CS 201
Computational Physics
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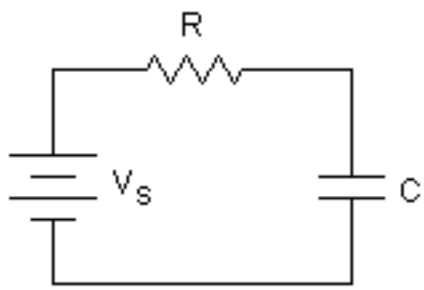
Assignment 8

Analysis of RC, RL and RLC Circuits

Q1. RC Circuit with a DC Source - Charging and Discharging

Consider the following series RC Circuit.



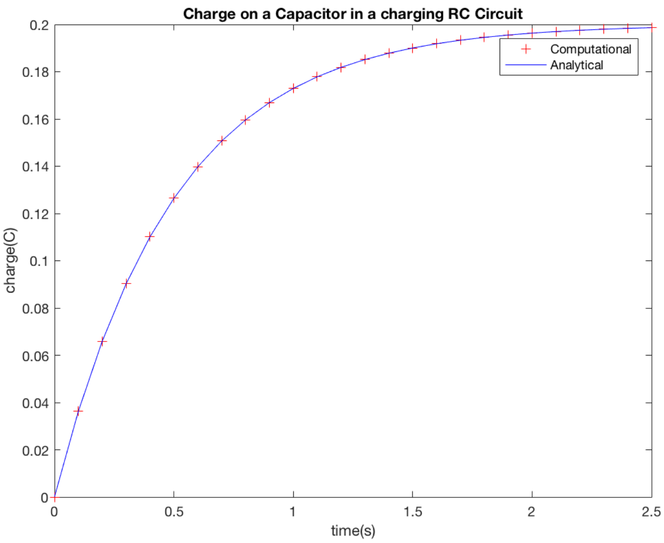
If **q** is the charge on the Capacitor then the circuit follows the following differential equation according to Kirchhoff's Voltage Law -

