1 Supplementary Material: Simulation Results

Table 1: Comparison of Weibull Logit (true model) fitted with Logit, SLogit, RPlogit, and FGLogit link functions. Parameter estimates along with their 95% HPD.

Parameter	True	Logit (True)		SLogit		RPLogit		FGLogit	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	1.00	[0.86, 1.15]	0.99	[0.84, 1.15]	0.99	[0.84, 1.15]	0.99	[0.84, 1.15]
β_1	−1.00	−1.00	[−1.12, −0.87]	−1.00	[−1.13, −0.87]	−1.00	[−1.13, −0.87]	−1.00	[−1.12, −0.87]
β_2	−2.00	−1.98	[−2.11, −1.84]	−1.99	[−2.14, −1.84]	−1.99	[−2.14, −1.84]	−1.99	[−2.14, −1.85]
γ_1	0.50	0.43	[0.18, 0.68]	0.51	[0.25, 0.77]	0.51	[0.25, 0.78]	0.65	[0.22, 1.31]
γ_2	0.60	0.64	[0.38, 0.90]	0.61	[0.35, 0.88]	0.61	[0.35, 0.89]	0.78	[0.31, 1.51]
σ	0.667	0.66	[0.58, 0.74]	0.67	[0.59, 0.76]	0.67	[0.59, 0.76]	0.67	[0.59, 0.76]
λ	–	–	–	–	–	1.00	[0.83, 1.19]	0.78	[0.18, 2.03]
α	–	–	–	1.00	[0.82, 1.19]	–	–	0.87	[0.31, 2.51]
LOO		545.96	–	545.82	–	545.61	–	545.44	–
WAIC		545.85	–	545.71	–	545.50	–	545.33	–
DIC		546.35	–	547.60	–	547.41	–	550.52	–

Table 2: Comparison of Weibull SLogit (true model) fitted with Logit, SLogit, RPlogit, and FGLogit link functions. Parameter estimates along with their 95% HPD.

Parameter	True	Logit		SLogit (True)		RPLogit		FGLogit	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.99	[0.84, 1.15]	1.00	[0.87, 1.14]	0.99	[0.86, 1.13]	0.99	[0.86, 1.13]
β_1	−1.00	−1.00	[−1.13, −0.87]	−0.98	[−1.08, −0.88]	−0.99	[−1.11, −0.88]	−1.00	[−1.10, −0.89]
β_2	−2.00	−1.99	[−2.14, −1.84]	−2.00	[−2.13, −1.87]	−1.99	[−2.13, −1.86]	−1.99	[−2.13, −1.86]
γ_1	0.50	0.51	[0.25, 0.77]	0.55	[0.32, 0.78]	0.65	[0.33, 1.00]	0.70	[0.27, 1.35]
γ_2	0.60	0.61	[0.35, 0.88]	0.57	[0.34, 0.80]	0.80	[0.47, 1.16]	0.85	[0.35, 1.59]
σ	0.667	0.67	[0.59, 0.76]	0.68	[0.61, 0.76]	0.67	[0.60, 0.75]	0.67	[0.60, 0.75]
λ	–	–	–	–	–	0.64	[0.50, 0.78]	0.69	[0.16, 1.78]
α	1.50	–	–	1.47	[1.24, 1.73]	–	–	1.15	[0.45, 3.06]
LOO		545.82	–	541.79	–	564.91	–	557.79	–
WAIC		545.71	–	541.69	–	564.81	–	557.68	–
DIC		547.60	–	543.55	–	567.43	–	562.99	–

Table 3: Comparison of Weibull RPLogit (true model) fitted with Logit, SLogit, and FGLogit link functions. Parameter estimates are shown with 95% HPD.

