ASSIGNMENT 3

Group 20:

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1. Simple Harmonic Motion:

Computationally investigate the motion of a **pendulum** and a **spring-mass system** as discussed in the class (without any damping). Draw phase plots to explain your observations.

Pendulum:

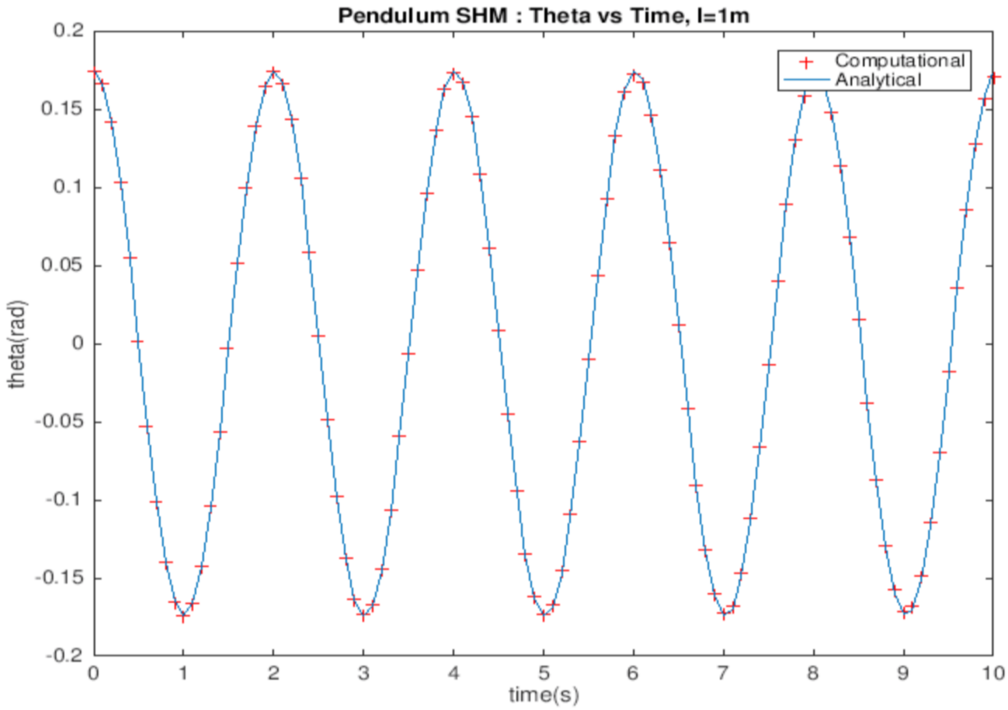
Approximations:

- No damping force
- Small angle displacement
- String is light, massless, and taut