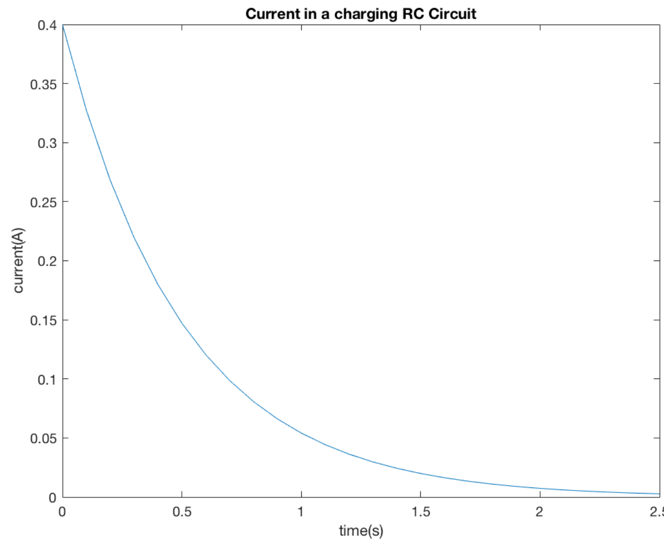
$$\frac{q}{C} + R\,.\frac{dq}{dt} = V$$

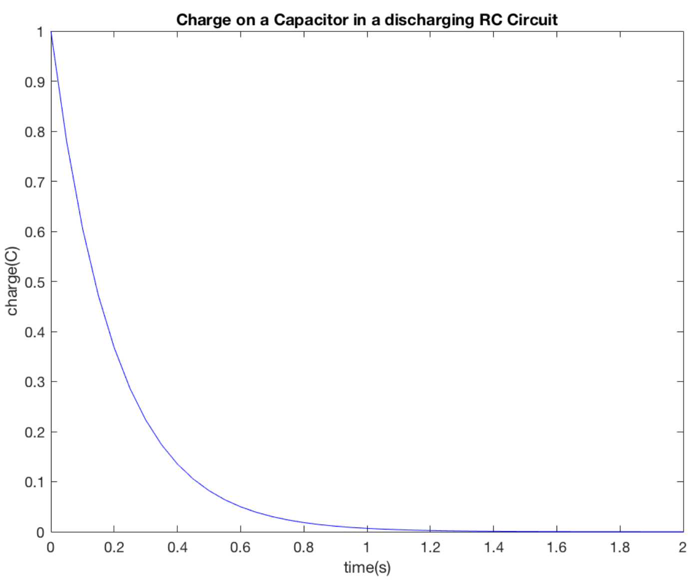
Assuming that the capacitor has 0 charge initially, and voltage V appears at t=0, we get the following equation for charge on the capacitor analytically -

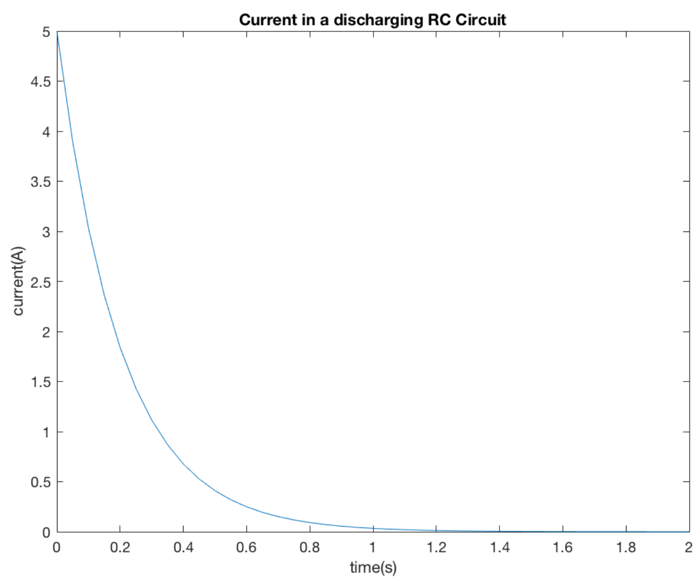
$$q(t) = C\,.\,V(1 - e^{\frac{-t}{R.C}})$$

For solving the differential equation computationally, we have used the ode45 solver in MATLAB. Graph attached on the next page shows that the computational solution agrees with the analytical solution.


Charge on a Capacitor in a charging RC Circuit
Computational
Analytical
charge(C)
time(s)

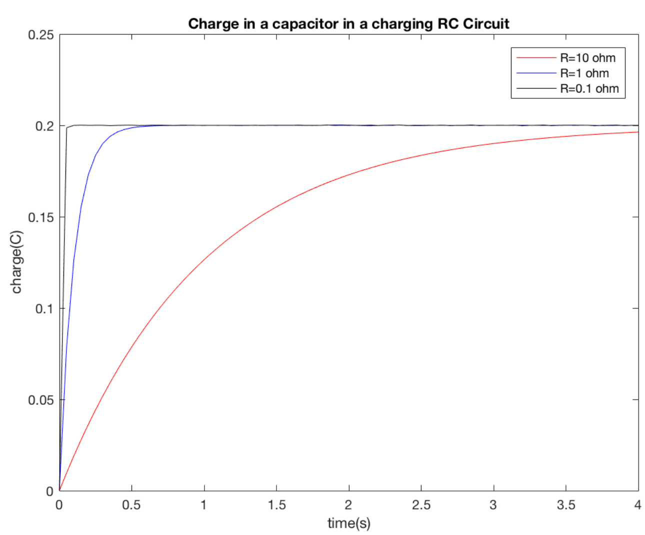

Current in a charging RC Circuit
current(A)
time(s)

