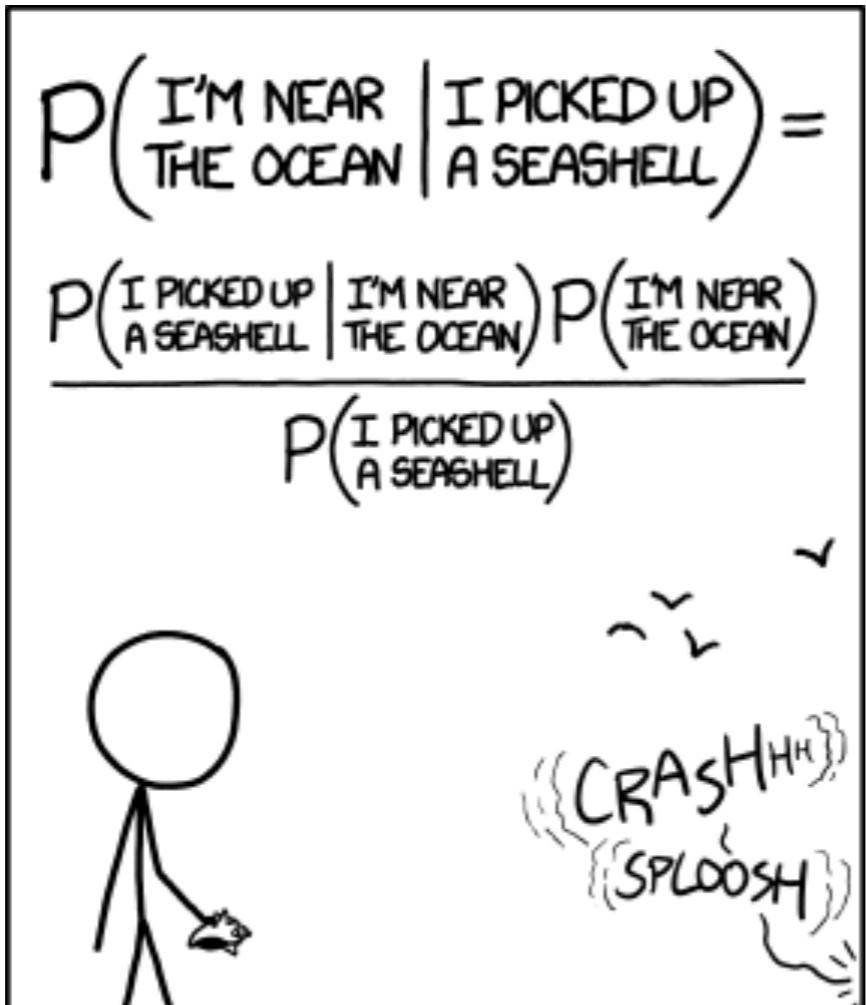


Bayesian data analysis 3



STATISTICALLY SPEAKING, IF YOU PICK UP A SEASHELL AND DON'T HOLD IT TO YOUR EAR, YOU CAN PROBABLY HEAR THE OCEAN.

MODIFIED BAYES' THEOREM:

$$P(H|X) = P(H) \times \left(1 + P(C) \times \left(\frac{P(x|H)}{P(x)} - 1 \right) \right)$$

H: HYPOTHESIS

X: OBSERVATION

P(H): PRIOR PROBABILITY THAT H IS TRUE

P(X): PRIOR PROBABILITY OF OBSERVING X

P(C): PROBABILITY THAT YOU'RE USING BAYESIAN STATISTICS CORRECTLY

Logistics

Your feedback

Your feedback

Little hard to follow today. Having one example (possibly more interesting than coin flips?) to follow throughout the lecture would help scaffold the flow.
But I'm still excited about Bayes!

Things that came up

Citations in RMarkdown

Can you point us to an R cookbook resource for integrating APA citations and a bibliography into our R Markdown scripts for the final project?

Yes!

W19-PSYCH-252-01 > Files > homework > project_report_template

The screenshot shows a file manager interface with the following details:

- Search for files:** A search bar at the top left.
- File Operations:** A toolbar with icons for search, view, download, upload, and delete.
- Selection:** A message "4 items selected" is displayed.
- Upload:** Buttons for "+ Folder" and "Upload".
- File List:** A table showing the contents of the "project_report_template" folder:

Name	Date Created	Date Modified	Modified By	Size
project_report.html	8:45pm	8:45pm	c	710 KB
project_report.pdf	8:45pm	8:45pm	c	176 KB
project_report.Rmd	8:45pm	8:45pm	c	2 KB
psych252-final-project.Rproj	8:45pm	8:45pm	c	205 bytes
references	8:46pm	--	c	
- File Structure:** A sidebar on the left shows the directory tree:
 - Statistical Methods for Be... (expanded)
 - homework (expanded)
 - 1_visualization
 - 2_wrangling
 - 4_linear_model
 - 5_modeling_data
 - 6_mixed_effects_mo...
 - 7_bayesian_data_an...
 - project_report_temp
 - midterm
- Storage Usage:** A progress bar at the bottom left indicates "15% of 5.2 GB used".
- All My Files:** A link at the bottom right.

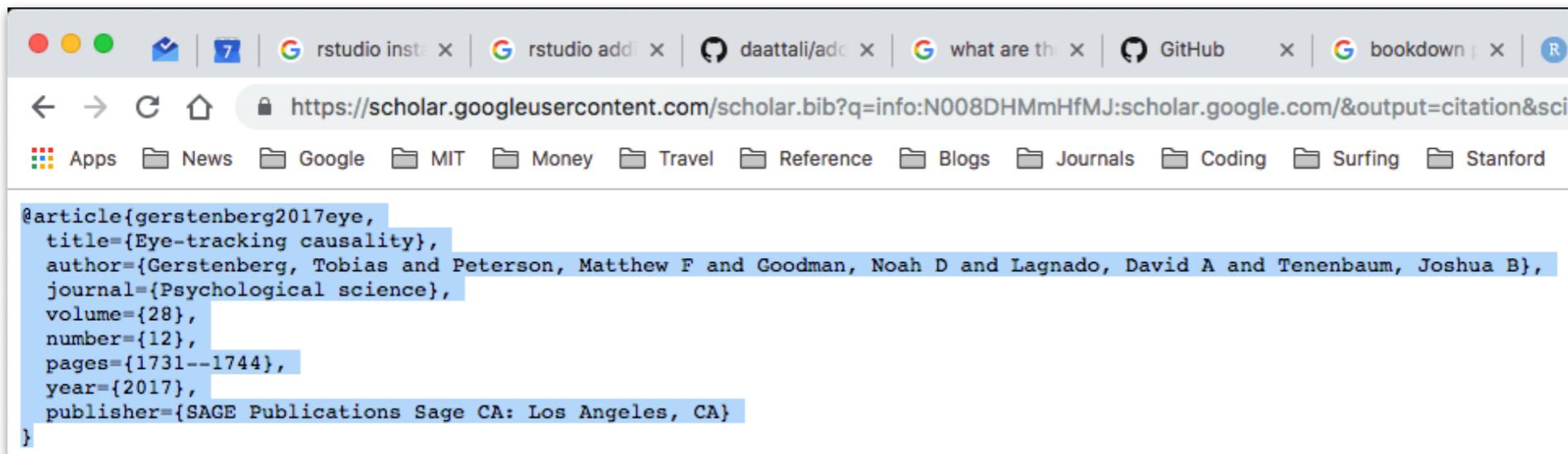
Citations in RMarkdown

Google Scholar search results for "tobias gerstenberg eye-tracking". The results page shows a single article titled "Eye-tracking causality" by T Gerstenberg, MF Peterson... from Psychological ... (2017). The "Import into BibTeX" link is highlighted with a blue oval.

If "Import into
BibTeX" link is
not shown

Google Scholar settings page. The "Bibliography manager" section is highlighted with a blue oval, showing the option "Show links to import citations into BibTeX".

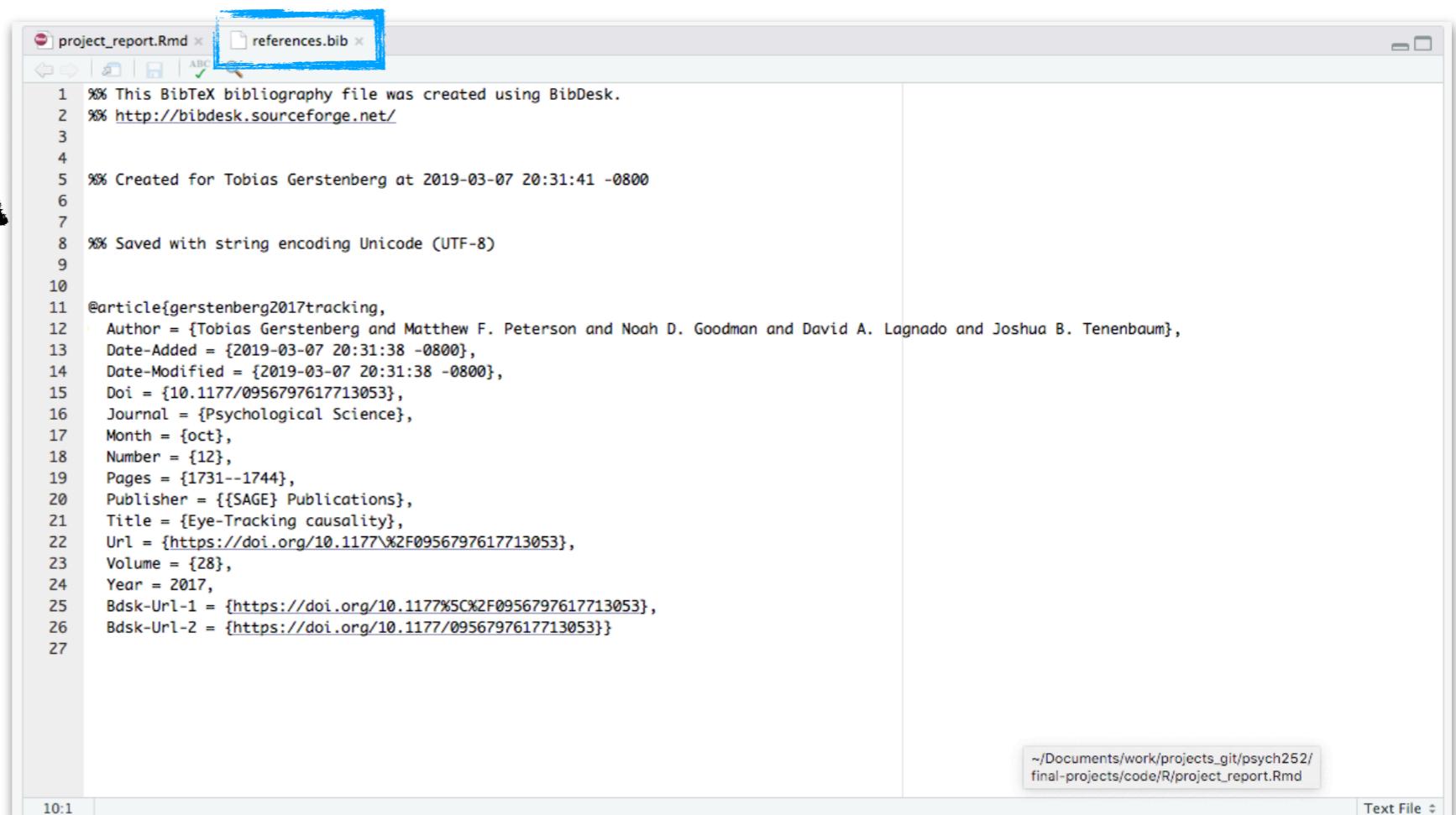
Citations in RMarkdown



A screenshot of a web browser window. The address bar shows the URL: <https://scholar.googleusercontent.com/scholar.bib?q=info:N008DHMmHfMJ:scholar.google.com&output=citation&scisrc=GGL&hl=en>. The page content displays a BibTeX entry:

```
@article{gerstenberg2017eye,
  title={Eye-tracking causality},
  author={Gerstenberg, Tobias and Peterson, Matthew F and Goodman, Noah D and Lagnado, David A and Tenenbaum, Joshua B},
  journal={Psychological science},
  volume={28},
  number={12},
  pages={1731--1744},
  year={2017},
  publisher={SAGE Publications Sage CA: Los Angeles, CA}}
```

copy and
paste



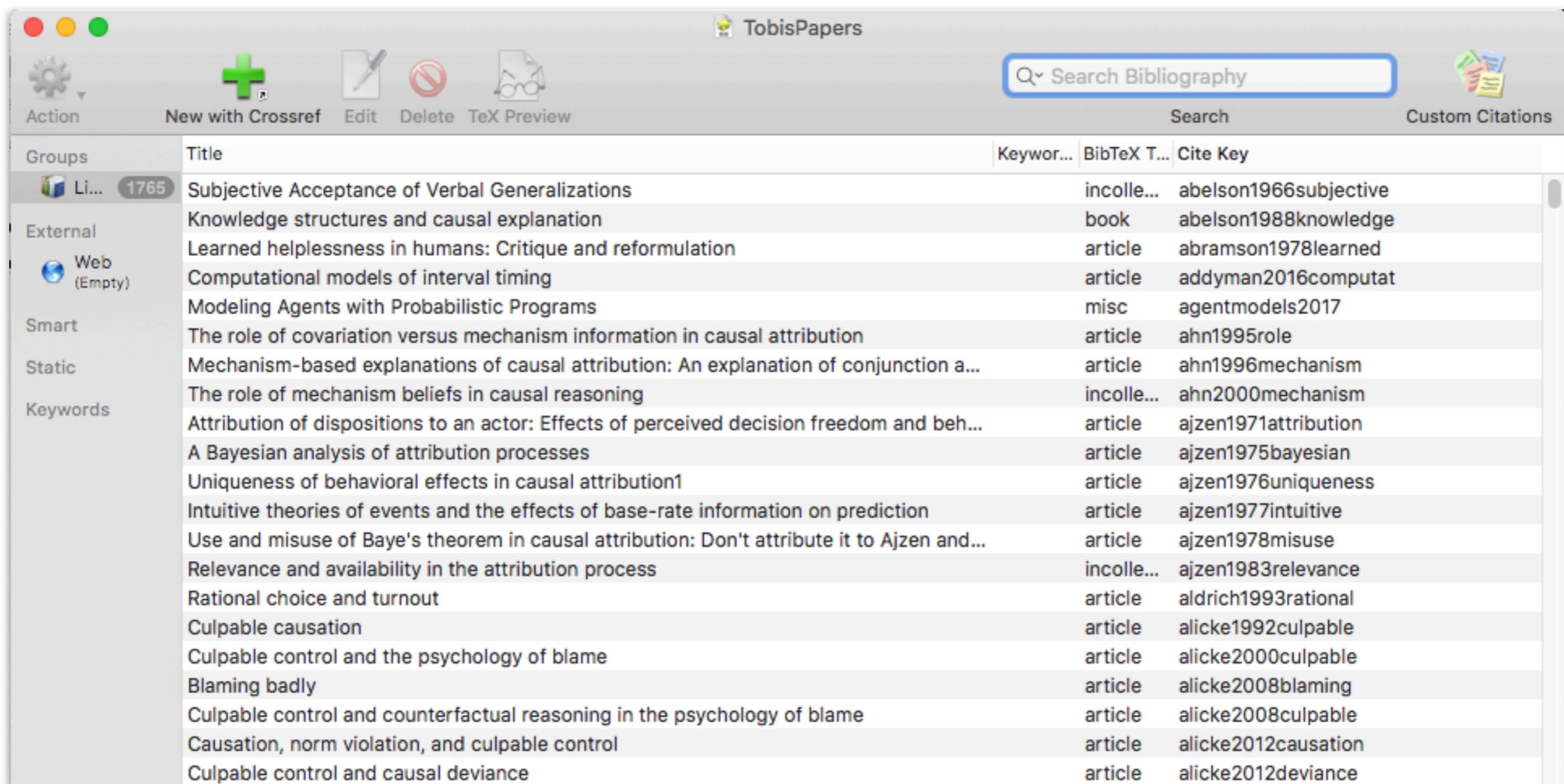
A screenshot of a text editor window titled "project_report.Rmd". The tab bar has two tabs: "project_report.Rmd" and "references.bib", with "references.bib" highlighted by a blue box. The main content area displays a BibTeX file:

```
%% This BibTeX bibliography file was created using BibDesk.
%% http://bibdesk.sourceforge.net/
%
% Created for Tobias Gerstenberg at 2019-03-07 20:31:41 -0800
%
% Saved with string encoding Unicode (UTF-8)
%
@article{gerstenberg2017tracking,
  Author = {Tobias Gerstenberg and Matthew F. Peterson and Noah D. Goodman and David A. Lagnado and Joshua B. Tenenbaum},
  Date-Added = {2019-03-07 20:31:38 -0800},
  Date-Modified = {2019-03-07 20:31:38 -0800},
  Doi = {10.1177/0956797617713053},
  Journal = {Psychological Science},
  Month = {oct},
  Number = {12},
  Pages = {1731--1744},
  Publisher = {{SAGE} Publications},
  Title = {Eye-Tracking causality},
  Url = {https://doi.org/10.1177/0956797617713053},
  Volume = {28},
  Year = 2017,
  Bdsk-Url-1 = {https://doi.org/10.1177%5C2F0956797617713053},
  Bdsk-Url-2 = {https://doi.org/10.1177/0956797617713053}}
```

The status bar at the bottom right shows the path: ~/Documents/work/projects_git/psych252/final-projects/code/R/project_report.Rmd

Citations in RMarkdown

Nice reference manager for bibtex files



The screenshot shows the BibDesk application window. The title bar reads "TobisPapers". The menu bar includes "Action", "New with Crossref", "Edit", "Delete", "TeX Preview", "Search", and "Custom Citations". A search bar at the top right says "Search Bibliography". The main area is a table with columns: "Groups", "Title", "Keywor...", "BibTeX T...", and "Cite Key". The table lists various academic papers categorized by source type (e.g., External, Web, Smart, Static, Keywords) and provides details like title, type, and citation key.

Groups	Title	Keywor...	BibTeX T...	Cite Key
Li... 1765	Subjective Acceptance of Verbal Generalizations	incolle...	abelson1966subjective	
External	Knowledge structures and causal explanation	book	abelson1988knowledge	
	Learned helplessness in humans: Critique and reformulation	article	abramson1978learned	
	Computational models of interval timing	article	addyman2016computat	
Web (Empty)	Modeling Agents with Probabilistic Programs	misc	agentmodels2017	
Smart	The role of covariation versus mechanism information in causal attribution	article	ahn1995role	
Static	Mechanism-based explanations of causal attribution: An explanation of conjunction a...	article	ahn1996mechanism	
	The role of mechanism beliefs in causal reasoning	incolle...	ahn2000mechanism	
Keywords	Attribution of dispositions to an actor: Effects of perceived decision freedom and beh...	article	ajzen1971attribution	
	A Bayesian analysis of attribution processes	article	ajzen1975bayesian	
	Uniqueness of behavioral effects in causal attribution1	article	ajzen1976uniqueness	
	Intuitive theories of events and the effects of base-rate information on prediction	article	ajzen1977intuitive	
	Use and misuse of Baye's theorem in causal attribution: Don't attribute it to Ajzen and...	article	ajzen1978misuse	
	Relevance and availability in the attribution process	incolle...	ajzen1983relevance	
	Rational choice and turnout	article	aldrich1993rational	
	Culpable causation	article	alicke1992culpable	
	Culpable control and the psychology of blame	article	alicke2000culpable	
	Blaming badly	article	alicke2008blaming	
	Culpable control and counterfactual reasoning in the psychology of blame	article	alicke2008culpable	
	Causation, norm violation, and culpable control	article	alicke2012causation	
	Culpable control and causal deviance	article	alicke2012deviance	

<https://bibdesk.sourceforge.io/>

Reference manager

zotero

Groups Documentation Forums Get Involved Log In Upgrade Storage

Use zotero (don't use Mendeley or Papers)

Your personal research assistant

Zotero is a free, easy-to-use tool to help you collect, organize, cite, and share research.