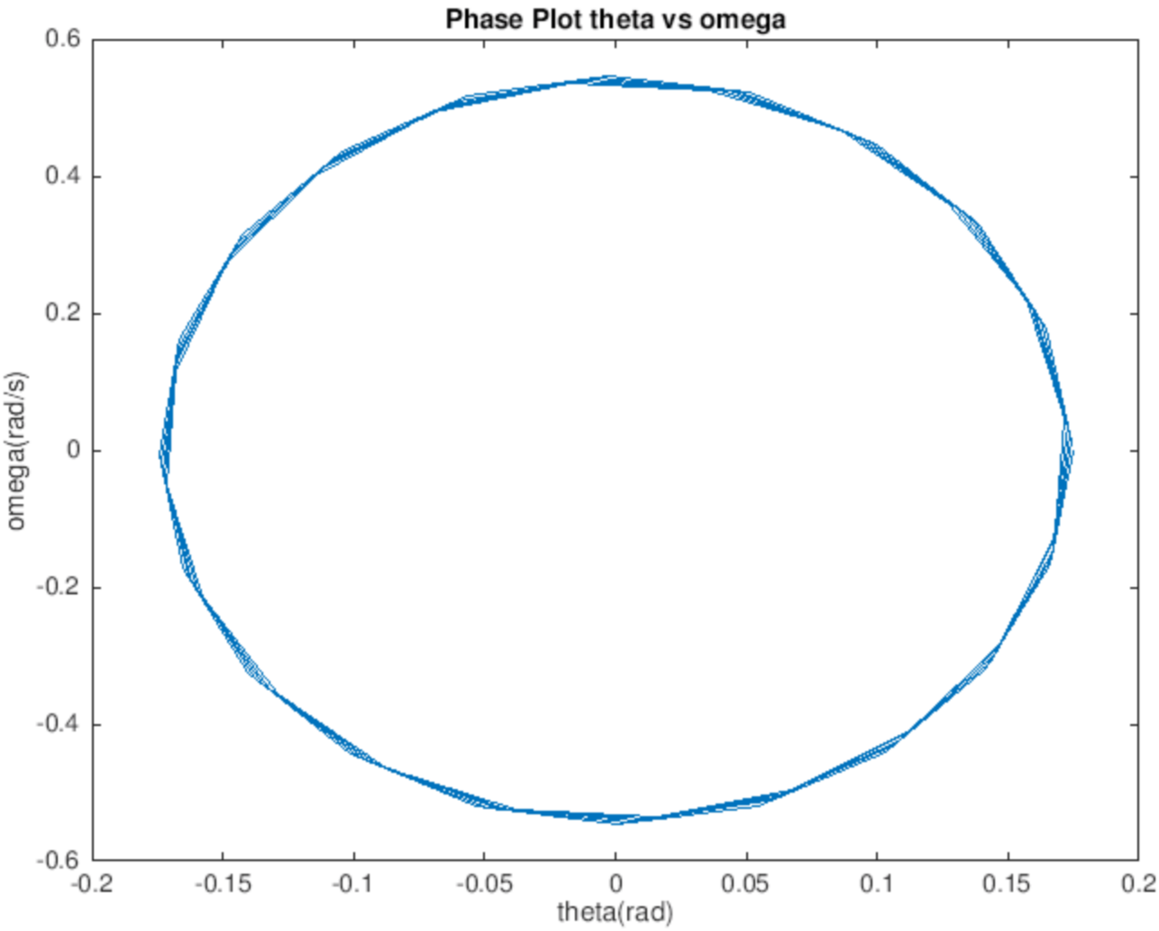
Equations governing the model:

- $\frac{d\theta}{dt} = w$
- $\frac{dw}{dt} = -\frac{g}{l}\theta$

Graph of position(θ) vs time(s):



Graph of position(θ) vs velocity(w):



<u>Mass Spring System:</u>

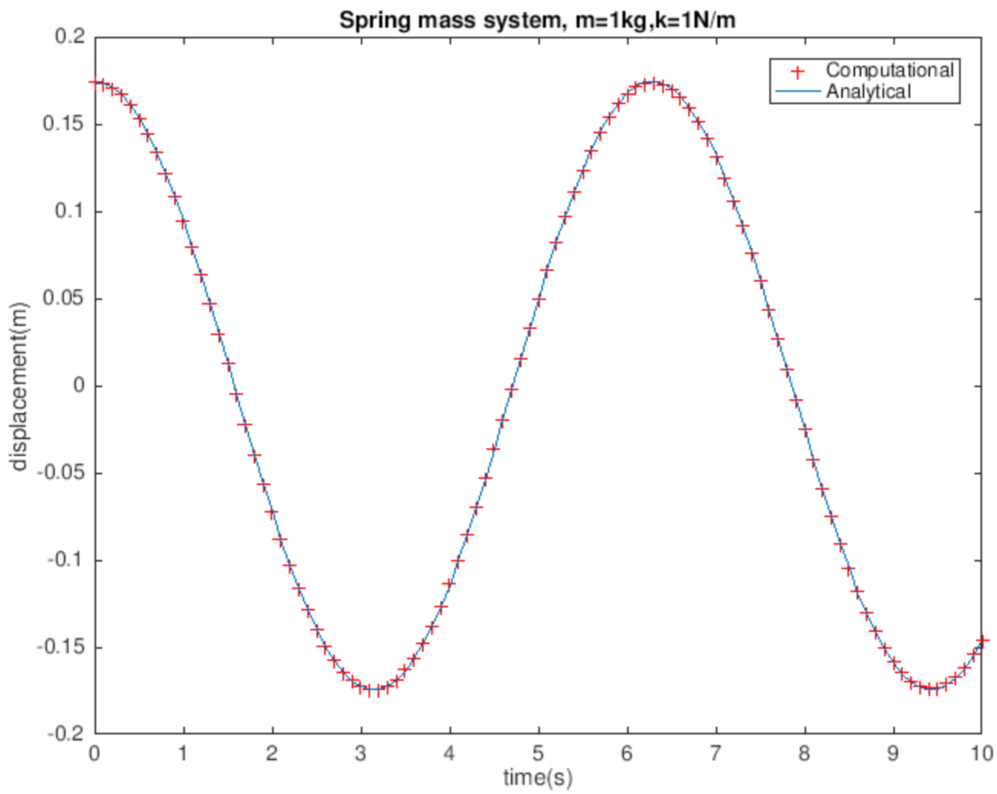
Approximations:

