Assignment 4

Group 21:

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Que 1: Charge Particle trajectories under Lorentz force: Write a general MATLAB code to reproduce charge particle motions under Lorentz force in the following cases (as shown in Lecture-16 slide, choice of correct initial conditions are important to reproduce the trajectories).

Report on the initial conditions and the rationale behind observed motion. Support your answer with several supporting graphs. (Try different 3D plotting schemes in MATLAB for better visualization, other than "plot3" as discussed in the class). Analyze the motion for t=0 to a reasonable value of t=t_final

i) Static and Uniform E field (for +ve and –ve charges)

Assumption : There is an electric field in Three directions (X-Y-Z)

Initial Condition: Charge particle has 0 initial velocity.

Analytic solution:

Force F;
Charge q = 1.6 x 10^{-19} C;
Electric field E ;

$$F = q * E;$$

$a = F/m;$

$a = \frac{q*E}{m}$

$v_x = a_x * t;$

$v_y = a_y * t;$

$v_z = a_z * t;$

x = $\frac{1}{2} * a_x * t^2;$

y = $\frac{1}{2} * a_y * t^2;$

z = $\frac{1}{2} * a_z * t^2;$

Computational Approach:

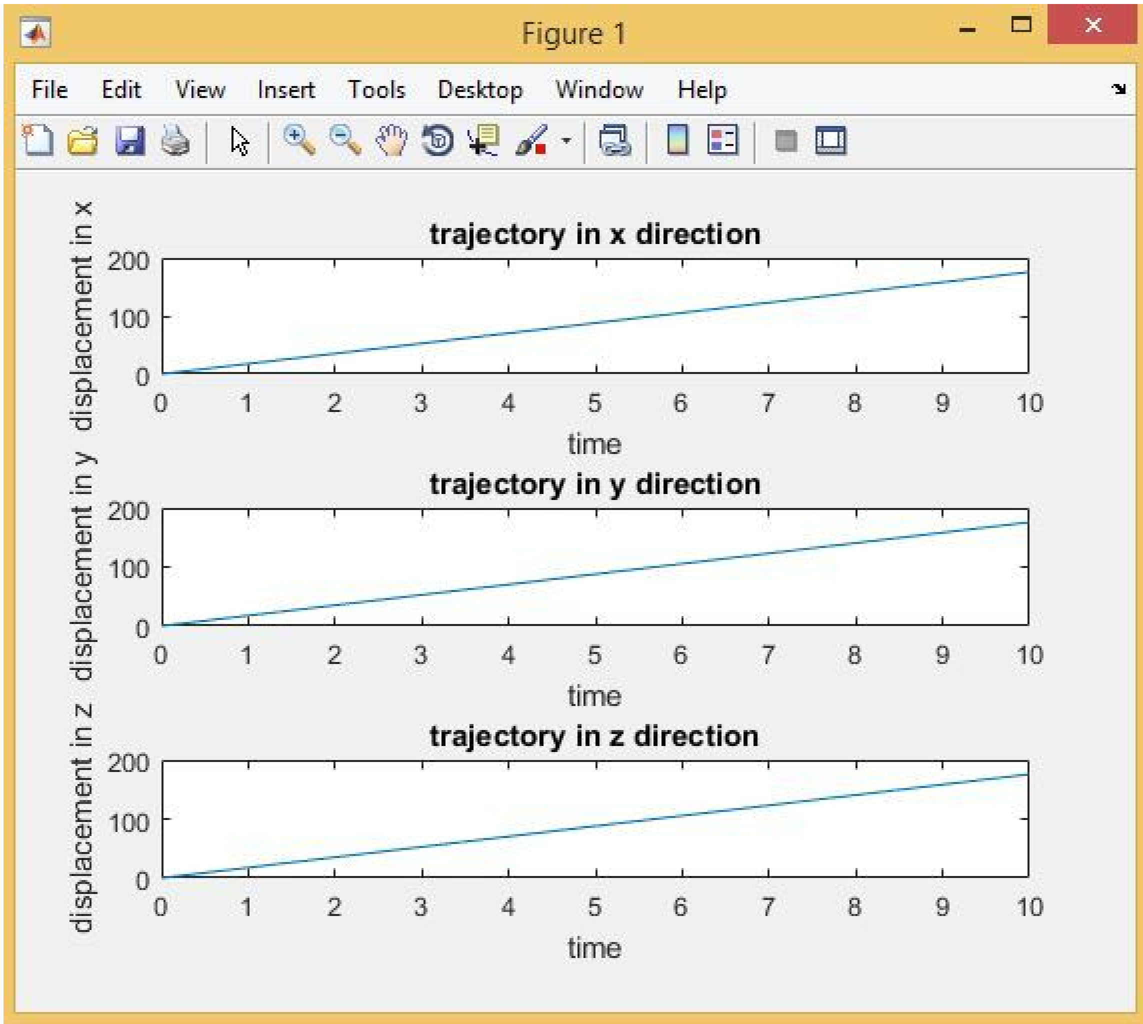
```
f = zeros(3,1);
q = 1.6e-19;
m = 1e-10;

f(1) = cmr*E(1);
f(2) = cmr*E(2);
f(3) = cmr*E(3);
```

Assumption: Charge to mass ratio (q/m) for a charged particle is take in order of 10^0 to 10^4 Because our ODE45 solver works only for the order of 10^6 .So our parameters (acceleration and velocity) should be in range of 10^6 ;

