Chapter 4

Image Enhancement in the Frequency Domain

◇ Preview

- Fourier transform of the images
- Understanding of the spatial techniques through the frequency domain
- Image enhancement in the frequency domain

4.1 Background(1)

Sines and cosines with variable freq.

Weighted sum

Any function

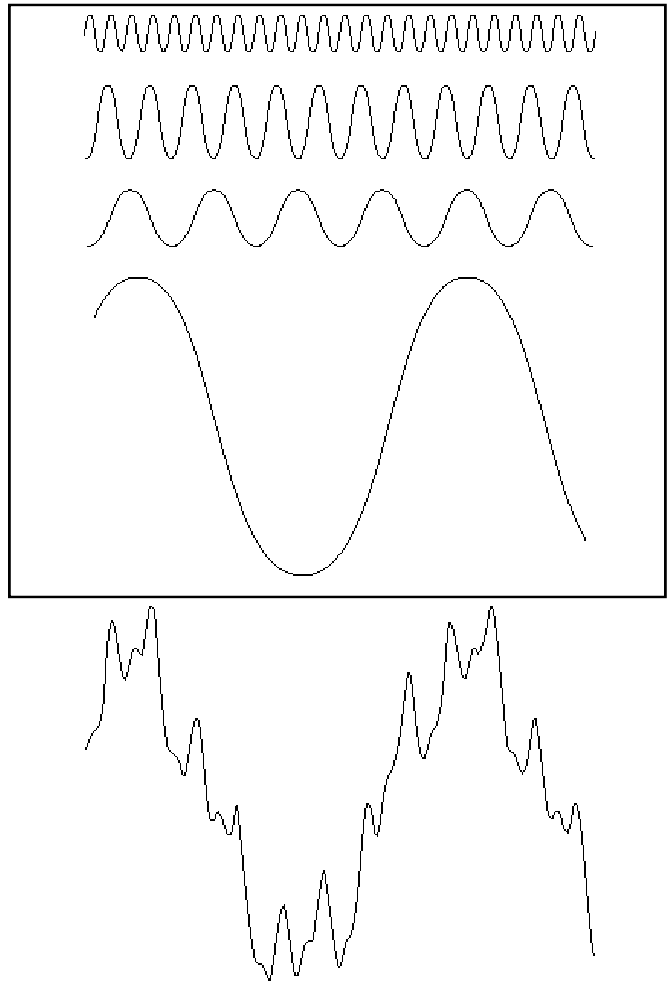


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

4.1 Background(2)

- Fourier was born in France in 1786. In 1807,
  - Invented ***Fourier series*** for periodic function
  - Invented ***Fourier transform*** for nonperiodic, but finite under-curve area function

- Late 1950s
  - Fast Fourier transform (FFT) algorithm
  - Revolution in signal processing

4.2 Introduction to the Fourier Transform and the Frequency Domain

- The One-Dimensional Fourier transform (FT) and its inverse
- The Two-Dimensional DFT and its inverse
- Filtering in the frequency domain
- Correspondence between filtering in the spatial and frequency domain

4.2.1 1D Fourier Transform and Its Inverse (1)

- f(x) : a continuous function of a real variable x.
- Fourier transform of function f(x)

$$\Im\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x)\exp[-j2\pi ux]dx, \quad \text{where } j = \sqrt{-1}$$

- Inverse Fourier transform

$$\Im^{-1}\{f(u)\} = f(x) = \int_{-\infty}^{\infty} F(u)\exp[j2\pi ux]du$$

Fourier transform pair

◇ 1D Fourier Transform and Its Inverse (2)

- Discrete Fourier transform (DFT)

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \qquad for \;\; u = 0, 1, 2, \ldots, M-1$$

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \qquad for \;\; x = 0, 1, 2, \ldots, M-1$$

- Using Euler's formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\underbrace{F(u)}_{\textit{Frequency domain}} = \frac{1}{M} \sum_{x=0}^{M-1} \underbrace{f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]}_{\textit{Frequency component}}$$

◇ 1D Fourier Transform and Its Inverse (3)

- Complex coordinate

$$F(u) = R(u) + jI(u)$$

- Polar coordinate

$$F(u) = |F(u)|\, e^{-j\phi(u)}$$

- Magnitude (Fourier spectrum)

$$|F(u)| = [R^2(u) + I^2(u)]^{1/2}$$

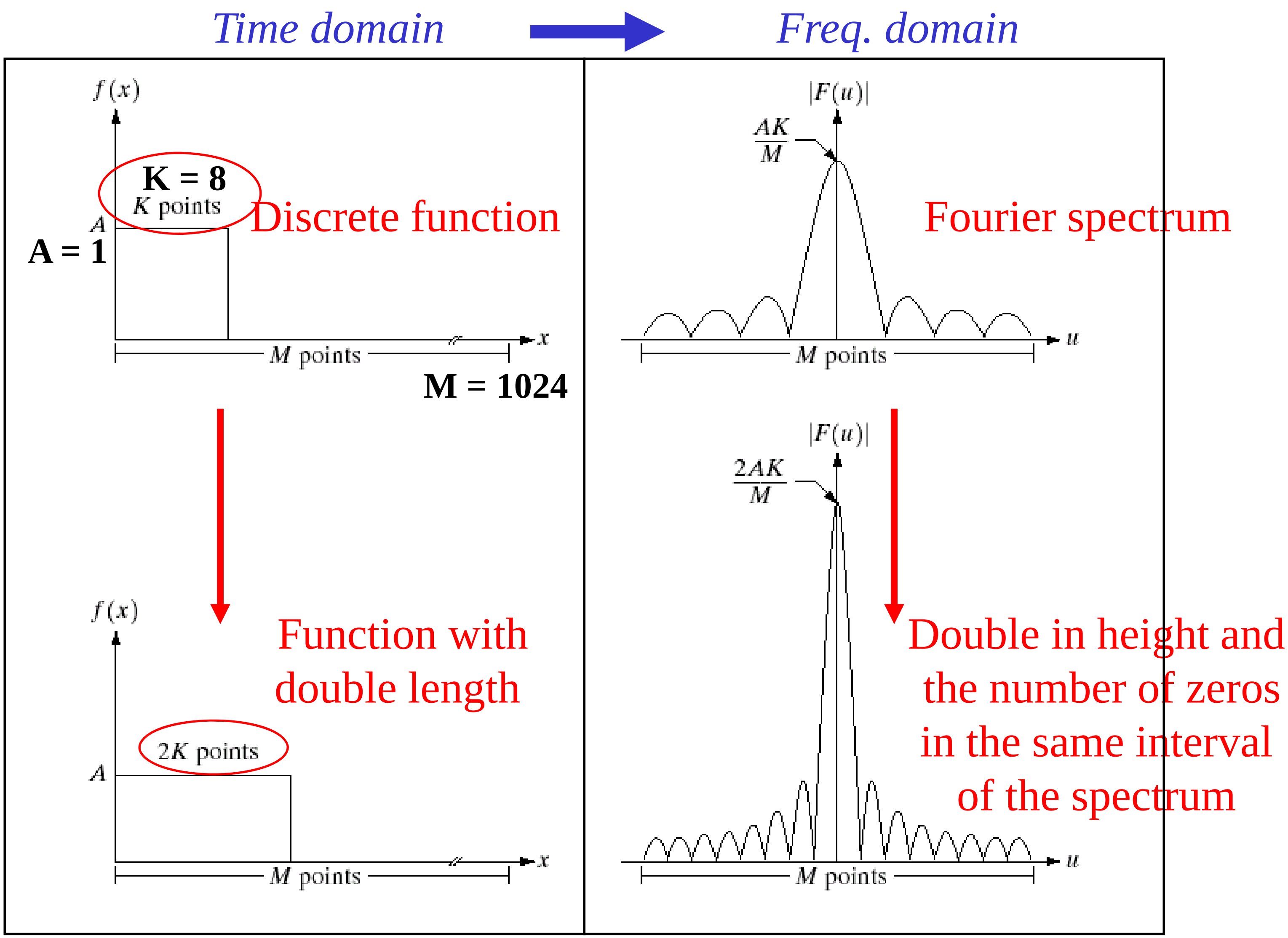
- Phase angle (Phase spectrum)

$$\phi(u) = \tan^{-1}\left[I(u) \;/\; R(u) \right]$$

- Power spectrum (Spectral density)

$$P(u) = |\mathrm{F(u)}|^2 = \mathrm{R}^2(u) + I^2(u)$$

◇ 1D Fourier Transform and Its Inverse (4)



◇ 1D Fourier Transform and Its Inverse (5)

- Notation of the time domain
  - Start sample : $f(x_0)$
  - Next sample : $f(x_0 + \Delta x)$
  - *k*th sample : $f(x_0 + k\Delta x)$
  - Final sample : $f(x_0 + [M-1]\Delta x)$
  - Shorthand notation : $f(x) \stackrel{\Delta}{=} f(x_0 + x\Delta x)$
- Frequency domain

$$F(u) \stackrel{\Delta}{=} F(u\Delta u)$$

- Relationship between two domains

$$\Delta u = \frac{1}{M\Delta x}$$

4.2.2 The Two-Dimensional DFT and Its Inverse (1)

- 2D Fourier transform : extension of 1DFT

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+uy)}dxdy$$

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+uy)}dudv$$

- DFT (Discrete Fourier Transform)

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

- Fourier spectrum, phase angle, and power spectrum

$$|F(u,v)| = [R^2(u,v)+I^2(u,v)]^{1/2}$$

$$\phi(u,v) = \tan^{-1}\left[I(u,v)/R(u,v)\right]$$

$$P(u,v)=|\mathrm{F(u,v)}|^2=\mathrm{R}^2(u,v)+I^2(u,v)$$

◇ 2D DFT and Its Inverse (2)

- Origin shift in the frequency domain : *F(0,0) → F(M/2, N/2)* *for representation of the Fourier spectrum in image*

$$\Im\left[f(x,y)(-1)^{x+y} \right] = F(u - M/2, v - N/2)$$

- Average gray level (DC component)

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

- Complex conjugate (Symmetric spectrum)

$$F(u,v) = F^*(-u,-v), \quad |F(u,v)| = |F(-u,-v)|$$

- Relationship between spatial and frequency domains

$$\Delta u = \frac{1}{M\Delta x}, \quad \Delta v = \frac{1}{N\Delta y}$$

◇ 2D DFT and Its Inverse (3)

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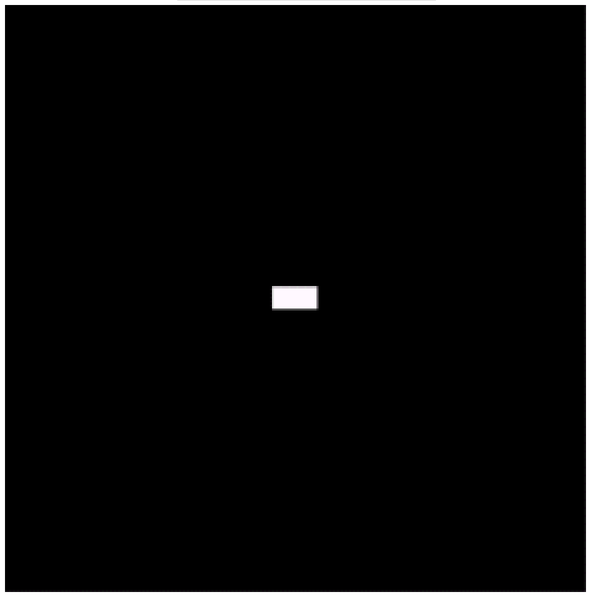
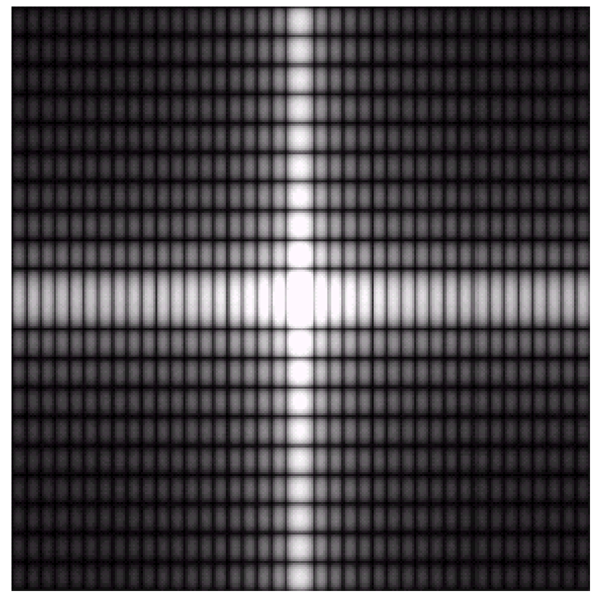
Image



