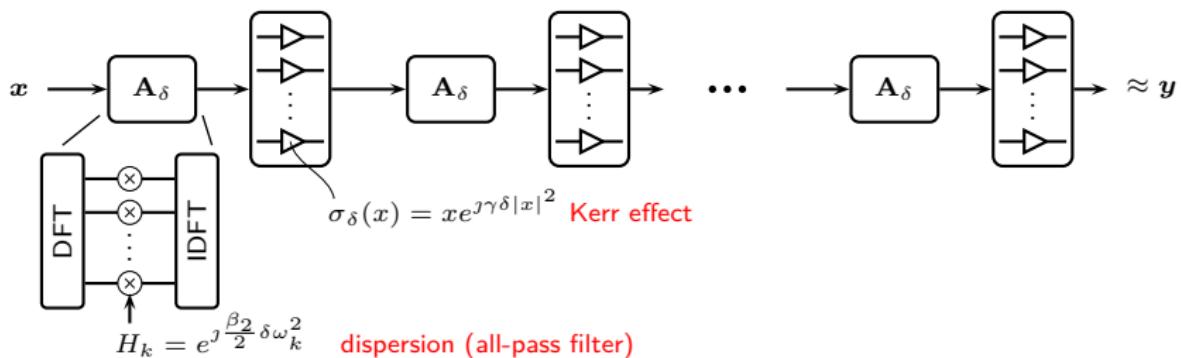


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  - Learned parameter configurations are interpretable
  - Satisfactory explanations for benefits over previous handcrafted solutions

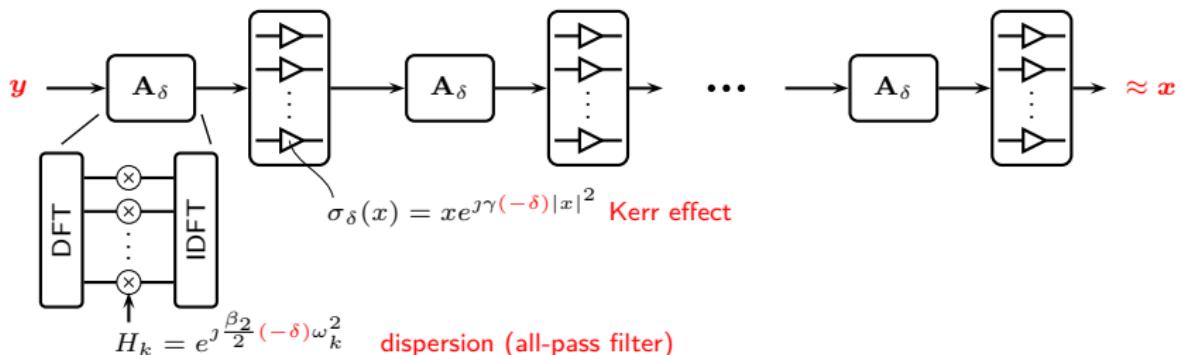
# Outline

1. Machine Learning and Neural Networks for Communications
2. Physics-Based Machine Learning for Fiber-Optic Communications
3. Learned Digital Backpropagation
4. Polarization-Dependent Effects
5. Conclusions

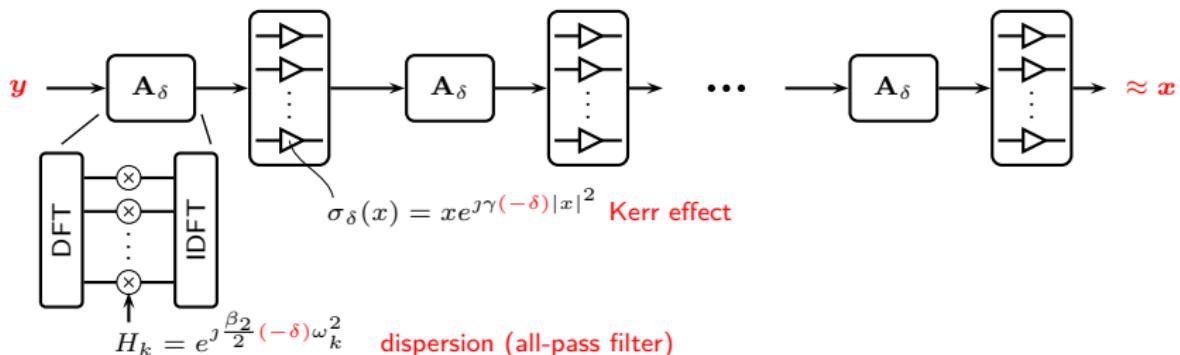
## Digital Backpropagation



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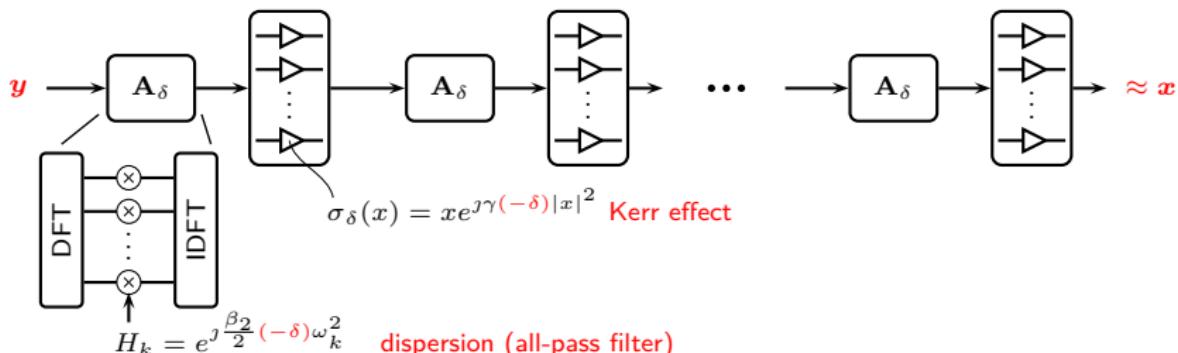


## Digital Backpropagation



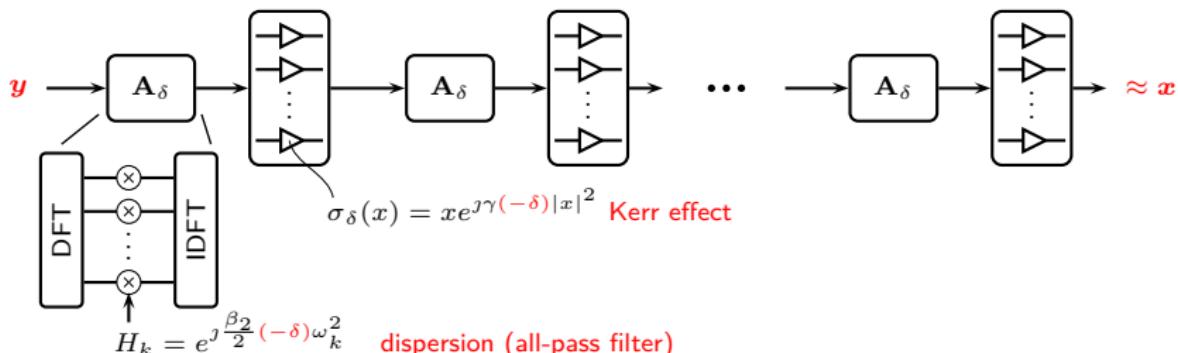
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## Digital Backpropagation



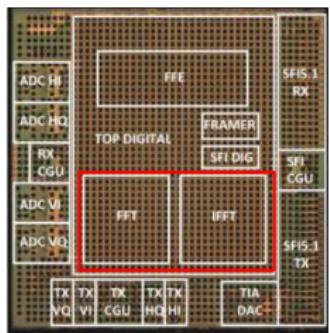
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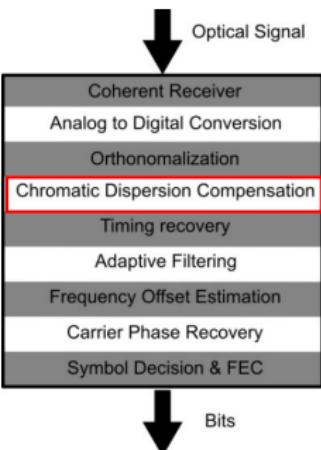


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- Widely considered to be impractical (**too complex**): linear equalization is already one of the **most power-hungry DSP blocks** in coherent receivers

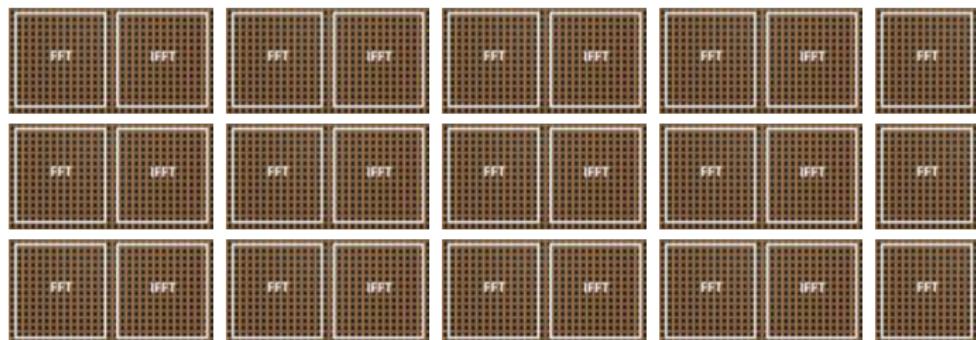
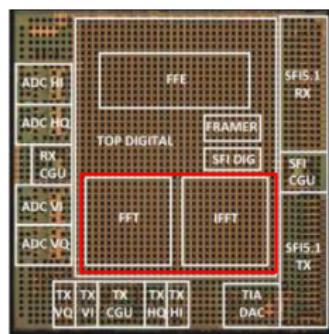
# Real-Time Digital Backpropagation