Parameter	True	Logit		SLogit		RPLogit (True)		FGLogit	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	1.14	[0.95, 1.35]	0.98	[0.80, 1.18]	1.05	[0.87, 1.23]	0.99	[0.82, 1.18]
β_1	−1.00	−1.05	[−1.22, −0.89]	−0.99	[−1.15, −0.84]	−0.98	[−1.12, −0.84]	−1.00	[−1.14, −0.85]
β_2	−2.00	−2.09	[−2.28, −1.91]	−1.98	[−2.17, −1.80]	−2.02	[−2.19, −1.85]	−1.99	[−2.17, −1.81]
γ_1	0.50	0.58	[0.31, 0.86]	0.66	[0.35, 0.98]	0.51	[0.29, 0.75]	0.72	[0.28, 1.42]
γ_2	0.60	0.66	[0.38, 0.94]	0.79	[0.47, 1.13]	0.58	[0.35, 0.82]	0.87	[0.36, 1.67]
σ	0.667	0.69	[0.59, 0.81]	0.67	[0.58, 0.78]	0.68	[0.58, 0.79]	0.67	[0.58, 0.78]
λ	1.50	–	–	–	–	1.48	[1.26, 1.73]	1.04	[0.27, 2.53]
α	–	–	–	0.63	[0.51, 0.77]	–	–	0.74	[0.24, 2.31]
LOO		493.35	–	474.55	–	470.17	–	475.40	–
WAIC		493.23	–	474.42	–	470.06	–	475.26	–
DIC		495.08	–	477.01	–	472.42	–	480.68	–

Table 4: Comparison of Weibull FGLogit as the true model fitted with Logit, SLogit, RPLogit, and FGLogit links. True values are listed in the second column for reference.

Parameter	True	Logit		SLogit		RPLogit		FGLogit (True)	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.95	[0.82, 1.09]	1.00	[0.85, 1.14]	1.00	[0.86, 1.14]	0.98	[0.83, 1.14]
β_1	−1.00	−0.98	[−1.10, −0.86]	−1.00	[−1.12, −0.88]	−1.00	[−1.12, −0.88]	−0.99	[−1.12, −0.87]
β_2	−2.00	−1.96	[−2.10, −1.83]	−1.99	[−2.13, −1.85]	−1.99	[−2.13, −1.86]	−1.96	[−2.10, −1.82]
γ_1	0.50	0.67	[0.40, 0.95]	0.66	[0.40, 0.93]	0.77	[0.45, 1.11]	0.86	[0.39, 1.56]
γ_2	0.60	0.82	[0.54, 1.11]	0.79	[0.53, 1.07]	0.92	[0.61, 1.27]	0.99	[0.45, 1.75]
σ	0.667	0.67	[0.59, 0.75]	0.67	[0.60, 0.75]	0.67	[0.60, 0.76]	0.70	[0.63, 0.79]
λ	1.50	–	–	–	–	0.78	[0.63, 0.95]	0.90	[0.28, 2.11]
α	2.00	–	–	1.27	[1.05, 1.51]	–	–	1.18	[0.46, 3.27]
LOO		542.12	–	536.73	–	536.80	–	541.84	–
WAIC		542.03	–	536.63	–	536.70	–	541.72	–
DIC		543.32	–	539.03	–	539.60	–	546.85	–

Table 5: Comparison of Lognormal Logit as the true model fitted with Logit, SLogit, RPLogit, and FGLogit links. True values are included in the second column.

Parameter	True	Logit (True)		SLogit		RPLogit		FGLogit	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.99	[0.71, 1.30]	0.97	[0.64, 1.35]	0.99	[0.66, 1.37]	0.98	[0.64, 1.35]
β_1	−1.00	−0.98	[−1.24, −0.73]	−0.98	[−1.25, −0.72]	−0.99	[−1.25, −0.73]	−0.98	[−1.23, −0.73]
β_2	−2.00	−1.97	[−2.23, −1.72]	−1.96	[−2.24, −1.69]	−1.96	[−2.25, −1.70]	−1.96	[−2.24, −1.69]
γ_1	0.50	0.49	[0.22, 0.78]	0.51	[0.22, 0.81]	0.52	[0.22, 0.85]	0.66	[0.22, 1.34]
γ_2	0.60	0.60	[0.32, 0.90]	0.61	[0.32, 0.92]	0.62	[0.32, 0.96]	0.79	[0.29, 1.54]
σ	1.50	1.51	[1.32, 1.73]	1.51	[1.30, 1.75]	1.52	[1.31, 1.76]	1.50	[1.30, 1.75]
λ	–	–	–	–	–	1.00	[0.77, 1.24]	0.79	[0.19, 2.05]
α	–	–	–	1.00	[0.79, 1.26]	–	–	0.88	[0.31, 2.55]
LOO		643.96	–	632.88	–	632.86	–	643.43	–
WAIC		643.88	–	632.79	–	632.77	–	643.34	–
DIC		643.73	–	633.55	–	633.58	–	647.35	–

Table 6: Lognormal SLogit as the true model, fitted with Logit, RPLogit, and FGLogit link functions. True parameter values are shown for reference.