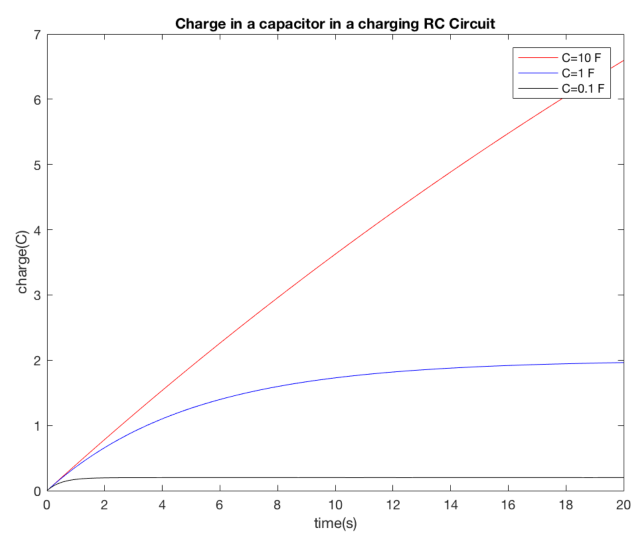

Charge on a Capacitor in a discharging RC Circuit
charge(C)
time(s)


Current in a discharging RC Circuit
current(A)
time(s)

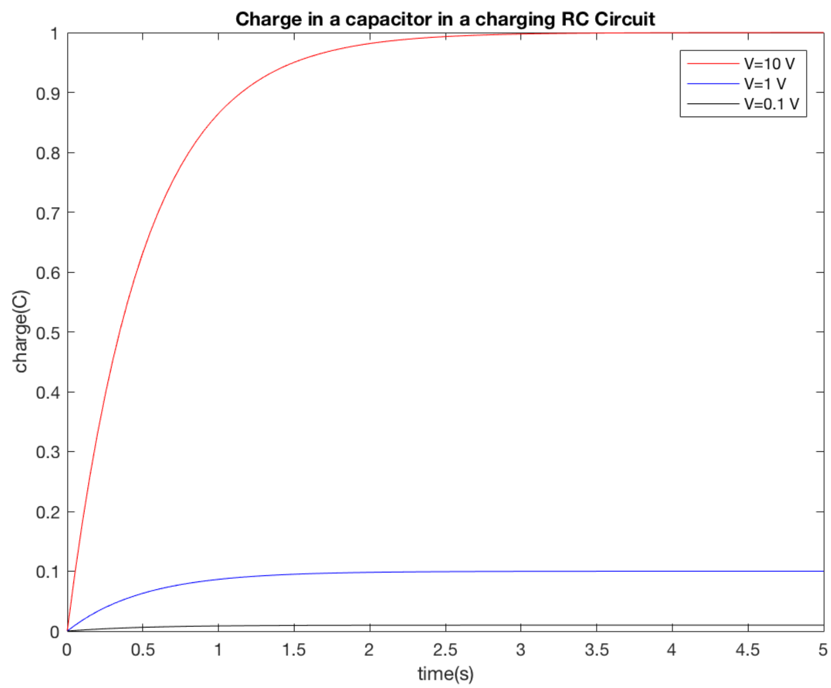
Looking at the effects of R,C and V on the current and the charge as a function of time, we make the following observations -

- **Effect of R -** We can look at R as a parameter which acts like a time delay in reaching the steady state i.e. higher the value of R, more is the time taken for reaching the steady state. This is understandable because, higher the value of R, the lesser the current will flow at a given time, hence a greater time will be taken for moving the charges from or to the capacitor. Another property of R is that its value does not change the steady state condition. The amount of charge on C finally is independent of R. It only changes the time taken to reach the steady state.
- **Effect of C -** Just like in the case of R, we can look at C as a parameter which acts like a time delay in reaching the steady state i.e. higher the value of C, more is the time taken for reaching the steady state. We can understand this by remembering that the potential across a capacitor for a given charge is directly proportional to C, hence to reach the required steady state potential, a higher value of C needs more charge to be accumulated or removed. Hence takes a longer time. Changing the value of C also changes the steady state charge on the capacitor.

