LAB ASSIGNMENT-2

Group 20:

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Q1. Computationally analyze the motion of freely falling body using Euler's method as discussed during the lecture. Consider realistic initial conditions height,initial velocity etc.) Compare your result with analytical solution and study the effect of discretization (time step) on computational result. Plot the results showing the velocity of the body and the distance travelled by it at different instant of time. The above problem is not realistic from Earth's viewpoint, use the code to analyze the motion of falling body on the moon (there is hardly any atmosphere, so in reality also we can neglect the effect of atmosphere, however initial conditions will be different).

Approximations:

1. We neglect the effect of atmosphere, drag force/air resistance.
2. We neglect the variation in g due to height difference.
3. we consider the value of g on the moon g/6;

Mathematical Model:

$d^2x/dt^2=g/6$.

$dv/dt=g/6$.

$dx/dt=v$.

The net acceleration faced by the freely falling body is equal to g/6.

Computational Formulation:

We have considered the following:

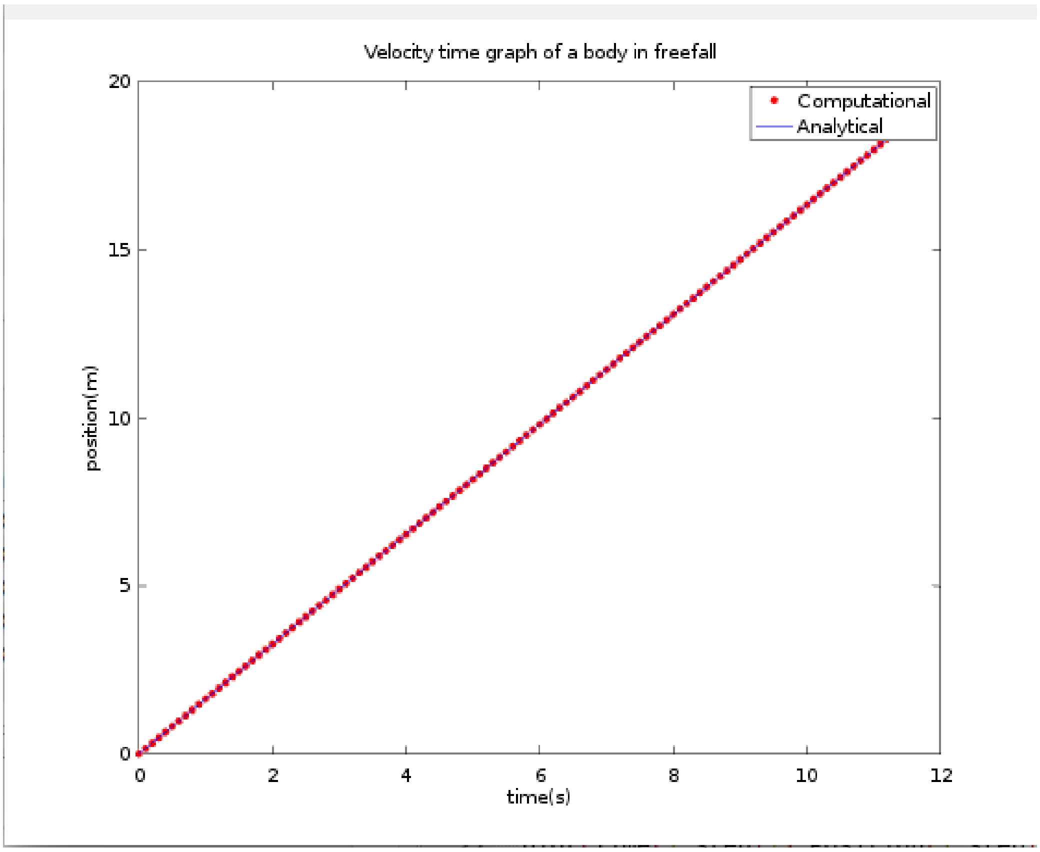
```
time=zeros(npoints,1);
position=zeros(npoints,1);
velocity=zeros(npoints,1);
```

We defined an array for time , position and velocity for the motion of the object and initialized them.

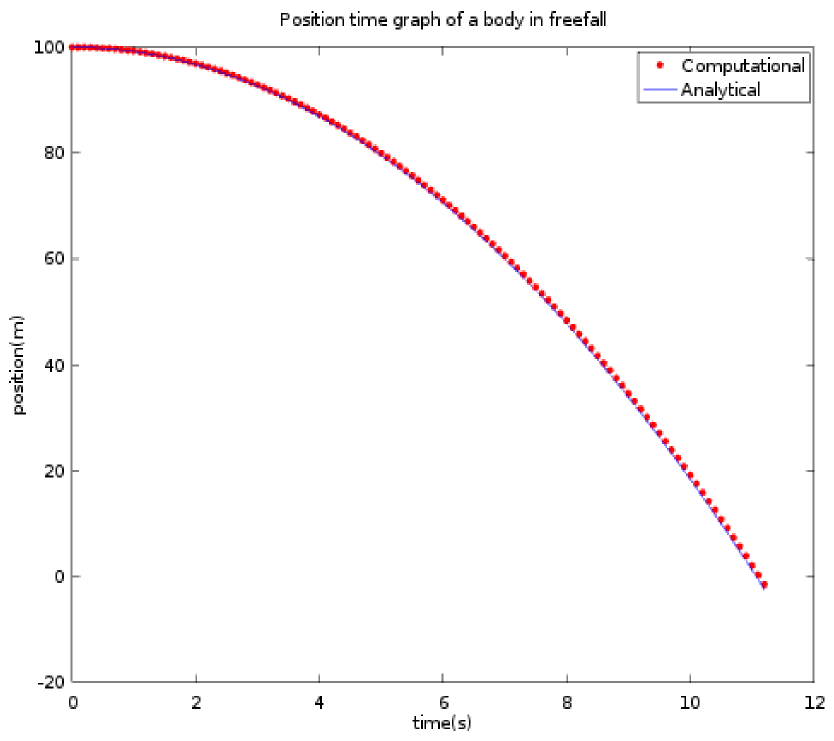
```
for step=1:npoints-1
 position(step+1)=position(step) + velocity(step)*dt;
 velocity(step+1)=velocity(step) + g*dt;
 time(step+1)=time(step)+dt;
end
```

Applying euler’s method , we computed the values of position and velocity at all the points(we considered 1000 points in the period of 10 seconds) in time.

Velocity vs Time graph:



Position Vs Time graph:



Results and Discussions :

The velocity of the body increases linearly with time as the body falls through the free space. The displacement of the body decreases in quadratic manner.

Accuracy:

The analytical and computational graphs of velocity and position almost coincide with each other. Thus, we can say that the computational model is fairly accurate.

Q2. Write down the equation for position of an object moving horizontally with a constant velocity “v”. Assume v=50 m/s, use the Euler method (finite difference) to solve the equation as a function of time. Compare your computational result with the exact solution. Compare the result for different values of the time-step

Approximations :

Velocity remains constant throughout

Mathematical model :

$x = x_0 + vt$

Equation of motion for displacement given constant velocity.

