Chapter 3

Image Enhancement in the Spatial Domain

Preview

• **Objective** : Process an image to make it more suitable than the original for a "**specific**" application

↓

Problem-oriented

• **Methods**

- Spatial domain methods (chap.3)
  - Point processing
  - Mask processing
- Frequency domain methods (chap.4)

3.1 Background

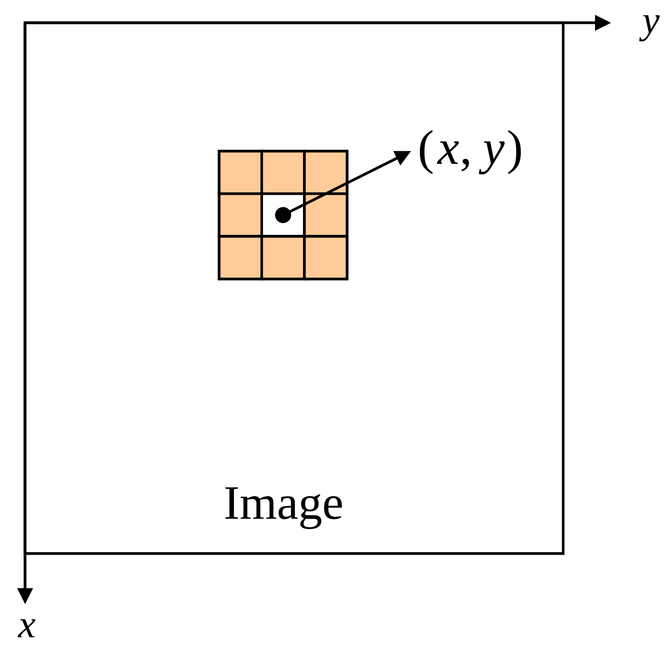
- Spatial domain
- Image processing function

$$g(x, y) = T[f(x, y)]$$

  - g(x,y): processed image
  - f(x,y): input image
  - T: operator

◇ Spatial Convolution Mask

- A 3×3 neighborhood about a point (x, y) in an image
  - Masks
  - Templates
  - Windows
  - Filters



◇ Simplest Form of T

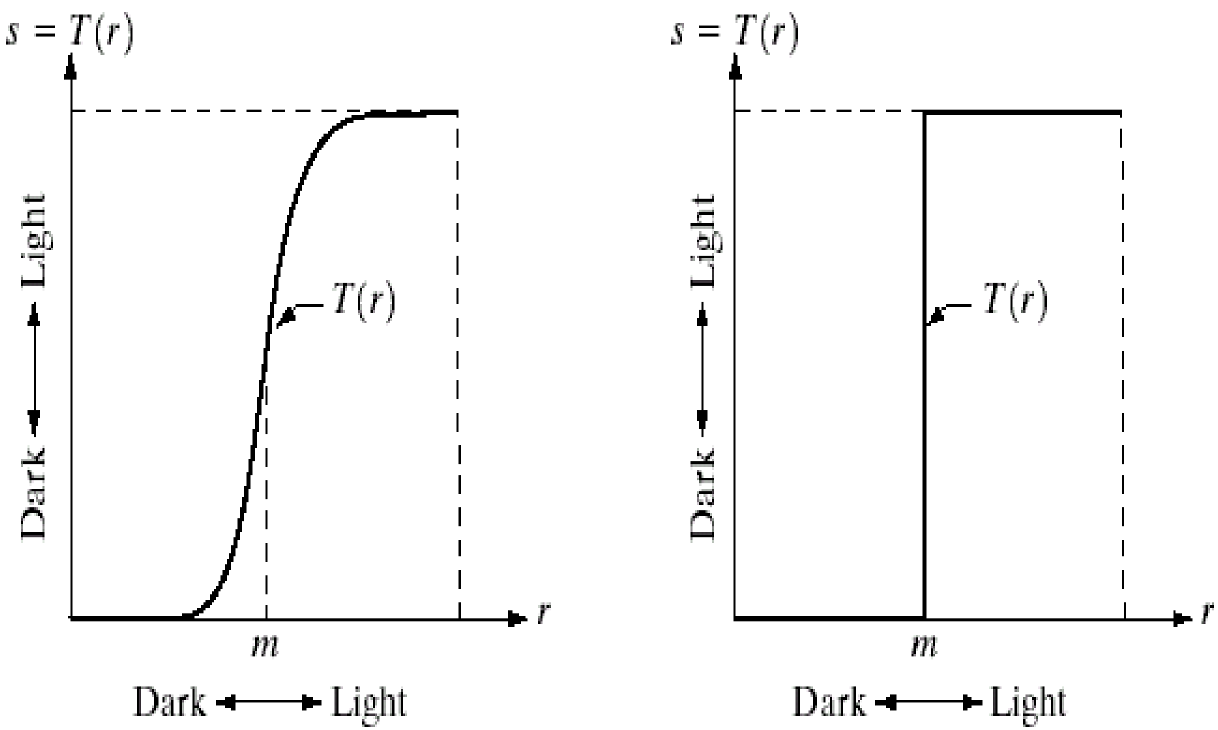
- Point processing
  - Gray-level transformation function / Mapping function
  - with 1×1 neighborhood

$$s = T[r]$$

- s: gray-level of g(x,y)
- r: gray-level of f(x,y)
- T: operator

◇ Example

– Contrast stretching



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

3.2 Some Basic Gray Level Transformations

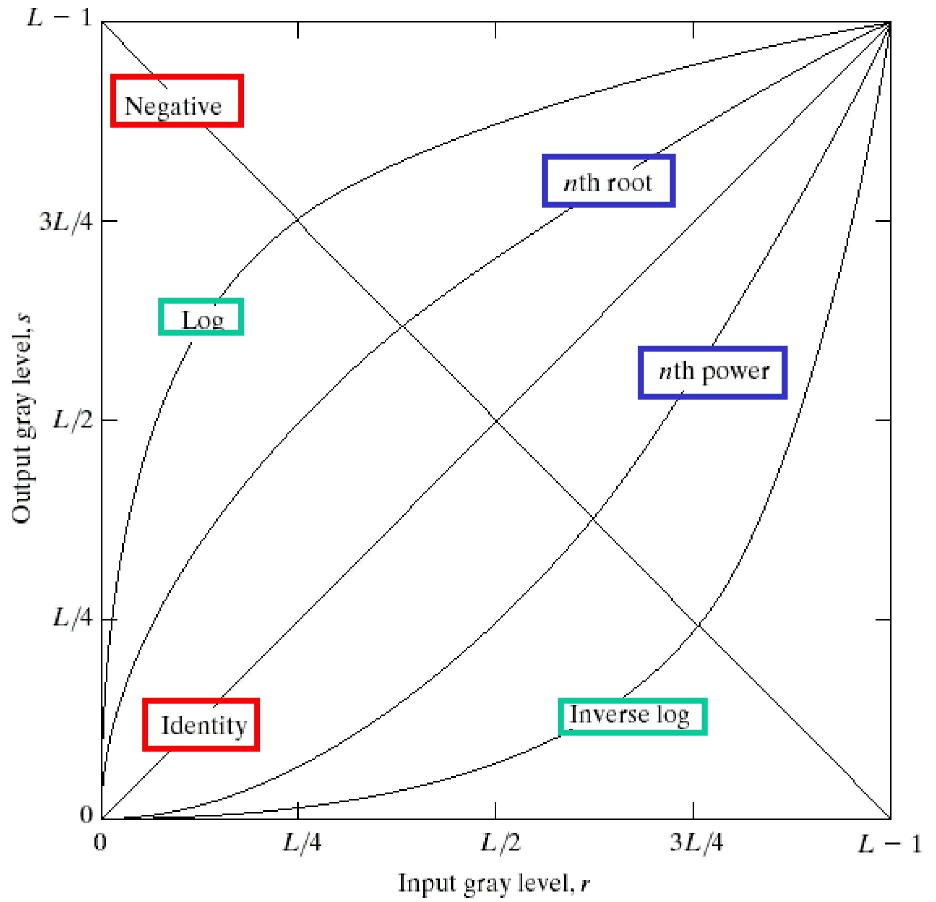
- Some simple image enhancement techniques

  - Linear (Negative and Identity)

  - Logarithmic (Log and Inverse Log)

  - Power-law (*n*th power and *n*th root)

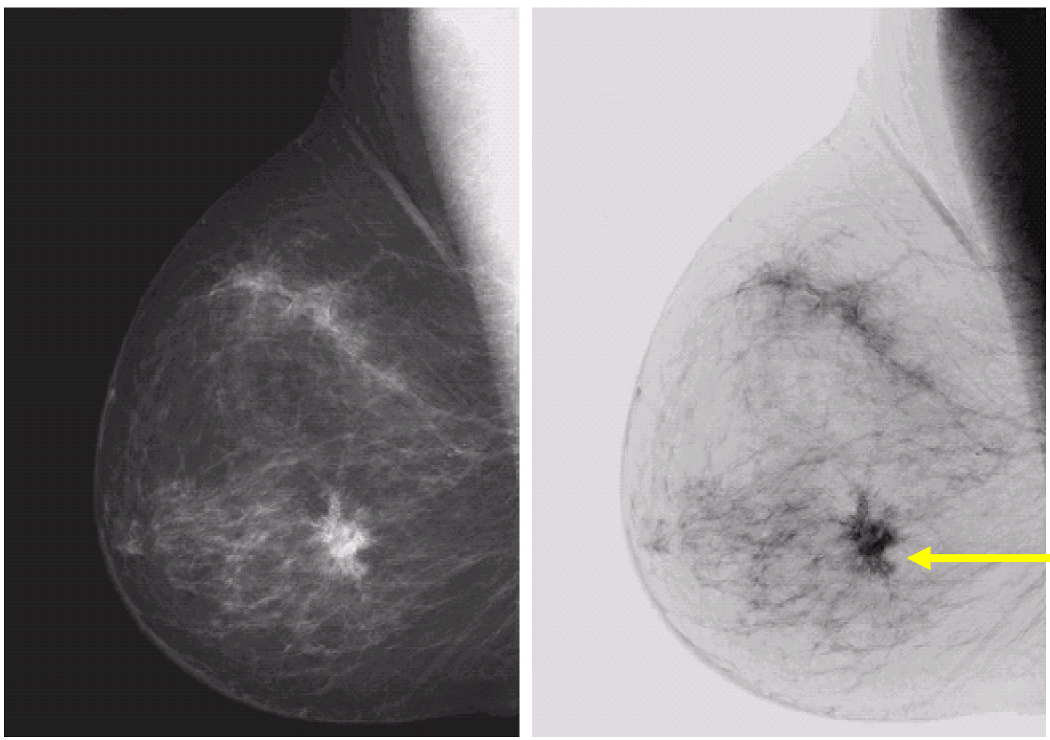
FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



3.2.1 Image Negatives

프로그램 실습(20분)

- Transformation function : $s = L - 1 - r$



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Lesion

3.2.2 Log Transformation

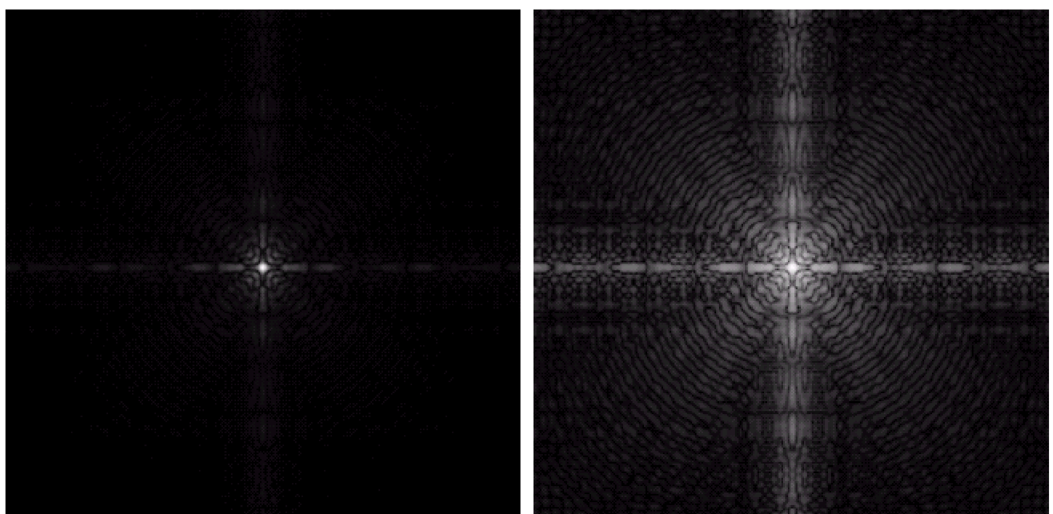
- General form

$$s = c \log(1 + |r|)$$

  – c: scaling constant (= 256 / 6.2)
- Spreading/Compressing of gray levels
- Example : Compressing of Fourier spectrum range
  - Fig.3.5(a) : [0 ~ 1.5 * 10^6] → Fig.3.5(b) : [0 ~ 6.2]

a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



3.2.3 Power-Law Transformations

- Basic form : $s = c\,r^{\gamma}$
  - c, γ : positive constant (if both are 1, then identity transform)
- Devices used for image capture, printing, and display respond according to a power law.

  - Intensity-to-voltage response in CRT is a power function with γ from 1.8 to 2.5.
- Gamma(γ) correction is used to correct the response.

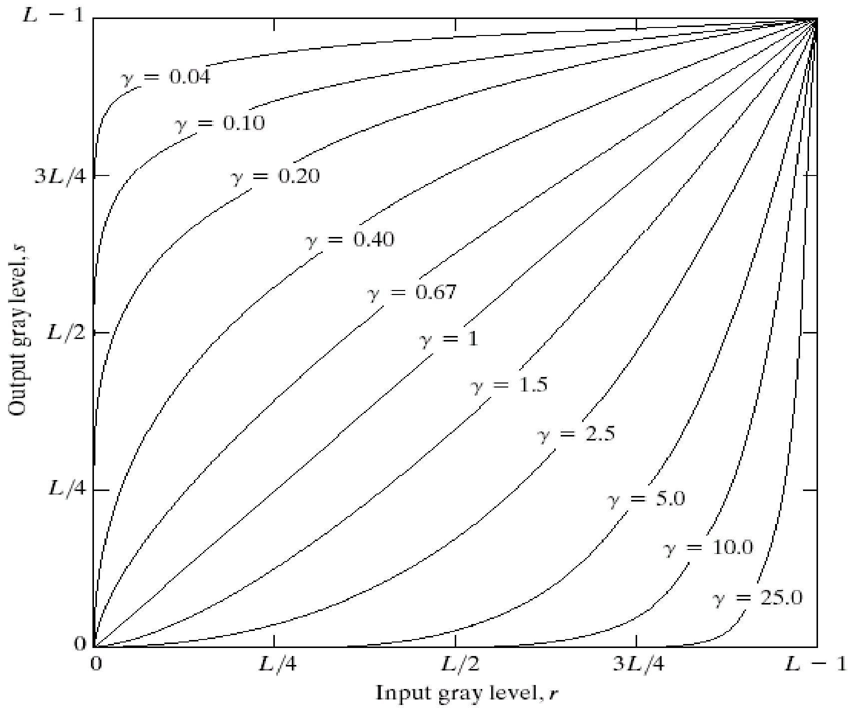


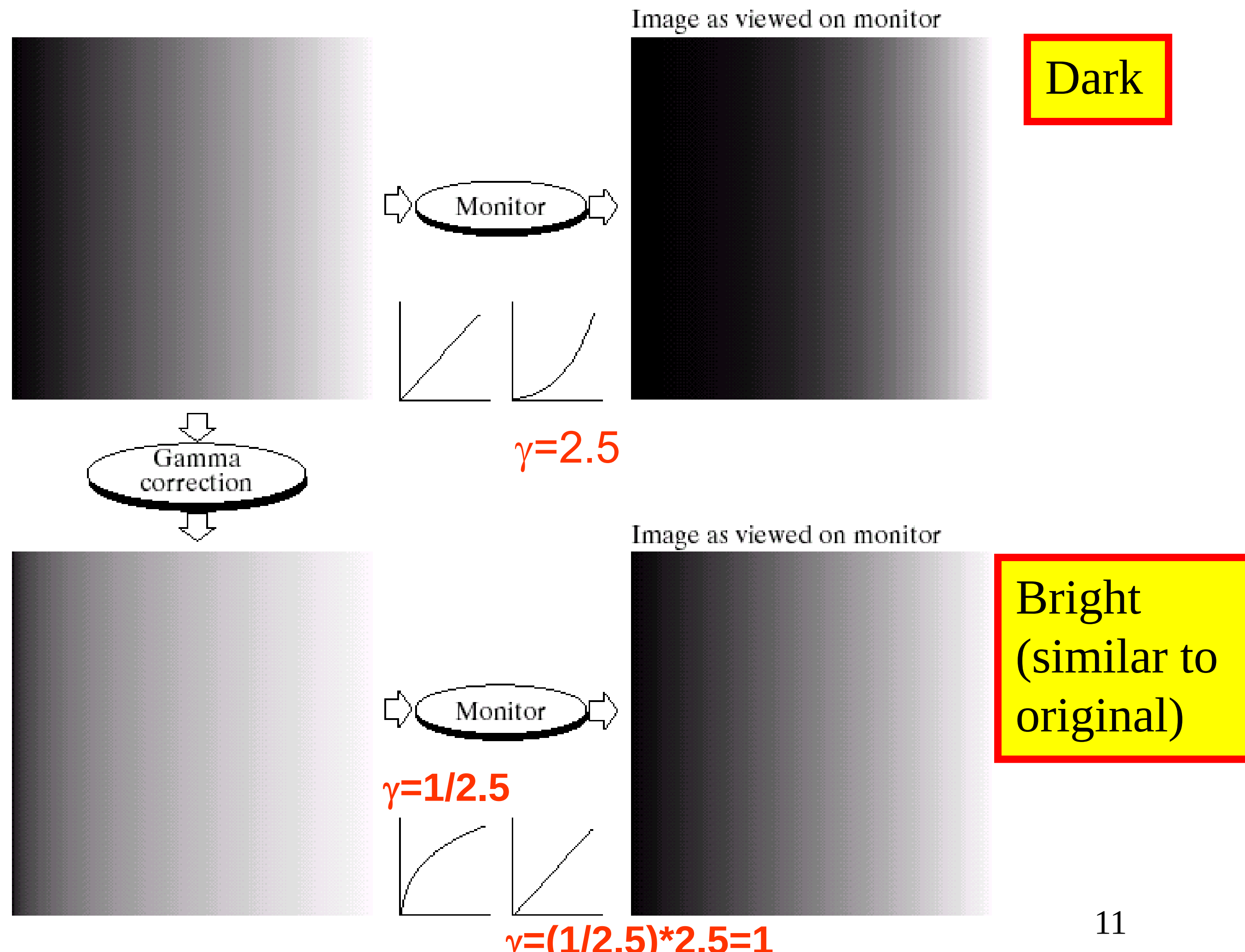
FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

◇ Example(1) : CRT Monitor

- $s = r^{\gamma}$

a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.