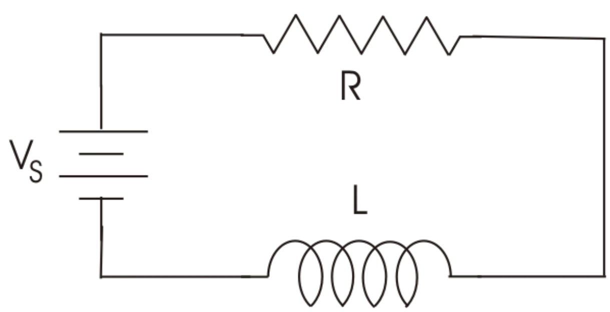




- **Effect of V -** V defines what the steady state of the RC circuit will be, because the steady state charge on the capacitor is CV.



Q2. RC Circuit with a DC Source

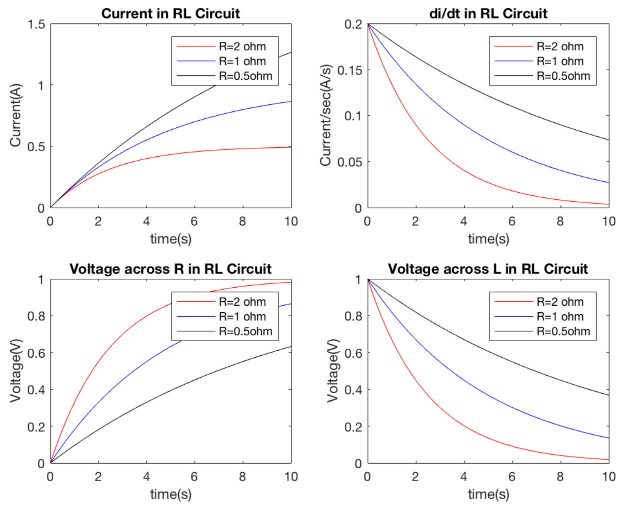


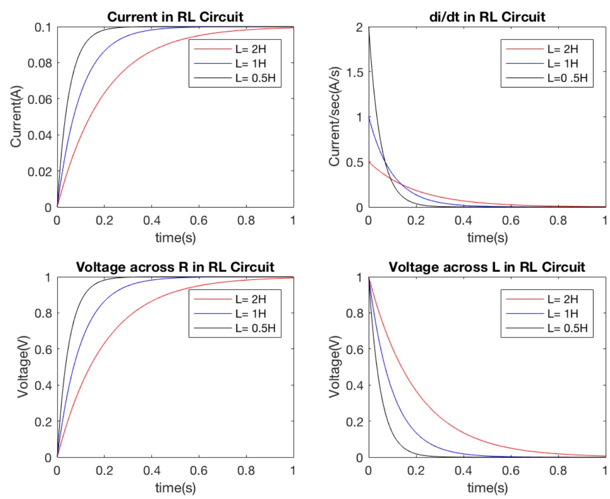
Consider the circuit shown above.

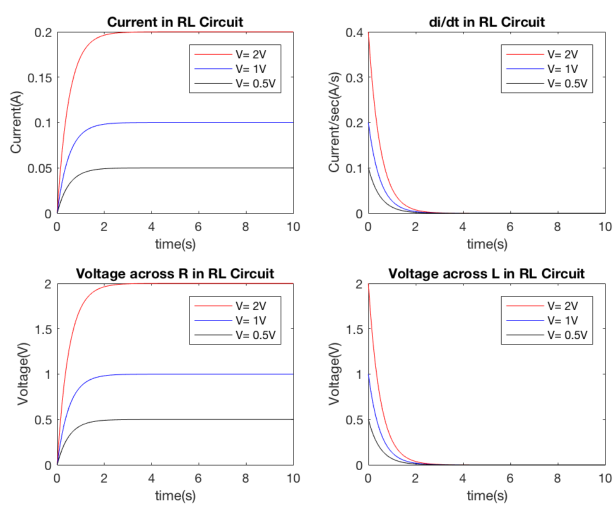
Applying Kirchhoff's Voltage Law, we get the following differential equation in terms of **i**, the current in the circuit.

