Assignment 6

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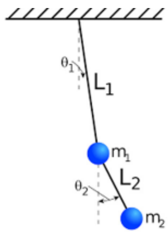
March 9, 2017

1 Double Pendulum

1.1 Euler-Lagrange Equations:



$x_1 = l_1 \sin\theta_1 \qquad x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$
$y_1 = l_1 \cos\theta_1 \qquad y_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2$

$\dot{x_1} = l_1\dot{\theta_1}\cos\theta_1 \qquad \dot{x_2} = l_1\dot{\theta_1}\cos\theta_1 + l_2\dot{\theta_2}\cos\theta_2$
$\dot{y_1} = -l_1\dot{\theta_1}\sin\theta_1 \qquad \dot{y_2} = -l_1\dot{\theta_1}\sin\theta_1 - l_2\dot{\theta_2}\sin\theta_2$

Now, L = T-V

Kinetic energy T is,

$$
\begin{aligned}
T &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\
&= \frac{1}{2}m_1(\dot{x_1}^2 + \dot{y_1}^2) + \frac{1}{2}m_2(\dot{x_2}^2 + \dot{y_2}^2) \\
T &= \frac{1}{2}m_1l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2(l_1^2\dot{\theta_1}^2 + l_2^2\dot{\theta_2}^2 + 2l_1l_2\cos(\theta_1-\theta_2)) \qquad ...(1)
\end{aligned}
$$

Also, Potential energy is given by,

$$
\begin{aligned}
V &= m_1gy_1 + m_2gy_2 \\
V &= -m_1gl_1\cos\theta_1 - m_2g(l_1\cos\theta_1 + l_2\cos\theta_2) \qquad ...(2)
\end{aligned}
$$

Now, the Lagrangian L can be found by,

$$L = T - V$$

Hence,

$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2(l_2^2\dot{\theta_2}^2 + 2l_1l_2\dot{\theta_1}\dot{\theta_2}\cos(\theta_1 - \theta_2)) + (m_1 + m_2)gl_1\cos\theta_1 + m_2gl_2\cos\theta_2$$

$Now, using\ the\ equations$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta_1}}\right) = \frac{\partial L}{\partial\theta_1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta_2}}\right) = \frac{\partial L}{\partial\theta_2}$$

We can derive the equations of motion for the double pendulum using the aboce equations. On solving, these equations come out to be,

$$\frac{du_2}{dt} = \frac{ed - bf}{ad - cb}$$

$$\frac{dv_2}{dt} = \frac{af - ce}{ad - cb}$$

where,

$$u_1(t) = \theta_1(t)$$
$$u_2(t) = \dot{\theta_1}(t)$$
$$v_1(t) = \theta_2(t)$$
$$v_2(t) = \dot{\theta_2}(t)$$

$$a = (m_1 + m_2)l_1$$
$$b = m_2l_2\cos(u_1 - v_1)$$
$$c = m_2l_1\cos(u_1 - v_1)$$
$$d = m_2l_2$$
$$e = -m_2l_2(v_2)^2\sin(u_1 - v_1) - g(m_1 + m_2)\sin(u_1)$$
$$and,\ f = m_2l_1(u_2)^2\sin(u_1 - v_1) - m_2g\sin(v_1)$$