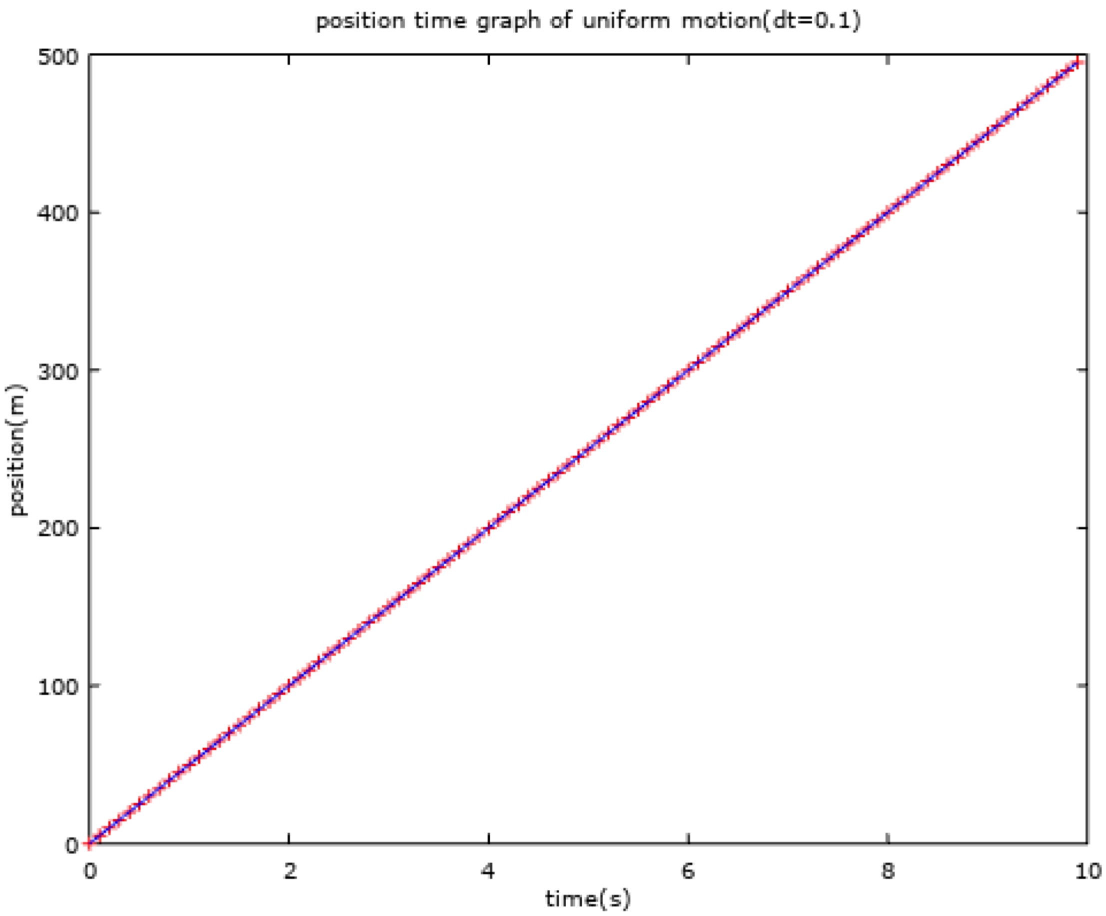
Computational model :

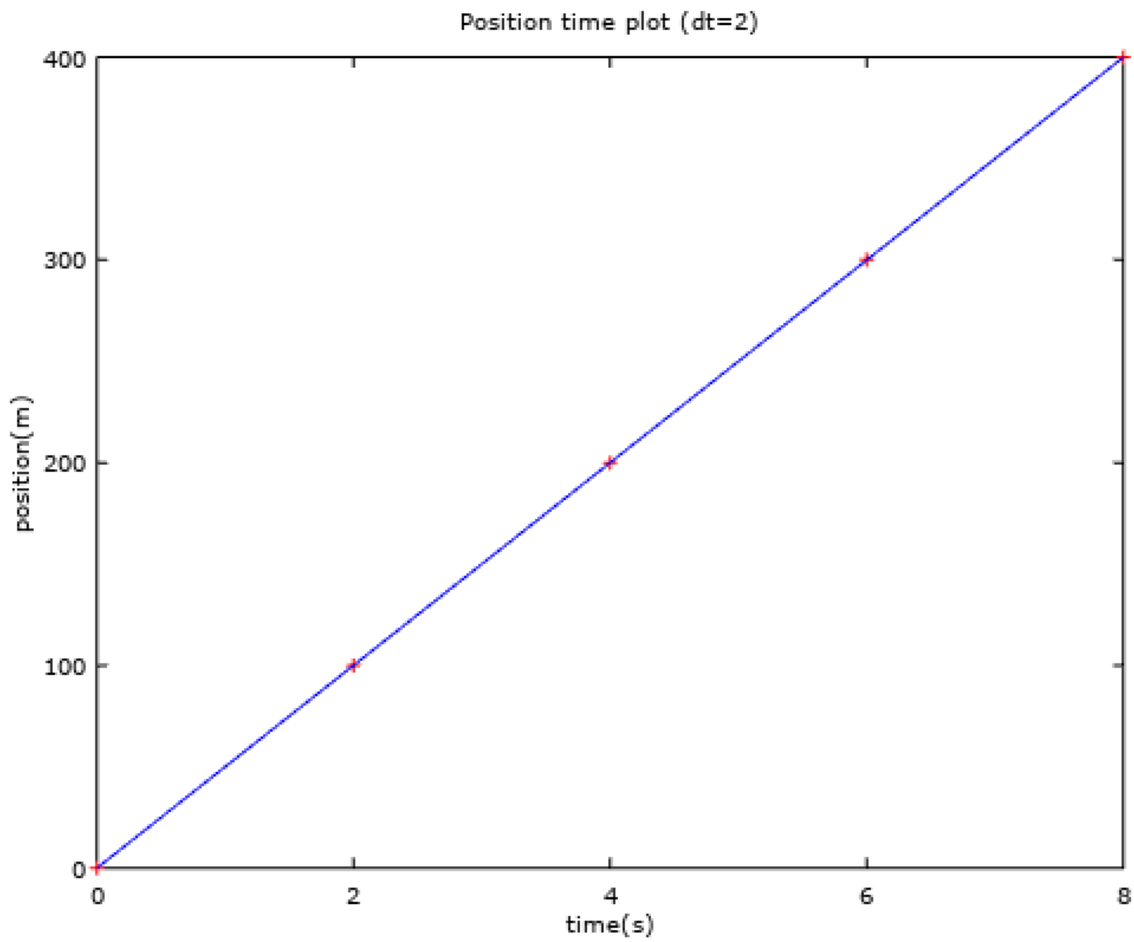
position(step+1) = position(step) + v_0*dt

Results and conclusions :

The computational and analytical model coincide, hence the solution is accurate given the approximations.

Positions Vs Time plot:





Q3.(a) Add the effect of atmosphere to problem 1 (still neglecting viscosity and drag). Suppose the falling object is a sphere of radius "r",computationally study the effect of buoyancy on the motion of the object. Net force needs to be modeled properly (as discussed during lecture); choose proper density of air. Study the effect of "r" and"mass". You can assume constant "g".

Approximations:

1. We neglect the air resistance and drag.

2. We consider the net force on the object to be due to gravity and buoyancy.

Mathematical Model:

d^2x/dt^2=g - 4/3*pi*r^3*g*density(air)

dx/dt=gt - 4/3*pi*r^3*g*density(air)*t

x=vt

The net force on the body is the force due to gravity minus the buoyant force. The differential equation modelling the given situation is mentioned above.