	True	Logit		SLogit (True)		RPLogit		FGLogit	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.73	[0.49, 0.98]	0.98	[0.70, 1.31]	0.98	[0.70, 1.30]	0.98	[0.69, 1.30]
β_1	−1.00	−0.93	[−1.14, −0.71]	−0.99	[−1.21, −0.77]	−0.98	[−1.21, −0.75]	−0.99	[−1.21, −0.77]
β_2	−2.00	−1.86	[−2.09, −1.65]	−1.97	[−2.22, −1.74]	−1.97	[−2.22, −1.74]	−1.98	[−2.22, −1.74]
γ_1	0.50	0.47	[0.20, 0.75]	0.51	[0.23, 0.79]	0.67	[0.29, 1.11]	0.69	[0.24, 1.37]
γ_2	0.60	0.59	[0.32, 0.88]	0.61	[0.33, 0.91]	0.82	[0.42, 1.28]	0.84	[0.33, 1.61]
σ	1.50	1.40	[1.25, 1.57]	1.51	[1.33, 1.72]	1.50	[1.32, 1.71]	1.50	[1.32, 1.71]
λ	–	–	–	–	–	0.64	[0.46, 0.83]	0.69	[0.16, 1.80]
α	1.50	–	–	1.52	[1.21, 1.92]	–	–	1.16	[0.44, 3.16]
LOO		705.72	-	678.53	-	694.04	-	701.26	-
WAIC		705.65	-	678.44	-	693.95	-	701.16	-
DIC		704.45	-	679.41	-	695.55	-	705.37	-

Table 7: Comparison of Lognormal RPLogit as the true model fitted with Logit, SLogit, and FGLogit links. Parameter estimates and 95% credible intervals are shown. True values are added for reference.

Parameter	True	Logit		SLogit		RPLogit (True)		FGLogit	
	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	1.50	[1.10, 1.95]	0.96	[0.57, 1.40]	0.98	[0.59, 1.43]	0.97	[0.58, 1.41]
β_1	−1.00	−1.04	[−1.37, −0.71]	−0.97	[−1.28, −0.66]	−0.96	[−1.26, −0.67]	−0.98	[−1.27, −0.69]
β_2	−2.00	−2.07	[−2.43, −1.73]	−1.94	[−2.28, −1.63]	−1.95	[−2.28, −1.65]	−1.94	[−2.27, −1.63]
γ_1	0.50	0.66	[0.32, 1.04]	0.66	[0.32, 1.02]	0.51	[0.26, 0.77]	0.72	[0.26, 1.43]
γ_2	0.60	0.77	[0.42, 1.16]	0.80	[0.44, 1.17]	0.61	[0.35, 0.90]	0.87	[0.35, 1.67]
σ	1.50	1.76	[1.47, 2.11]	1.50	[1.26, 1.79]	1.51	[1.28, 1.81]	1.50	[1.26, 1.78]
λ	1.50	–	–	–	–	1.50	[1.21, 1.81]	1.04	[0.27, 2.54]
α	–	–	–	0.63	[0.48, 0.82]	–	–	0.75	[0.24, 2.33]
LOO		538.80	–	529.05	–	537.57	–	536.91	–
WAIC		538.72	–	528.96	–	537.48	–	536.81	–
DIC		540.88	–	530.39	–	538.36	–	541.04	–

Table 8: Lognormal FGLogit as the true model, with comparative model fitting using Logit, SLogit, and RPLogit links. True parameter values are provided for reference.

