Assignment 5

Group 21:

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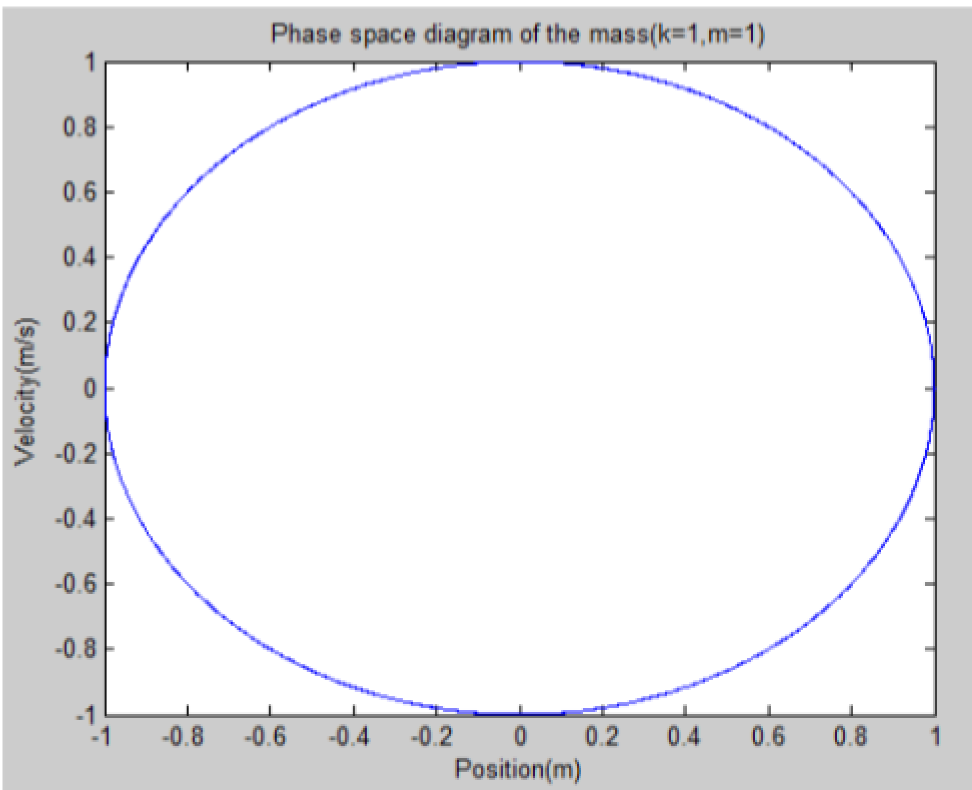
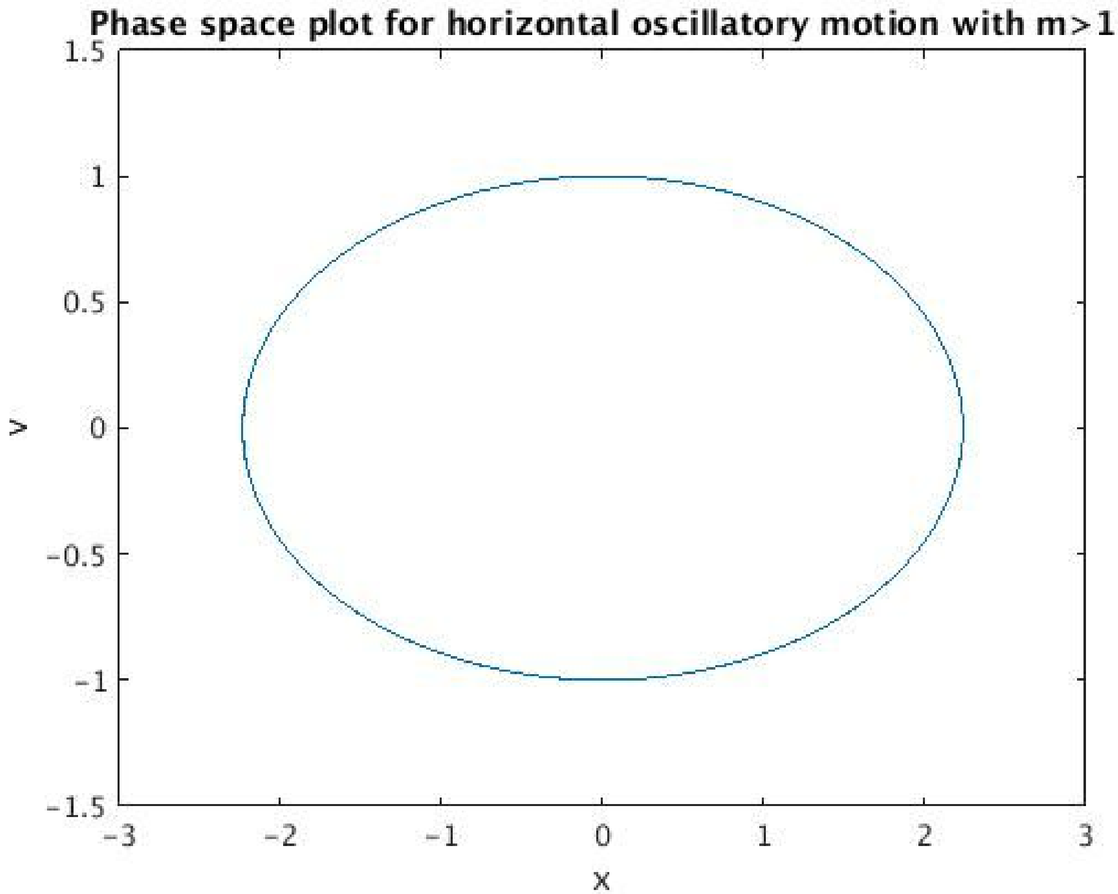
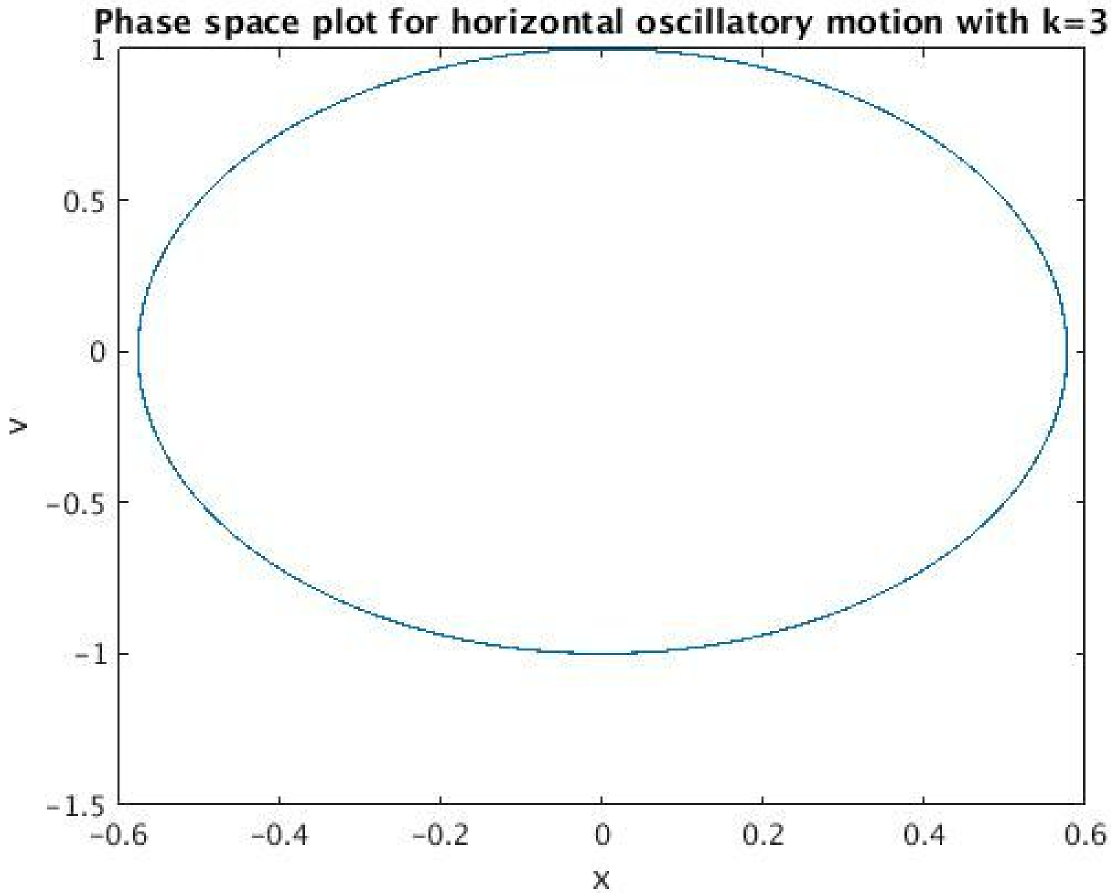
Simple Harmonic Oscillations (Mass-spring system):

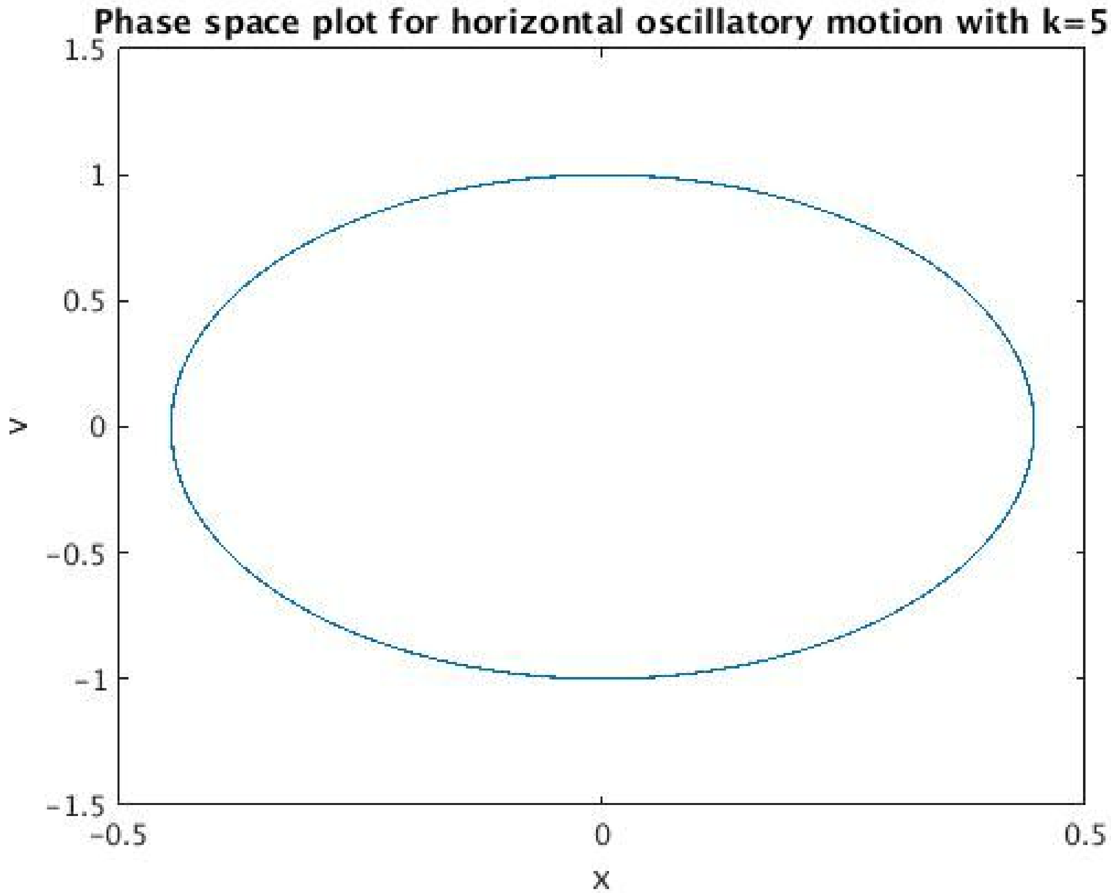


Phase space plot for the motion of the object is elliptical and in clockwise direction.