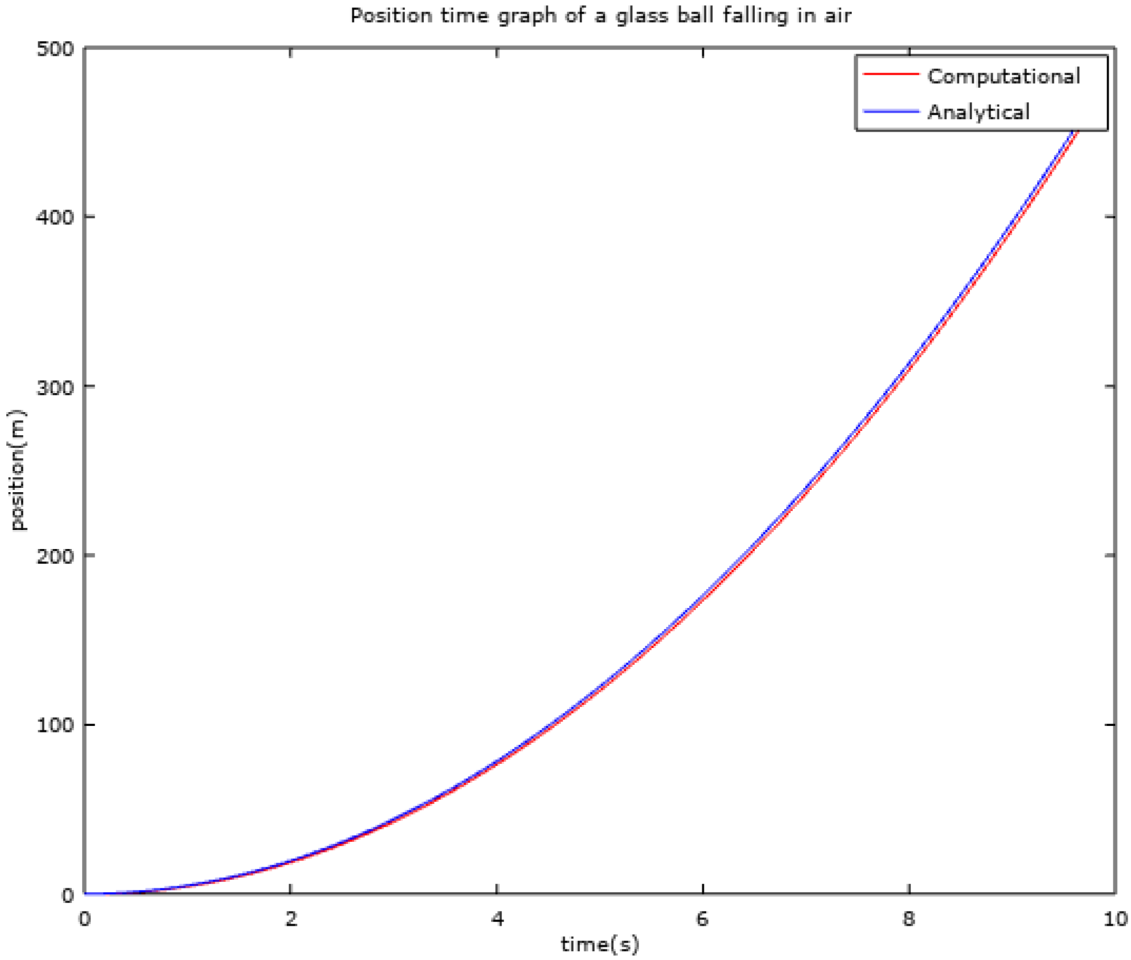
Computational Formulation:

position(step+1)=position(step) + velocity(step)*dt;

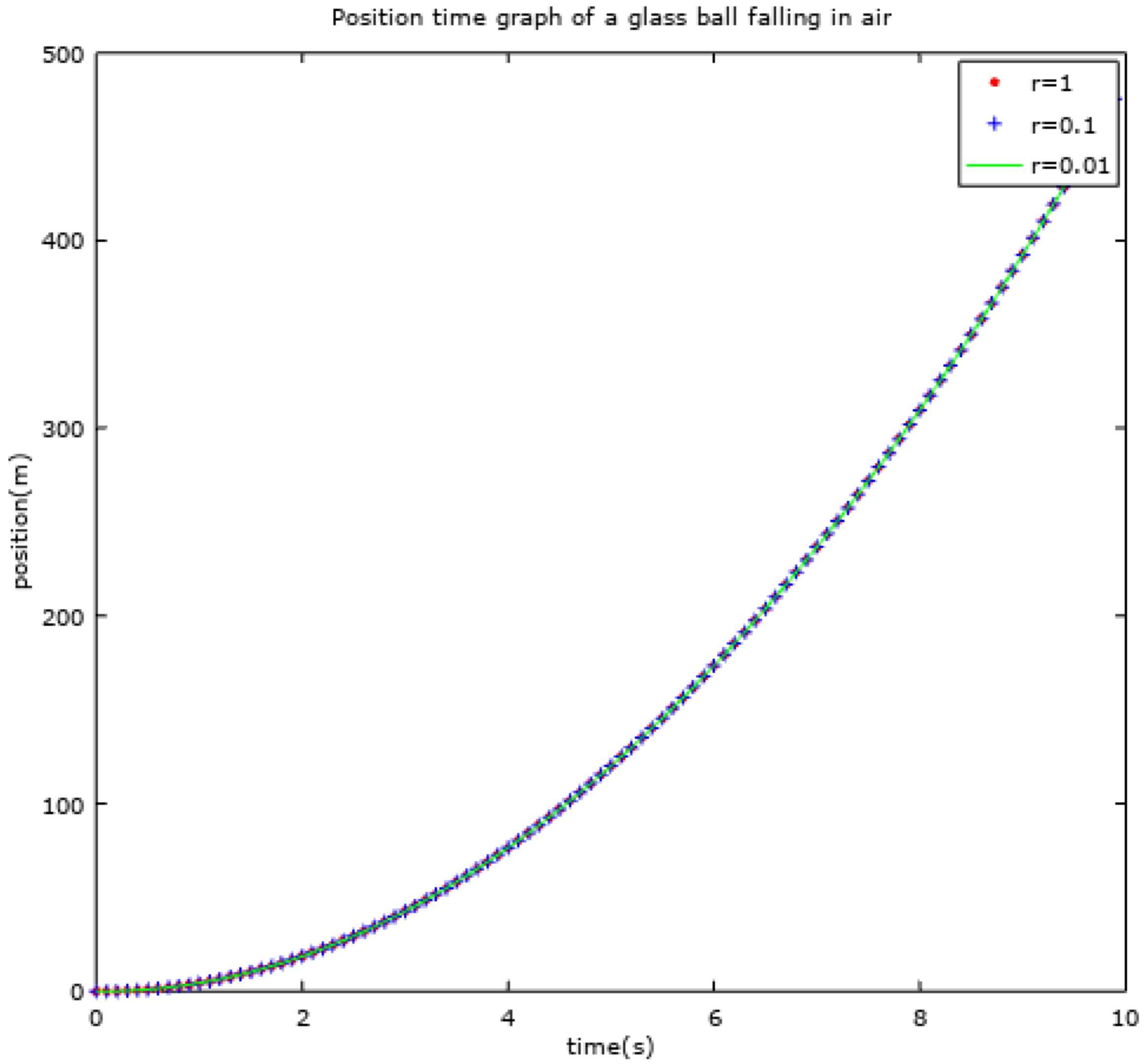
velocity(step+1)=velocity(step) + (g*dt -

4/3*pi*r^3*1.225*g*dt/mbody);

Computational model vs Analytical model



Position vs Time for r=1, r=0.1m and r=0.001m.



Conclusions and Discussions:

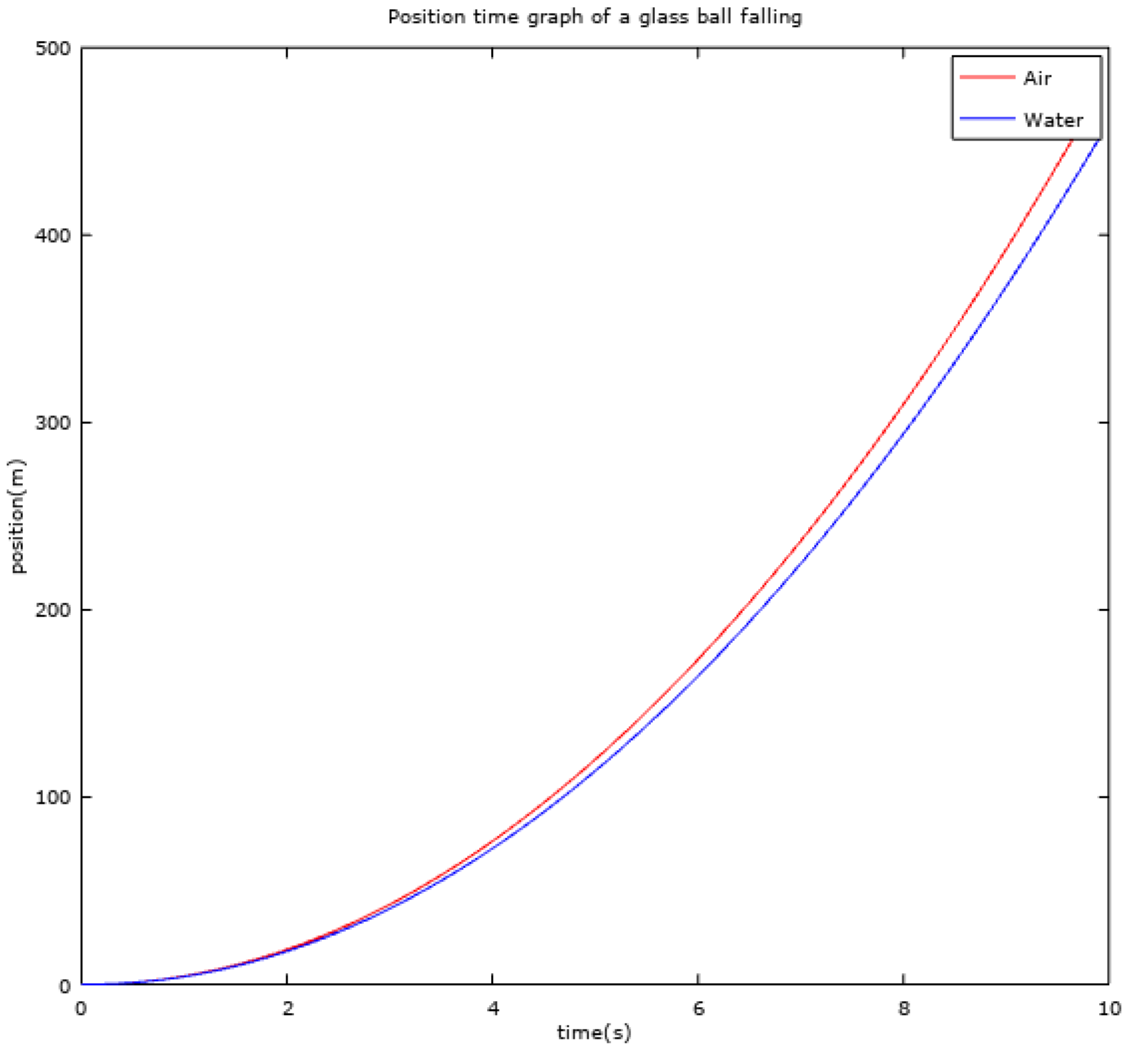
As the radius of the object increases, the buoyant force on the object increases. Hence , there is more decrease in its velocity with time.