Spectrum



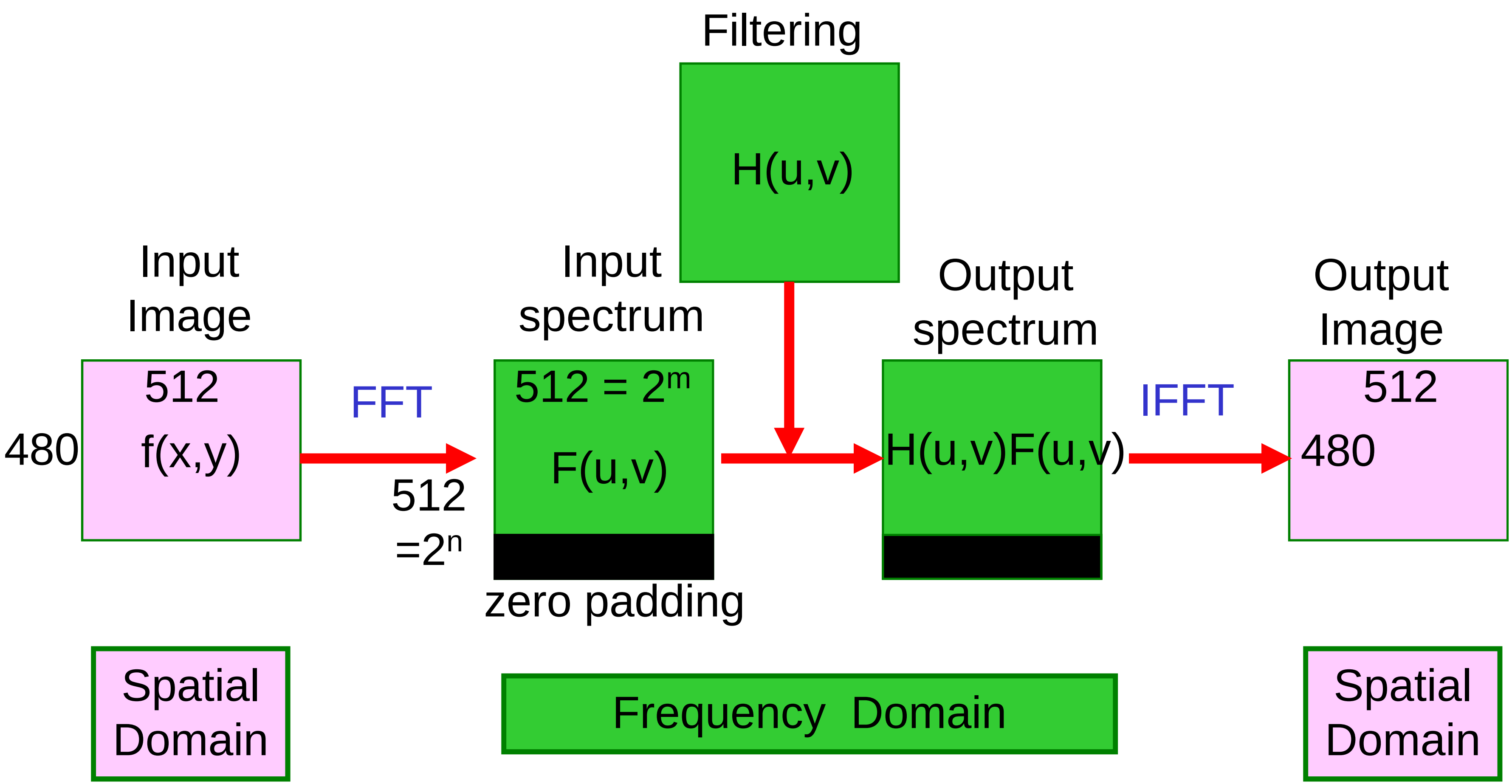
a b

FIGURE 4.3 (a) Image of a 20 × 40 white rectangle on a black background of size 512 × 512 pixels. (b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

- **White : 20 x 40 pixels**
- **Black : 512 x 512 pixels**
- **Multiply image by $(-1)^{x+y}$**

- **Origin shift : (0,0) →(256, 256)**
- **u :v = 1:2 in zero separations**
- **Log transform on spectrum**

Filtering Process using FFT

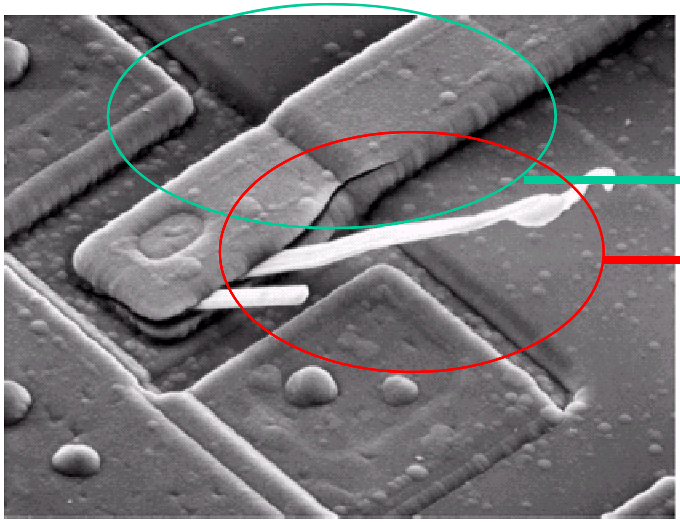


4.2.3 Filtering in the Frequency Domain

◇ Some Basic Properties (1)

- **Near origin of the Fourier transform**
  - Smooth gray-level variations
- **Move away from origin**
  - Slow gray-level changes
- **Higher frequency**
  - Faster gray-level changes
- **Very high frequency**
  - Edge, abrupt change, noise, …

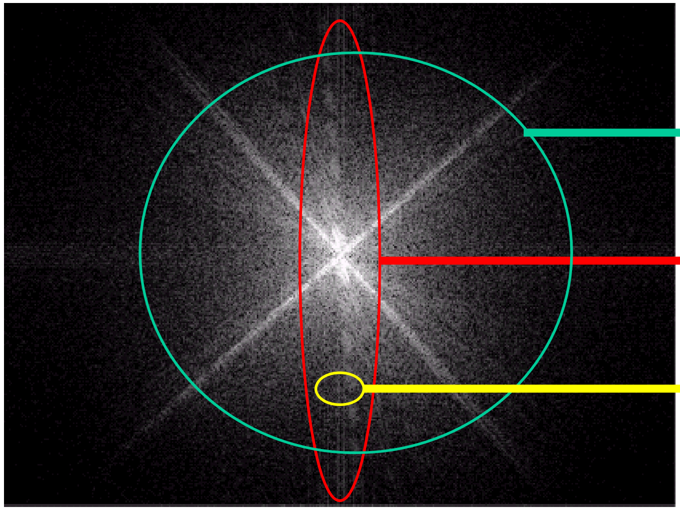
◇ Some Basic Properties (2)



Example

(a) Image of an IC

- Strong Edges running at 45^o direction
- Two white oxide protrusions



(b) Fourier spectrum

- Strong edge → Prominent components along 45^o
- Edge of oxide protrusions → Vertical component
- Narrow vertical span of oxide protrusions → Zeros of Vertical component

◇ Basics of Filtering in Freq. Domain (1)

1. Multiply the input image by $(-1)^{x+y}$ to center Fourier transform

   $$\Im[f(x,y)(-1)^{x+y}] = F(u - M/2, v - N/2)$$

2. Compute $F(u,v)$, the DFT of the image from (1)
3. Multiply $F(u,v)$ by a *filter* function $H(u,v)$

   Filtered image $G(u,v) = H(u,v)\,F(u,v)$

4. Compute the inverse DFT of the result from (3)

   Filtered image g(x,y) = $\Im^{-1}[G(u,v)]$

5. Obtain the real part of the result in (4)
6. Multiply the result in (5) by $(-1)^{x+y}$

   $g(x,y)\,(-1)^{x+y}$

◇ Basics of Filtering in freq. domain (2)

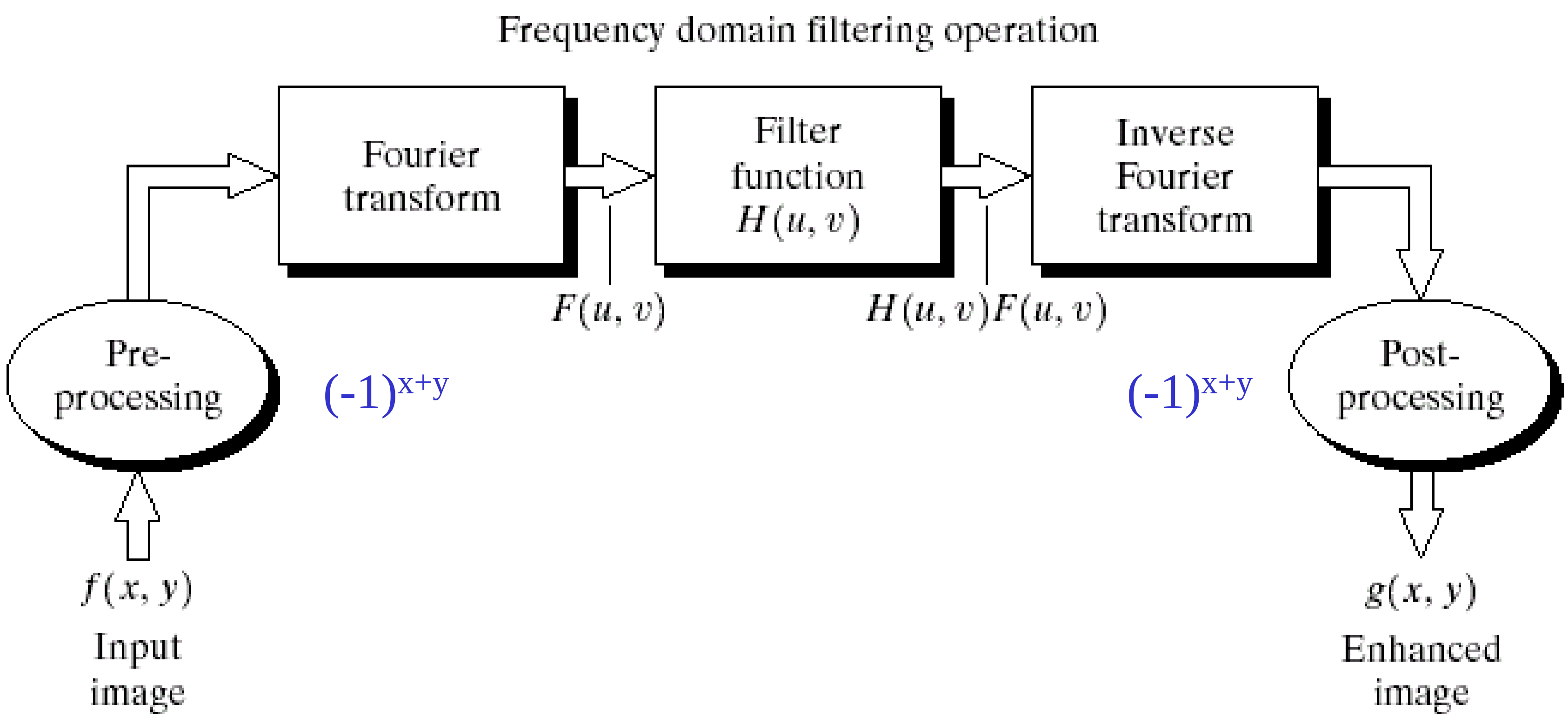


FIGURE 4.5 Basic steps for filtering in the frequency domain.

◇ Some basic filter and their properties (1)

❑ *Notch Filter*

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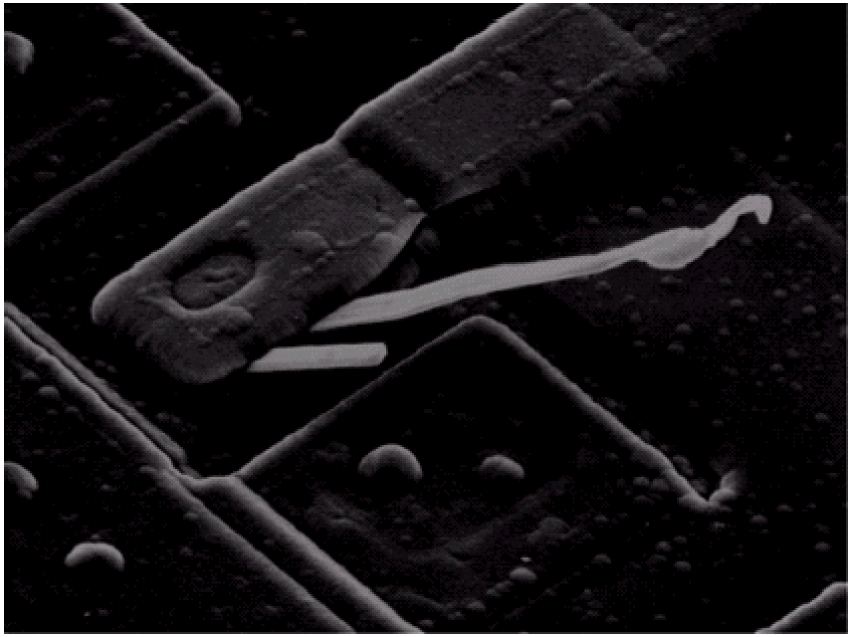
$$H(u,v) = \begin{cases} 0 & \text{if } (u, v) = (M/2, N/2) \\ 1 & \text{otherwise} \end{cases}$$

Eq. (4.2-29)

- **Role of Notch Filter**
  - Set F(0,0) to Zero
  - Leave all other frequency components
  - Drop in overall average gray Level



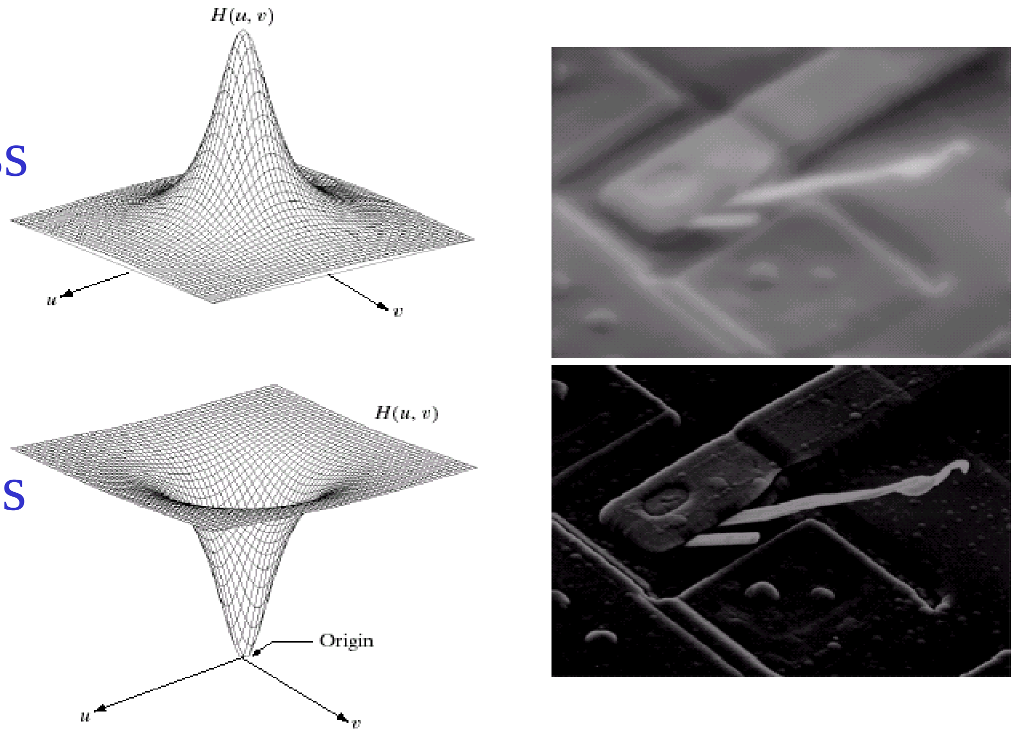
◇ Some basic filter and their properties (2)