[Crivelli et al., 2014]

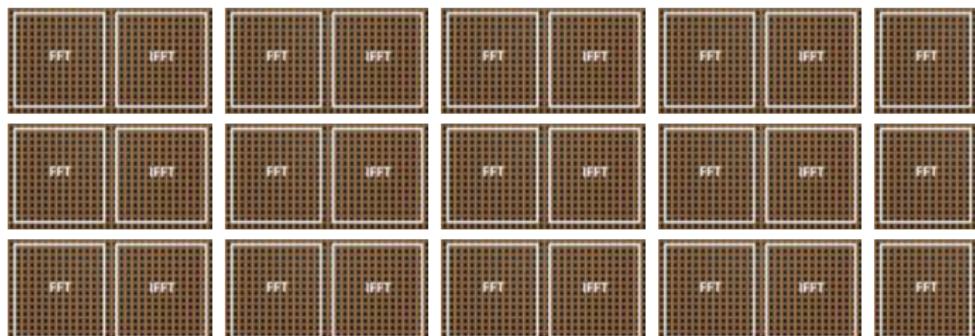
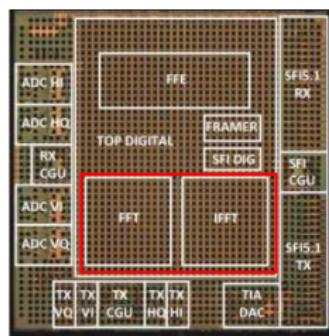


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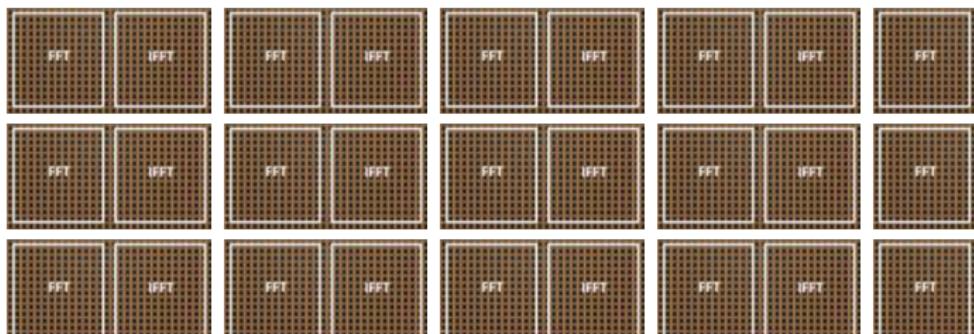
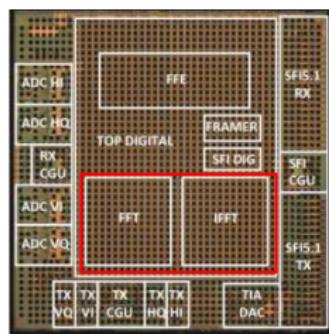
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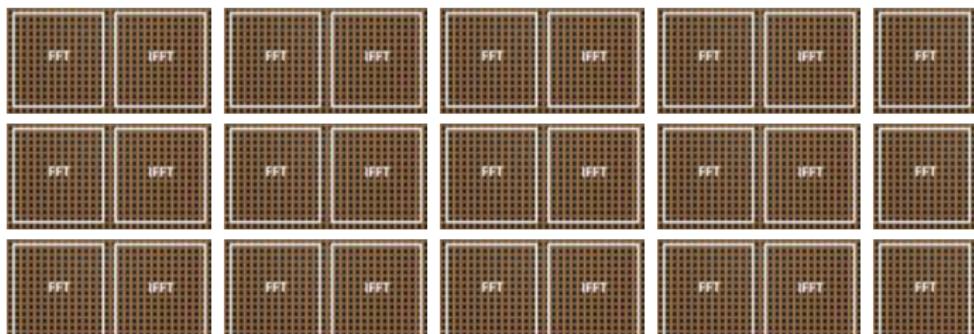
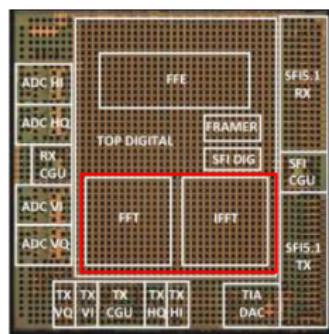
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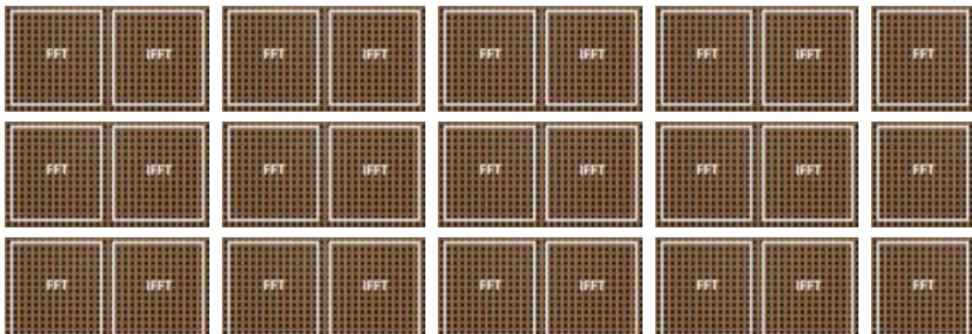
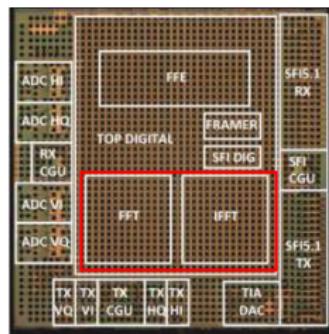
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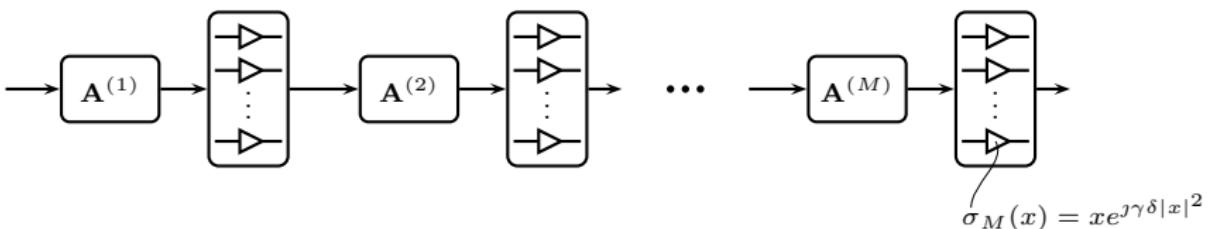
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Our approach: many steps but model compression

Joint optimization, pruning, and quantization of all linear steps  $\Rightarrow$  hardware-efficient digital backpropagation

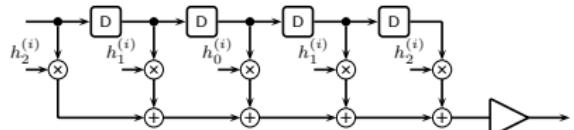
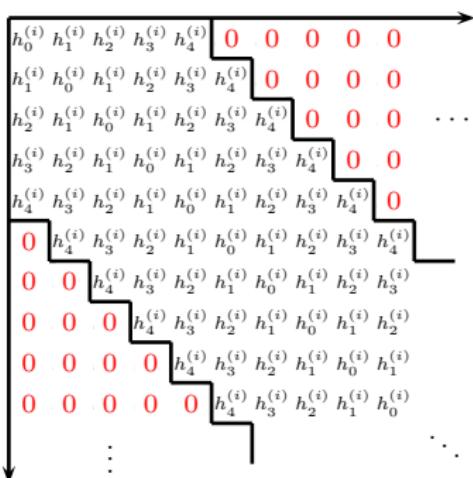
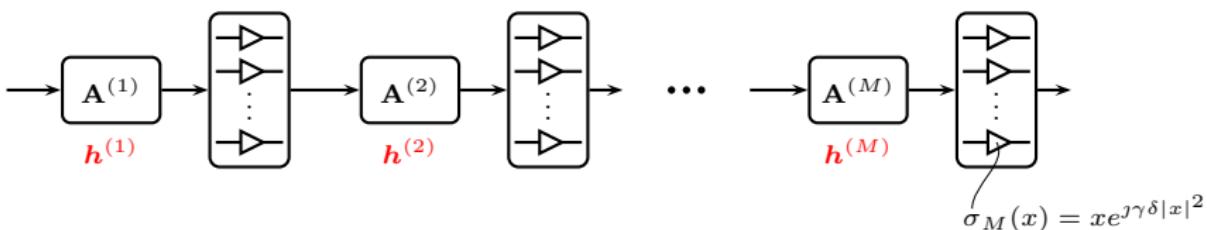
## Learned Digital Backpropagation