- No damping force
- Small displacement so that the spring obeys Hook's Law
- No friction
- Spring is attached to a rigid support

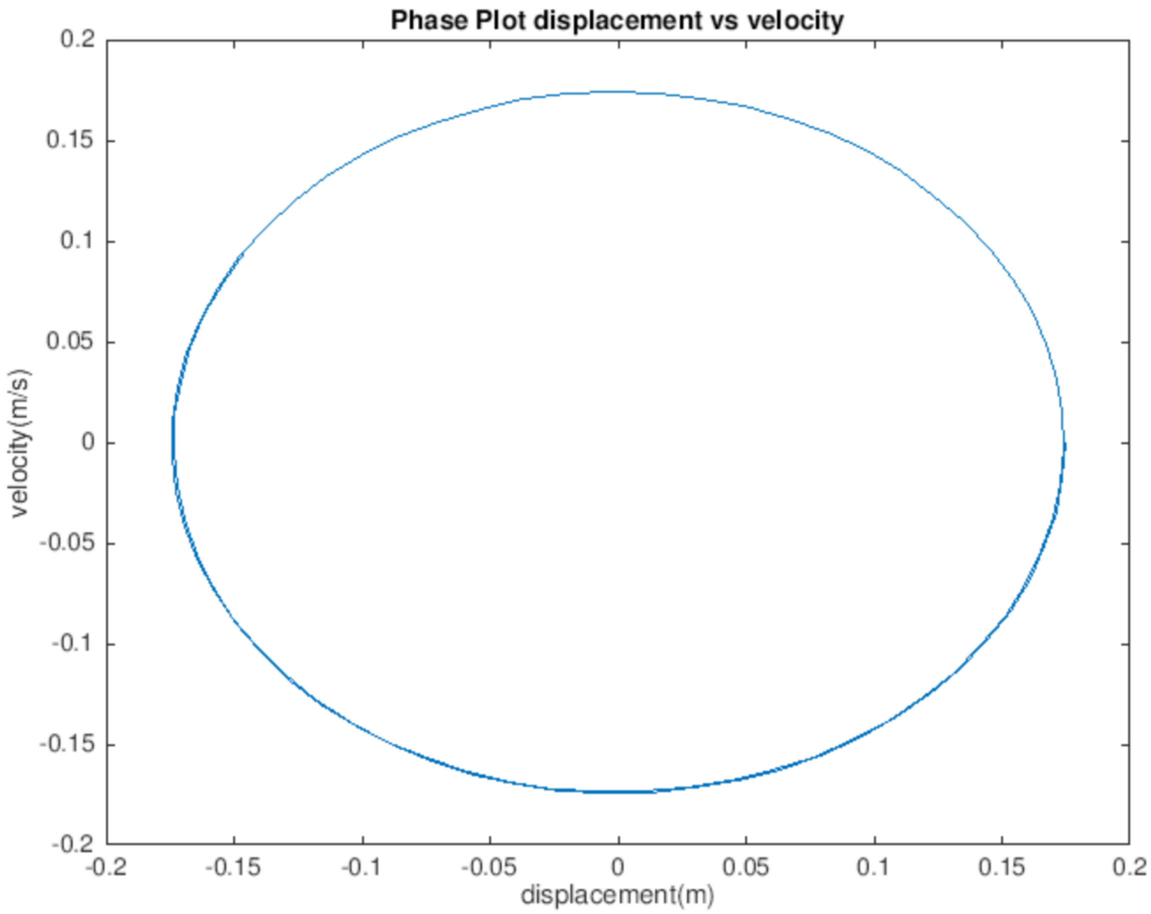
Equations governing the model:

- $\frac{dx}{dt} = v$
- $\frac{dv}{dt} = -w_0^2 x$

Graph of position(θ) vs time(s):



Graph of position(θ) vs velocity(w):



Conclusion:

The position and velocity of the pendulum/mass are governed by the sinusoidal equation $\boldsymbol{A\, sin(wt + \phi) + B\, cos(wt + \phi)}$**,** $(B\, cos(wt + \phi)\ \ in\ this\ case)$. Furthermore, the phase difference between position and velocity is π/2. Also, the energy of the system remains constant at $\frac{1}{2}kB^2$.

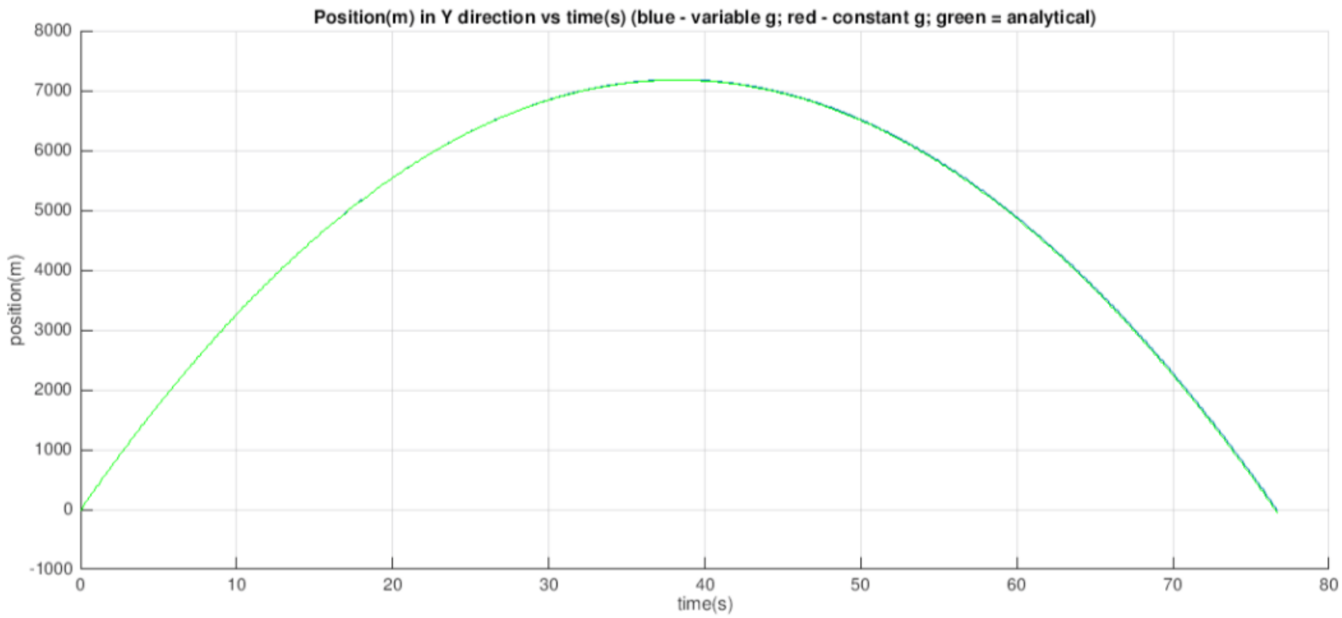
2. Projectile Motion: Cannon Shell/ Missile Problem

(a) Investigate (computationally) the cannon-shell trajectories ignoring both air drag and the effect of air density. Compare your result with exact solutions. Acceleration due to gravity depends on altitude; include this effect in your computational model by making some rational assumption.

Approximations:

- No air drag
- No effect of air density

Position(m) in y direction vs Time(s):



Conclusion:

Due to variable 'g', the object requires more time to reach the ground than it requires when 'g' is constant.

b) Investigate the trajectory of the canon shell including both air drag (proportional to square of velocity) and reduced air density at high altitudes. Perform your calculation for different firing angles; and determine the value of the angle that gives the maximum range.

$$F_{drag} = -Bv^2$$

Density of atmosphere varies as follows:

$$\rho = \rho_0 \exp(-y/y_0)$$

y is the altitude; y_0 =1000 m.