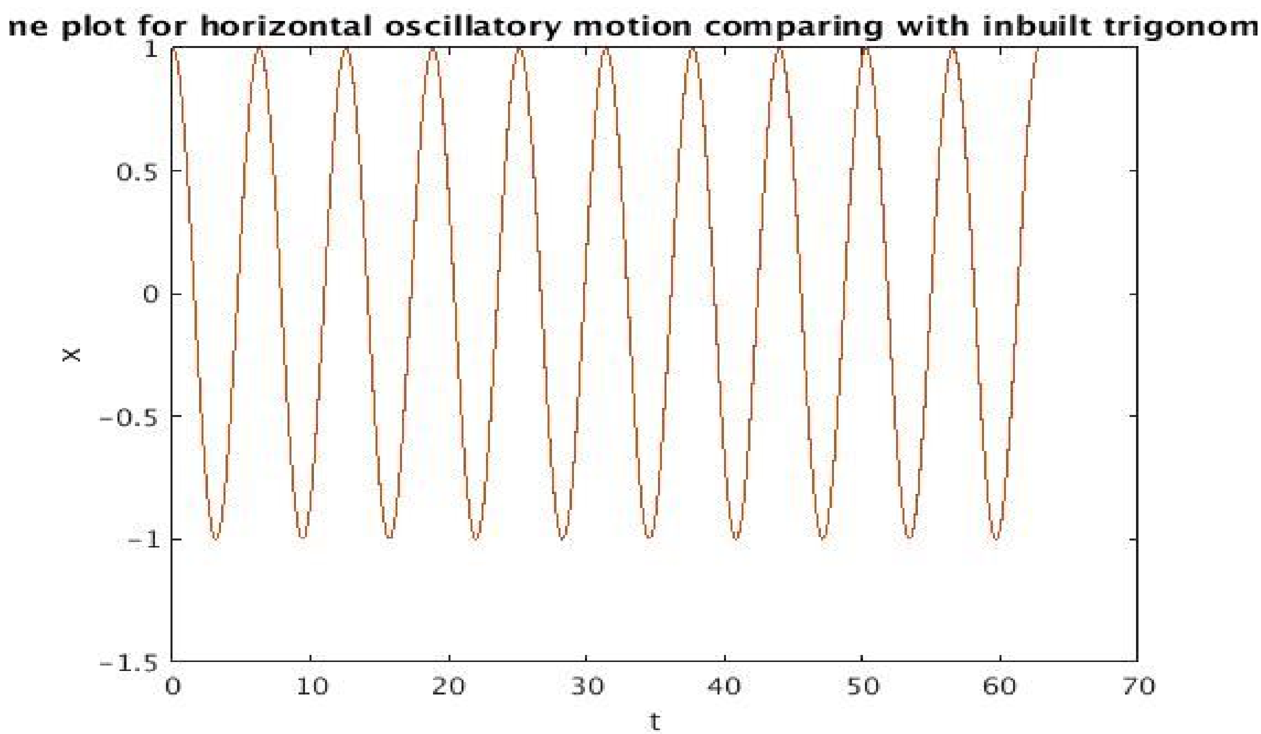
With increase in mass of the object, the amplitude of oscillation increases and the time period also increases.

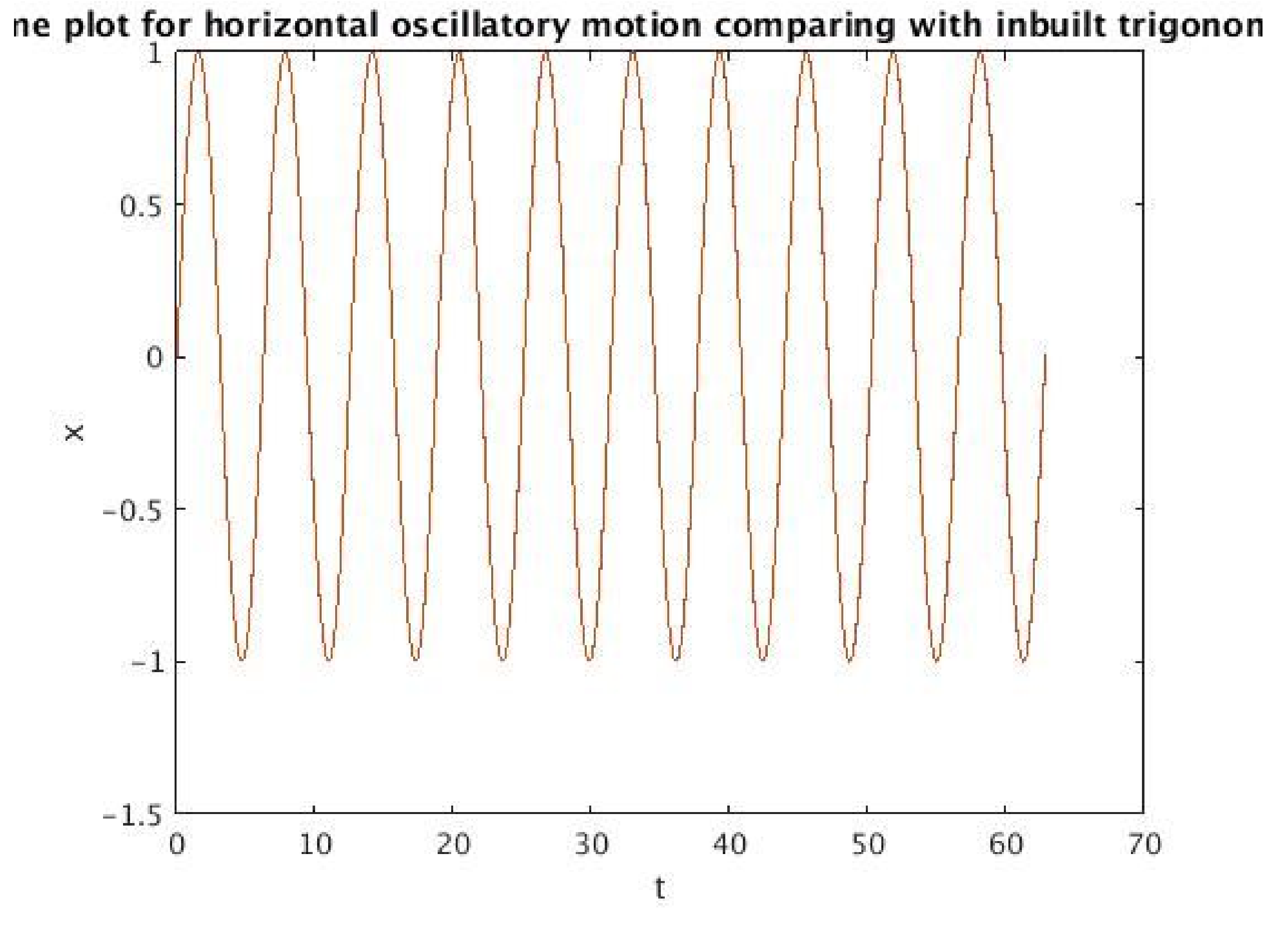

Phase space plot for horizontal oscillatory motion with k=3
v
x
1
0.5
0
−0.5
−1
−1.5
−0.6
−0.4
−0.2
0
0.2
0.4
0.6



With increase in the value of spring constant k, the amplitude decreases and time period of oscillation also decreases. For the above plots we assumed, initial velocity as 1 m/s and initial position as 1 m to the right.

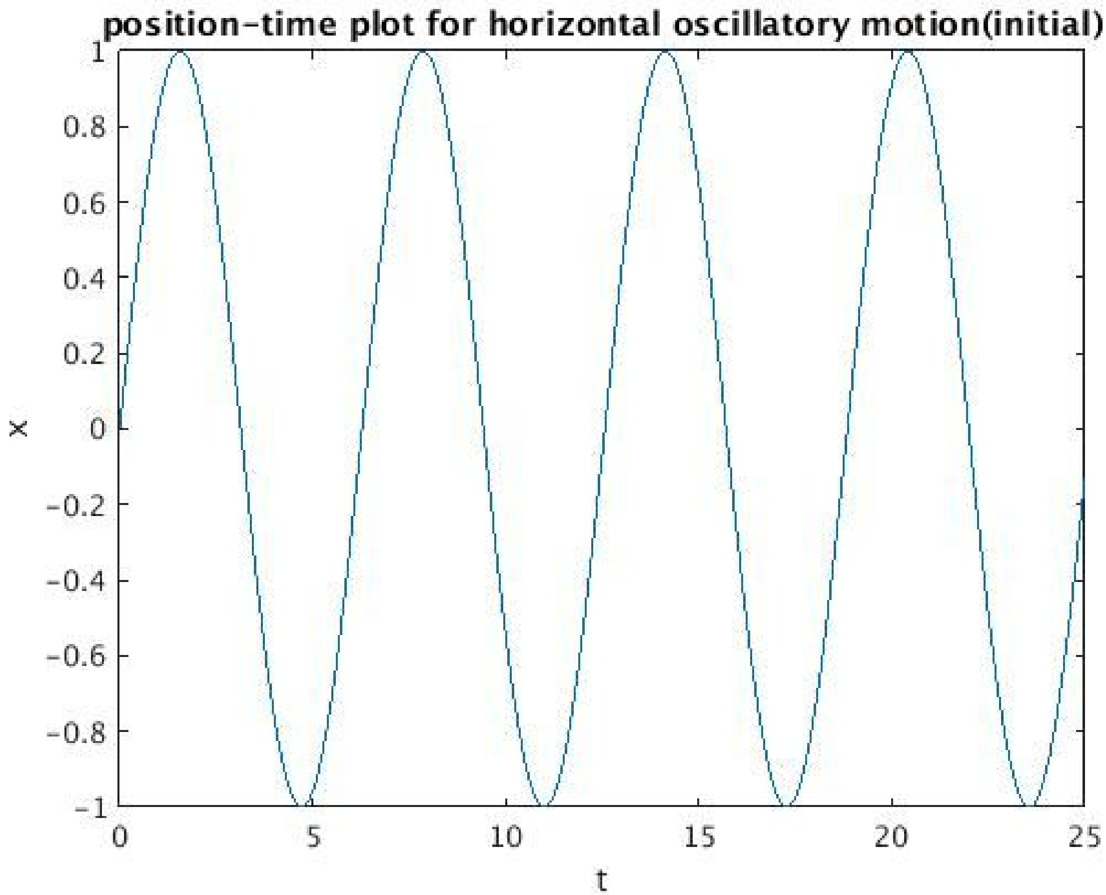
The plots of the numerically obtained sinusoidal function did not completely overlap with the analytical solution, but on decreasing the time-step appropriately, the plots overlapped perfectly as shown below.



