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CSCI-C 290 6W2 Summer 2022[*]

Programming Quantum Computers
Assignment 01

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Problem 0.1

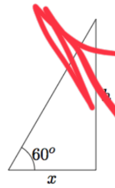
Consider the quadratic equation

$$ax^2 + bx + c = 0$$

Please provide a root to this equation. If there are more, just provide one of them.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Problem 0.2



$\tan 60^\circ = \dfrac{h}{x}$

$h = \tan 60^\circ * x$

What is the value of h, in terms of x?

Problem 0.3

Rewrite $1 + i$ in the form $re^{i\phi}$. Please make sure that your r is positive and that your ϕ is in the range $[0, 2\pi]$.

$z = \sqrt{2}\, e^{\frac{i\pi}{4}}$

Answer true or false to the following statements[1]:

Problem 1

$$|0\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

FALSE. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

[*] https://legacy.cs.indiana.edu/classes/c290-quantum-dgerman/sum2022/

[1] If the answer is false, please indicate the correct answer.

Problem 2

$$|1\rangle = \begin{pmatrix}1\\1\end{pmatrix}$$

FALSE. $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

Problem 3

TRUE

A quantum state is a unit vector in a complex vector space.

Problem 4

FALSE

Measurements can only be performed in the computational[2] basis.

Problem 5

FALSE. Probability that outcome is x is equal to the absolute value of the probability amplitude squared.

The probability amplitude of $|x\rangle$ is equal to the probability that the outcome of a measurement is x.

Problem 6

FALSE

The inner product of $|+\rangle$ and $|-\rangle$ is 1.

$\langle +|-\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$

Now indicate the correct answer and how you determined it:

Problem 7

In $\mathbb{C}^2$, how many **real** unit vectors are there whose projection onto $|1\rangle$ has length $\frac{\sqrt{3}}{2}$?

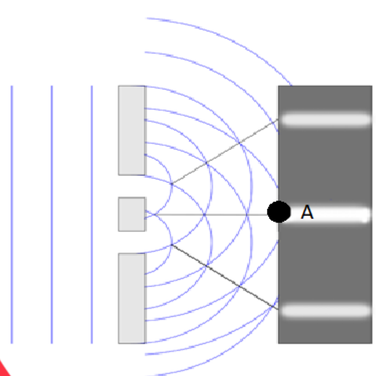
4. $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$, $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$, $-\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$, & $-\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$.

Problem 8

In $\mathbb{C}^2$, how many complex unit vectors are there whose projection onto $|1\rangle$ has length $\frac{\sqrt{3}}{2}$?

Infinite

Problem 9



In a double slit experiment with three different incident objects we

[2]Recall that the computational (standard) basis is comprised of $|0\rangle$ and $|1\rangle$.

find that the centre of the detector (A) as shown has an intensity 1, when only one of the slits is open. What would be the intensity of the same point when both slits are open in each of the following cases: (a) bullet, (b) wave, (c) quantum mechanics (wave or photon)

(a) bullets are discrete $P_{12}(x) = P_1(x) + P_2(x) = 1 + 1 = 2.$

(b) continous $I_{12}(x) = (h_1(x) + h_2(x))^2 = (1+1)^2 = 4$

(c) Photons - discrete $P_{12}(x) = |a_{12}(x)|^2 = (1+1)^2 = 4$

Problem 10

A qubit exists in state, $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$
If we measure this in the following $|u\rangle$ basis

$$|u\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

$$|u^{\perp}\rangle = \frac{-1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

What is the probability that $|\psi\rangle$ in state $|u\rangle$ when measured in orthonormal bases u and $u^{\perp}$?

$$\langle u|\psi\rangle = \left(\frac{\sqrt{3}}{2}\ \ \frac{1}{2}\right)\begin{pmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} = 0.75$$

Problem 11

For a state $|\psi\rangle = a|0\rangle + b|1\rangle$, where a and b satisfy all the rules of a quantum state. After measuring $|\psi\rangle$ we find probability of $|0\rangle$ is $\frac{9}{25}$. What are the numerical values of a and b?

$a = \frac{3}{5}$ $b = \frac{4}{5}$

$|a|^2 + |b|^2 = 1$

$|a|^2 = \frac{9}{25} = \left(\frac{3}{5}\right)^2$

$a = \frac{3}{5}$ $b = \sqrt{1 - |a|^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$|\theta\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

Problem 12

A qubit exists in state, $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$
If we measure this in the following $|u\rangle$ basis

$$|u\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

$$|u^{\perp}\rangle = \sin\theta\,|0\rangle - \cos\theta\,|1\rangle$$

In order for u and $u^{\perp}$ to be equally probable, what should be the value of θ? (consider a value between 0 and 90 degrees)