	True	Logit		SLogit		RPLogit		FGLogit (True)	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.83	[0.58, 1.10]	0.98	[0.67, 1.31]	0.99	[0.69, 1.31]	0.97	[0.67, 1.30]
β_1	−1.00	−0.95	[−1.19, −0.72]	−0.98	[−1.23, −0.75]	−0.98	[−1.23, −0.75]	−0.98	[−1.21, −0.75]
β_2	−2.00	−1.91	[−2.16, −1.67]	−1.97	[−2.23, −1.72]	−1.97	[−2.23, −1.72]	−1.96	[−2.21, −1.72]
γ_1	0.50	0.64	[0.35, 0.95]	0.66	[0.36, 0.97]	0.78	[0.41, 1.20]	0.81	[0.34, 1.53]
γ_2	0.60	0.78	[0.48, 1.10]	0.79	[0.49, 1.11]	0.93	[0.55, 1.38]	0.97	[0.43, 1.78]
σ	1.50	1.43	[1.27, 1.63]	1.50	[1.31, 1.72]	1.50	[1.31, 1.72]	1.50	[1.32, 1.72]
λ	1.50	–	–	–	–	0.78	[0.58, 1.00]	0.85	[0.24, 2.05]
α	2.00	–	–	1.27	[1.00, 1.61]	–	–	1.18	[0.44, 3.27]
LOO		653.52	–	650.30	–	650.55	–	665.53	–
WAIC		653.44	–	650.21	–	650.46	–	665.44	–
DIC		653.15	–	651.50	–	652.30	–	669.74	–

Table 9: Loglogistic Logit as the true model, compared with SLogit, RPLogit, and FGLogit link functions. True parameter values are included for reference.

	True	Logit (True)		SLogit		RPLogit		FGLogit	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.99	[0.77, 1.24]	0.99	[0.74, 1.26]	0.99	[0.75, 1.26]	0.99	[0.74, 1.25]
β_1	−1.00	−0.99	[−1.19, −0.79]	−0.99	[−1.18, −0.80]	−0.98	[−1.18, −0.79]	−0.98	[−1.18, −0.79]
β_2	−2.00	−1.99	[−2.21, −1.78]	−1.98	[−2.21, −1.75]	−1.98	[−2.21, −1.75]	−1.97	[−2.21, −1.75]
γ_1	0.50	0.50	[0.23, 0.77]	0.50	[0.23, 0.78]	0.51	[0.23, 0.81]	0.67	[0.23, 1.34]
γ_2	0.60	0.60	[0.33, 0.89]	0.60	[0.33, 0.89]	0.61	[0.33, 0.92]	0.79	[0.30, 1.54]
σ	0.667	0.67	[0.58, 0.78]	0.67	[0.58, 0.78]	0.67	[0.58, 0.79]	0.67	[0.58, 0.79]
λ	–	–	–	–	–	0.99	[0.79, 1.21]	0.79	[0.18, 2.04]
α	–	–	–	1.00	[0.81, 1.21]	–	–	0.88	[0.31, 2.53]
LOO		612.63	–	604.69	–	625.40	–	616.00	–
WAIC		612.55	–	604.60	–	625.31	–	615.91	–
DIC		613.58	–	606.49	–	627.19	–	621.10	–

Table 10: Loglogistic SLogit as the true model, compared with Logit, RPLogit, and FGLogit link functions. True parameter values are included for reference.

	True	Logit		SLogit (True)		RPLogit		FGLogit	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.83	[0.64, 1.03]	0.99	[0.78, 1.22]	0.83	[0.64, 1.03]	0.99	[0.78, 1.22]
β_1	−1.00	−0.95	[−1.12, −0.78]	−0.99	[−1.16, −0.81]	−0.95	[−1.12, −0.78]	−0.99	[−1.17, −0.82]
β_2	−2.00	−1.89	[−2.09, −1.71]	−1.98	[−2.17, −1.79]	−1.89	[−2.09, −1.71]	−1.99	[−2.19, −1.79]
γ_1	0.50	0.50	[0.25, 0.77]	0.51	[0.26, 0.76]	0.50	[0.25, 0.77]	0.69	[0.24, 1.37]
γ_2	0.60	0.61	[0.35, 0.88]	0.61	[0.35, 0.88]	0.61	[0.35, 0.88]	0.84	[0.34, 1.60]
σ	0.667	0.64	[0.56, 0.72]	0.67	[0.59, 0.77]	0.64	[0.56, 0.72]	0.67	[0.58, 0.77]
λ	–	–	–	–	–	0.69	[0.16, 1.80]	0.69	[0.16, 1.80]
α	1.50	–	–	1.52	[1.24, 1.85]	–	–	1.16	[0.45, 3.15]
LOO		689.79	–	669.98	–	689.79	–	663.49	–
WAIC		689.72	–	669.90	–	689.72	–	663.40	–
DIC		690.08	–	671.97	–	690.08	–	668.77	–