TensorFlow implementation of the computation graph  $f_\theta(y)$ :



# Learned Digital Backpropagation

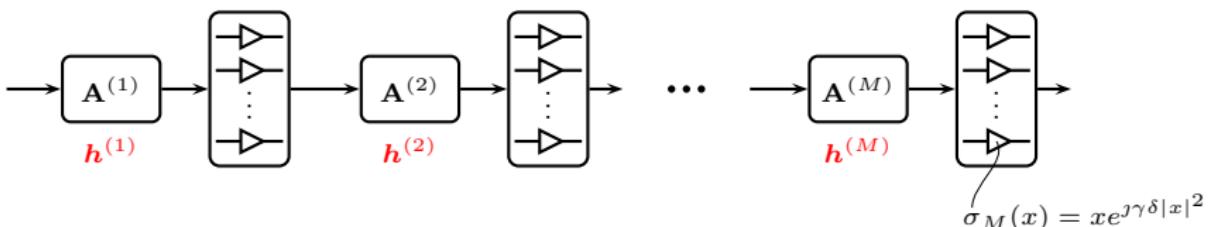
TensorFlow implementation of the computation graph  $f_\theta(y)$ :



finite impulse response (FIR)  
complex & symmetric coefficients

## Learned Digital Backpropagation

TensorFlow implementation of the computation graph  $f_\theta(\mathbf{y})$ :



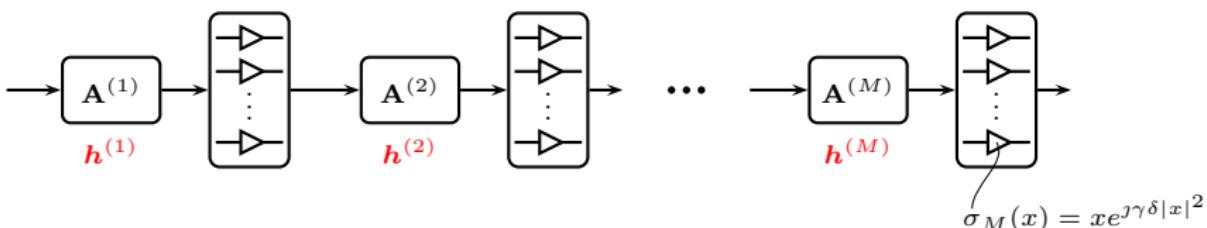
Deep learning of all FIR filter coefficients  $\theta = \{\mathbf{h}^{(1)}, \dots, \mathbf{h}^{(M)}\}$ :

$$\min_{\theta} \sum_{i=1}^N \text{Loss}(f_\theta(\mathbf{y}^{(i)}), \mathbf{x}^{(i)}) \triangleq g(\theta) \quad \begin{array}{l} \text{using } \theta_{k+1} = \theta_k - \lambda \nabla_{\theta} g(\theta_k) \\ \text{Adam optimizer, fixed learning rate} \end{array}$$

mean squared error

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Iteratively prune (set to 0) outermost filter taps during gradient descent

## Iterative Filter Tap Pruning

$$\theta = \left\{ \begin{array}{l} \boldsymbol{h}^{(1)} \\ \boldsymbol{h}^{(2)} \\ \vdots \\ \boldsymbol{h}^{(M)} \end{array} \right\}$$

## Iterative Filter Tap Pruning

←———— starting length  $2K' + 1$  —————→

$$\theta = \left\{ \begin{array}{l} \mathbf{h}^{(1)} = ( h_{K'}^{(1)} \dots h_K^{(1)} \dots h_1^{(1)} h_0^{(1)} h_1^{(1)} \dots h_K^{(1)} \dots h_{K'}^{(1)} ) \quad \text{step 1} \\ \mathbf{h}^{(2)} = ( h_{K'}^{(2)} \dots h_K^{(2)} \dots h_1^{(2)} h_0^{(2)} h_1^{(2)} \dots h_K^{(2)} \dots h_{K'}^{(2)} ) \quad \text{step 2} \\ \vdots \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots \\ \mathbf{h}^{(M)} = ( h_{K'}^{(M)} \dots h_K^{(M)} \dots h_1^{(M)} h_0^{(M)} h_1^{(M)} \dots h_K^{(M)} \dots h_{K'}^{(M)} ) \quad \text{step } M \end{array} \right.$$

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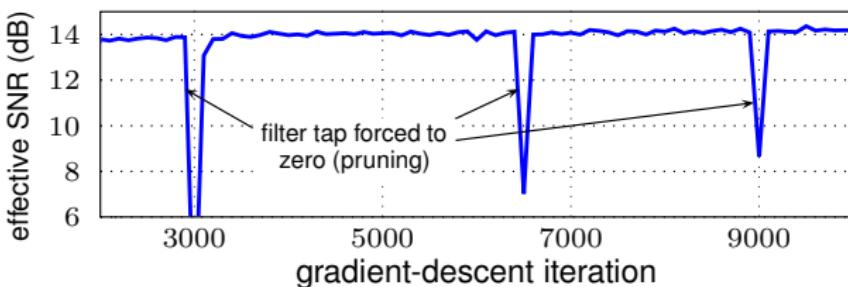
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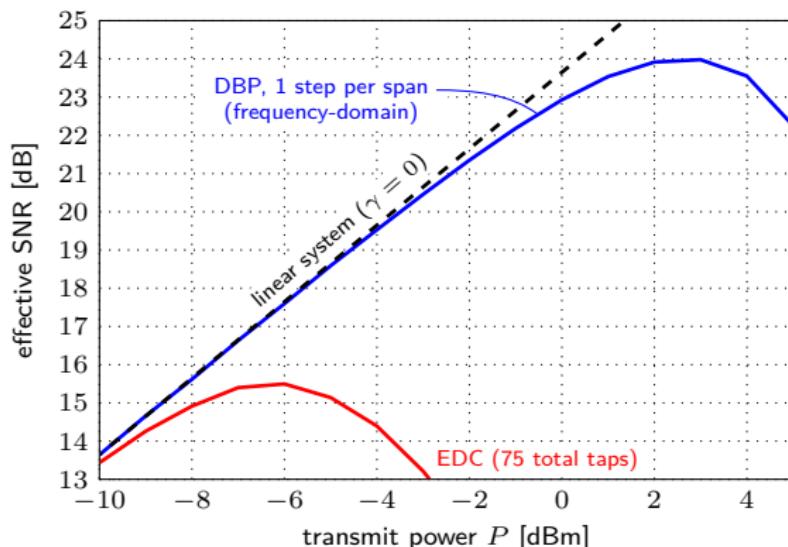
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- Typical learning curve:



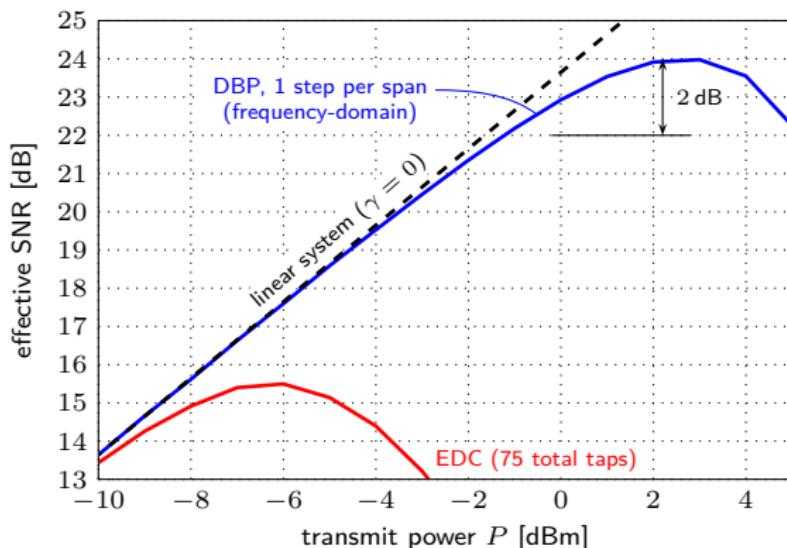
## Revisiting Ip and Kahn (2008)



Parameters similar to [Ip and Kahn, 2008]:

- $25 \times 80$  km SSFM
- Gaussian modulation
- RRC pulses (0.1 roll-off)
- 10.7 Gbaud
- 2 samples/symbol processing
- single channel, single pol.