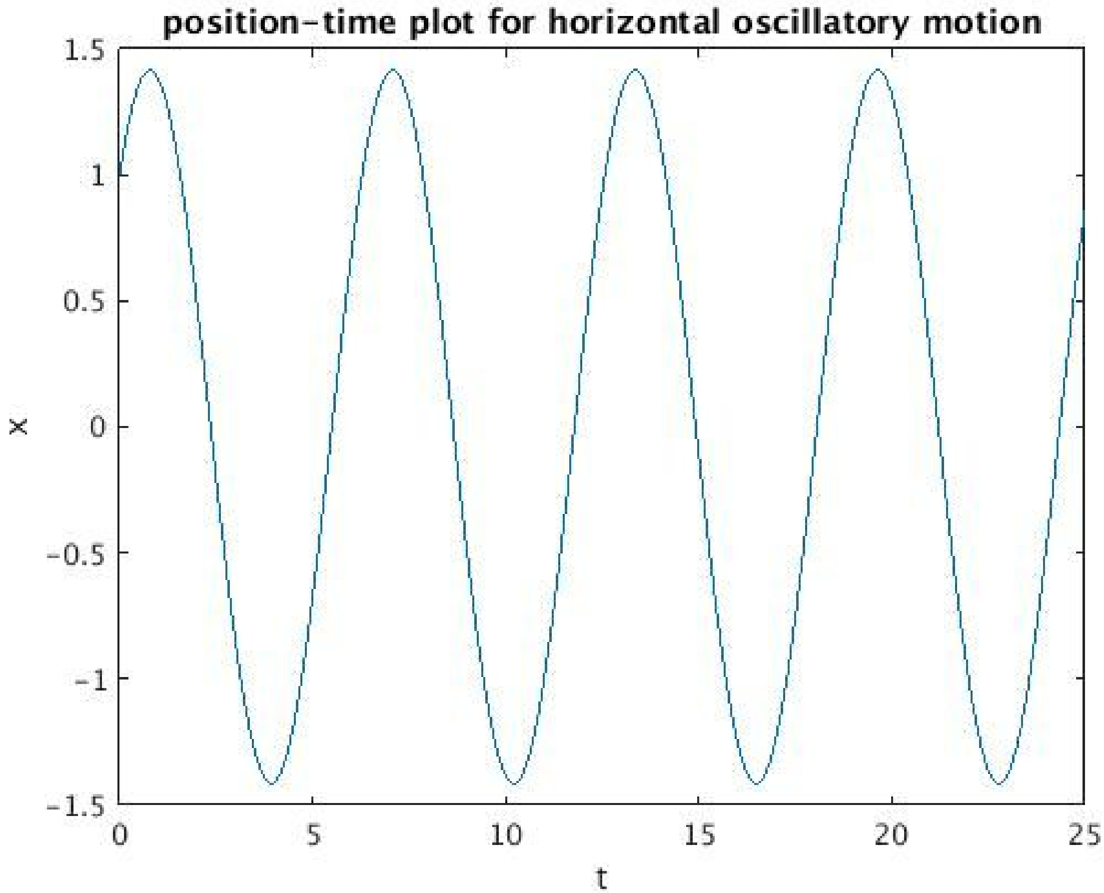


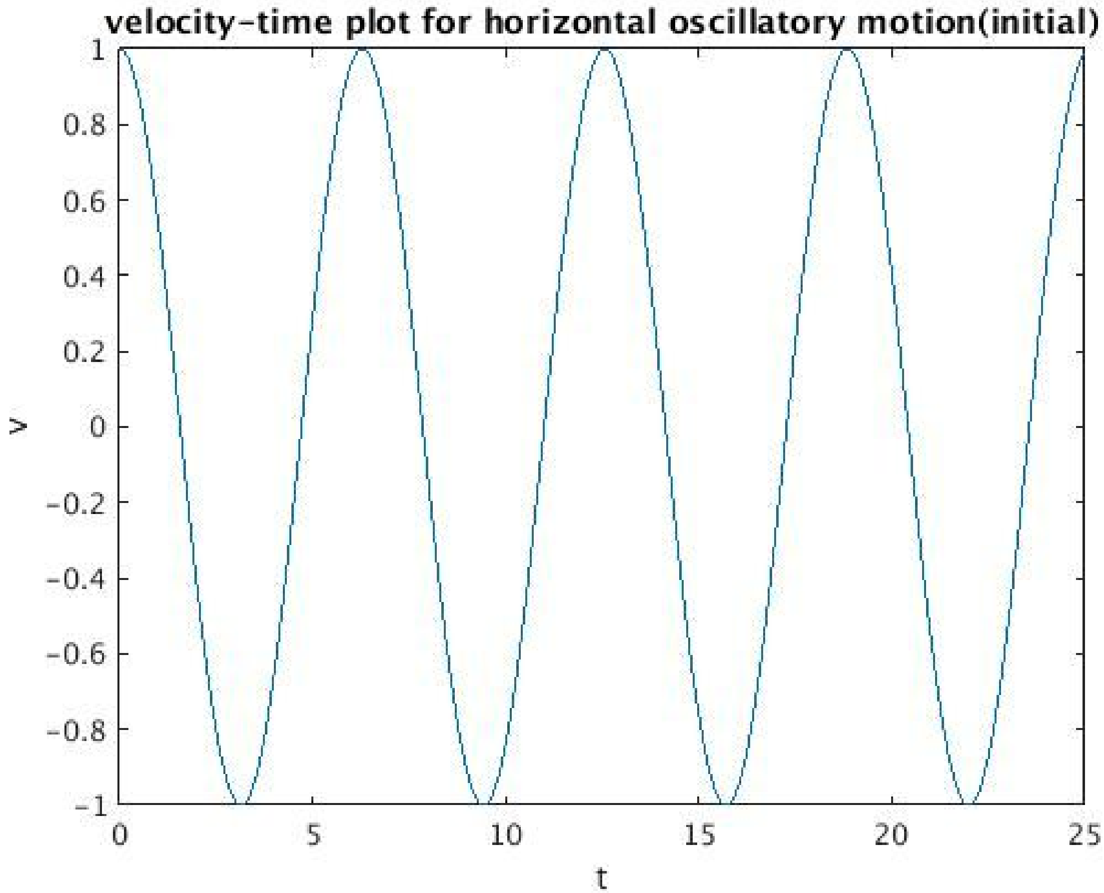
ne plot for horizontal oscillatory motion comparing with inbuilt trigonon
1
0.5
0
-0.5
-1
-1.5
x
0
10
20
30
40
50
60
70
t

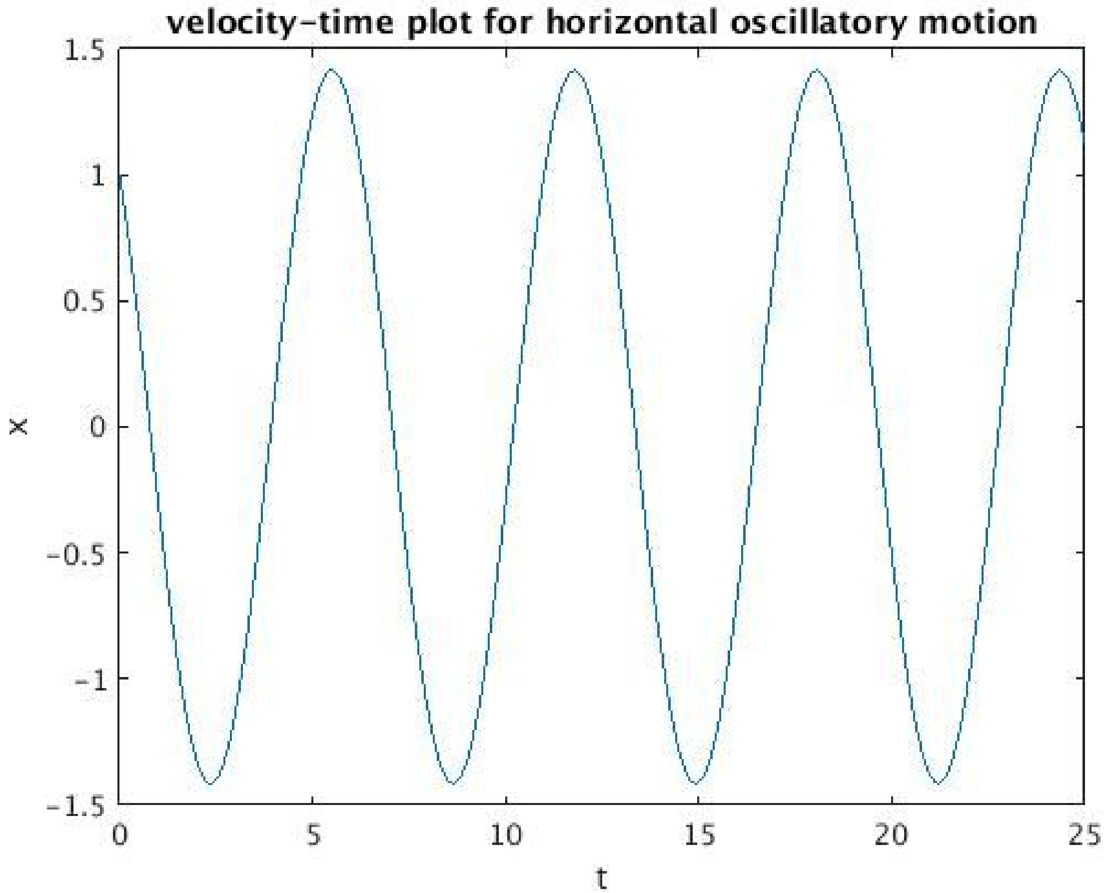
$x_0 = 1 \;\; v_0 = 1$:



position-time plot for horizontal oscillatory motion(initial)
x
t


position-time plot for horizontal oscillatory motion
1.5
1
0.5
0
-0.5
-1
-1.5
x
0
5
10
15
20
25
t


velocity-time plot for horizontal oscillatory motion(initial)
v
t
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25


velocity-time plot for horizontal oscillatory motion
x
t
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25

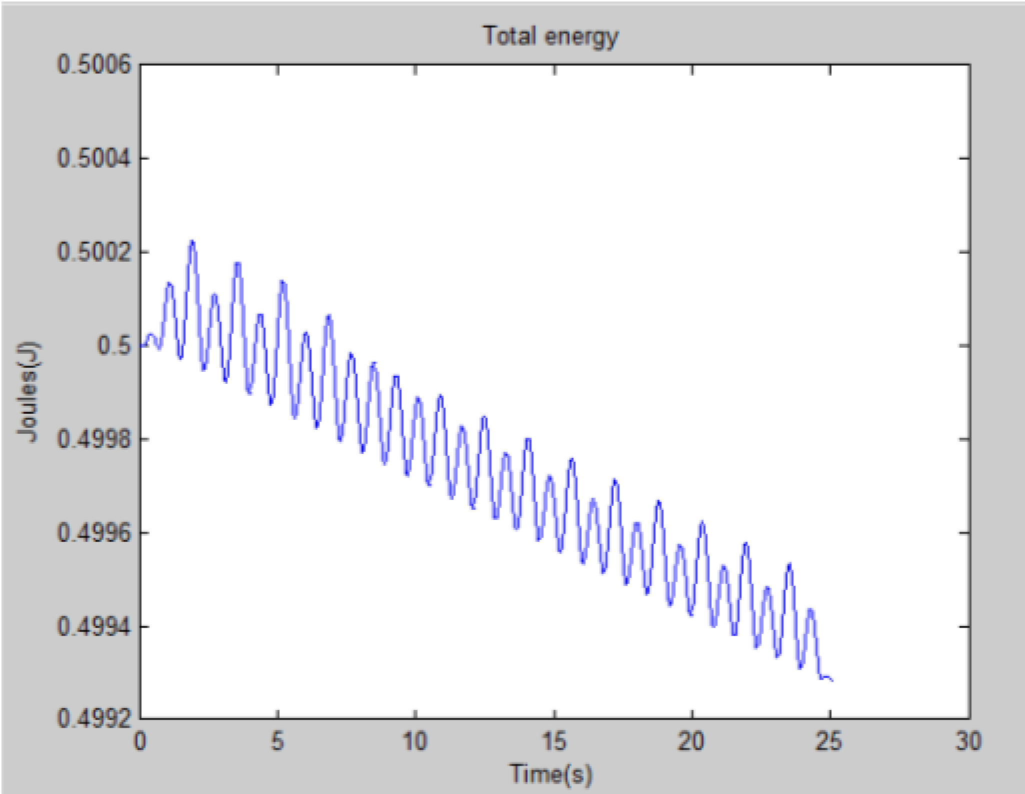
For k=1 and m=1,
Amplitude=1 m
Time period=7 seconds

Analytically, the amplitude is 1 m and time period is 6.283 seconds.