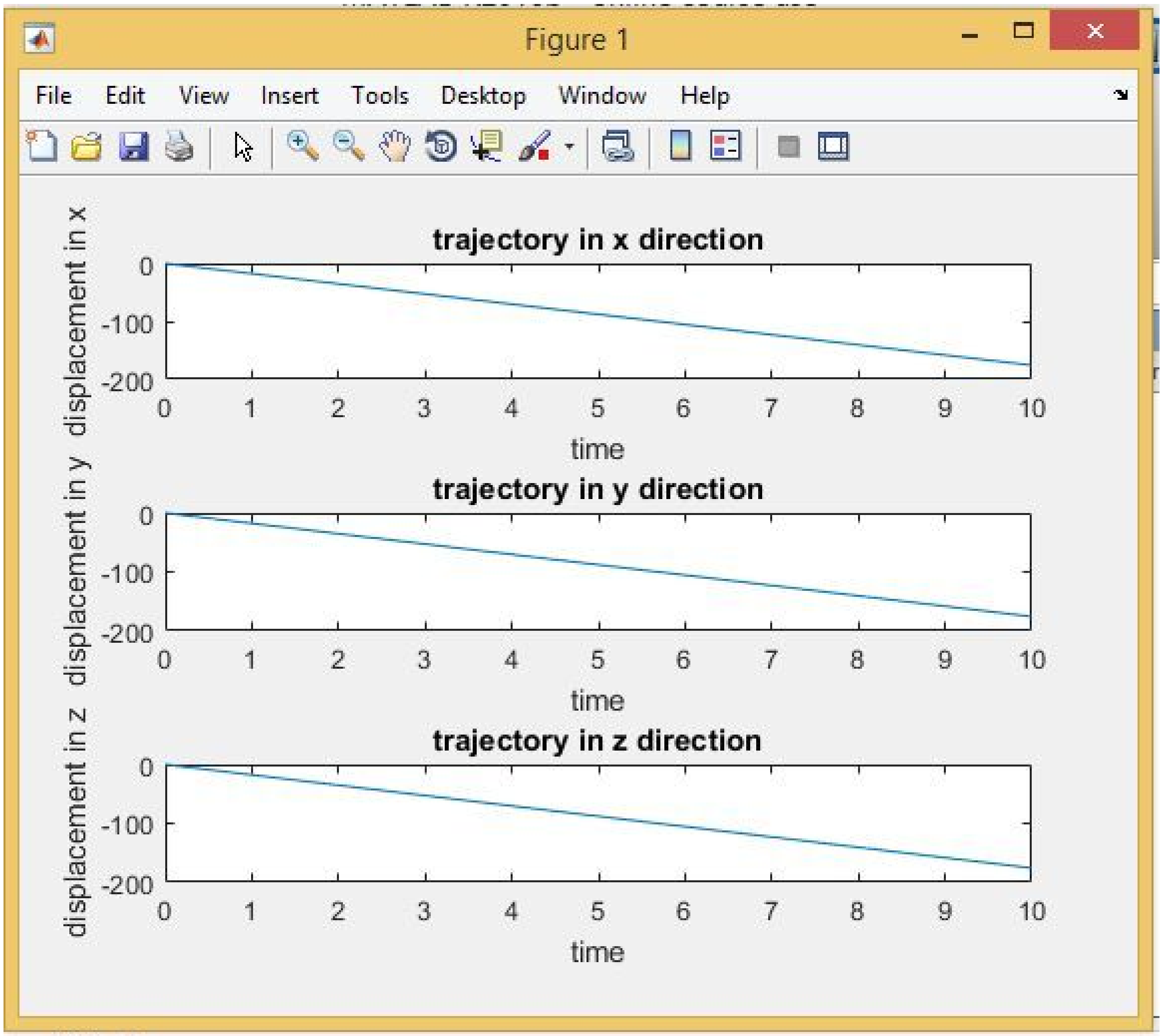
ODE:

Plots:

1) **+ve charge:-**



2) **-ve charge:-**

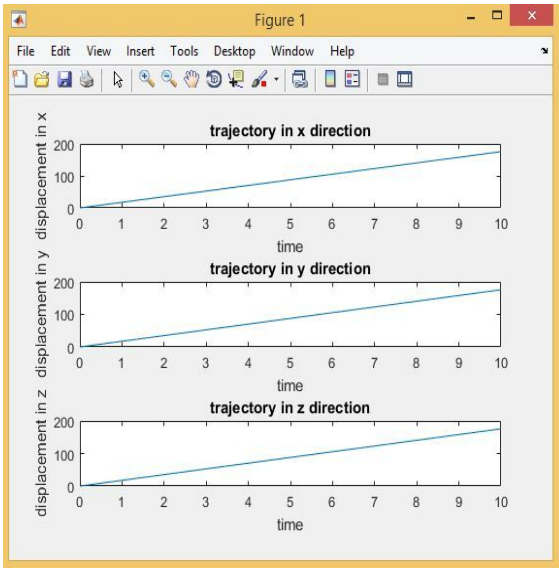


Conclusion :

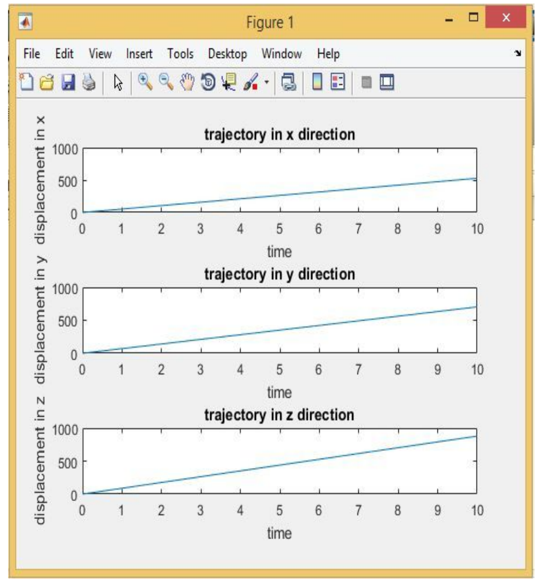
Charge particle will be exerted a force due to presence of Electric field.positive charge will move in direction of Electric field . Motion of negative charge will be opposite to direction of Electric field. Value of force will be dependent on value of charge.

Comparison with initial conditions:-

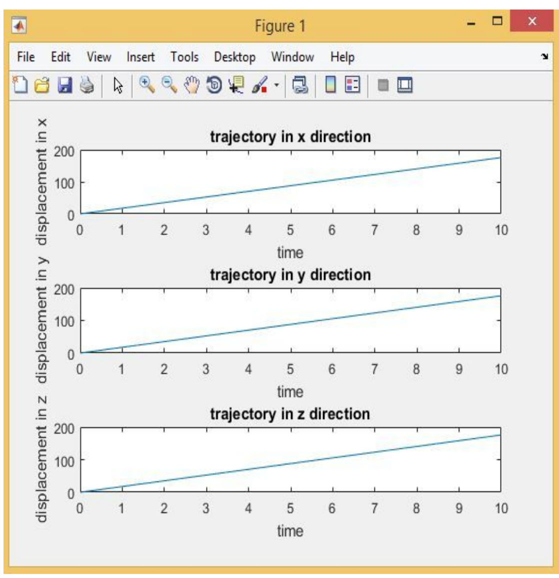
1)Ex=Ey=Ez=10,m=9.1e-20



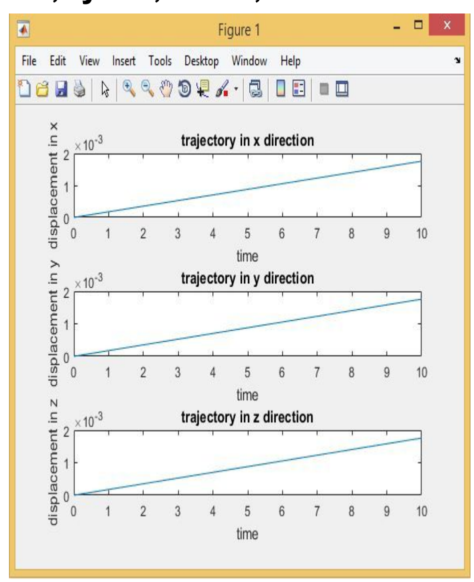
2)Ex=30,Ey=40,Ez=50,m=9.1e-20



1)Ex=Ey=Ez=10,m=9.1e-20



2)Ex=10,Ey=10,Ez=10,m=9.1e-15



ii)Static and uniform B field. (for +ve and –ve charges)

Assumption : There is Static and uniform magnetic field in Z - direction.