## Revisiting Ip and Kahn (2008)

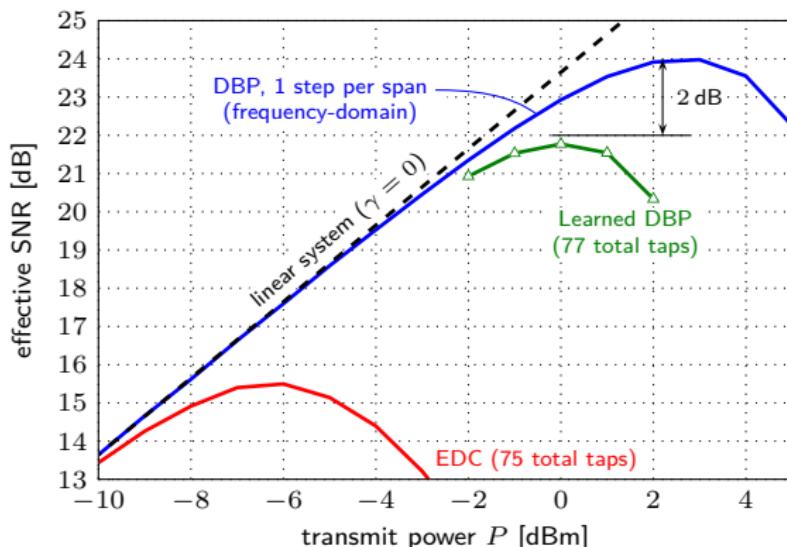


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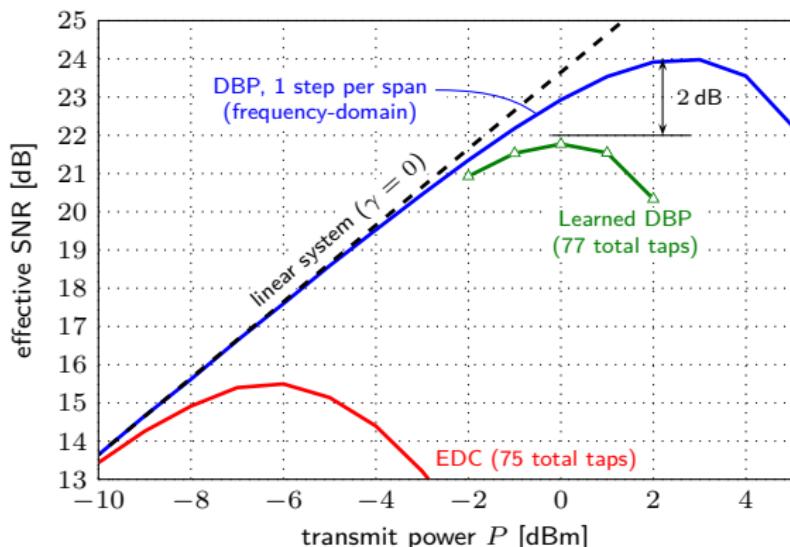


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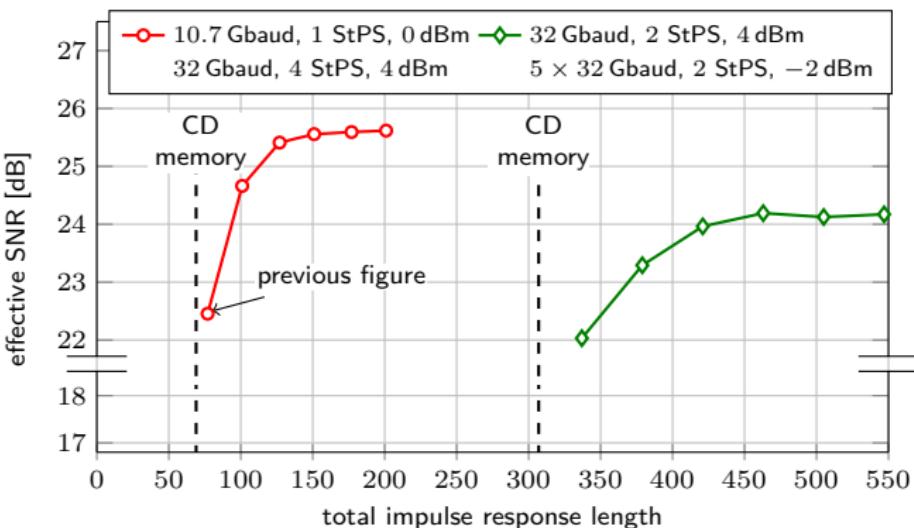


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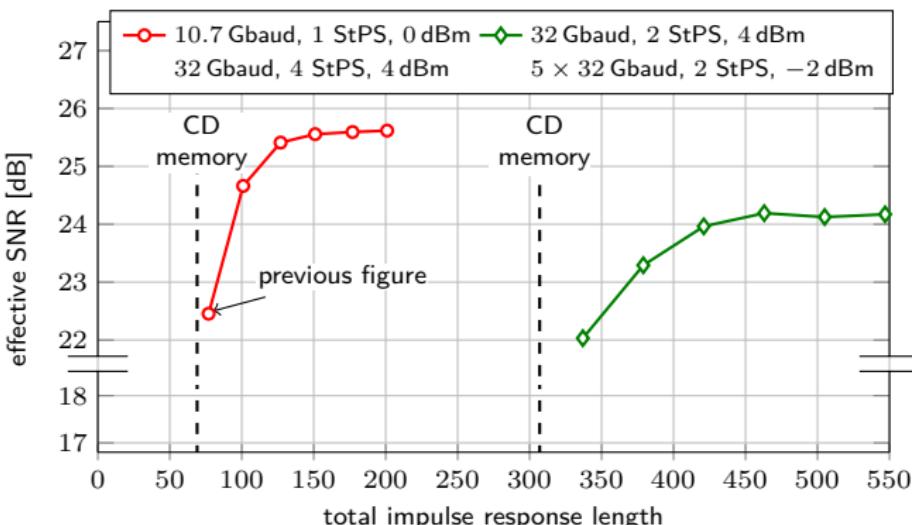
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- Learned approach uses **only 77 total taps**: alternate 5 and 3 taps/step and use **different** filter coefficients in all steps [Häger and Pfister, 2018a]
- Can **outperform** “ideal DBP” in the nonlinear regime [Häger and Pfister, 2018b]

## Performance–Complexity Trade-off



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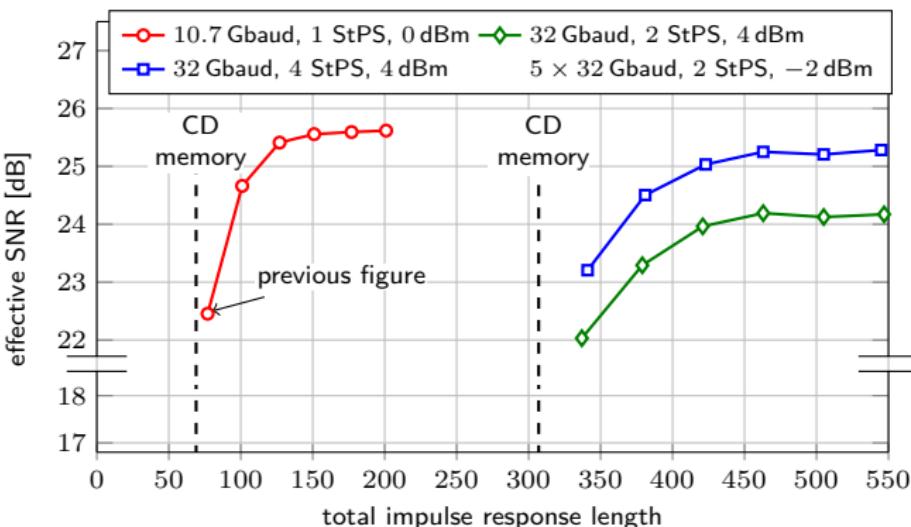
Conventional wisdom: Steps are inefficient  $\implies$  reduce as much as possible

Complexity

?  
≈

Number of  
Steps

## Performance–Complexity Trade-off



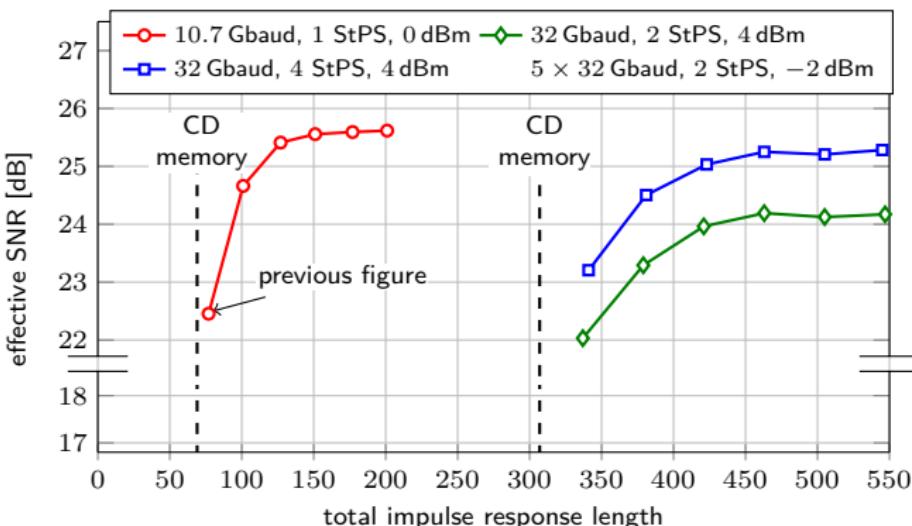
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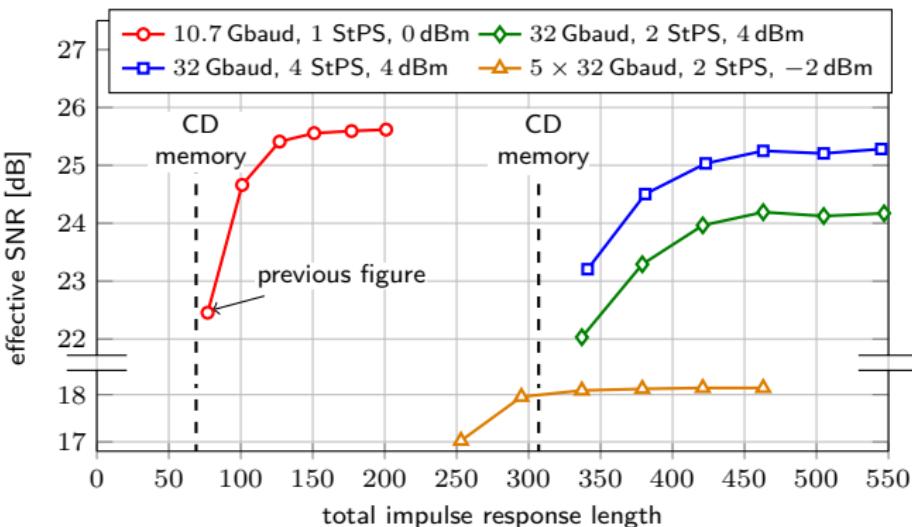
=