Drag force with air resistance:

$$F^*_{\text{drag}} = \frac{\rho}{\rho_0} F_{\text{drag}}(y=0)$$

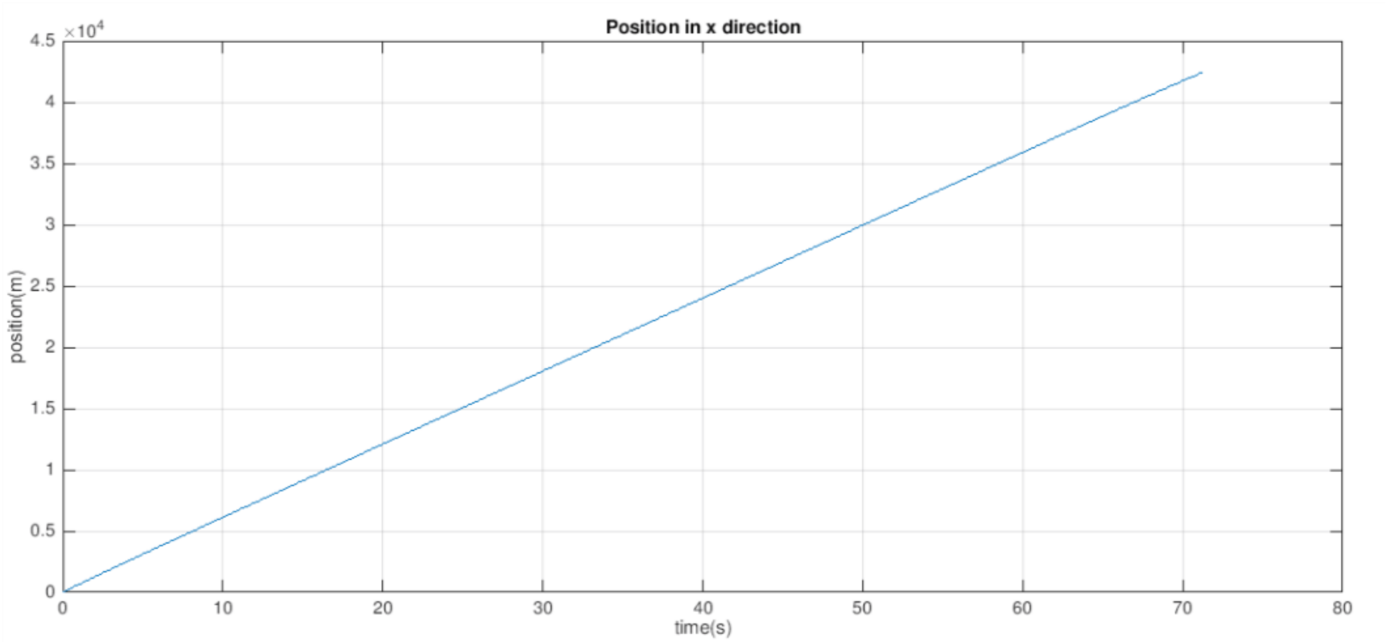
Take initial speed=750 m/s;

B/m= 4E-5 m^{-1}.