Initialization : Charge particle has some initial velocity ($\neq 0$) in X or Y direction

$v_y = 10$;

Analytic solution:

Force F;
Charge q;
Magnetic Field B;

F = q* (**v** x **B**);

a = $\frac{q * (v\ x\ B)}{m}$;

a_x = q * ($v_y * B_z - v_z * B_y$)/m;

a_y = q * ($v_x * B_z - v_z * B_x$)/m;

$a_z = 0$;

$v_x = a_x * t$;

$v_y = a_y * t$;

$v_z = a_z * t$;

x = $\frac{1}{2} * a_x * t^2$;

y = $\frac{1}{2} * a_y * t^2$;

z = $\frac{1}{2} * a_z * t^2$;

Computational Approach:

Assumption: Charge to mass ratio (q/m) for a charged particle is take in order of 10^0 to 10^4 Because our ODE45 solver works only for the order of 10^6.So our parameters (acceleration and velocity) should be in range of 10^6;

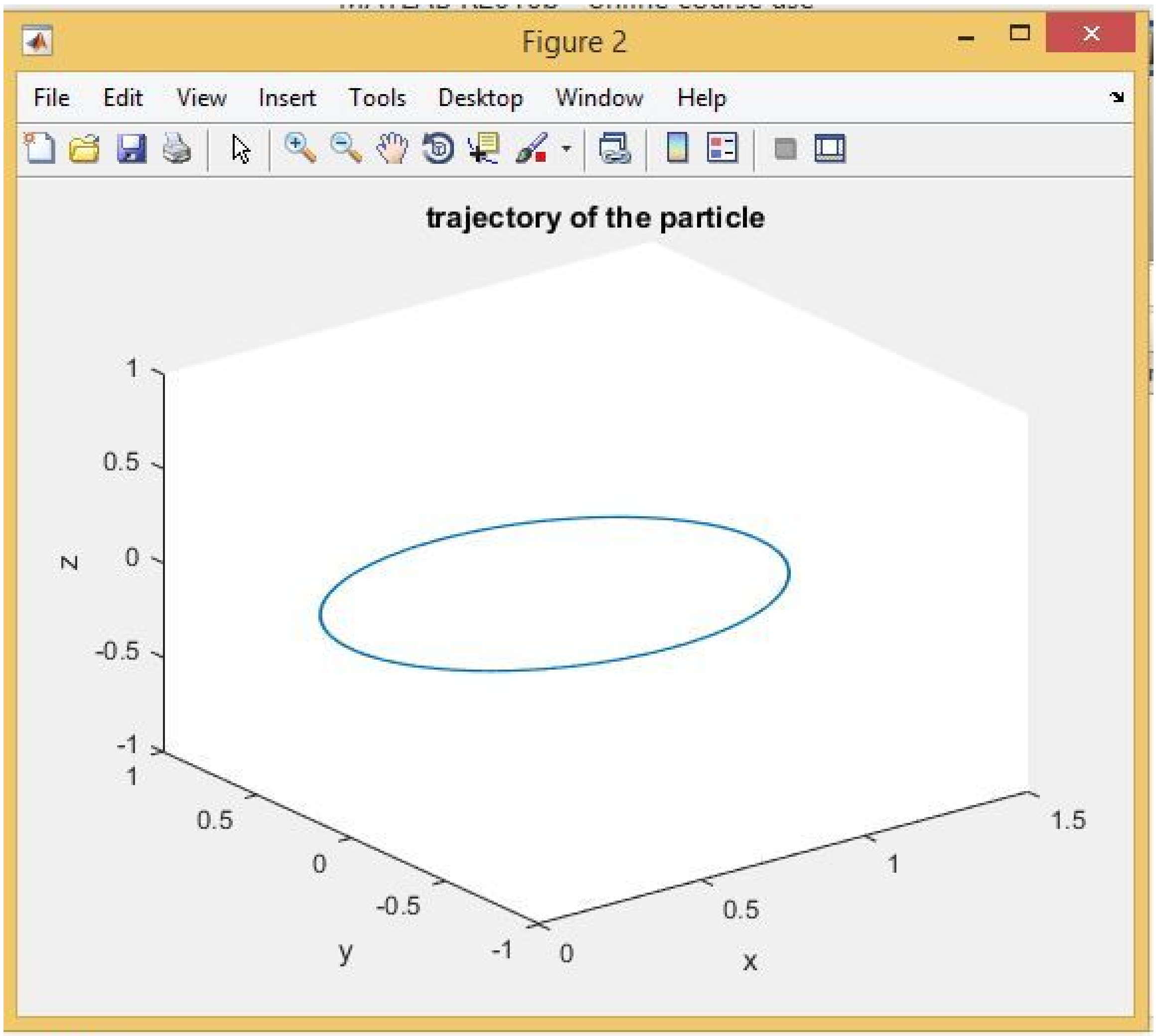
RHS:

```
f = zeros(6,1);
q = 1.6e-19;
m = 1e-19;

f(1) = cmr*(u(2)*B(3)-B(2)*u(3));