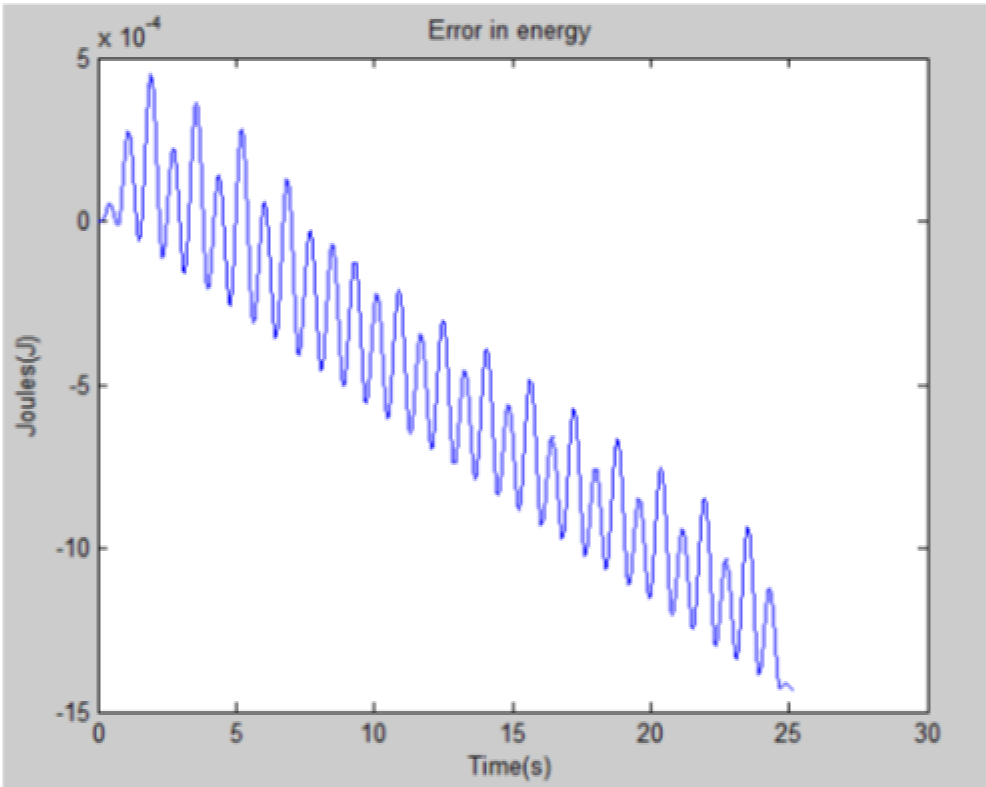
Phase-space trajectory: do you always get similar curves with different initial conditions. Do these always start from the same point? Does the trajectory trace itself in the same direction for different initial conditions? Comment on the observations.

- No, we do not get get similar curves with different initial conditions. The eccentricity of the ellipse in the phase diagram depends on the values of k and m.
- The starting point of the plots also change with change in initial values.
- The direction of trajectory for different initial conditions does not change.

There arises an error in energy in the total energy of the oscillator ($(E_i - E_o)/E_o$)which can be seen in the following graphs



Total energy
0.5006
0.5004
0.5002
0.5
0.4998
0.4996
0.4994
0.4992
Joules(J)
0
5
10
15
20
25
30
Time(s)



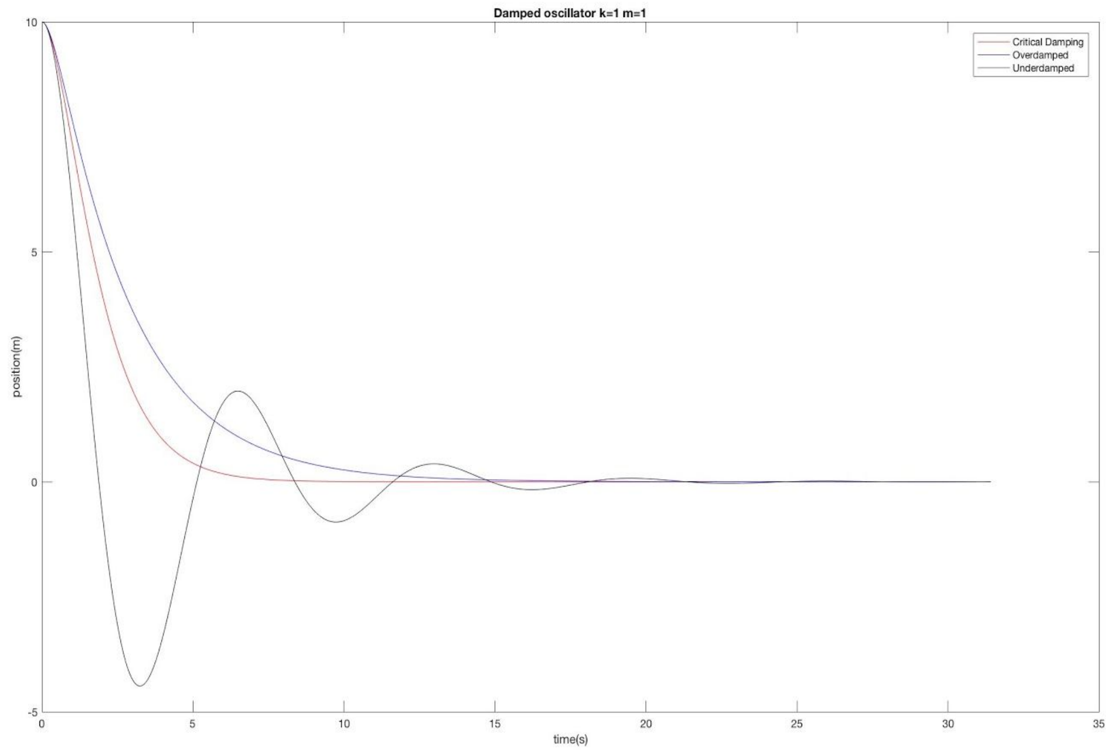
On decreasing the time step, this error decreases.

Damped Oscillations

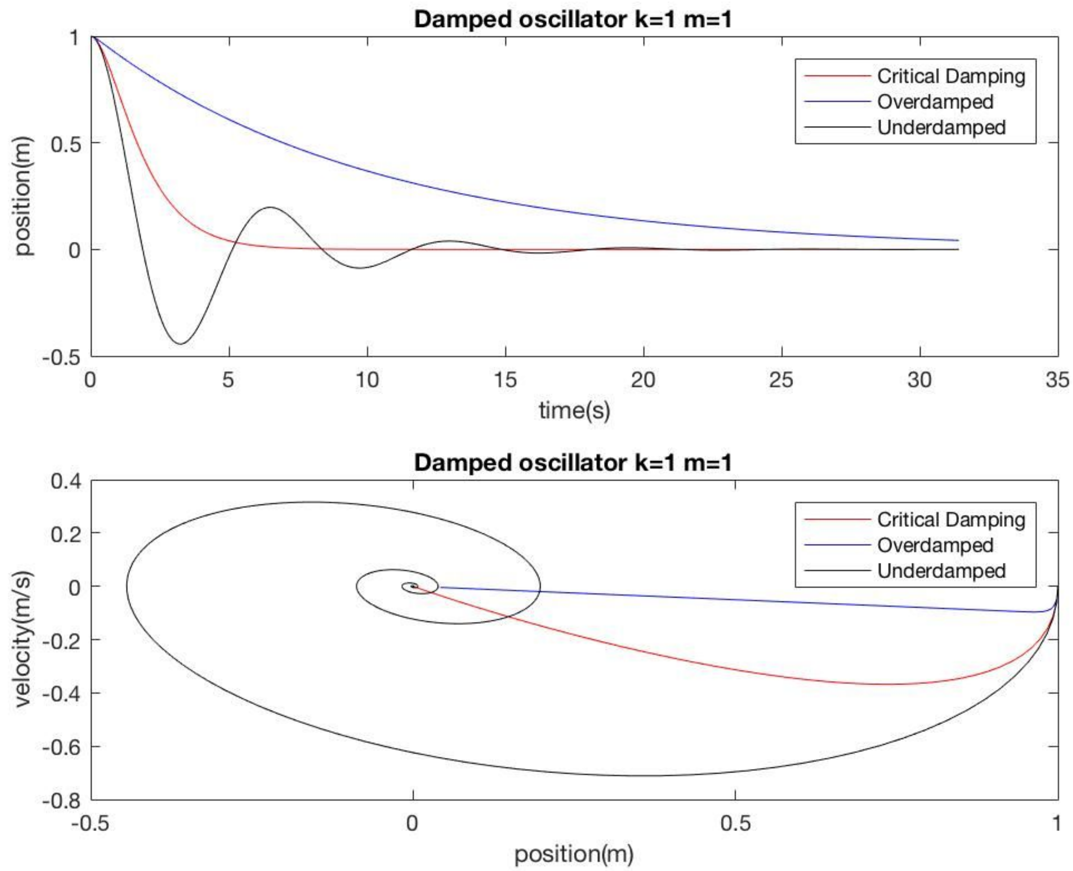
Damping Force = cv, where c=0.5

Damping force is always opposite to that of velocity. Damping reduces the total energy of the system gradually, thus affecting the amplitude.

In underdamping, we see oscillations whose amplitude decays gradually with time. As we increase the value of c, we see that the



decay becomes faster. At one particular value of c there is no oscillation occuring, just an exponential decay of position. This point is known as Critical Damping. At this value of c, the system attains equilibrium fastest. See the figure below -



We already know that there is an exponential decay of position in case of overdamping and Critical Damping. The same is true for Amplitude of oscillations in Underdamping. If we plot the maximum amplitude of a cycle vs the number of cycles, we get an exponentially decaying graph shown below -

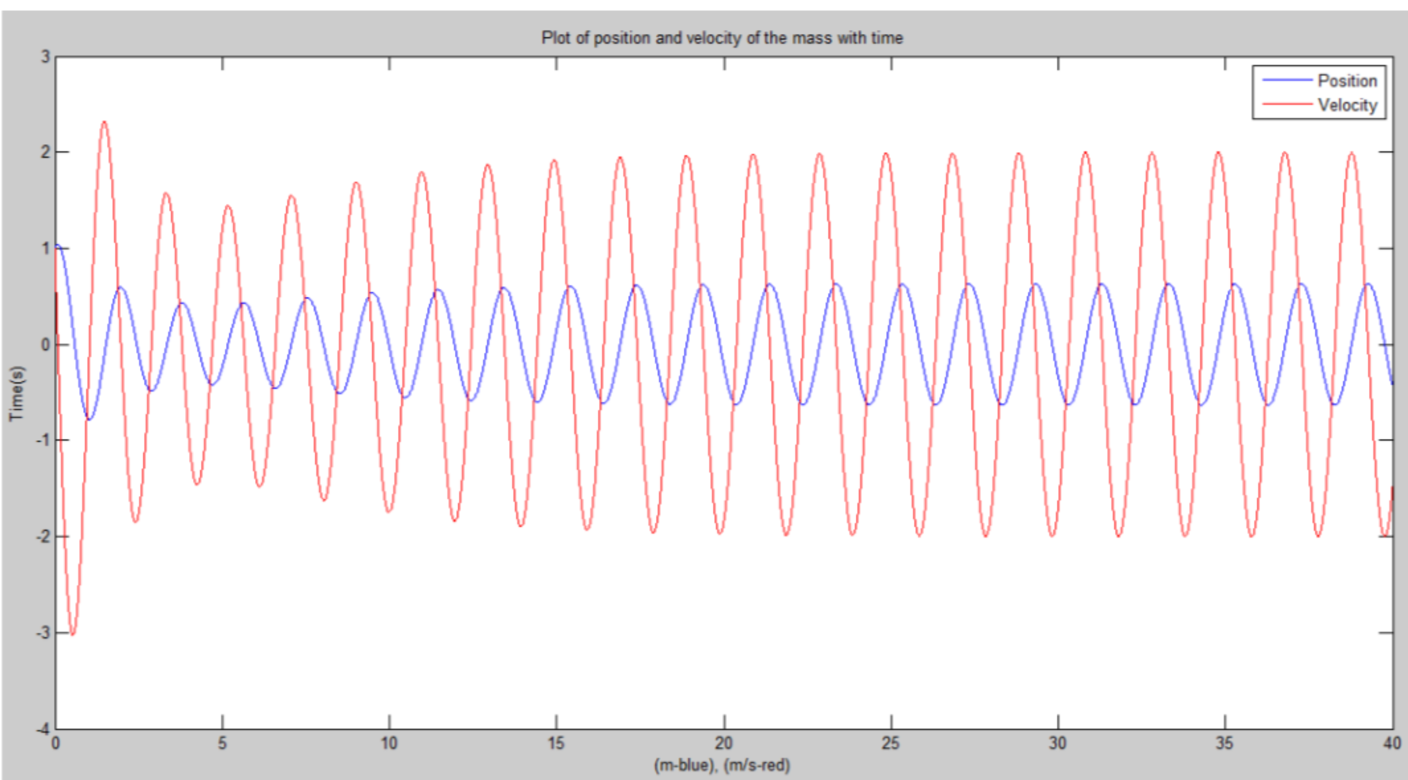
Now let us see the effect of frictional force on the oscillations. Frictional force is constant unlike the damping force, but just like the damping force, it opposes the velocity of the oscillator. This leads the oscillator to stop at some other position which might not be the initial mean position.
See the plot below -