Download

Available for Mac, Windows, and Linux

New: Just need to create a quick bibliography? Try [ZoteroBib](#).

The screenshot shows the Zotero desktop application. On the left is a sidebar with a tree view of 'My Library' containing folders like 'Book Reviews', 'Colonial Medicine' (which is selected), 'Dissertation', 'Science and Empire', 'Teaching', and 'My Publications'. The main area displays a table of research items with columns for 'Title', 'Creator', and 'Year'. The 'Colonial Medicine' folder is expanded, showing several entries. A specific entry for 'Circulation of Medicine in the Early Modern Atlantic World' by Cook and Walker (2013) is selected. The right panel provides a detailed view of this item, including its type (Journal Article), title, authors (Cook, Harold J. and Walker, Timothy D.), abstract ('The search for powerful drugs has caused people and commodities to move around the globe for many centuries, as it still does...'), publication information ('Social History of Medicine' Volume 26), and note fields for 'Info', 'Notes', 'Tags', and 'Related'.

<https://www.zotero.org/>

Homework 6

Mixed effects models

How many hours did it take you to complete Homework 6?



Final presentations

QUESTIONS RESPONSES **19**

19 responses

SUMMARY INDIVIDUAL

Accepting responses

When/how will you present?

19 responses

When/How	Percentage
On March 21st (Final presentations day)	84.2%
On March 15th (Final class)	10.5%
On March 19th at 2pm (Stats instructors meeting)	5.3%
I will record the presentation and submit a video.	0%

● On March 21st (Final presentations day)
● On March 15th (Final class)
● On March 19th at 2pm (Stats instructors meeting)
● I will record the presentation and submit a video.

<https://tinyurl.com/psych252presentation>

Plan for today

- Doing Bayesian data analysis
 - A simple linear regression
 - Measuring uncertainty: Confidence interval vs. credible interval
- Building Bayesian models with `brms`
 - Model evaluation:
 - Visualizing and interpreting results
 - Testing hypotheses
 - Inference evaluation: Did things work out?
- Some `goob` examples
 - Evidence for null results
 - Dealing with unequal variance
 - Zero-one inflated beta binomial model
 - Ordinal logistic regression

Doing Bayesian data analysis

A simple linear regression

Model specification

```
1 library("greta")
2 library("tidybayes")
3
4 # variables & priors
5 b0 = normal(0, 10) ← priors
6 b1 = normal(0, 10)
7 sd = cauchy(0, 3, truncation = c(0, Inf))
8
9 # linear predictor
10 mu = b0 + b1 * attitude$complaints ← linear combination
11
12 # observation model (likelihood)
13 distribution(attitude$rating) = normal(mu, sd)
14
15 # define the model
16 m = model(b0, b1, sd)
```

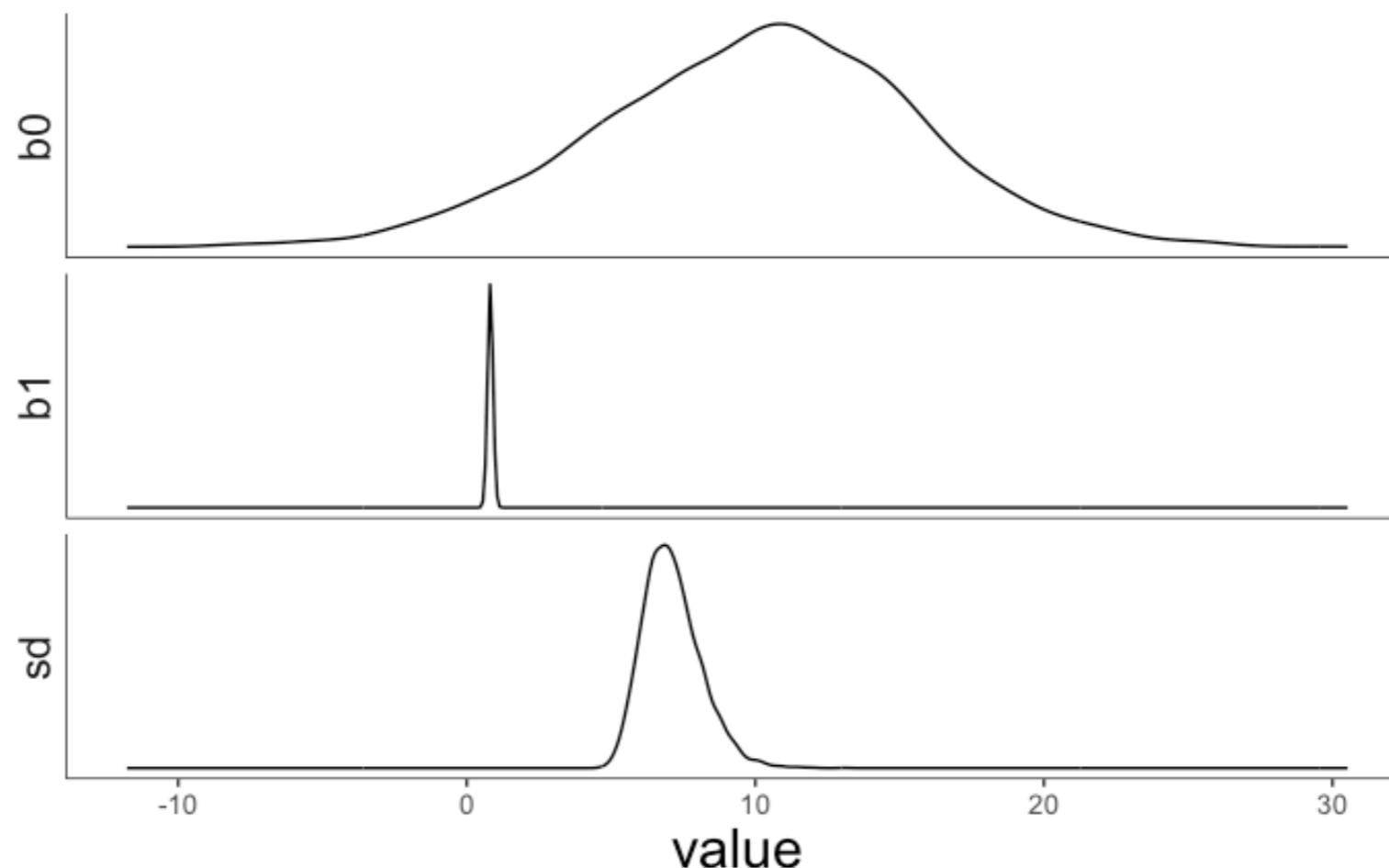
← **build the model**

← **Gaussian likelihood**

← **linear combination**

← **priors**

Summarizing results



- Posterior over each parameter is the result of the Bayesian data analysis.
- no p-values
- no confidence intervals

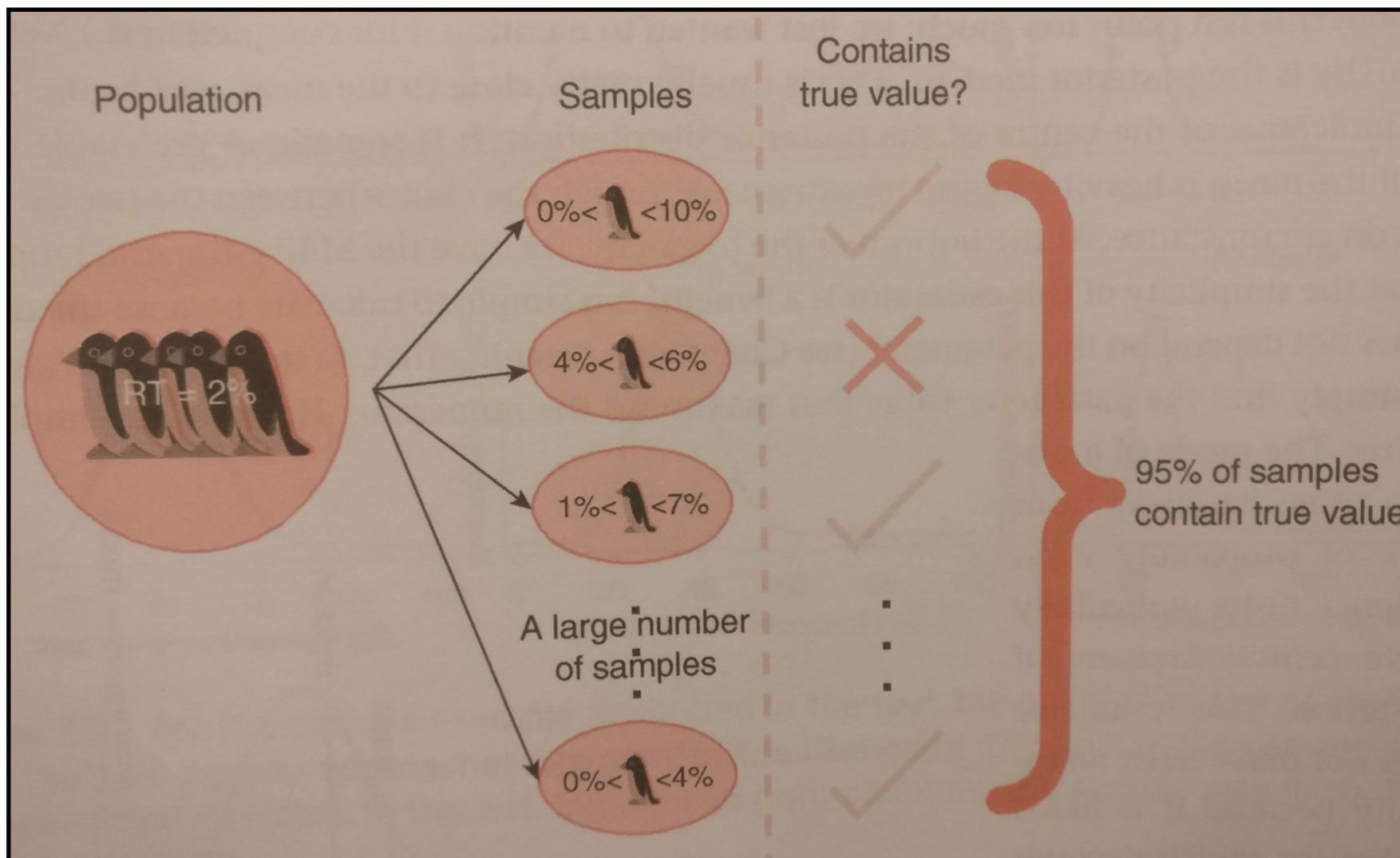
Measuring uncertainty: Confidence interval vs. credible interval

booh!

yay!

Confidence interval vs. credible interval

"From our research, we concluded that the percentage of penguins with red tails, RT, has a 95% **confidence interval** of $1\% < RT < 5\%$."



For 95% of the (hypothetical) samples, the confidence interval contains the true value.

Confidence interval vs. credible interval

"From our research, we concluded that the percentage of penguins with red tails, RT, has a 95% **credible interval** of $0\% < RT < 4\%$."

Straightforward interpretation

There is a 95% probability that the percentage of penguins with red tails lies in the range of $0\% < RT < 4\%$.

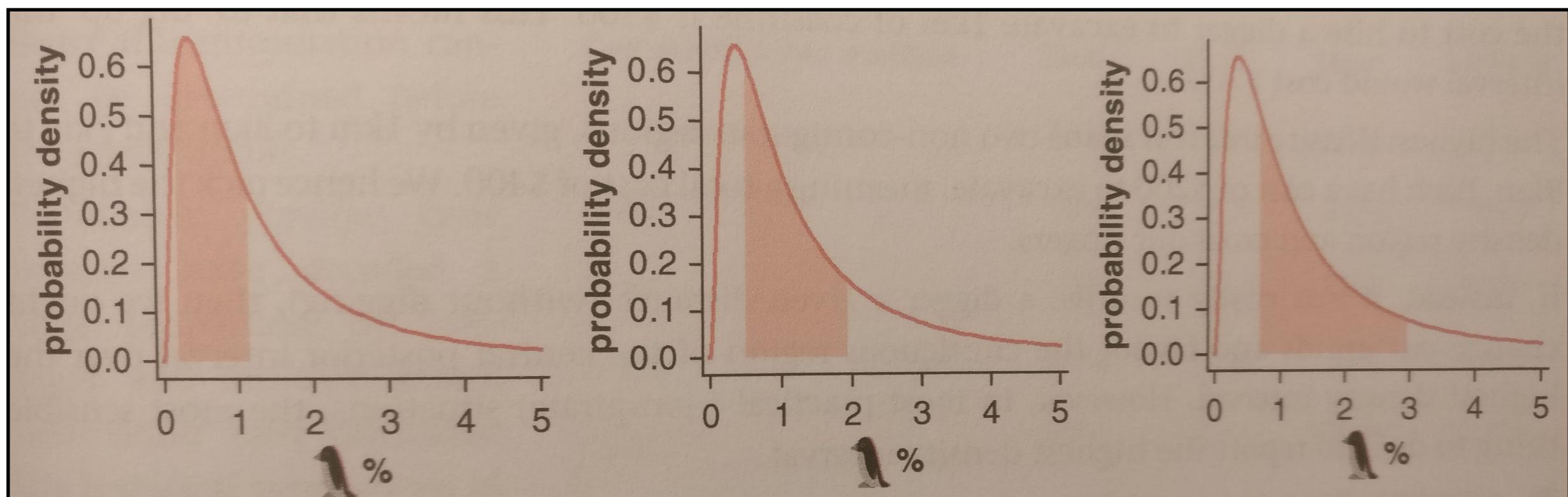
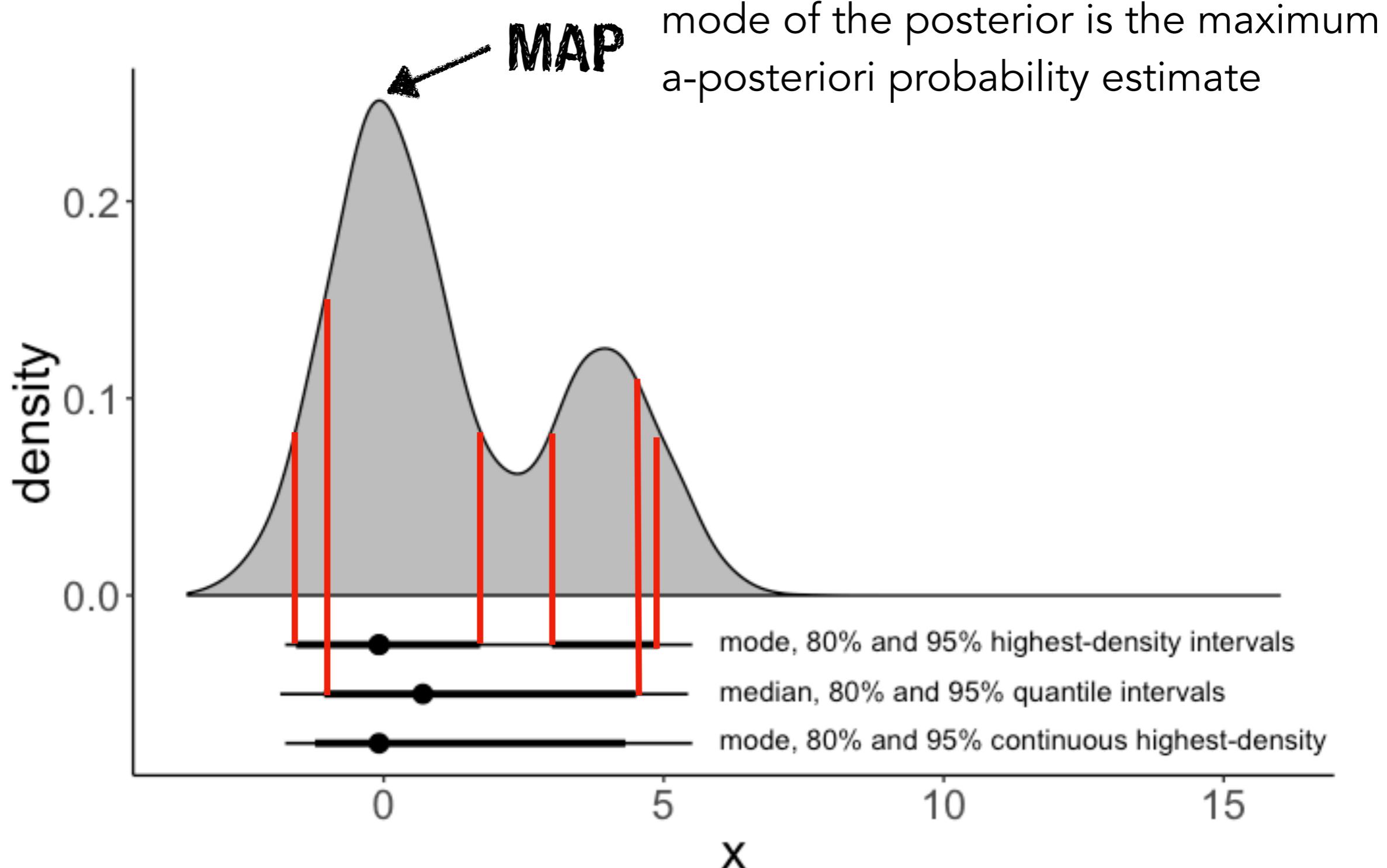


Figure 7.8 Three examples of 50% credible intervals for a parameter representing the proportion of penguins with red tails.