Accuracy:

The analytical and computational graphs of velocity and position almost coincide with each other. Thus , we can say that the computational model is fairly accurate.

(b) Also computationally investigate the motion of
the same object traveling through a liquid (say water), and compare the motion with the case of air. Use computational data and plots to explain your answer
(motion as a function of time)

Position vs Time for motion in water and motion in air



Q4. Now add the effect of viscous drag to the problem 3(b) assuming a small sphere is falling through the liquid with low speed. Model the system using viscous force given by Stokes law as discussed during the class. Choose proper coefficient of viscosity (look at the unit), and analyze the phenomena of terminal velocity.

Approximations:

1. We neglect variation of g

2. We consider only gravitational, viscous and buoyant force acting on the object

Mathematical model:

Net force acting on the object,

F= mg – (4/3)*∏r3ρg - 6∏ηrv

a = dv/dt = g – (4/3)*∏r3ρg/m - 6∏ηrv/m

integrating, we get

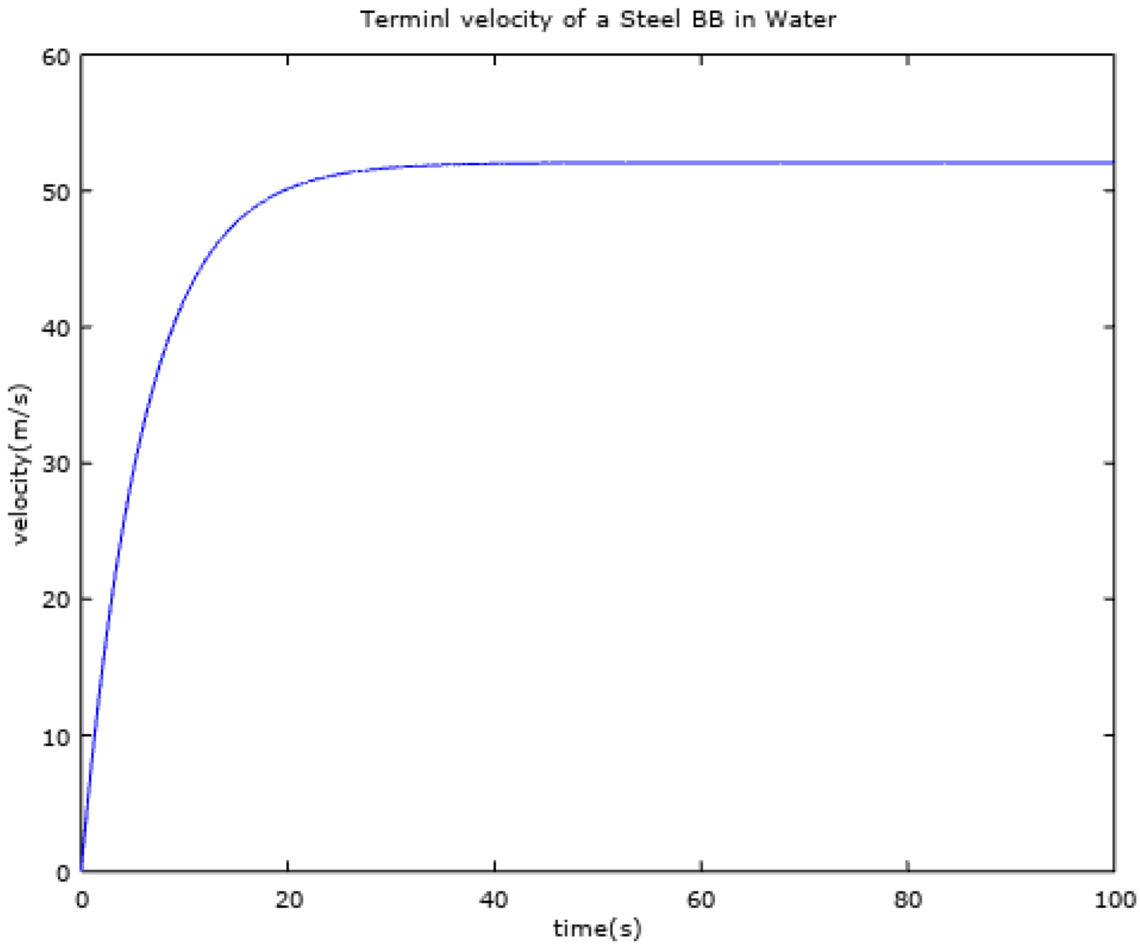
v= (g – (4/3)*∏r3ρg/m – e6∏ηrvt/m)*m/6∏ηr

Initially, velocity is zero and acceleration is maximum. As velocity increases, acceleration decreases and the object will achieve its terminal velocity as acceleration becomes zero.

Computational Formulation:

X(step+1) = X(step) + V(step)*dt;
V(step+1) = V(step) +
g*(1-((4/3*densityHoney*pi*r^3)/mass))*dt-
((6*pi*etaHoney*r*V(step))/mass)*dt;

Velocity Vs Time graph for water:



Results and Discussion:

Ploting the graph, we can see that the object achieves its terminal velocity after some time, which is supportive of the analytic solution.

As the viscosity of the fluid increases, lesser time is taken to attain terminal velocity.

Q5. Modify the program (problem 3) and include the variation of “g” with height. Use the program to computationally investigate the motion of a body dropped from a height of 20 KM (assume constant air density). How will you use the above program to investigate free fall in a deep mine (by taking proper initial conditions Google).

Approximations:

1. We neglect the air resistance and drag.
2. We consider the net force on the object to be due to gravity and buoyancy.
3. We do not treat gravitational force to be constant, but rather the effect of height is considered while calculating the value of ‘g’.