<u>Forced/Driven Oscillations:</u>

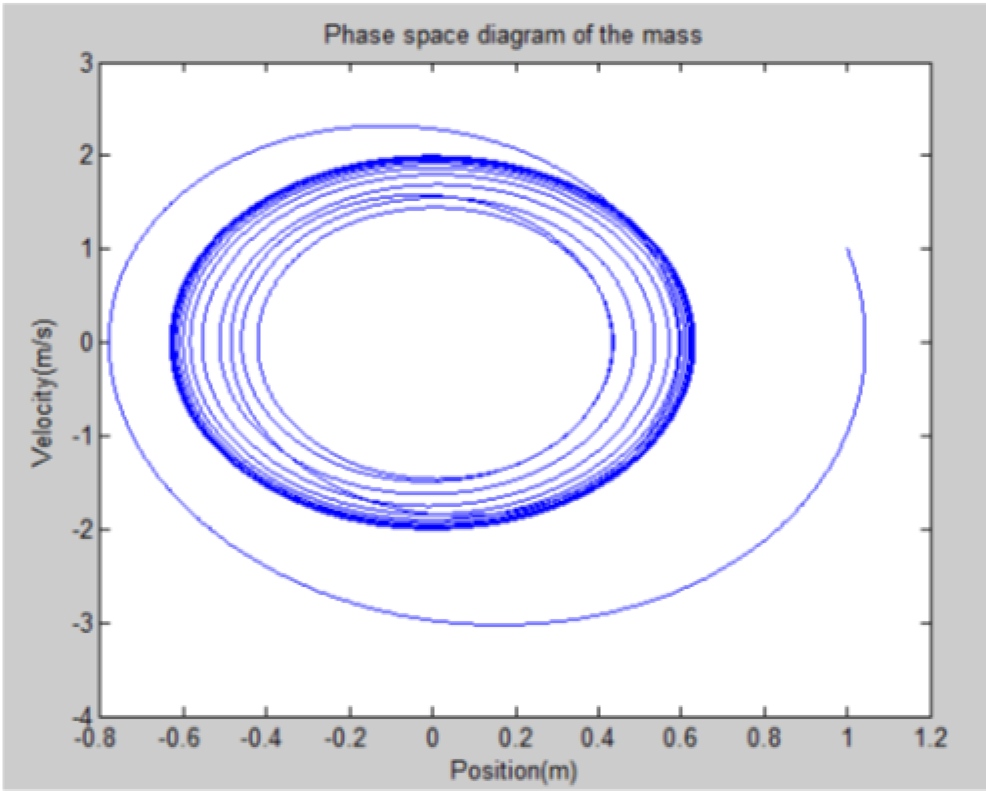
Damping force = cv. c=1 unless specified
Force acting on the oscillator:

$$F = -kx - cv - F_0.cos(wt) \quad (F_0=1)$$

Graph of Position and Velocity of the mass:

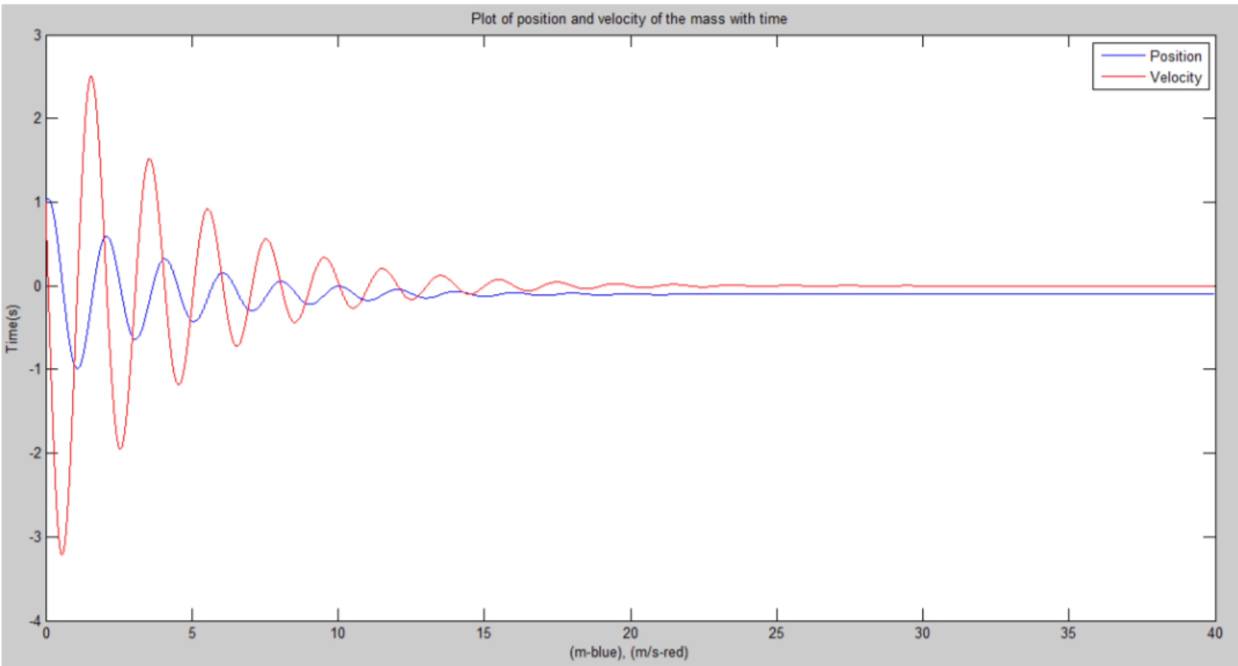


Phase diagram of the mass undergoing driven oscillations:



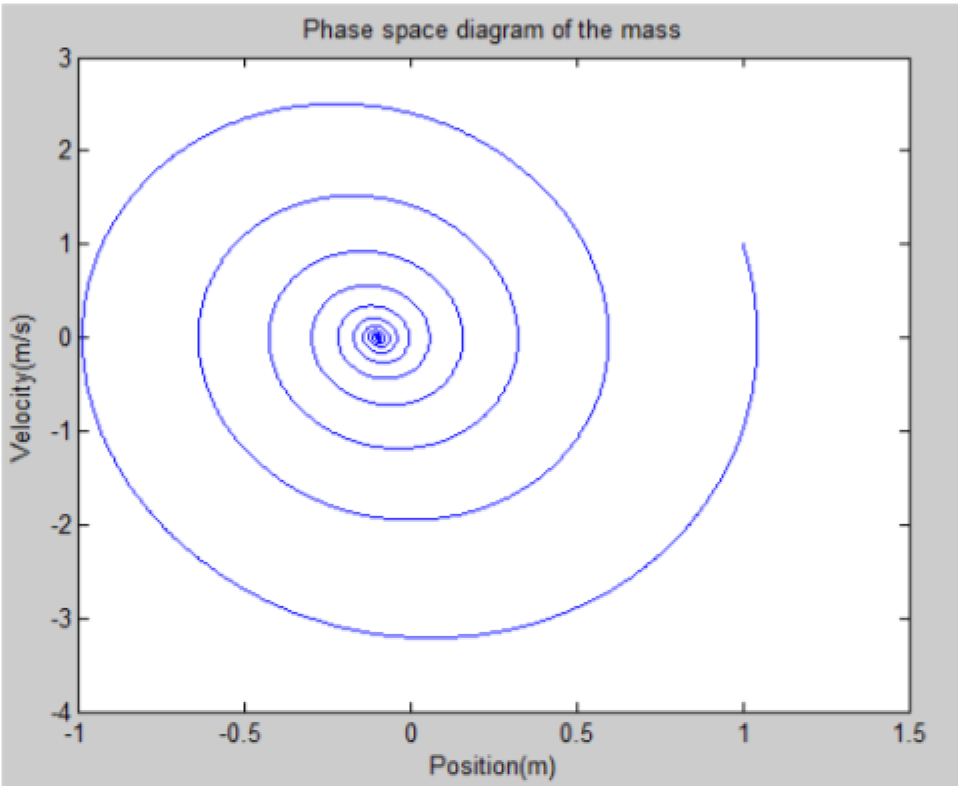
As we can see from the above graphs, there exists a transient state in the beginning after which the oscillations become stable with the frequency not equal to the natural frequency of the spring because of the periodic force acting on the mass.

$wo >> w$:



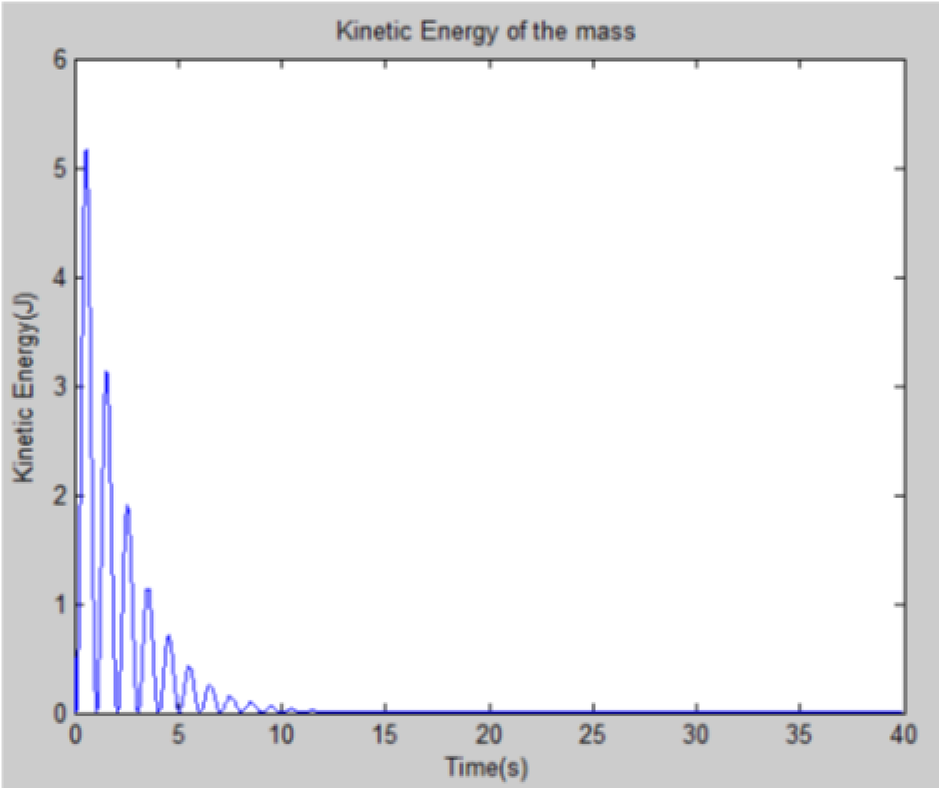
Plot of position and velocity of the mass with time
Position
Velocity
Time(s)
(m-blue), (m/s-red)
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
35
40