◇ Example(2) : Magnetic Resonance Image

- Human spine

Fig.3.8(a) : Original

Fig.3.8(b) : $\gamma = 0.6$

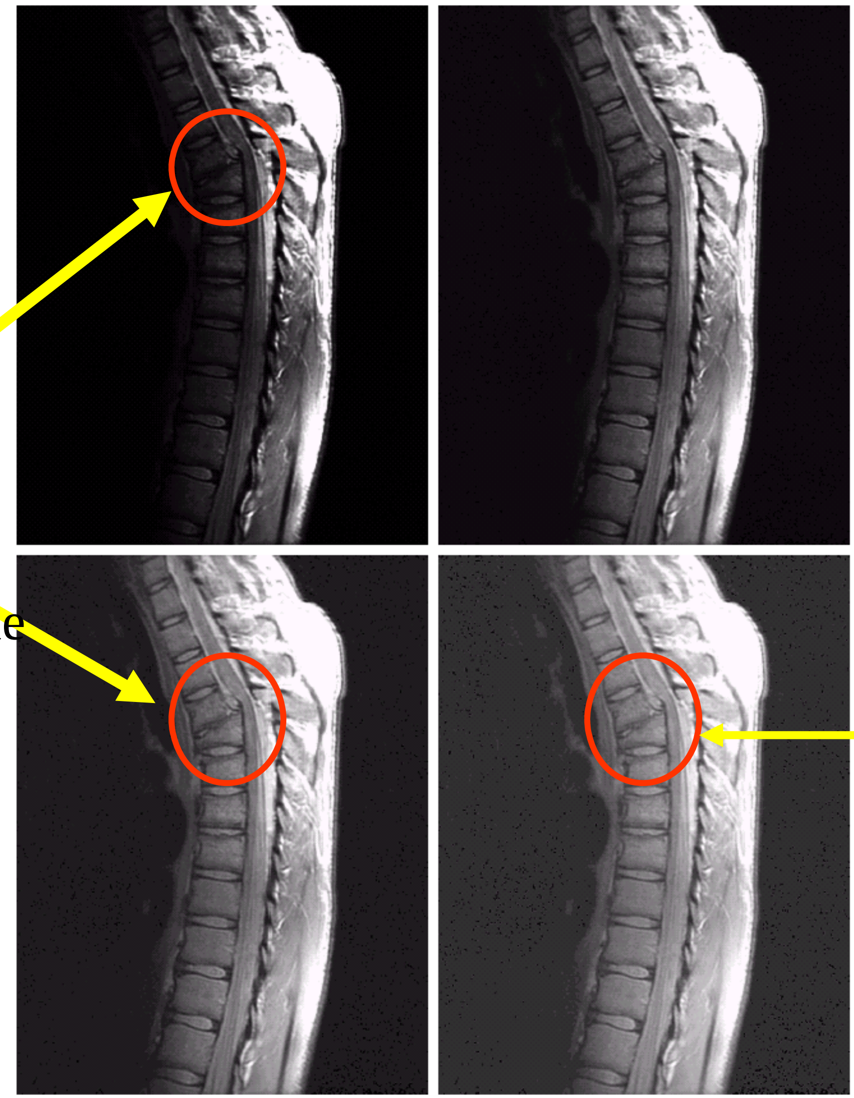
Fig.3.8(c) : $\gamma = 0.4$

Fig.3.8(d) : $\gamma = 0.3$

dark

Fracture dislocation

visible



a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Corrected too much (washed-out look)

◇ Example(3) : Aerial Image

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

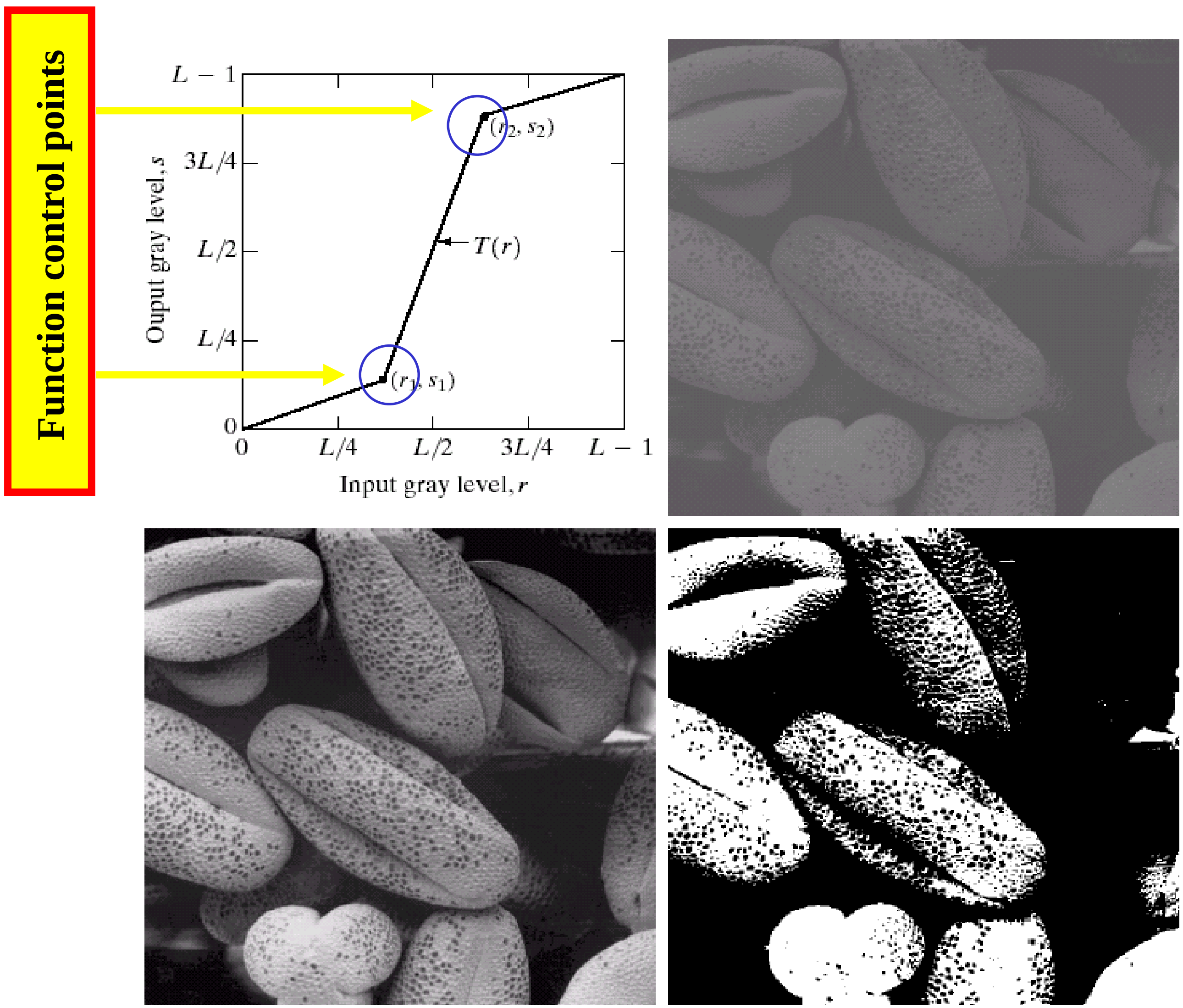
Too bright

More visible

Corrected too much

3.2.4 Piecewise-Linear Transformations

- Contrast stretching

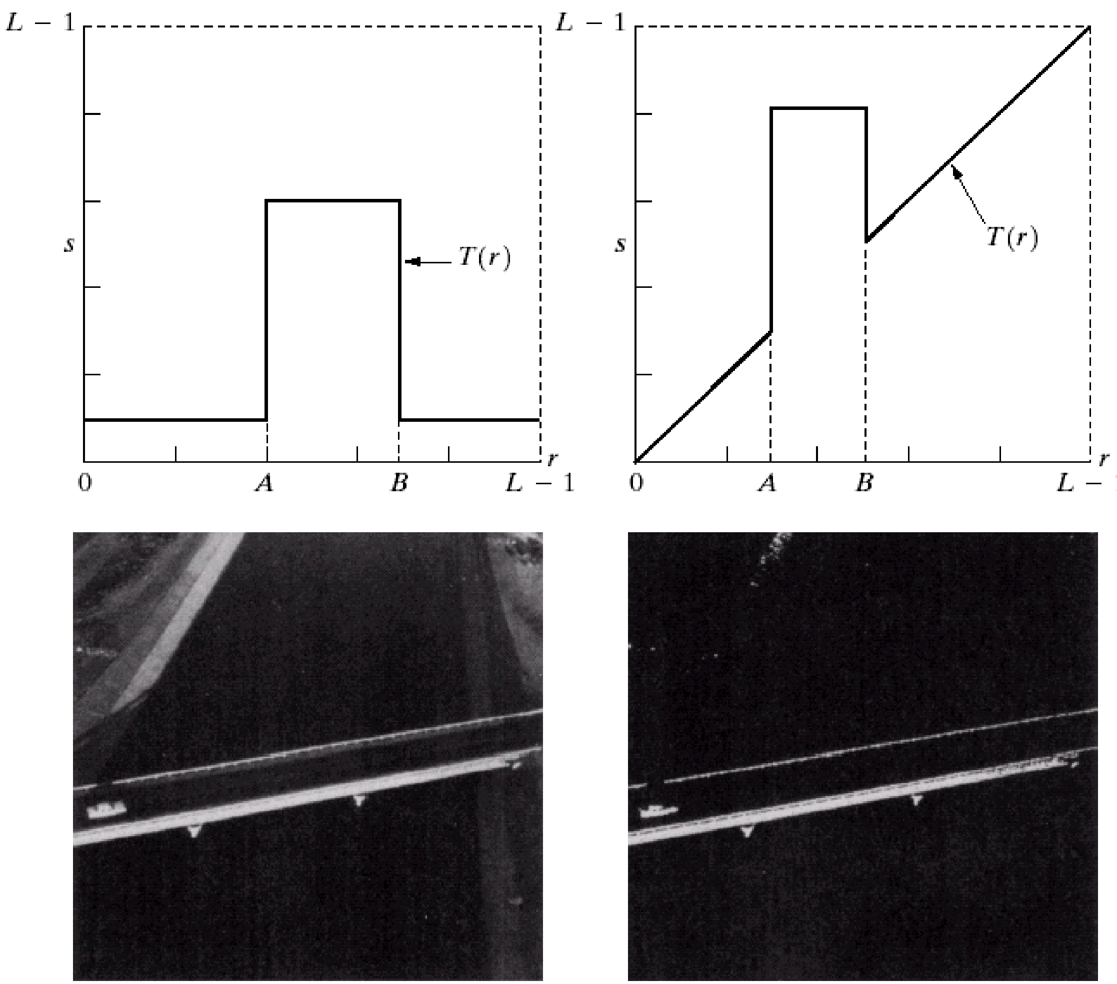


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

◇Gray-Level Slicing

- Intensity-Level slicing



a b
c d

FIGURE 3.11
(a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level.
(b) This transformation highlights range [A, B] but preserves all other levels.
(c) An image.
(d) Result of using the transformation in (a).

◇Bit-Plane Slicing (1)

- Bit-Plane representation
  - 8-bit Image

7	6	5	4	3	2	1	0

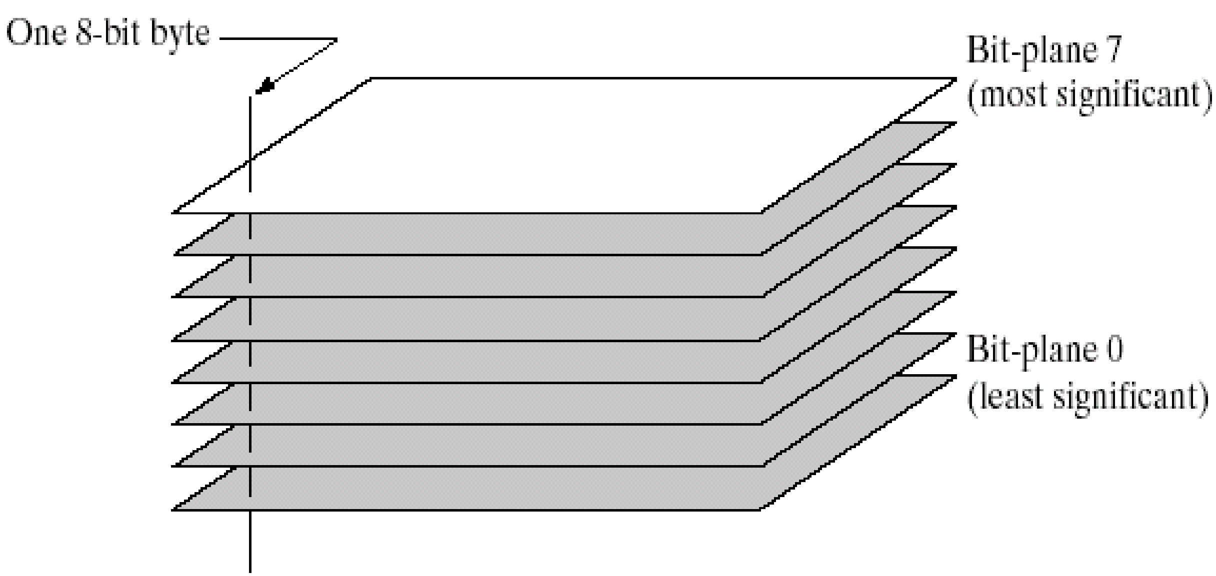


FIGURE 3.12 Bit-plane representation of an 8-bit image.

◇Bit-Plane Slicing (2)

- 8-bit fractal image

프로그램 숙제

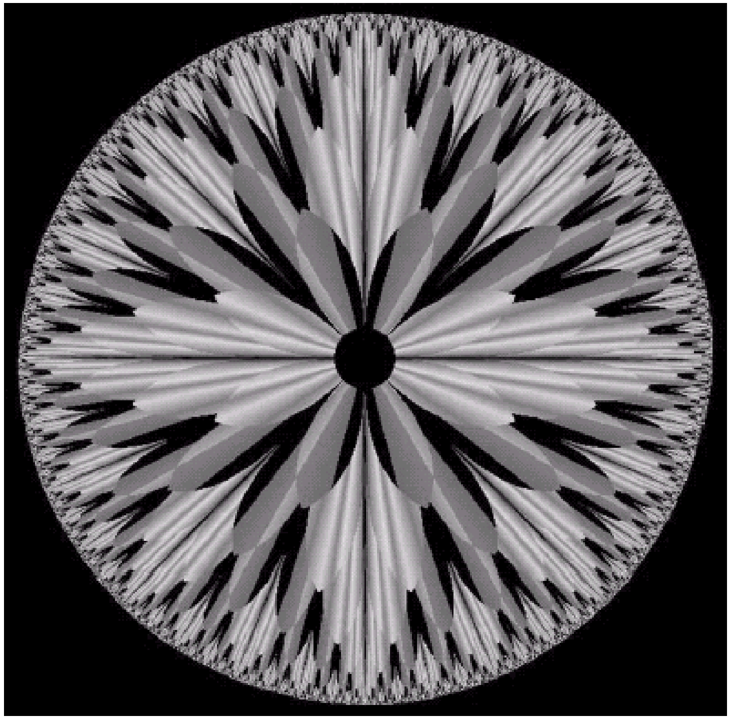


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

◇Bit-Plane Slicing (3)

- 8-bit planes

Bit plane 7

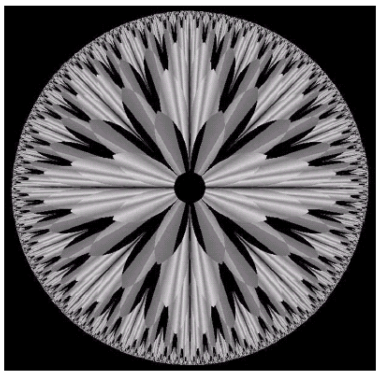
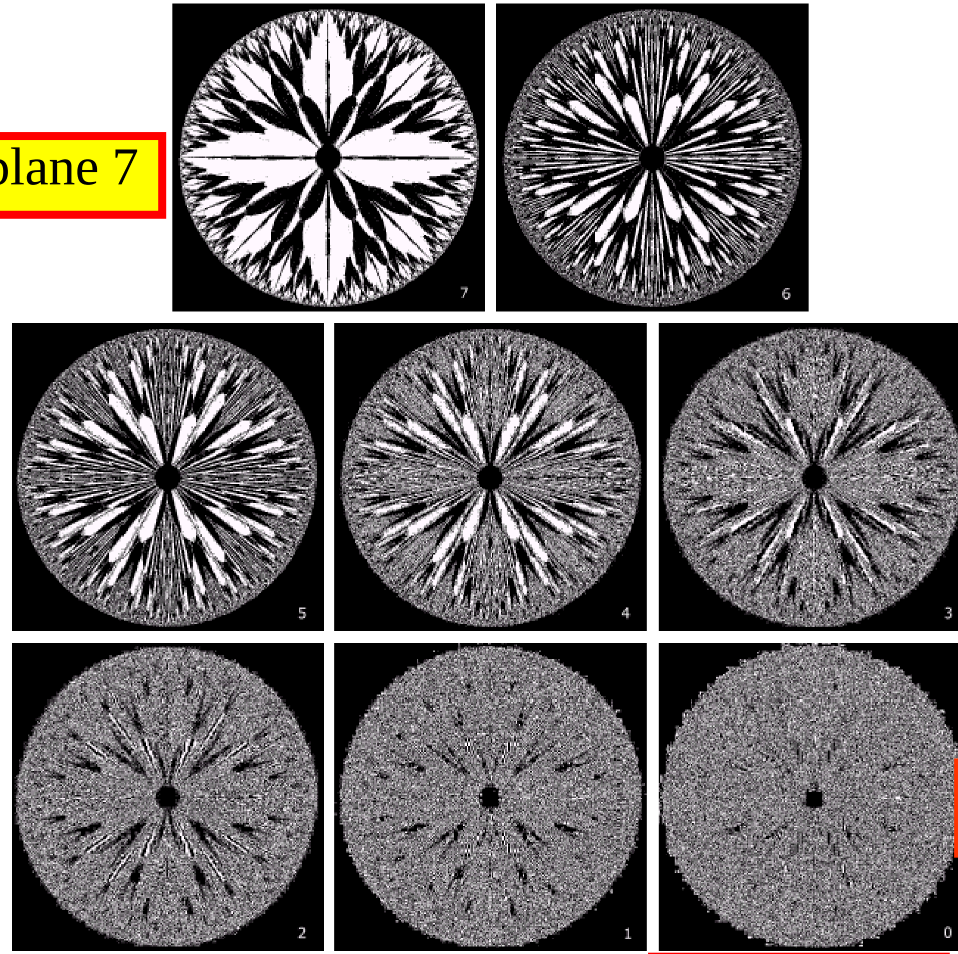


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

Original image



Random

FIGURE 3.14 The eight bit planes of the image in Fig. 3.1 ... n, right of each image identifies the bit plane.

Bit plane 0

What can you observe?