Mathematical Model:

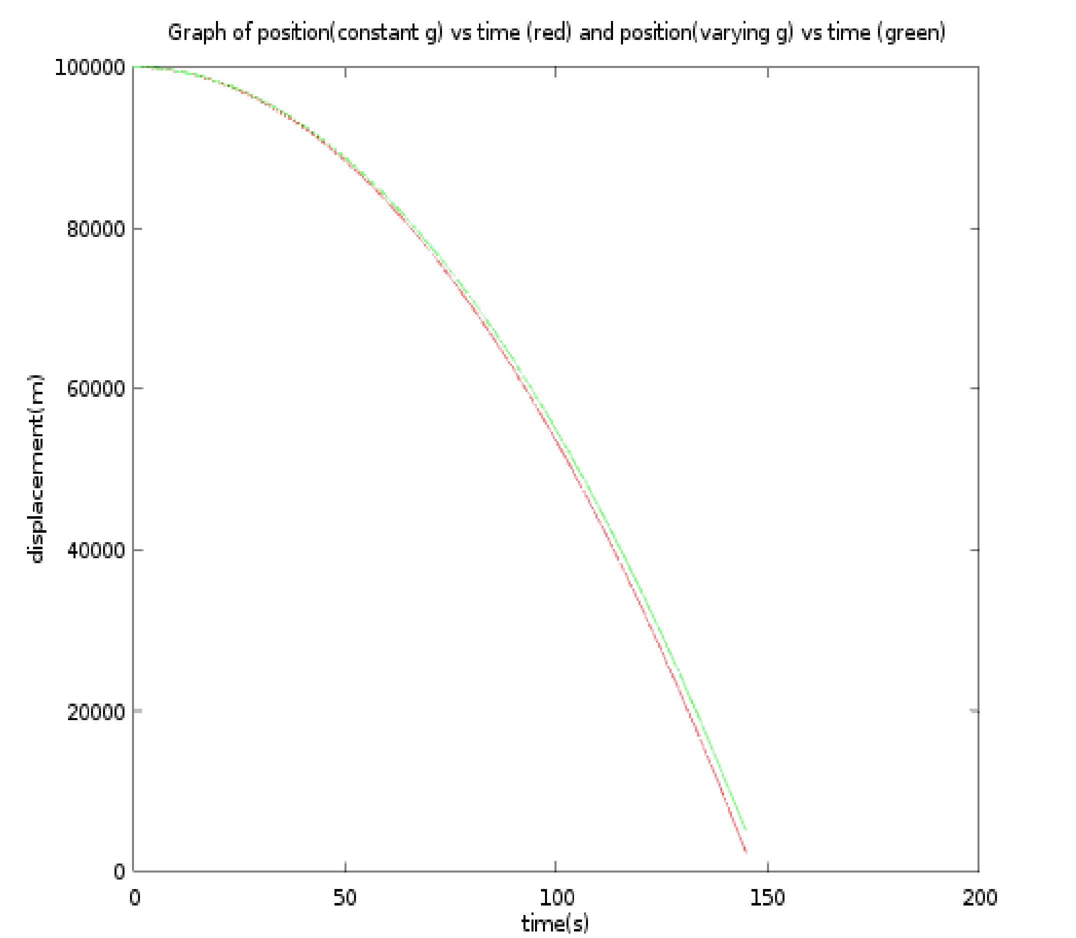
$g(h) = g(1+h/r)^2$ where r= radius of earth.

The above equation represents the variation of g with height.

$d^2x/dt^2 = g - 4/3*pi*r^3*g(h)*density(air)$

$dx/dt = g(h)t - 4/3*pi*r^3*g(h)*density(air)*t$

$x = vt$



Q6. A stone is thrown vertically upwards from the ground with some initial velocity in vacuum (choose a proper realistic velocity). Track the complete motion till it comes down to the ground (computationally). What is the velocity when it strikes the ground, compare with analytical result?

Approximations :

1. g remains constant throughout.
2. There is no drag force. Zero air resistance.
3. Initial velocity is 50 m/s.

Mathematical model :

$v = v_0 + gt$

$h = 1/2gt^2$

$v^2 - u^2 = 2gh$

Applying equations of motion, find out the time at which velocity becomes zero. Then consider that height and using acceleration due to gravity, find final velocity.

Analytical Solution :

0 = 50 + 9.8t

At peak, velocity becomes zero.

T = -5.1 s

Time it takes to reach maximum height.

h= 127.45 m

Find maximum height.

$v^2 = 2498.02$

Final velocity using height and acceleration.

v= 49.98 m/s

Computational Formulation :

V_init = 50

Velocity = zeros(npoints,1)

Position = zeros(npoints,1)

Time = zeros(npoints,1)

for step = 1: npoints-1

position(step+1) = position(step+velocity(step)*dt;

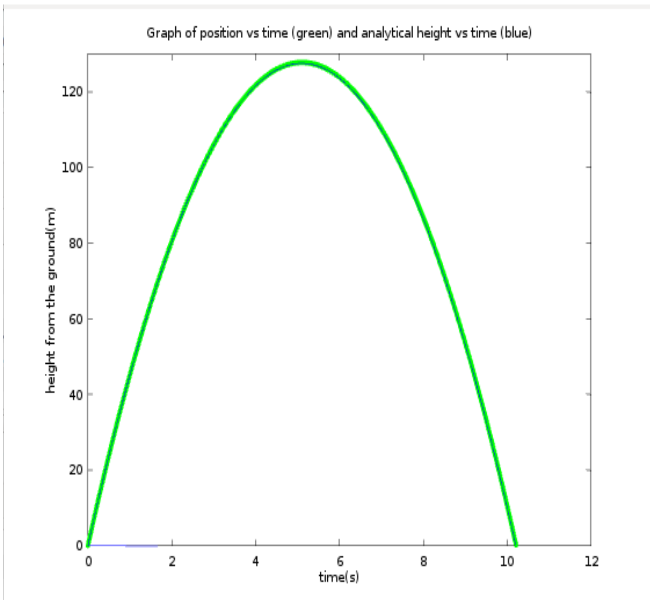
velocity(step+1) = velocity(step) – g*dt;

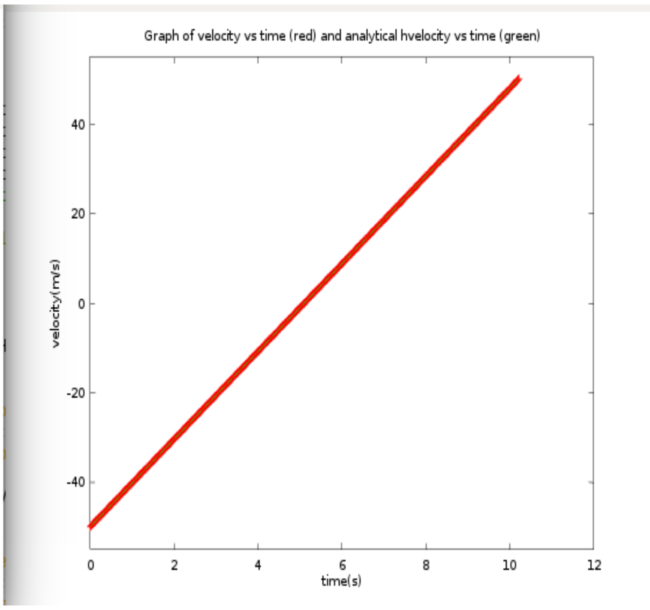
time(step+1) = time(step) + dt;

An array for position, time and velocity is made and initialised. Update values of position and velocity using previous values.

Results and conclusion :

Value obtained from graph matches that of analytical solution. i.e. 50 m/s. Point at which position becomes zero, coincides with the point at which velocity is -50 m/s.





Q7. Computationally study the motion of a balloon filled with Helium (use realistic data from Google). Also study the same for 3-4 different gases of your choice. Will the balloon rise up or fall down? Vary the size of the balloon (5 diff size) to study its effect on velocity and distance travelled as a function of time. All conclusions should be based on the plots from your computational data.

Approximation :

1.Here we will want to simulate the more realistic case by considering the air drag and buoyancy.

2.The value of gravitational constant doesn't varies with height.

Computational Formulation :

In this question we are asked to simulate motion of helium balloon , now balloon goes upward as against downward, we can see the physics through computation.

A body moving downwards (We imagine the balloon to come down) so it will have three forces acting on it.

Forces acting:

1. mg acting downwards.
2. buoyant force acting upwards.
3. viscous force acting upwards.