Different kinds of credible intervals



Building Bayesian models with brms

Software packages



Bayesian regression
modeling with Stan

```
library("brms")
```

- very powerful package that makes it easy to run Bayesian regression models
- we specify models using the same syntax we've already learned based on **lm()**, **glm()**, and **lmer()**
- brms turns this into Stan code and fits the model
- we can then use tidybayes to investigate the posterior

Software packages



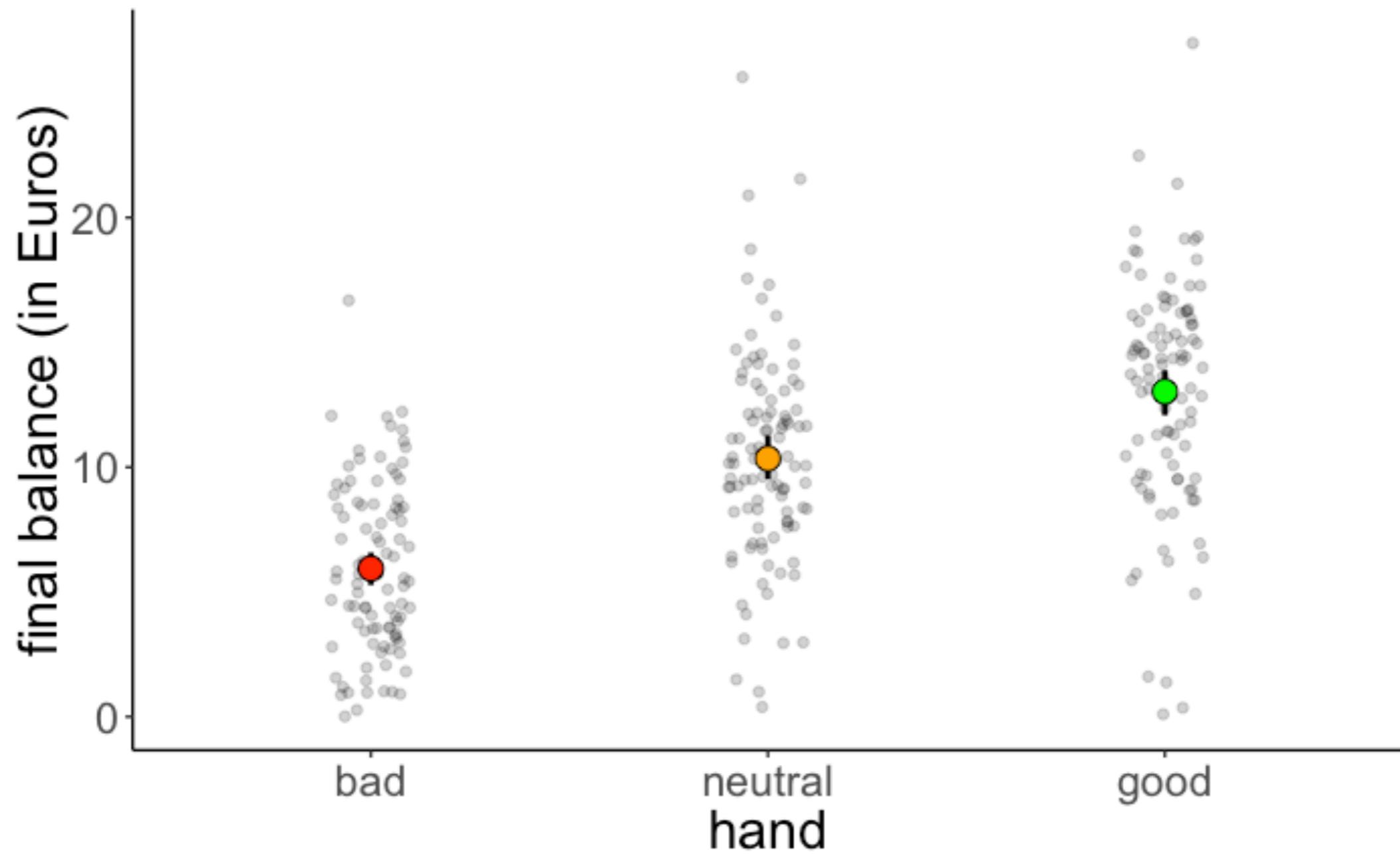
The Stan logo features a large, stylized red letter 'S' with a white diagonal line through it. Behind the 'S' are several thin, light-colored, swirling lines in shades of red and white.

Stan

Stan® is a state-of-the-art platform for statistical modeling and high-performance statistical computation. Thousands of users rely on Stan for statistical modeling, data analysis, and prediction in the social, biological, and physical sciences, engineering, and business.

<https://mc-stan.org/>

Poker data



Using lm()

```
1 fit.lm = lm(formula = balance ~ 1 + hand,  
2               data = df.poker)  
3  
4 fit.lm %>% summary()
```

```
Call:  
lm(formula = balance ~ 1 + hand, data = df.poker)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-12.9264 -2.5902 -0.0115  2.6573 15.2834  
  
Coefficients:  
              Estimate Std. Error t value Pr(>|t|)  
(Intercept) 5.9415    0.4111 14.451 < 2e-16 ***  
handneutral 4.4051    0.5815  7.576 4.55e-13 ***  
handgood    7.0849    0.5815 12.185 < 2e-16 ***  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
Residual standard error: 4.111 on 297 degrees of freedom  
Multiple R-squared:  0.3377, Adjusted R-squared:  0.3332  
F-statistic: 75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

Using brm()

```
1 fit.brm = brm(formula = balance ~ 1 + hand,  
2                  data = df.poker)  
3  
4 fit.brm %>% summary()
```

```
Family: gaussian  
Links: mu = identity; sigma = identity  
Formula: balance ~ 1 + hand  
Data: df.poker (Number of observations: 300)  
Samples: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;  
         total post-warmup samples = 4000
```

Population-Level Effects:

	Estimate	Est.Error	l-95%	CI	u-95%	CI	Eff.Sample	Rhat
Intercept	5.94	0.41	5.14	6.74	3495	1.00		
handneutral	4.41	0.59	3.23	5.53	3125	1.00		
handgood	7.09	0.59	5.92	8.24	3830	1.00		

Family Specific Parameters:

	Estimate	Est.Error	l-95%	CI	u-95%	CI	Eff.Sample	Rhat
sigma	4.12	0.17	3.81	4.47	3745	1.00		

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

Comparison between lm() and brm()

lm()

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.9415	0.4111	14.451	< 2e-16 ***
handneutral	4.4051	0.5815	7.576	4.55e-13 ***
handgood	7.0849	0.5815	12.185	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

brm()

Population-Level Effects:

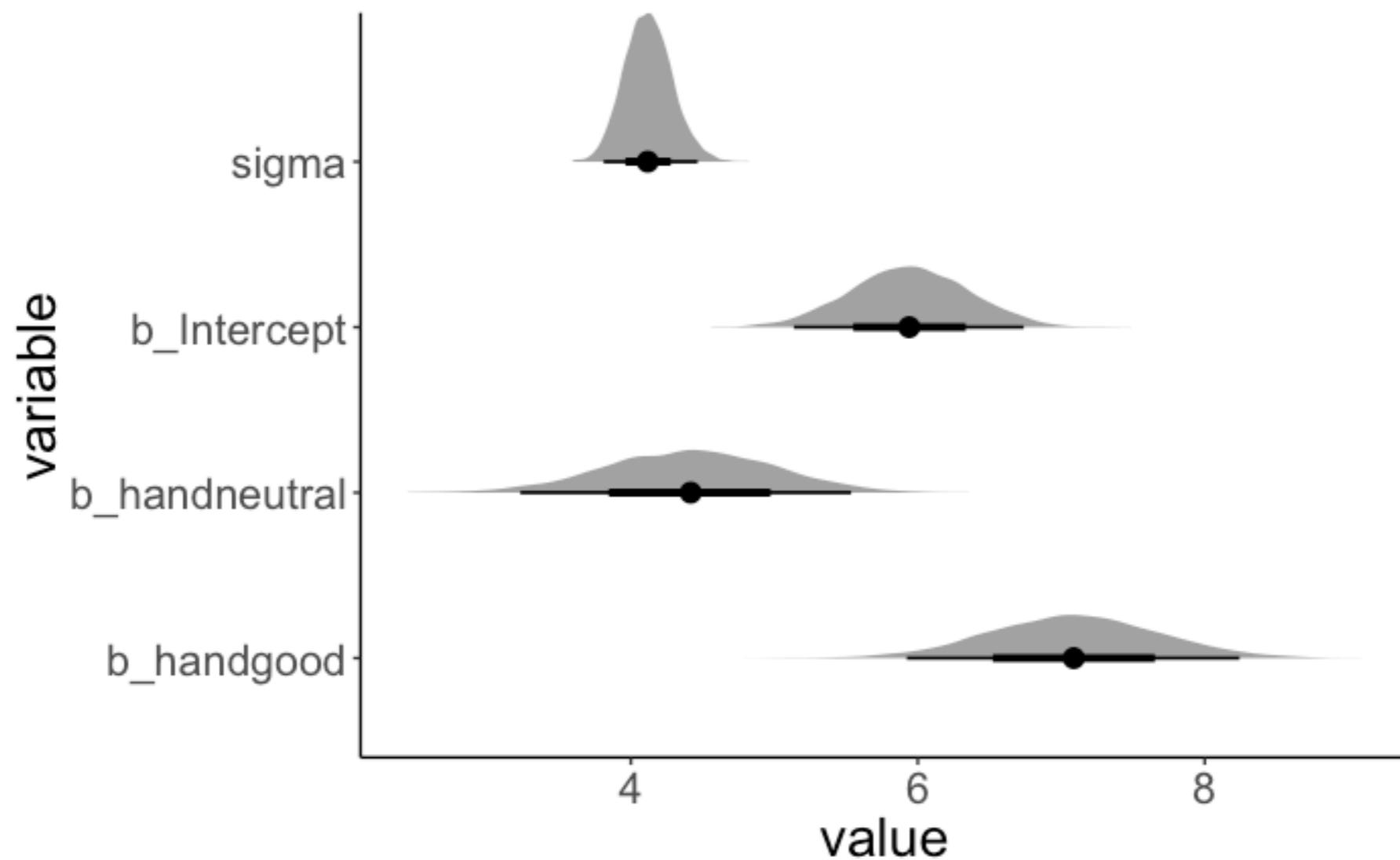
	Estimate	Est.Error	l-95%	CI	u-95%	CI	Eff.Sample	Rhat
Intercept	5.94	0.41	5.14		6.74		3495	1.00
handneutral	4.41	0.59	3.23		5.53		3125	1.00
handgood	7.09	0.59	5.92		8.24		3830	1.00

**almost identical
results!**

Model evaluation

Visualizing and interpreting results

Summary of the posterior

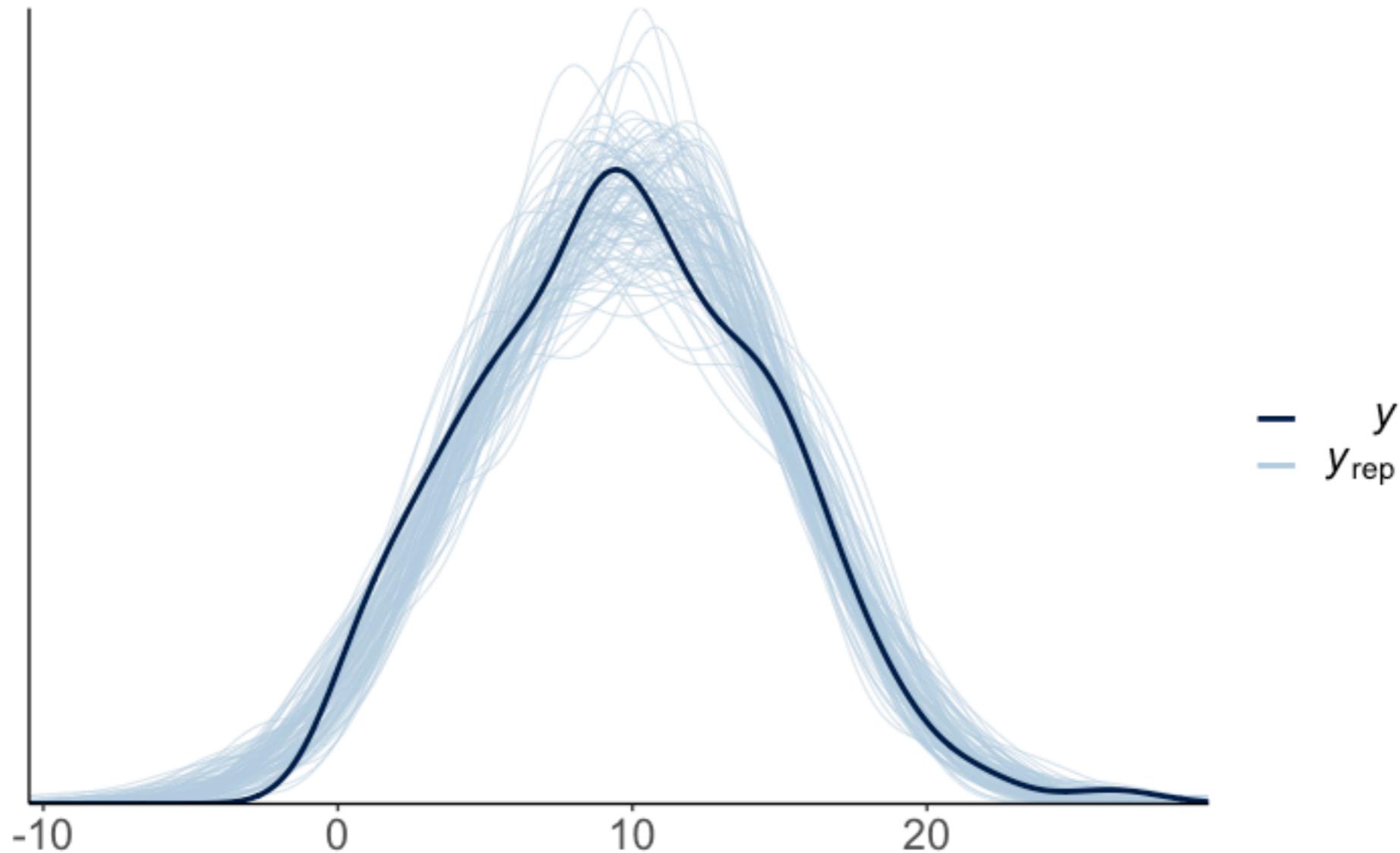


MAP estimate and 95%
highest density interval

parameter	lower	mode	upper
b_handgood	5.97	7.07	8.27
b_handneutral	3.21	4.43	5.51
b_intercept	5.17	5.95	6.77
sigma	3.81	4.12	4.47

Posterior predictive check