f(2) =  -cmr*(u(1)*B(3)-B(1)*u(3));
f(3) =  cmr*(u(1)*B(2)-B(1)*u(2));
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

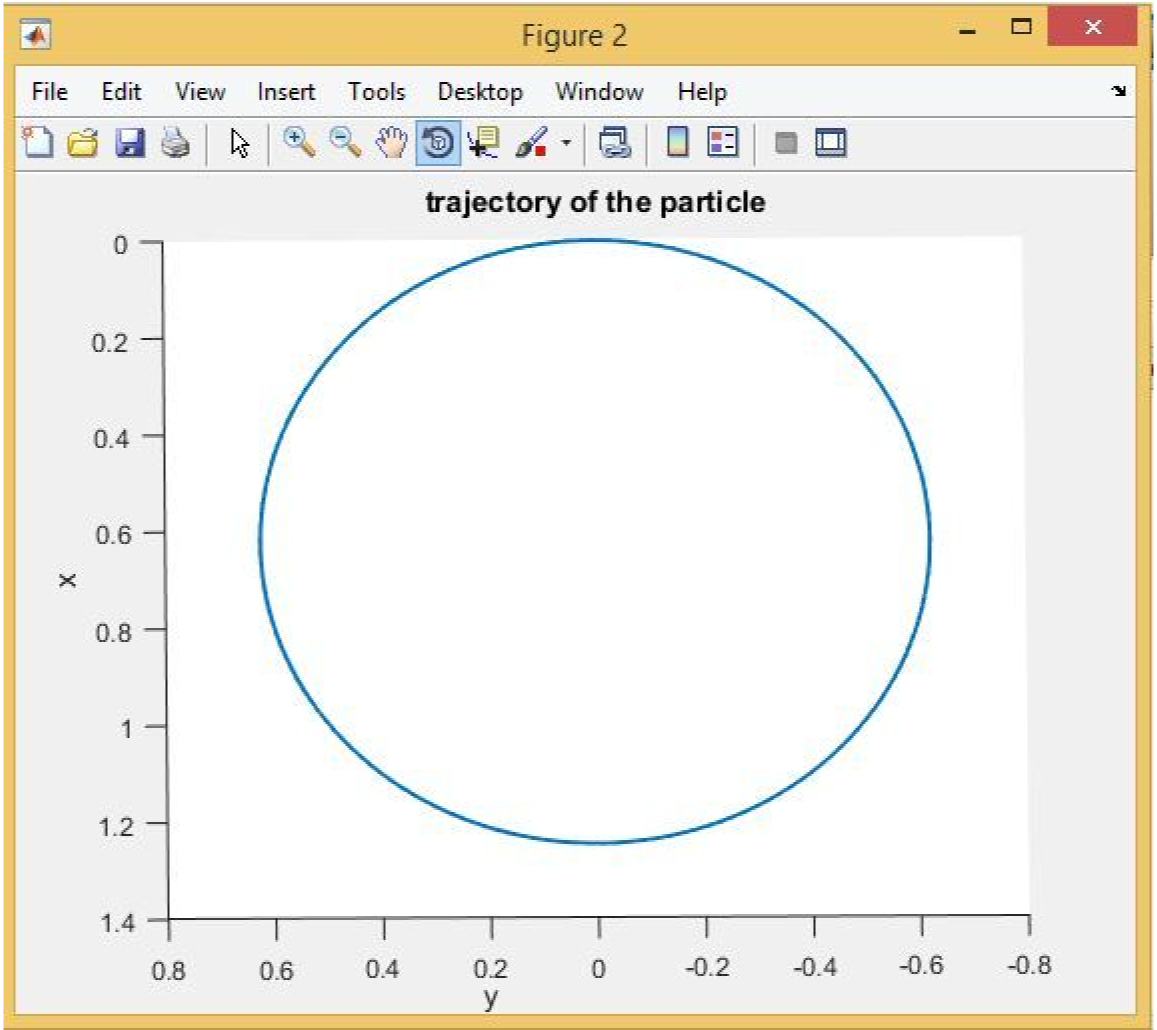
Plots:

1) **+ve particle:**

Trajectory of particle in 3-D:

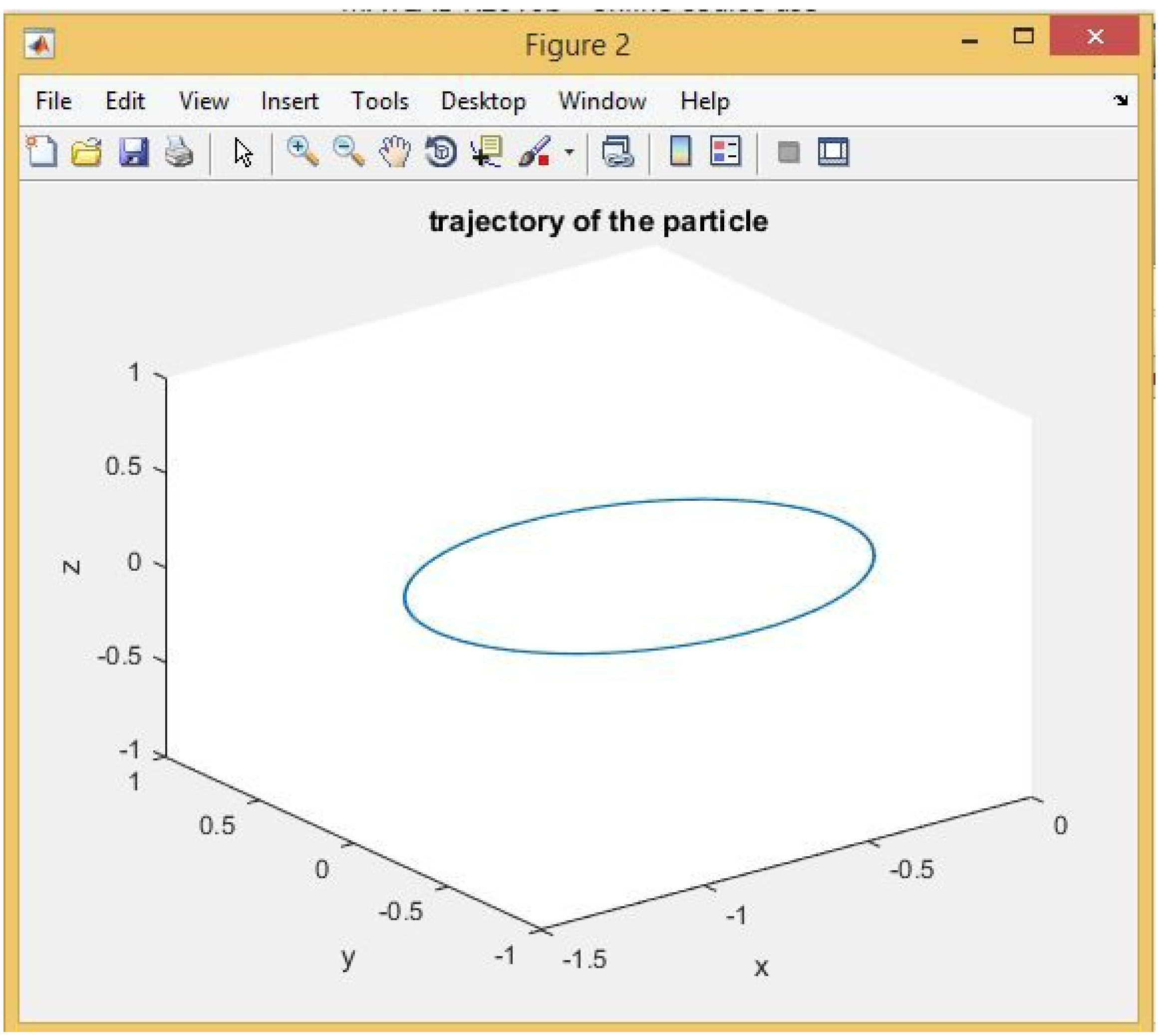


Trajectory view in 2-D:

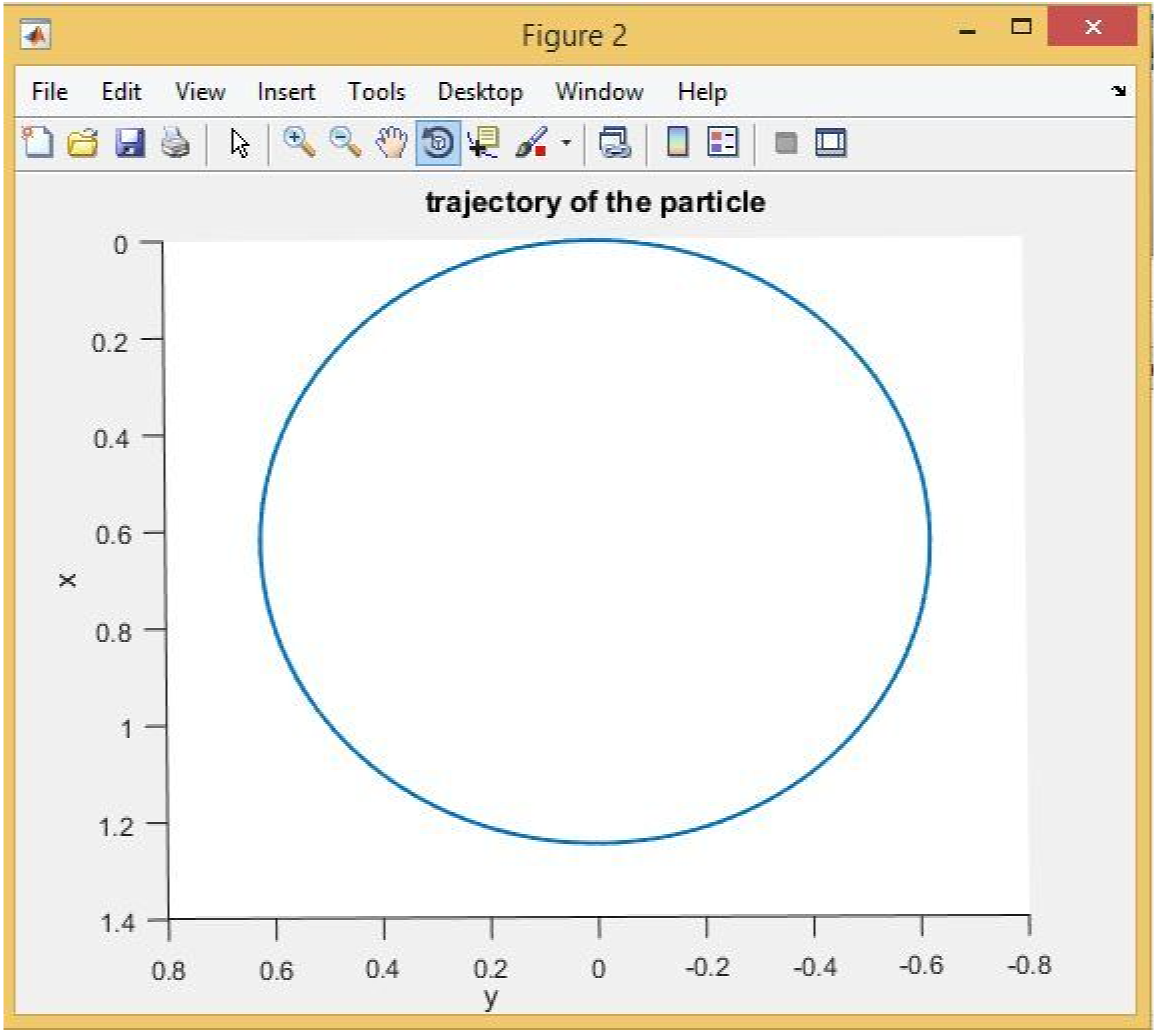


2) -ve charge:

Trajectory of particle in 3-D:



Trajectory view in 2-D:



Conclusion :

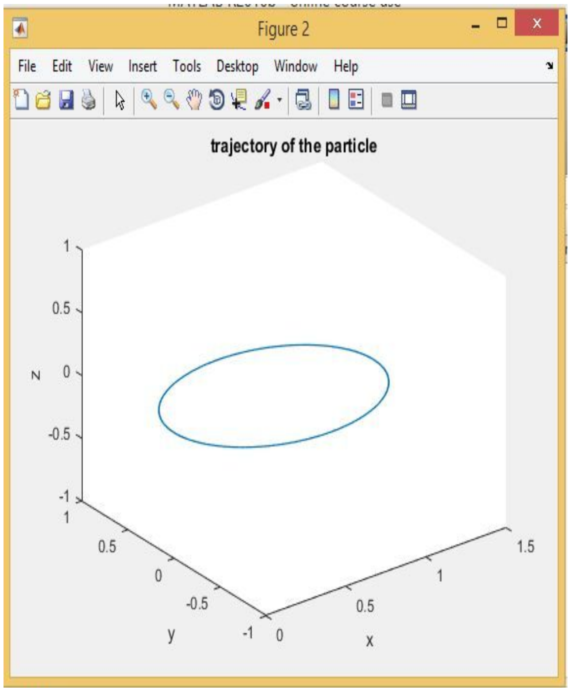
Particle has initial velocity in Y direction. So force will be exerted on the particle due to presence of Magnetic field in Z direction . Velocity and Field are mutually perpendicular to each other so force will also perpendicular to motion. And this will create circular motion of charge particle in plane perpendicular to field.