So the equation of motion is

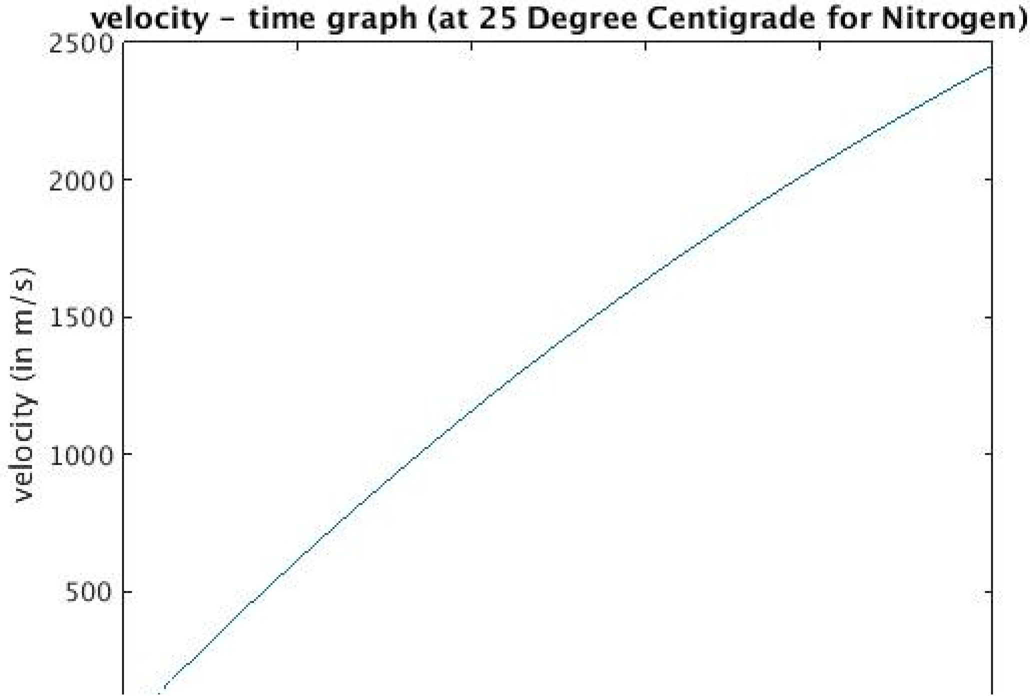
One important thing to consider here is the density of helium is less than air so the nature of velocity is that it increases with time and then attains a terminal velocity. In contrast with nitrogen the terminal velocity attains much later.

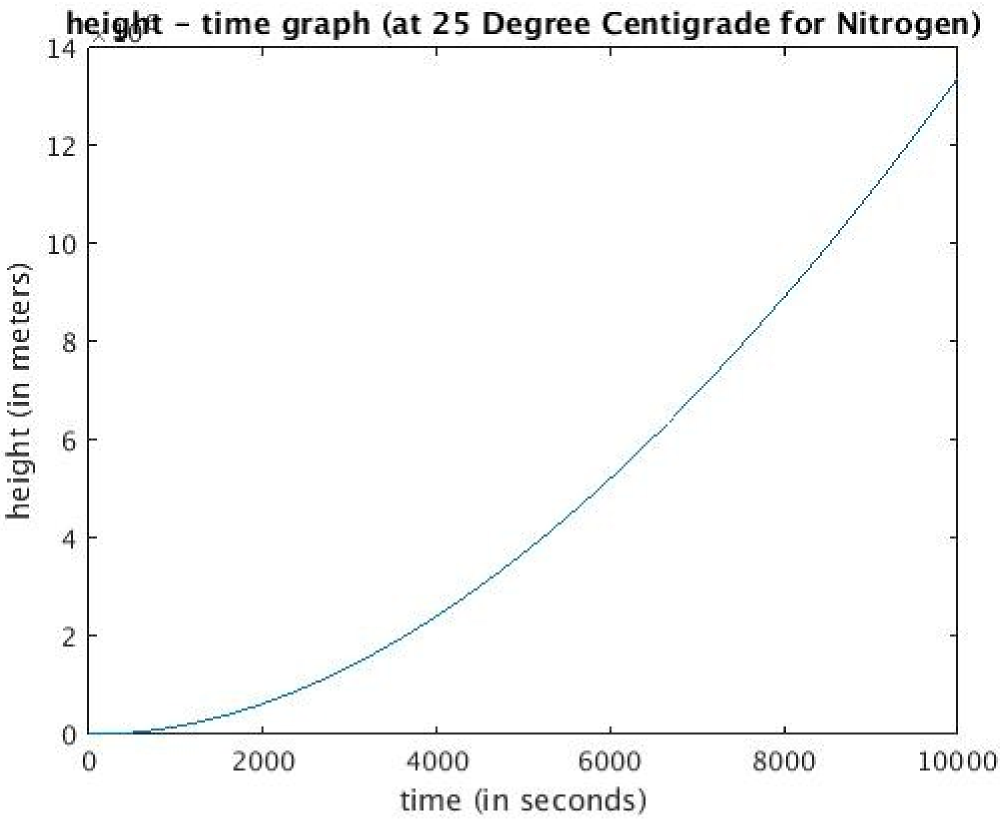
So the Euler's equation for the velocity becomes

elocity(n+1)=velocity(n)-(s_cons*stepsize*velocity(n)/m)-g*(1 - rho_med/rho_body)*stepsize;

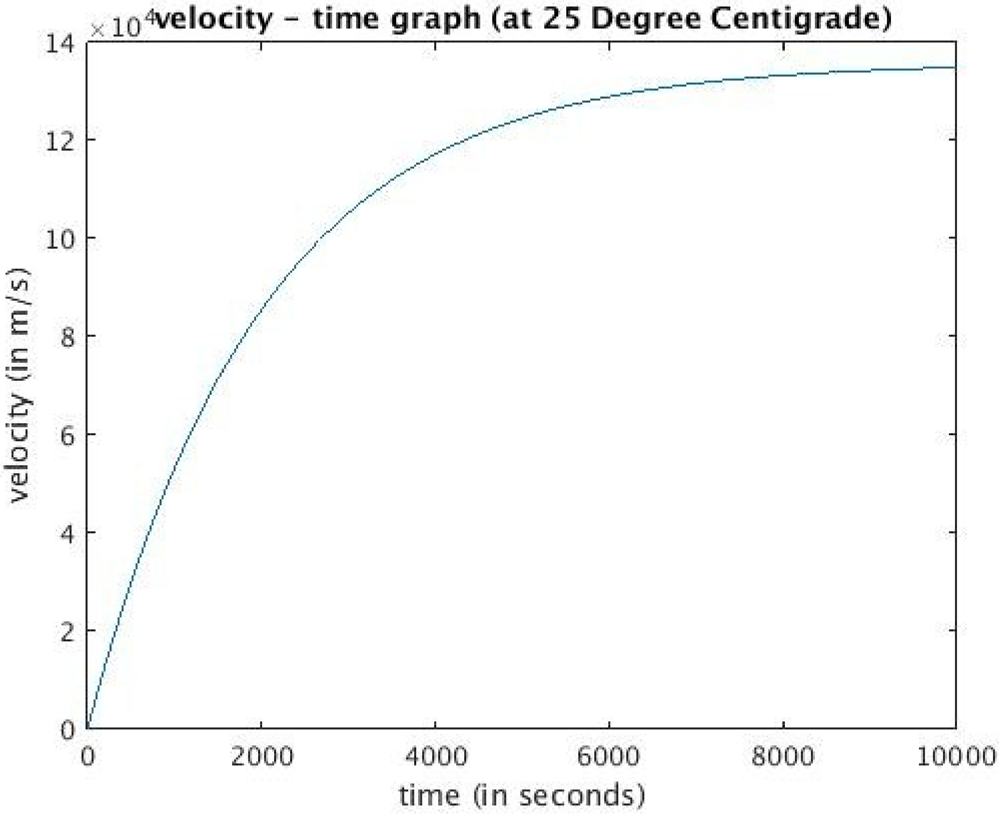
where s_cons = 6*pi*nita*r and rho_body is the density of helium and rho_med is density of air.