Number of  
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$\times$

Complexity  
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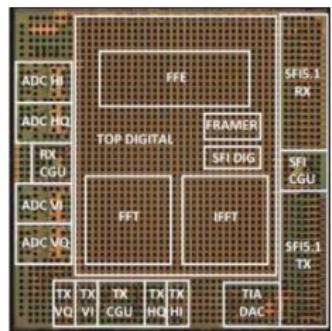
## Performance–Complexity Trade-off



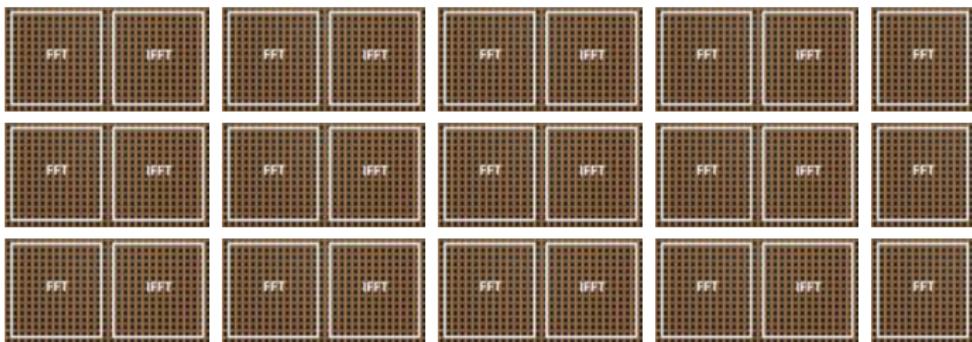
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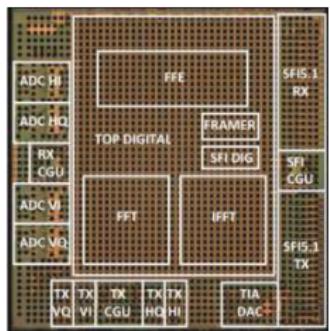
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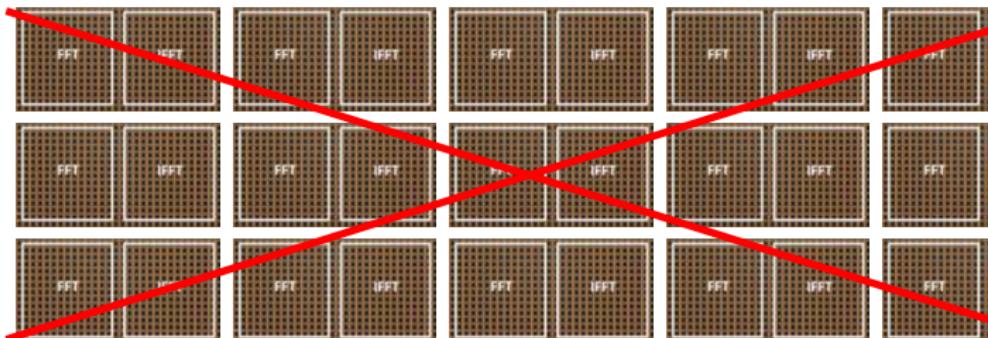
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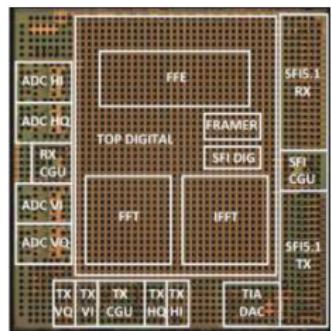


[Fougstedt et al., 2017], Time-domain digital back propagation: Algorithm and finite-precision implementation aspects, (*OFC*)

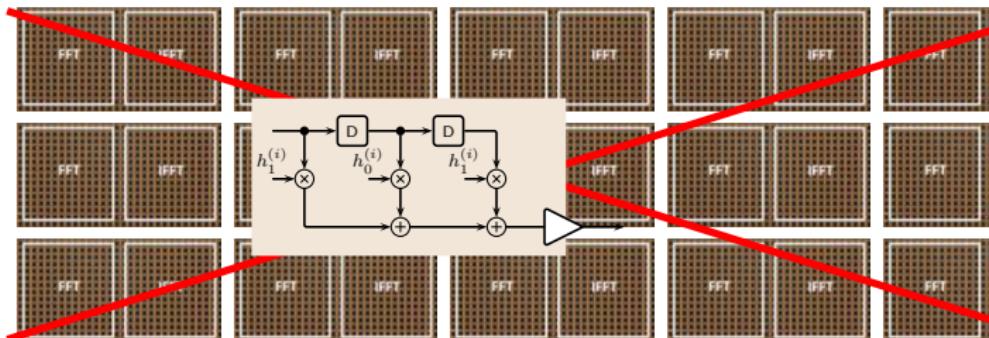
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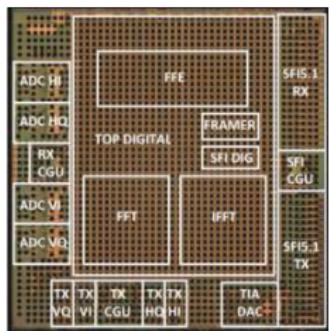


[Crivelli et al., 2014]

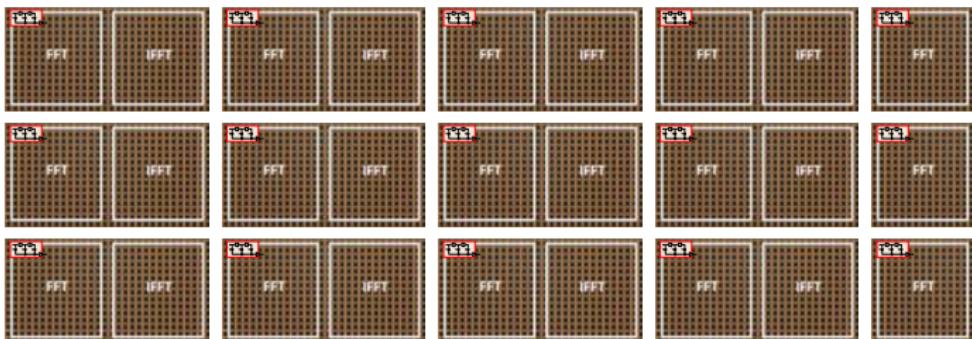


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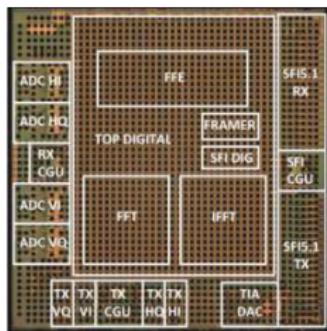
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- Our linear steps are **very short symmetric FIR filters** (as few as **3 taps**)
- 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
  - Only **5-6 bit** filter coefficients via **learned quantization**
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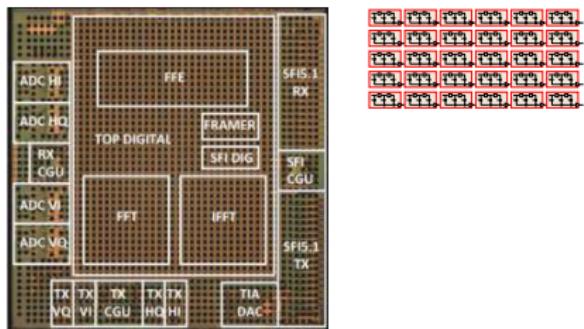


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- < 2× power compared to EDC [Crivelli et al., 2014, Pillai et al., 2014]

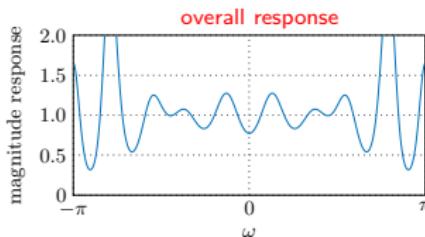
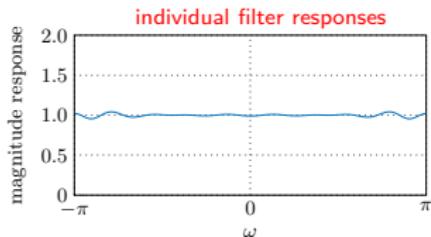
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# Why Does The Learning Approach Work?

