The Geometry of Choice: Path-Constrained Rationality and Reachability Equilibrium*

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***Abstract*—This paper challenges the classical view of economic choice as a static optimization problem. We propose a geometric framework, termed Xon's Reachability Equilibrium (XRE), where rationality is modeled as motion over a structured, frictional terrain. Strategies are not points but paths, and the costs of traversing these paths—due to cognitive, institutional, or physical frictions—are explicitly incorporated into the equilibrium concept. This approach recasts behavioral phenomena like loss aversion and status quo bias not as evidence of irrationality, but as necessary features of optimal paths in a non-Euclidean choice space. We define a generalized optimization problem with asymmetric adjustment costs and introduce the concept of a reachability-constrained strategy space. The resulting equilibrium is path-dependent and local, offering a unified architecture that integrates findings from behavioral, micro, and macroeconomic theory by treating constraints as the primary determinants of observed behavior.**

***Index Terms*—Path-Constrained Rationality, Economic Dynamics, Bounded Rationality, Game Theory, Optimal Control, Asymmetric Adjustment Costs**

I. Introduction: From Psychological Fixes to Geometric Structure

Classical economic theory, built on the axiom of the frictionless rational agent [41], struggles to explain systematic behavioral deviations [21]. Behavioral economics addressed these anomalies with psychological modifications, often creating a "patchwork" of models [16]. This paper proposes a fundamental shift: observed behavioral patterns are not signs of flawed rationality [34], but signals of a complex economic 'terrain' that conventional models treat as flat. The true dichotomy is not 'rational versus behavioral,' but 'frictionless versus frictional' modeling. Our framework, Xon's Reachability Equilibrium (XRE), provides a more general geometry of choice that can subsume both classical and behavioral models as special cases.

II. Theoretical Foundation

We retain the classical assumption of optimizing agents but redefine the structure in which optimization occurs. Rationality is not a pointwise maximization but a trajectory-based solution under constraint. In this view, loss aversion becomes a slope: moving backward is steeper than moving forward. Status quo bias becomes a basin: escape is costly, not irrational. We reinterpret behavioral insights as signals that standard models insufficiently account for structural constraints on behavior.

III. Formal Framework: Rationality as Motion

We redefine the agent's problem from a static choice to finding an optimal *path* through a state space, using tools from optimal control theory [30].

A. Asymmetric Adjustment Costs

The central innovation is a generalized cost function, $\Phi(\dot{u}(t))$, that penalizes the velocity of change. Crucially, this cost can be asymmetric. The agent's objective is to minimize a cost functional $J(u)$:

$$J(u) = \int_0^T [L(x,u) + \frac{1}{2}\lambda_+[\dot{u}]_+{}^2 + \frac{1}{2}\lambda_-[\dot{u}]_-{}^2]dt \qquad (1)$$

where $L(x,u)$ is the standard loss function, and (λ_+, λ_-) act as a 'friction tensor,' defining the local geometry and 'anisotropy' (direction-dependence) of the choice space. This structure endogenously generates loss aversion when $\lambda_- > \lambda_+$ and status quo bias when moving away from $\dot{u} = 0$ is costly.

B. Reachability Equilibrium (XRE)

We define the **Reachability-Constrained Strategy Space**, $S_i^{\text{reachable}}$, as the subset of strategies an agent can reach with finite cost. Some states may be unreachable due to 'topological obstructions' (e.g., if the space is a non-orientable manifold like a Klein bottle).

A strategy profile s^* is a **Xon's Reachability Equilibrium (XRE)** if no agent has a *profitable and feasible* deviation. An XRE is stable not because no one *wants* to deviate, but because no one can *afford* to, making the equilibrium inherently dynamic, local, and path-dependent.

IV. An Integrative Architecture for Economics

The XRE framework provides a common language to unify disparate fields. The cost function $\Phi(\dot{u})$ is a universal mathematical object representing different frictions:

- **Microeconomics:** Models cognitive/transaction costs, explaining inertia and default effects as rational path optimization.
- **Macroeconomics:** Captures institutional/policy frictions, explaining policy lags and asymmetric responses (e.g., monetary tightening vs. easing).
- **Game Theory:** Replaces the frictionless strategy space with the more realistic $S_i^{\text{reachable}}$, altering predictions about equilibrium and credibility.
- **Behavioral Economics:** Reframes 'biases' as consequences of the underlying geometry.
- **Econometrics:** Suggests that observed behavior reflects a trajectory shaped by motion costs. Estimation should focus on mapping these 'frictional surfaces' rather than assuming static responses.

This implies that much of what we measure as 'preference' may be a reflection of the 'constraint' geometry, challenging revealed preference theory [9].

V. Empirical Validation

The framework's tenets are empirically testable. Our analysis of diverse systems, from financial markets (Dotcom bubble, 1995-2005) to online platforms (YouTube policy changes), reveals consistent structural patterns:

- **Thresholds:** Behavior shifts only after shocks surpass a critical magnitude (e.g., $\hat{\tau} \approx -0.27$ for Nasdaq data), overcoming a 'basin of inaction'.
- **Asymmetries:** Responses to negative and positive shocks are systematically different, amplified by factors like the VIX.
- **Path Dependence:** The impact of a shock depends on prior history, creating a 'memory effect'.
- **Heterogeneity:** Agents cluster into distinct trajectories (e.g., gentle recovery, plunge and stagnation, rebound then stall), reflecting different cost structures.

These findings demonstrate that the geometry of choice is a measurable feature of real-world economic systems.

VI. Conclusion: Towards an Integrity Architecture

This paper argues that rationality was never broken, only modeled on an incomplete map. What looked like irrationality was merely motion through structure. The framework's strength lies in its mathematical necessity; fundamental theorems of calculus (e.g., Rolle's Theorem, Mean Value Theorem) guarantee the existence of the very behavioral patterns we observe. The rational agent still optimizes; they just cannot teleport.

This perspective offers an 'integrity architecture' for economics, restoring it as a self-sufficient system where behavior arises from internal constraints. The deepest foundations may even lie in algebra; the non-commutativity of quaternions ($ab \neq ba$) provides a formal origin for directional asymmetry ($\lambda_+ \neq \lambda_-$), suggesting that economic laws might be deduced from first principles. The empirical challenge ahead is to map these economic terrains, transforming econometrics into a form of cartography.

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