❑ *Lowpass Filter, Highpass Filter*

- High frequencies : Details such as edges and noise
- Low frequencies : General gray-level appearance

Lowpass

Highpass



Blurred

Sharp with little smooth gray level detail

a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Some Basic Filter and Their Properties (3)

Filter function = Highpass + Constant

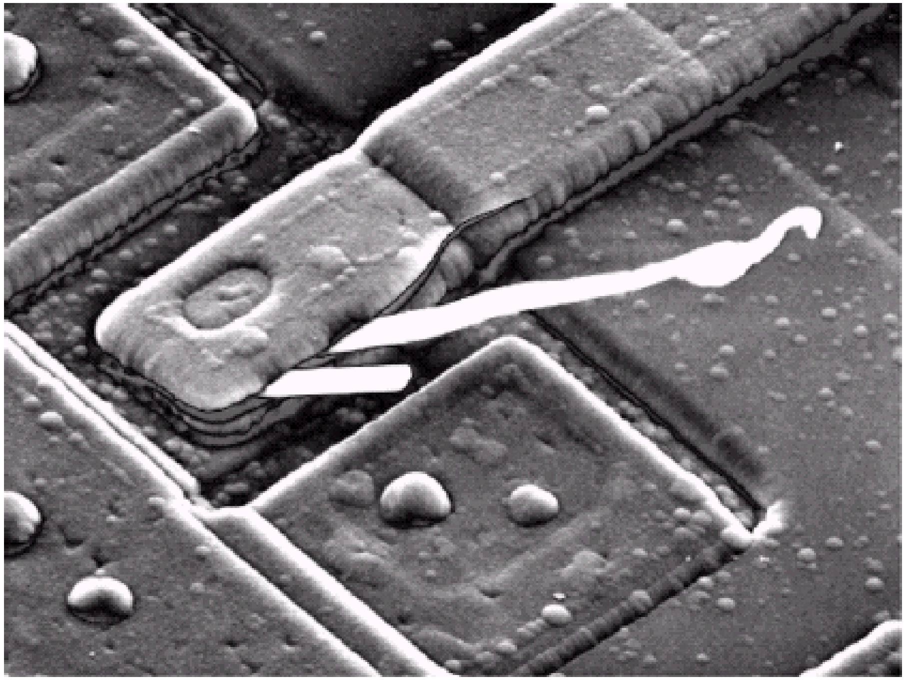


FIGURE 4.8 Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).

The improvement is evident over previous result

4.2.4 Correspondence between Filtering in the Spatial and Freq. Domains(1)

- Relationship between spatial and frequency domains is established by the convolution theorem

- Filtering using mask is the convolution process
  - Move mask from pixel to pixel
  - Compute sum of products of the mask coeff. and the image pixels

- Definition of the discrete convolution

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$$

$f(x,y): image\ function$

$h(x,y): mask\ function$

$M\times N: image\ size$

- Fourier transform pair of the convolution

$$f(x,y)*h(x,y)\Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y)\Leftrightarrow F(u,v)*H(u,v)$$

◇ Correspondence between Filtering in the Spatial and Freq. Domains(2)

- Summation of a function $s(x,y)$ multiplied by an impulse

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)A\delta(x-x_0, y-y_0) = As(x_0, y_0)$$

$s(x,y)$: a function
A: strength
$\delta(x,y)$: Impulse function

- The case of a unit impulse located at the origin

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)\delta(x,y) = s(0,0)$$

- **Fourier transform of a unit impulse at the origin. Suppose** $f(x,y)=\delta(x,y)$

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\delta(x,y)e^{-j2\pi(ux/M+vy/N)} \quad =\frac{1}{MN}$$

- **Convolution of a unit impulse at the origin**

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta(m,n)h(x-m,y-n)= \quad \frac{1}{MN}h(x,y)$$

◇ Correspondence between Filtering in the Spatial and Freq. Domains(3)

- By combining the above results

$$f(x,y)*h(x,y) \Leftrightarrow F(u,v)H(u,v)$$
$$\delta(x,y)*h(x,y) \Leftrightarrow \Im[\delta(x,y)]H(u,v)$$
$$h(x,y) \Leftrightarrow H(u,v).$$

$f(x,y): image\ function$

$h(x,y): mask\ function$

- Filters in spatial domain is obtained by the inverse Fourier transform of the filter defined in frequency domain.

◇ Correspondence between Filtering in the Spatial and Freq. Domains(4)

- Example : 1D Gaussian filter function in freq. domain

$$H(u) = Ae^{-u^2/2\sigma^2} \qquad \sigma : \text{ standard deviation}$$

- Corresponding filter in spatial domain

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$

- Importance of the Gaussian filter function

(1) Both components of FT pair are Gaussian and real.

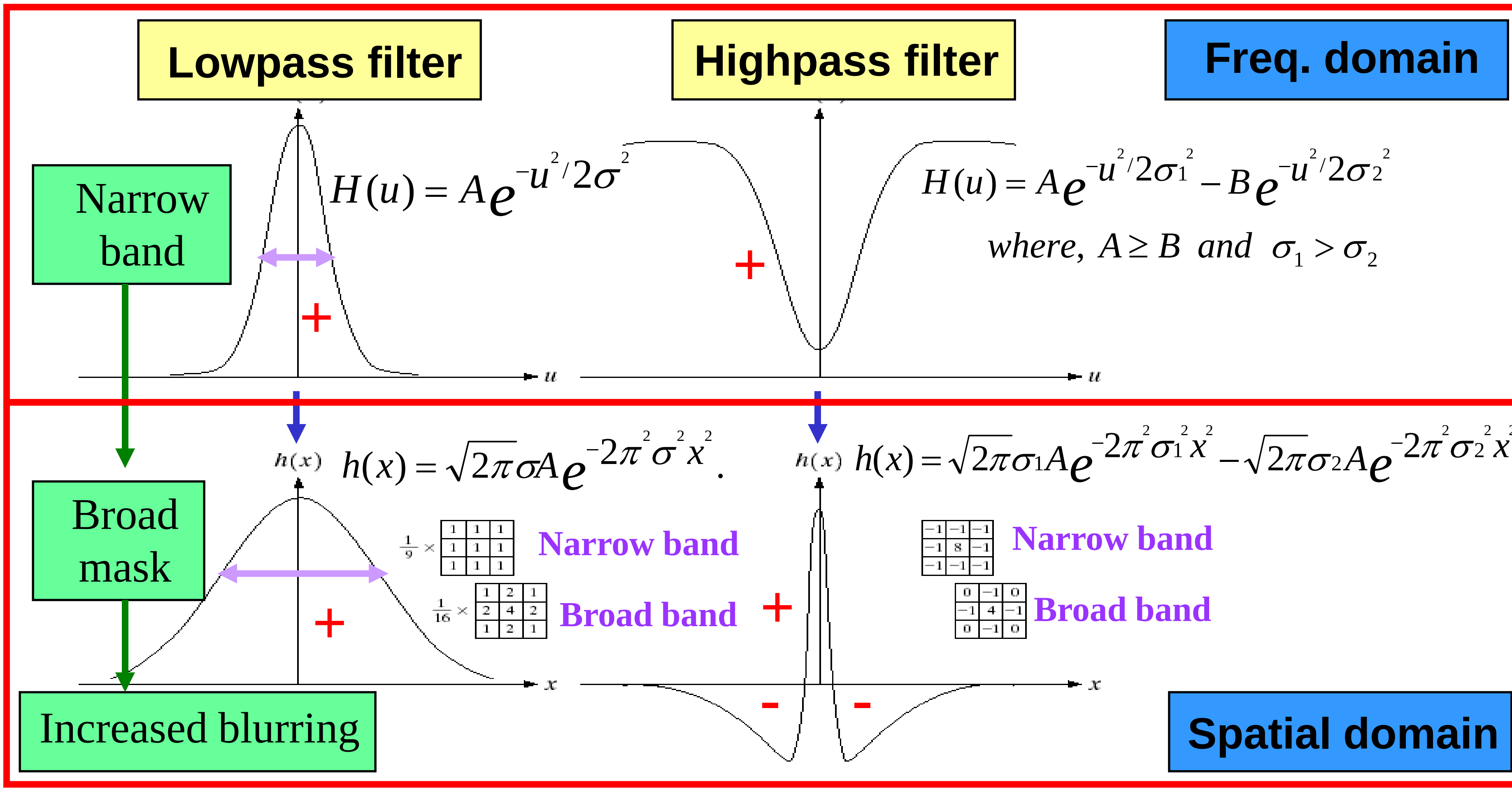
- This facilitates analysis considerably because we do not have to be concerned with complex numbers.
- Gaussian curves are intuitive and easy to manipulate.

(2) These function behave reciprocally with respect to one another.

- If *H(u)* has a broad(narrow) profile, then *h(x)* has a narrow(broad) profile
- When σ approaches infinity, *H(u)* tends toward a constant and *h(x) tends* toward an impulse

◇ Correspondence between Filtering in the Spatial and Freq. Domains(5)

- **The shapes of *h(x)* are used as a guide to specify the mask coefficients in the spatial domain**



◇Correspondence between Filtering in the Spatial and Freq. Domains(6)

- **Why do in the frequency domain what could be done in the spatial domain?**
  - More intuitive to specify filters in freq. domain
  - Size of the spatial mask is implementation-dependant

- **Implementation of convolution**
  - Spatial domain : mask processing
  - Freq. domain : inverse transform of $F(u,v)H(u,v)$ (using FFT)
  - Freq. domain is faster if N(the number of points) > 32

4.3 Smoothing Frequency-Domain Filters

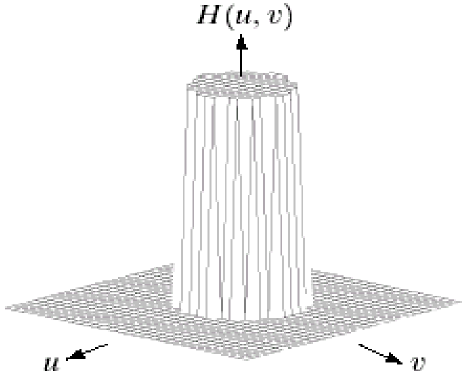
- Ideal lowpass filters
- Butterworth lowpass filters
- Gaussian lowpass filters
- Additional examples of lowpass filtering

◇ Basic model

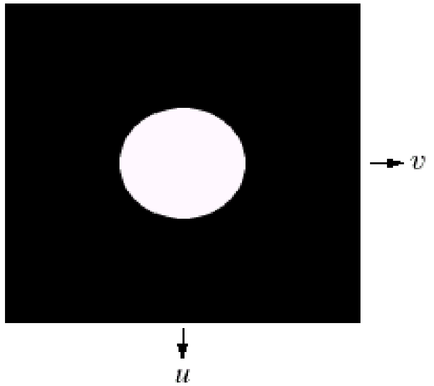
- $G(u, v) = H(u, v)\, F(u, v)$
  - $F(u, v)$: the Fourier transform of an image to be smoothed
  - $H(u, v)$: a filter transfer function
  - $G(u, v)$: attenuating the high-frequency components of $F(u,v)$

4.3.1 Ideal Lowpass Filters (1)

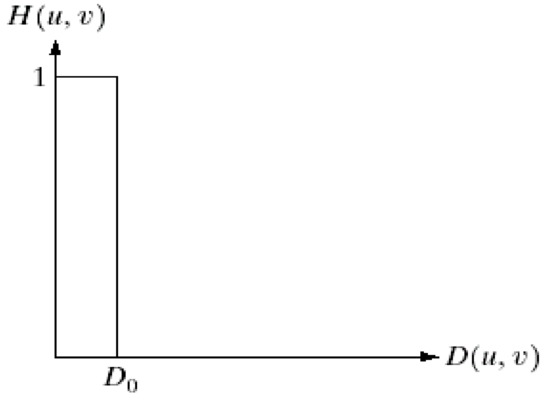
Perspective Plot



Display as image



Radial cross section



- 2D ideal lowpass filter (ILPF)

$$\mathrm{H(u,v)} = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

$D(u, v)$: distance from point (u, v) to the origin $(M/2, N/2)$

$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

D_0 : cutoff frequency

◇ Ideal Lowpass Filters (2)

- Total image power

$$P_T = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} P(u,v)$$

P(u,v): power at each point (*u, v*)

- Ratio of the filtered image power

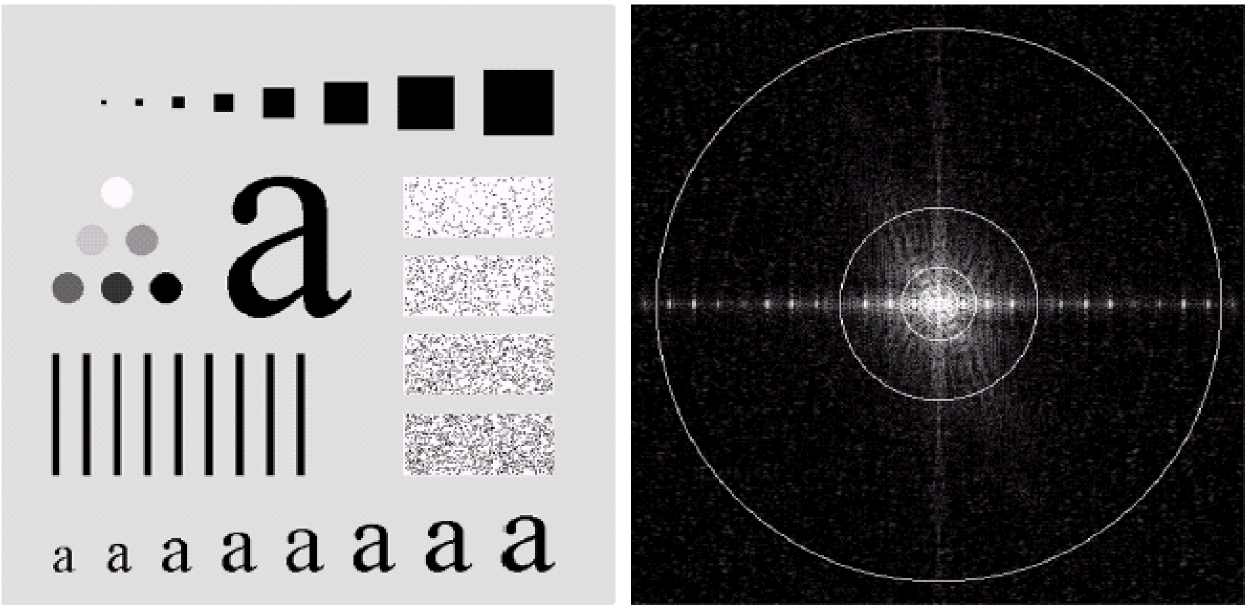
$$\alpha = 100[\sum_{u} \sum_{v} P(u,v)/P_T]$$

$$where, \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2} < r$$

$$r : radius\ of\ spectrum$$

◇ Ideal Lowpass Filters (3)