$$i\,.\,R + L\,.\,\frac{di}{dt} = V$$

For solving the differential equation computationally, we have used the ode45 solver in MATLAB. Graph attached on the next page shows the solution for the current and various voltages in the circuit.


Current in RL Circuit
Current(A)
time(s)
R=2 ohm
R=1 ohm
R=0.5ohm
di/dt in RL Circuit
Current/sec(A/s)
Voltage across R in RL Circuit
Voltage(V)
Voltage across L in RL Circuit


Current in RL Circuit
Current(A)
time(s)
L= 2H
L= 1H
L= 0.5H
di/dt in RL Circuit
Current/sec(A/s)
L=0 .5H
Voltage across R in RL Circuit
Voltage(V)
Voltage across L in RL Circuit


Current in RL Circuit
Current(A)
time(s)
V= 2V
V= 1V
V= 0.5V
di/dt in RL Circuit
Current/sec(A/s)
Voltage across R in RL Circuit
Voltage(V)
Voltage across L in RL Circuit

Differential equation of a body in free fall is

$$\frac{dv}{dt} = g - k \, . \, v$$

We can see that this equation is similar to the equations of an RC Circuit -

$$\frac{dq}{dt} = \frac{V}{R} - (\frac{1}{R \, . \, C}) \, . \, q$$

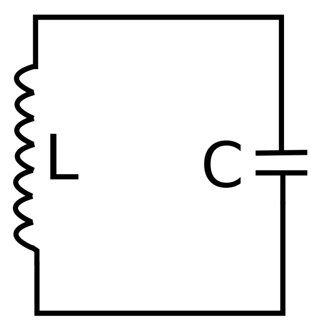
And an RL Circuit -

$$\frac{di}{dt} = \frac{V}{L} - (\frac{R}{L}) \, . \, i$$

Notice that all the above differential equations are same with respect to the independent and dependent variables. Therefore we can conclude that all the 3 have similar solutions and thereby also proving that RL, RC, and free fall are analogous systems mathematically.

Q3. LC oscillating circuit

Consider the following circuit -



If **q** is the charge on the capacitor, then **q** is a function of time, and by Kirchhoff's Voltage Law we can see that it follows the following differential equation -

$$\frac{d^2q}{dt^2} = -\frac{1}{L \, . \, C} \, . \, q$$

Comparing it with the equation of Simple Harmonic Motion -

$$\frac{d^2x}{dt^2} = -\omega_o^2 \, . \, x$$

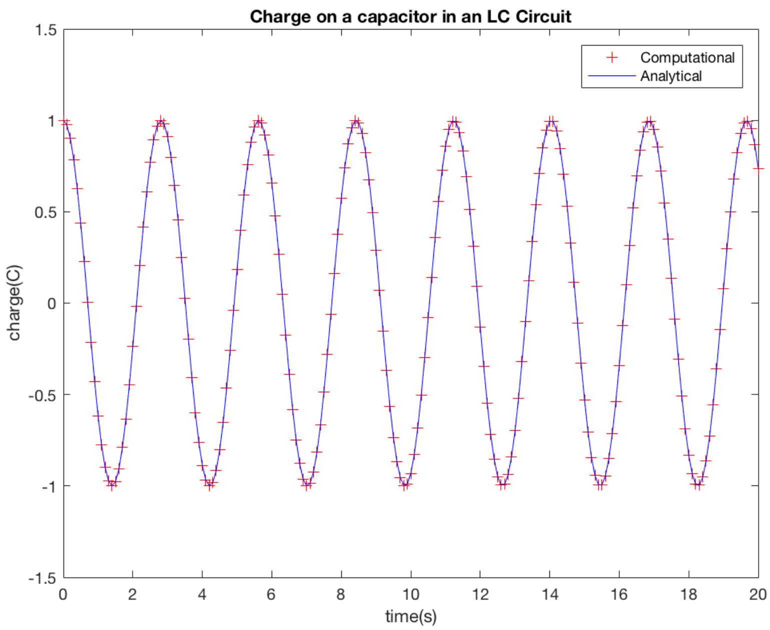
We see that the analytical solution will be of the form -

