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Question 1: Constraint Satisfaction

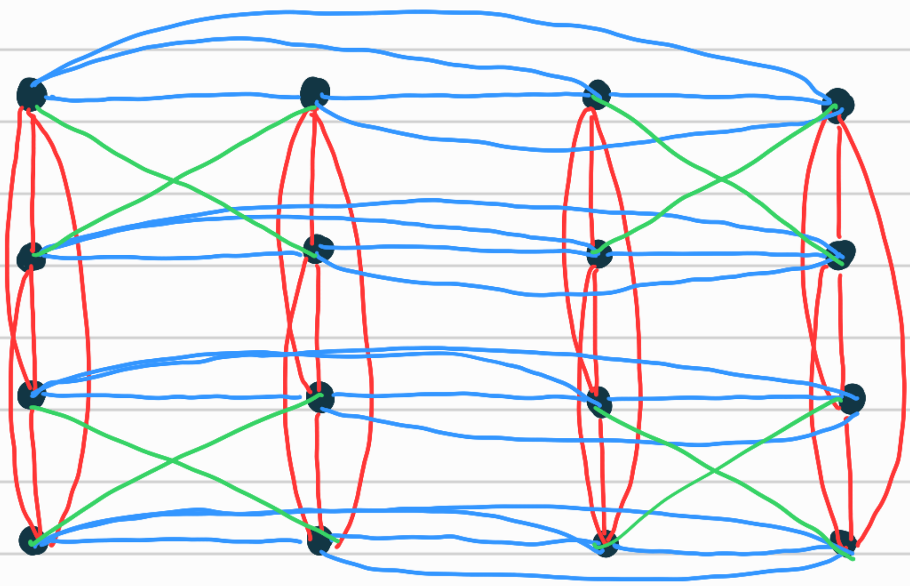
a) Formulate this problem as a CSP. List the variables, their domains, and the constraints. Draw the constraint graph.

Variables: Every empty cell in the Sudoku board. Can be denoted as (col, row)

Domains: For every cell, the domain is [1, 2, 3, 4]

Constraints: No repeating numbers in every row, every column and every defined 2x2 square.

Each node represents a variable



red: Every column must have unique numbers

blue: Every row must have unique numbers

green: Every defined 2x2 square must have unique numbers.

b) Find the first ten steps of backtracking search on this problem, where you order the variables in increasing order first by row, then by column, and the values from lowest to highest.

①

1			4
			2
1		4	
			1

②

2			4
			2
1		4	
			1

③

2	1		4
			2
1		4	
			1

④

2	1	3	4
			2
1		4	
			1

⑤

2	1	3	4
3			2
1		4	
			1

⑥

2	1	3	4
3	4		2
1		4	
			1

⑦

2	1	3	4
3	4	1	2
1		4	
			1

⑧

2	1	3	4
3	4	1	2
1	2	4	
			1

⑨

2	1	3	4
3	4	1	2
1	2	4	3
			1

⑩

2	1	3	4
3	4	1	2
1	2	4	3
4			1

c) Show the first ten steps of backtracking search on this problem with one-step forward checking, where you order the variables in increasing order first by row, then by column, and the values from lowest to highest.

c1	c2	c3	4
c4	c5	c6	2
1	c7	4	c8
c9	c10	c11	1

Start	Nothing Assigned
c1	[2, 3]
c2	[1, 2, 3]
c3	[1, 3]
c4	[3, 4]
c5	[1, 3, 4]
c6	[1, 3]
c7	[2, 3]
c8	[3]
c9	[2, 3, 4]
c10	[2, 3, 4]
c11	[2, 3]

1	Assign c1
c1	[2, 3]
c2	[1, 3]
c3	[1, 3]
c4	[3, 4]
c5	[1, 3, 4]
c6	[1, 3]
c7	[2, 3]
c8	[3]
c9	[3, 4]
c10	[2, 3, 4]
c11	[2, 3]

c1	c2	c3	4
c4	c5	c6	2
1	c7	4	c8
c9	c10	c11	1

②	Assign c2
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[3, 4]
c6	[1, 3]
c7	[2, 3]
c8	[3]
c9	[3, 4]
c10	[2, 3, 4]
c11	[2, 3]

③	Assign c3
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[3, 4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[3, 4]
c10	[2, 3, 4]
c11	[2]

④	Assign c4
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[2, 3, 4]
c11	[2]

⑤	Assign c5
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[2, 3]
c11	[2]

⑥	Assign c6
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[2, 3]
c11	[2]