```
pp_check(fit.brm, nsamples = 100)
```

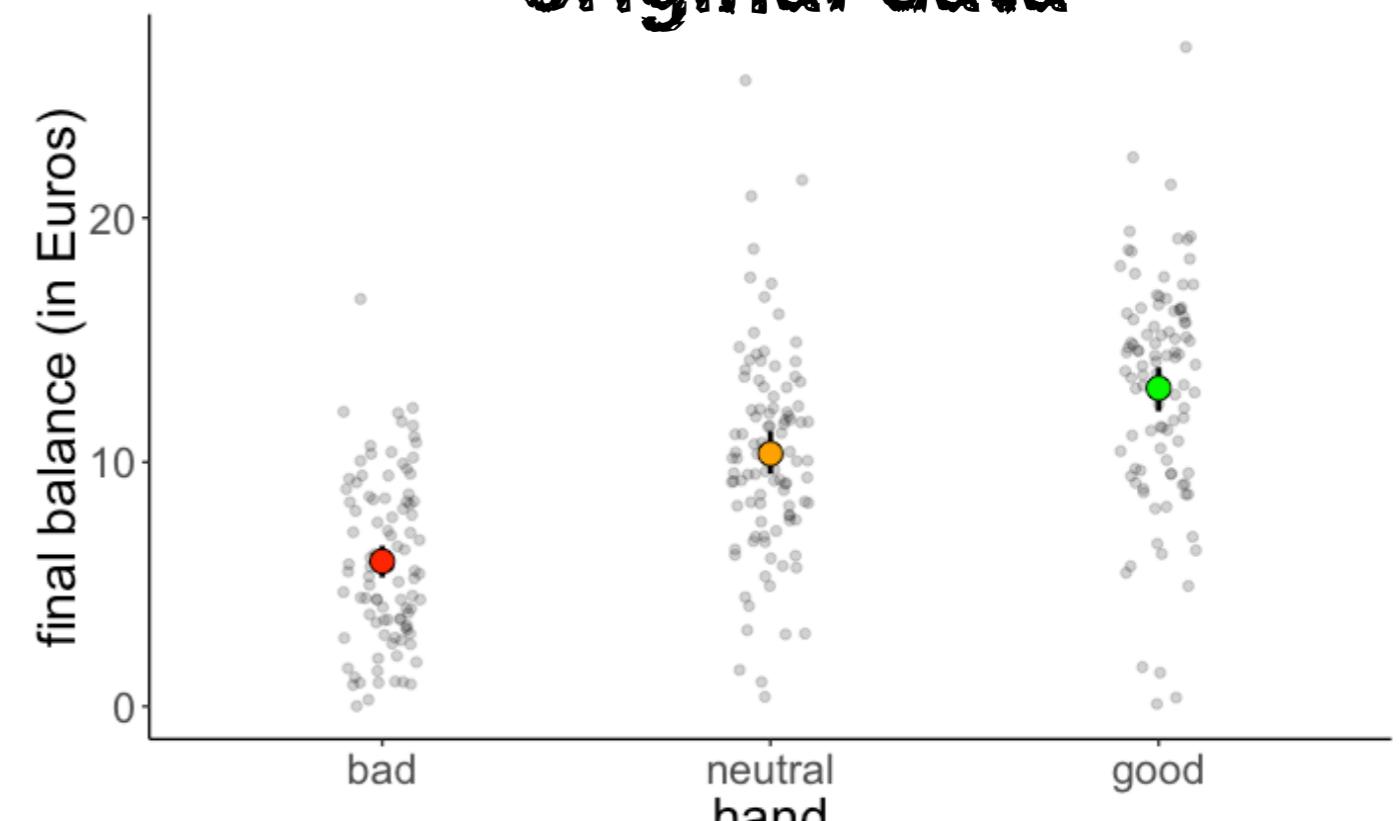
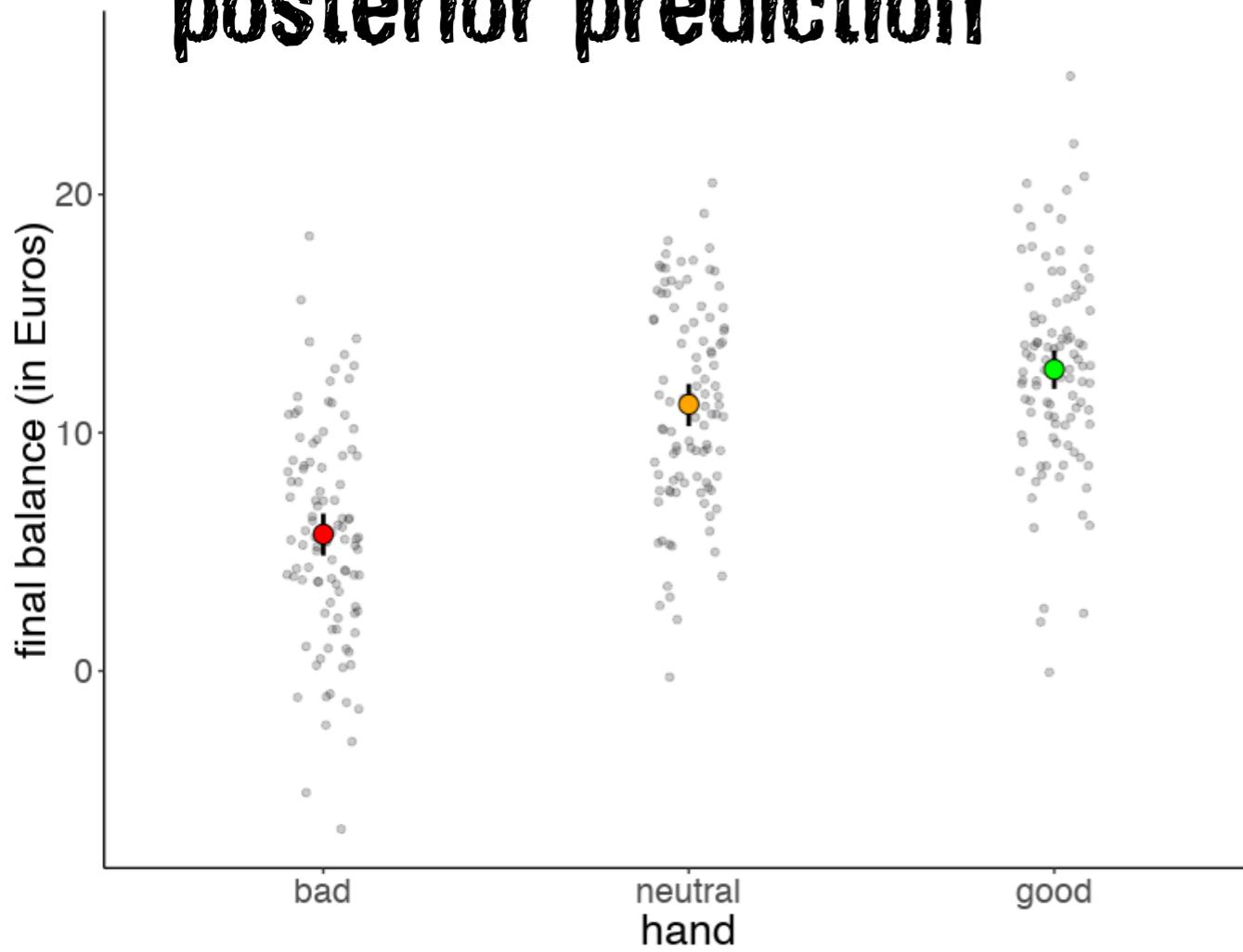


The model accurately captures the distribution of the response variable

Posterior predictive check

original data

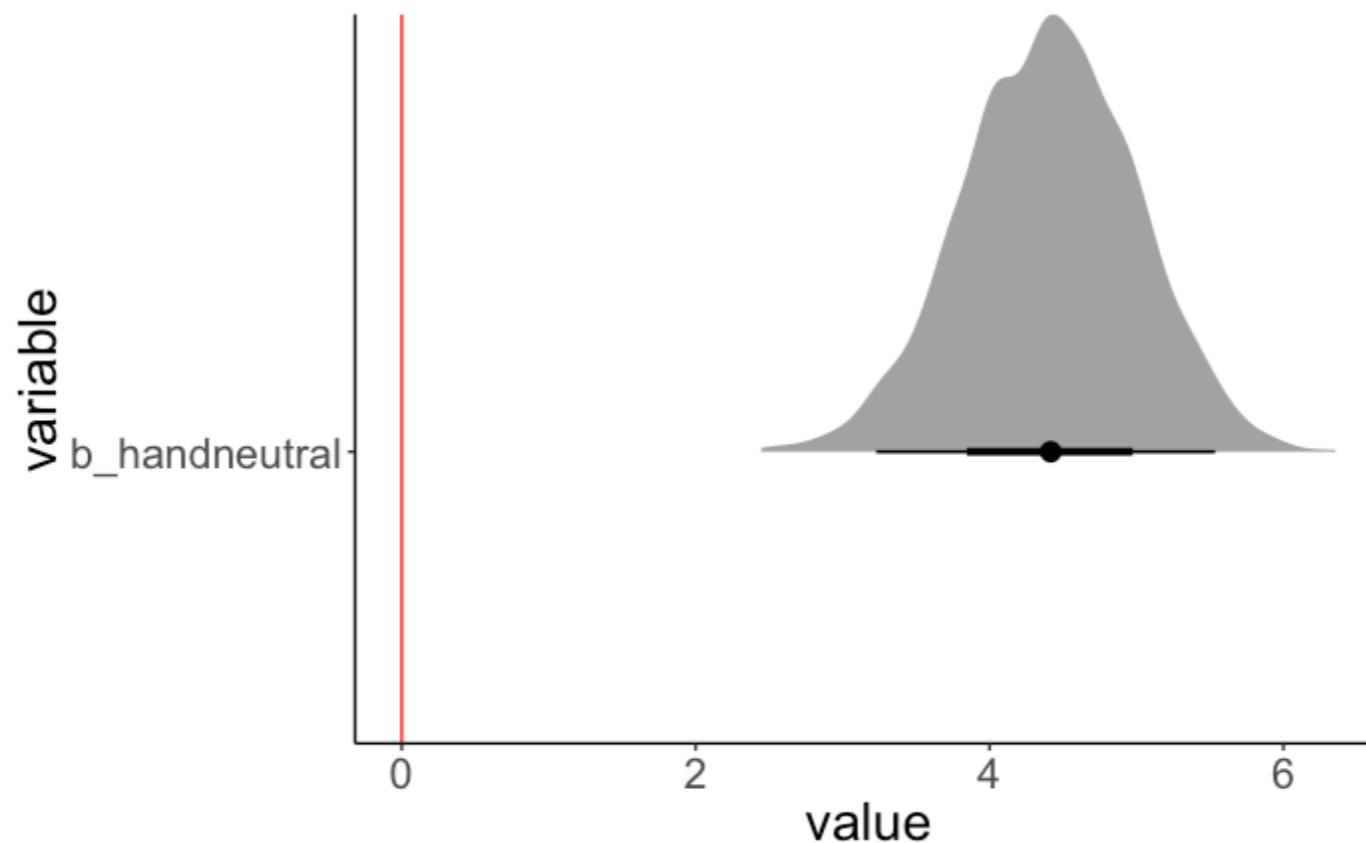
posterior prediction



Testing hypotheses

Asking questions based on the posterior

Do neutral hands earn more money than bad hands?



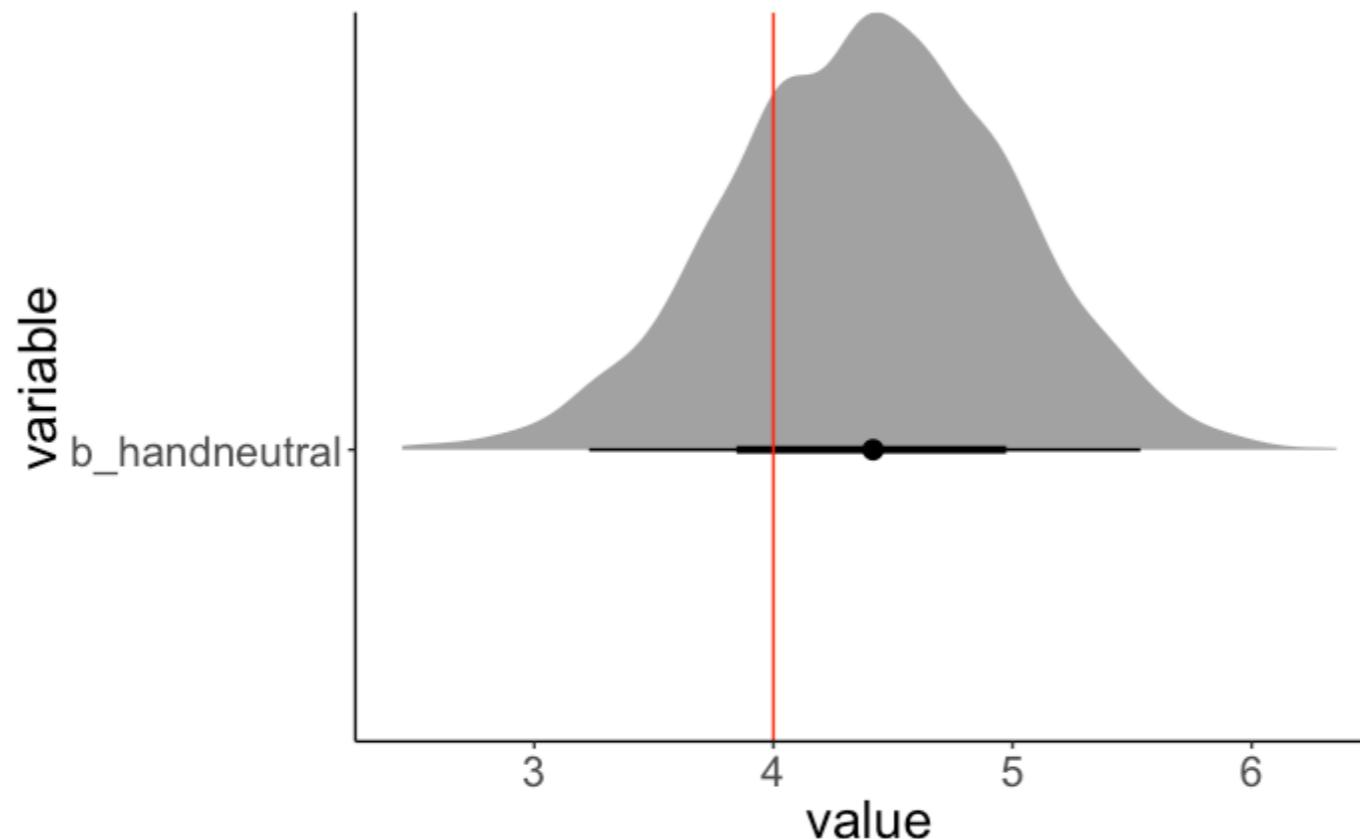
What's the probability that handneutral is less than 0?

```
1 hypothesis(fit.brn,  
2             hypothesis = "handneutral < 0")
```

$$p = 0$$

Asking questions based on the posterior

Do neutral hands earn much more money than bad hands?



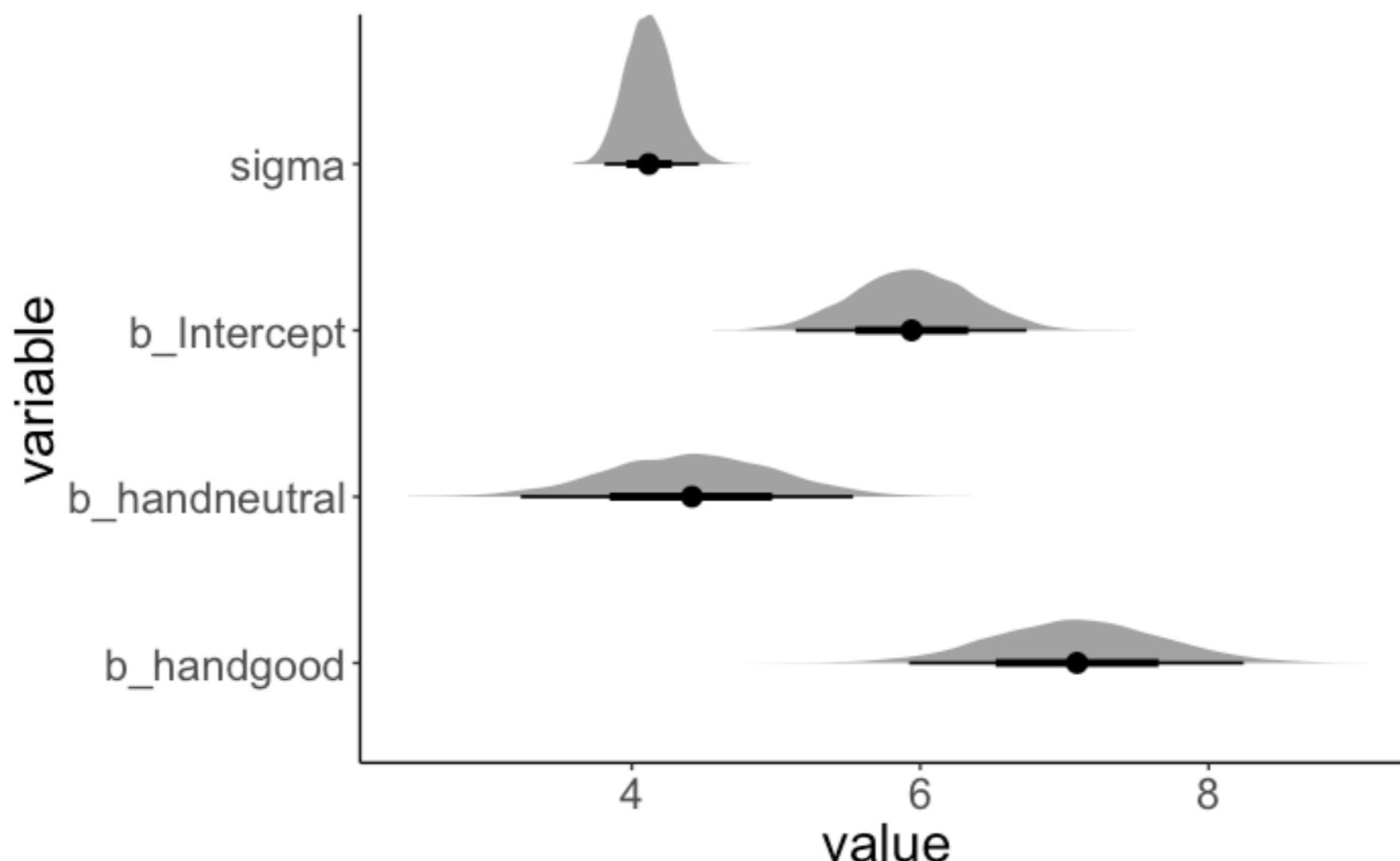
What's the probability that `handneutral` is **more than 4**?

```
1 hypothesis(fit.brm,  
2             hypothesis = "handneutral > 4")
```

$$p = 0.75$$

Asking questions based on the posterior

Do good hands make twice as much as bad hands?

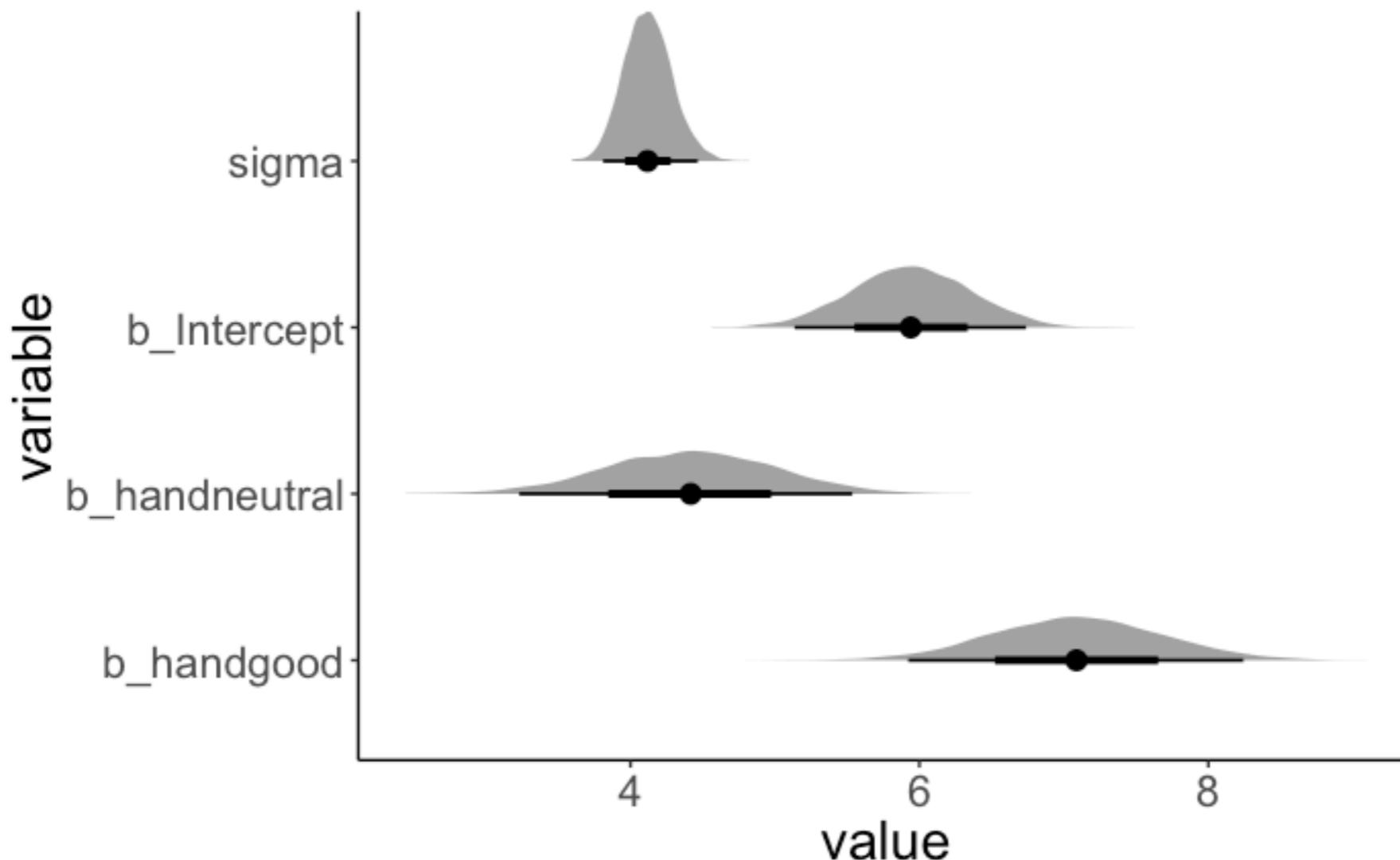


```
1 hypothesis(fit.brm,  
2           hypothesis = "handgood + Intercept > 2 * Intercept")
```

$p = 0.89$

Asking questions based on the posterior

Are neutral hands worse than bad and good hands combined?