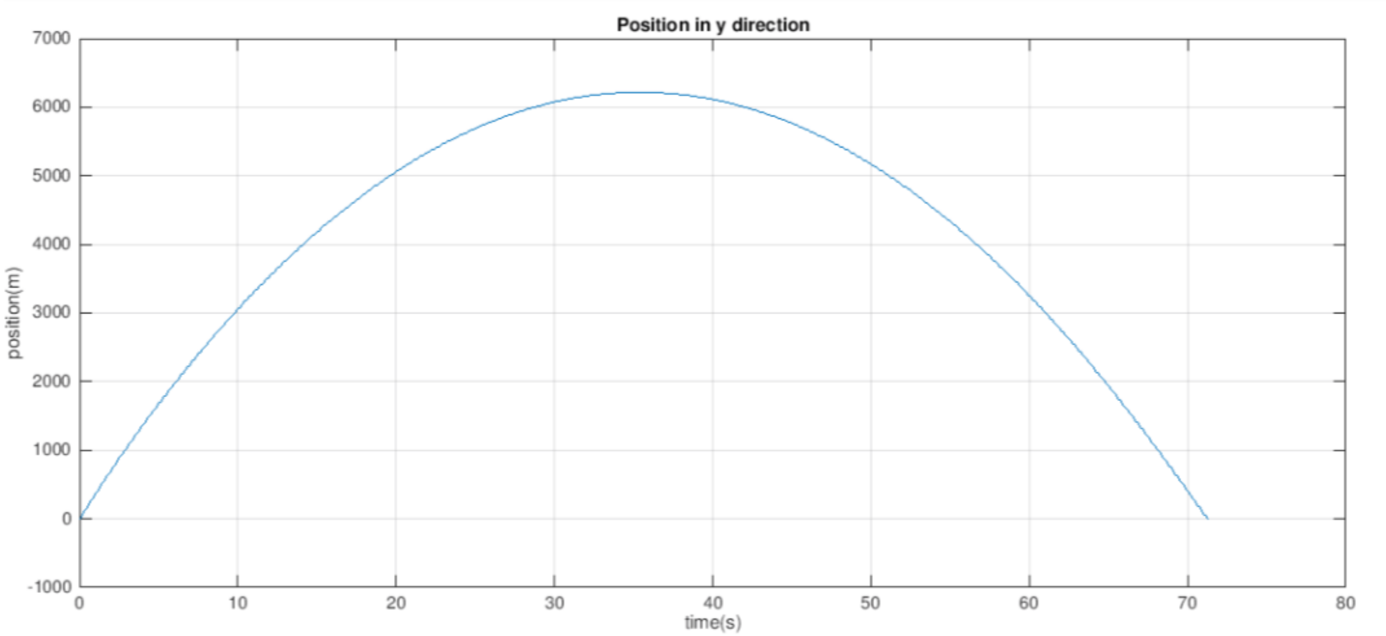
Approximations:

- Assumed constant value of g since there isn't much change even at an altitude of 10kms

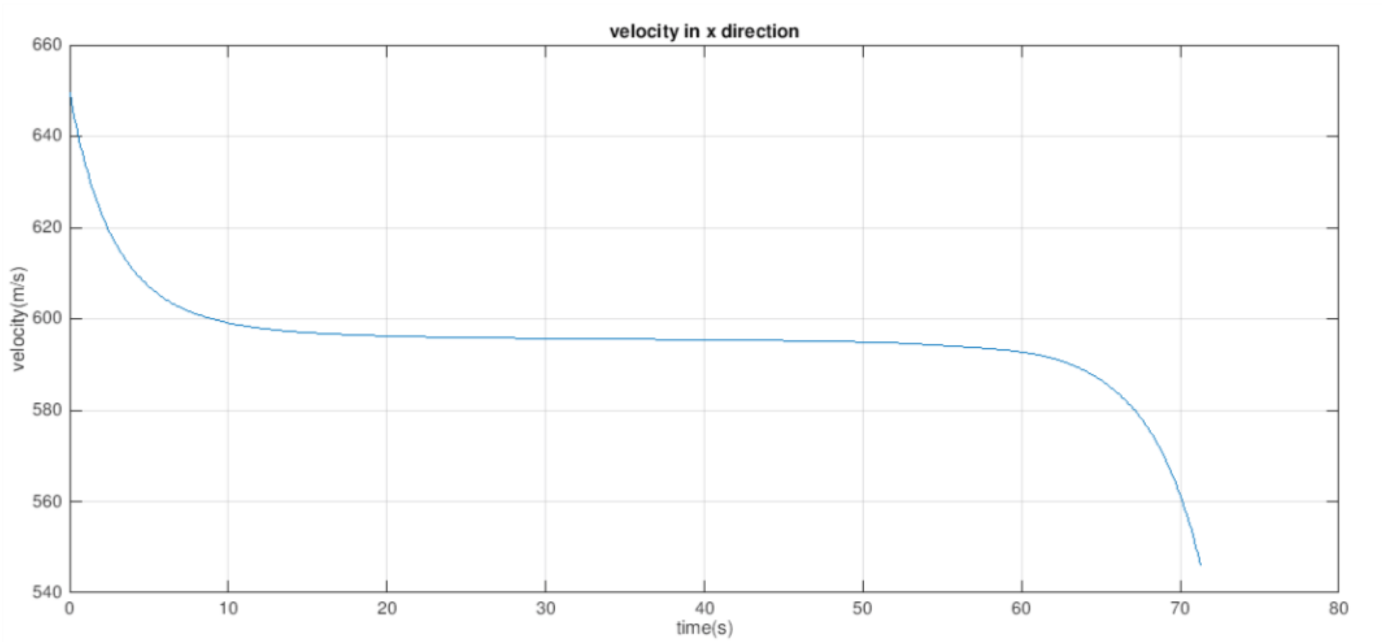
Position in x direction(m) vs time(s):