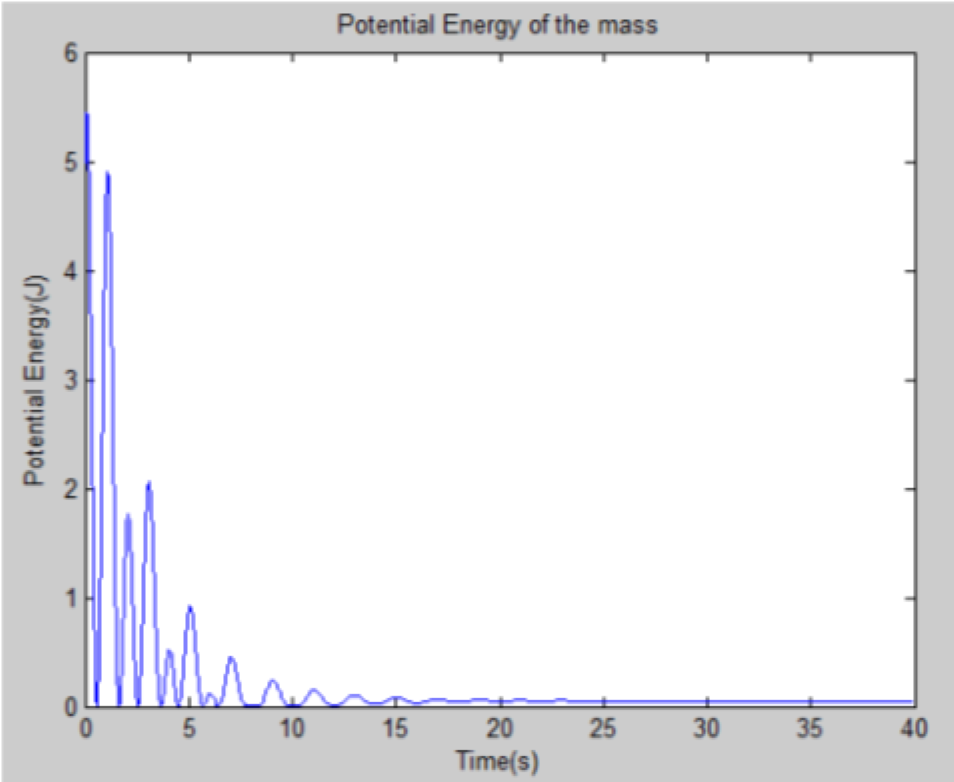
Phase diagram:

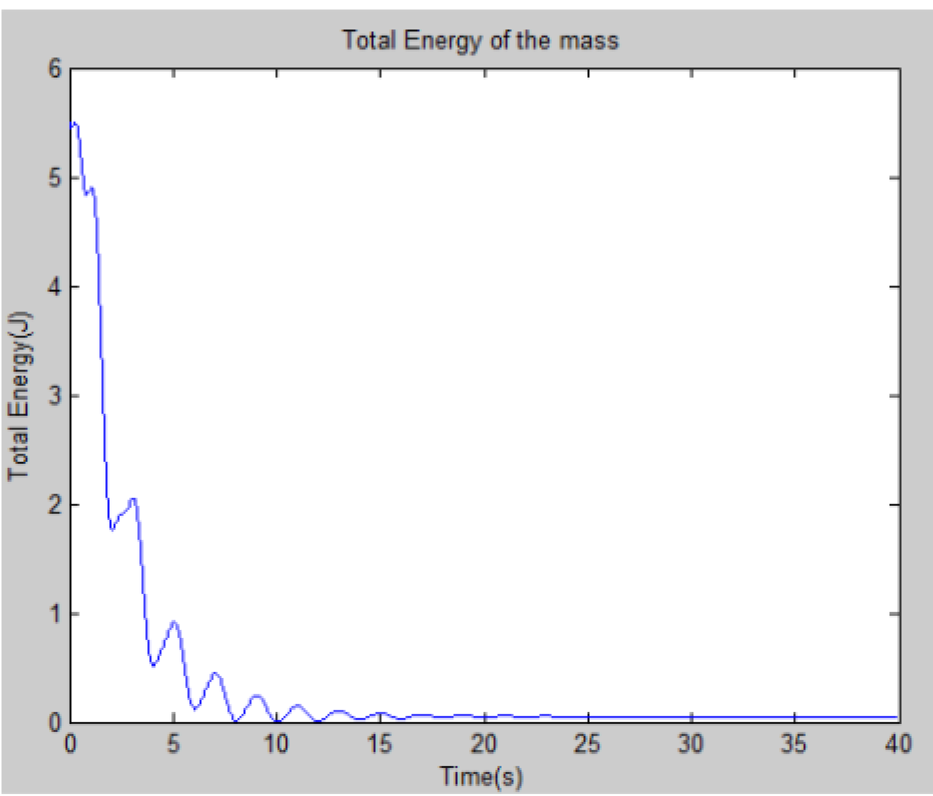


Furthermore, when w (frequency of the driving force) is taken to be very small when compared to w_o (natural frequency of the system), the force acts almost as a damping force which can be seen in the above graphs. The amplitude of the oscillations also decrease.

The kinetic energy, potential energy and the total energy of the system can be seen in the following graphs.


Kinetic Energy of the mass
Kinetic Energy(J)
Time(s)
0
1
2
3
4
5
6
0
5
10
15
20
25
30
35
40

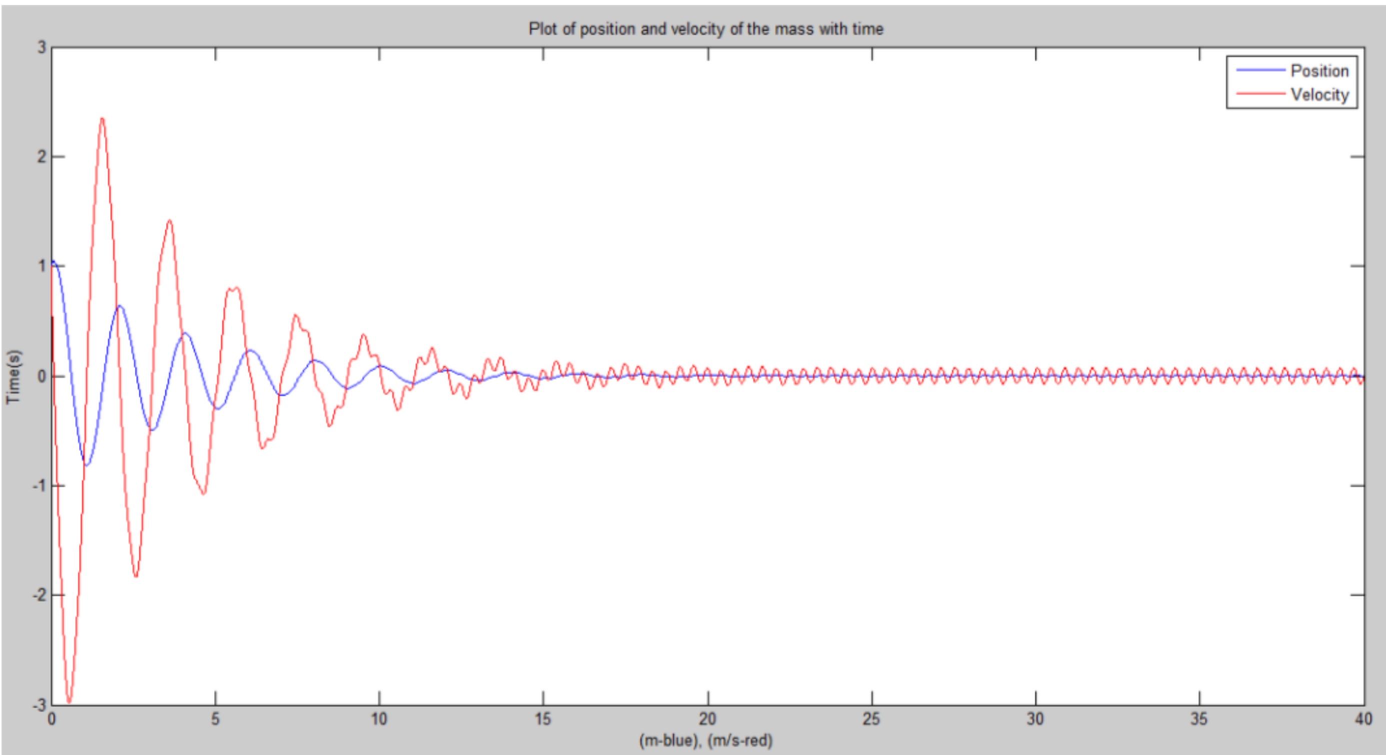

Potential Energy of the mass
Potential Energy(J)
Time(s)
0
1
2
3
4
5
6
0
5
10
15
20
25
30
35
40


Total Energy of the mass
Total Energy(J)
Time(s)
0
1
2
3
4
5
6
0
5
10
15
20
25
30
35
40

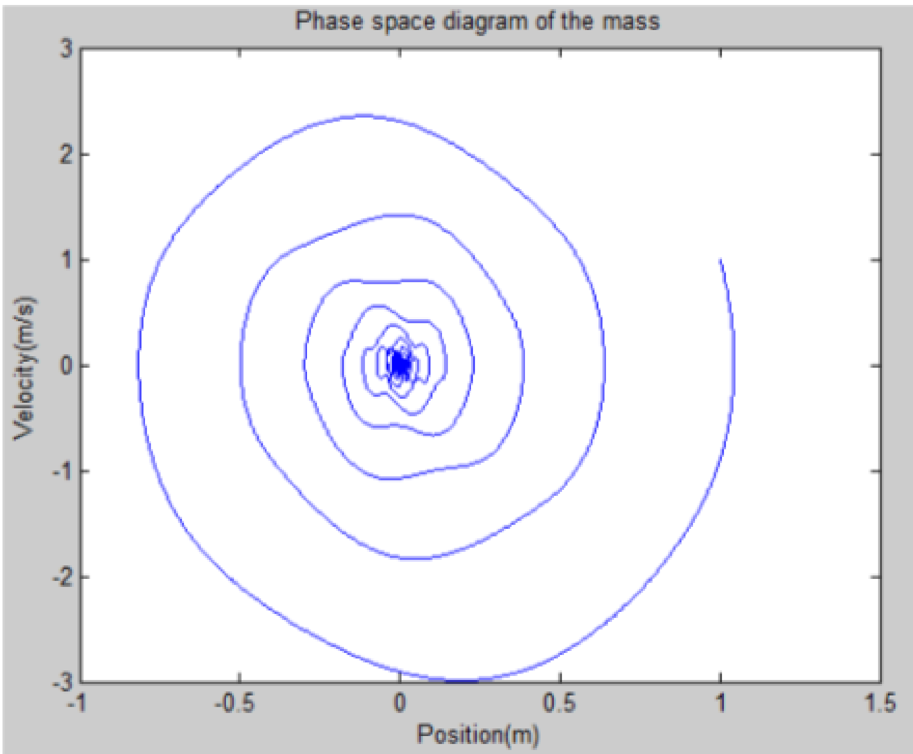
We can see that the total energy of the system decreases with time and eventually become zero.