```
1 hypothesis(fit.brm,  
2 hypothesis = "Intercept + handneutral < (Intercept + Intercept + handgood) / 2")
```

$p = 0.04$

Testing hypothesis

```
1 df.hypothesis = fit.brm %>%
2   posterior_samples() %>%
3   clean_names() %>%
4   select(starts_with("b_")) %>%
5   mutate(neutral = b_intercept + b_handneutral,
6         bad_good_average = (b_intercept + b_intercept + b_handgood)/2,
7         hypothesis = neutral < bad_good_average)
```

samples
from the
posterior



b_intercept	b_handneutral	b_handgood	neutral	bad_good_average	hypothesis
6.07	4.10	7.20	10.17	9.67	FALSE
6.06	4.44	6.95	10.49	9.53	FALSE
5.88	5.00	6.73	10.87	9.24	FALSE
5.85	4.78	6.18	10.63	8.94	FALSE
5.86	4.46	7.68	10.32	9.70	FALSE

```
1 df.hypothesis %>%
2   summarize(p = sum(hypothesis) / n())
```

$$p = 0.04$$

Testing hypotheses

- Having a posterior distribution allows us to ask questions about the data in a very flexible way
- No need to control for multiple comparisons (since we don't use a heuristic criterion for rejecting null hypotheses)!
- No need for setting up planned linear contrasts!

Bayes factor

$$p(H|D) = \frac{p(D|H) \cdot p(H)}{\sum_{i=1}^n p(D|H_i) \cdot p(H_i)}$$

Two hypotheses

$$p(H_1|D) = \frac{p(D|H_1) \cdot p(H_1)}{p(D|H_1) \cdot p(H_1) + p(D|H_2) \cdot p(H_2)}$$

Odds form = Bayes factor

If priors are equal

$$\frac{p(H_1|D)}{p(H_2|D)} = \frac{p(D|H_1)}{p(D|H_2)} \cdot \frac{p(H_1)}{p(H_2)}$$

$$BF = \frac{p(D|H_1)}{p(D|H_2)}$$

Fit two models and compare

```
1 fit.brm1 = brm(formula = balance ~ 1 + hand,  
2                   data = df.poker,  
3                   save_all_pars = T,  
4                   file = "brm_factor1")  
5  
6 fit.brm2 = brm(formula = balance ~ 1 + hand + skill,  
7                   data = df.poker,  
8                   save_all_pars = T,  
9                   file = "brm_factor2")
```

bayes_factor(fit.brm2, fit.brm1)

$$BF = \frac{p(D | H_1)}{p(D | H_0)} = 3.81$$

relative evidence for one model over the other

Bayes factor BF_{10}			Interpretation
	>	100	Decisive evidence for H_1
30	-	100	Very Strong evidence for H_1
10	-	30	Strong evidence for H_1
3	-	10	Substantial evidence for H_1
1	-	3	Anecdotal evidence for H_1
	1		No evidence
1/3	-	1	Anecdotal evidence for H_0
1/10	-	1/3	Substantial evidence for H_0
1/30	-	1/10	Strong evidence for H_0
1/100	-	1/30	Very Strong evidence for H_0
	<	1/100	Decisive evidence for H_0

**Inference evaluation:
Did things work out?**

What about the priors?

```
1 fit.brm = brm(formula = balance ~ 1 + hand,  
2                  data = df.poker)
```

By default, brms uses weakly informative priors for the model parameters.

There are quite a few other defaults, let's take a look under the hood ...

"Full" specification of the model

```
1 fit.brm2 = brm(  
2   formula = balance ~ 1 + hand,  
3   family = "gaussian",  
4   data = df.poker,  
5   prior = c(  
6     prior(normal(0, 10), class = "b", coef = "handgood"),  
7     prior(normal(0, 10), class = "b", coef = "handneutral"),  
8     prior(student_t(3, 3, 10), class = "Intercept"),  
9     prior(student_t(3, 0, 10), class = "sigma")  
10 ),  
11   inits = list(  
12     list(Intercept = 0, sigma = 1, handgood = 5, handneutral = 5),  
13     list(Intercept = -5, sigma = 3, handgood = 2, handneutral = 2),  
14     list(Intercept = 2, sigma = 1, handgood = -1, handneutral = 1),  
15     list(Intercept = 1, sigma = 2, handgood = 2, handneutral = -2)  
16   ),  
17   iter = 4000, ← how many runs in the inference chain  
18   warmup = 1000, ← how long for the warmup  
19   chains = 4, ← how many chains  
20   file = "brm2",  
21   seed = 1  
22 )
```

make reproducible

priors

initialization

how many runs in the inference chain

how long for the warmup

how many chains

save the model result

fitting Bayesian models takes some time, so storing results is key

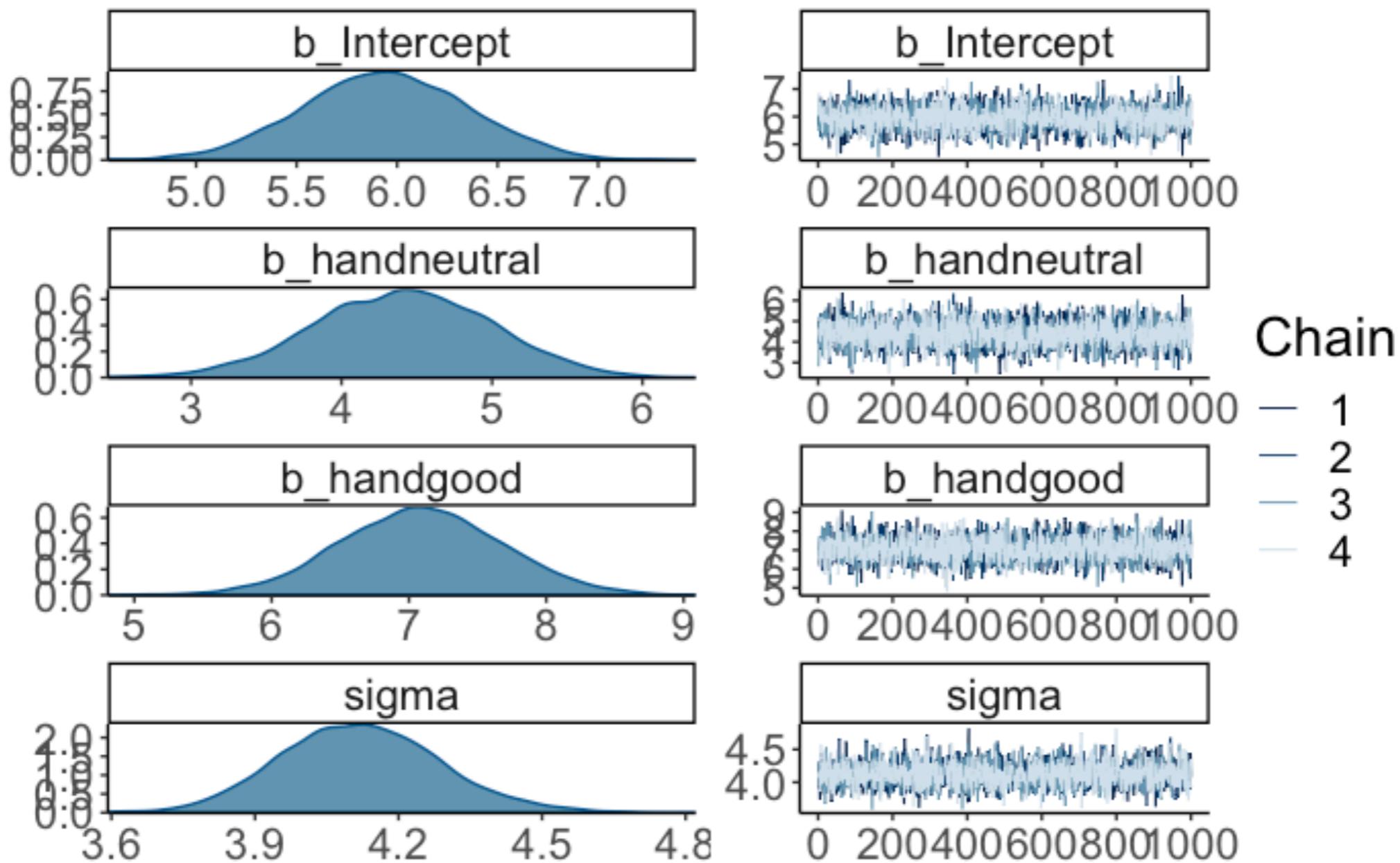
Turned into Stan code

```
// generated with brms 2.7.0
functions {
}
data {
  int<lower=1> N; // total number of observations
  vector[N] Y; // response variable
  int<lower=1> K; // number of population-level effects
  matrix[N, K] X; // population-level design matrix
  int prior_only; // should the likelihood be ignored?
}
transformed data {
  int Kc = K - 1;
  matrix[N, K - 1] Xc; // centered version of X
  vector[K - 1] means_X; // column means of X before centering
  for (i in 2:K) {
    means_X[i - 1] = mean(X[, i]);
    Xc[, i - 1] = X[, i] - means_X[i - 1];
  }
}
parameters {
  vector[Kc] b; // population-level effects
  real temp_Intercept; // temporary intercept
  real<lower=0> sigma; // residual SD
}
transformed parameters {
}
model {
  vector[N] mu = temp_Intercept + Xc * b;
  // priors including all constants
  target += normal_lpdf(b[1] | 0, 10);
  target += normal_lpdf(b[2] | 0, 10);
  target += student_t_lpdf(temp_Intercept | 3, 3, 10);
  target += student_t_lpdf(sigma | 3, 0, 10)
    - 1 * student_t_lccdf(0 | 3, 0, 10);
  // likelihood including all constants
  if (!prior_only) {
    target += normal_lpdf(Y | mu, sigma);
  }
}
generated quantities {
  // actual population-level intercept
  real b_Intercept = temp_Intercept - dot_product(means_X, b);
}
```

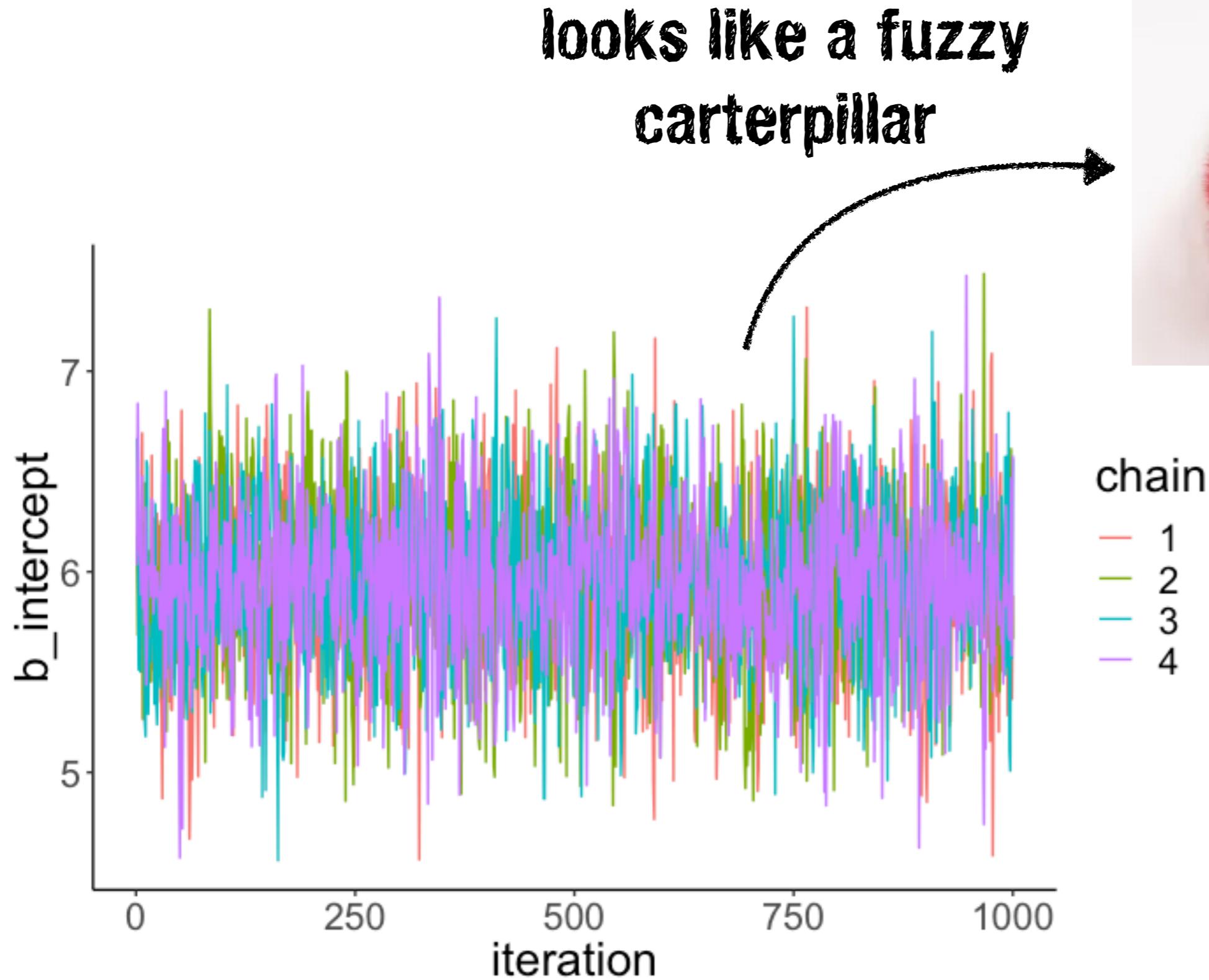
- probabilistic programming language
- flexible construction of Bayesian models
- ports have been written for R, Python, Julia, ...
- implements a fast inference algorithm

Can we trust the model results?

`plot(fit.brm)`



Can we trust the model results?



When things don't work out

```
1 df.data = tibble(y = c(-1, 1)) ← only two data points!
2
3 fit.brml = brm(
4   data = df.data,
5   family = gaussian,
6   formula = y ~ 1,
7   prior = c(
8     prior(uniform(-1e10, 1e10), class = Intercept),
9     prior(uniform(0, 1e10), class = sigma)
10   ),
11   inits = list(
12     list(Intercept = 0, sigma = 1),
13     list(Intercept = 0, sigma = 1)
14   ),
15   iter = 4000,
16   warmup = 1000,
17   chains = 2,
18   file = "fit_brml"
19 )
```

incredibly wide uniform priors

When things don't work out

summary(fit.brml)

The model has not converged (some Rhats are > 1.1). Do not analyse the results!
We recommend running more iterations and/or setting stronger priors. There were 362 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help.
See <http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup> Family: gaussian
Links: mu = identity; sigma = identity
Formula: y ~ 1
Data: list(y = c(-1, 1)) (Number of observations: 2)
Samples: 2 chains, each with iter = 4000; warmup = 1000; thin = 1;
total post-warmup samples = 6000

Population-Level Effects:

	Estimate	Est.Error	1-95% CI	u-95% CI	Eff.Sample	Rhat
Intercept	56599494.07	142352414.04	-36858534.50	548777591.78	12	1.20

Family Specific Parameters:

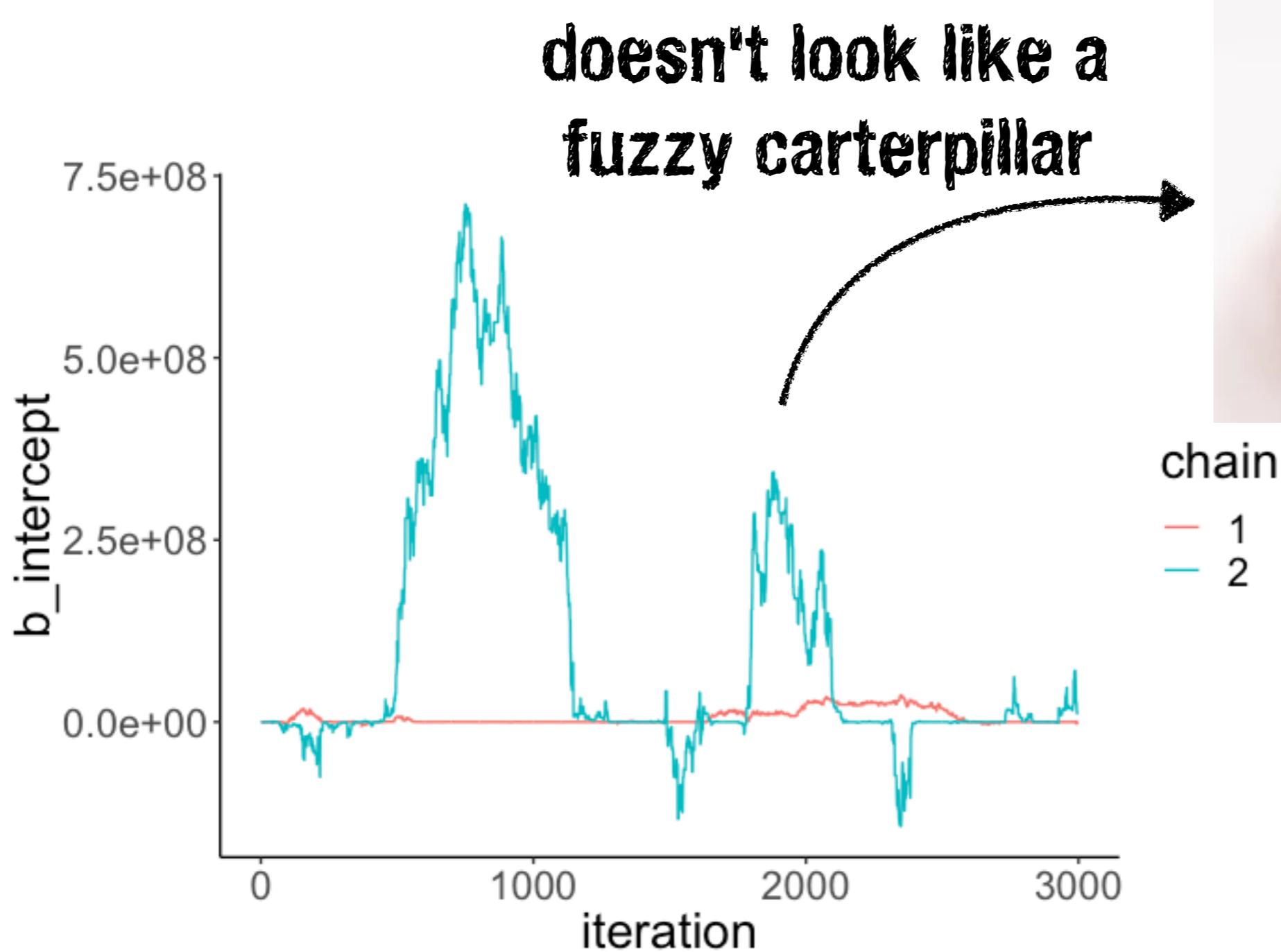
	Estimate	Est.Error	1-95% CI	u-95% CI	Eff.Sample	Rhat
sigma	269115068.56	931020177.84	757.39	2652989409.36	37	1.06

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

should
be 1

should be
much
higher

When things don't work out



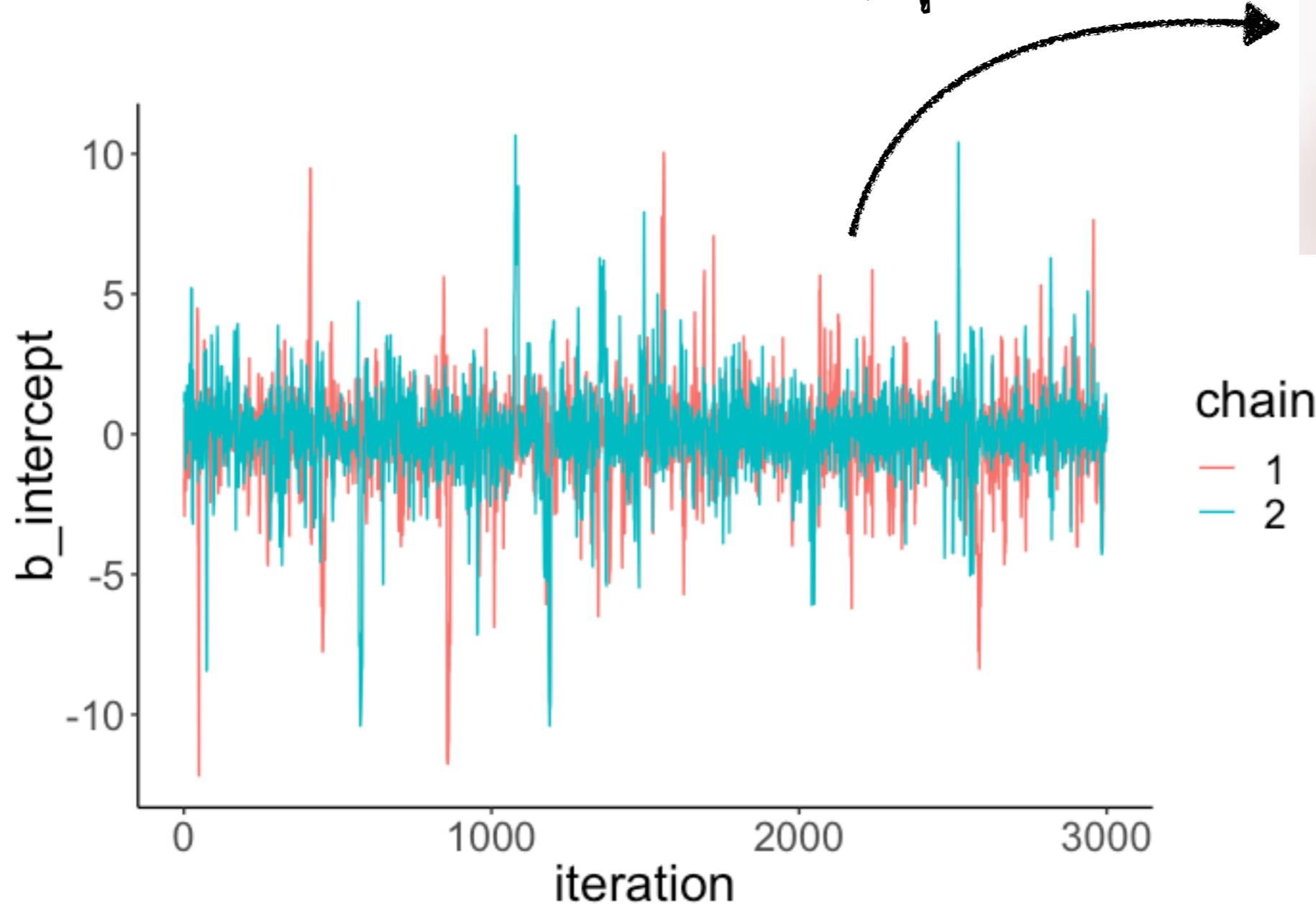
Having somewhat informative priors fixes things