Previous work: design a single filter or filter pair and **use it repeatedly**.

⇒ Good overall response only possible with **very long** filters.



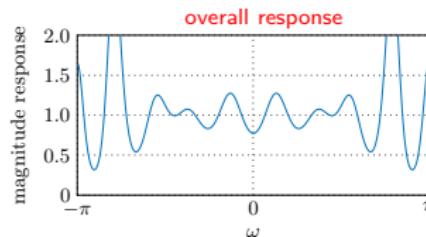
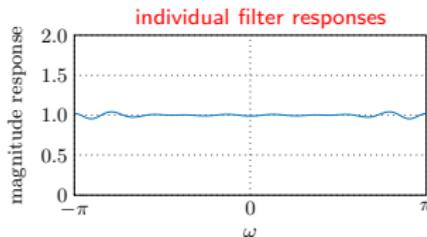
From [Ip and Kahn, 2009]:

- “We also note that [...] 70 taps, is much larger than expected”
- “This is due to amplitude ringing in the frequency domain”
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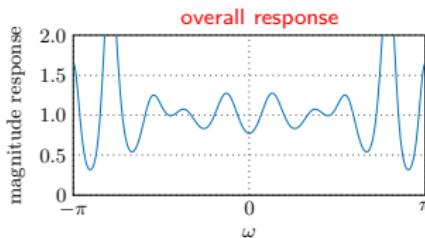
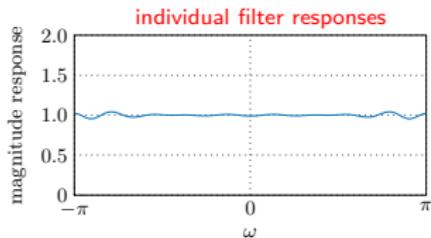
The learning approach uncovered that there is no such requirement!

[Lian, Häger, Pfister, 2018], What can machine learning teach us about communications? (ITW)

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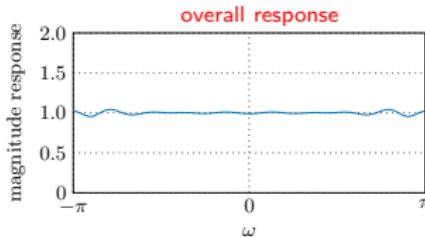
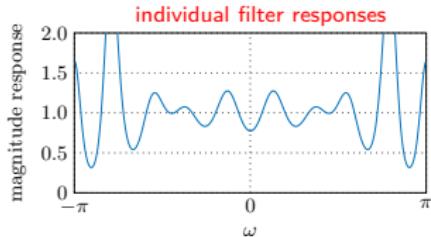
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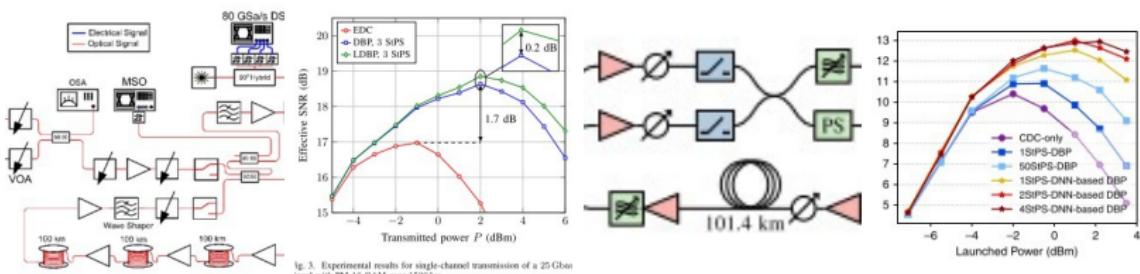


Sacrifice **individual filter accuracy**, but **different response per step**.

⇒ **Good overall response** even with **very short filters** by joint optimization.



# Experimental Investigations



Training with **real-world data sets** including presence of various **hardware impairments** (phase noise, timing error, frequency offset, etc.)

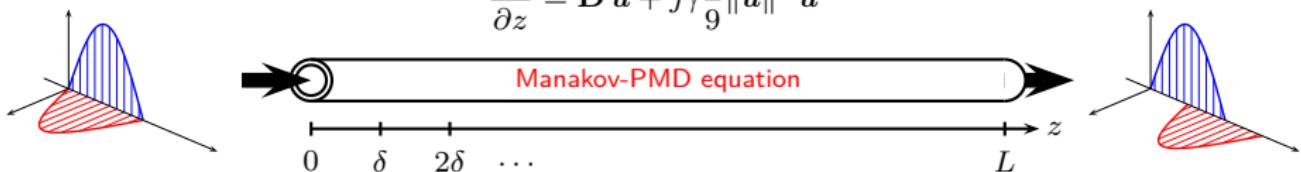
- [Oliari et al., 2020], Revisiting Efficient Multi-step Nonlinearity Compensation with Machine Learning: An Experimental Demonstration, (*J. Lightw. Technol.*)
- [Sillekens et al., 2020], Experimental Demonstration of Learned Time-domain Digital Back-propagation, (*Proc. IEEE Workshop on Signal Processing Systems*)
- [Fan et al., 2020], Advancing Theoretical Understanding and Practical Performance of Signal Processing for Nonlinear Optical Communications through Machine Learning, (*Nat. Commun.*)
- [Bitachon et al., 2020], Deep learning based Digital Back Propagation Demonstrating SNR gain at Low Complexity in a 1200 km Transmission Link, (*Opt. Express*)

# Outline

1. Machine Learning and Neural Networks for Communications
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4. Polarization-Dependent Effects
5. Conclusions

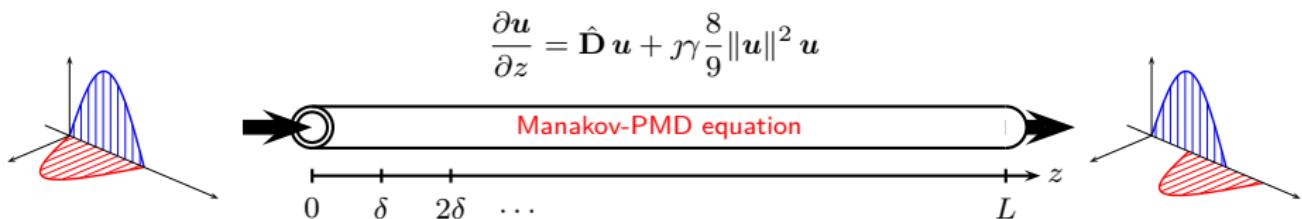
## Evolution of Polarization-Multiplexed Signals

$$\frac{\partial \mathbf{u}}{\partial z} = \hat{\mathbf{D}} \mathbf{u} + j\gamma \frac{8}{9} \|\mathbf{u}\|^2 \mathbf{u}$$

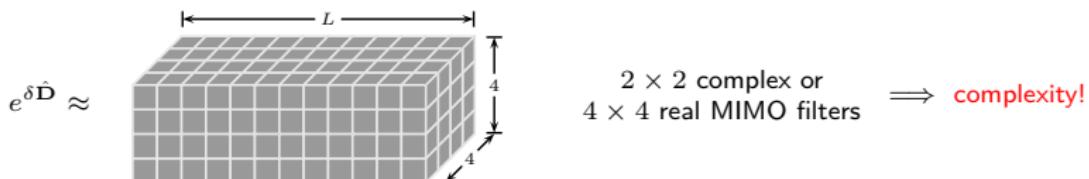
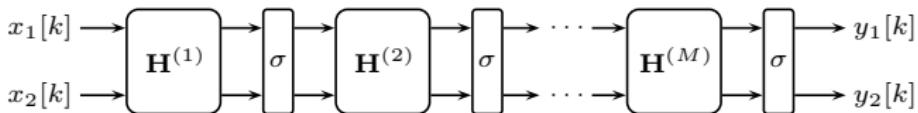


- Jones vector  $\mathbf{u} \triangleq (u_1(t, z), u_2(t, z))^\top$  with complex baseband signals
- linear operator  $\hat{\mathbf{D}}$ : attenuation, chromatic & polarization mode dispersion

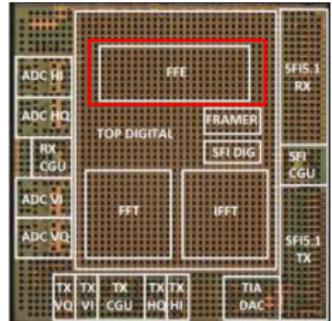
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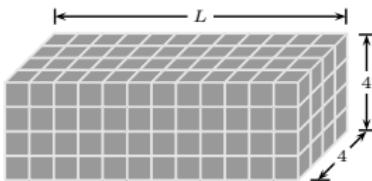
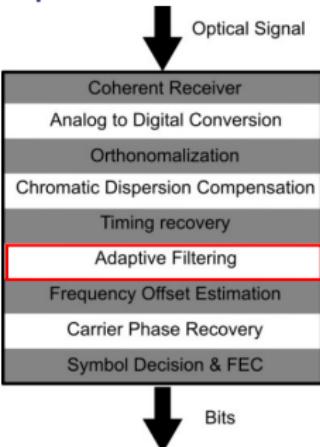
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- Split-step method: alternate linear and nonlinear steps  $\sigma(x) = xe^{j\gamma \frac{8}{9} \delta \|\mathbf{x}\|^2}$