Image　　　　Fourier spectrum



a b

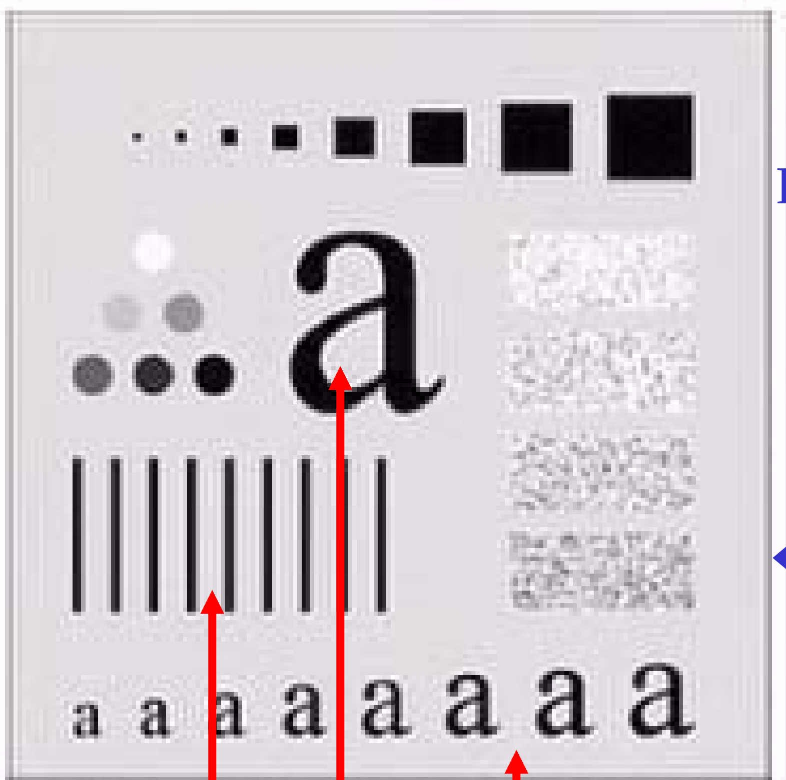
FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

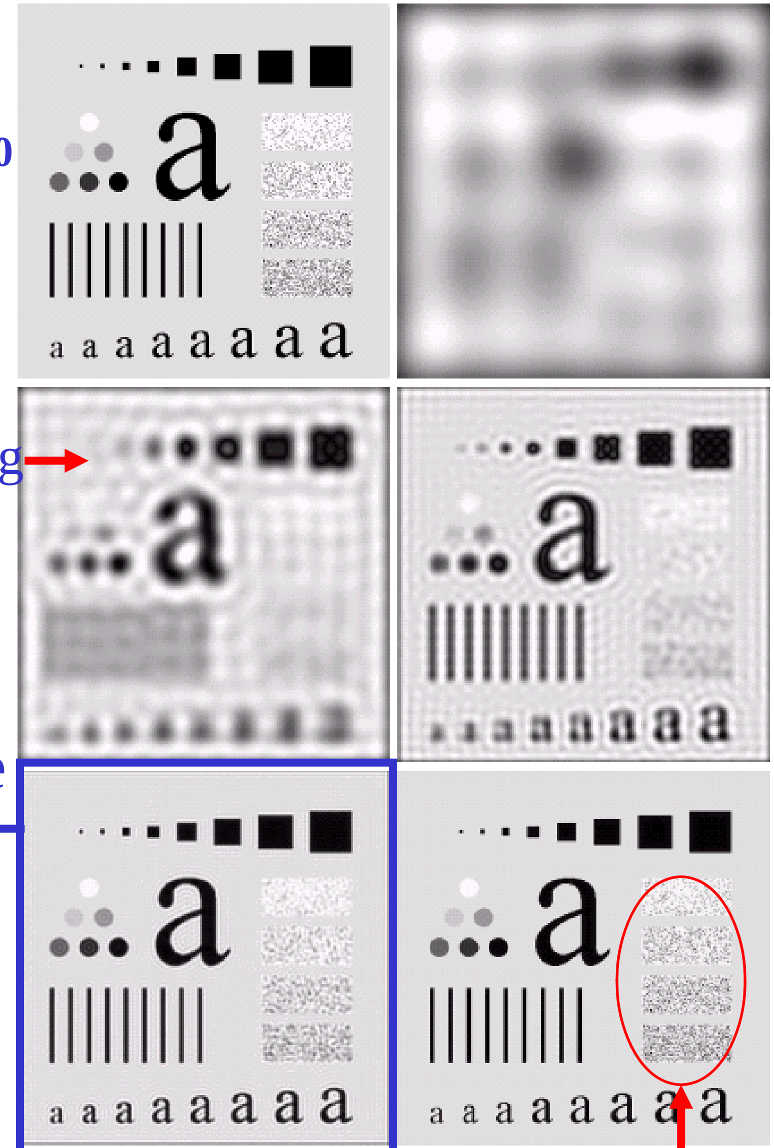
Radii(r)	Ratio(%)
5	92.0
15	94.6
30	96.4
80	98.0
230	99.5

◇ Ideal Lowpass Filters (4)

• Results of applying ideal lowpass filters

(a)Origin. (b) r=5
(c) r=15 (d) r=30
(e) r=80 (f) r=230





Ringing

Scale

Slight blurring

Ringing is evident

Ringing becomes finer as we go from (c) to (e).

◇ Ideal Lowpass Filters (5)

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- Fig.4.12(b)
  - Useless for all practical purposes
  - Eliminate all details in the image except the “blobs” representing the largest objects
  - Sharp detail is contained in the 8% power removed by the filter
- Fig. 4.12(e)
  - Ringing is evident in the image in which only 2% of the total power was removed
- Fig. 4.12(f)
  - Very slight blurring in the noisy squares, but quite close to the original
  - Little edge information is contained in the upper 0.5% of the power spectrum

◇ Ideal Lowpass Filters (6)

Blurring and ringing properties

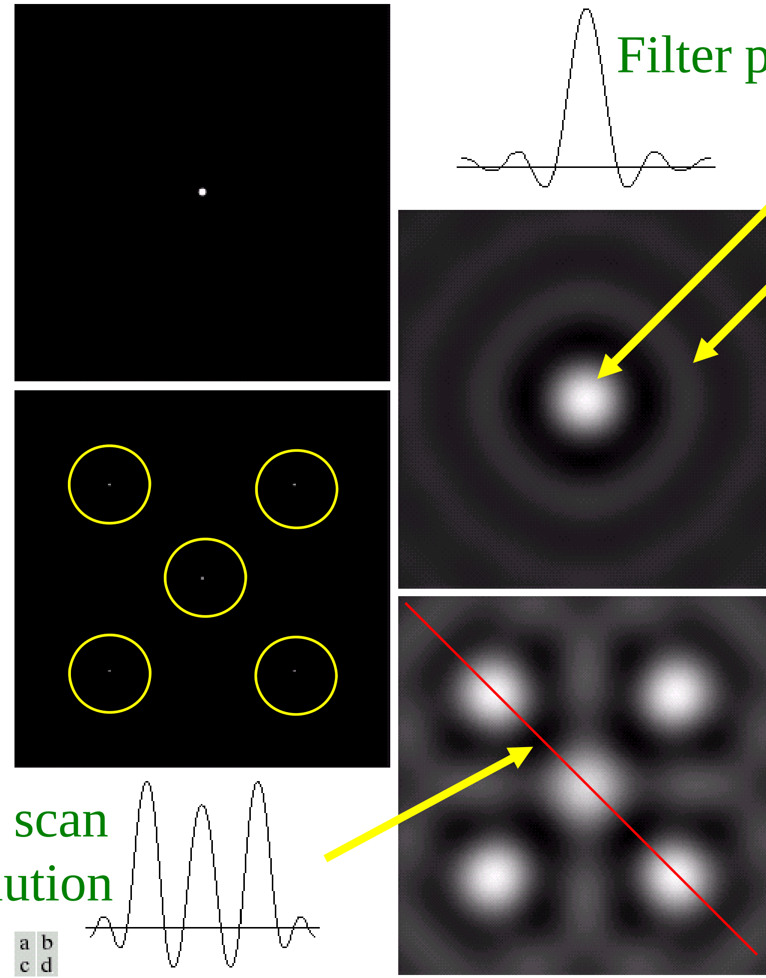

FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

◇ Ideal Lowpass Filters (7)

- Blurring and ringing properties can be explained by the convolution.

$$G(u,v) = H(u,v)F(u,v)$$

$$g(x,y) = h(x,y) * f(x,y)$$

$f(x,y)$: Original image
$g(x,y)$: Blurred image
$h(x,y)$: Filter function

- Convolution of *h(x,y)* and *f(x,y)* is simply a process of "copying" *h(x,y)* at the location of each impulse.

- Filter function *h(x,y)* has two distinctive characteristics.
  - Center component at the origin is primarily responsible for blurring.
  - Concentric circle is responsible primarily for ringing.

- Ringing is so severe and is caused by their interference with one another.

- Radius of the center component and the number of circles are inversely proportional to the value of the cutoff frequency.

- Blurring and ringing are more severe in the narrower cutoff frequency.

4.3.2 Butterworth Lowpass Filters (1)

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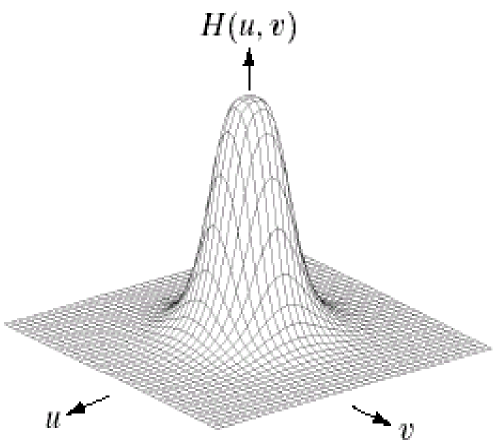
- The transfer function
  - smooth transition without sharp discontinuity

$$\boxed{H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}}$$

D_0 : cutoff frequency, when H(u, v) = 0.5

n : filter order

$$where\ D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$



Perspective plot

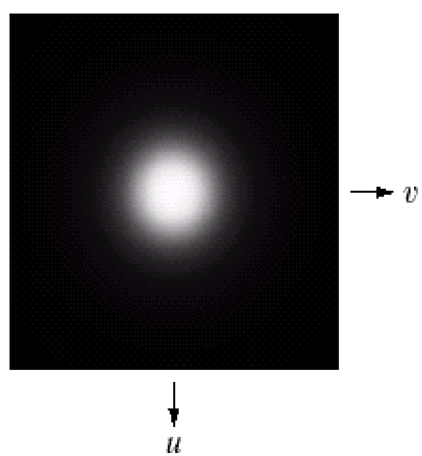
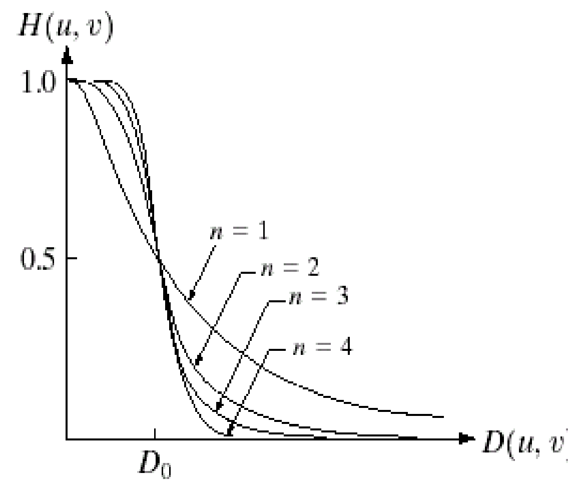


Image display

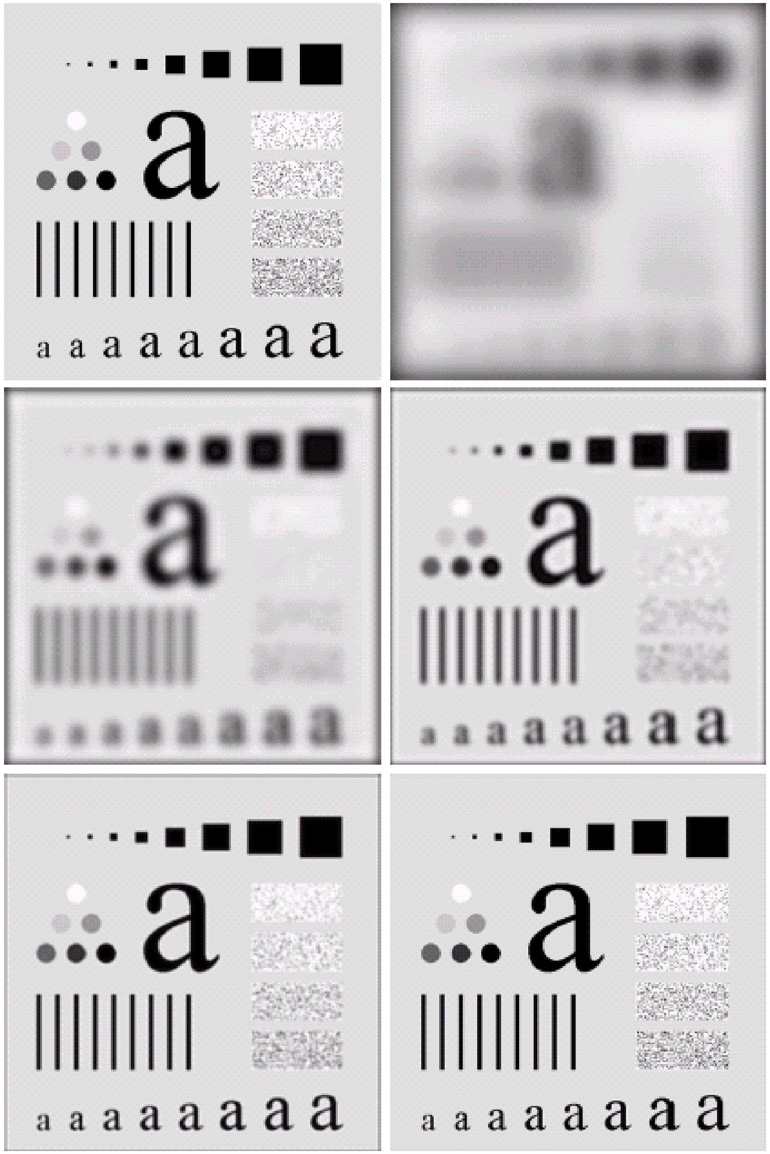


Radial cross section

◇ Butterworth Lowpass Filters (2)