⑦	Assign c7
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[3]
c11	[2]

c1	c2	c3	4
c4	c5	c6	2
1	c7	4	c8
c9	c10	c11	1

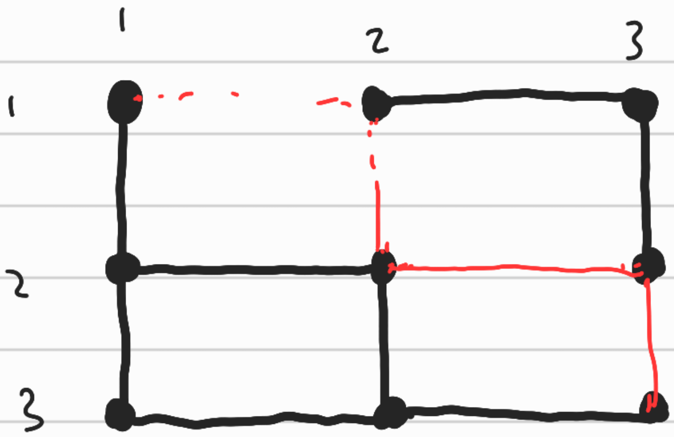
8	Assign c8
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[3]
c11	[2]

9	Assign c9
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[3]
c11	[2]

10	Assign c10
c1	[2, 3]
c2	[1, 3]
c3	[3]
c4	[3, 4]
c5	[4]
c6	[1]
c7	[2, 3]
c8	[3]
c9	[4]
c10	[3]
c11	[2]

Question 2: Search and Game Playing

You are playing the dots and box game on a 3×3 grid shown below. Each player has to draw an edge connecting two dots, if it doesn't already exists. The player who draws the 4th line making a unit square receives a +1 score. The player with the largest score after none of the players can draw edges anymore wins the game.



(init state)

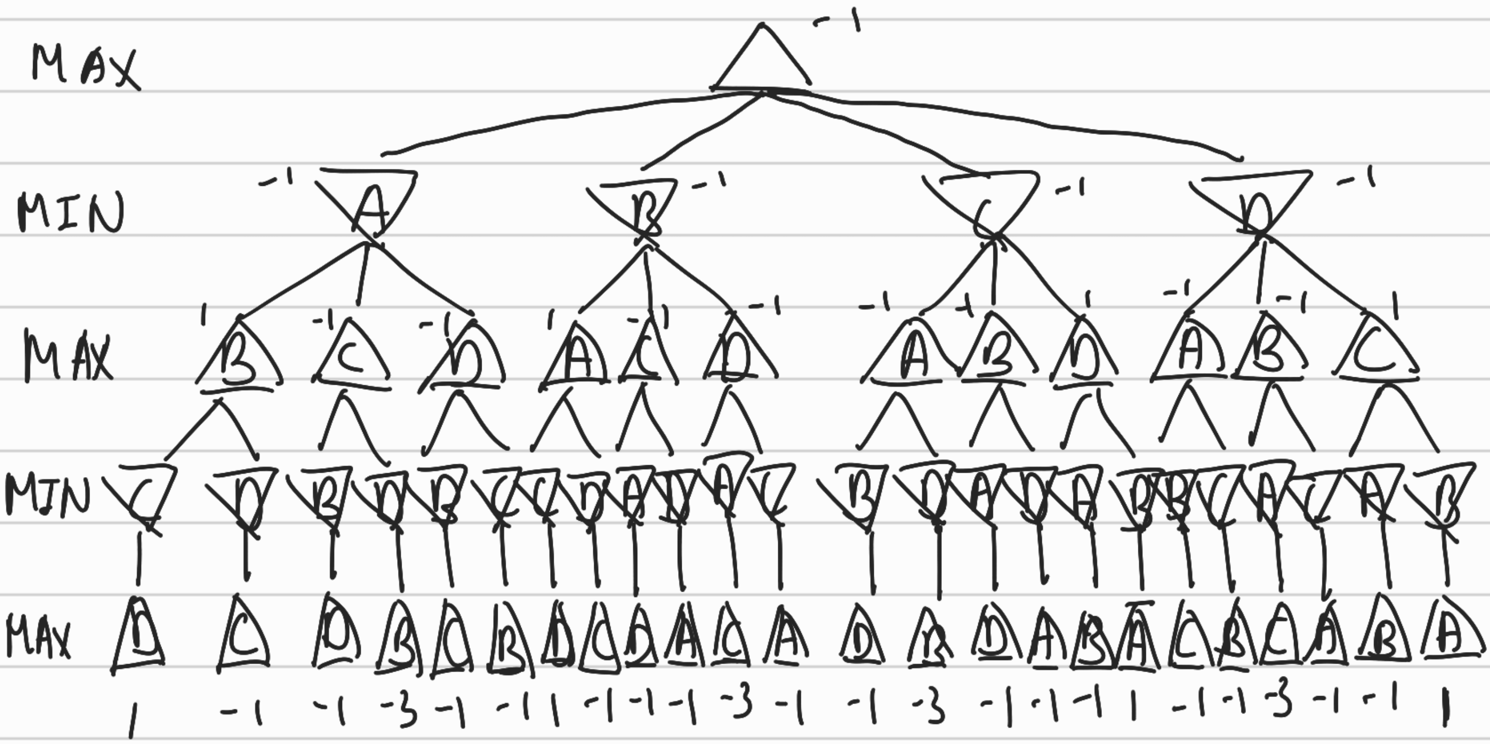
It's the max player's turn.