$w >> wo$:

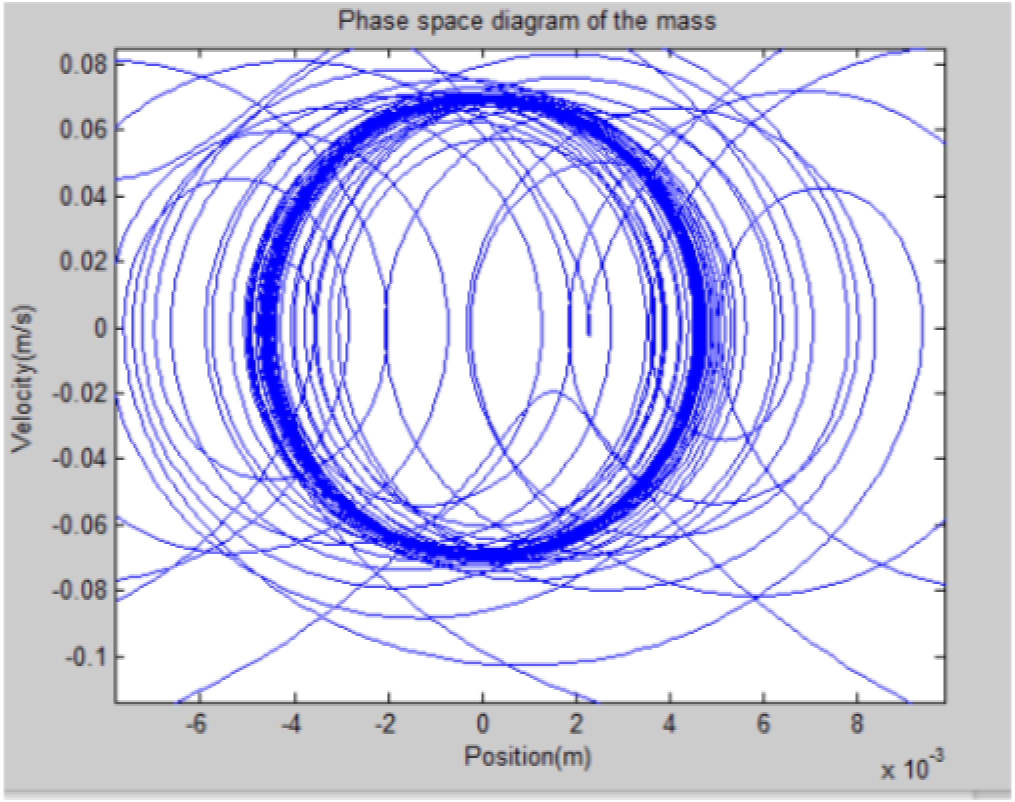
Graph of position and velocity of the mass:



Phase diagram:

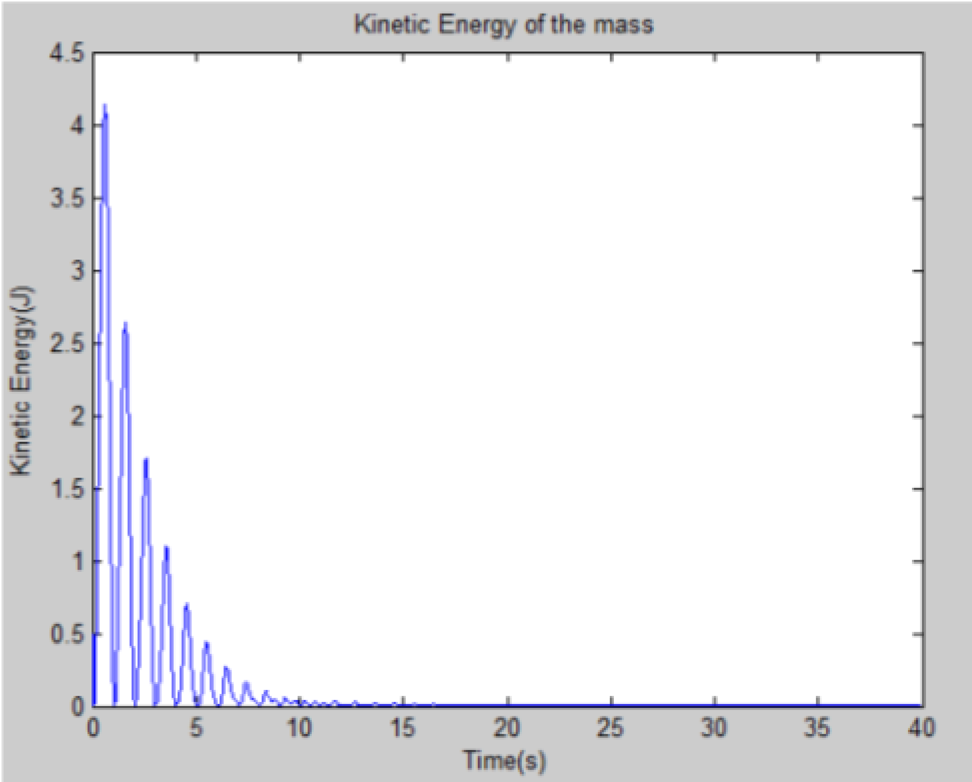


Phase diagram zoomed in near the centre:

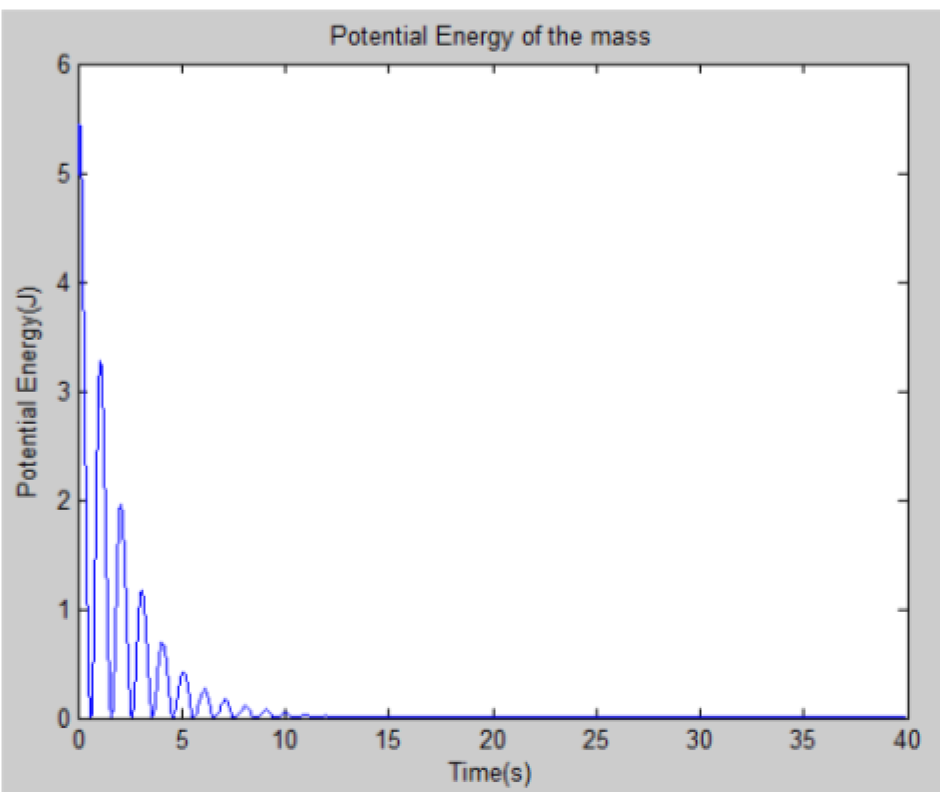


As we can see from the graph of position of the mass, if the frequency of the driving force is greater than the natural frequency of the system, the amplitude of the oscillations decrease and the frequency of the system increases.

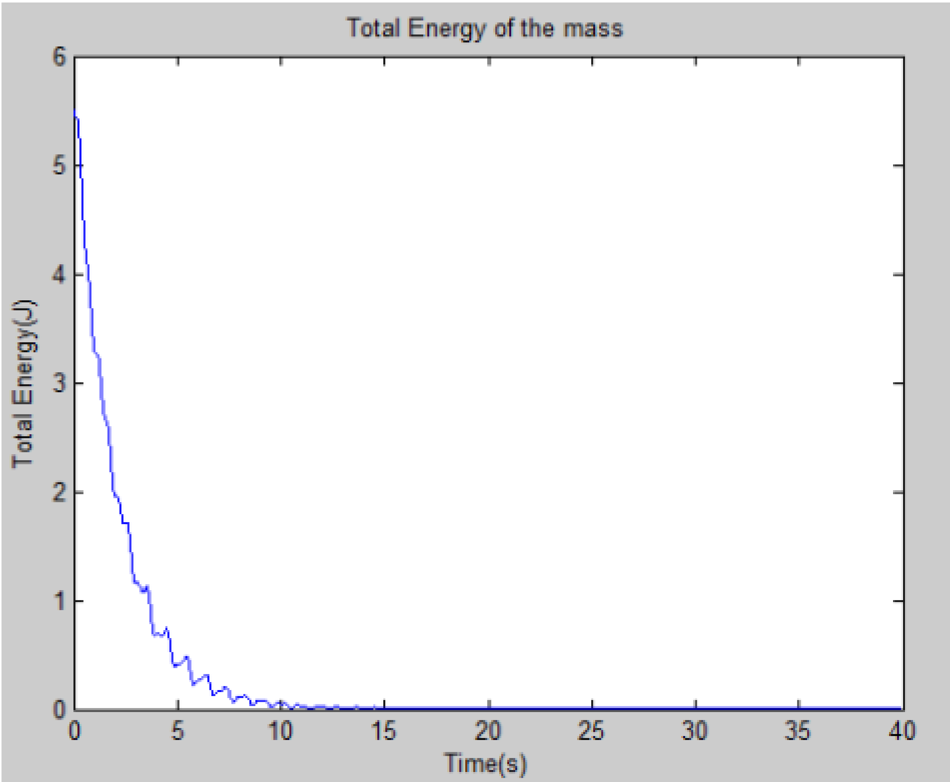
The kinetic energy, potential energy and the total energy of the system can be seen in the following graphs.



Kinetic Energy of the mass
Kinetic Energy(J)
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
10
15
20
25
30
35
40



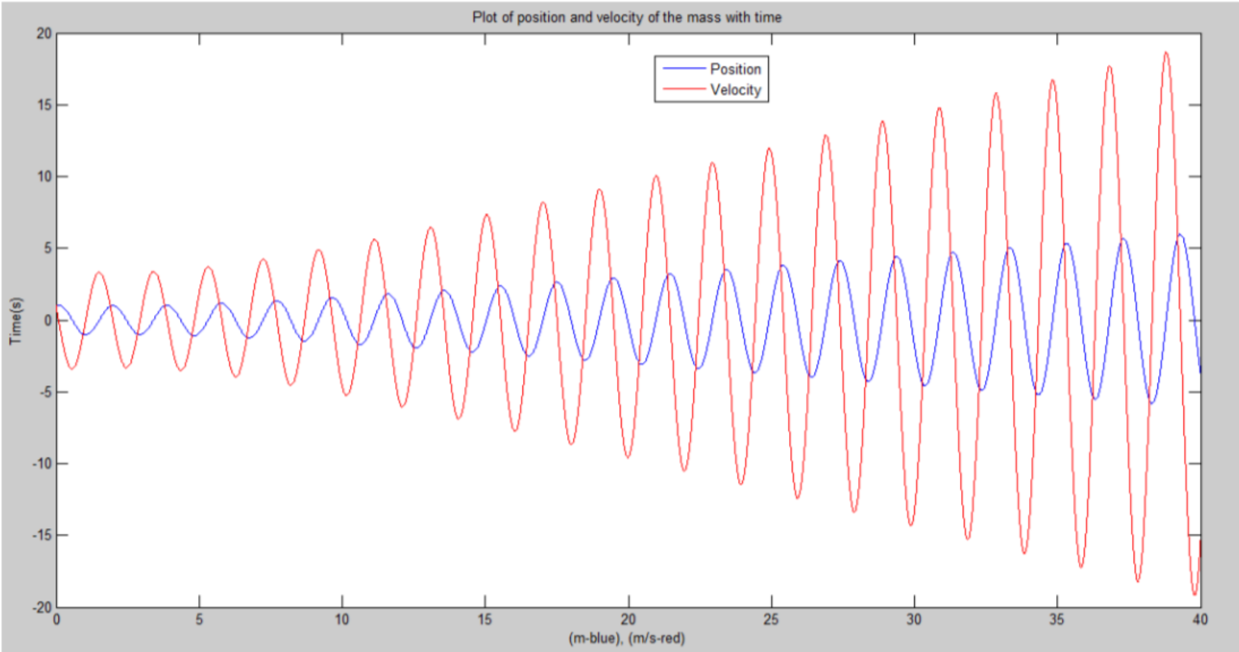
Potential Energy of the mass
Potential Energy(J)
Time(s)
0
1
2
3
4
5
6
10
15
20
25
30
35
40