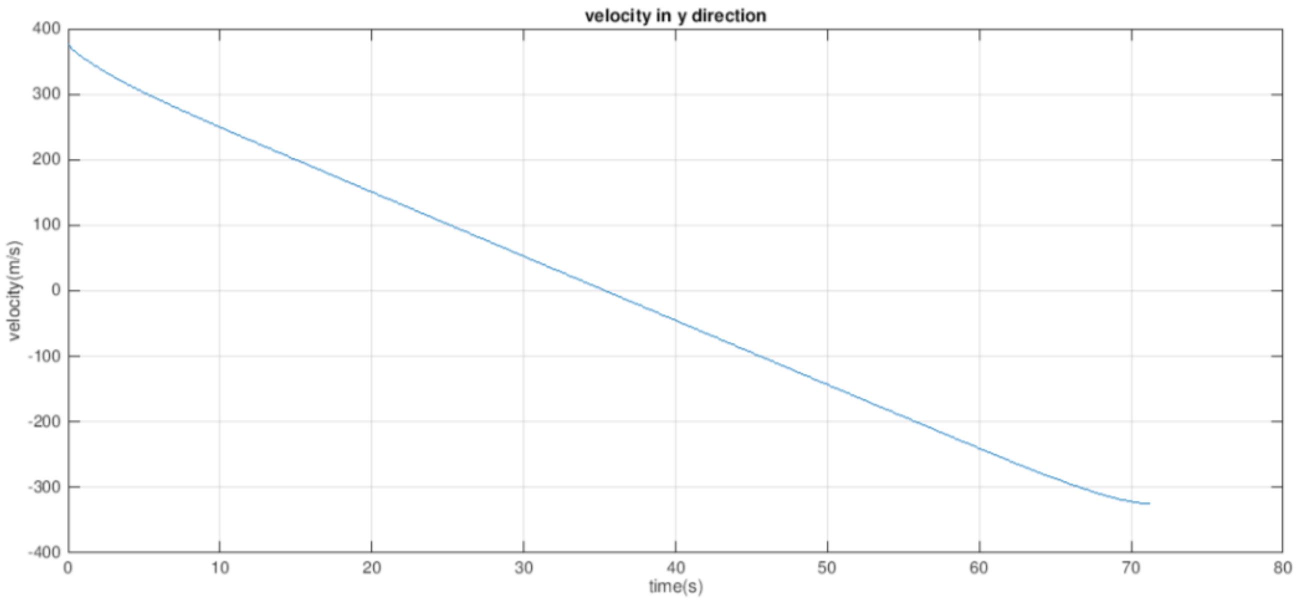
Position in y direction(m) vs time(s):



Velocity in x direction (m/s) vs time (s):



Velocity in x direction (m/s) vs time(s):



Conclusion:

The object reaches less height due to the resistance provided by the changing density of air and the drag force. Also, it reaches the maximum distance when the firing angle is equal to 47° (instead of 45° because of the resistance).