```
1 fit.brm2 = brm(  
2   data = df.data,  
3   family = gaussian,  
4   formula = y ~ 1,  
5   prior = c(  
6     prior(normal(0, 10), class = Intercept), # more reasonable priors  
7     prior(cauchy(0, 1), class = sigma)  
8   ),  
9   iter = 4000,  
10  warmup = 1000,  
11  chains = 2,  
12  seed = 1,  
13  file = "fit_brm2"  
14 )
```

```
Family: gaussian  
Links: mu = identity; sigma = identity  
Formula: y ~ 1  
Data: list(y = c(-1, 1)) (Number of observations: 2)  
Samples: 2 chains, each with iter = 4000; warmup = 1000; thin = 1;  
         total post-warmup samples = 6000  
  
Population-Level Effects:  
Estimate Est.Error 1-95% CI u-95% CI Eff.Sample Rhat  
Intercept -0.06      1.72    -3.78     3.27      1033 1.00  
  
Family Specific Parameters:  
Estimate Est.Error 1-95% CI u-95% CI Eff.Sample Rhat  
sigma    2.21      6.99     0.61     6.92      1006 1.00  
  
Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample  
is a crude measure of effective sample size, and Rhat is the potential  
scale reduction factor on split chains (at convergence, Rhat = 1).
```

Having somewhat informative priors fixes things

looks like a fuzzy
caterpillar



Interim summary

- Bayesian models are very flexible: They support asking many different kinds of questions about the data
- Asking these questions is easy: we just directly use the samples from the posterior
- We can use the Bayes factor to compare different models (they need not be nested)
- There are ways for us to check whether the inference worked
- If we have little data, then weakly informative priors help

Some cool examples

Evidence for null results

[Front Psychol.](#) 2014; 5: 781.

PMCID: PMC4114196

Published online 2014 Jul 29. doi: [10.3389/fpsyg.2014.00781](https://doi.org/10.3389/fpsyg.2014.00781)

PMID: [25120503](#)

Using Bayes to get the most out of non-significant results

[Zoltan Dienes*](#)

► Author information ► Article notes ► Copyright and License information [Disclaimer](#)

[HTML] [Using Bayes to get the most out of non-significant results](#)

[Z Dienes - Frontiers in psychology, 2014 - frontiersin.org](#)

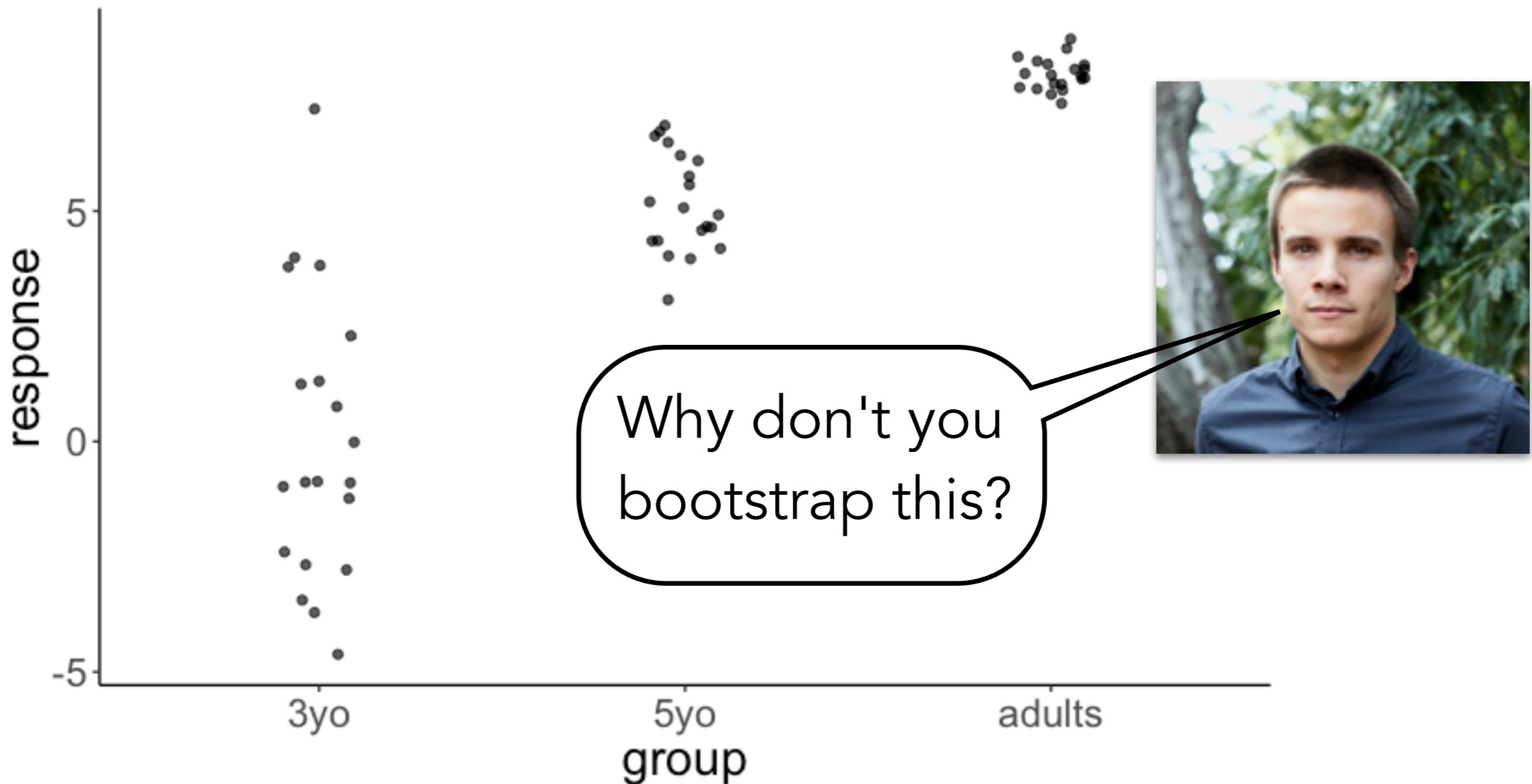
No scientific conclusion follows automatically from a statistically non-significant result, yet people routinely use non-significant results to guide conclusions about the status of theories (or the effectiveness of practices). To know whether a non-significant result counts against a theory, or if it just indicates data insensitivity, researchers must use one of: power, intervals (such as confidence or credibility intervals), or else an indicator of the relative evidence for one theory over another, such as a Bayes factor. I argue Bayes factors allow theory to be ...

☆ 99 Cited by 649 Related articles All 14 versions Import into BibTeX »

- There is nothing special about H_0 compared to H_1 in Bayesian inference
- We can get evidence of H_0 over H_1 (e.g. using the Bayes factor approach)

Dealing with unequal variance

Unequal variance aka heteroscedasticity



Unequal variance aka heteroscedasticity

```
1 fit.variance = brm(  
2   formula = bf(response ~ group,  
3     sigma ~ group),  
4   data = df.variance,  
5   file = "variance")
```