# Real-Time Compensation of Polarization Impairments

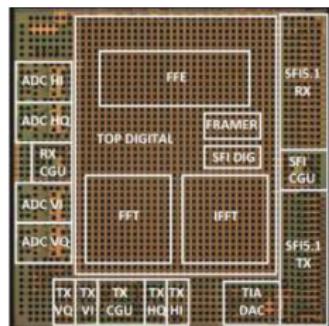


[Crivelli et al., 2014]

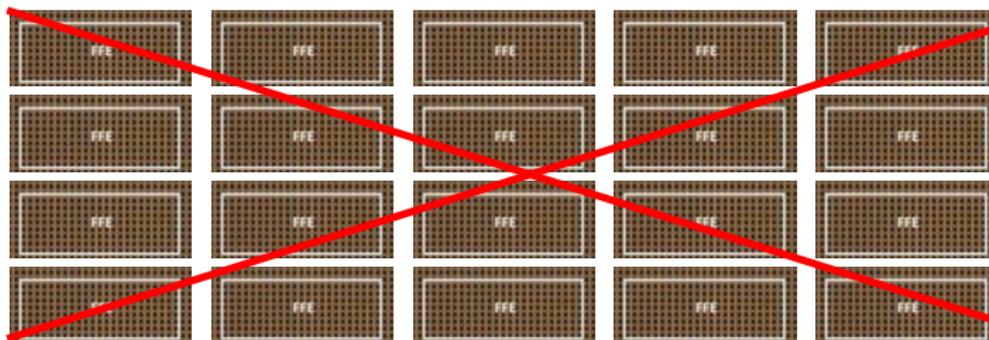


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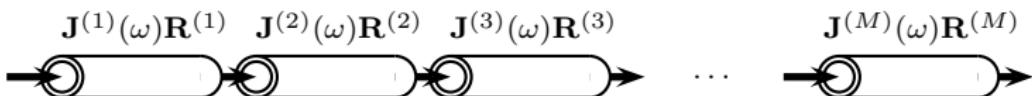


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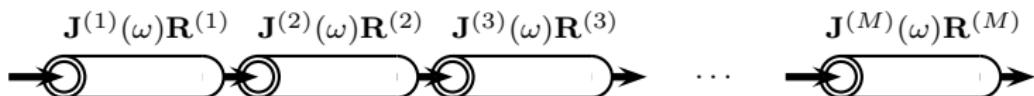
- time-varying effects (e.g., drifts) & apriori unknown realizations
  - $\Rightarrow$  adaptive filtering (via stochastic gradient descent) required
- 
- Using (and updating) full MIMO filters in each step is not feasible.
  - We propose a hardware-efficient machine-learning model based on the propagation characteristics

## Modeling of Polarization Mode Dispersion (PMD)



The overall PMD is modeled via *M sections*, where each section introduces

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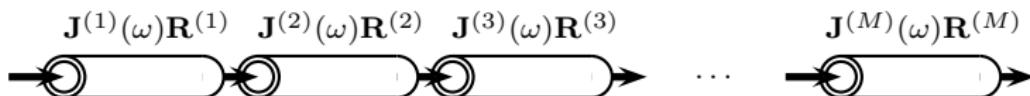


The overall PMD is modeled via  **$M$  sections**, where each section introduces

1. a **differential group delay (DGD)**  $\tau^{(k)}$ , described by

$$\mathbf{J}^{(k)}(\omega) = \begin{pmatrix} \exp\left(-j\omega\frac{\tau^{(k)}}{2}\right) & 0 \\ 0 & \exp\left(j\omega\frac{\tau^{(k)}}{2}\right) \end{pmatrix}$$

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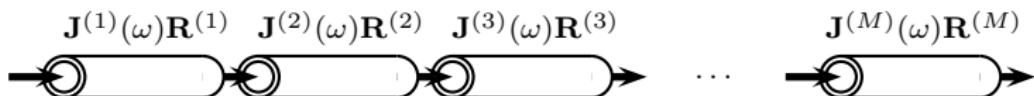
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$$\text{SU}(2) = \left\{ \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} : a, b \in \mathbb{C}, |a|^2 + |b|^2 = 1 \right\}$$

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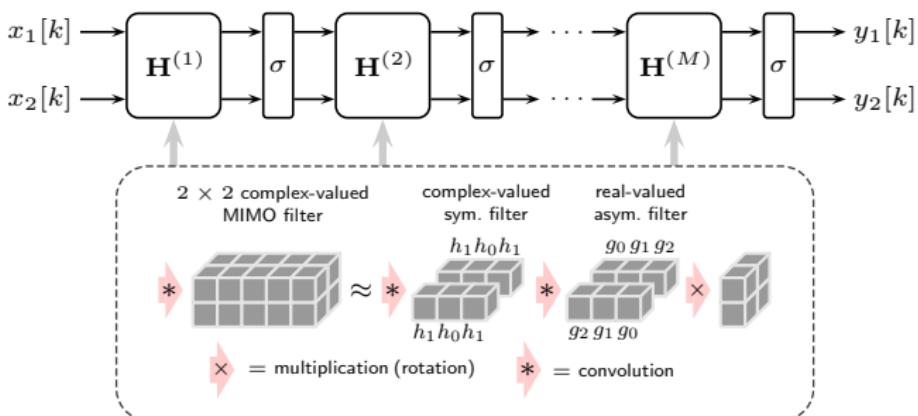
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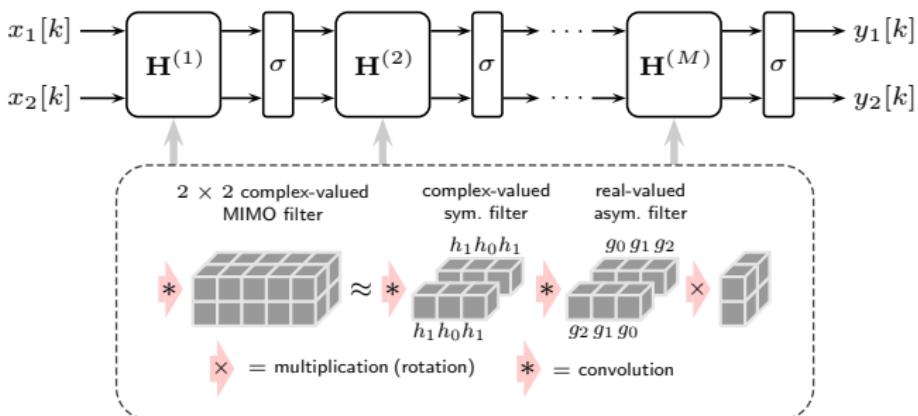
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- (i) We **integrate** these operations in each step/layer
- (ii) We use real-valued (asymmetric) FIR filters to approximate DGD

## The Final Machine-Learning Model



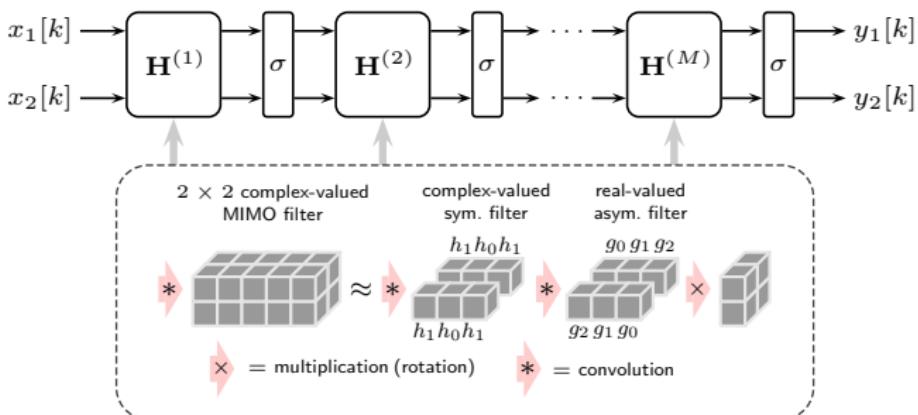
## The Final Machine-Learning Model



Each linear step consists of 3 trainable components

1. complex-valued (symmetric) filters that mainly account for dispersion
2. real-valued (asymmetric) filters for DGD
3. memoryless "rotation" matrices  $\begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix}$ , where  $a, b \in \mathbb{C}$  (4 real parameters)