Change in value of initial velocity change the radius of circular motion but will not change the frequency of motion.Radius increases as increment in velocity and vice versa.

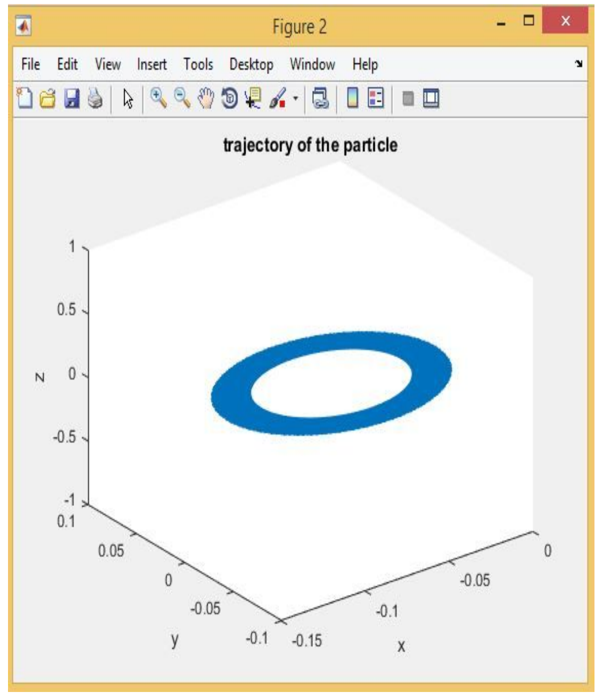
Change in value of charge and mass will change the value of frequency.

Comparison in initial value:

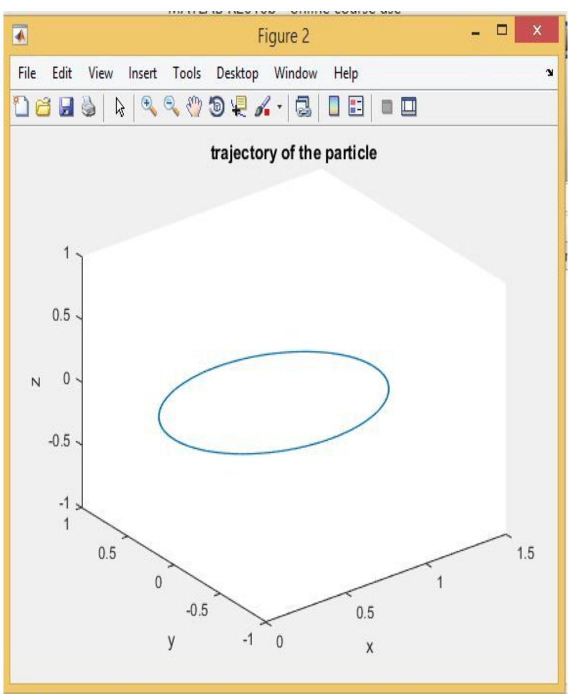
1)Bz=10,vx=10



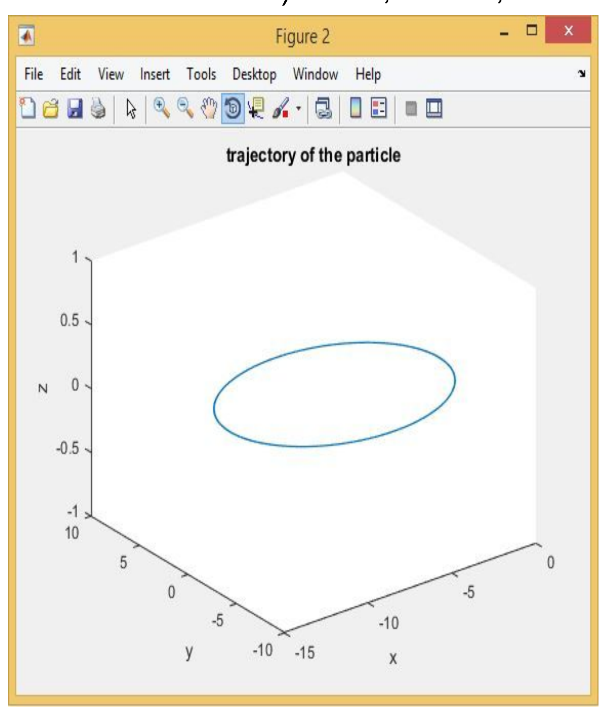
2)Bz=100,vx=10;



1)Bz=10,vx=10



2)Bz=10,vx=100;



iii) Static and uniform E and B. (ExB drift) (what happens when v=v0x; B=B0z; and E=E0y; and v0=E0/B0).

Assumptions: There is Static and uniform electric field E in Y direction and Static and uniform Magnetic field B in Z direction and Charge particle is moving in X direction.

Initialization : charge particle is initially moving in X direction with some finite velocity $v_x = v_{x0}$.