(a) Original

(b) n = 2
D_0 = 5



(c) D_0 = 15

(d) D_0 = 30

(e) D_0 = 80

(f) D_0 = 230

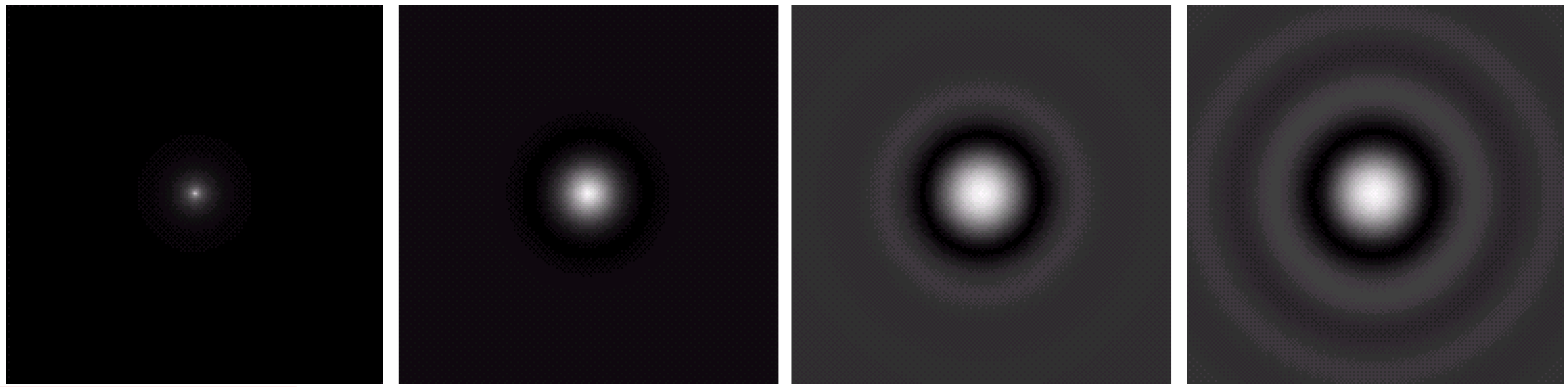
- no ringing
- smooth transition in blurring as a function of cutoff freq.

a b
c d
e f

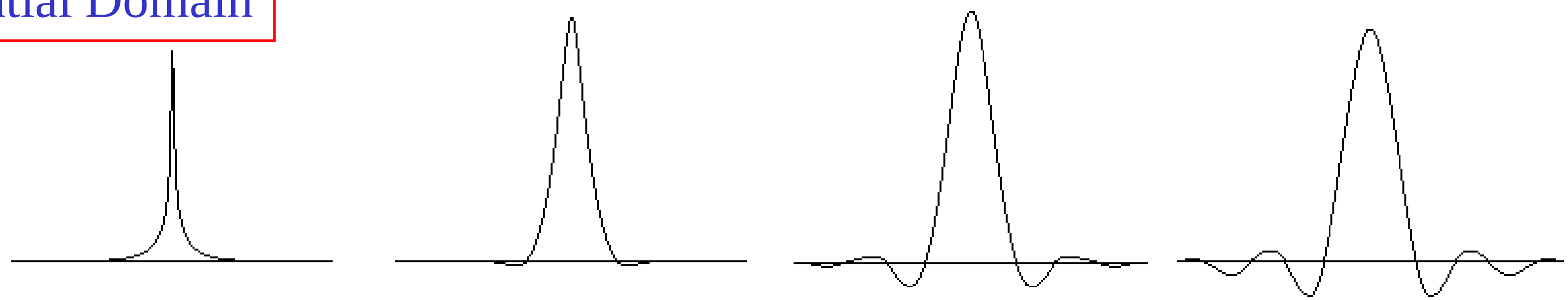
FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

◇ Butterworth Lowpass Filters (3)

- Comparison between BLPFs with the various orders



Spatial Domain



(a) $N = 1, D_0 = 5$
No ringing
No negative value

(b) $N = 2, D_0 = 5$
Mild ringing
Small negative value

(c) $N = 5, D_0 = 5$
Significant ringing
Some negative value

(d) $N = 20, D_0 = 5$
Identical to ILPF

- BLPFs of order 2 are good compromise between effective lowpass filtering and acceptable ringing characteristics

4.3.3 Gaussian Lowpass Filters (1)

- 2D transfer function

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

$D(u,v)$: Distance of (u, v) from the origin of the FT

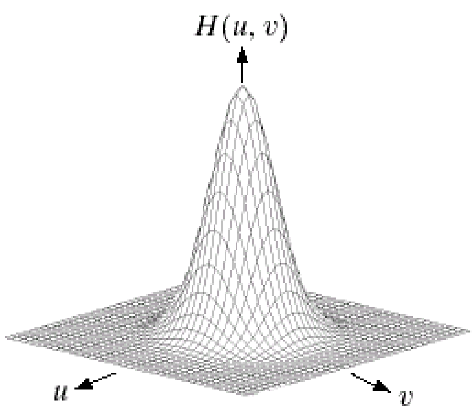
σ : Measure of the spread of the Gaussian curve

D_0 : Cutoff frequency

- Let $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

프로그램 숙제



Perspective plot

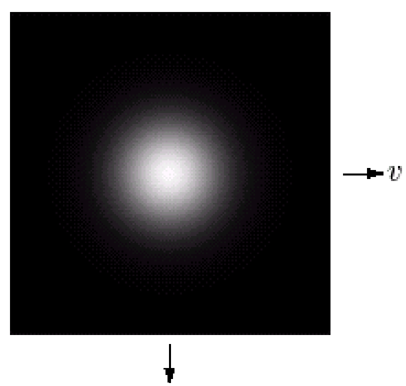
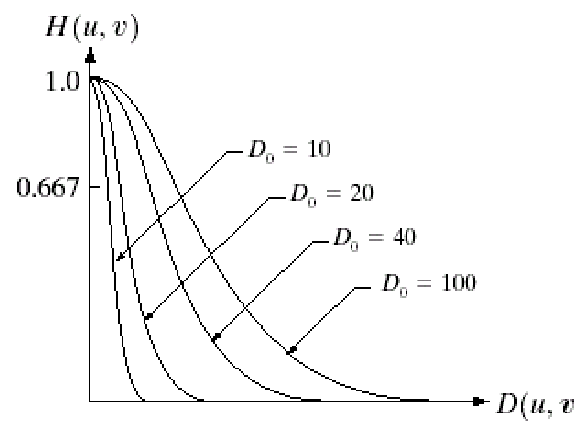


Image display



Radial cross section

◇ Gaussian Lowpass Filters (2)

(a) Original

(b) $D_0 = 5$

(c) $D_0 = 15$

(d) $D_0 = 30$

(e) $D_0 = 80$

(f) $D_0 = 230$

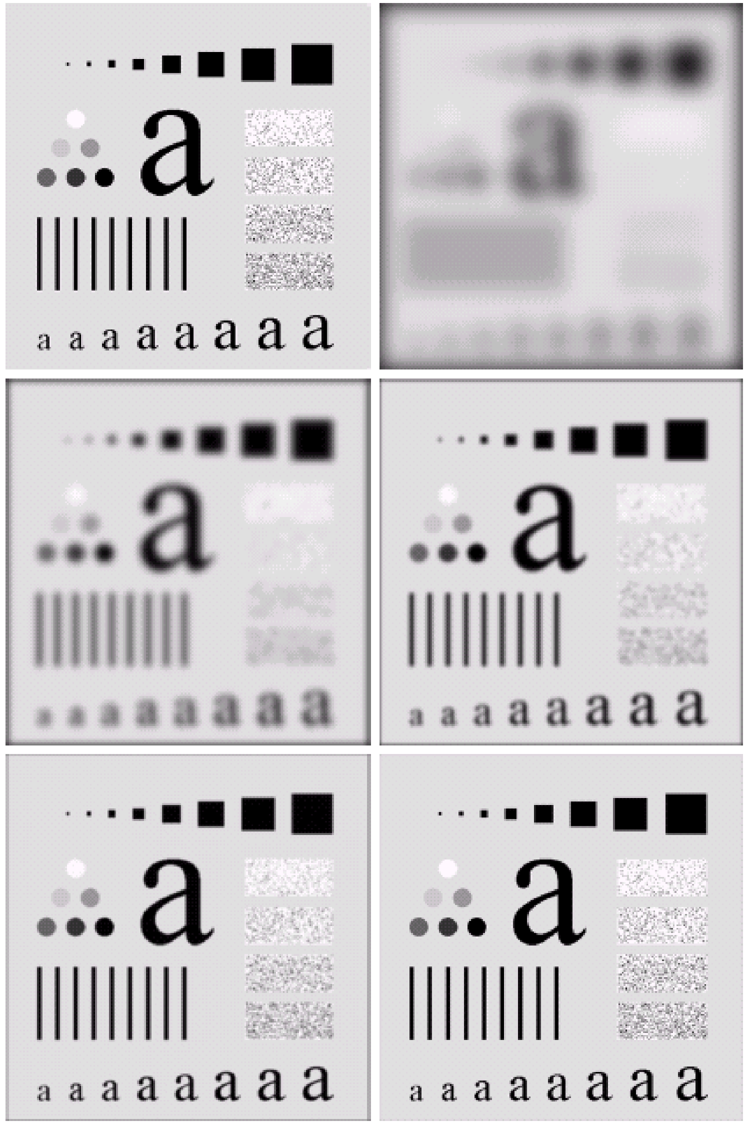


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

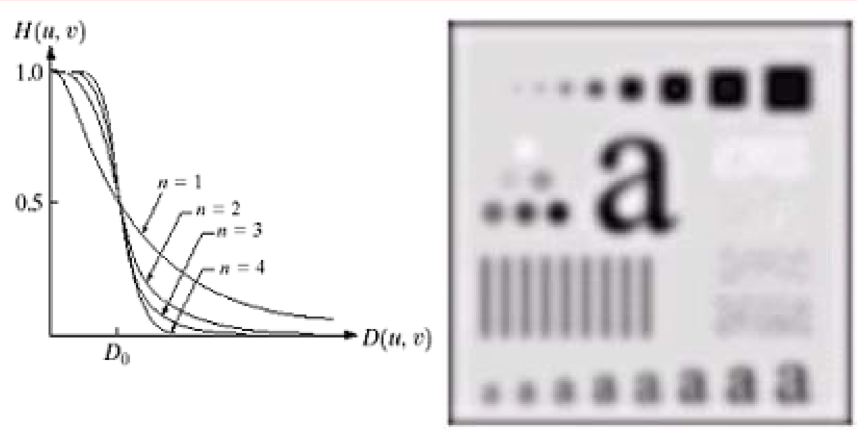
a	b
c	d
e	f

◇ Gaussian Lowpass Filters (3)

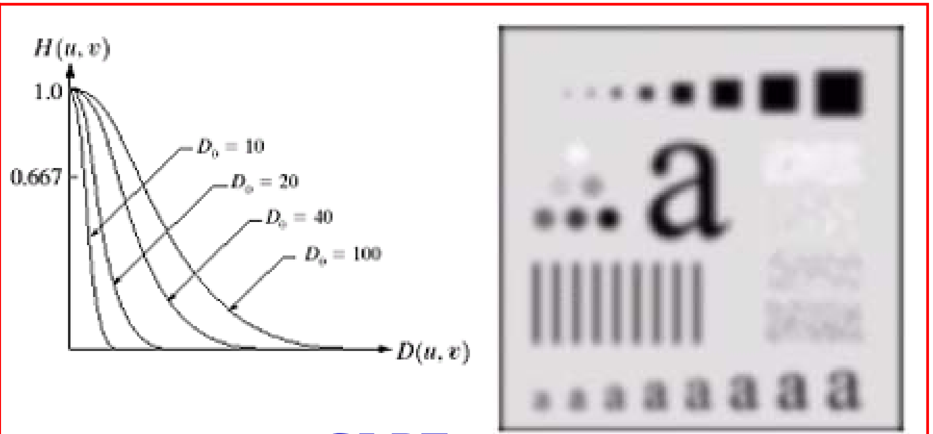
- **Characteristics of the GLPF against the BLPF of order 2 for same cutoff frequency**
  - **- the profile is less tight**
  - **- less smoothing**
  - **- no ringing**

- **No ringing is important characteristic in practice, especially in situation where any type artifact (e.g, in medical imaging) is not acceptable**

- **BLPF can control transition tightly over filter profile**
  - The price of this control is the possibility of ringing



BLPF of order 2



GLPF

4.3.4 Additional Examples of Lowpass Filtering (1)

- A few practical applications of LPF
  - machine perception
  - printing and publishing industry
  - processing satellite and aerial images

◇ Additional Examples of Lowpass Filtering (2)