1.2 Computational results

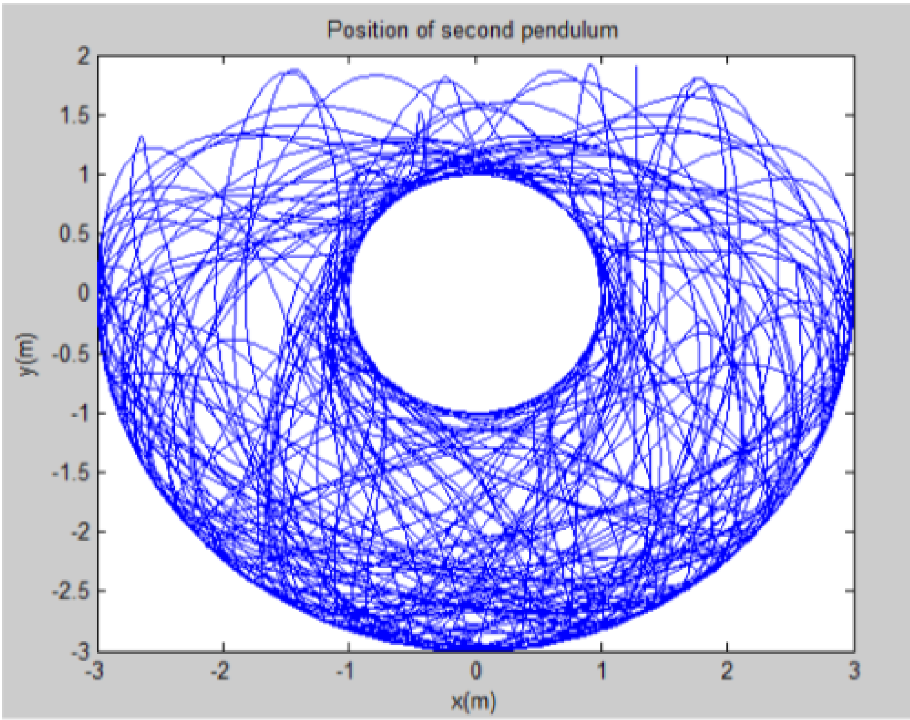


Figure 1:

For the values $m_1 = 2, m_2 = 1, l_1 = 1, l_2 = 2$ and for simulation time T=100 s, the position of the second pendulum in the x-y plane can be seen in the above figure

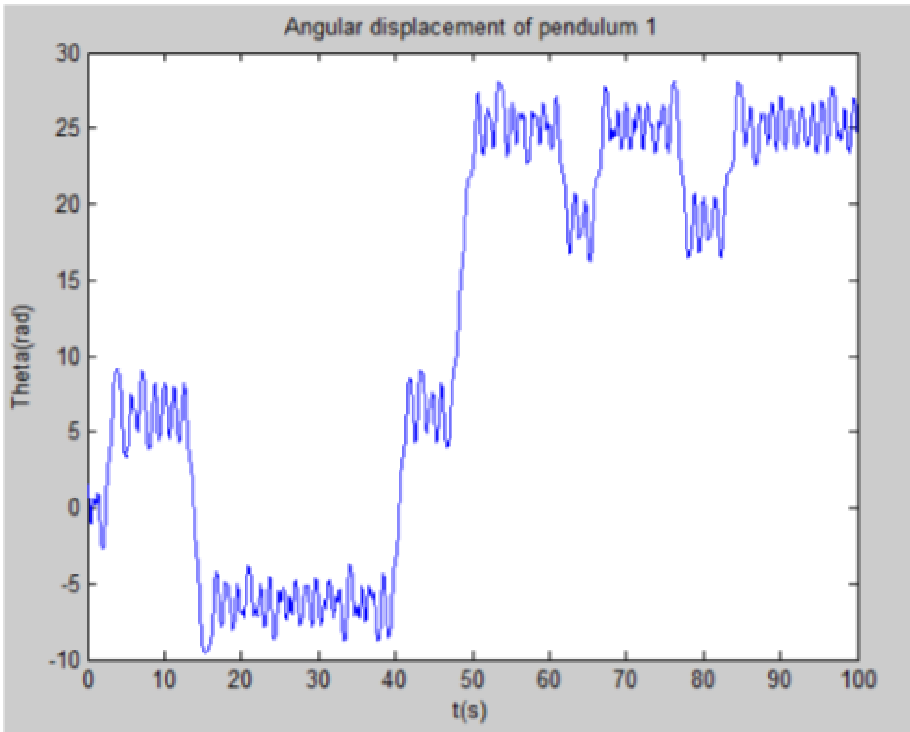


Figure 2:

In the above graph, we can see the angular displacement of the first pendulum. As can be seen, this motion is in some ways sinusoidal for some intervals of time.