a) Apply the Minimax algorithm to the above state, by preferring horizontal moves to vertical moves, and expanding in increasing order first by row, then by column. For convenience, you can represent the moves as (start row, start col, end row, end col) eg drawing a vertical line in the bottom right corner can be represented as the move (3,3,2,3) or equivalently (2,3,3,3). Draw the corresponding search tree.

→

We only have 4 possible moves:

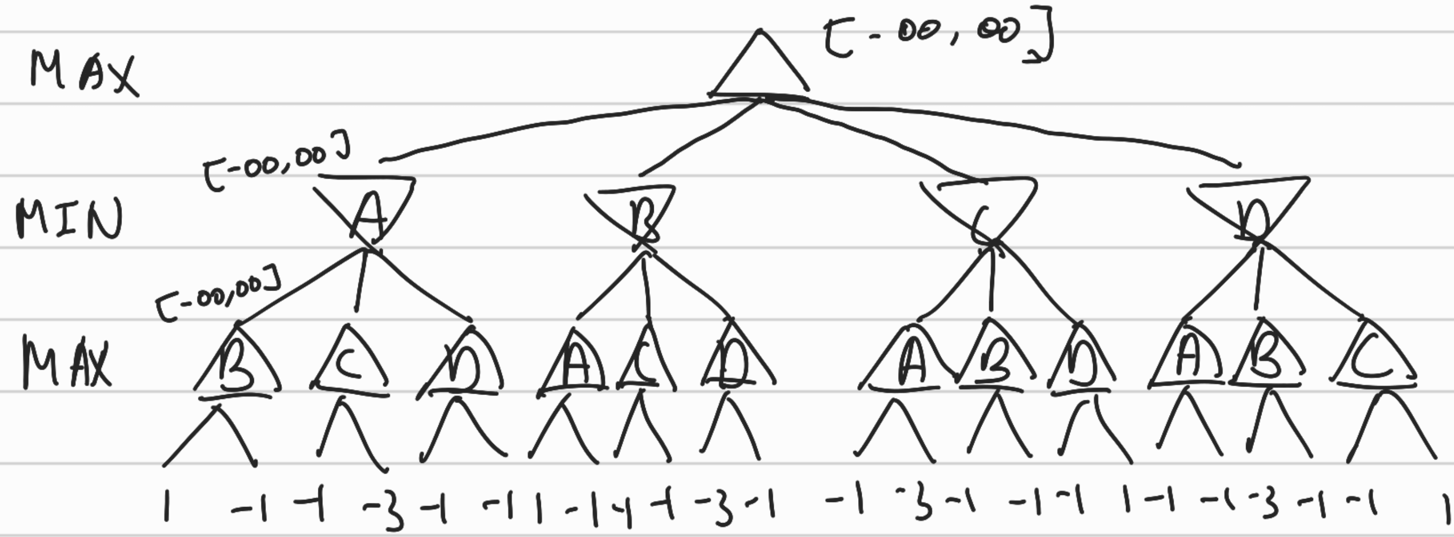
A: (1,1,1,2)
B: (2,2,2,3)
C: (1,1,2,2)
D: (2,3,3,3)