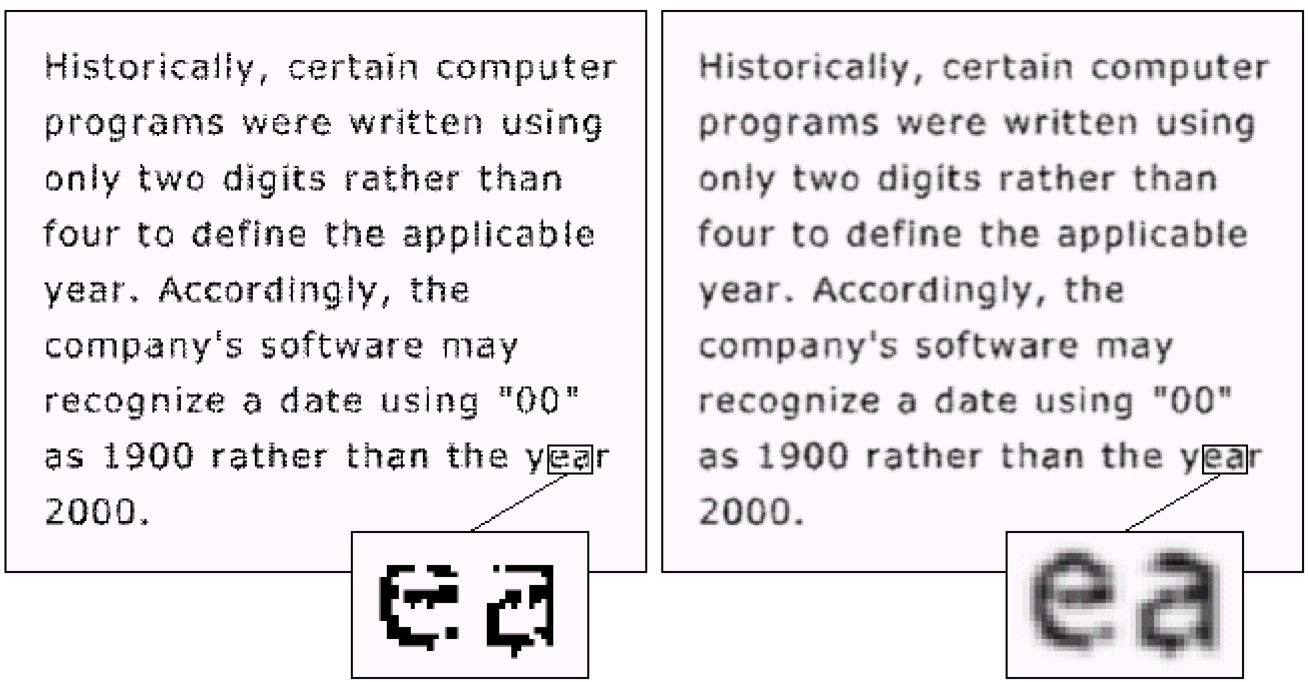
- A machine recognition system has real difficulties reading broken characters.
- Broken characters are "repaired" by blurring.

(a) Text of poor resolution

(b) GLPF with $D_0 = 80$

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



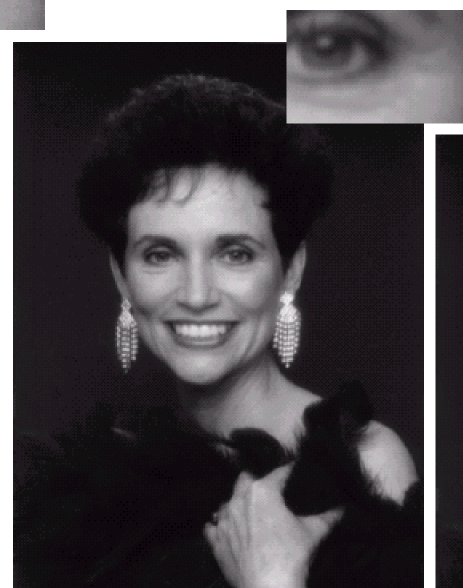
broken characters

repaired characters

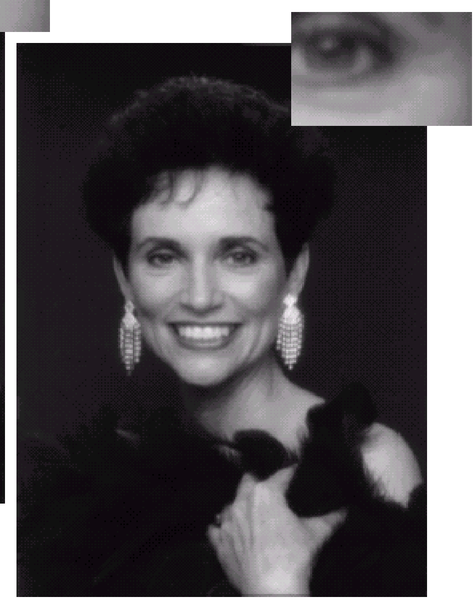
◇ Additional Examples of Lowpass Filtering (3)

- “Cosmetic” processing is another use of lowpass filtering prior to printing.
- A significant reduction in fine skin lines around the eyes
- The smoothed images look quite soft and pleasing

(a) Original



(b) GLPF with $D_0 = 100$



(c) GLPF with $D_0 = 80$

Figure 4.20

◇ Additional Examples of Lowpass Filtering (4)

- Fig. 4.21(a) : remotely sensed images with pronounce scan lines
- Fig. 4.21(b) : reduction in the effect of the scan lines
- Fig. 4.21(c) : blur out as much detail as possible
  while leaving large features recognizable
  - preprocessing stage for an image analysis system
  - LPF helps the analysis by averaging out features smaller than the ones of interest

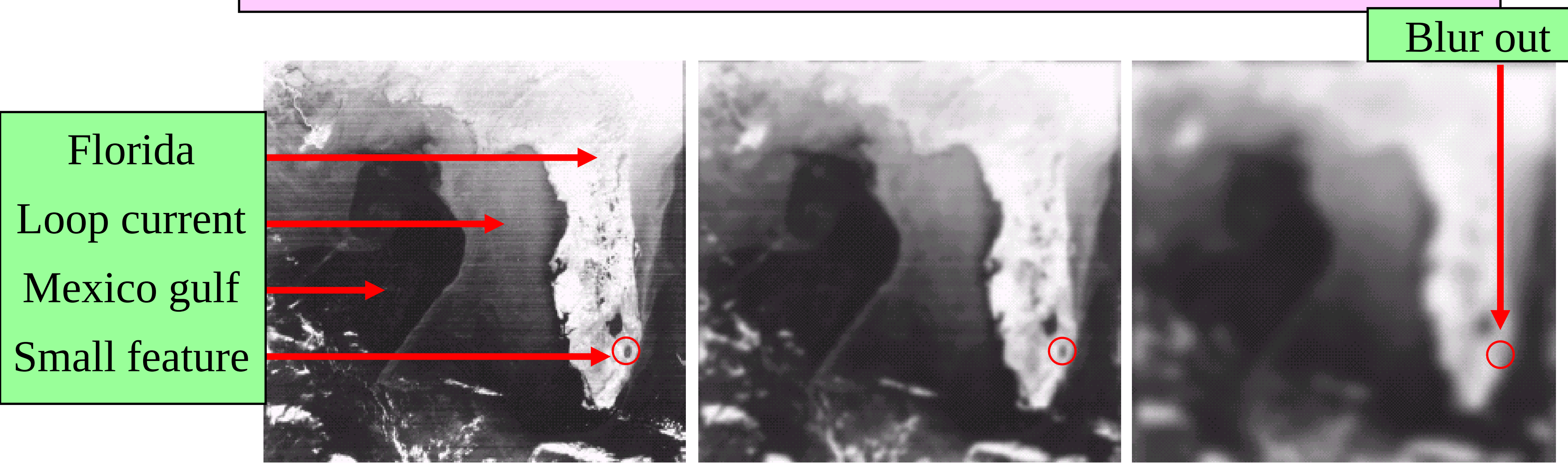


Figure 4.21

(a) Original with horizontal scan lines

(b) GLPF with $D_0 = 30$

(c) GLPF with $D_0 = 10$

4.4 Sharpening Frequency Domain Filters

- Ideal highpass filters
- Butterworth highpass filters
- Gaussian highpass filters
- The Laplacian in the frequency domain
- Unsharp masking, High-Boost filtering, and High-Frequency emphasis filtering

◇ Highpass Filters(1)

- Edge and other abrupt changes in gray levels are associated with high-frequency components
- Image sharpening can be achieved by highpass filtering
- Highpass filtering attenuates the low-frequency components without disturbing high-frequency information
- Highpass filtering is the reverse operation of low-pass filtering

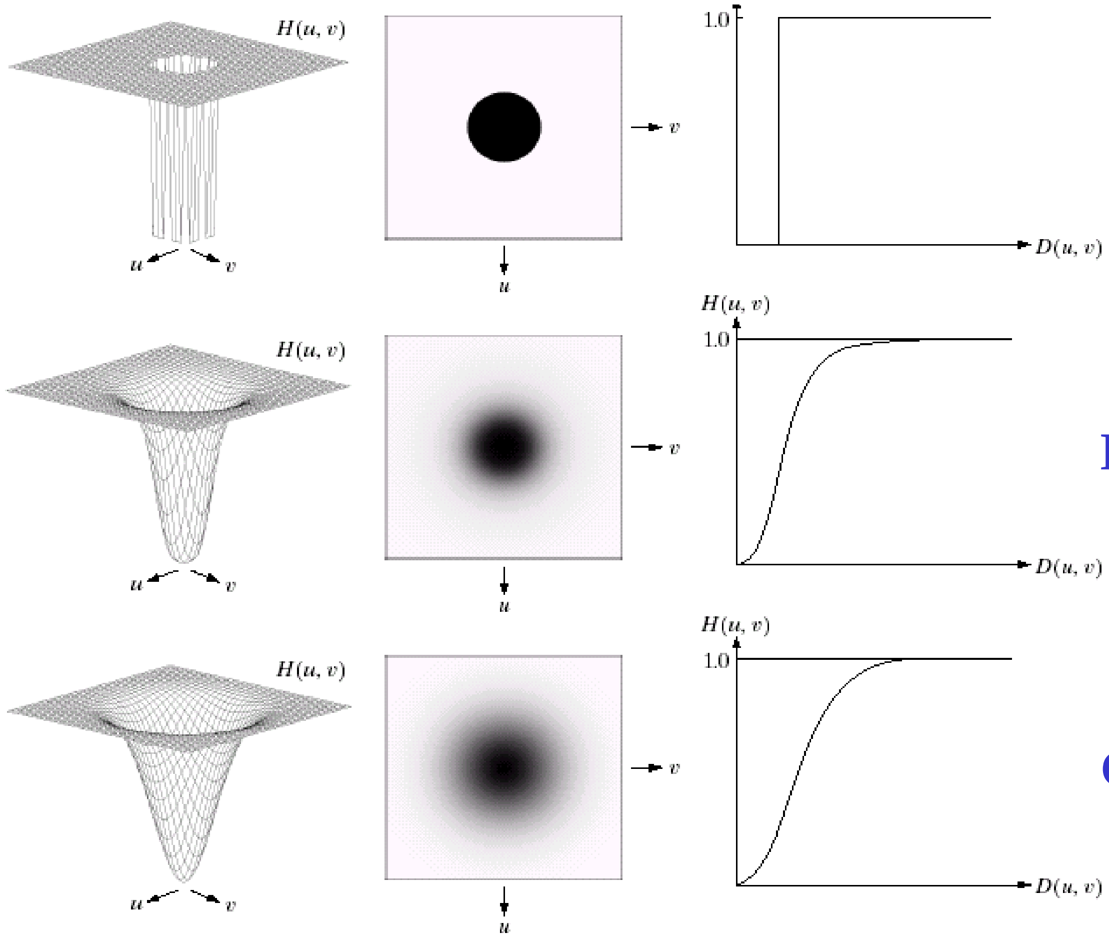
- The transfer function of the highpass filters

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

$H_{lp}(u,v)$: The transfer function of the lowpass filters

◇ Highpass Filters(2)

Perspective plot Image display Radial cross section



Ideal HPF

Butterworth HPF

Gaussian HPF

Butterworth filter represents a transition between the sharpness of the ideal filter and the smoothness of the Gaussian filter

4.4.1 Ideal Highpass Filters (1)

프로그램 읽기

- The transfer function of 2D ideal highpass filter (IHPF)

$$H(u,v) = \begin{cases} 0 & \text{if } D(u, v) \le D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