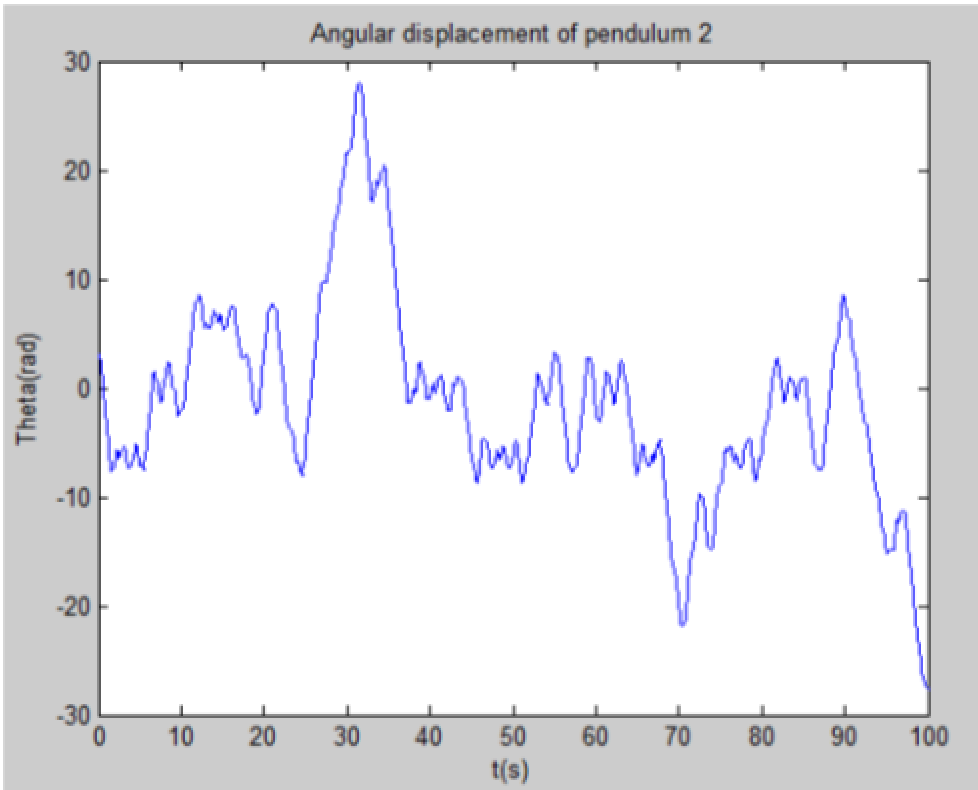
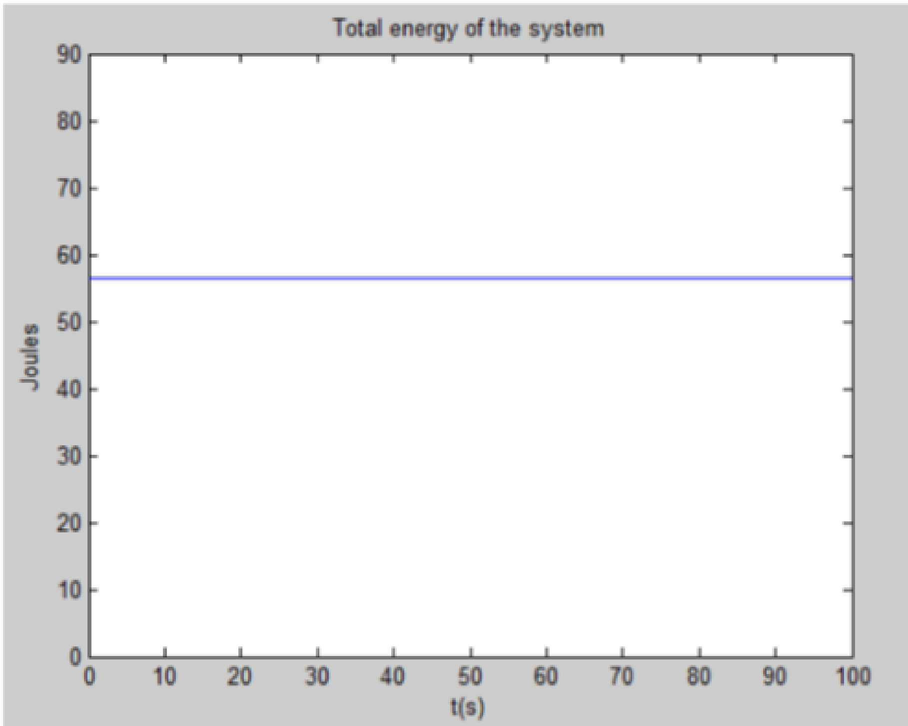


Figure 3:

The above graph shows the angular displacement of the second pendulum.



To check whether our numerical computation is true or not, we plot the Total energy of the system ,which should be constant. As can be seen from the graph, the total energy of the system remains constant.

From the above graphs, we can see that the motion of the double pendulum is "chaotic". But this is not true for all the cases as the degree of chaoticity also depende on the initial angular displacements of each pendulums and whether they are very large or small.

For very small initial displacements, the motion of the pendulum and the system is less chaotic. This can be seen in the following figures.