3.3 Histogram Processing

- Histogram equalization
- Histogram matching (specification)
- Local enhancement
- Use of histogram statistics for image enhancement

◇Histogram Types

프로그램 읽기

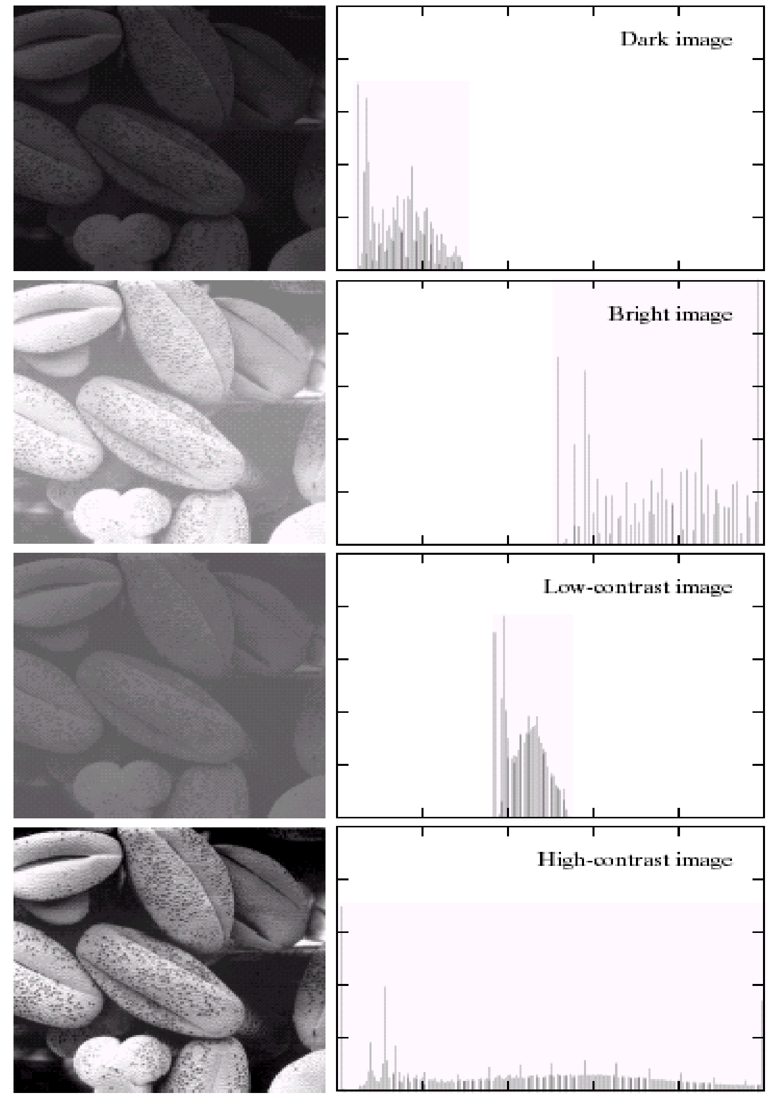
- Histogram : function for the number of pixels with gray-level r_k

$$h(r_k) = n_k$$

  - r_k : k th gray level, k = 0, 1, … , L-1
  - n_k : number of k th gray-level

- 4 Basic image types by histogram

  a) Dark
  b) Bright
  c) Low-Contrast
  d) High-Contrast



3.3.1 Histogram Equalization (1)

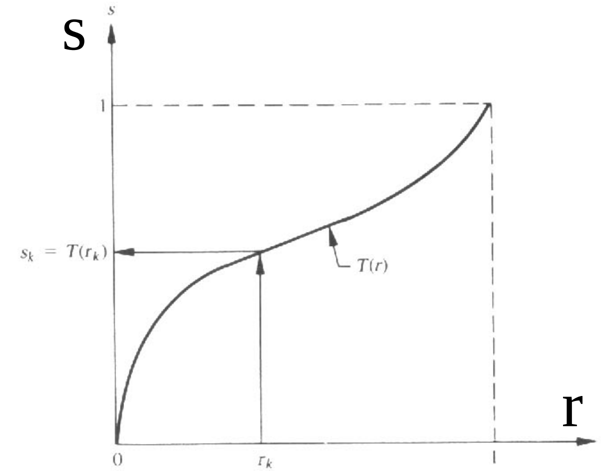
– Transformation function (Continuous function)

$$s = T(r) \Leftrightarrow r = T^{-1}(s)$$

r : Original gray levels

s : Transformed gray level

T : Transformation Function



- Conditions of transformation function T(r) at 0 ≤ r ≤ 1

(a) Single-valued and monotonically increasing

(b) 0 ≤ T(r) ≤ 1

◇Histogram Equalization (2)

– Probability Density Function (PDF)

$$p_s(s) = p_r(r)\,\frac{dr}{ds}\bigg|_{r=T^{-1}(s)}$$

$p_r(r)$: PDF of r

$p_s(s)$: PDF of s

– Suppose

$$s = T(r) = \boxed{\int_0^r p_r(w)\,dw}$$

CDF(cumulative density function) of *r*

then

$$\frac{ds}{dr} = \boxed{p_r(r)}$$

$$\therefore \quad p_s(s) = \boxed{p_r(r)\,\frac{1}{p_r(r)} = 1}$$

"Uniform density"

☞ Histogram equalization
Histogram linearization
Histogram flattening

◇Histogram Equalization (3)

- Example : Uniform density transformation function

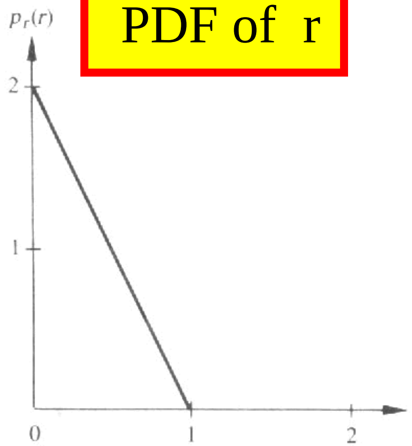
$$p_r(r) = \begin{cases} -2r+2 & 0 \le \mathrm{r} \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = T(r) = \int_0^r (-2w+2)\,dw$$
$$= -r^2 + 2r$$

$$r = T^{-1}(s) = 1 - \sqrt{1-s} \qquad (0 \le r \le 1)$$

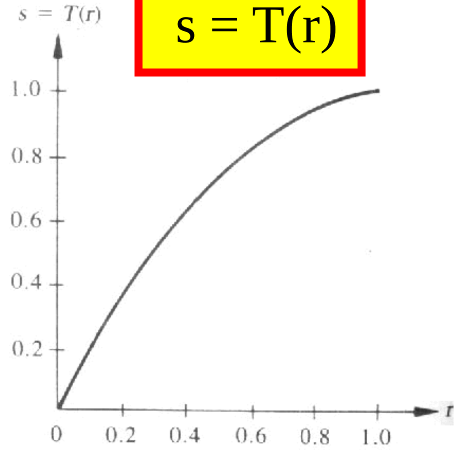
$$p_s(s) = \left[p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[(-2r+2) \frac{dr}{ds} \right]_{r=1-\sqrt{1-s}}$$
$$= \left[(2\sqrt{1-s}) \frac{d}{ds} (1-\sqrt{1-s}) \right] = 1 \qquad (0 \le s \le 1)$$

PDF of r



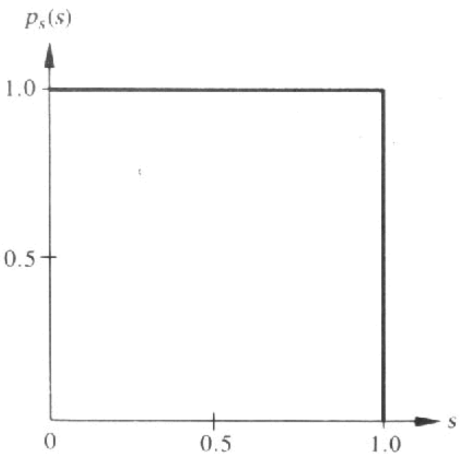
(a)

s = T(r)



(b)

PDF of s



(c)

◇ Histogram Equalization (4)

– For discrete function

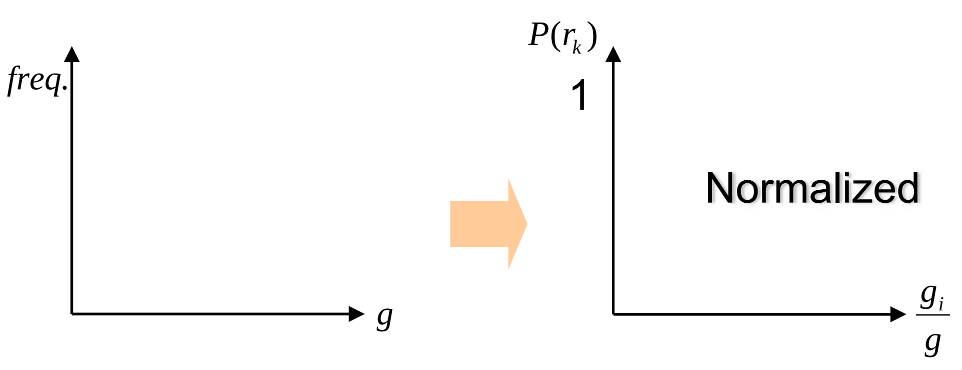
– Histogram : function for the number of pixels with gray-level r_k

$$h(r_k) = n_k$$

– Probability of occurrence of gray-level r_k

$$p_r(r_k) = n_k / n$$

- r_k : k th gray level, k = 0, 1, … , L-1
- n_k : number of k th gray-level
- n : total number of pixels

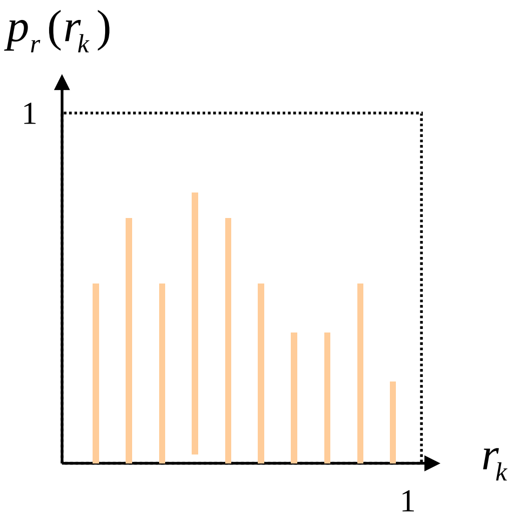


◇Histogram Equalization (5)

- Histogram linearization / Histogram flattening
  - Histogram: Plot of $P_r(r_k)$ versus r_k

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n} \quad (0 \le r \le 1 \text{ and } k = 0, 1, \ldots, \mathrm{L}-1)$$



$$r_k = T^{-1}(s_k) \quad (0 \le s_k \le 1)$$

◇Histogram Equalization (6)

- Example

- 64×64, 3bit (8 gray-level)

r_k	n_k	$P_r(r_k)=n_k/n$
$r_0=0$	790	0.19
$r_1=1/7$	1023	0.25
$r_2=2/7$	850	0.21
$r_3=3/7$	656	0.16
$r_4=4/7$	329	0.08
$r_5=5/7$	245	0.06
$r_6=6/7$	122	0.03
$r_7=1$	81	0.02

Transformation function

$$s_0 = T(r_0) = \sum_{j=0}^{0} p_r(r_j)$$

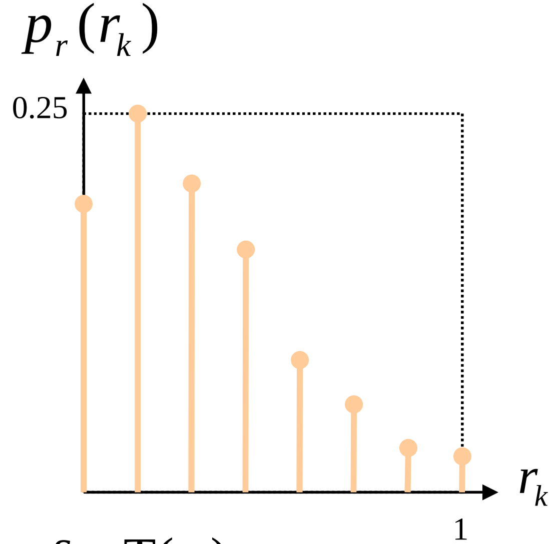
$$= p_r(r_j) = 0.19$$

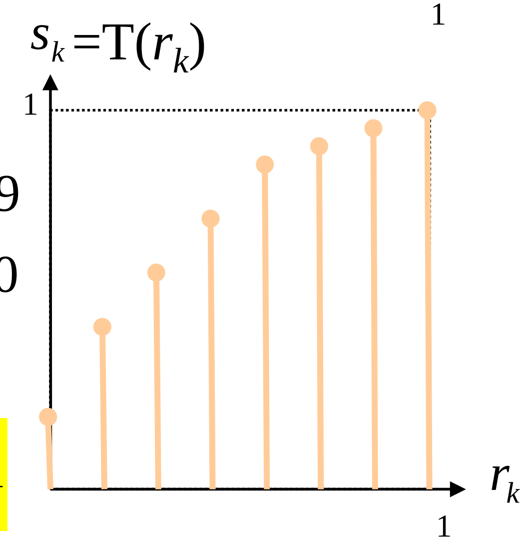
$$s_1 = T(r_1) = \sum_{j=0}^{1} p_r(r_j)$$

$$= p_r(r_0) + p_r(r_1) = 0.44$$

$$s_2 = 0.65, \quad s_3 = 0.81, \quad s_4 = 0.89$$

$$s_5 = 0.95, \quad s_6 = 0.98, \quad s_7 = 1.00$$





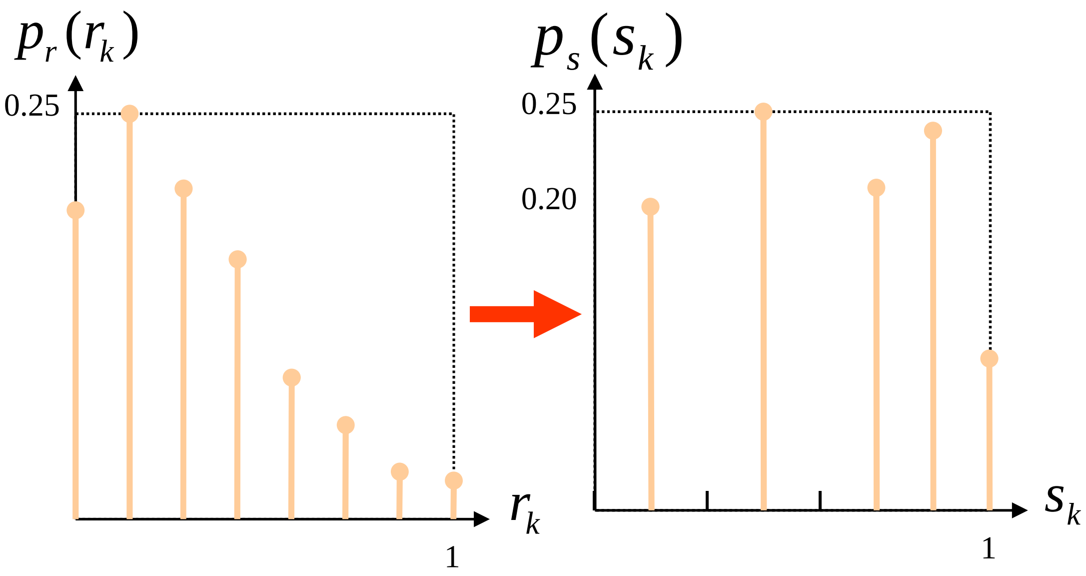
$$s_0 \approx \frac{1}{7}, \quad s_1 \approx \frac{3}{7}, \quad s_2 \approx \frac{5}{7}, \quad s_3 \approx \frac{6}{7}, \quad s_4 \approx \frac{6}{7}, \quad s_5 \approx 1, \quad s_6 \approx 1, \quad s_7 = 1$$

◇Histogram Equalization (7)

– Approximation of flat histogram

$$s_0 \approx \frac{1}{7},\quad s_1 \approx \frac{3}{7},\quad s_2 \approx \frac{5}{7},\quad s_3 \approx \frac{6}{7},\quad s_4 \approx \frac{6}{7},\quad s_5 \approx 1,\quad s_6 \approx 1,\quad s_7 = 1$$

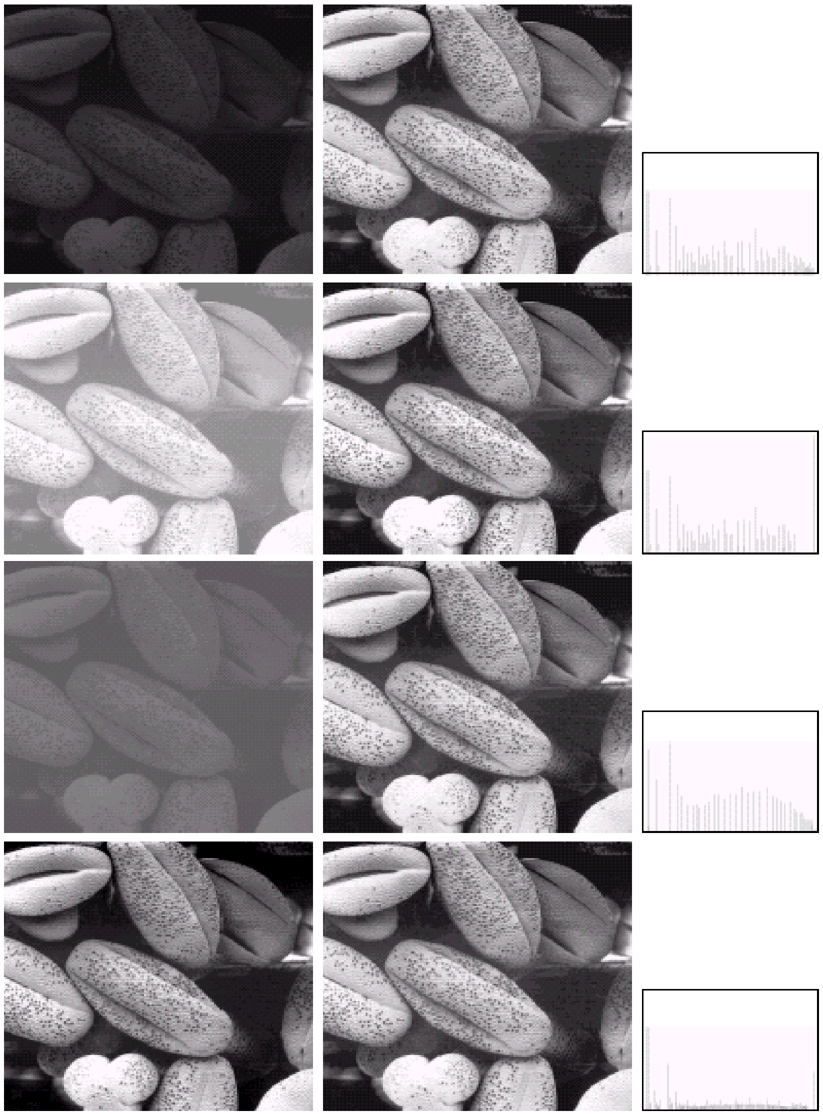
$$\begin{aligned} \therefore\quad & r_0 \rightarrow 0.19\,(s_0 = 1/7) \\ & r_1 \rightarrow 0.25\ (s_1 = 3/7) \\ & r_2 \rightarrow 0.21\ (s_2 = 5/7) \\ & r_3, r_4 \rightarrow 0.24\ (s_3 = 6/7) \\ & r_5, r_6, r_7 \rightarrow 0.11\ (s_4 = 1) \end{aligned}$$



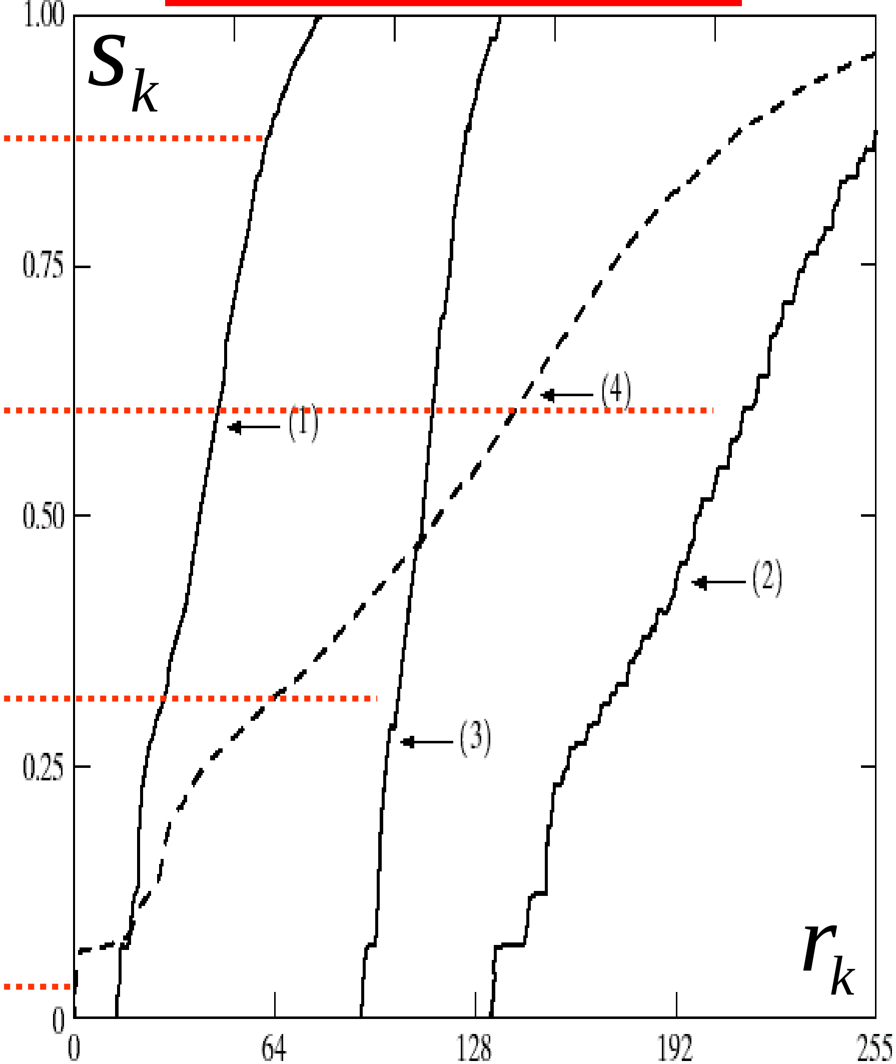
◇Histogram Equalization (8)

프로그램 읽기

(a) Original (b) Equalized (c) Hist.



