Shaded Exponential Discounting: A Structural Interpretation of Present Bias*

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***Abstract*—Standard exponential discounting (ED) is the canonical model for intertemporal choice, yet it fails to explain robust empirical anomalies like present bias and time inconsistency. This paper introduces Shaded Exponential Discounting (Shaded-ED), a structural refinement that preserves the ED framework while accounting for these behaviors. We introduce a structural activation function, $\lambda(t) \in [0,1]$, which dictates whether utility at a future time t is behaviorally realized. The agent's utility is reformulated as $U = \sum_t \lambda(t) \cdot \delta^t u(c_t)$. This minimal modification reinterprets anomalies not as irrationality or preference reversals, but as "structurally timed silences" in the utility function. By separating the discounting of value from the activation of utility, Shaded-ED offers a parsimonious, tractable, and empirically testable bridge between classical theory and observed intertemporal choice.**

***Index Terms*—Intertemporal Choice, Exponential Discounting, Present Bias, Structural Activation, Time Inconsistency, Behavioral Economics**

I. Introduction: The Silence in Time

Exponential discounting (ED) is valued for its time-consistent and analytically tractable representation of intertemporal choice. However, persistent empirical findings, such as present bias and preference reversals, have led to alternative models (e.g., hyperbolic discounting) that abandon ED's structural elegance. This paper argues that the issue may not lie in the discounting function itself, but in the unstated assumption that future utility is always evaluated. We propose a reinterpretation: what if behavioral anomalies arise not from how future utility is valued, but from *when* it is structurally permitted to be considered?

II. The Shaded Exponential Discounting (Shaded-ED) Model

We introduce a minimal, structure-preserving extension to the classical ED model. The core innovation is a **structural activation function**, $\lambda(t) \in [0,1]$, which acts as a gate for the realization of future utility.

A. The Conditionally Activated Utility Function

The standard ED utility function, $U = \sum_t \delta^t u(c_t)$, is reformulated as:

$$U = \sum_t \lambda(t) \cdot \delta^t u(c_t) \tag{1}$$

This formulation preserves the constant discount factor δ and the instantaneous utility function $u(c_t)$. The new term, $\lambda(t)$, represents the degree to which utility at time t is structurally activated or behaviorally relevant.

- If $\lambda(t) = 1$, future utility is fully realized and discounted exponentially.
- If $\lambda(t) = 0$, future utility is structurally silent and excluded from the agent's consideration, not due to its low value, but due to its inaccessibility.

This modification reframes anomalies as outcomes of non-evaluation rather than mis-evaluation.

III. Reinterpreting Intertemporal Anomalies

The Shaded-ED framework provides a unified structural explanation for key behavioral patterns without altering the core logic of exponential discounting.

- **Present Bias:** This is modeled as a selective activation profile where the present is always active ($\lambda(0) = 1$), while the future is often structurally suppressed ($\lambda(t) < 1$ for $t > 0$). The observed bias towards the present is therefore a consequence of the future's structural silence, not a different form of discounting.
- **Time Inconsistency:** Apparent preference reversals are explained as shifts in the activation profile, $\lambda(t)$, over time. An option that was structurally inaccessible at time t_0 may become activated at t_1, leading to a change in choice without any change in underlying preferences or the discount factor δ.
- **Hyperbolic-like Curvature:** The model can generate behavior that appears hyperbolic if the activation function $\lambda(t)$ declines in a convex, non-exponential manner over time. The observed curvature in discounting is thus attributed to the structure of activation, not the function of valuation.

IV. Conclusion: A Structure-Preserving Reinterpretation

Shaded-ED does not reject the exponential discounting framework; it provides a structural envelope that defines its operative domain. By introducing a conditional activation function, the model accounts for a wide range of intertemporal choice anomalies in a parsimonious and tractable way. It reframes behavioral patterns as coherent responses to a non-uniform temporal decision geometry, where the key question shifts from *how* agents discount the future to *when* the future is structurally allowed to count.

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