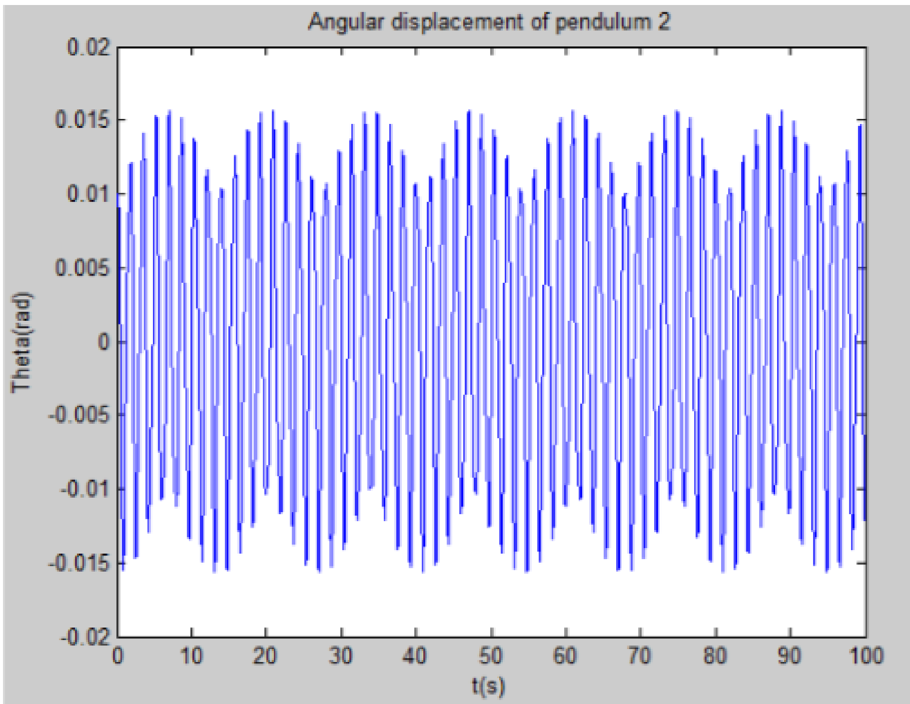


Figure 4: Very small initial angular displacements

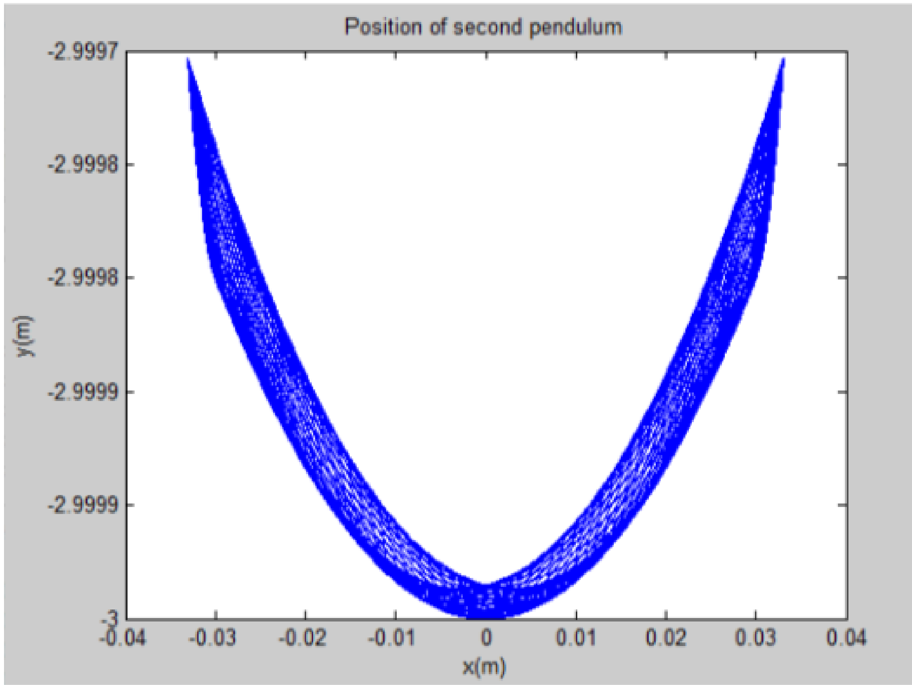


Figure 5: Position of second pendulum for small thetas

Similarly,even when initial angular displacements very large, the system is less chaotic. This can be seen in the following graphs:

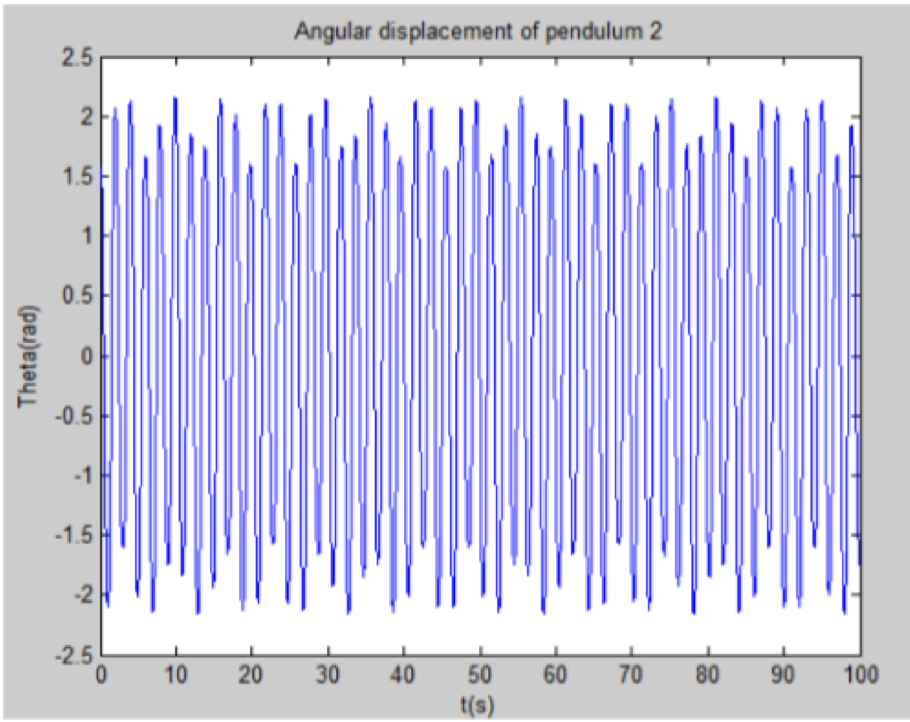


Figure 6: Very large initial angular displacements

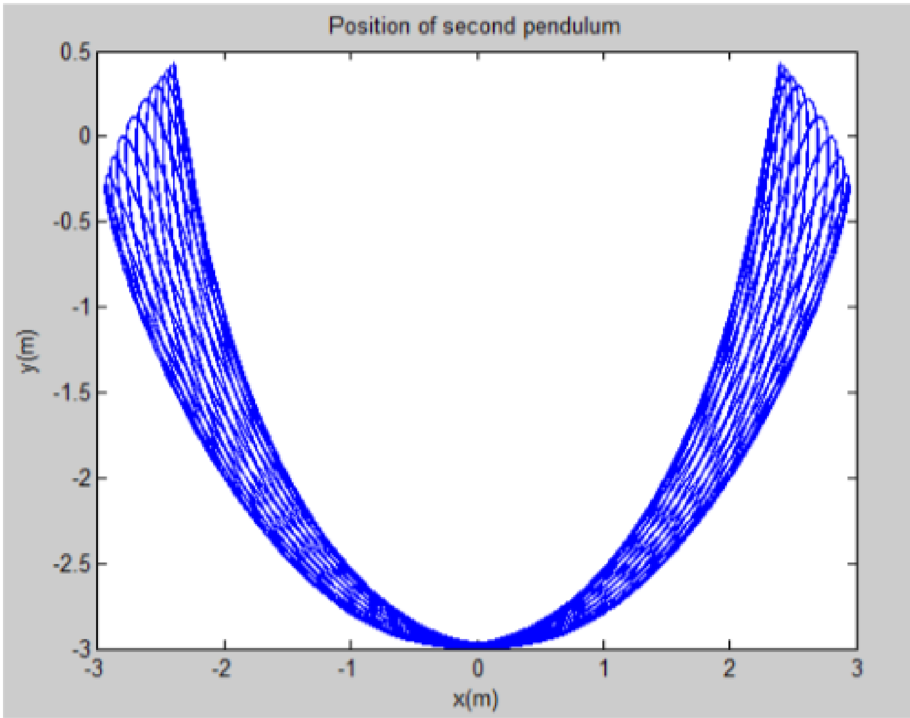


Figure 7: Position of second pendulum for large thetas

From the above graphs, we can observe that a system of double pendulum behaves more chaotically for moderate values of initial displacements but for very small or very large initial angular displacements, the system is less chaotic. Therefore, the chaoticity of the system and the chaotic orbitals, both depend on the initial values of the angular displacements and the characteristics of the masses and lengths of the strings.