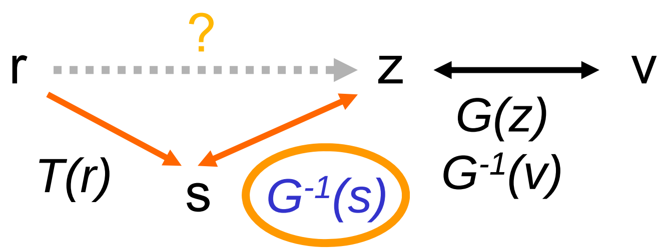
Transformation function



3.3.2 Histogram Matching (Specification)(1)

◇ Development of the method

- Directly specified histogram
- User-defined histogram



r : gray-level of original image
s : histogram eq. of r
z : specified histogram
v : histogram eq. of specified histogram

- r, z : given
- T(r), G(z) : can be computed
- s & v should be the same theoretically (∵ Flattened histogram)

- For histogram specification
  Use known $G^{-1}(s)$ instead of $G^{-1}(v)$

◇ Histogram Matching (2)

- Transformation function (Continuous)

$$s = T(r) = \int_0^r p_r(w)dw \qquad p_r(r) \text{ : computed from input image}$$

$$G(z) = \int_0^r p_z(t)dt = s \qquad p_z(z) \text{ : specified}$$

- Mapping function

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

◇ Histogram Matching (3)

- Transformation function (Discrete)

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \qquad \mathrm{k} = 0,1,2,\ldots,\mathrm{L}\text{ -1}$$

$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j) = s_k \qquad \mathrm{k} = 0,1,2,\ldots,\mathrm{L}\text{ -1}$$

- Mapping function

$$z_k = G^{-1}[T(r_k)] \;\; \text{or} \;\; z_k = G^{-1}[s_k] \qquad \mathrm{k} = 0,1,2,\ldots,\mathrm{L}\text{ -1}$$

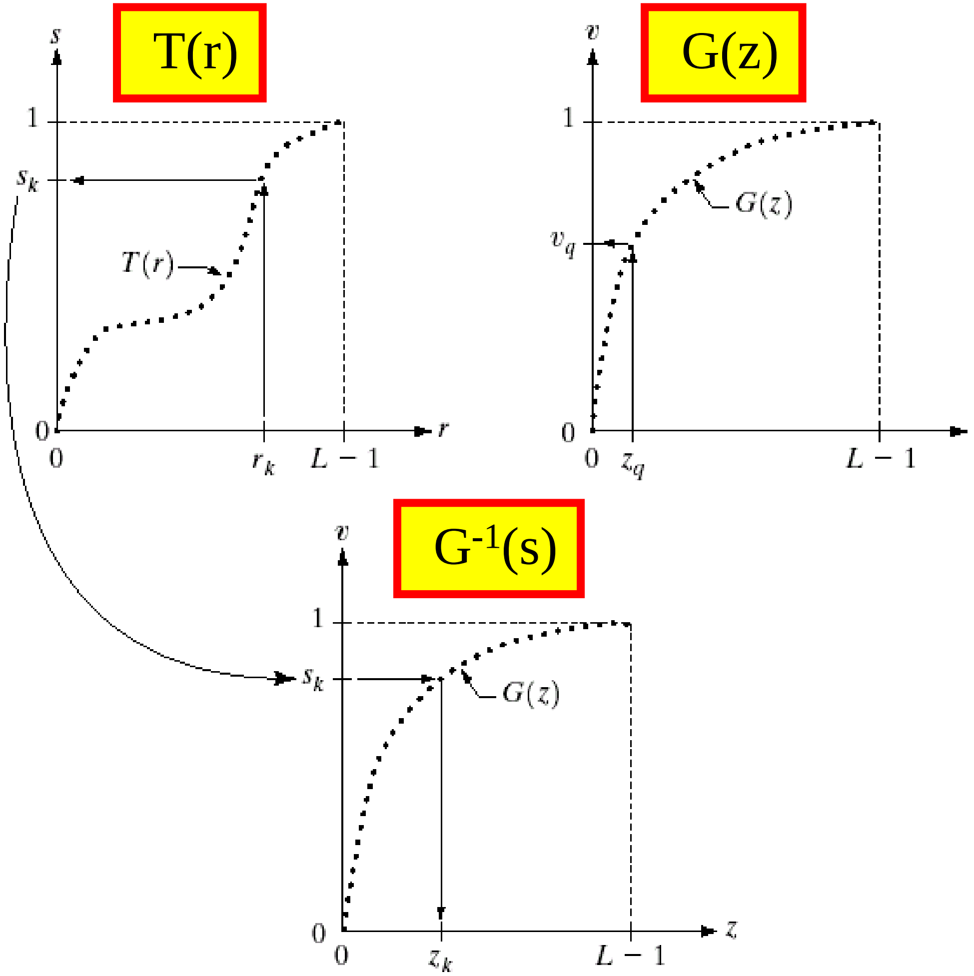
◇ Histogram Matching (4)

- Implementation

- Obtain histogram of the given image
- Precompute mapped level s_k for r_k
- Obtain G(z) from the given $p_z(z)$
- Precompute z_k for s_k
- Map s_k into the final z_k

$\hat{z}$ is the smallest integer such that

$$(G(\hat{z}) - s_k) \geq 0 \quad k = 0,1,2,\ldots,L-1$$

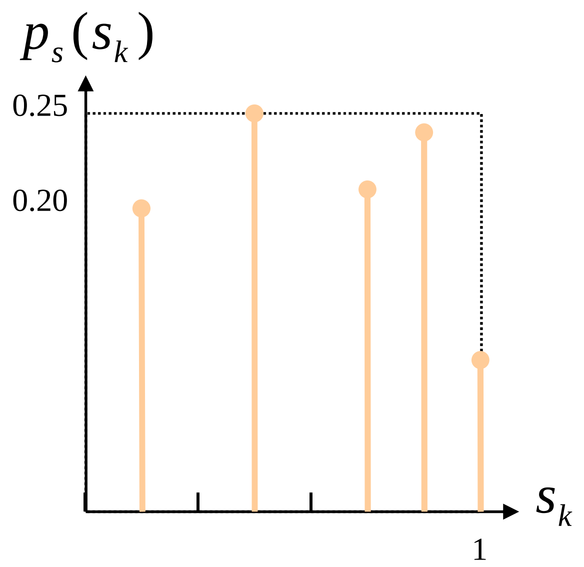


◇Histogram Matching (5)

- Example

  - r : same as before

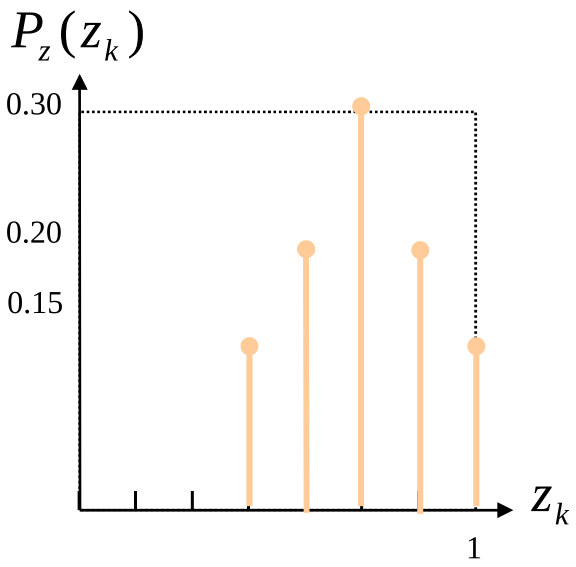
$r_j \rightarrow s_k$	n_k	$P_s(s_k)$
$r_0 \rightarrow s_0 = 1/7$	790	0.19
$r_1 \rightarrow s_1 = 3/7$	1023	0.25
$r_2 \rightarrow s_2 = 5/7$	850	0.21
$r_3, r_4 \rightarrow s_3 = 6/7$	985	0.24
$r_5, r_6, r_7 \rightarrow s_4 = 1$	448	0.11



◇Histogram Matching (6)

- Specified histogram

z_k	$P_z(z_k)$
$z_0=0$	0.00
$z_1=1/7$	0.00
$z_2=2/7$	0.00
$z_3=3/7$	0.15
$z_4=4/7$	0.20
$z_5=5/7$	0.30
$z_6=6/7$	0.20
$z_7=1$	0.15



◇Histogram Matching (7)

- *G*

$$v_0 = G(z_0) = 0.00 \quad v_4 = G(z_4) = 0.35$$
$$v_1 = G(z_1) = 0.00 \quad v_5 = G(z_5) = 0.65$$
$$v_2 = G(z_2) = 0.00 \quad v_6 = G(z_6) = 0.85$$
$$v_3 = G(z_3) = 0.15 \quad v_7 = G(z_7) = 1.00$$

- *G*$^{-1}$

$$s_0 = 1/7 \approx G(z_3) \qquad \therefore \quad G^{-1}(s_0) = z_3 = 3/7$$
$$s_1 = 3/7 \approx G(z_4) \qquad G^{-1}(s_1) = z_4 = 4/7$$
$$s_2 = 5/7 \approx G(z_5) \qquad G^{-1}(s_2) = z_5 = 5/7$$
$$s_3 = 6/7 \approx G(z_6) \qquad G^{-1}(s_3) = z_6 = 6/7$$
$$s_4 = 1 \quad \approx G(z_7) \qquad G^{-1}(s_4) = z_7 = 1.0$$

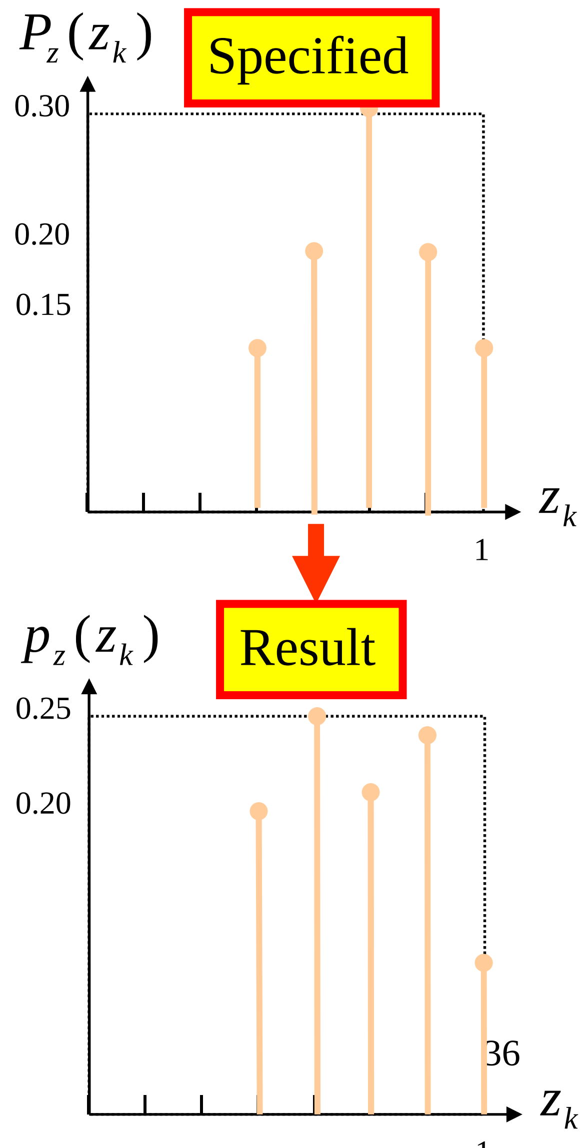
◇Histogram Matching (8)

- Specified histogram

z_k	$P_z(z_k)$
$z_0=0$	0.00
$z_1=1/7$	0.00
$z_2=2/7$	0.00
$z_3=3/7$	0.15
$z_4=4/7$	0.20
$z_5=5/7$	0.30
$z_6=6/7$	0.20
$z_7=1$	0.15

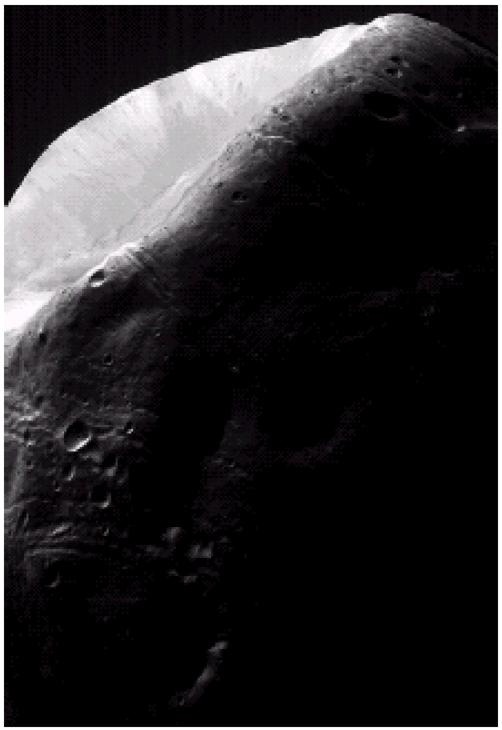
- Result

z_k	n_k
$z_0=0$	0
$z_1=1/7$	0
$z_2=2/7$	0
$z_3=3/7$	790
$z_4=4/7$	1023
$z_5=5/7$	850
$z_6=6/7$	985
$z_7=1$	448

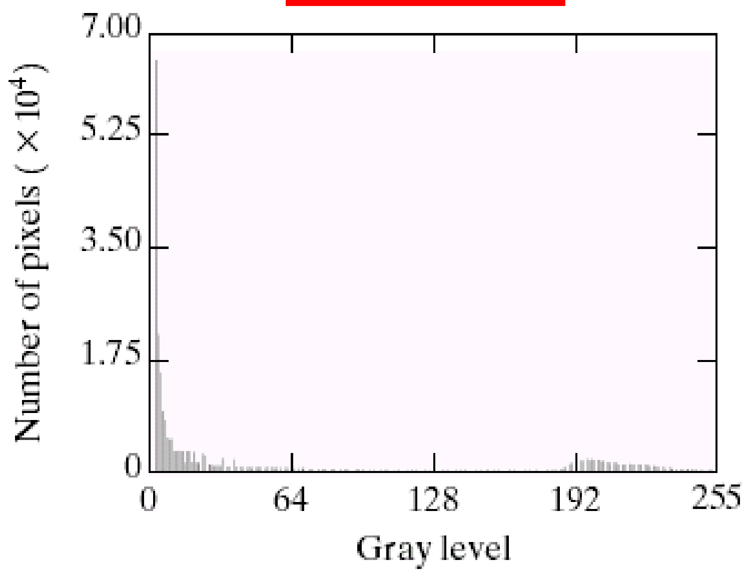


◇Histogram Matching (9)

Original image



Histogram

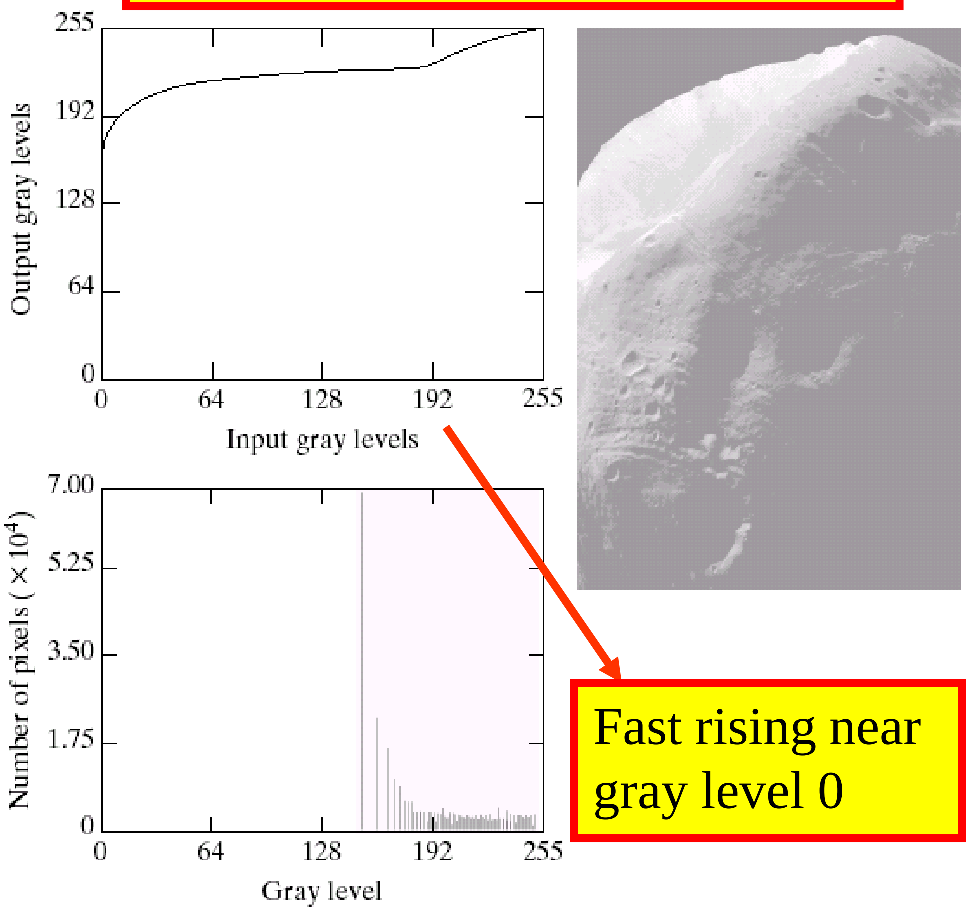


a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

◇Histogram Matching (10)