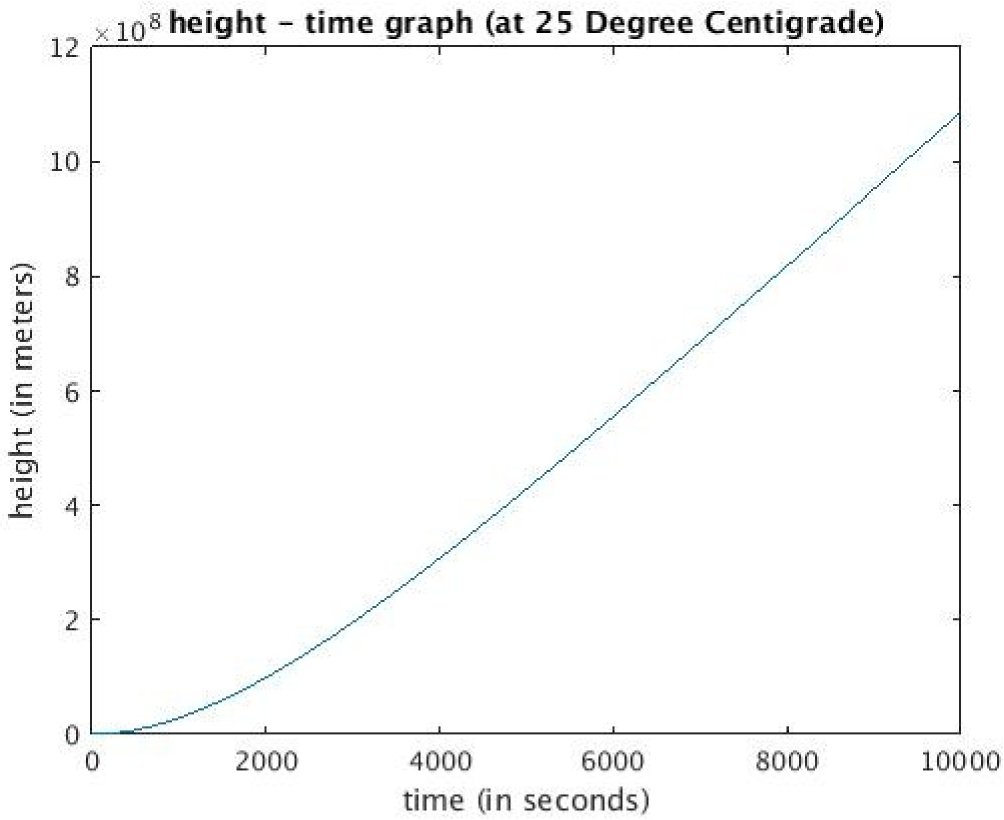
For Nitrogen(r=1)



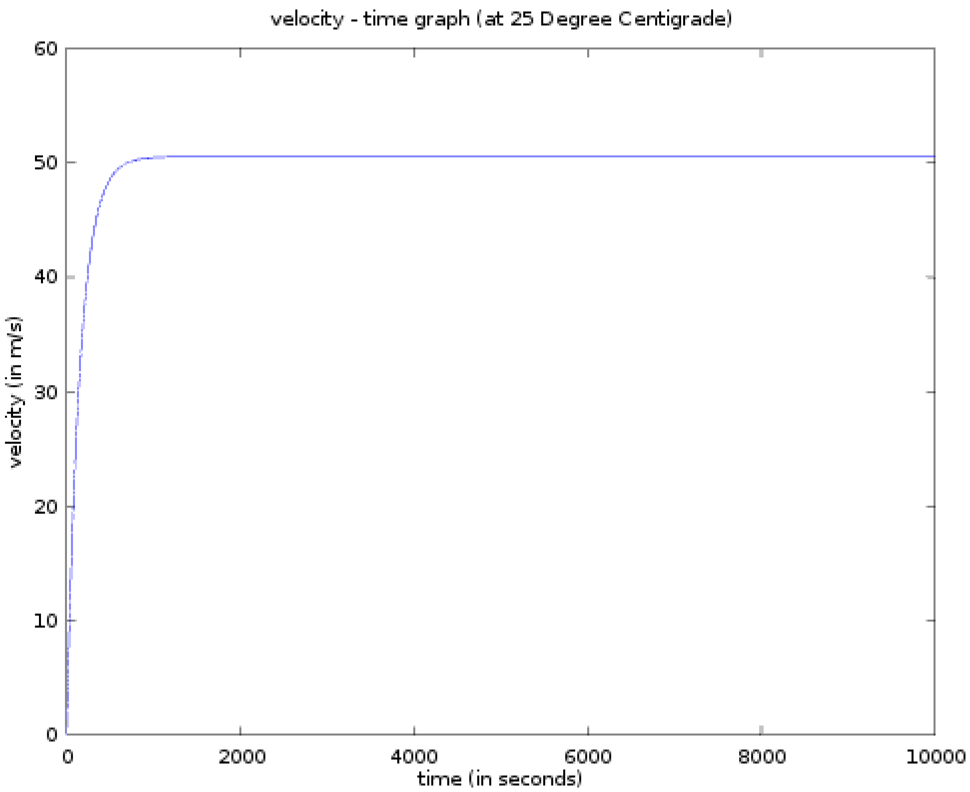


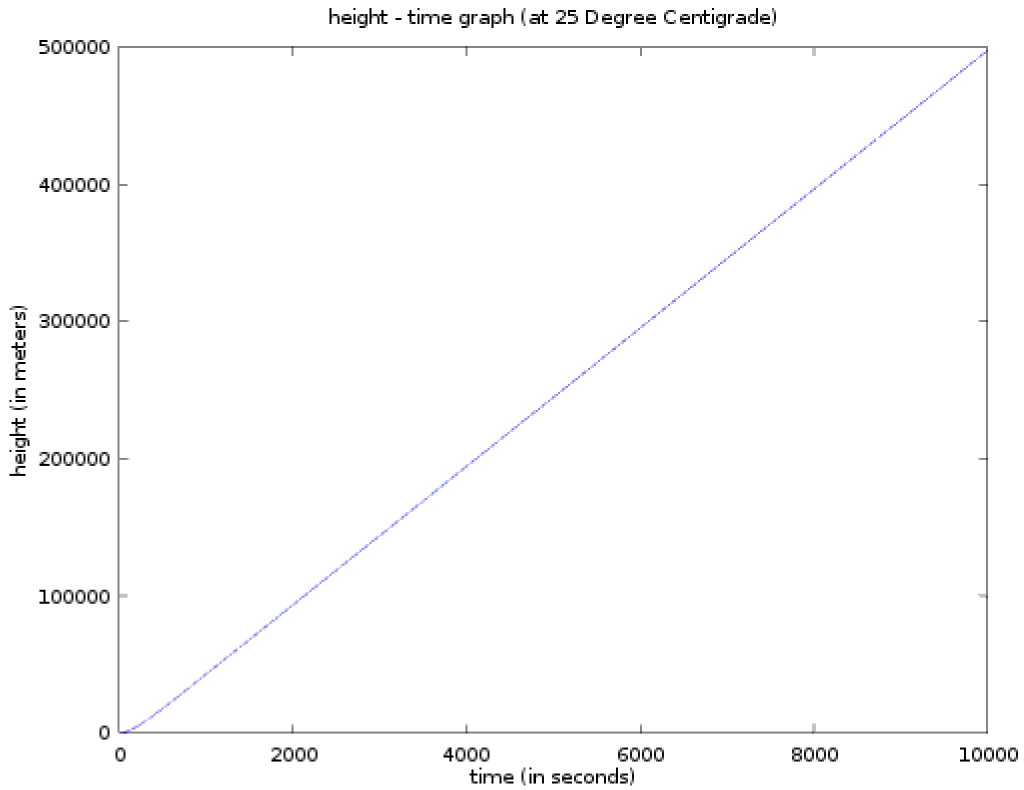
For Helium(r=1)