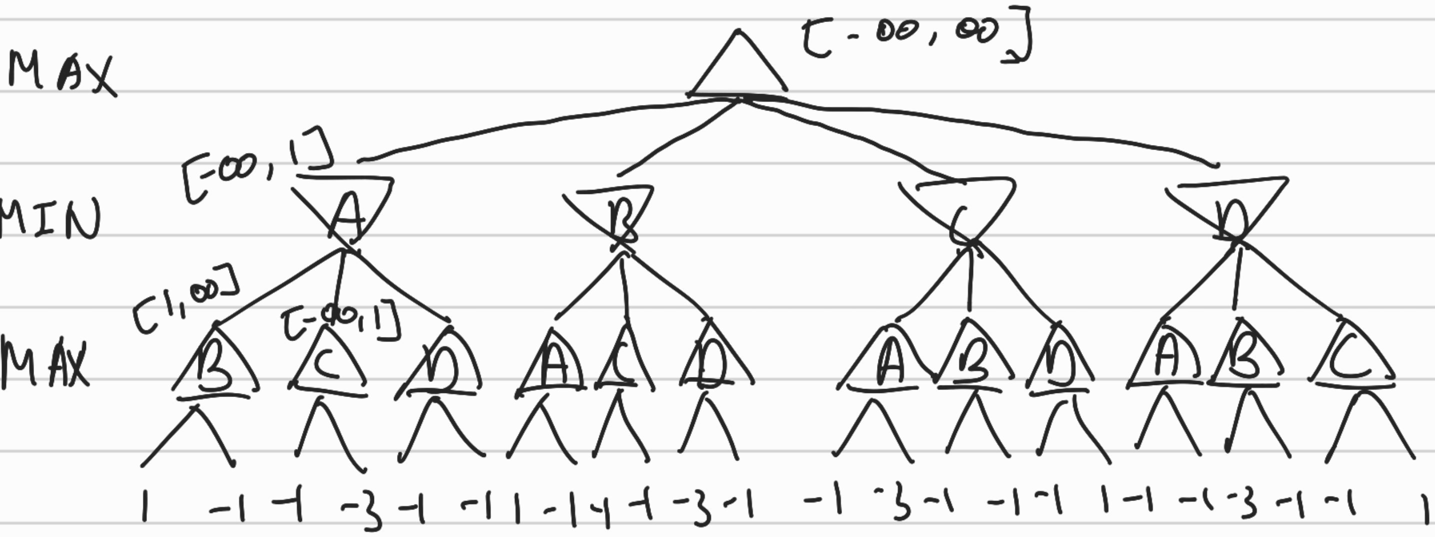


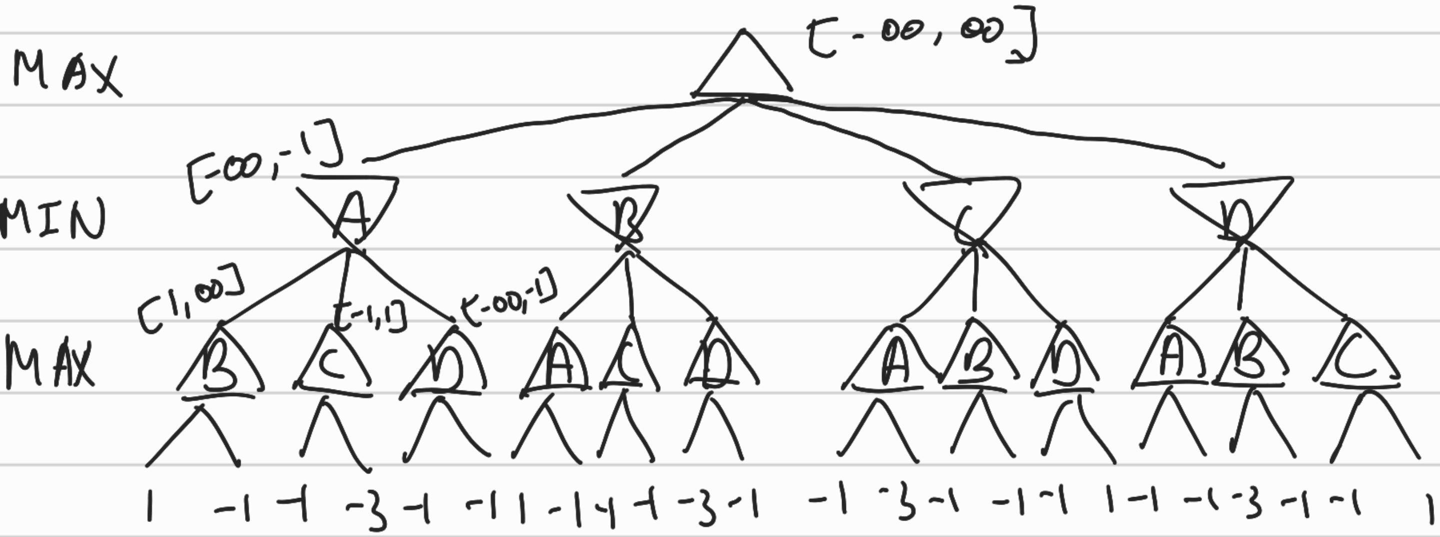
b) Apply the alpha-beta pruning method using the same order of node expansion and show the alpha-beta values for all nodes. Is there any advantage to using alpha-beta pruning?

