Chua, Aaron Eldreich L.   COM22I   Exercise 3   08/20/2025

$R_{school} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ $R_{stay} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ $P_{school} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$ $P_{stay} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$

Step 1:

1. Find r_{π} for sunny =?

$r_{\pi} = 0.5 \times (5) + 0.5 \times (-5) = 2.5 + 2.5 = \boxed{0}$

2. Find r_{π} for cloudy

$r_{\pi} = 0.5 \times (1) + 0.5 \times (3) = 0.5 + 1.5 = \boxed{2}$

3. Find r_{π} matrix. $r_{\pi} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Step 2:

Row 1 (sunny):

$P_{\pi}(1,1) = ??$ $0.5 \times 0.8 + 0.5 \times 0.9 = 0.4 + 0.45 = \boxed{0.85}$
$P_{\pi}(1,2) = ??$ $0.5 \times 0.2 + 0.5 \times 0.1 = 0.1 + 0.05 = \boxed{0.15}$

Row 2 (cloudy):

$P_{\pi}(2,1) = ??$ $0.5 \times 0.4 + 0.5 \times 0.3 = 0.2 + 0.15 = \boxed{0.35}$
$P_{\pi}(2,2) = ??$ $0.5 \times 0.6 + 0.5 \times 0.7 = 0.3 + 0.35 = \boxed{0.65}$

4. $P_{\pi} = \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$

Step 3

$V_1 = 0 + 0.9 (0.85 v_1 + 0.15 v_2)$

5. $\boxed{V_1 = 0.765 v_1 + 0.135 v_2}$

$V_2 = 2 + 0.9 (0.35 v_1 + 0.65 v_2)$

$V_2 = 2 + 0.315 v_1 + 0.585 v_2$

$V_2 - 0.315 v_1 - 0.585 v_2 = 2$

6. $\boxed{-0.315 v_1 + 0.595 v_2 = 2}$

Step 4

$\dfrac{0.765 v_1 + 0.135 v_2}{0.765} \qquad \dfrac{}{0.765}$

$\boxed{V_1 = \dfrac{0.135 v_2}{0.765}}$

$-0.315 v_1 + 0.595 v_2 = 2$

$-0.315 \left(\dfrac{0.135 v_2}{0.765}\right) + 0.595 v_2 = 2$

$\left(-0.315 \times \dfrac{0.135}{0.765} v_2\right) + 0.595 v_2 = 2$

$-0.056 + 0.595 v_2 = 2$

$0.595 v_2 = 2 + 0.056$

7. $V_{\pi} (\text{cloudy}) = \dfrac{2 + 0.056}{0.595} = \dfrac{2.056}{0.595} = \boxed{3.456}$

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Step 4

$$\boxed{V_1 = \dfrac{0.135\,V_2}{0.765}}$$

$$V_1 = \frac{0.135(3.456)}{0.765} = \frac{0.46656}{0.765} = 0.6099$$

8. $\boxed{V_{\#}(\text{sunny}) = 0.6099}$

Step 5

9. For sunny (v_1) using ~~program~~ school:

$$V_{\#}(\text{sunny}) = 5 + 0.9(0.8\,v_1 + 0.2\,v_2)$$

10. For cloudy (v_2) using school

$$V_{\#}(\text{cloudy}) = 3 + 0.9(0.4\,v_1 + 0.6\,v_2)$$

Step 6

Sunny: $v_1 = 5 + 0.72\,v_1 + 0.18\,v_2$

$v_1 - 0.72\,v_1 - 0.18\,v_2 = 5$

$$\boxed{0.28\,v_1 - 0.18\,v_2 = 5}$$

Cloudy $v_2 = -3 + 0.36\,v_1 + 0.54\,v_2$

$$\boxed{-0.36\,v_1 + 0.46\,v_2 = -3}$$

From the sunny equation:

$$0.28\,v_1 = 5 + 0.18\,v_2$$

$$v_1 = \frac{5 + 0.18\,v_2}{0.28}$$

11. Using the equation from cloudy $\boxed{-0.36\,v_1 + 0.46\,v_2 = -3}$

$$-0.36\left(\frac{5 + 0.18\,v_2}{0.28}\right) + 0.46\,v_2 = -3$$

$$-0.36 \times \frac{5}{0.28} = -6.429$$

$$-0.36 \times \frac{0.18}{0.28} =$$

$$-6.429 - 0.23\,v_2 + 0.46\,v_2 = -3$$

$$-6.429 + (0.46 - 0.23)\,v_2 = -3$$

$$-6.429 + 0.229\,v_2 = -3$$

$$0.229\,v_2 = -3 + 6.429 = 3.429$$

$$\to 0.229\,v_2 = 3.429$$

$$\boxed{V_{\#}(\text{cloudy}) = \frac{3.429}{0.229} = 14.974}$$

12. Solve for V_t (sunny)

$$V_1 = \frac{5 + 0.18v_2}{0.28}$$

$$V_1 = \frac{5 + 0.18 \times 14.974}{0.28}$$

$$V_1 = \frac{5 + 1.695}{0.28} = \frac{7.695}{0.28}$$

$$\boxed{V_t = 27.482} \text{ —sunny}$$

$$\boxed{V_t = 14.974} \text{ —cloudy}$$

Step 7: Solve for q

13. $q(1, \text{School}) = 5 + 0.9(0.8v_1 + 0.2v_2) = \boxed{27.478}$

14. $q(1, \text{Home}) = -5 + 0.9(0.9v_1 + 0.1v_2) = \boxed{18.606}$

15. $q(2, \text{School}) = 3 + 0.9(0.4v_1 + 0.6v_2) = \boxed{20.965}$

16. $q(2, \text{Home}) = 1 + 0.9(0.3v_1 + 0.7v_2) = \boxed{17.837}$