$$q(t) = A \, . \, cos(\omega_o \, . \, t + \phi)$$

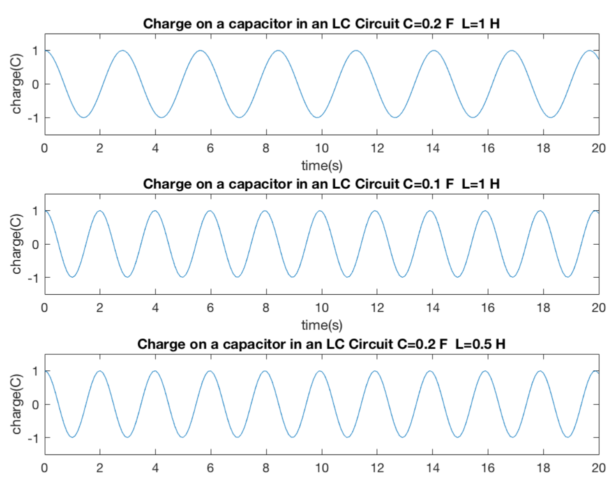
with frequency of oscillation -

$$\omega_o = \frac{1}{\sqrt{L \, . \, C}}$$

Computationally, we solve the differential equation by using ode45 differential equation solver. In the figure shown below we see that the computational solution is in agreement with the analytical solution.

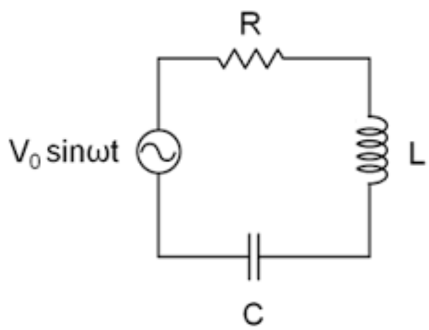


We see that on increasing the value of L or C, the frequency of oscillations decreases. This too is in accordance with our analytical solution.



Q4. Driven LCR circuit

Consider the following circuit -



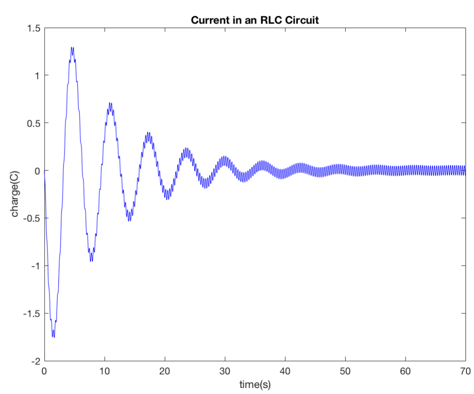
It follows the following differential equation -

$$L \, . \frac{d^2q}{dt^2} + R \, . \frac{dq}{dt} + \frac{q}{C} = V_o \, . sin(\omega \, . t)$$

The steady state analytical solution to the above differential equation is -

$$i(t) = i_o sin(\omega \, . t + \phi)$$

The actual solution is initially not same as the solution given above, but it rather converges to the above solution.



The differential equation of the RLC driven circuit is similar to the forced oscillations equation -

$$m \, . \frac{d^2x}{dt^2} - b \, . \frac{dx}{dt} + k \, . x = F_o sin(\omega \, . t)$$