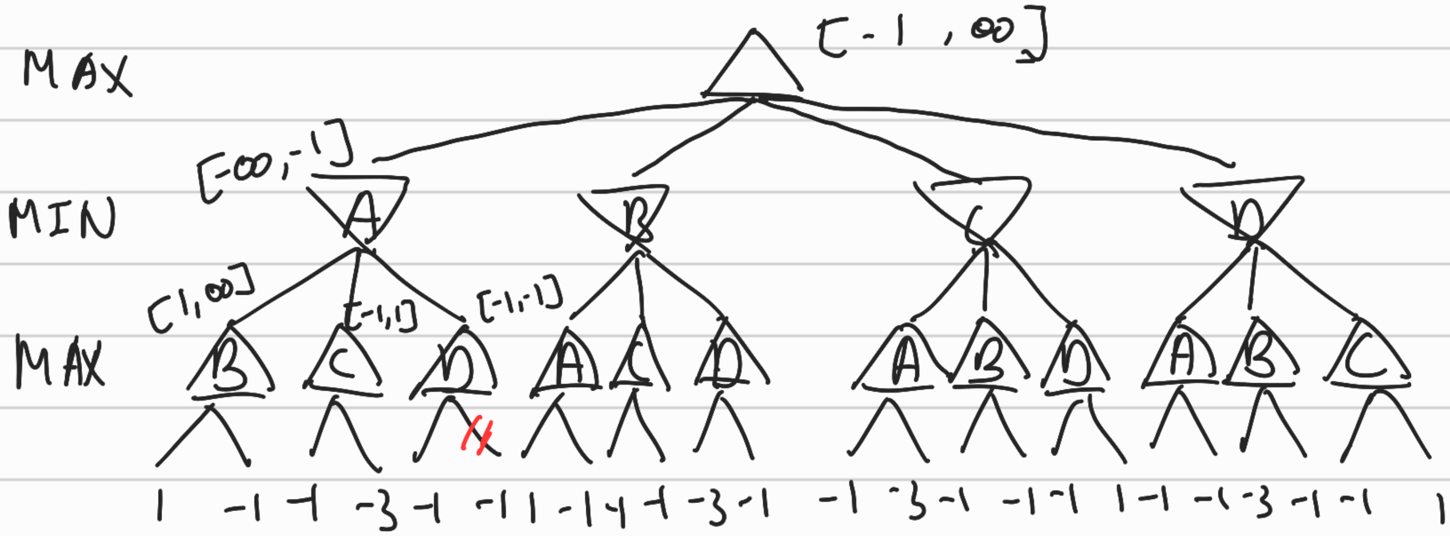
$[\alpha, \beta]$

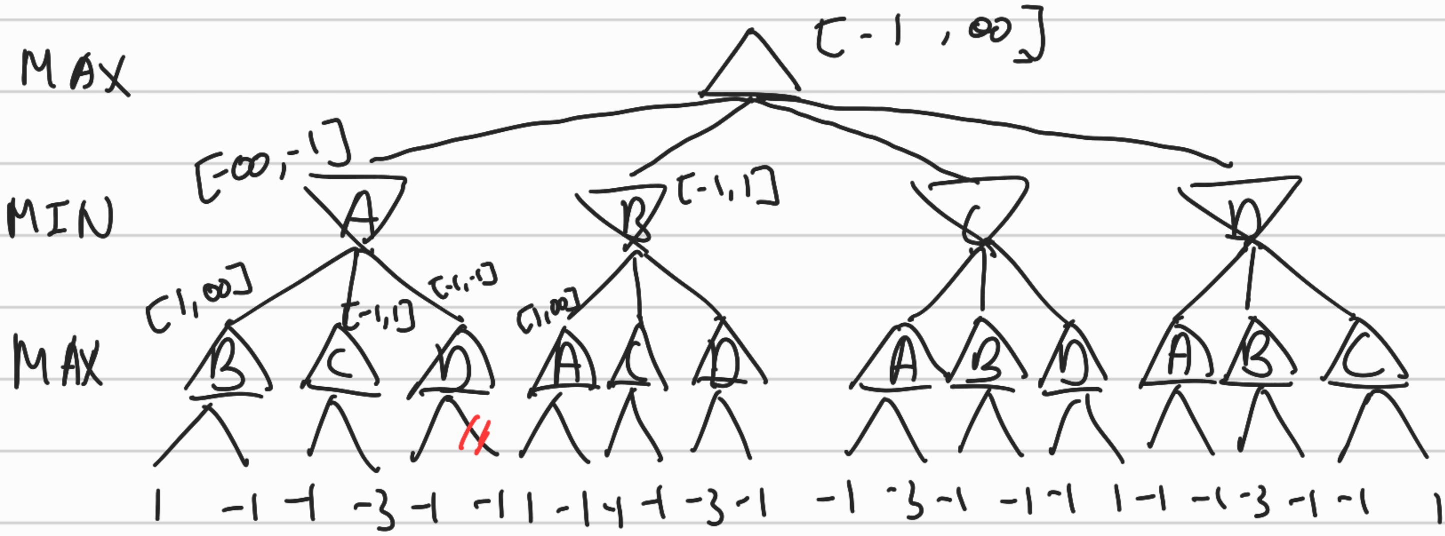
We prune when $\alpha \geq \beta$

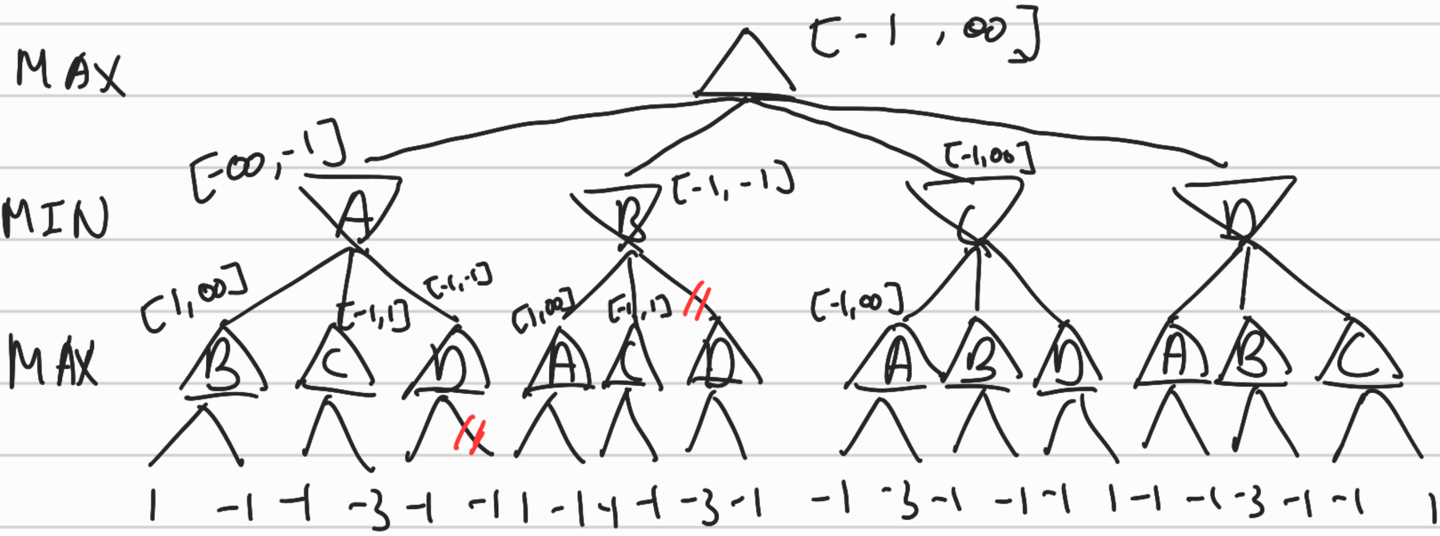

MAX
MIN
MAX
[-∞, ∞]
[-∞,∞]
[-∞,∞]
A
B
C
D

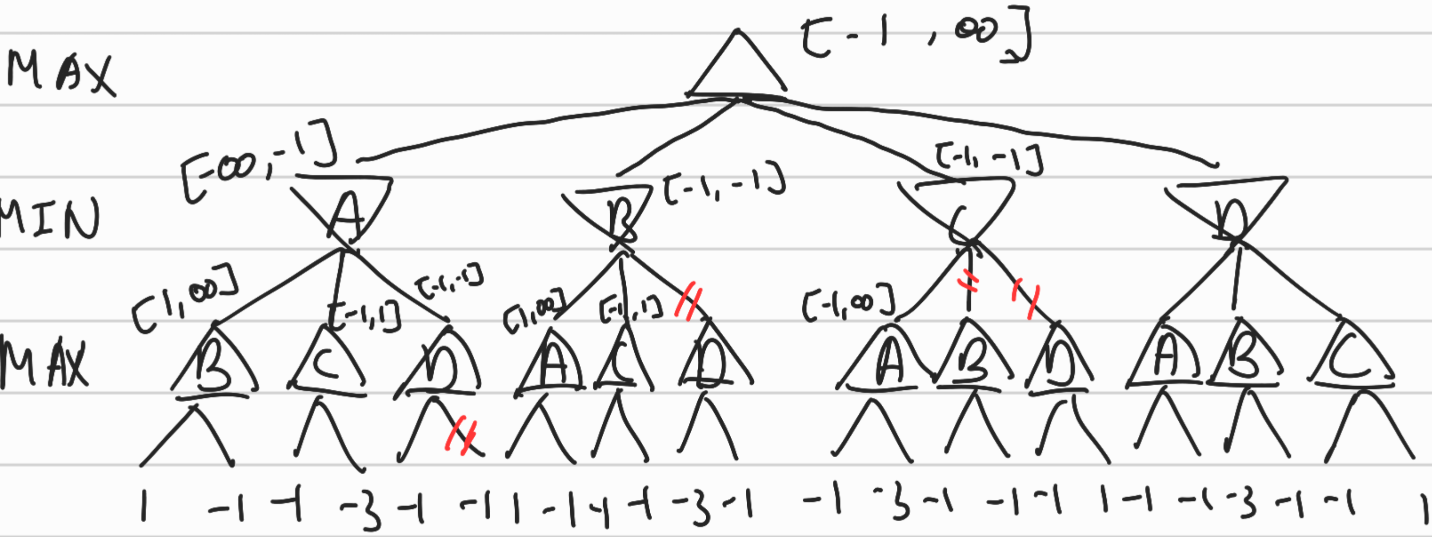

MAX
MIN
MAX
[-∞, ∞]
[-∞, 1]
[1,∞]
[-∞,1]
A
B
C
D

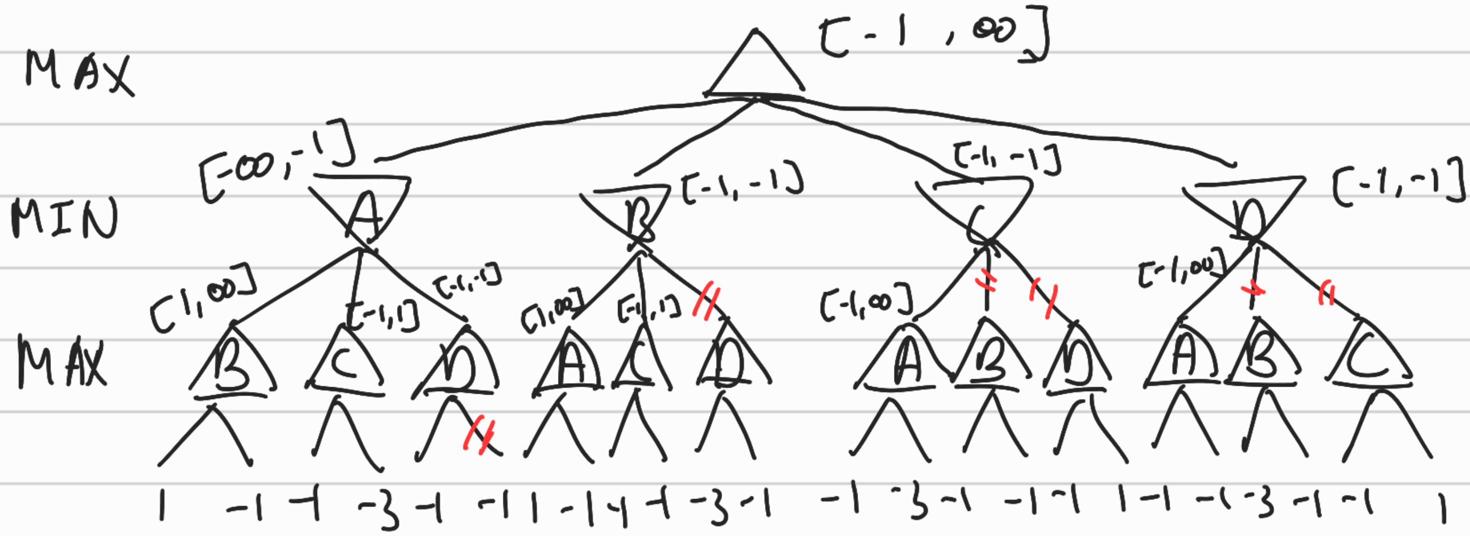

MAX
MIN
MAX
[-∞, ∞]
[-∞,-1]
[1,∞]
[-1,1]
[-∞,-1]
A
B
C
D


MAX
MIN
MAX
[-1 , ∞]
[-∞,-1]
[1,∞]
[-1,1]
[-1,-1]
A
B
C
D


MAX
MIN
MAX
[-1 , ∞]
[-∞, -1]
[-1, 1]
[1, ∞]
A
B
C
D
1 -1 -1 -3 -1 -1 1 -1 4 -1 -3 -1 -1 -3 -1 -1 -1 1 -1 -1 -3 -1 -1 1


MAX
MIN
MAX
[-1 , ∞]
[-∞, -1]
[-1, -1]
[-1, ∞]
[1, ∞]
[-1, 1]
A
B
C
D
1 -1 -1 -3 -1 -1 1 -1 4 -1 -3 -1 -1 -3 -1 -1 -1 1 -1 -1 -3 -1 -1 1


MAX
MIN
MAX
[-1 , ∞]
[-∞, -1]
[-1, -1]
[-1, ∞]
[1, ∞]
[-1, 1]
A
B
C
D
1 -1 -1 -3 -1 -1 1 -1 4 -1 -3 -1 -1 -3 -1 -1 -1 1 -1 -1 -3 -1 -1 1


MAX
MIN
MAX
[-1 , ∞]