Results of histogram equalization



a b
c

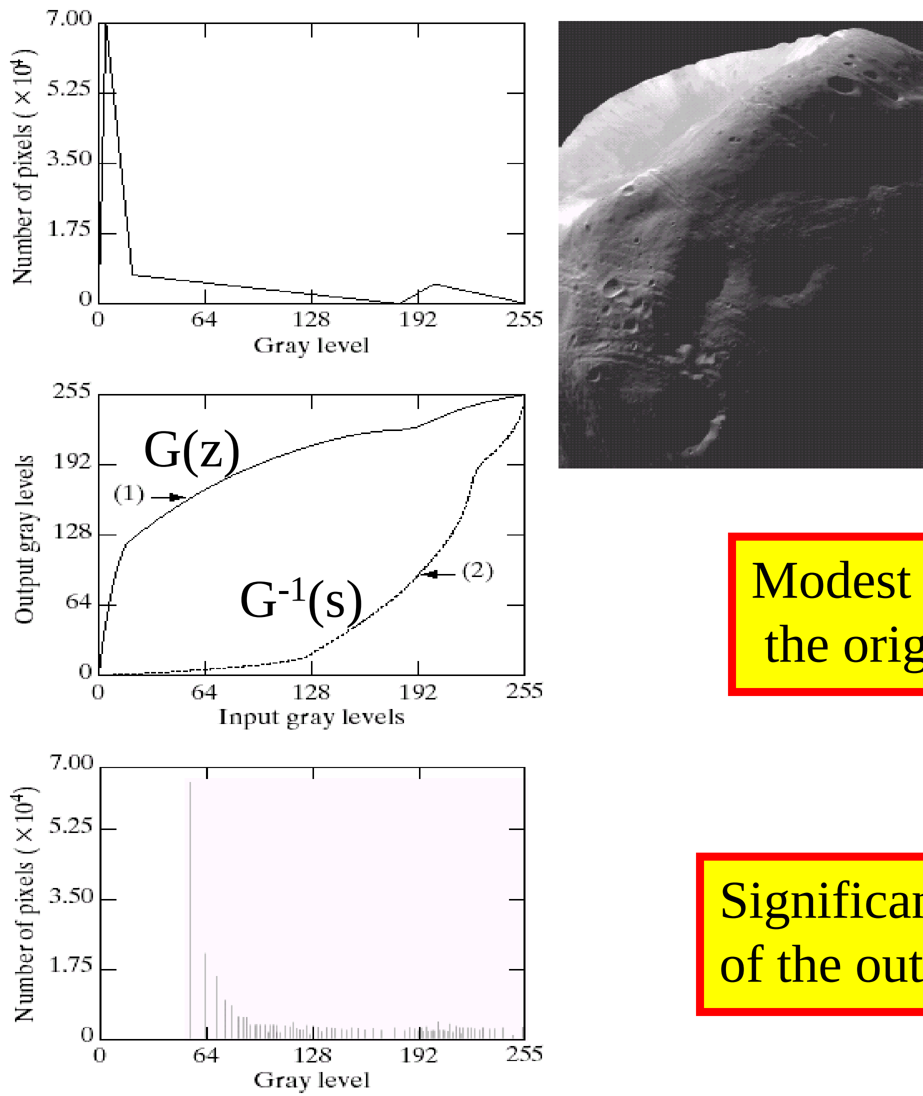
FIGURE 3.21
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

Fast rising near gray level 0 → Narrow interval of the gray scale

◇Histogram Matching (11)

a	c
b	
d	

FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).



Modest change of the original histogram

Significant improvement of the output image

3.3.3 Local Enhancement (1)

– Type I

  For each pixel *P*,

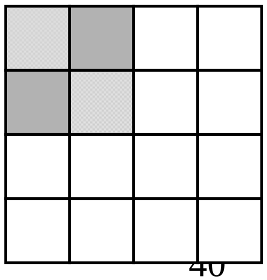
  - take local neighborhood
  - compute local histogram
  - obtain equalization mapping
  - change gray-value of *P* to mapped gray-value by obtained function

    ▷ *much computation time*

– Type II

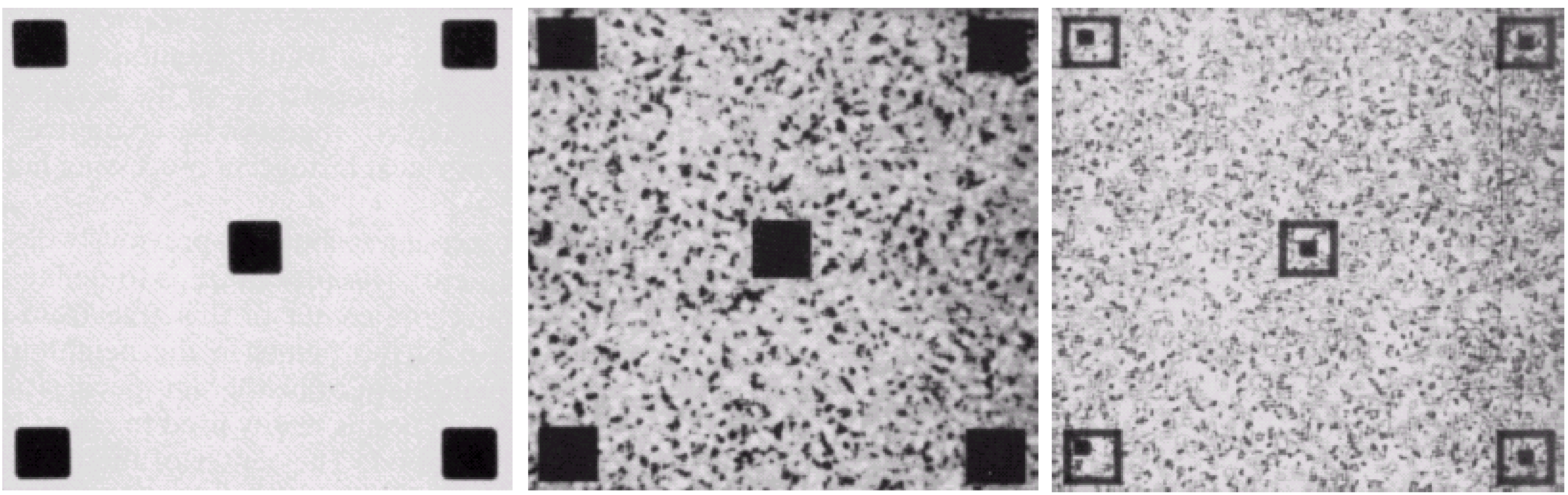
  - Divide N*N image M m*m non-overlapping regions
  - Compute M local histograms
  - Equalize M regions independently

    ▷ *less computation time, but checkerboard Effect*



◇ Local Enhancement (2)

a) Original : Slightly blurred to reduce noise
b) Global HE : No new structural detail,
considerable enhancement of noise
c) Local HE (7×7) : New structural detail, finer noise texture



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

3.3.4 Use of Histogram Statistics for Image Enhancement (1)

- Use statistical parameters instead of histogram itself
- Statistical parameters : Obtainable from histogram
  - Intensity mean
  - Variance (or Standard deviation)

- *n*th Moment of *r*

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \, p(r_i)$$

r_i : *i*th Gray Level

$p(r_i)$: Prob. of Occurrence of r_i

- Mean value

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i)$$

- Second moment : Standard deviation or variance, $\sigma^2(r)$

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 \, p(r_i)$$

◇ Use of Histogram Statistics for Image Enhancement (2)

- Mean

  - A Measure of average gray level

- Variance

  - A Measure of average contrast

- Global mean and variance

  - Adjust overall intensity and contrast

- Local mean and variance

  - More powerful in local enhancement,

  - Bases for changing local characteristics

◇ Use of Histogram Statistics for Image Enhancement (3)

- Subimage of specified size : S_{xy}
- Local mean

$$m_{s_{xy}} = \sum_{(s,t)\in s_{xy}} r_{s,t}\, p(r_{s,t})$$

$r_{s,t}$: Gray Level at Coordinates (s, t)

$p(r_{s,t})$: Normalized Histogram Component

- Local variance

$$\sigma^2_{s_{xy}} = \sum_{(s,t)\in s_{xy}} [r_{s,t} - m_{s_{xy}}]^2\, p(r_{s,t})$$

- Important aspect

  - Provide flexibility in developing simple, yet powerful enhancement techniques

◇ Use of Histogram Statistics for Image Enhancement (4)

- Image of a tungsten filament
- Filament in the center : Quite clear
- Another filament in the right side : Much darker
- Local enhancement of hidden features
  - Enhance only dark areas
  - Unchange the light areas

프로그램 숙제

◇ Use of Histogram Statistics for Image Enhancement (5)

- A Local enhancement method

  - Definition

    Local and global mean : $m_{s_{xy}}, M_G$

    Local and global variance : $\sigma_{s_{xy}}, D_G$

    Original and processed image : $f(x, y), g(x, y)$

  - Select dark areas with low contrast

    Dark area if $m_{s_{xy}} \leq k_0 M_G, 0 < k_0 < 1.0$

    Low contrast if $k_1 D_G \leq \sigma_{s_{xy}} \leq k_2 D_G, 0 < k_1 < k_2 < 1.0$

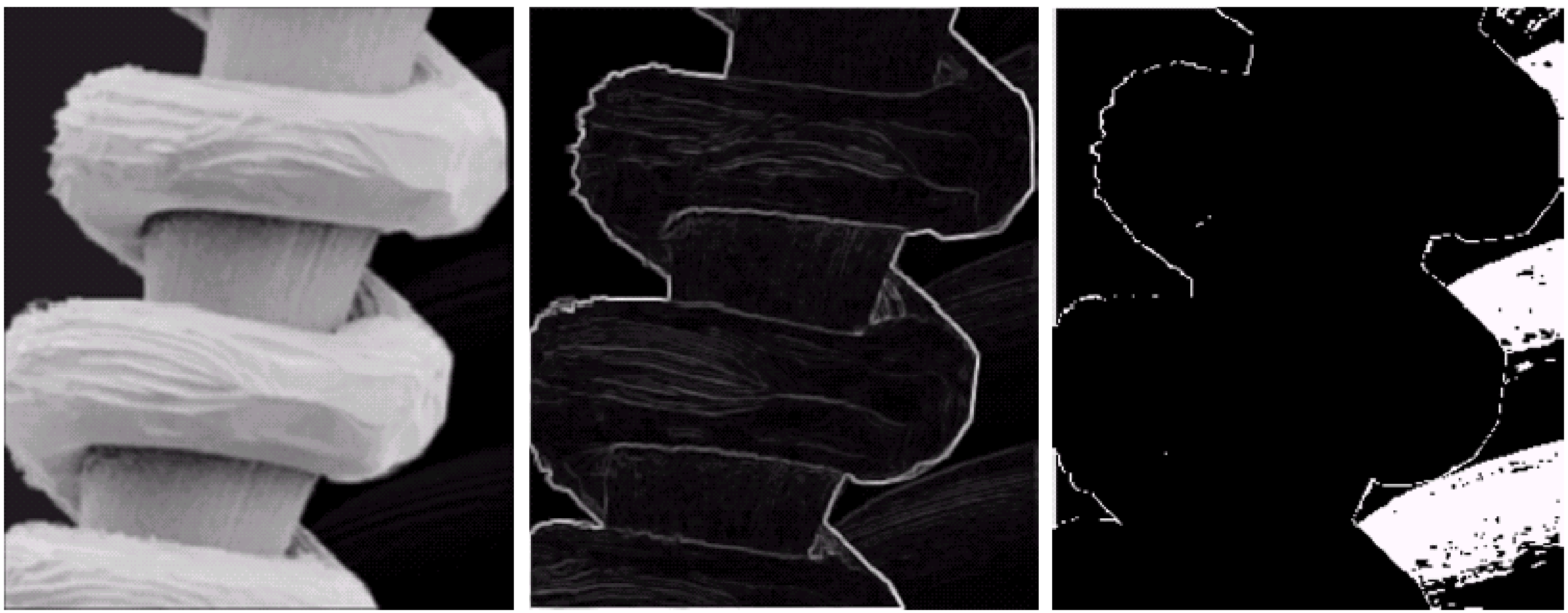
  - Multiply constant *E* in the *area* that meets all conditions

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{if } m_{s_{xy}} \leq k_0 M_G \text{ and } k_1 D_G \leq \sigma_{s_{xy}} \leq k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

◇ Use of Histogram Statistics for Image Enhancement (6)

-Successful selection of parameters

$E = 4.0$, $k_0 = 0.4$, $k_1 = 0.02$, $k_2 = 0.4$, small(3x3) region



(a) Local mean (b) Local variance (c) Multiplied by constant, 1 or E

◇ Use of Histogram Statistics for Image Enhancement (7)

- **Enhanced image**

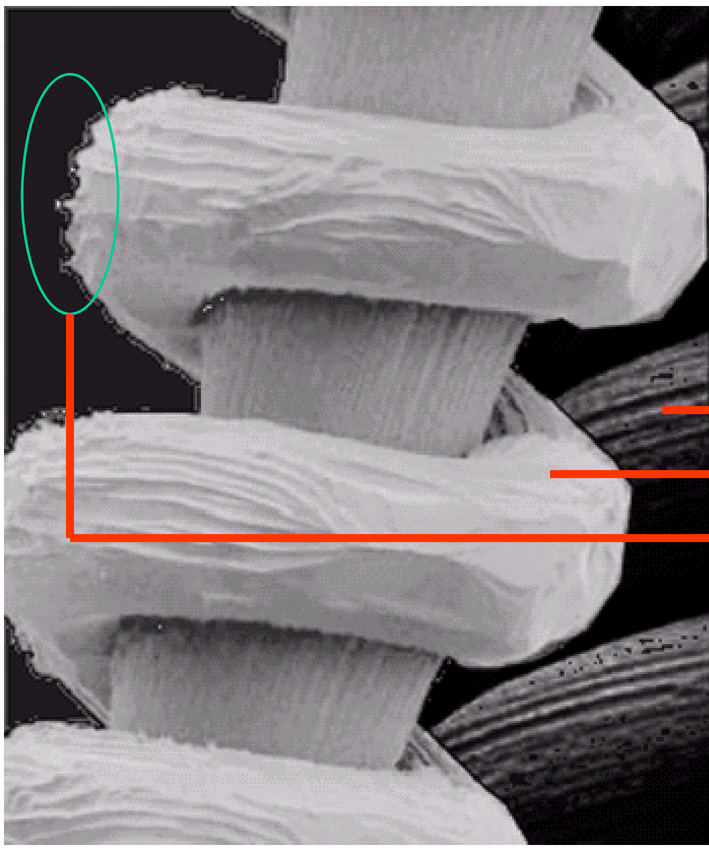


FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

Dark area : Obvious detail
Light area : Unchanged
Drawback : Small bright dots

3.4 Enhancement Using Arithmetic / Logic Operation

- Arithmetic operation

  - Image subtraction

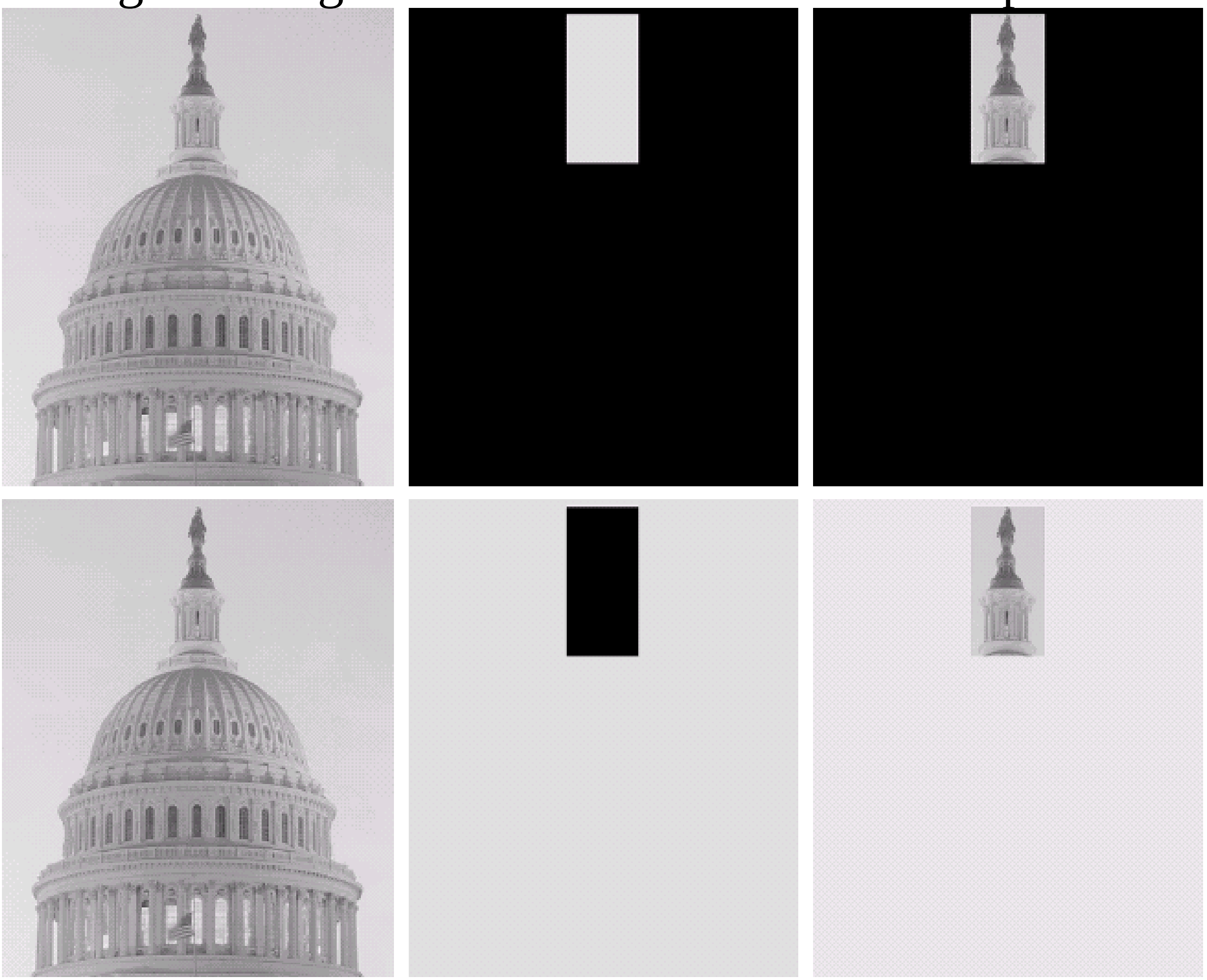
  - Image averaging

- Logic operation

  - AND, OR, NOT, etc.

