Curves of Strategy: Riemannian Models of Dynamic Economic Behavior*

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Chaeyeon Renée Eunice Xon
School of Economics
Yonsei University
Seoul, Republic of Korea
eunixon@yonsei.ac.kr

***Abstract*—This paper proposes a geometric framework where strategies are modeled not as static points but as smooth trajectories on a Riemannian manifold. This approach allows for a formal treatment of strategic behavior as a dynamic process, incorporating curvature-based costs, responsiveness to information, and phase transitions. We define a strategic response functional that penalizes excessive velocity and acceleration in strategy curves, deriving corresponding Euler-Lagrange equations. This framework offers a generalizable method for analyzing dynamic behavior in settings characterized by bounded rationality, limited attention, or adaptive learning, reinterpreting the cost of transitioning between strategies in geometric terms.**

***Index Terms*—Differential Geometry, Economic Dynamics, Bounded Rationality, Game Theory, Strategy, Riemannian Manifolds**

I. Introduction

Classical economic models predominantly treat strategy as a discrete, instantaneous choice [4]. This static framework, while tractable, overlooks the continuous, temporally extended process through which agents form, revise, and execute strategies. Real-world strategic decisions unfold not as isolated points, but as trajectories shaped by evolving beliefs and informational frictions.

To address this gap, we propose a differential geometric framework where strategy is represented as a smooth curve $S(t)$ on a Riemannian manifold. This formulation embeds strategic behavior within a continuous-time space that captures curvature, acceleration, and path-dependence. We aim to reconceptualize strategic decision-making not as a pointwise optimization problem, but as the traversal of a strategic manifold—a curved space defined by time, information, and structural constraints.

II. Mathematical Framework

We model the space of instantaneous strategies as a smooth Riemannian manifold (M, g), where each point $p \in M$ represents a strategic state. A strategic trajectory is a smooth curve $S : [0, T] \to M$, with its tangent vector $\dot{S}(t)$ representing the instantaneous velocity of strategic change.

To evaluate a trajectory, we define a strategic response functional $\mathcal{R}(S)$ that balances utility with penalties on velocity and acceleration:

$$\mathcal{R}(S) = \int_0^T [u(S(t)) - \lambda_1 \|\dot{S}(t)\|_g^2 - \lambda_2 \|\nabla_t \dot{S}(t)\|_g^2] dt \qquad (1)$$

where $u(S(t))$ is utility, and λ_1, λ_2 penalize the velocity and acceleration of the strategy curve, respectively. This functional captures a realistic behavioral constraint: agents may pursue high-payoff strategies, but sharp or volatile transitions incur costs, a concept central to models of bounded rationality [8].

III. Implications and Applications

This geometric formulation provides a new lens for analyzing decision-making in dynamic environments.

A. Bounded Rationality as Smooth Trajectories

Our framework replaces static heuristics with smooth trajectories, allowing rationality to vary as a function of velocity and acceleration. The energy penalties formalize the trade-off between responsiveness and stability, making rationality path-dependent rather than binary.

B. Strategy Dynamics in Game Theory

The framework extends classical game theory by embedding strategies as curves on a shared manifold. Instead of selecting fixed strategies, agents evolve trajectories $S_i(t)$. The geodesic distance $d(S_i, S_j)$ provides a basis for measuring strategic divergence or convergence in repeated interactions.

IV. Conclusion

This paper introduces a geometric reinterpretation of strategic behavior, replacing static optimization with smooth trajectories over differentiable manifolds. The framework yields novel measures of responsiveness and cost of change derived from first principles. By endogenizing not just choices but the motion between them, the model provides new tools for economic theorizing, connecting economics with differential geometry and the calculus of variations. The challenge ahead is empirical: measuring these curves and validating the framework across various domains.

References

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