```
Family: gaussian  
Links: mu = identity; sigma = log  
Formula: response ~ group  
         sigma ~ group  
Data: df.variance (Number of observations: 60)  
Samples: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;  
         total post-warmup samples = 4000
```

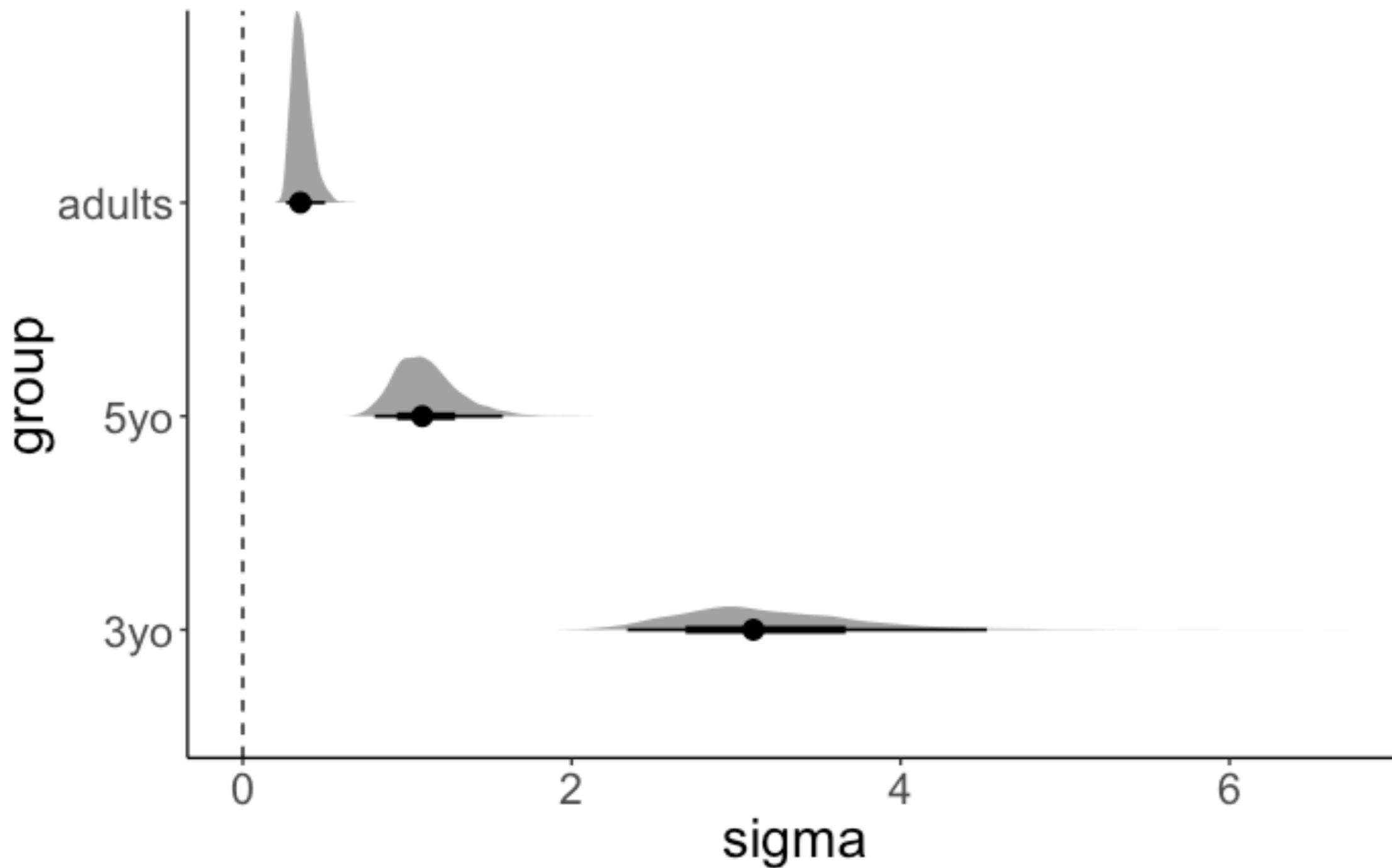
Population-Level Effects:

	Estimate	Est.Error	1-95% CI	u-95% CI	Eff.Sample	Rhat
Intercept	0.00	0.72	-1.38	1.45	1273	1.00
sigma_Intercept	1.15	0.17	0.85	1.51	2033	1.00
group5yo	5.16	0.77	3.60	6.62	1424	1.00
groupadults	7.96	0.72	6.49	9.37	1276	1.00
sigma_group5yo	-1.05	0.23	-1.51	-0.59	2355	1.00
sigma_groupadults	-2.19	0.23	-2.65	-1.74	2231	1.00

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

Model estimates both group effects **and** variance!

Unequal variance aka heteroscedasticity



Zero-one inflated beta binomial model

Example taken from ...

How to analyze visual analog (slider) scale data?

A reasonable choice might be the zero-one-inflated beta model

Feb 18, 2019 · 25 min read · psychology, statistics

- [Introduction](#)
- [The zero-one-inflated beta model](#)
- [ZOIB regression](#)
- [Simulation: Compare ZOIB and t-test performances](#)
- [Discussion](#)
- [References](#)

Introduction

In psychological experiments, subjective responses are often collected using two types of response scales: ordinal and visual analog scales. These scales are unlikely to provide normally distributed data. However, researchers often analyze responses from these scales with models that assume normality of the data.¹

Ordinal scales, of which binary ratings are a special case, provide ordinal data and are thus better analyzed using ordinal models (Bürkner and Vuorre 2018; Liddell and Kruschke 2018).

<https://vuorre.netlify.com/post/2019/02/18/analyze-analog-scale-ratings-with-zero-one-inflated-beta-models/#zoib-regression>

In general, what is more important to you?

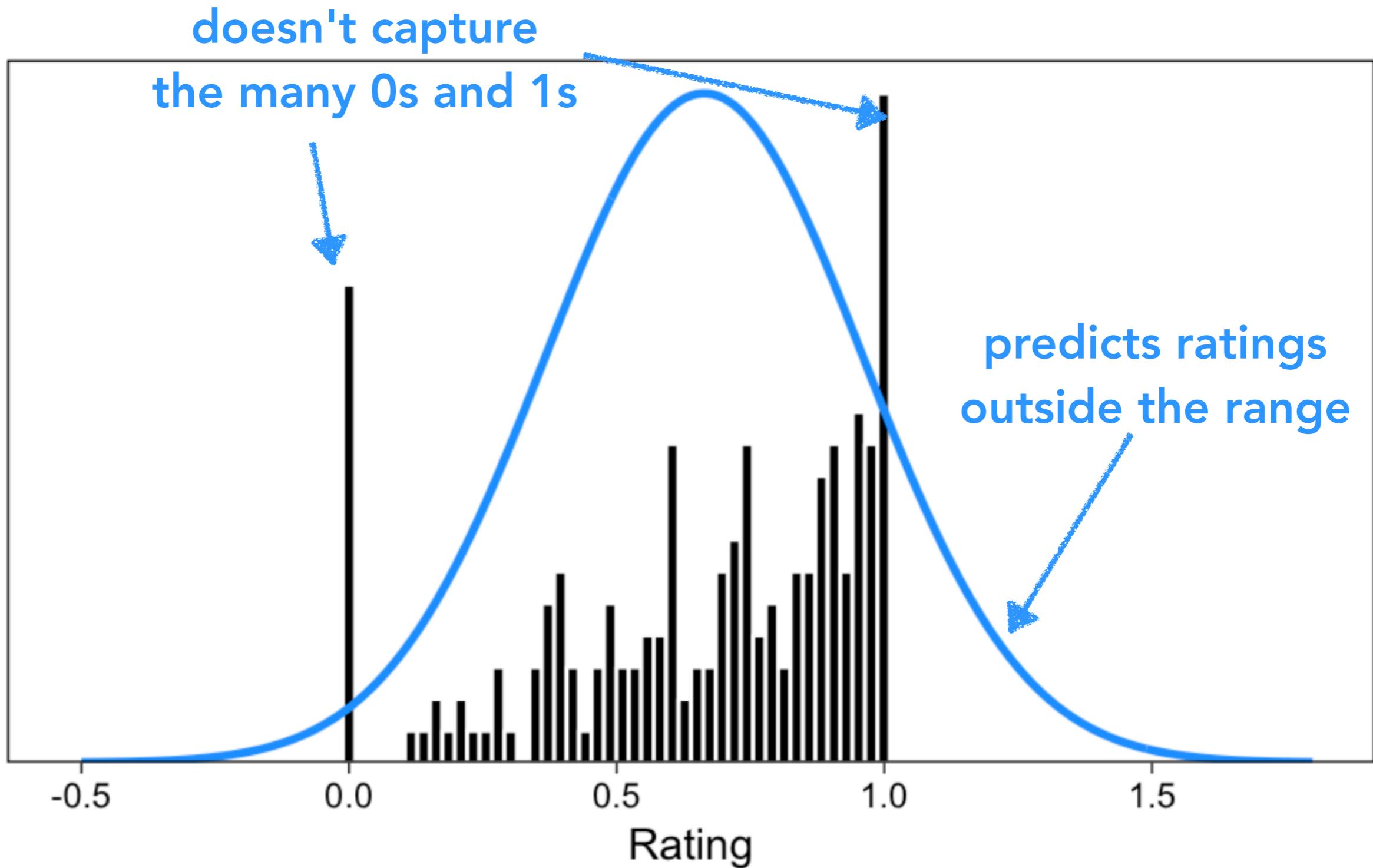
Leisure

Money



Next

Normality assumption is (almost always) violated



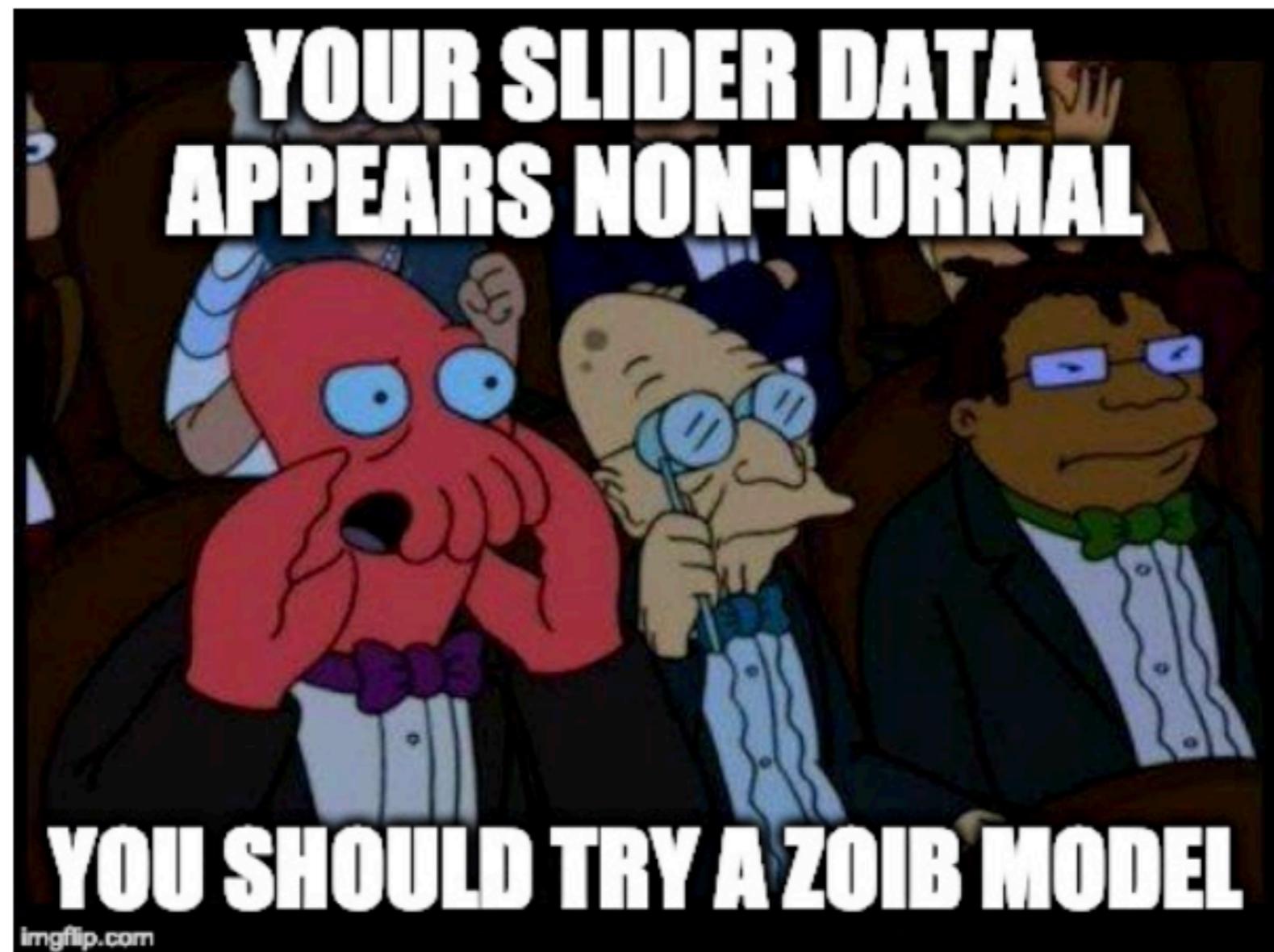
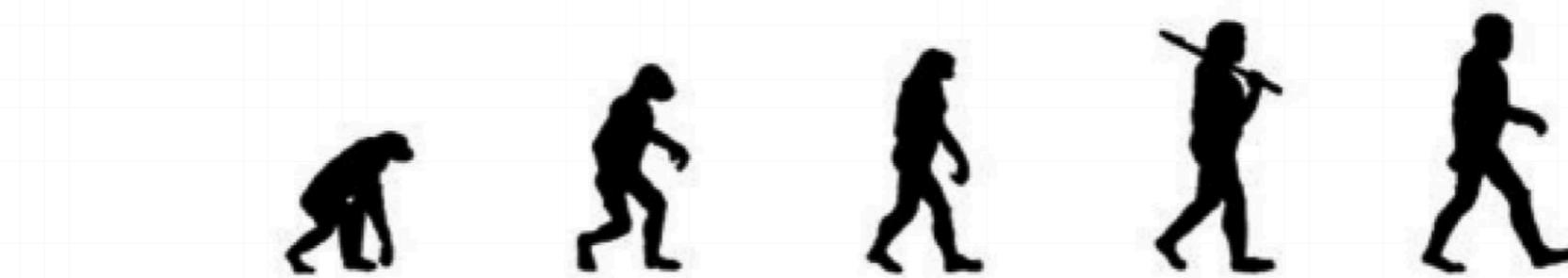


Figure 3: Dr. John A. Zoidberg thinks you should try a ZOIB model on your slider scale data.

From Nour Kteily's talk or an OIB model?!

People can vary in how human-like they seem. Some people seem highly evolved whereas others seem no different than lower animals. Using the image below, indicate using the sliders how evolved you consider the average member of each group to be:



Americans

Arabs

Canadians

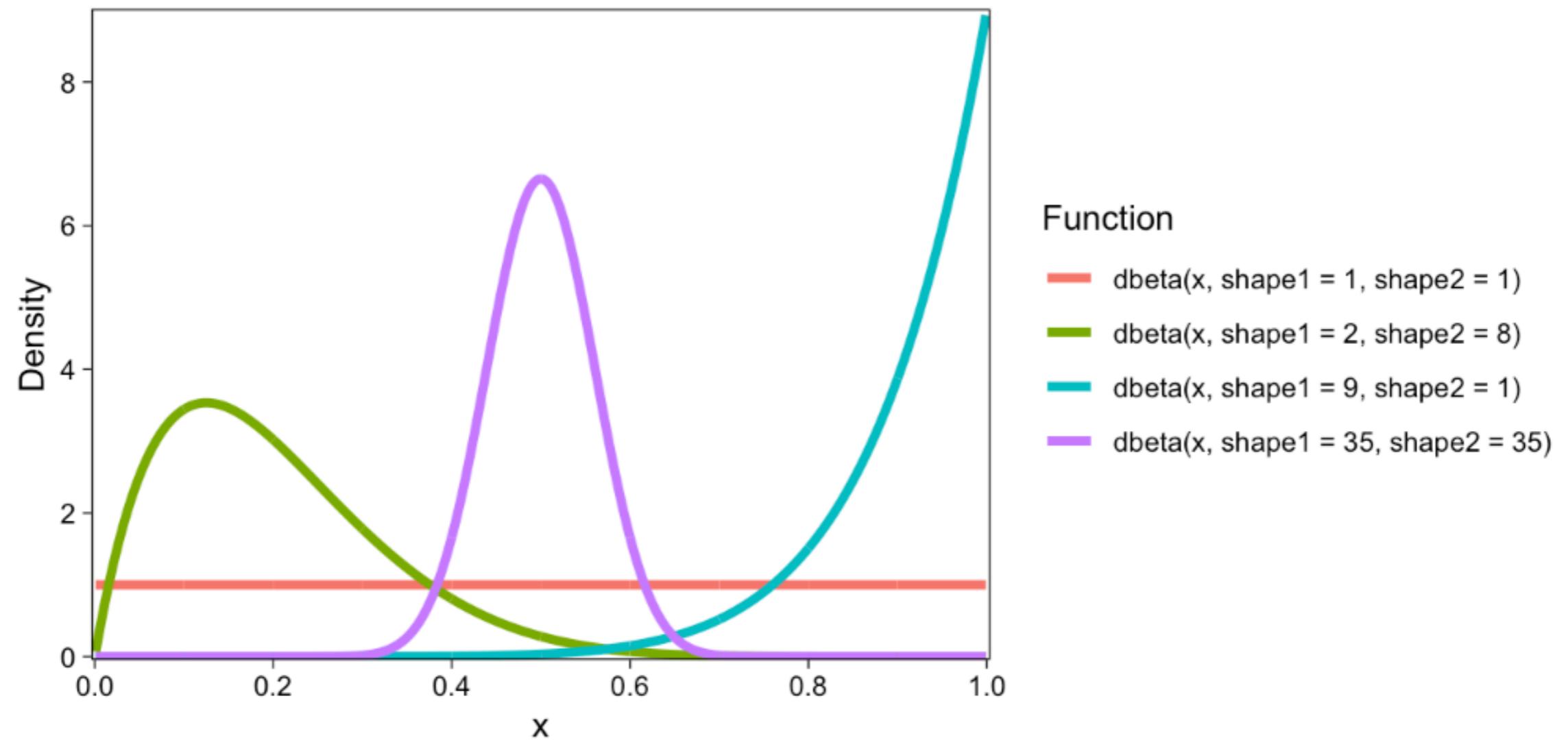
Chinese

Europeans

Muslims

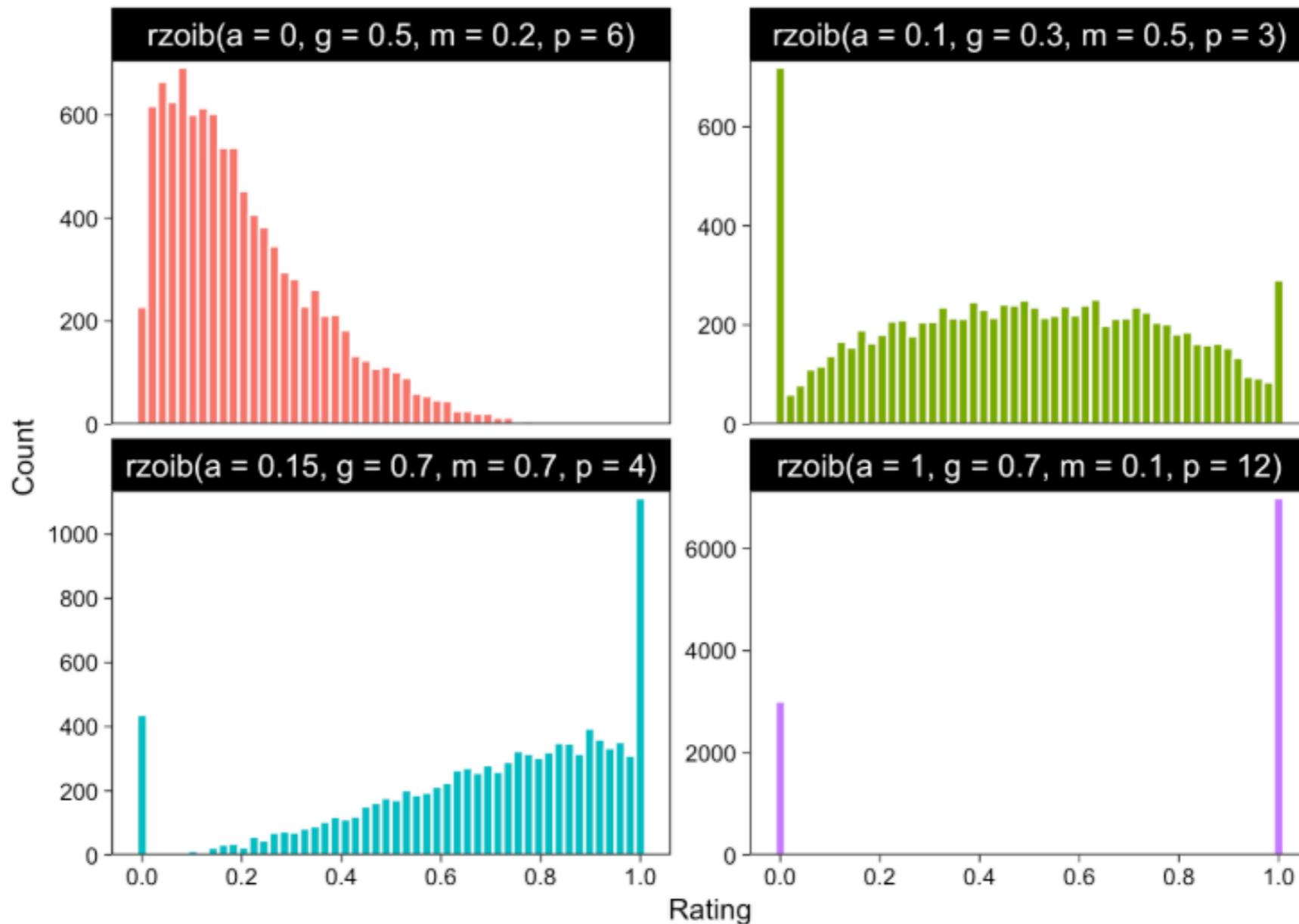
Figure 1. The Ascent measure of blatant dehumanization. Responses were made for each target group using the sliders next to the groups. Target group order was randomized across participants.

Beta distribution



Zero-one inflated **beta** binomial model

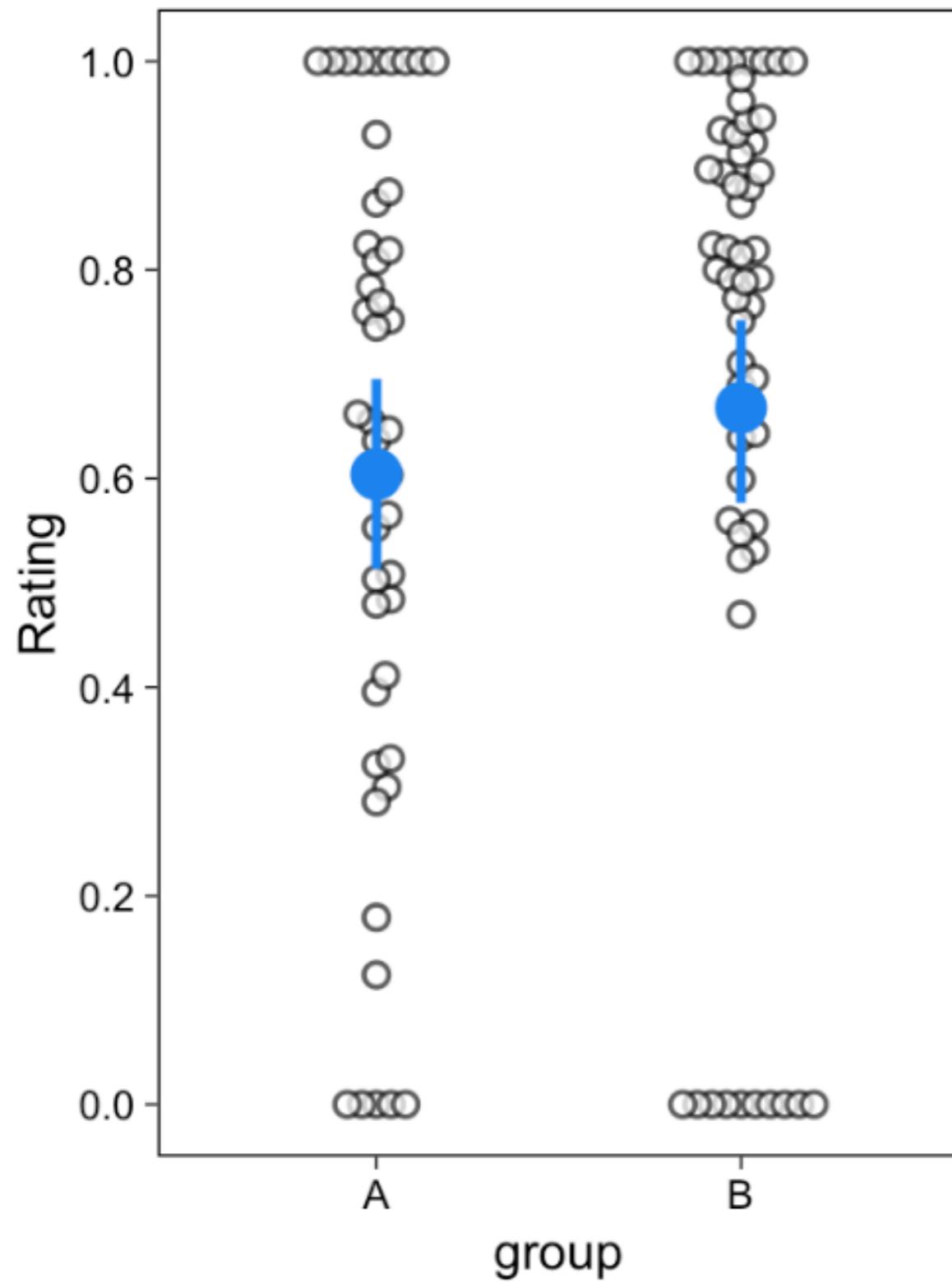
Zero-one inflated beta binomial model



Generative process

Some chance the a person will pick a 0 or 1, if not then response is determined by the beta distribution.

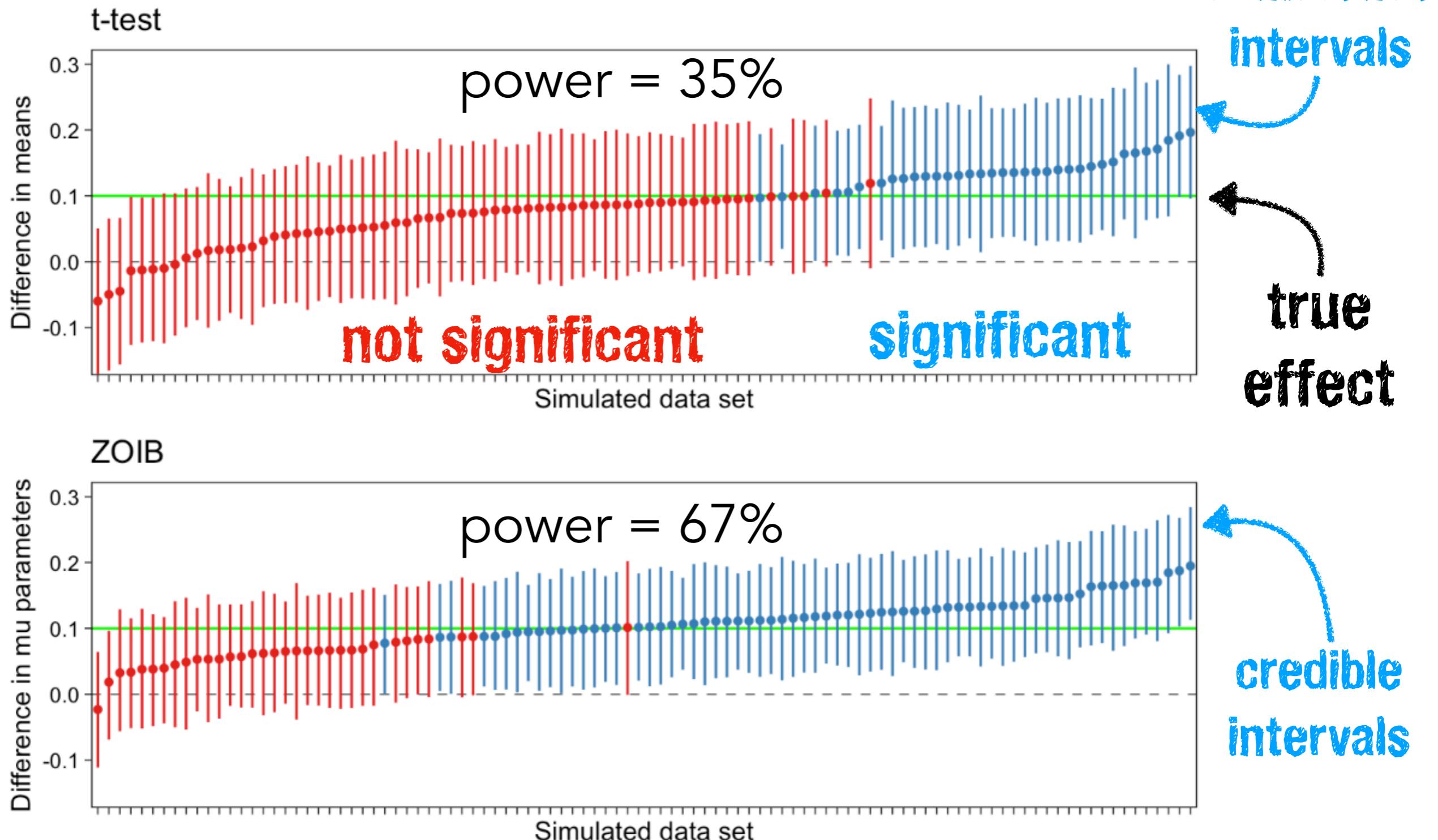
Example data



Fit the ZOIB