◇ AND, OR, NOT Operation

Original Image AND Mask AND operation



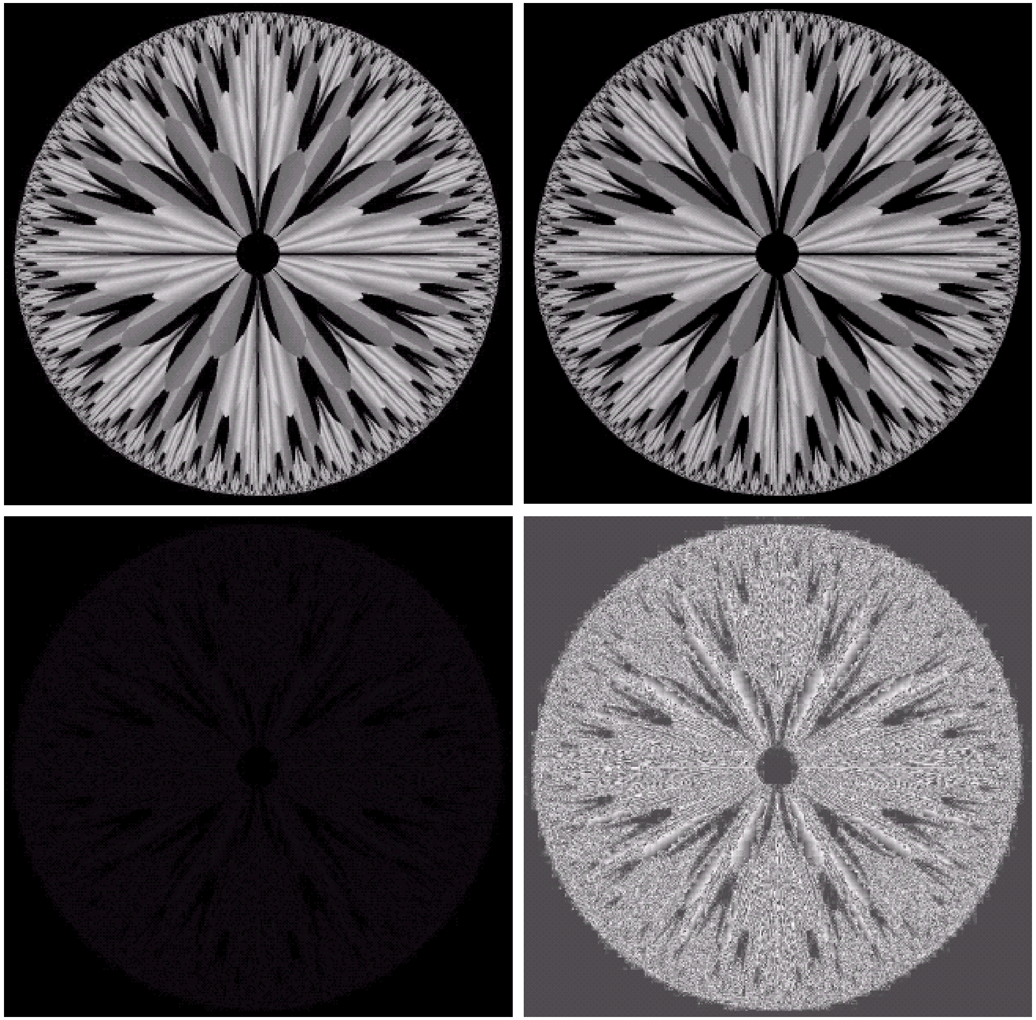
Original Image OR Mask OR operation

a b c
d e f

FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

3.4.1 Image Subtraction (1)

(a)Original Image (b) 4 lower-order bit planes = 0



(c) Difference [(a)-(b)] (d)Histogram equalization of (c)

a b
c d

FIGURE 3.28 (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).

$$g(x,y) = f(x,y) - \boxed{h(x,y)}$$

mask

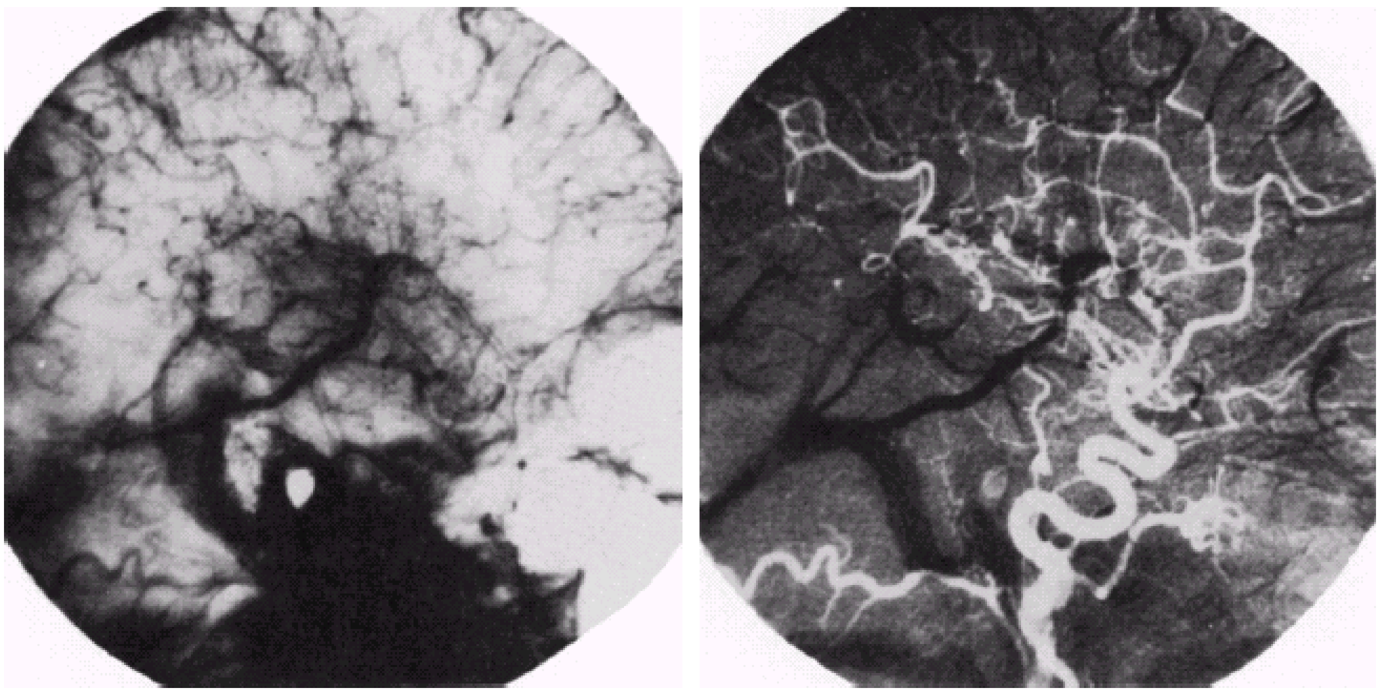
프로그램 실습(20분)

◇ Image Subtraction (2)

- Medical imaging(Mask mode radiography)

a) Mask image

b) Dye-injected image with mask subtracted out



a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

3.4.2 Image Averaging (1)

– Noisy image $$g(x,y) = f(x,y) + \boxed{\eta(x,y)}$$

noise

noise is uncorrelated with zero mean

– *Noisy Reduction by adding a set of noisy images*, $\{g_i(x,y)\}$

- An image formed by averaging K different noisy images

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

- Expected value & variance

$$E\{\bar{g}(x,y)\} = f(x,y) \qquad \sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

- Standard deviation at any point in the average image

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

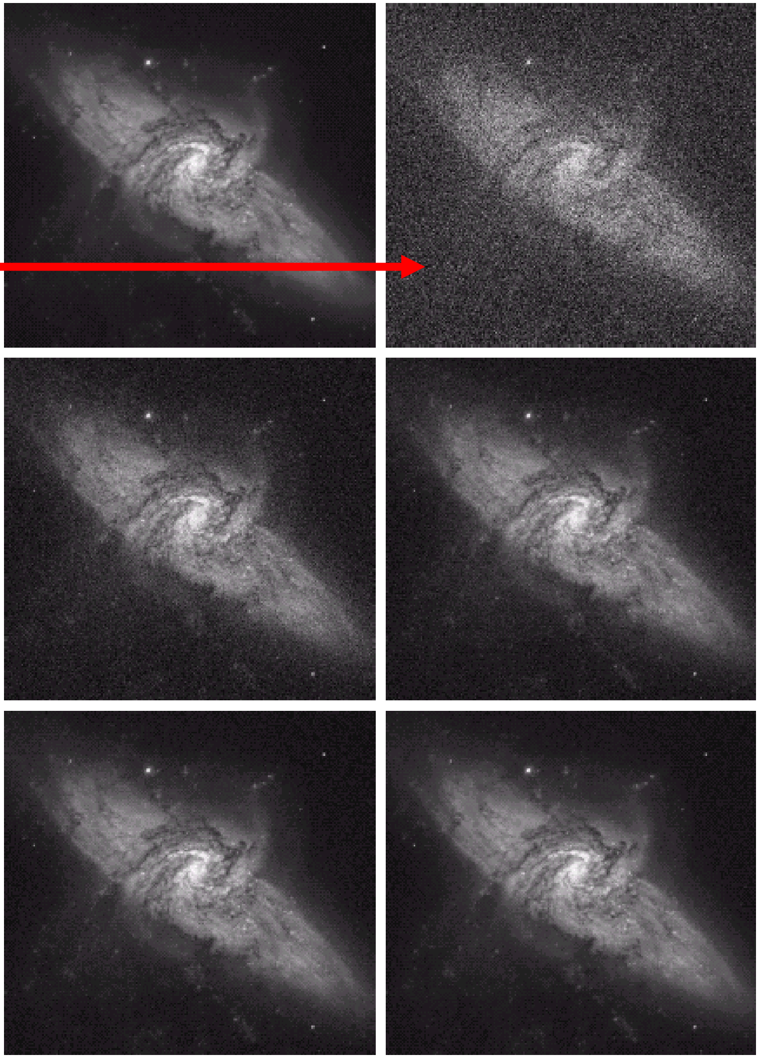
▷ If K ↑, Variability of the pixel values ↓

◇Image Averaging (2)

- A galaxy pair

(140 million light-years)

a) Original image
b) Gaussian noise added
c) averaging (K=8)
d) averaging (K=16)
e) averaging (K=64)
f) averaging (K=128)



➢ Gaussian noise
- uncorrelated
- zero mean
- standard deviation of 64 gray levels

◇Image Averaging (3)

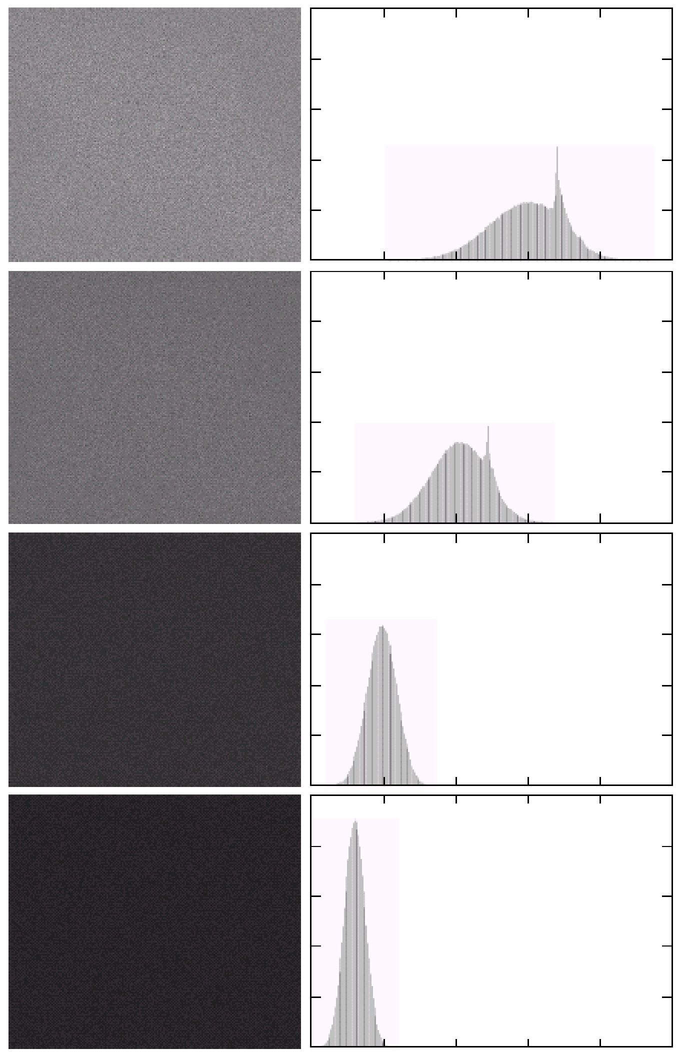
Image difference   Histogram

(a) – (c)

(a) – (d)

(a) – (e)

(a) – (f)



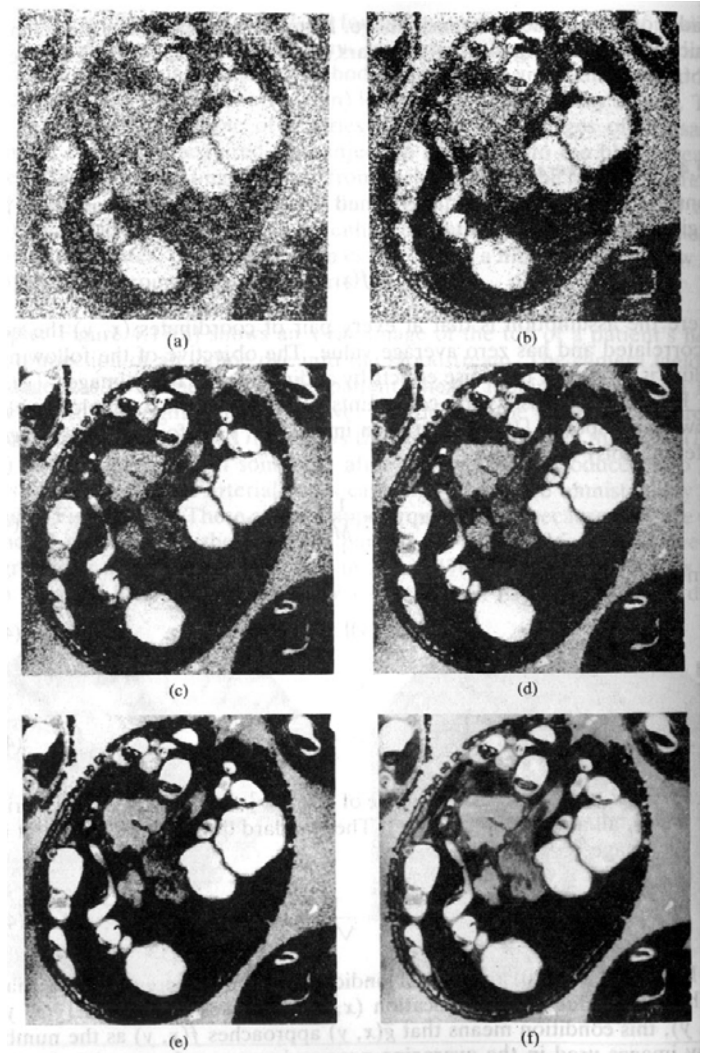
a b

FIGURE 3.31
(a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

What can you observe?

◇Image Averaging (4)

- Another example : cell
  - a) typical noisy image
  - b) averaging (M=2)
  - c) averaging (M=8)
  - d) averaging (M=16)
  - e) averaging (M=32)
  - f) averaging (M=128)



3.5 Basics of Spatial Filtering (1)

- Subimage
  - Filter
  - Mask
  - Kernel
  - Template
  - Window

$$
\begin{aligned}
R = {} & w(-1,-1)\,f(x-1, y-1) \\
& + w(-1,0)\,f(x-1, y) + \cdots \\
& + w(0,0)\,f(x, y) + \cdots \\
& + w(1,0)\,f(x+1, y) \\
& + w(1,1)\,f(x+1, y+1),
\end{aligned}
$$

