1) Sample space → S(set)

2) Event (A) : $A \subset S$

$P(A) = prob\,(\text{outcome} \in A)$: 확률에 대한 정의.

3) conditional probability

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

4) Total probability.

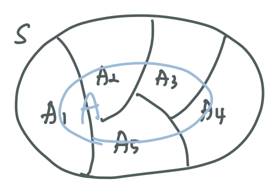
$P(A) = P(A_1) + P(A_2) \cdots + P(A_n)$

$\{A_1, A_2, \cdots A_n\}$ partition of S

$P(A_1) = P(A_1 \cap A)$

$= P(A|A_1)P(A_1)$.

$$P(A) = \sum_{i=1}^{n} P(A|A_i)P(A_i)$$



mutually exclusive 한 A들은 더해나가면

$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + \cdots$
$+ P(A_n \cap A)$

5) Bayesian Theorem

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$$

$$\frac{P(A|A_i)P(A_i)}{P(A)}$$

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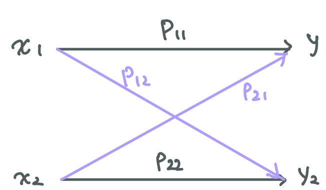
ex 1.7) Binary Symmetric Channel.

input symbol $\{x_1, x_2\}$. → transmitter

output symbol $\{y_1, y_2\}$. → receiver



(입력 x_1 에 대해 출력 y_1 이 나올 확률, P_{11})

P_{12}, P_{21} 은 error!

$P_{11} = P_{22}$, $P_{12} = P_{22}$ ⇒ Binary Symmetric

$P_{11} = P(y_1|x_1)$

$P_{22} = P(y_2|x_2)$

$P_{12} = P(y_2|x_1)$

$P_{21} = P(y_1|x_2)$

$\oplus = 1$

$\oplus = 1$

$P(x_1)$, $P x_2$

Priori

transmitter 에서 error가 나는 확률.

1) $P_{error} = Prob(x_1 \text{ trans}, y_2 \text{ receive}) + Prob(x_2 \text{ trans}, y_1 \text{ receive})$

$$= P(y_2|x_1)P(x_1) + P(y_1|x_2)P(x_2)$$

→ unconditioned error

ex 2) When y_2 received

→ what prob of x_1 transmission?

$$\Rightarrow P(x_1|y_2) = \frac{P(y_2|x_1)P(X_1)}{P(y_2)}$$

$$= \frac{P(y_2|x_1)P(X_1)}{P(y_2|x_1)P(X_1) + P(y_2|x_2)P(x_2)}$$

ex 3) $P(x_1|error)$

1.8 Independent Events

If A and B are (mutually) independent,

$P(B|A) \triangleq P(B)$ → A가 무엇이 오든 B에는 영향을 끼치지 않는다.

$P(A|B) \triangleq P(A)$

조건부 확률

$P(B|A) = \frac{P(A \cap B)}{P(A)}$ → $P(A \cap B) = P(B|A)P(A)$

repeated (restored) trials

Independent ≠ exclusive.

If A and B are independent. 이면 다음 조건도 성립!

$A \ \bar{B}$

$\bar{A} \ B$

$\bar{A} \ \bar{B}$

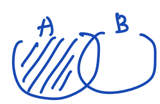
⇒ independent!

증명하는 방법!

$P(A \cap \bar{B}) = P(A)P(\bar{B})$

$\Rightarrow P(A) - P(A \cap B)$

$= P(A) - P(A)P(B)$



1.9 Combined Experiments.

for two experiments with S_1, S_2

$\Rightarrow S = S_1 \times S_2$

↳ Cartesian product.

$\{(x_i, y_j) \mid x_i \in S_1, y_j \in S_2\}$.

ex) 3 coins tossing.

$\Rightarrow$ 3 experiments of 1 coin tossing. $\underline{H}$ $\underline{T}$.

$S_1 = \{H, T\}$

$S_2 = \{H, T\}$

$S_3 = \{H, T\}$

$\Rightarrow S = S_1 \times S_2 \times S_3$

$= \{\underbrace{HHH \cdots TTT}_{8}\}$