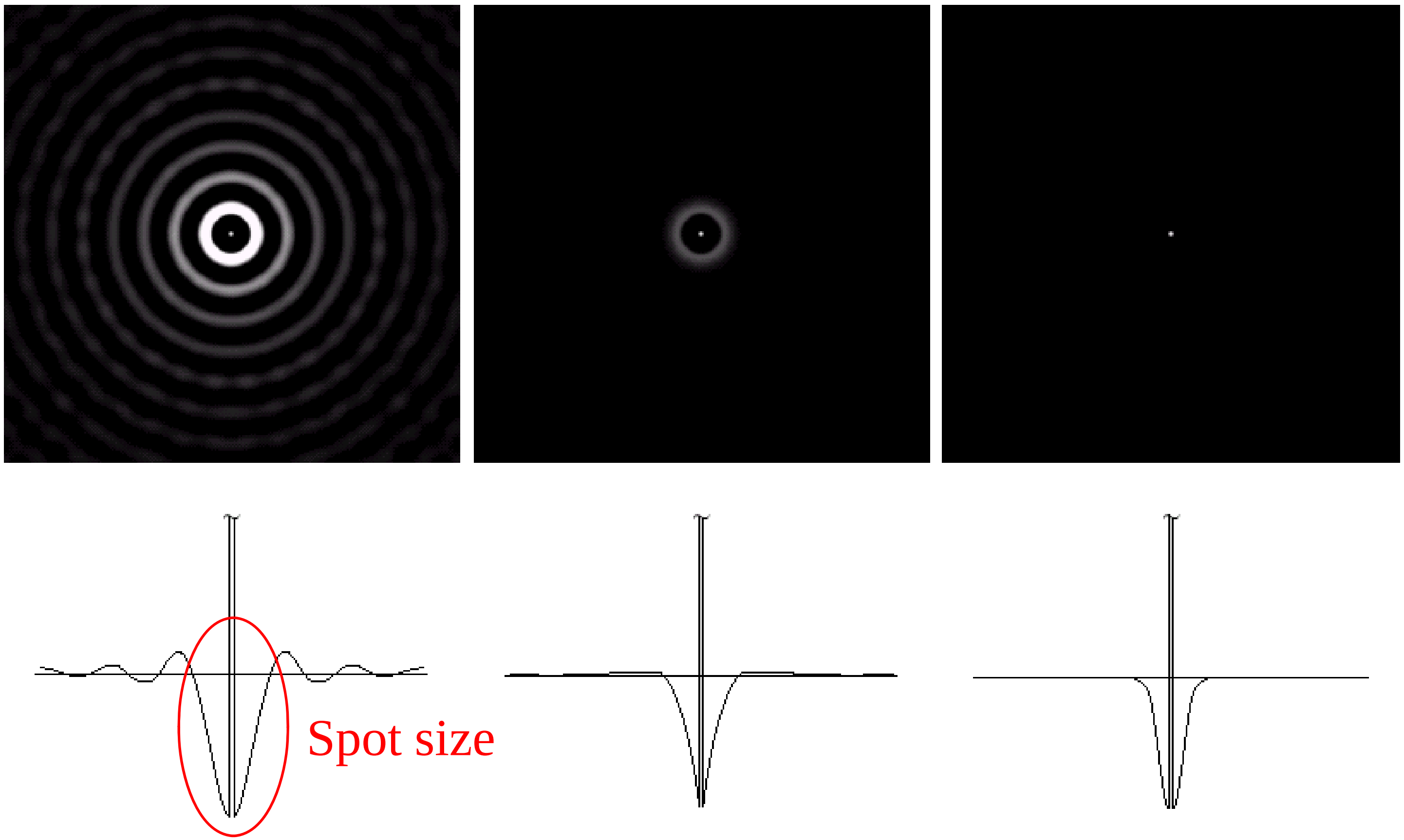
Eq. (4.4-2)

$D(u, v) = [(u-M/2)^2 + (v-N/2)^2)]^{1/2}$: distance from point (u, v) to the origin

D_0 : cutoff frequency

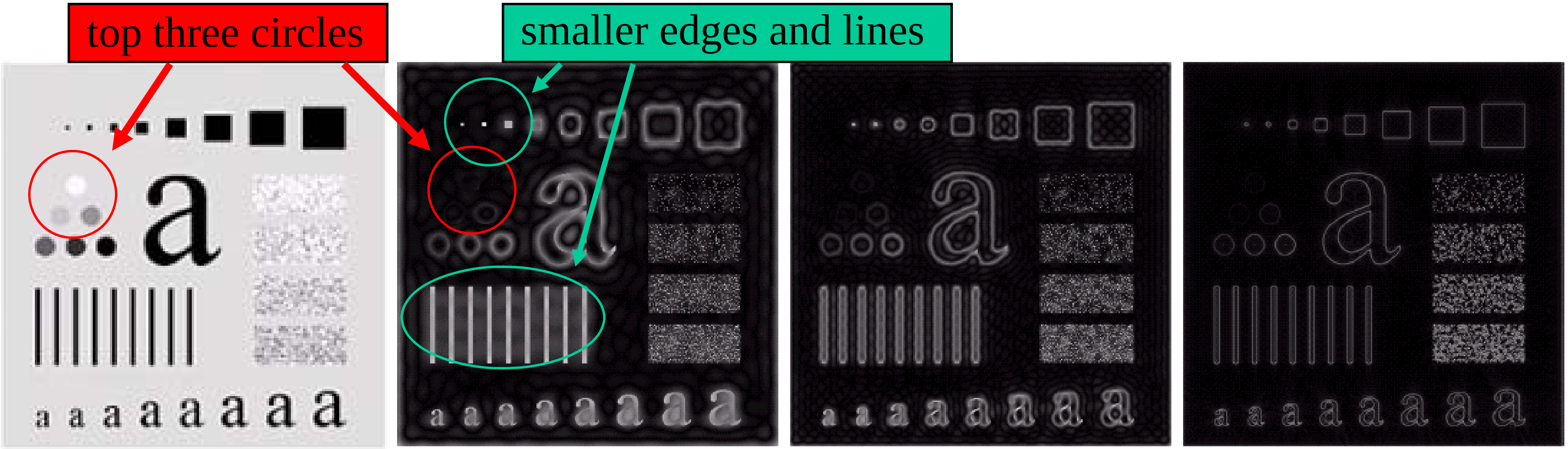
◇ Ideal Highpass Filters (2)

- Spatial representation
- IHPF has the same ringing properties as ILPF



a b c (a) Ideal (b) Butterworth (c) Gaussian

◇ Ideal Highpass Filters (3)



(a) Original (b) $D_0 = 15$ (c) $D_0 = 30$ (d) $D_0 = 80$

- Fig. 4.24(b) : the ringing is so severe that it produced distorted, thickened object boundaries
  - Edges of the top three circles do not show well because they are not as strong as the other edges
  - The smaller edges and lines appear almost solid white
- Fig. 4.24(c) : the situation improved somewhat with $D_0 = 30$
  - Edge distortion still is evident
- Fig. 4.24(d) : the edges are much cleaner and less distorted, and smaller object have been filtered properly
  - Reason : the filter with large D_0 has small spot size in the spatial domain

4.4.2 Butterworth Highpass Filters(BHPF) (1)

- **Transfer function**

프로그램 실습

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

Eq. (4.4-3)

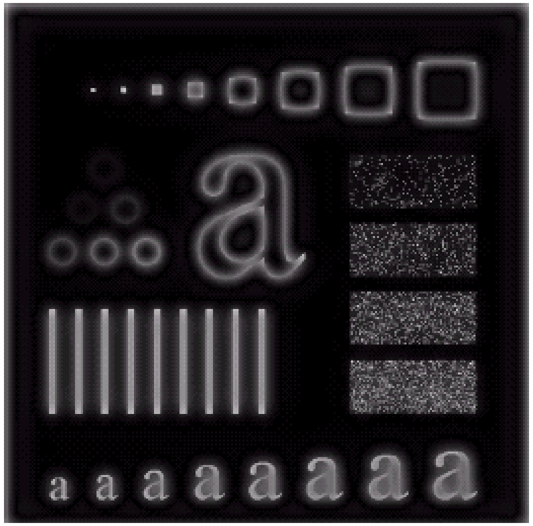
$$D(u,v) = [\left(u - M/2\right)^2 + \left(v - N/2\right)^2]^{1/2}$$

D_0 : cutoff frequency when $H(u, v) = 0.5$

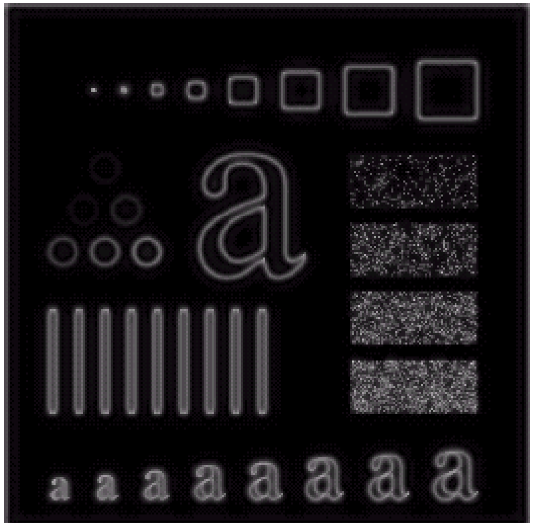
n : order

◇ Butterworth Highpass Filters (2)

- **BHPF behaves smoother than IHPF**
- **Example : BHPF of order 2**
- **The boundaries are much less distorted than IHPF**
- **Since the center spot sizes of the IHPF and BHPF is similar, their performance in filtering the small objects is comparable**



(a) $D_0 = 15$



(b) $D_0 = 30$

(c) $D_0 = 80$

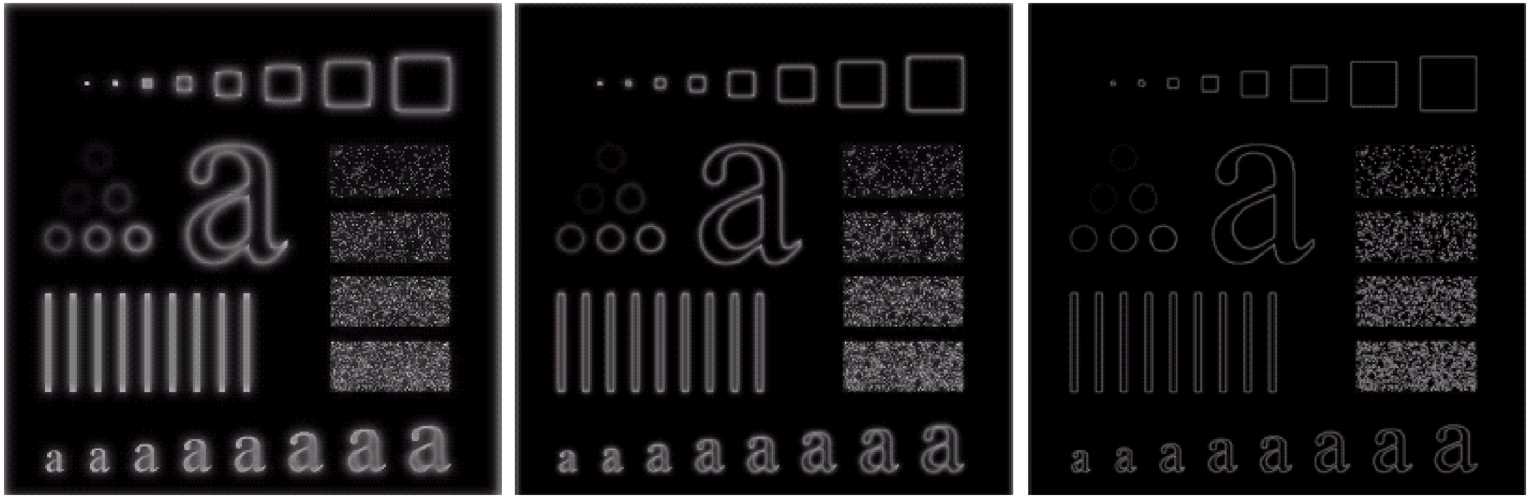
4.4.3 Gaussian Highpass Filters

프로그램 숙제

- The transfer function

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

$D(u,v)$: distance of (u, v) from the origin
D_0 : cutoff frequency



(a) $D_0 = 15$ (b) $D_0 = 30$ (c) $D_0 = 80$

- The results of the GHPF are smoother than with the previous two filters
- Even the filtering of the smaller objects and thin bars is cleaner with the GHPF

4.4.4 The Laplacian in the Freq. Domain(1)

- Fourier transform of derivative

$$\Im\left[\frac{d^n f(x)}{dx^n}\right] = (j2\pi u/M)^n F(u), \qquad f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M}$$

- Fourier transform of the Laplacian

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = (j2\pi u/M)^2 F(u,v) + (j2\pi v/N)^2 F(u,v)$$

$$\Im\left[\nabla^2 f(x,y)\right] = -(2\pi)^2\{(u/M)^2 + (v/N)^2\}F(u,v)$$

- Laplacian filter in the frequency domain

$$H(u,v) = -(2\pi)^2\{(u/M)^2 + (v/N)^2\}$$

- The shifted center of the filter : (0,0) → (M/2, N/2)

$$H(u,v) = -(2\pi)^2[\{(u-M/2)/M\}^2 + \{(v-N/2)/N\}^2]$$

◇ The Laplacian in the Freq. Domain(2)

- The Laplacian-filtered image in the spatial domain

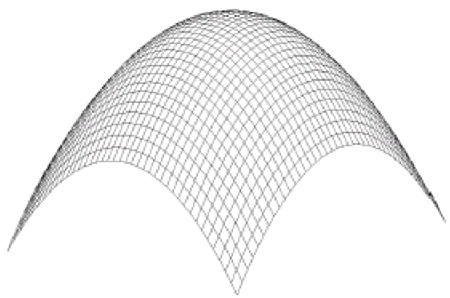
$$\nabla^2 f(x,y) = \Im^{-1}\{H(u,v)F(u,v)\}$$
$$= \Im^{-1}\left\{-(2\pi)^2[\{(u-M/2)/M\}^2 + \{(v-N/2)/N\}^2]F(u,v)\right\}$$

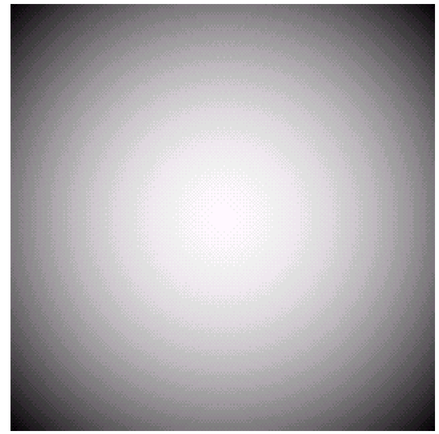
- Laplacian filter function in the spatial domain

$$h(x,y) = \Im^{-1}\{-(2\pi)^2[\{(u-M/2)/M\}^2 + \{(v-N/2)/N\}^2]\}$$

◇ The Laplacian in the Freq. Domain(3)

(a) 3D plot of H(u,v)

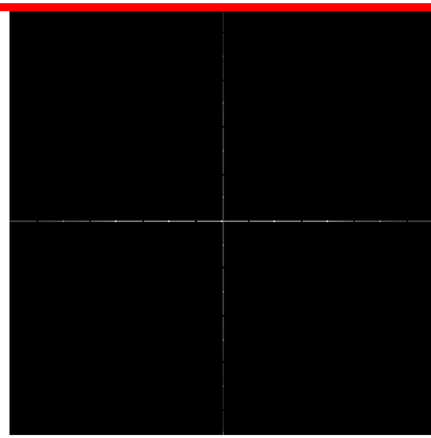


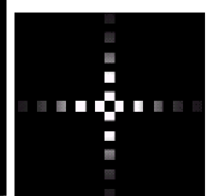


(b) Image of H(u,v)