[-∞, -1]
[-1, -1]
[-1, ∞]
[1, ∞]
[-1, 1]
A
B
C
D
1 -1 -1 -3 -1 -1 1 -1 4 -1 -3 -1 -1 -3 -1 -1 -1 1 -1 -1 -3 -1 -1 1

There is an advantage of using alpha-beta pruning because we pruned a total of 6 branches.

Question 3 Propositional Logic.

a) How many models are there for each of the following statements in propositional logic?

i) $\overline{(A \land B)} \lor \overline{(C \land \overline{B})}$

8 models

ii) $A \Rightarrow (A \land B) \land \overline{B} \lor \overline{C}$

6 models

iii) $\overline{(A \Rightarrow B \land C \land D)} \lor (B \Rightarrow \overline{C})$

13 models

b) State whether each of the following is a valid, unsatisfiable, or satisfiable. Support your answers with a truth table or a proof using rules of logical inference.

i) $(A \lor B) \Rightarrow C \Rightarrow (A \Rightarrow C) \land (B \Rightarrow C)$

It is valid because it is true in every model

truth table

$\longrightarrow$

A	B	C	(equation)
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

iii) $((A \Rightarrow B) \wedge (A \vee C)) \Rightarrow (B \vee C)$

A	B	C	Equation
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

It's valid because it's true in every model.

Question 4: First Order Logic
Consider a vocabulary with the following symbols:

Occupation (p, o) : Predicate. Person has occupation o
Customer (p1, p2) : Predicate. Person p1 is customer of person p2.
Boss (p1, p2) : Predicate. Person p1 is a boss of person p2.
Doctor, Surgeon, lawyer, Actor : Constants denoting occupations.
Emily, Joe : Constants denoting people.

a) Use these symbols to write the following assertions in FOL
i) Emily is ~~either~~ a surgeon or a lawyer.
Occupation (Emily, Surgeon) ∨ Occupation (Emily, lawyer)

ii) Joe Is an actor, but he also holds another job