```
zoib_model = bf(  
    Rating ~ group,  
    phi ~ group,  
    zoi ~ group,  
    coi ~ group,  
    family = zero_one_inflated_beta()  
)  
  
fit = brm(  
    formula = zoib_model,  
    data = dat  
)
```

Capturing the data-generating process gives you power

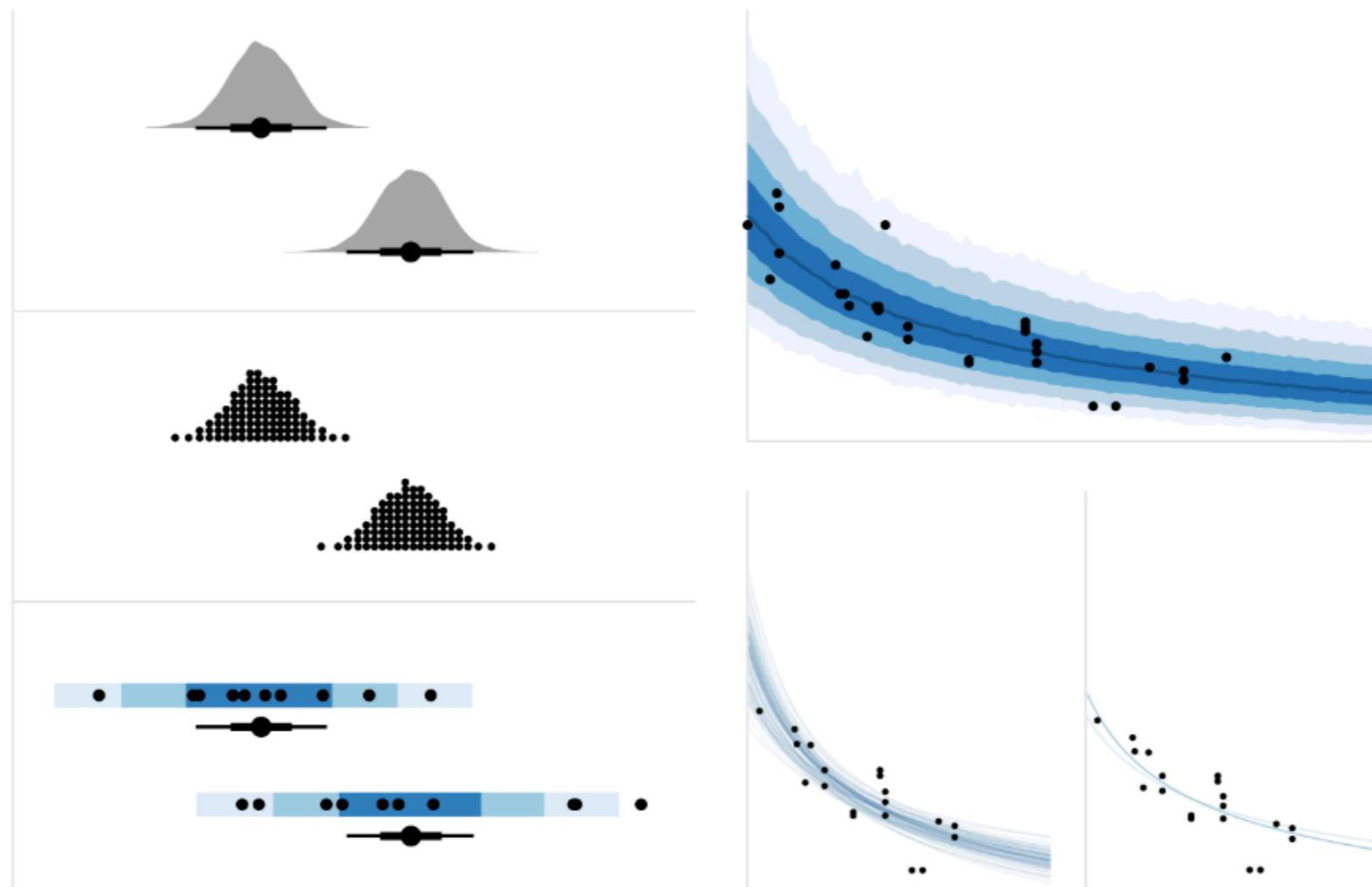


Ordinal logistic regression

Example taken from ...

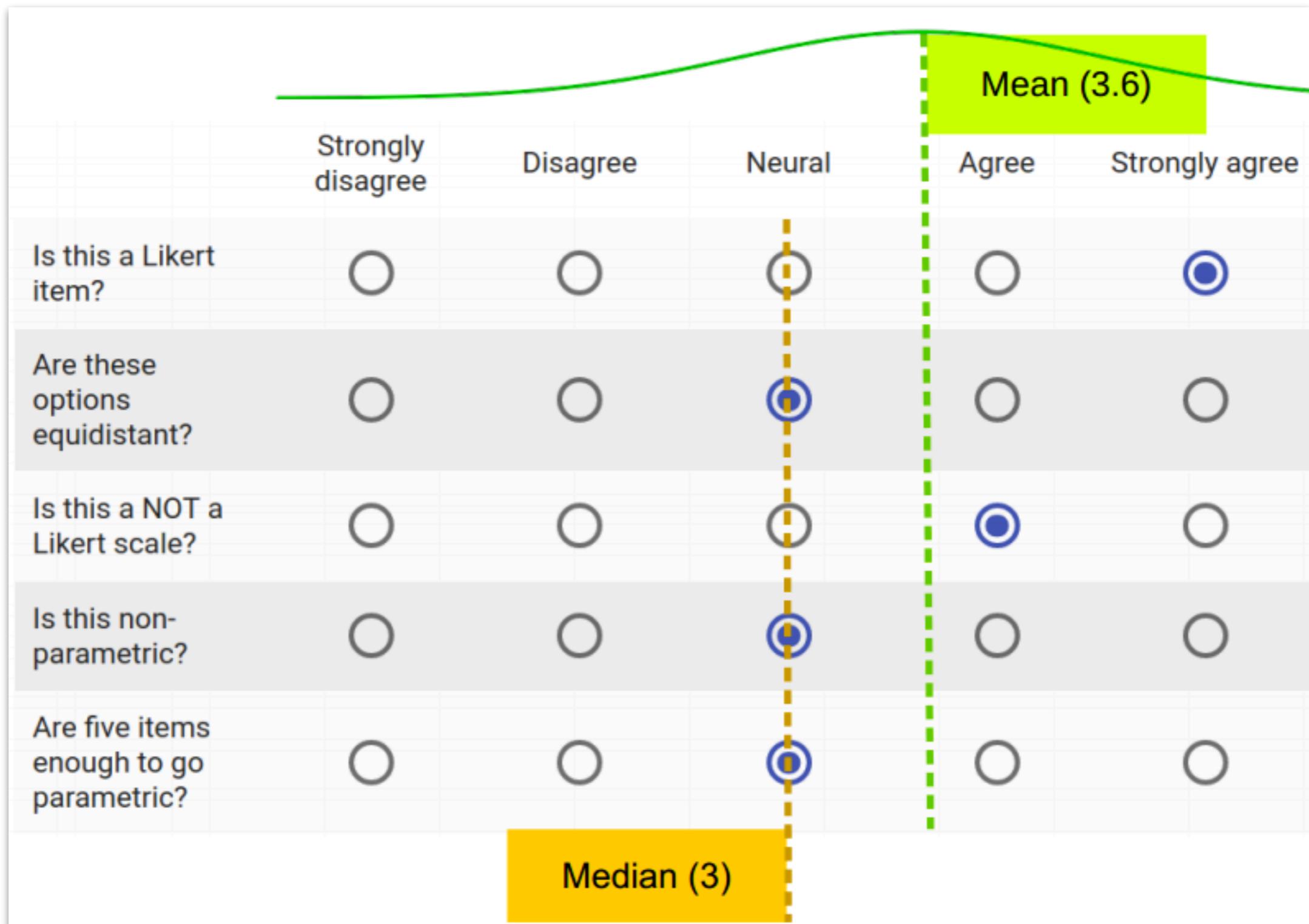
tidybayes: Bayesian analysis + tidy data + geoms

build passing codecov 92% CRAN 1.0.4 downloads 1373/month DOI 10.5281/zenodo.1468151



<https://mjskay.github.io/tidybayes/articles/tidy-brms.html#ordinal-models>

Likert scale data



mtcars -- a classic R data set



We want to predict how many cylinders a car's engine has based on the car's efficiency (miles per gallon).

Bayesian ordinal regression

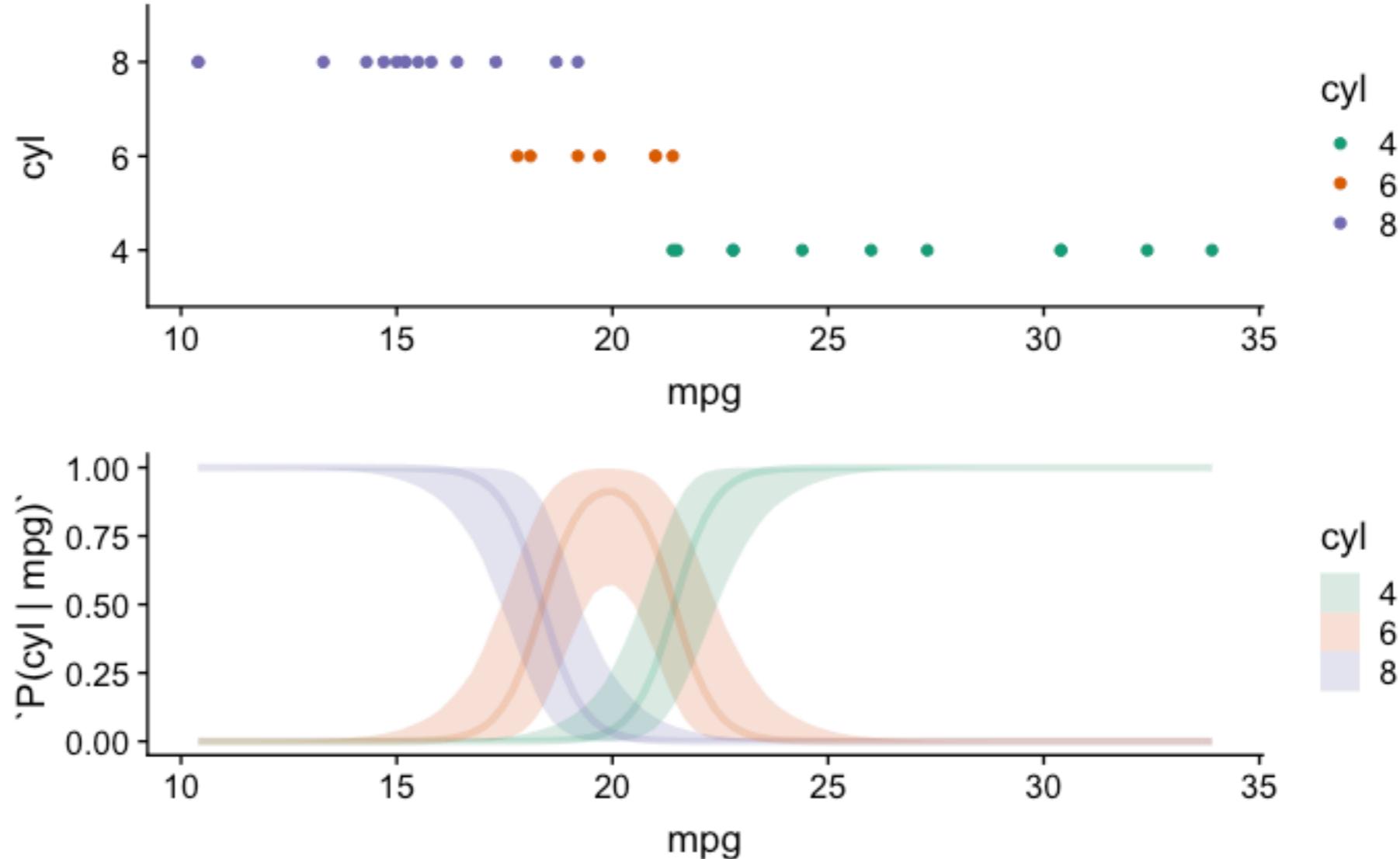
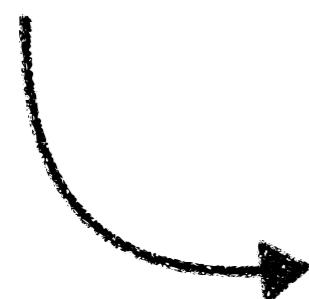
```
1 df.cars = mtcars %>%
2   mutate(cyl = ordered(cyl)) # creates an ordered factor
```

```
'data.frame': 32 obs. of 11 variables:
 $ mpg : num 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
 $ cyl  : Ord.factor w/ 3 levels "4"<"6"<"8": 2 2 1 2 3 2 3 1 1 2 ...
 $ disp: num 160 160 108 258 360 ...
 $ hp   : num 110 110 93 110 175 105 245 62 95 123 ...
 $ drat: num 3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
 $ wt   : num 2.62 2.88 2.32 3.21 3.44 ...
 $ qsec: num 16.5 17 18.6 19.4 17 ...
 $ vs   : num 0 0 1 1 0 1 0 1 1 1 ...
 $ am   : num 1 1 1 0 0 0 0 0 0 0 ...
 $ gear: num 4 4 4 3 3 3 3 4 4 4 ...
 $ carb: num 4 4 1 1 2 1 4 2 2 4 ...
```

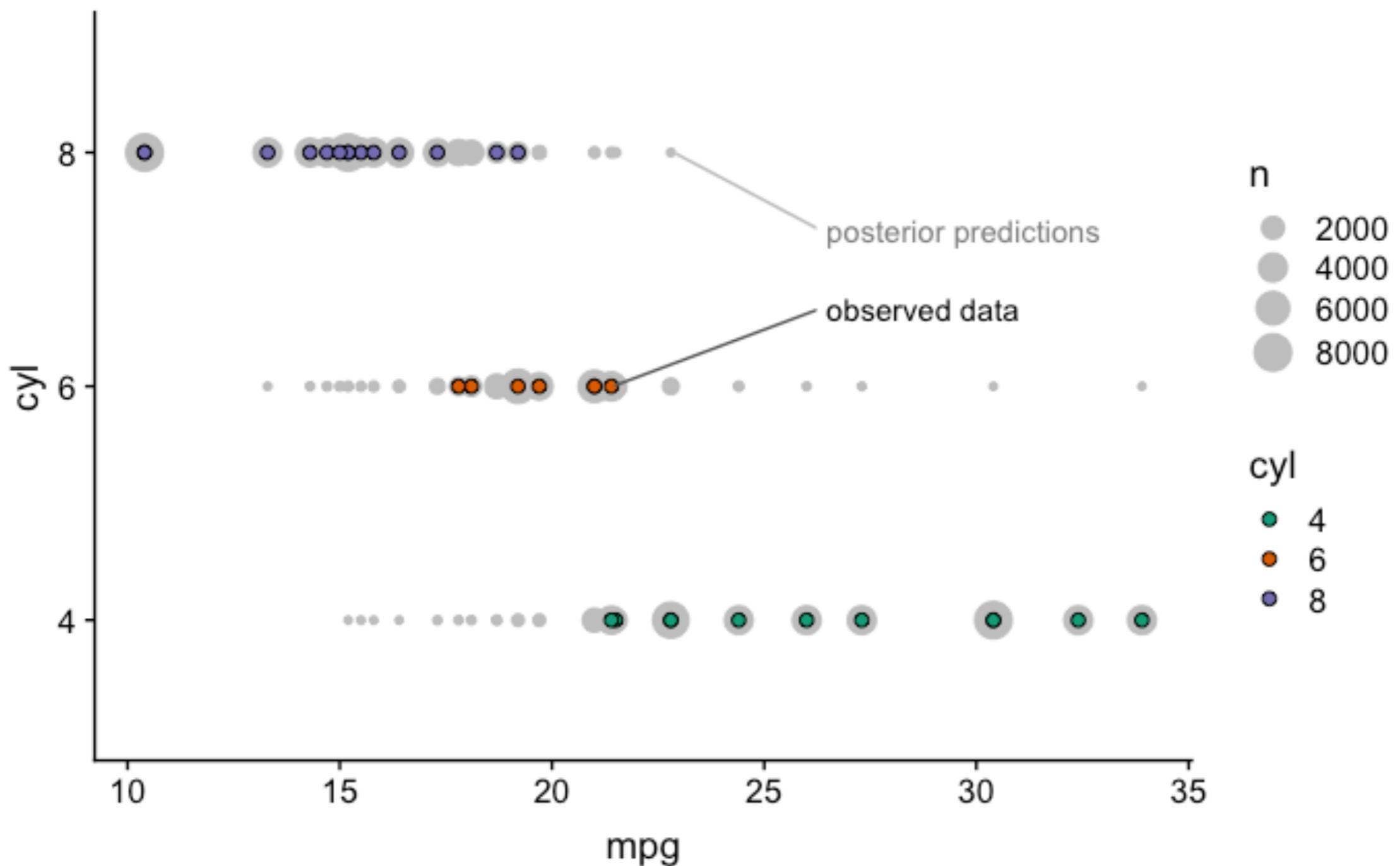
Bayesian ordinal regression

```
1 fit.cars = brm(formula = cyl ~ mpg,  
2                   data = df.cars,  
3                   family = "cumulative",  
4                   file = "cars",  
5                   seed = 1)
```

probability of
having 4, 6, or
8 cylinders



Posterior predictive check



Summary

- Doing Bayesian data analysis
 - A simple linear regression
 - Measuring uncertainty: Confidence interval vs. credible interval
- Building Bayesian models with `brms`
 - Model evaluation:
 - Visualizing and interpreting results
 - Testing hypotheses
 - Inference evaluation: Did things work out?
- Some `goob` examples
 - Evidence for null results
 - Dealing with unequal variance
 - Zero-one inflated beta binomial model
 - Ordinal logistic regression

Feedback

How was the pace of today's class?

much a little just a little much
too too right too too
slow slow

How happy were you with today's class overall?



What did you like about today's class? What could be improved next time?

Thank you!