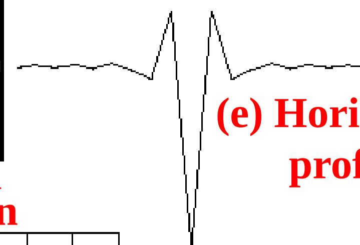
Frequency domain

(c) Laplacian h(x,y)





(d) Zoom near origin



(e) Horizontal profile

0	1	0
1	–4	1
0	1	0

(f) Mask

Spatial domain

a b
c d e
f

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

◇ The Laplacian in the Freq. Domain(4)

- The enhanced image in the spatial domain from chapter 3

$$g(x,y) = f(x,y) - \nabla^2 f(x,y) \qquad \text{for the negative center coeff.}$$

- The enhanced image in the frequency domain

$$g(x,y) = \Im^{-1}\{\underbrace{[1-(2\pi)^2\{((u-M/2)/M)^2+((v-N/2)/N)^2]}_{H(u,v)}F(u,v)\}$$

- Filter function for the enhanced image

$$H(u,v) = 1-(2\pi)^2[\{(u-M/2)/M\}^2+\{(v-N/2)/N\}^2]$$

◇ The Laplacian in the Freq. Domain(5)

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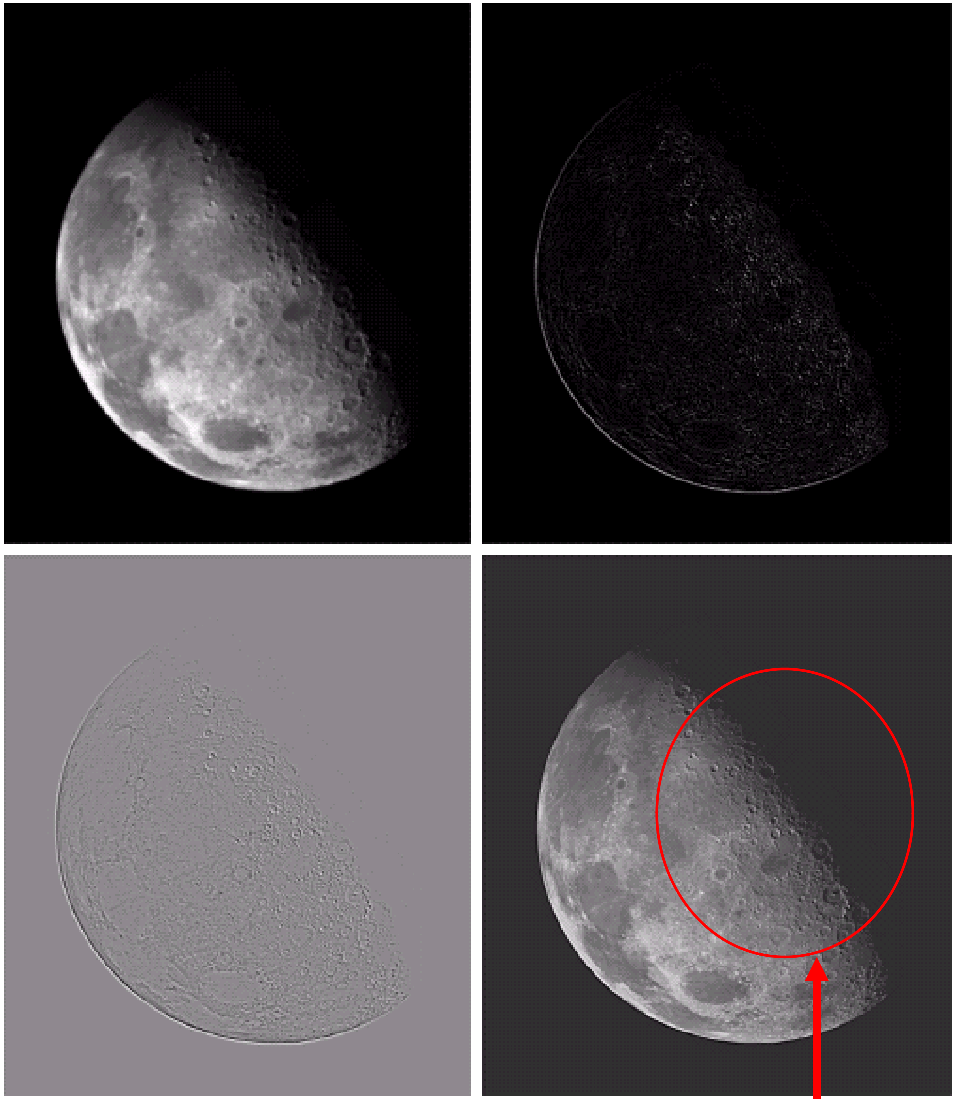
(a) Original

(b) Laplacian image



(c) Scaled Laplacian for display

(d) Enhancement (a) + (b)

The increase in sharpness of small features is unmistakable.

4.4.5 Unsharp Masking, High-Boost Filtering, High-Frequency Emphasis Filtering

- Image enhancement using the highpass filters
  - Adding a portion of the original image to the filtered image
- Example : enhancement using Laplacian
- Unsharp Masking,
- High-Boost Filtering,
- High-Frequency Emphasis Filtering

◇ Unsharp masking, High-Boost Filtering(1)

- Unsharp masking generates a sharp image (Highpass-filtered image)
- Highpass-filtered image = the original image - a lowpass-filtered image

$$f_{hp}(x, y) = f(x, y) - f_{lp}(x, y)$$

$f_{hp}(x, y)$: *highpass-filtered image*

$f_{lp}(x, y)$: *lowpass-filtered image*

- **High-boost filtering is a generalization of unsharp masking**

$$f_{hb} = Af(x, y) - f_{lp}(x, y)$$

$f_{hb}(x, y)$: *high-boost-filtered image*

$A \geq 1$: high-boost filtering

$= 1$: regular HPF

- The constant A is a flexible factor for obtaining the enhanced image

$$f_{hb}(x, y) = (A-1)f(x, y) + f(x, y) - f_{lp}(x, y)$$

$$f_{hb}(x, y) = (A-1)f(x, y) + f_{hp}(x, y)$$

◇ Unsharp masking, High-Boost Filtering(2)

- Implementation of unsharp masking in the frequency domain

$$F_{hp}(u,v) = F(u,v) - F_{lp}(u,v)$$

$$where \quad , F_{lp}(u,v) = H_{lp}(u,v)\ F(u,v)$$

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

$$where, H_{lp}(u,v) : transfer\ \ function\ of\ LPF$$

- Implementation of **high-boost filtering**

$$H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$$

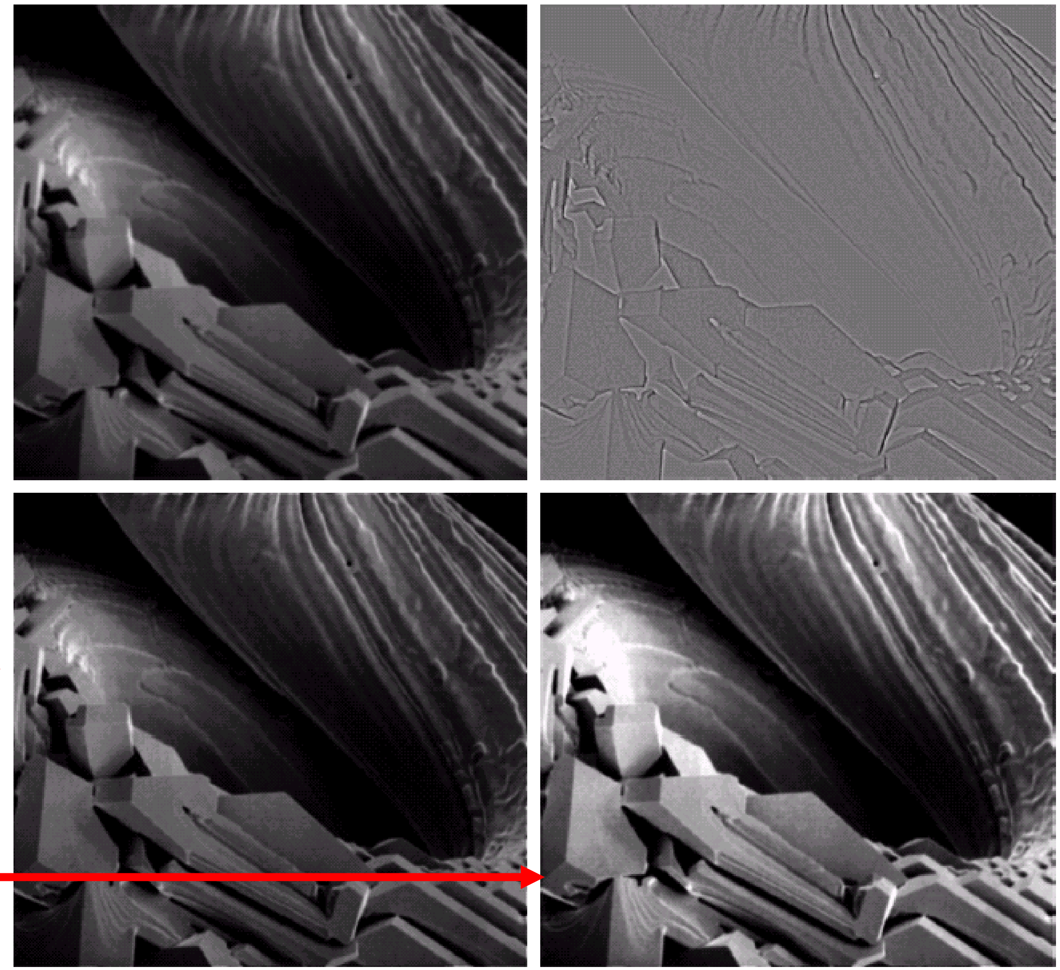
프로그램 읽기

◇ Unsharp masking, High-Boost Filtering(3)

a b
c d

FIGURE 4.29
Same as Fig. 3.43,

- **(a) Input image**
- **(b) Laplacian of (a)**
- **(c) HBF with A=2**
- **(d) HBF with A=2.7**



sharper but still too dark

improved results

◇ High-Frequency Emphasis Filtering(1)

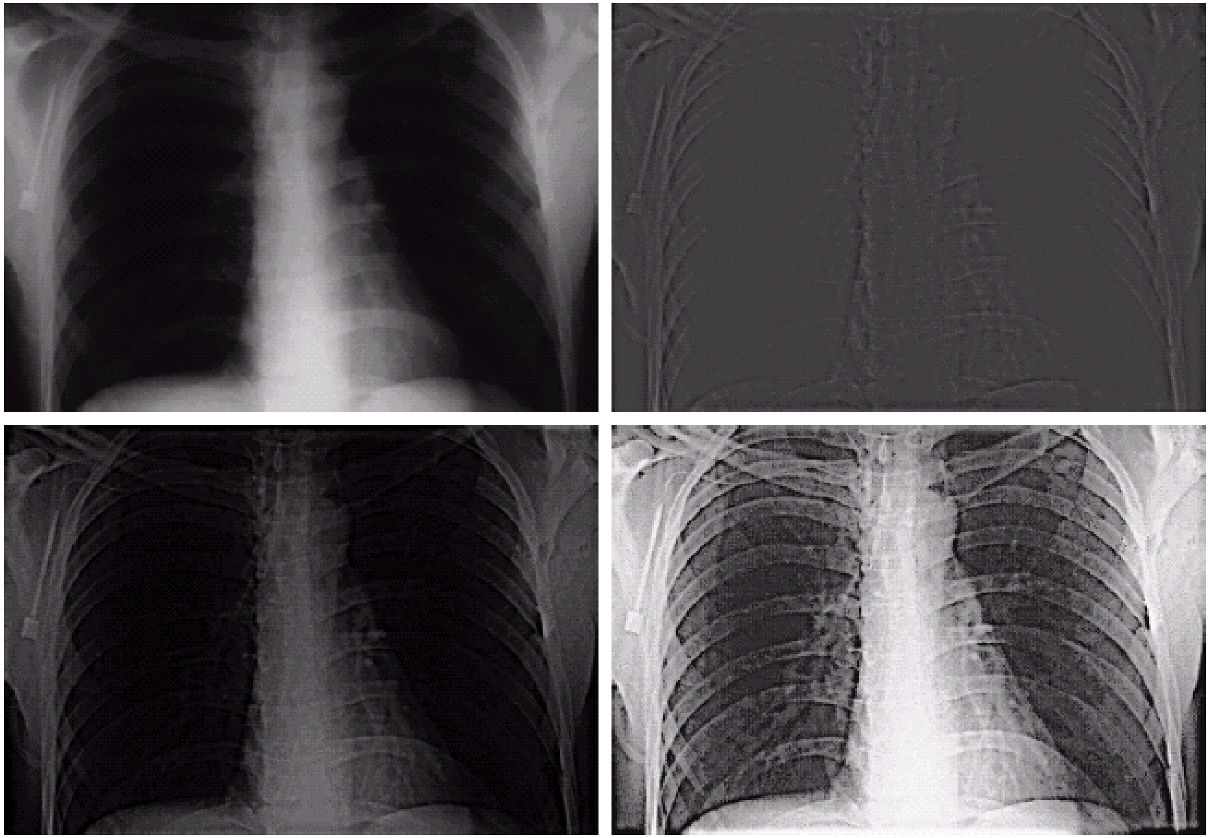
- Sometimes it is advantageous to accentuate the high-frequency components of an image
- A transfer function for high frequency emphasis filter

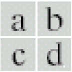
$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$

$$where \quad , b \geq a > 0$$

- Typical values for a and b
  - a = 0.25~0.5
  - b = 1.5~2.0
- Relationship
  - a = (A-1), b = 1 : high-boost filtering
  - a = (A-1), b > 1 : high frequency emphasis filtering
  - a = 0, b = 1 : regular highpass filtering

◇ High-Frequency Emphasis Filtering(2)





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(a) Chest X-ray Image
(b) Butterworth highpass filtering
(c) High-frequency emphasis filtering (a=0.5, b=2.0)
(d) Histogram equalization on (c).

- Fig. 4.30(c) : The advantage of high frequency emphasis filter is shown.
- Fig. 4.30(d) : Note clarity of bone structure and other details that are not visible in other images.
  - The enhanced image is a little noisy, but this is typical of X-ray images when their gray scale is expanded.