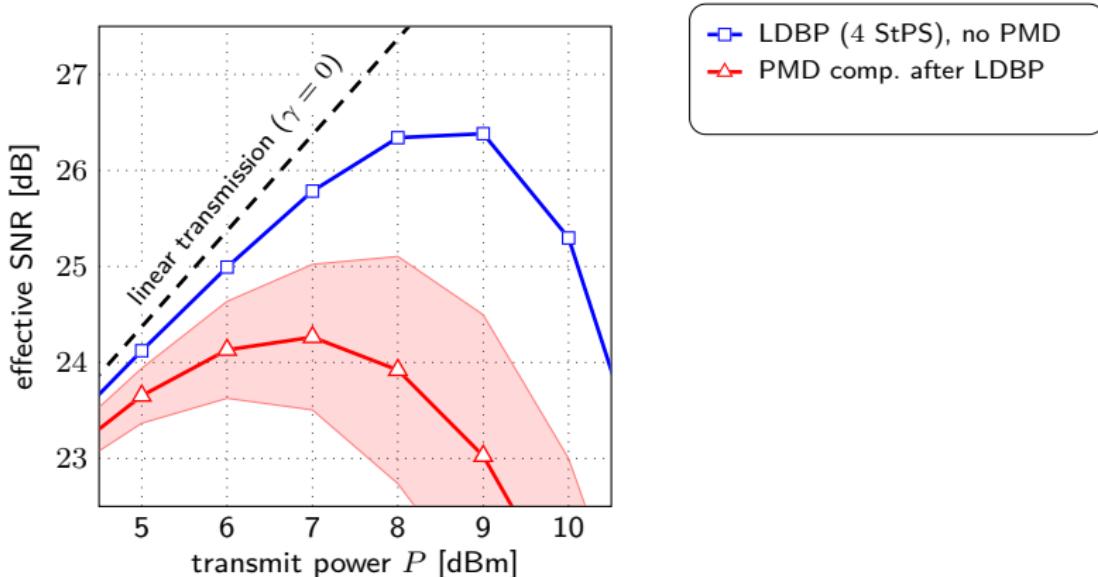
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Compared to prior work, our learning-based approach ...

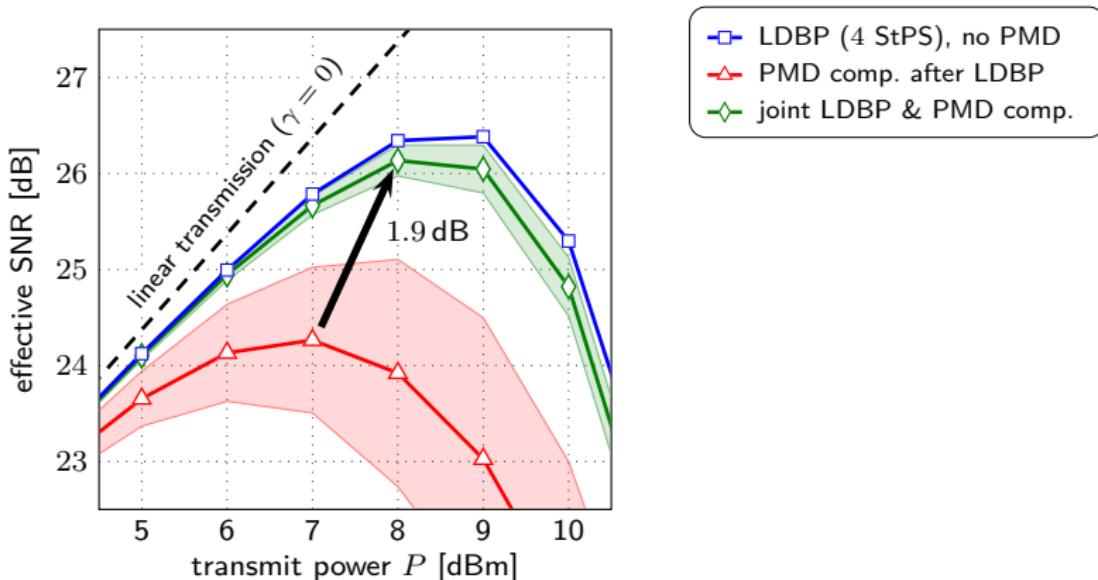
- assumes no knowledge about PMD realizations or accumulated PMD
- is FIR-filter based! Avoids frequency-domain (FFT-based) filtering

[Goroshko et al., 2016], Overcoming performance limitations of digital back propagation due to polarization mode dispersion, (*CTON*)  
[Czegledi et al., 2017], Digital backpropagation accounting for polarization-mode dispersion, (*Opt. Express*)  
[Liga et al., 2018], A PMD-adaptive DBP receiver based on SNR optimization, (*OFC*)

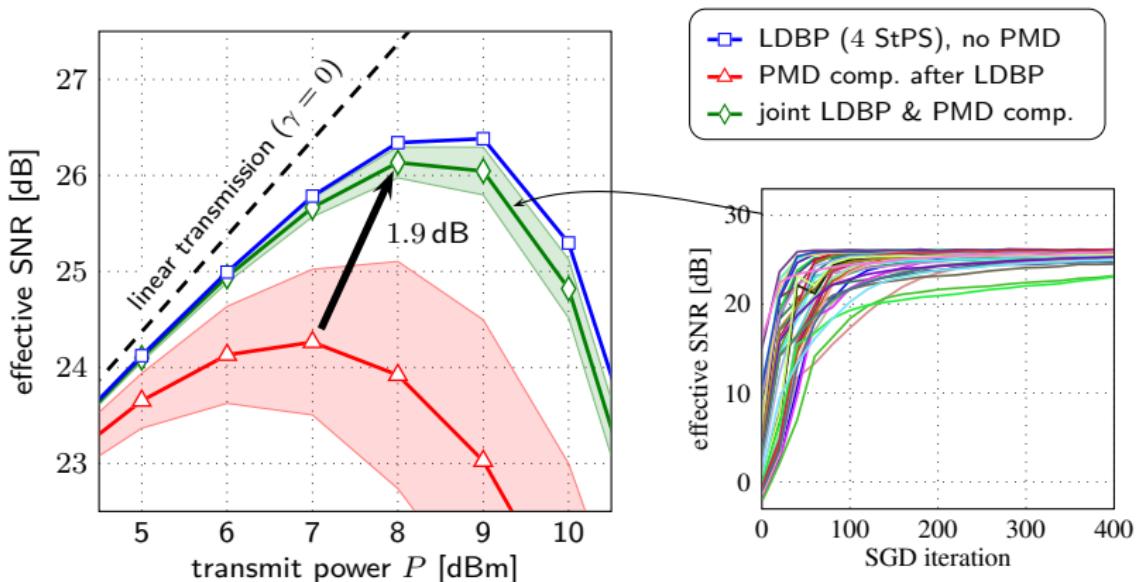
Results (32 Gbaud,  $10 \times 100$  km,  $0.2 \text{ ps}/\sqrt{\text{km}}$  PMD)

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- Similar parameters & simulation setup compared to [Czegledi et al., 2016], results averaged over 40 PMD realizations
- **Reliable convergence** “from scratch” + only 9 real parameters per step

[Bütler et al., 2021]. Model-based Machine Learning for Joint Digital Backpropagation and PMD Compensation, *(J. Lightw. Technol.)*, see arXiv:2010.12313

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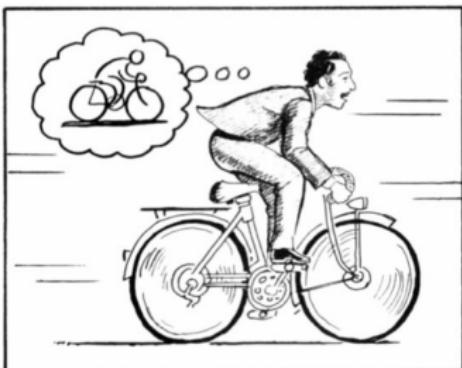
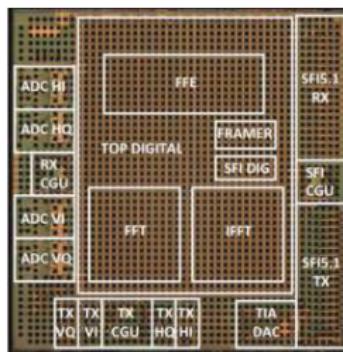


Figure 1. A World Model, from Scott McCloud's *Understanding Comics*. (McCloud, 1993; E, 2012)



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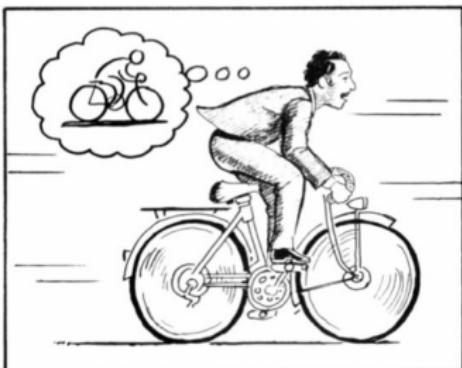
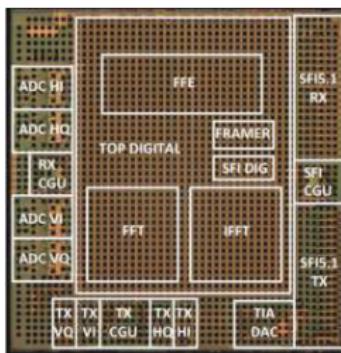


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[Crivelli et al., 2014]

- Optical receivers build models of their "environment"
- Currently these models are **linear** and/or **rigid** (non-adaptive)
- Interpretable **physics-based “multi-layer” models** for machine learning can be obtained by exploiting our existing domain knowledge

Machine Learning  
ooooo

Physics-Based Models  
oooooooo

Learned DBP  
oooooooooooo

Polarization Effects  
oooooo

Conclusions  
oo●

CHALMERS

## Conclusions

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## Conclusions

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universal function approximators

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Thank you!

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