$$
g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\,f(x+s, y+t)
$$

a = (m-1)/2, b=(n-1)/2

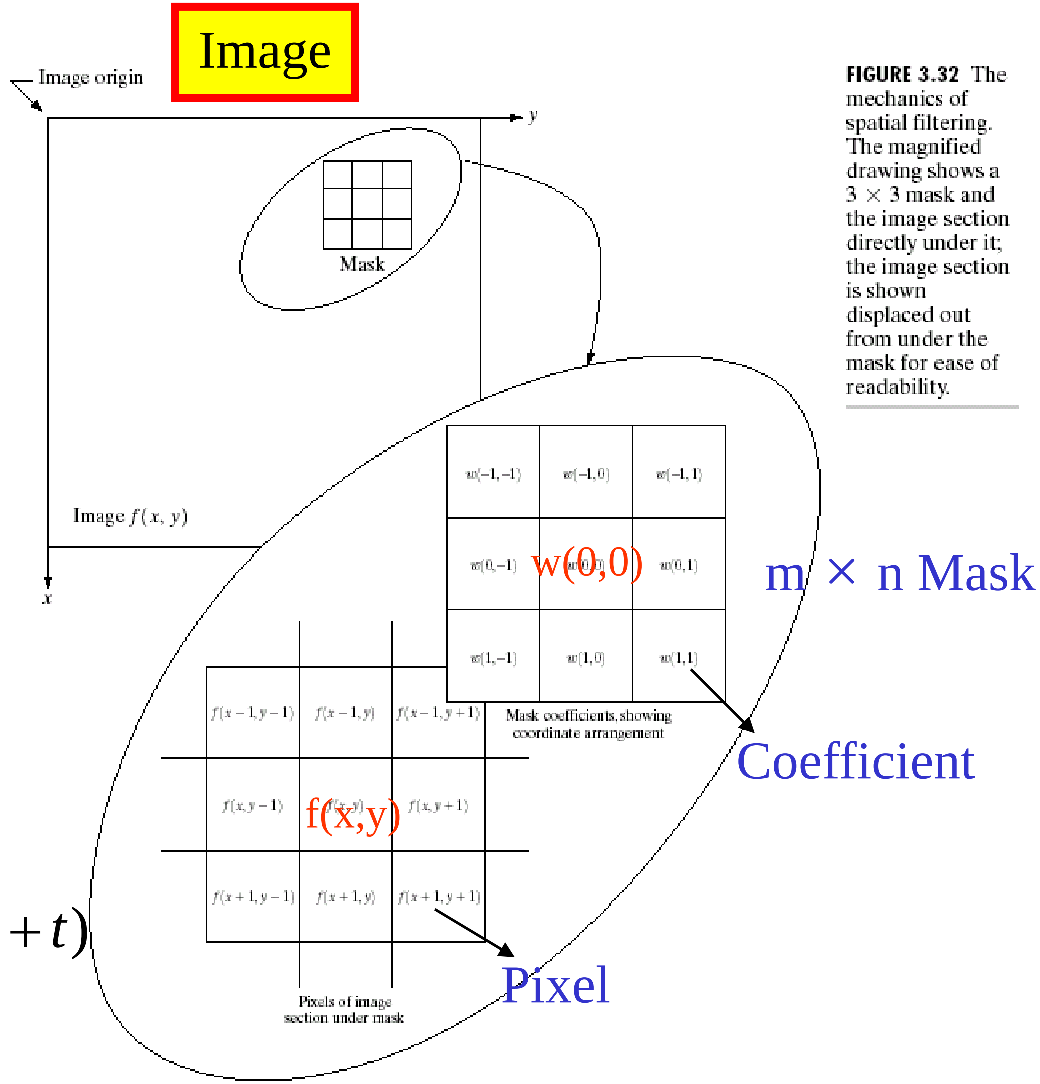
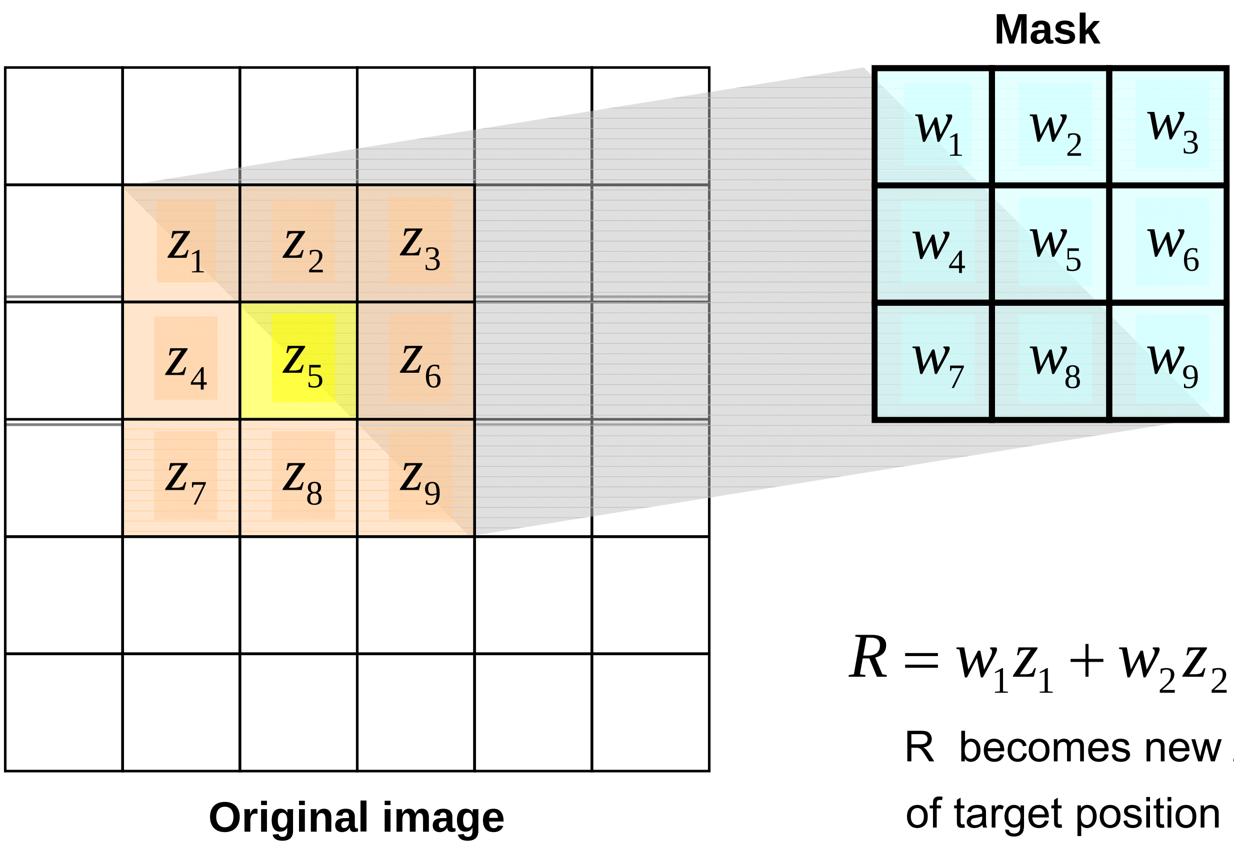


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

◇ Basics of Spatial Filtering (2)

- 3×3 mask with arbitrary coefficients (weights)



$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9$$

R becomes new z_5, gray-value of target position (x,y)

◇ Mask Processing / Spatial Filtering

- Smoothing / blurring
  - Lowpass spatial filtering
  - Median filtering

▷ *Noise reduction*

- Sharpening / highlighting
  - Highpass spatial filtering
  - High-boost filtering
  - Derivative filters

▷ *Edge detection*

3.6 Smoothing Spatial Filters

- Blurring
  - Removal of small details
  - Bridging of small gaps in lines or curves
- Noise reduction

3.6.1 Smoothing Linear Filters(mask)

- Average of the pixels using the filter mask
  - Average filter
  - Lowpass filter

프로그램 읽기

Standard average(Box filter)

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Weighted average

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

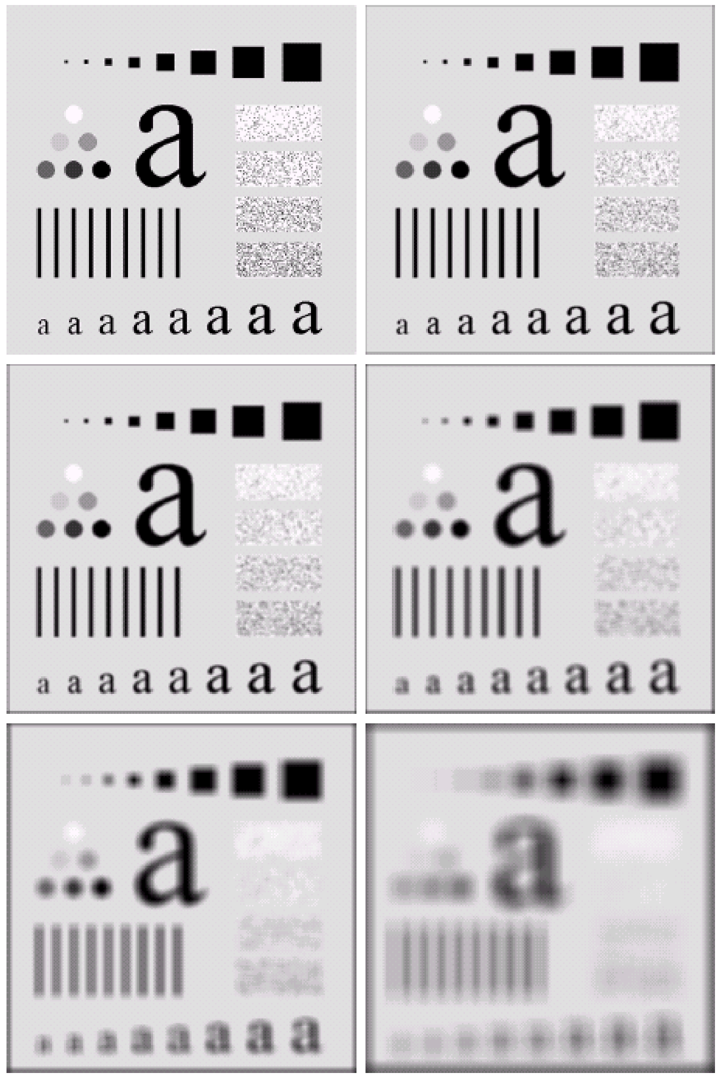
$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

◇ Smoothing Linear Filters(Ex.1)

Original image

Mask size
n = 3



n = 5

n = 9

n = 15

n = 35

◇ Smoothing Linear Filters(Ex. 2)

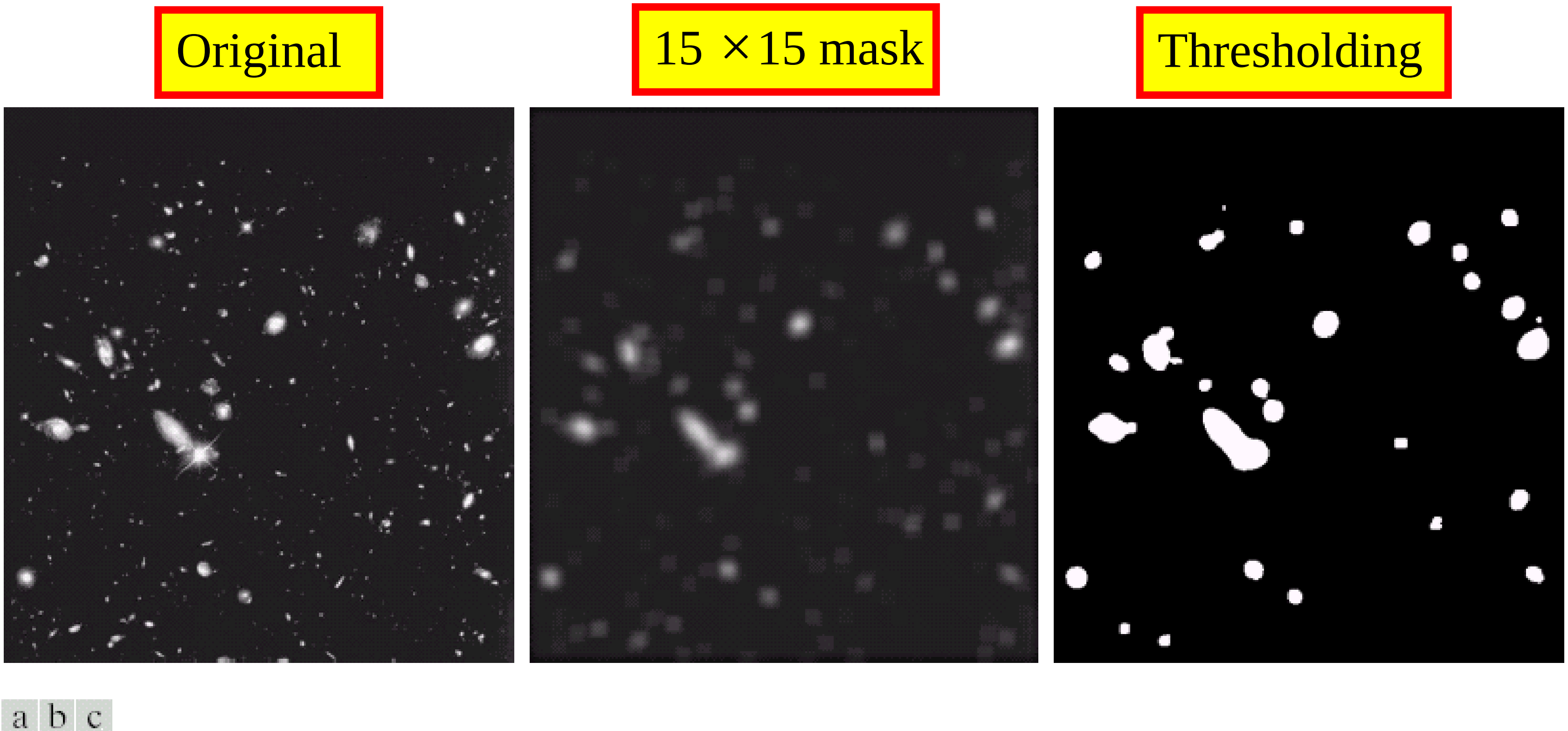


FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

3.6.2 Order-Statistics Filters

- Linear spatial filter
  - Lowpass filter
  - Highpass filter
  - Bandpass filter
- Nonlinear spatial filter (Order-statistics filters)
  - Median filter
  - Max filter
  - Min filter
- Usage of median filters
  - Reduction of impulse noise or salt-and-pepper noise

◇Median Filtering (1)

- Example

프로그램 숙제

  - Gray-values of 3×3 neighborhood

10	20	20
20	15	20
20	25	100

  - Sorted values

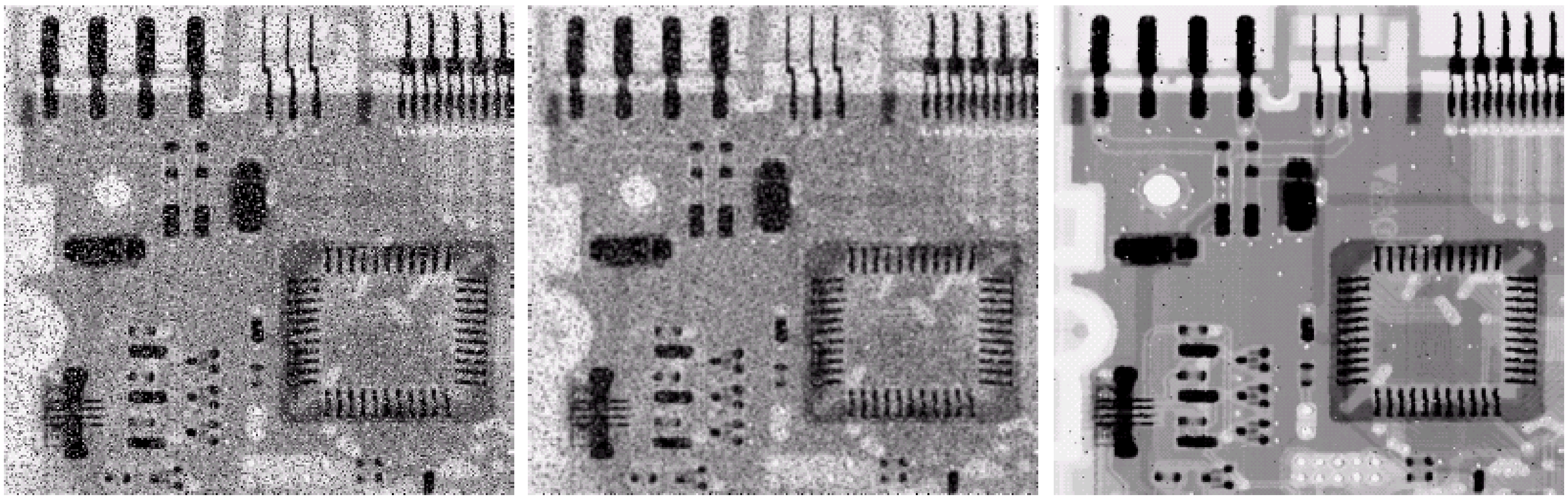
(10,15,20,20,*20*,20,20,25,100)

↑ 5th index

  - Median value = 20

◇Median Filtering (2)

X-ray image 3× 3 average mask 3× 3 median filter



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

◇Filtering: Average vs. Median

- Difference between average and median filter

  – Average filter

100	100	50
`100	100	50
100	100	50

→

100	100	50
100	83	50
100	100	50

  – Median filter

100	100	50
`100	100	50
100	100	50

→

100	100	50
100	100	50
100	100	50

3.7 Sharpening Spatial Filters

- Objective : highlight fine detail
- Foundation
- Use of first derivatives for enhancement

  - the Gradient
- Use of second derivatives for enhancement

  - the Laplacian

3.7.1 Foundation(1)

- 1-D first-order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

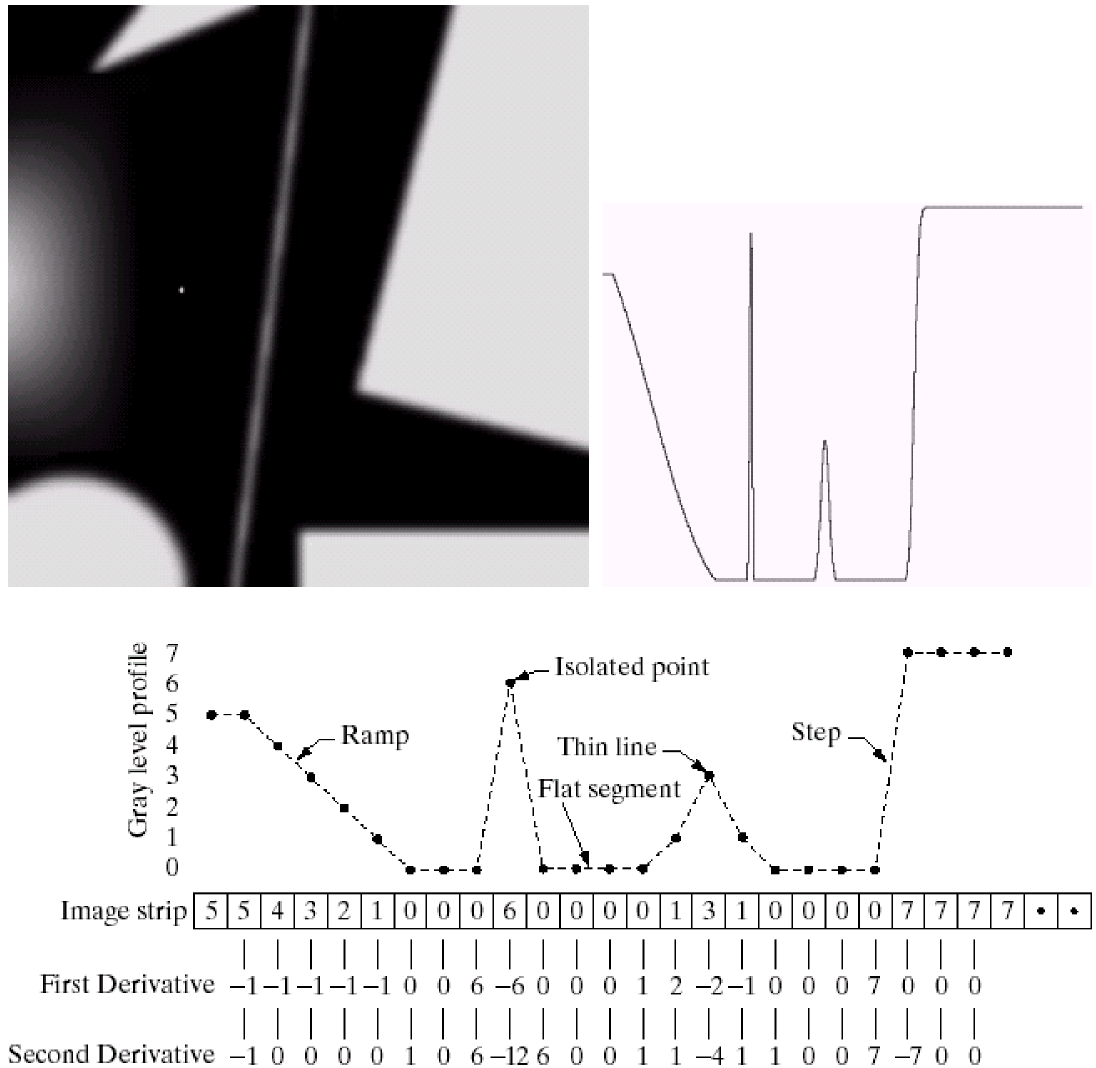
- 1-D second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

◇ Foundation(2) - Example

a b
c

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



3.7.2 Second Derivatives - The Laplacian (1)

- Method

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$ Eq. (3.7-1)

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$ Eq. (3.7-2)

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$ Eq. (3.7-3)

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$ Eq. (3.7-4)

◇ The Laplacian (2)

- A Basic 3 x 3 Highpass Filter

Digital Laplacian

Laplacian including diagonal neighbors

프로그램실습(30분)

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

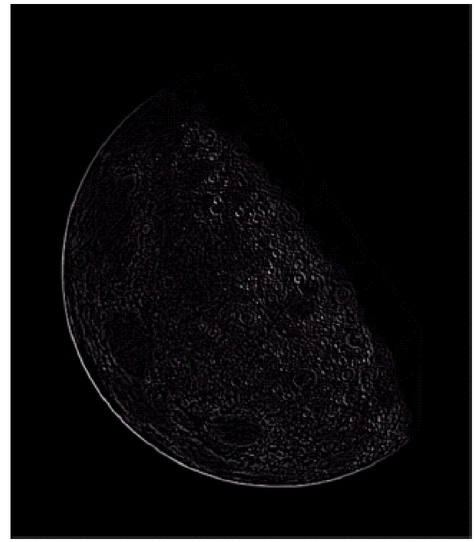
◇ The Laplacian (3)