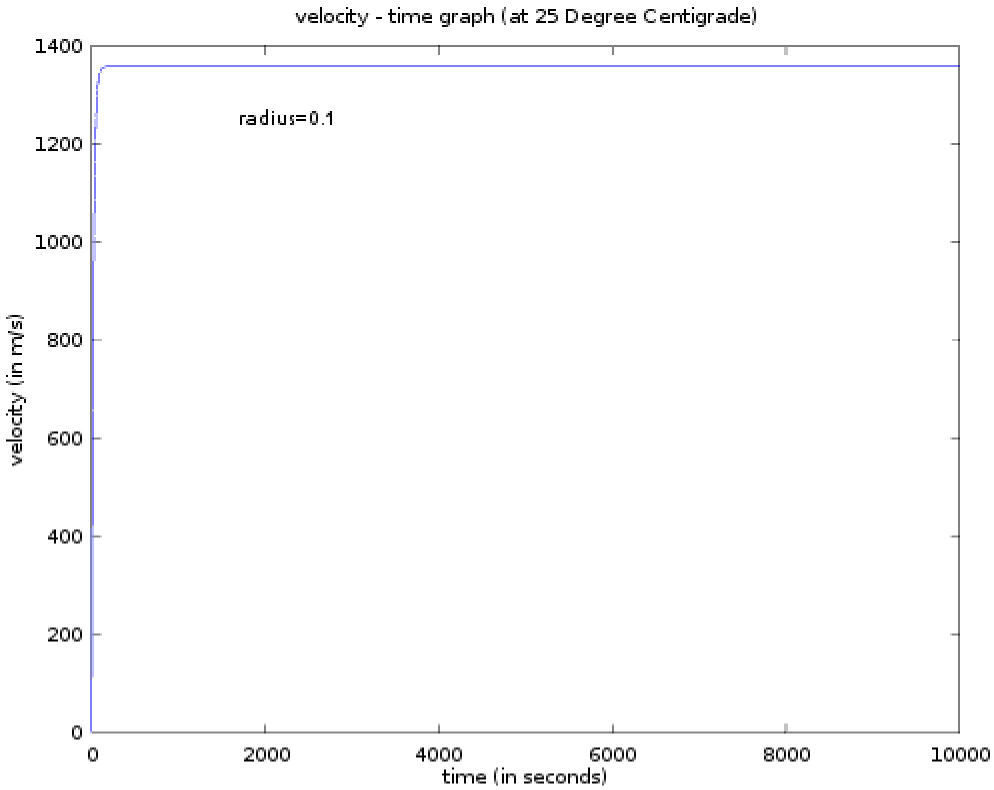


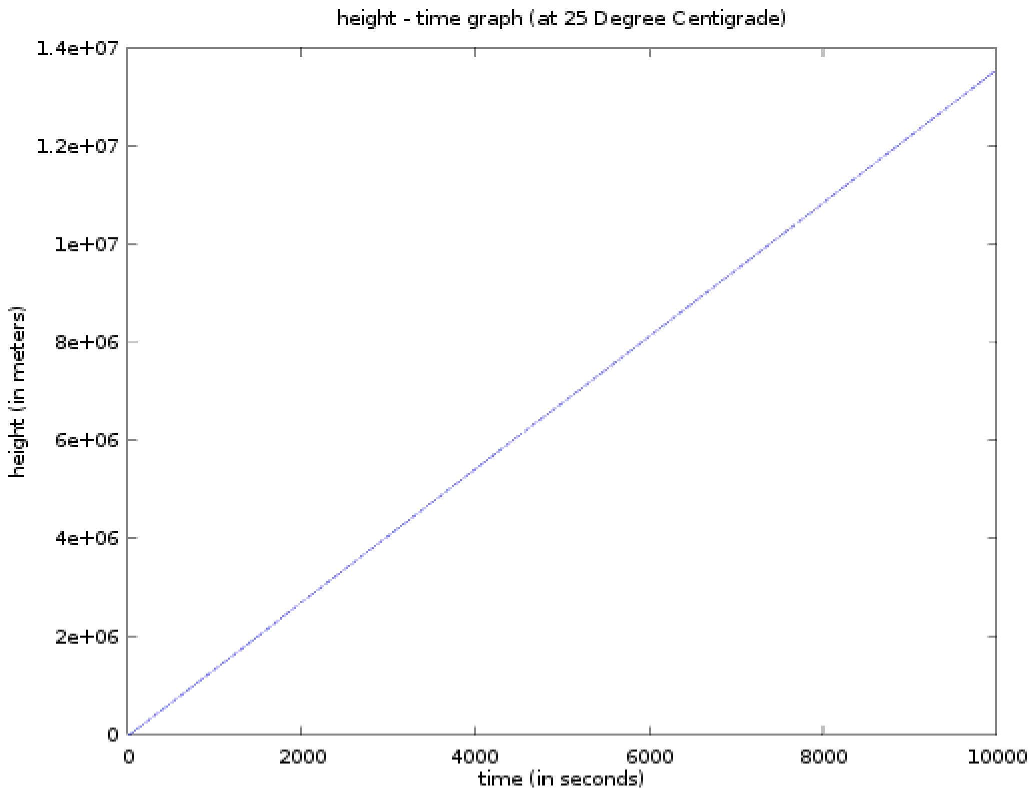
For Nitrogen(r=0.1)





For Helium(r=0.1)





Q8.Parachute problem: frictional force on the object increases as the objects moves faster (as we learned today in the class). Role of parachute is to

produce the frictional force in the form of air drag. Consider the most simple form, so the equation for velocity : dv/dt = a - bv where a (from applied force), b (from friction) are constants.

Use Euler's method to solve for "v" as a function o

f time. Choose a=10 and b=1.

What is the terminal velocity in this case ?

Approximations :

1. Constant gravitational acceleration.

2. Parachute moving slowly, hence Fdrag is proportional to velocity.

Mathematical model :

dv/dt = a - bv

Fdrag is proportional to velocity

Hence, v changes as,

v= v_0 + (a-bv)*dt

Integrating at taking t tends to infinity,

A - bv = e^{-bt}

v= a/b = 10 m/s

Computational Formulation :

v(step+1)=v(step)+(a-b*v(step))*dt

Results and conclusion :

Observation from graph coincides that of analytical solution i.e. at 10 m/s, velocity becomes constant.

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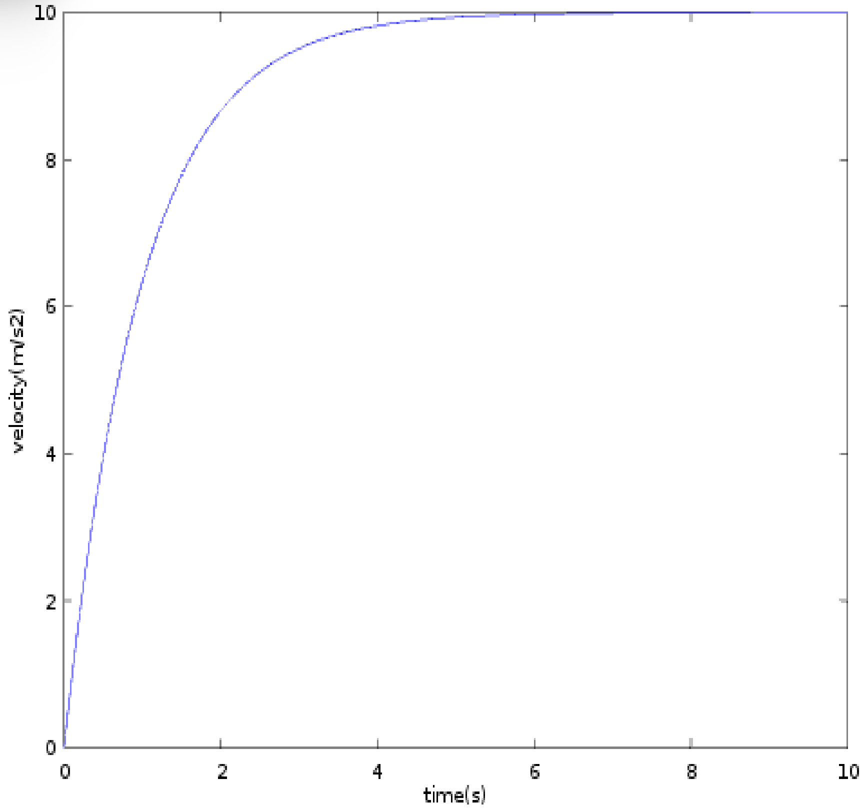

10
8
6
4
2
0
0
2
4
6
8
10
velocity(m/s2)
time(s)