Table 11: Loglogistic RPLogit as the true model, compared with Logit, SLogit, and FGLogit link functions. True parameter values are included for reference.

	True	Logit		SLogit		RPLogit (True)		FGLogit	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	1.34	[1.03, 1.70]	0.98	[0.69, 1.30]	0.99	[0.70, 1.30]	0.98	[0.70, 1.30]
β_1	−1.00	−1.05	[−1.30, −0.81]	−0.98	[−1.21, −0.76]	−0.98	[−1.22, −0.75]	−0.99	[−1.22, −0.76]
β_2	−2.00	−2.11	[−2.43, −1.82]	−1.96	[−2.24, −1.70]	−1.97	[−2.24, −1.73]	−1.97	[−2.24, −1.71]
γ_1	0.50	0.63	[0.33, 0.96]	0.66	[0.33, 1.01]	0.51	[0.26, 0.76]	0.72	[0.26, 1.43]
γ_2	0.60	0.75	[0.43, 1.10]	0.80	[0.47, 1.15]	0.61	[0.36, 0.88]	0.87	[0.35, 1.67]
σ	0.667	0.78	[0.64, 0.95]	0.67	[0.56, 0.80]	0.68	[0.56, 0.81]	0.67	[0.56, 0.81]
λ	1.50	–	–	–	–	1.51	[1.24, 1.79]	1.04	[0.27, 2.53]
α	–	–	–	0.63	[0.49, 0.79]	–	–	0.74	[0.24, 2.30]
LOO		549.32	–	524.87	–	516.79	–	518.07	–
WAIC		549.25	–	524.79	–	516.70	–	517.98	–
DIC		551.82	–	527.42	–	518.72	–	523.37	–

Table 12: Loglogistic FGLogit as the true model, compared with Logit, SLogit, and RPLogit link functions. True parameter values are provided for reference.

	True	Logit		SLogit		RPLogit		FGLogit (True)	
Parameter	Value	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD	Estimate	95% HPD
β_0	1.00	0.90	[0.69, 1.11]	0.99	[0.77, 1.23]	0.99	[0.77, 1.23]	0.98	[0.76, 1.22]
β_1	−1.00	−0.97	[−1.14, −0.80]	−0.99	[−1.17, −0.81]	−0.99	[−1.17, −0.81]	−0.99	[−1.17, −0.80]
β_2	−2.00	−1.93	[−2.15, −1.73]	−1.98	[−2.20, −1.76]	−1.98	[−2.19, −1.78]	−1.97	[−2.18, −1.77]
γ_1	0.50	0.65	[0.38, 0.95]	0.67	[0.40, 0.95]	0.77	[0.45, 1.13]	0.82	[0.34, 1.53]
γ_2	0.60	0.81	[0.53, 1.11]	0.80	[0.52, 1.10]	0.93	[0.59, 1.31]	0.98	[0.43, 1.78]
σ	0.667	0.65	[0.57, 0.74]	0.67	[0.58, 0.78]	0.67	[0.58, 0.77]	0.67	[0.58, 0.78]
λ	1.50	–	–	–	–	0.79	[0.62, 0.97]	0.84	[0.24, 2.03]
α	2.00	–	–	1.27	[1.03, 1.56]	–	–	1.17	[0.44, 3.18]
LOO		619.86	–	654.82	–	631.99	–	630.72	–
WAIC		619.79	–	654.73	–	631.90	–	630.63	–
DIC		620.90	–	657.15	–	634.80	–	636.09	–