3. Bicycle Problem

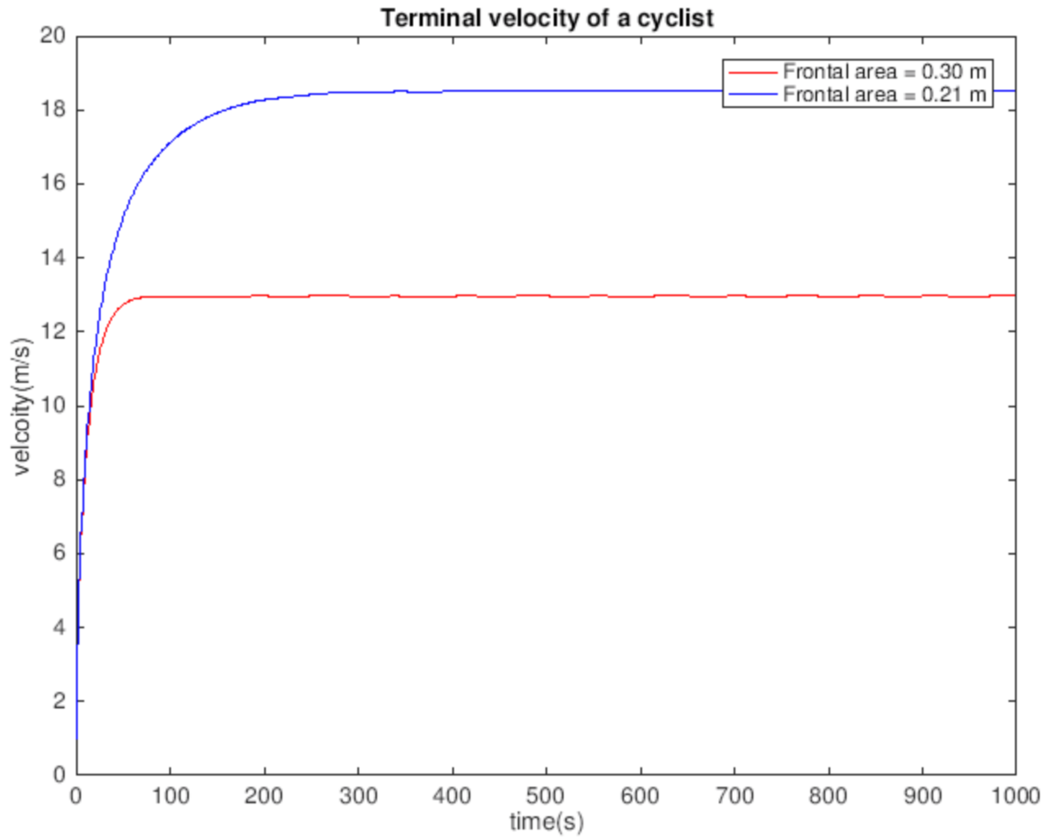
a) Rewrite the bicycle problem/code as discussed in the class. Investigate the effect of rider's power, mass, and frontal area on the ultimate velocity. Generally, for a rider in the middle of a group the effective frontal area is about 30% less than the rider at the front. How much less energy does a rider in the group expend than one at the front (assuming both moving at 12.5 m/s).

Approximations:

- The rider can give a constant power.
- Bicycle wheels are not skidding.
- Cyclist is travelling in a straight line

Governing equations:

- $\frac{dx}{dt} = v$
- $\frac{dv}{dt} = \frac{P}{mv}$ (if no drag is considered)
- $F_{drag} = \frac{\rho A v^2}{2}$ where ρ is the density of air, A is the frontal area



Conclusion:

We see here that the rider attains a Terminal Velocity if drag is present. This is because the more the velocity increases, the more is the retarding drag force, which ultimately brings down the net acceleration to 0. For an average cyclist, this velocity is about 12.5 m/s. Also, since for the rider in the middle of the group, the effective frontal area is 30% less than the rider in the front and if both they are moving with a velocity of 12.5 m/s, the rider in the group must expend 30% less energy than the rider in the front.

b) Run your code (case (a) discussed during class) with initial v=0; observe the output and give possible explanation. Explain why it is important to give a non-zero initial velocity.

- We cannot run the code in (a) with 0 initial velocity. This is because the acceleration would become infinite (a=P/v) and thus non-computable.

c) As discussed in the class, we have assumed that the bicyclist maintains a constant power. What about the assumption when the bicycle has a very small velocity? (instantaneous power=product of force and velocity).

- For very low velocities, this assumption is not valid because to deliver the same power, infinitely large force would be needed to be applied (Humans cannot do this)