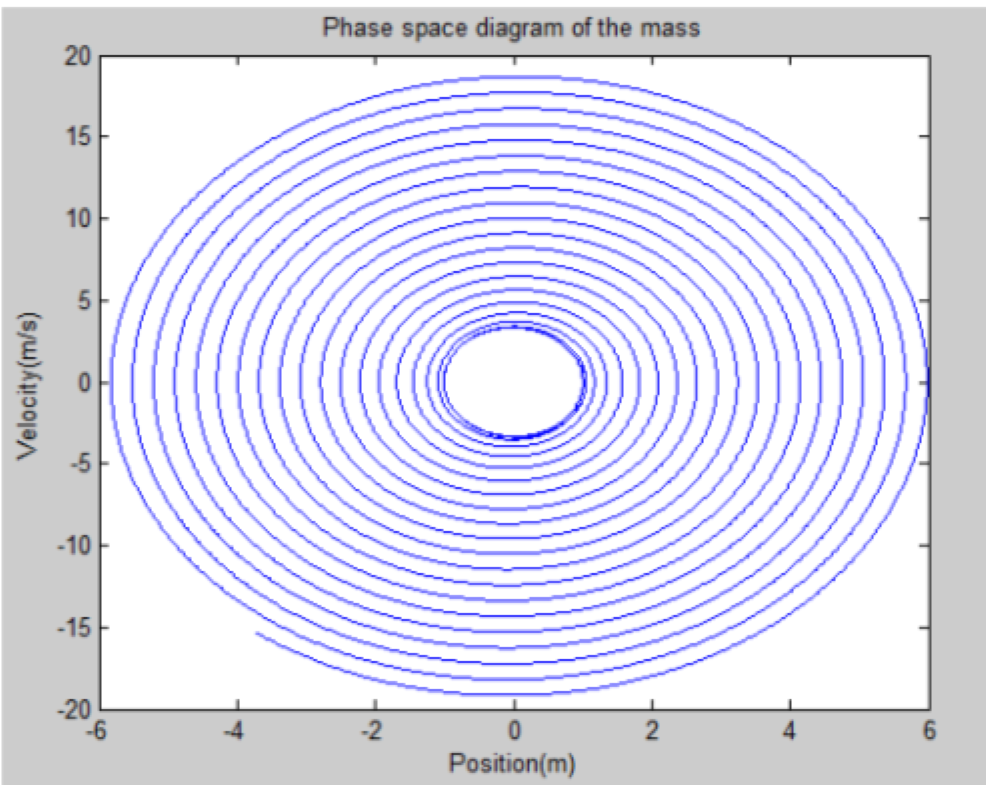
We can see that the total energy of the system will decrease, but not become zero and will continue to oscillate.

$wo = w$:

Graph of position and velocity of the mass:

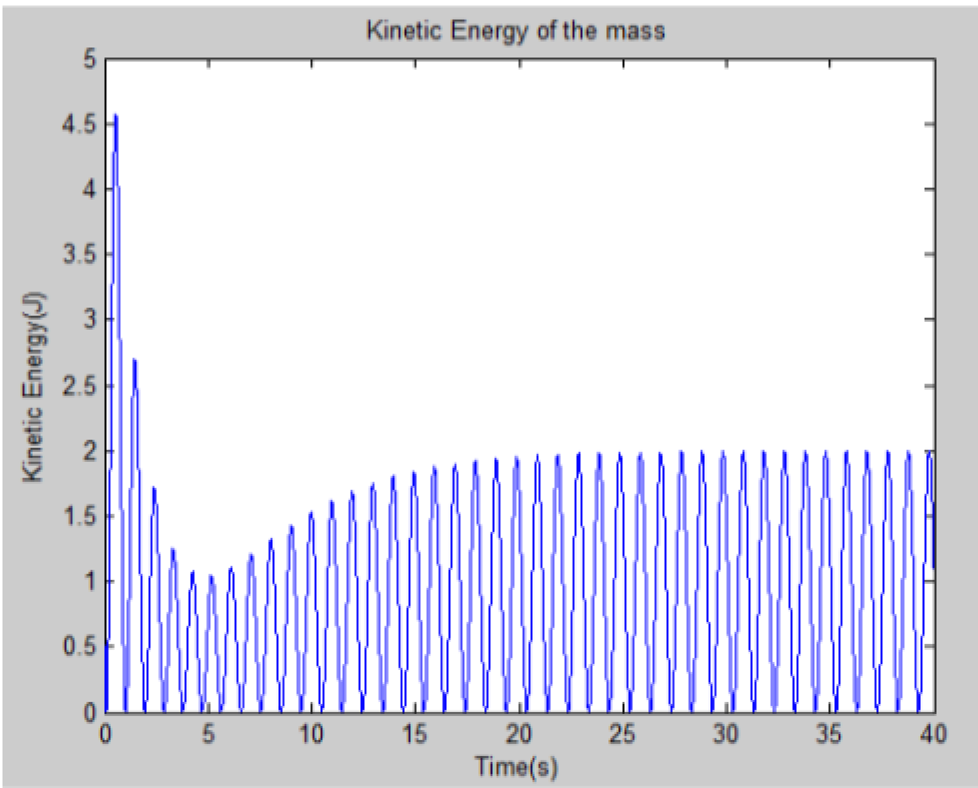


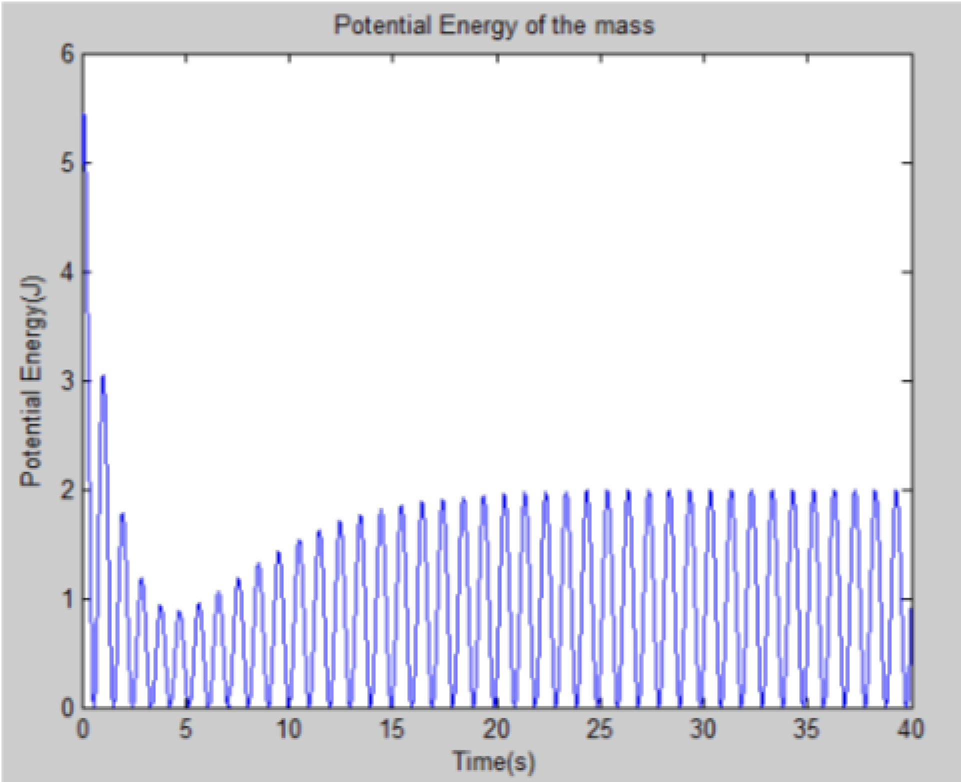
Phase diagram:

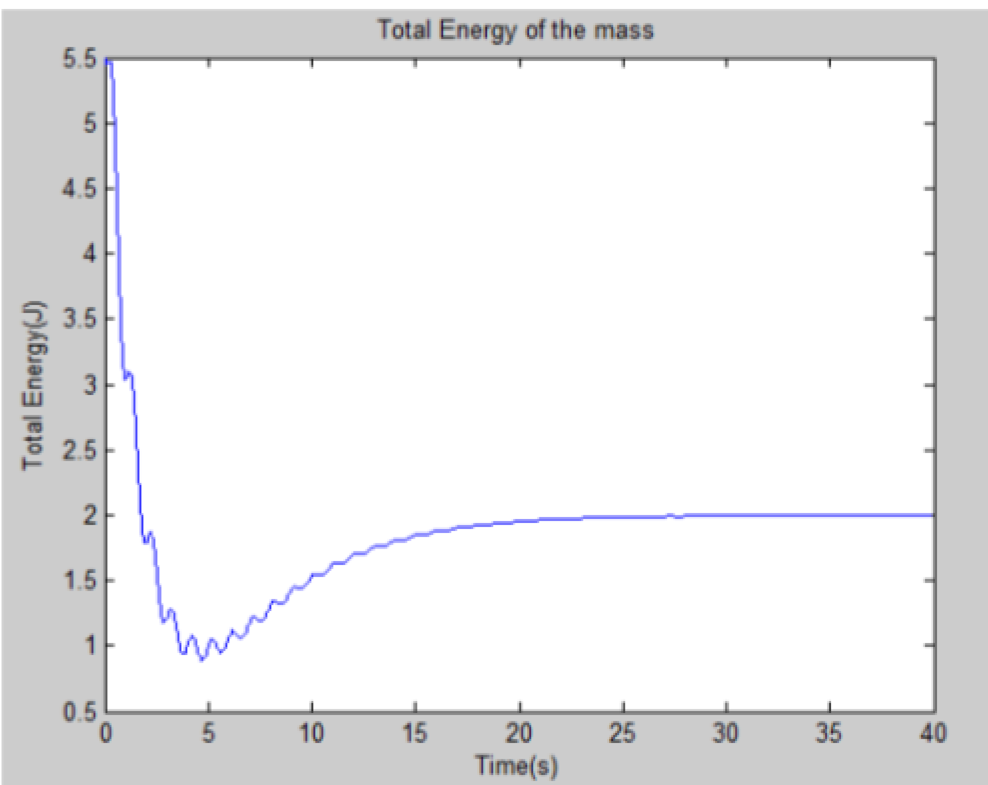


We can see that when the frequency of the driving force is equal to the natural frequency of the system and there is no damping force(c=0), the amplitude increases infinitely with time,i.e. The system achieves resonance.

The Kinetic energy, potential energy and total energy for a system here the driving frequency is equal to the natural frequency and there exists a damping force(c=1) is shown in the following graphs:




Potential Energy of the mass
Potential Energy(J)
Time(s)
0
1
2
3
4
5
6
0
5
10
15
20
25
30
35
40


Total Energy of the mass
Total Energy(J)
Time(s)
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
0
5
10
15
20
25
30
35
40

We can see that the total energy of the system becomes constant after the transient state. This is because the mass starts to perform stable oscillations.

From the above experiments, we can say that the amplitude increases as driving frequency tends to become equal to the natural frequency and then starts to decrease again as the driving frequency becomes more than that of the natural frequency.