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta$

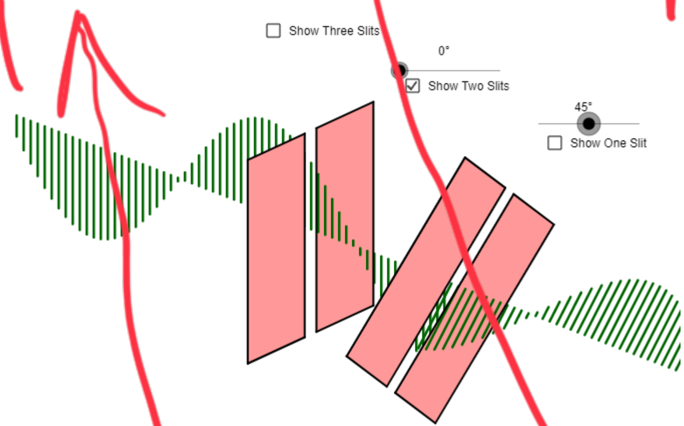
$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta$

$\left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)^2 =$

$\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)^2$

$\theta = \frac{5\pi}{12} = 75°$

Problem 13

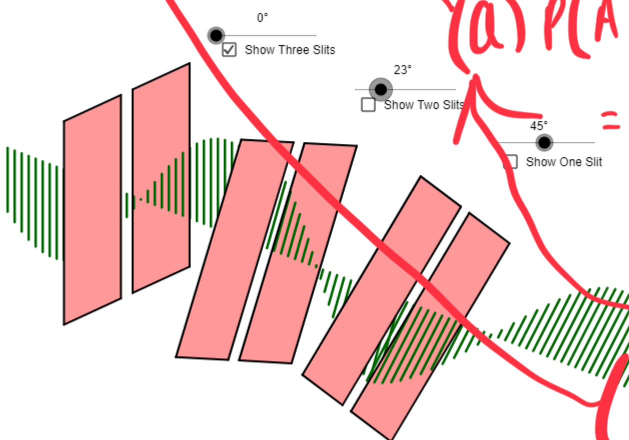


Vertically aligned light is incident on the left most polarizing filter as shown above. If a second filter is placed at 45 degrees, what is the probability that photon is transmitted through both filters?

(a) $P(A \text{ given } B) = P(A) \cdot P(B)$

$= |\cos(45°)^2| \cdot 1 = \left(\frac{\sqrt{2}}{2}\right)^2 \cdot 1$

$= \frac{1}{2} = 0.5$



(b) $P(x) = 1 \cdot \cos^2(90-x) \cdot \cos^2(x-45)$

$\sin(135-2x) = 0 \rightarrow x = \frac{135}{2} = 67.5°$

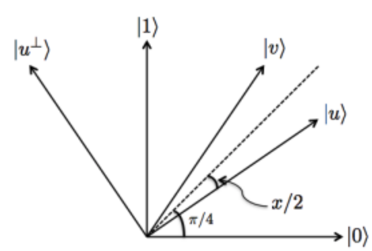
Now we place a third filter between these two as shown. At what angle should we place these filters such that probability of photons exiting at the final filter is maximum? In the final setup, what is the probability of photon transmitting through all three filters? (Hint: an interactive animation may help you prove your solution[3].)

(c) $\cos((90-67.5)°)^2$ $\cos((67.5-45)°)^2$

~ 0.729

Problem 14

A qubit is either in the state $|u\rangle = \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)|0\rangle + \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)|1\rangle$ or $|v\rangle = \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)|0\rangle + \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)|1\rangle$ and we want to determine which state it is in by measuring it. One of the following two measurements is optimal in terms of the probability of success.



Measurement I: Measure in the basis $|u\rangle = \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)$, $|u^{\perp}\rangle = -\sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$. If the outcome is u, guess that the qubit was the state $|u\rangle$, and if the outcome was $u^{\perp}$, guess that it was in the state $|v\rangle$.

[3] https://ophysics.com/l3.html

Measurement II: Measure in the standard basis and interpret $|0\rangle$ as $|u\rangle$ and $|1\rangle$ as $|v\rangle$.

The probability of success is defined as $\frac{1}{2}p_u + \frac{1}{2}p_v$, where

$$p_u = \Pr[\text{we guess } |u\rangle \text{ given that the qubit was in the state } |u\rangle]$$

and

$$p_v = \Pr[\text{we guess } |v\rangle \text{ given that the qubit was in the state } |v\rangle]$$

(a) What is the probability of success of Measurement I as a function of x?

(b) What is the probability of success of Measurement II as a function of x?

(c) Now note that the following become good approximations as $\theta \to 0$:

$$\sin\theta \approx \theta$$

$$\sin^2(\frac{\pi}{4} - \theta) \approx \frac{1}{2} - \theta$$

Use these approximations to estimate the probability of success of the two measurements as $x \to 0$ as a function of x.

(a) $\frac{1+\sin^2 x}{2}$

(b) $\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

(C) Measurement I: $\frac{1+\sin^2(x)}{2} \approx \frac{1+x^2}{2}$

Measurement II: $\cos^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = 1 - \sin^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right) \approx \frac{1}{2} + \frac{x}{2} = \frac{1+x}{2}$

d) if x close to 0, $\frac{1+x^2}{2} < \frac{1+x}{2}$. Measurement II is better at distinguishing