Occupation (Joe, actor) ∧ ∃ job, job ≠ Actor ∧ Occupation (Joe, job)

iii) All surgeons are doctors.
∀x Surgeon (x) -> Doctor (x)

iv) Joe does not have a lawyer
∀x lawyer (x) -> ¬ Customer (Joe, x)

v) Emily has a boss who Is a lawyer
∃x Boss (x, Emily) -> Occupation (x, lawyer)

vi) There exists a lawyer all of whose customers are doctors
∃ x Occupation (x, lawyer) ∧ ∀ c Customer (c, x) -> Occupation (c, Doctor)

viii) Every Surgeon has a lawyer

$\forall x$ Occupation (x, Surgeon) -> $\exists c$ Customer (x, Occupation (c, lawyer))

b) Give a model of the above FOL such that clauses i) - viii) above are satisfied.

People domain: { Emily, Joe, EmilyBoss, ADoctor, ALawyer }
Occupation domain: { Doctor, Surgeon, lawyer, Actor }.

Interpretations:
Occupations: (Emily, lawyer), (Joe, Actor), (Joe lawyer)
(EmilyBoss, lawyer), (ADoctor, Doctor),
(ALawyer, lawyer)

Customer: (ADoctor, ALawyer)
Boss: (EmilyBoss, Emily)