Analytic Solution:

Force **F**;
Charge q;
electric field **E**;
Magnetic field **B**;

F = q*__E__ + q*(**v*B**);

a = (q*__E__ + q*(**v x B**))/m;

a_x= q*(E_x **+** $v_y * B_z - v_z * B_y$)/m;

a_y= q*(E_y **+** $v_z * B_x - v_x * B_z$)/m;

a_z= q*(E_z **+** $v_y * B_x - v_x * B_y$)/m;

$v_x = a_x * t;$

$v_y = a_y * t;$

$v_z = a_z * t;$

x = $\frac{1}{2} * a_x * t^2;$

y = $\frac{1}{2} * a_y * t^2;$

z = $\frac{1}{2} * a_z * t^2$;

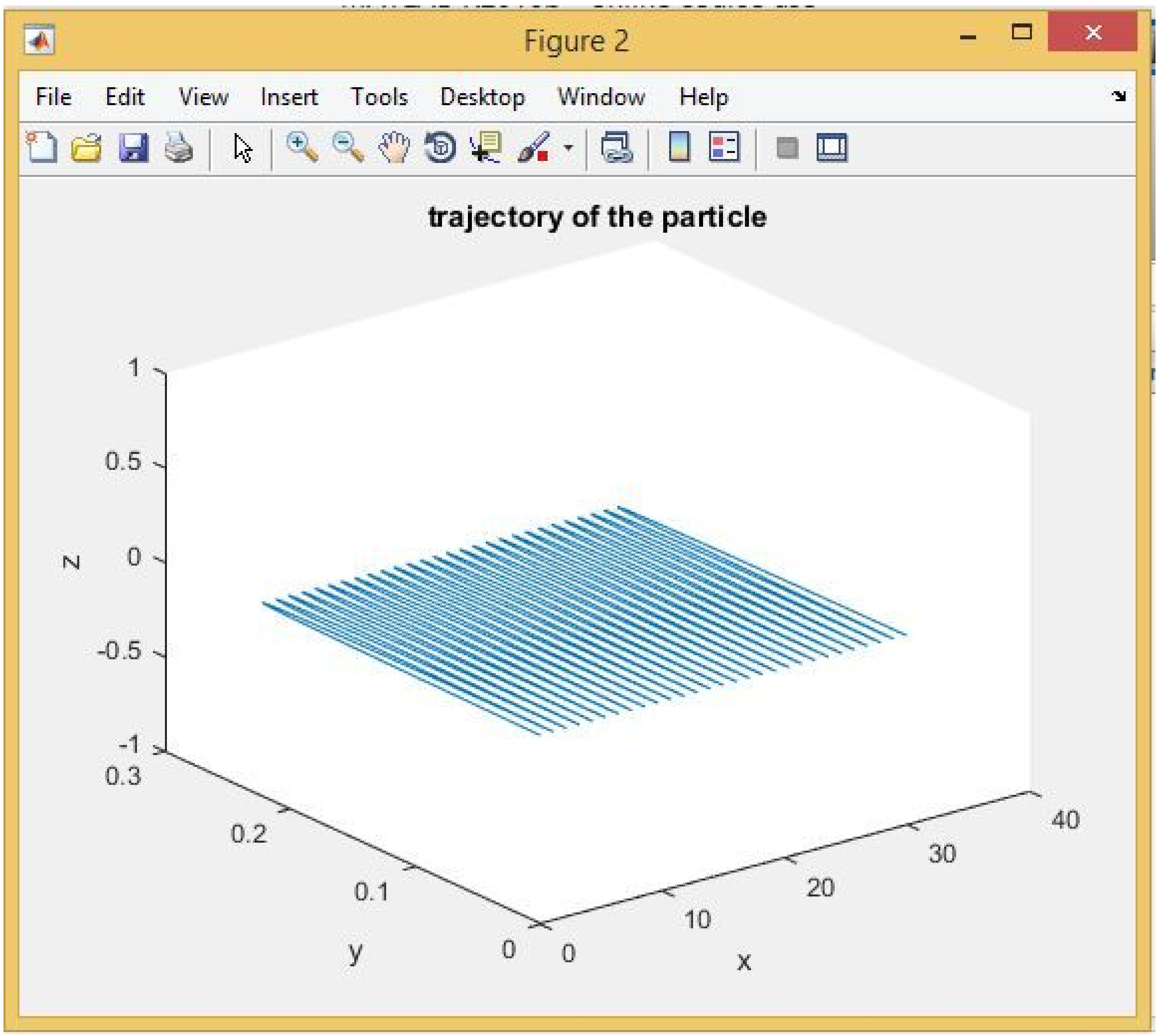
Computational Approach:

Assumption: Charge to mass ratio (q/m) for a charged particle is take in order of 10^0 to 10^4 Because our ODE45 solver works only for the order of 10^6.So our parameters (acceleration and velocity) should be in range of 10^6;

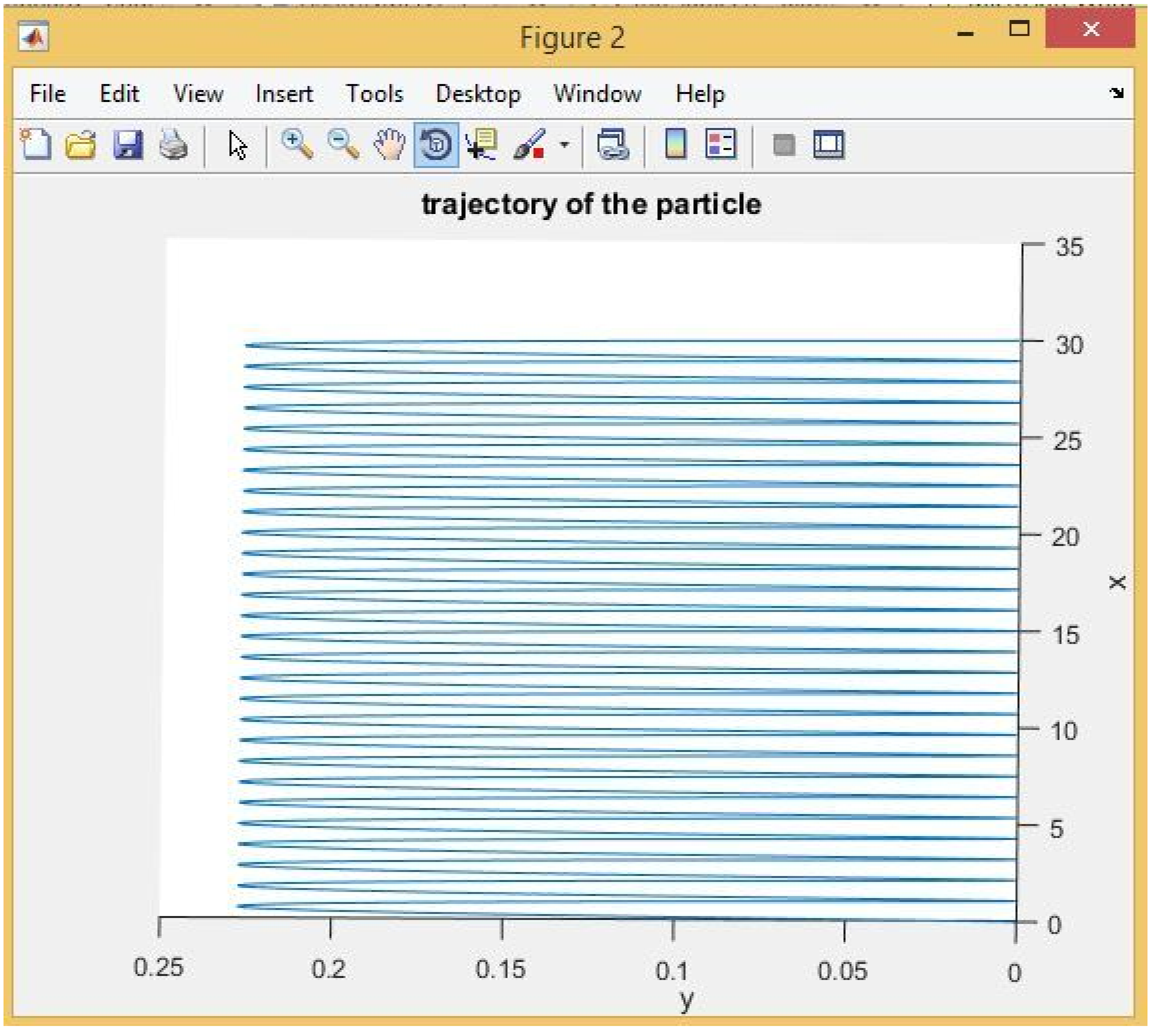
```
f = zeros(6,1);
q = 1.6e-19;
m = 1e-19;