- The sharpening effect of the Laplacian

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) & \text{for the negative center coefficient} \\ \\ f(x,y)+\nabla^2 f(x,y) & \text{for the positive center coefficient} \end{cases}$$

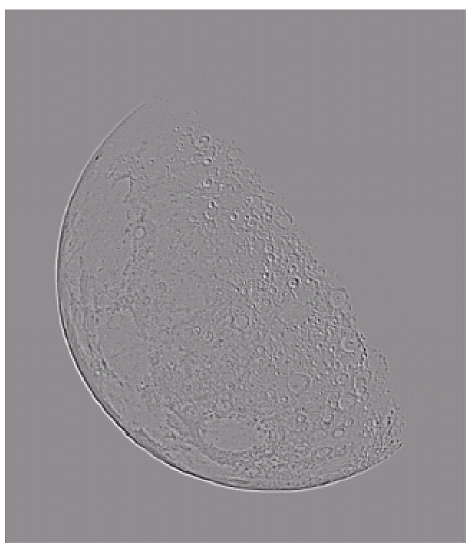
◇ The Laplacian (4)

(a)Original



(b)Laplacian image

(c)Scaled Laplacian for display



(d)Sharpening Effect (a) + (b)

◇ The Laplacian (5)

❑ Simplifications of sharpening effect

f(x,y) : Original image
g(x,y) : Sharpened image

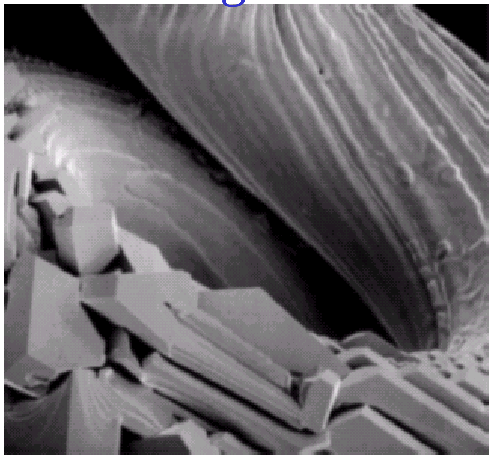
$$
\begin{aligned}
g(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\quad + f(x,y+1) + f(x,y-1)] + 4f(x,y) \\
&= 5f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\quad + f(x,y+1) + f(x,y-1)]
\end{aligned}
$$

Mask

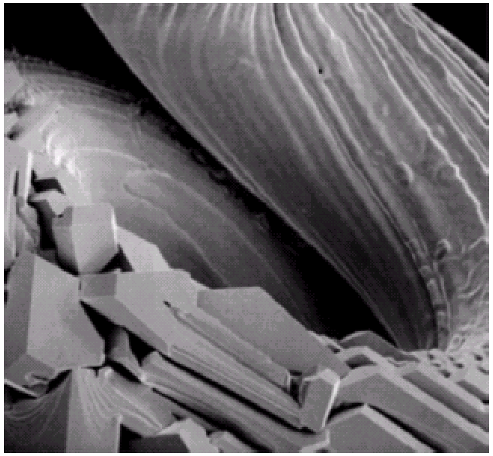
0	−1	0
−1	5	−1
0	−1	0

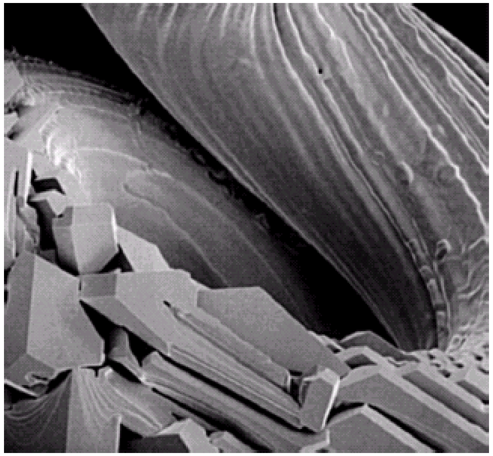
−1	−1	−1
−1	9	−1
−1	−1	−1

Original



Sharpened images





a	b	c
	d	e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

프로그램실습(30분)

◇ Unsharp Masking and High-Boost Spatial Filtering (1)

프로그램 숙제

– Unsharp masking

$$f_s(x,y) = f(x,y) - \overline{f}(x,y)$$

$f_s(x,y)$: the sharpened image, $\overline{f}(x,y)$: blurred image

– High-Boost filtering : Generalization of unsharp masking

$$f_{hb}(x,y) = Af(x,y) - \overline{f}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - \overline{f}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f_s(x,y)$$

- A: Amplification factor, $A \geq 1$
  If $A = 1$, Standard highpass
  If $A > 1$, High-Boost
  (☞ Edge enhancement)

◇ Unsharp Masking and High-Boost Spatial Filtering (2)

- Sharpening effect

$$f_{hb} = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{for the negative center coefficient} \\ \\ Af(x,y) + \nabla^2 f(x,y) & \text{for the positive center coefficient} \end{cases}$$

0	−1	0
−1	$A+4$	−1
0	−1	0

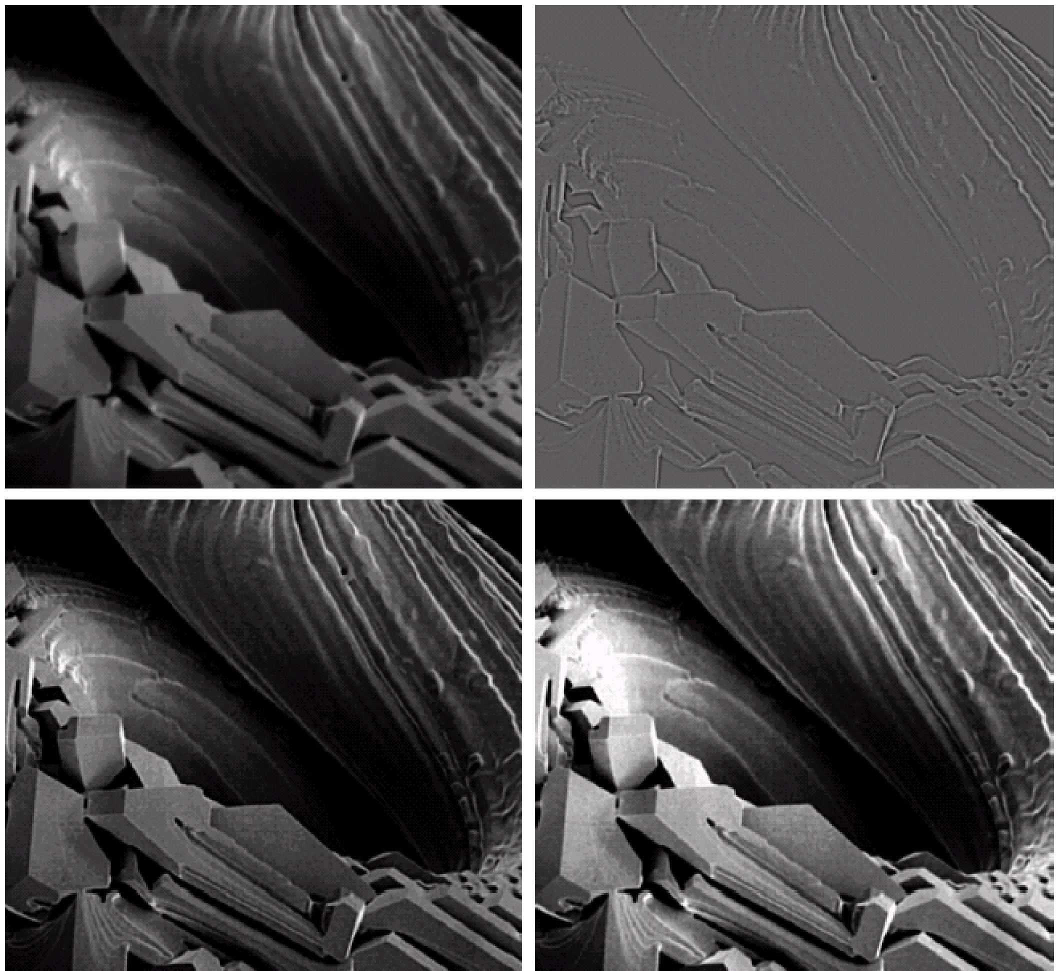
−1	−1	−1
−1	$A+8$	−1
−1	−1	−1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

◇ Unsharp Masking and High-Boost Spatial Filtering (3)

- Application : Brighten the dark image



Dark image

Laplacian

Sharpened image
A=1
Still dark

Sharpened image
A=1.7
More natural

3.7.3 Use of First Derivatives for Enhancement – The Gradient (1)

- Gradient techniques
  - *Magnitude* of gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(a)

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \end{bmatrix}$$

$$\nabla f = \mathrm{mag}(\nabla \mathrm{f}) = \left[G_x^{\;2} + G_y^{\;2} \right]^{1/2} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$$\nabla f \approx \left| G_x \right| + \left| G_y \right| = \left| z_5 - z_8 \right| + \left| z_5 - z_6 \right|$$

  - *Direction angle* of the vector $\nabla \mathrm{f}$

$$\alpha(x, y) = \tan^{-1} \begin{pmatrix} G_y \\ G_x \end{pmatrix}$$

◇ The Gradient (2)

- Gradient filter mask

a	
b	c
d	e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(a) 3 × 3 Mask

−1	0
0	1

0	−1
1	0

(b,c) Roberts Cross-Gradient Operators

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

−1	−2	−1
0	0	0
1	2	1

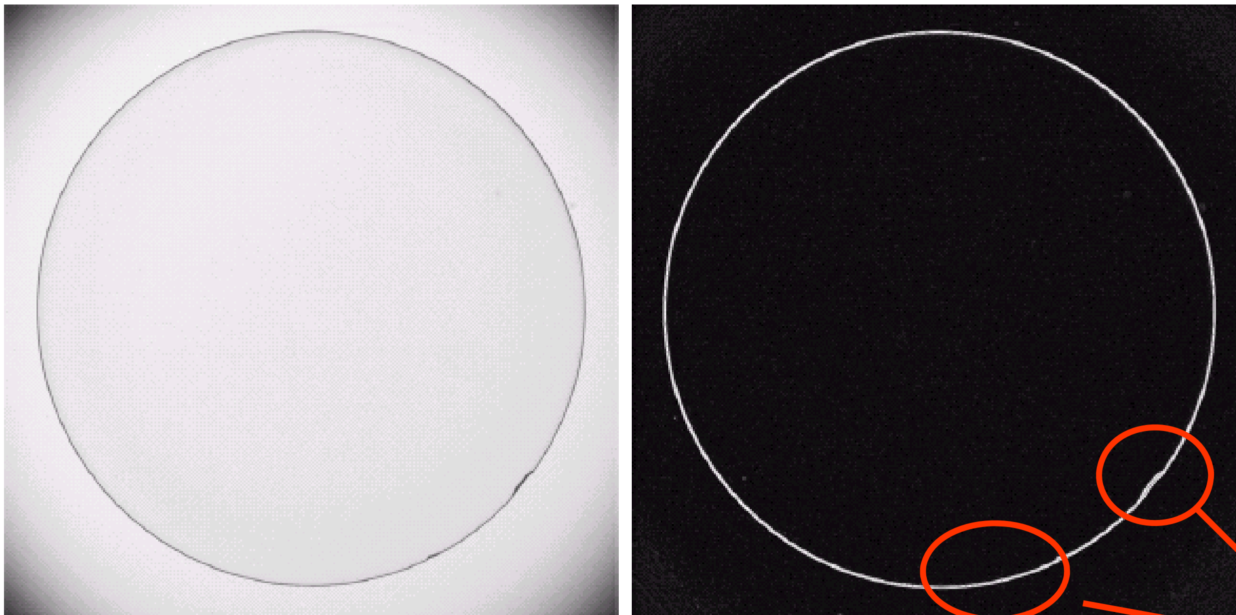
−1	0	1
−2	0	2
−1	0	1

(d,e) Sobel Operators

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

프로그램실습(20분)

◇ The Gradient (3)



a b

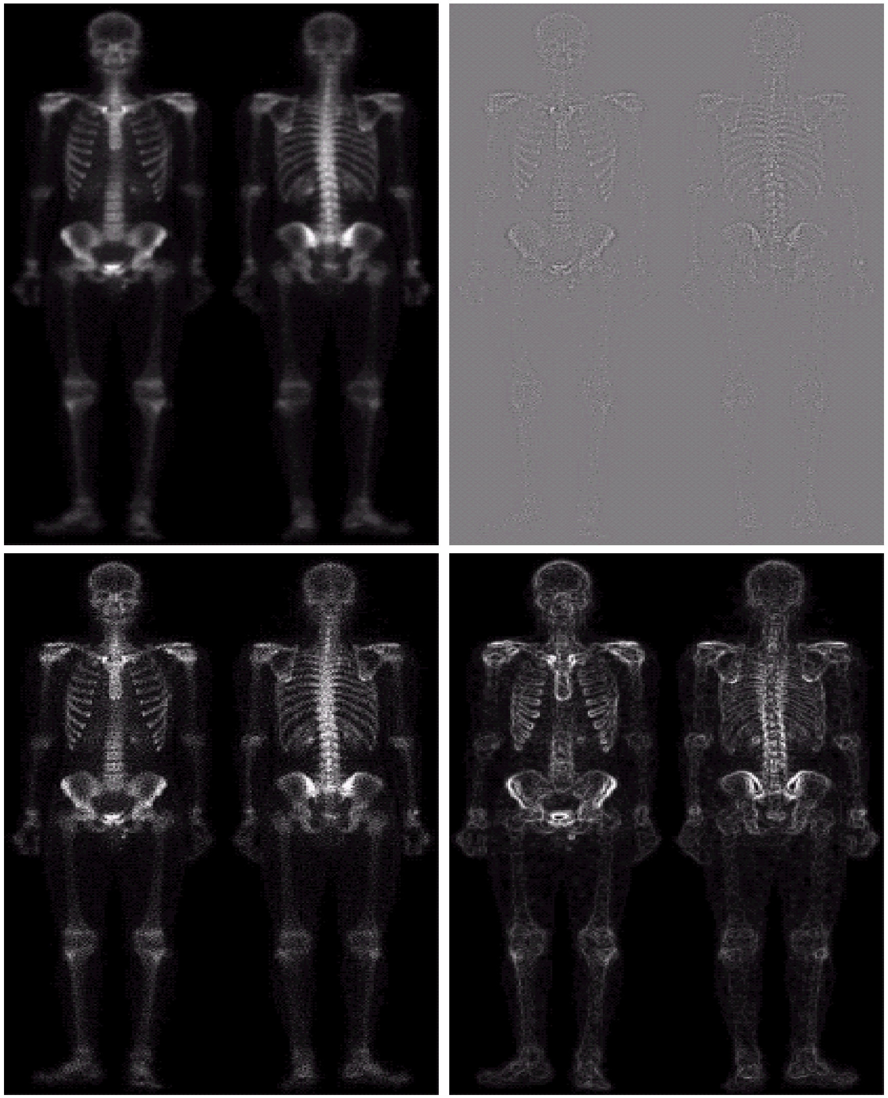
FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Edge defects are quite visible

An Optical Image of Contact Lens

Gradient Using Sobel Mask

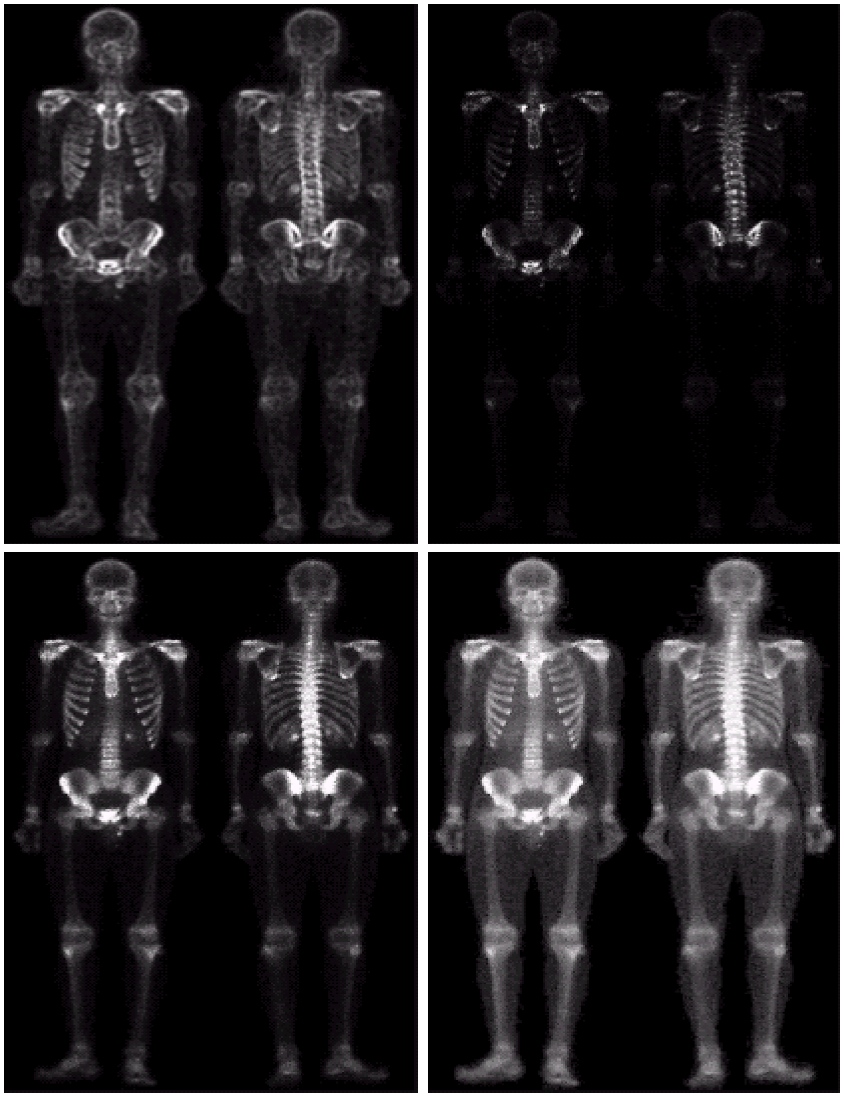
3.8 Combining Spatial Enhancement Methods (1)



a b
c d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).

◇ Combining Spatial Enhancement Methods(2)



e	f
g	h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Chap. 3 Homework

■ 숙제 샘플에 다음의 기능을 추가한 프로그램을 설계한다.

1. Bit-Plane Slicing(PPT P.16-18)
2. Use of Histogram Statistics for Image Enhancement(PPT P.44-48)
3. Median Filtering (PPT P.65-67)
4. High-boost filtering(PPT P.76-78). 단, 필터 마스크는 Fig.3.42(b) (PPT P.77 오른쪽)를 이용한다. A의 값은 슬라이더로 조정한다.

■ 프로그램 제출방법

1. 폴더명은 제3장(학생이름,학년)으로 한다.
2. 다른 사람의 프로그램을 복사한 경우는, 보여준 사람과 복사한 사람 모두 숙제 점수만큼 마이너스 점수를 준다.
3. 프로그램 방법을 교수 또는 다른 학생에게 질문을 할 수는 있지만, 프로그램 코드는 자기가 직접 작성한다
4. 실행 가능한 프로그램에서 Debug 폴더는 삭제하고 제출한다.
5. 제출기한 : 2주후