f(1) = cmr*(E(1)+(u(2)*B(3)-B(2)*u(3)));
f(2) =  cmr*(E(2)-(u(1)*B(3)-B(1)*u(3)));
f(3) =  cmr*(E(3)+(u(1)*B(2)-B(1)*u(2)));
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

Plots:

Trajectory of particle in 3-D:



Drift in 2-D (x-y plane):-

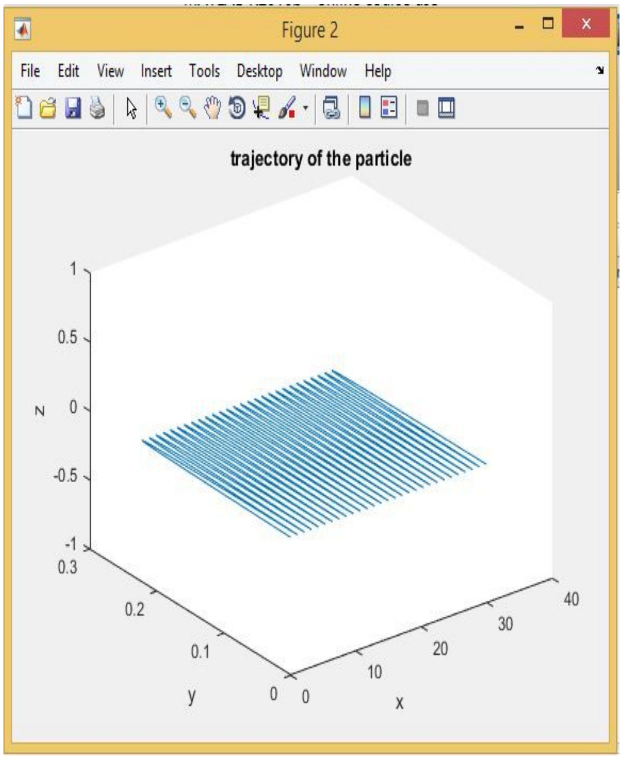


Conclusion:

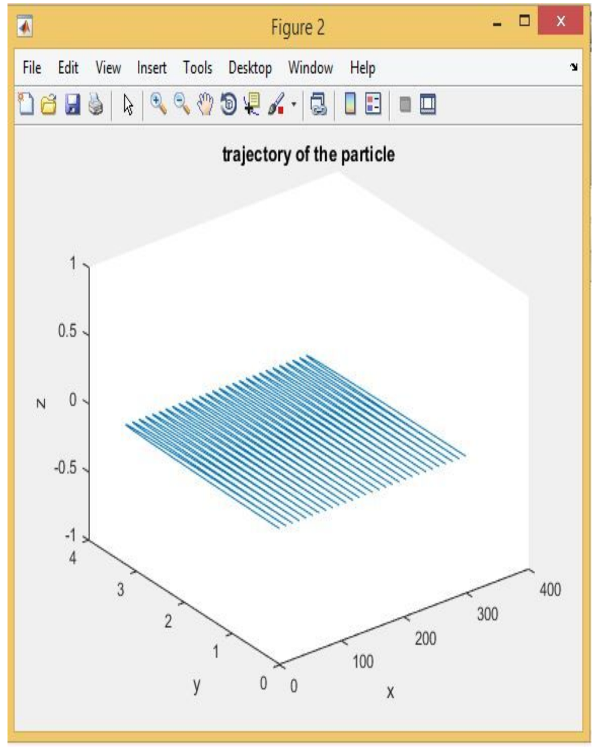
The force on charge particle acts due to Electric and Magnetic field. Presence of Magnetic field creates circular motion in plane perpendicular to field . Whereas presence of Electric field cause drift in motion in same plane. This drift will be in direction perpendicular to both fields.Drift velocity depends upon both fields , electric and magnetic.

Comparison for different initial conditions:

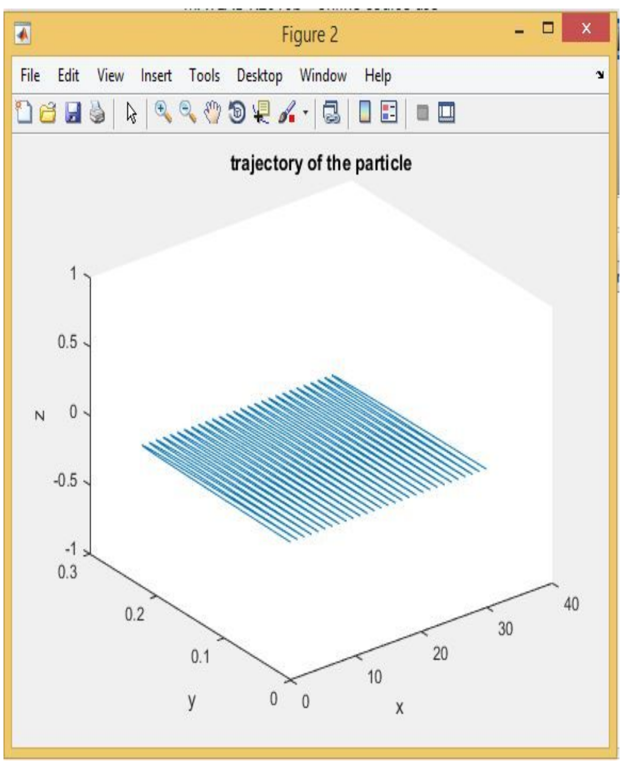
1)Bz=10,Ey=30,vx=1;



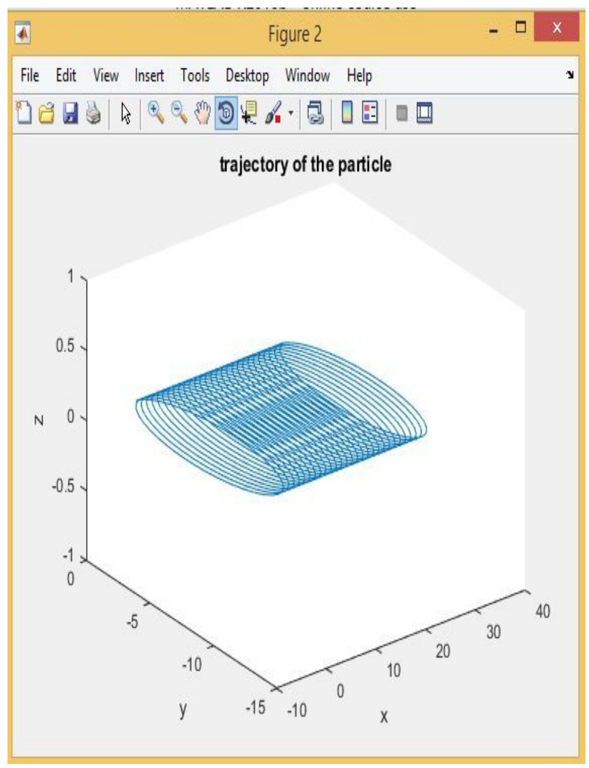
2)Bz=10, Ey=300,vx=1;



1)Bz=10,Ey=30,vx=1;



2)Bz=10, Ey=30,vx=100;



iv) Static and uniform B, and under gravitational force (for different mass)

Assumptions: There Static and uniform Magnetic Field in Z direction and constant gravitational force in Y direction downwards.The value of gravitational acceleration (g) is constant.

Initial conditions : Charge particle is moving in X direction with $v_x =$ 10 m/s.

Analytic solution:

Force **F**;
Charge q;
Magnetic field **B**;
Gravitational acceleration g = 9.8 m/s^2;
Gravitational Force F_g= m*g;

a_x= q*($v_y * B_z - v_z * B_y$)/m ;

a_y= q*($v_z * B_x - v_x * B_z$)/m - g;

a_z= q*($v_y * B_x - v_x * B_y$)/m;

$v_x = a_x * t;$

$v_y = a_y * t;$

$v_z = a_z * t;$

x = $\frac{1}{2} * a_x * t^2;$

y = $\frac{1}{2} * a_y * t^2;$

z = $\frac{1}{2} * a_z * t^2;$

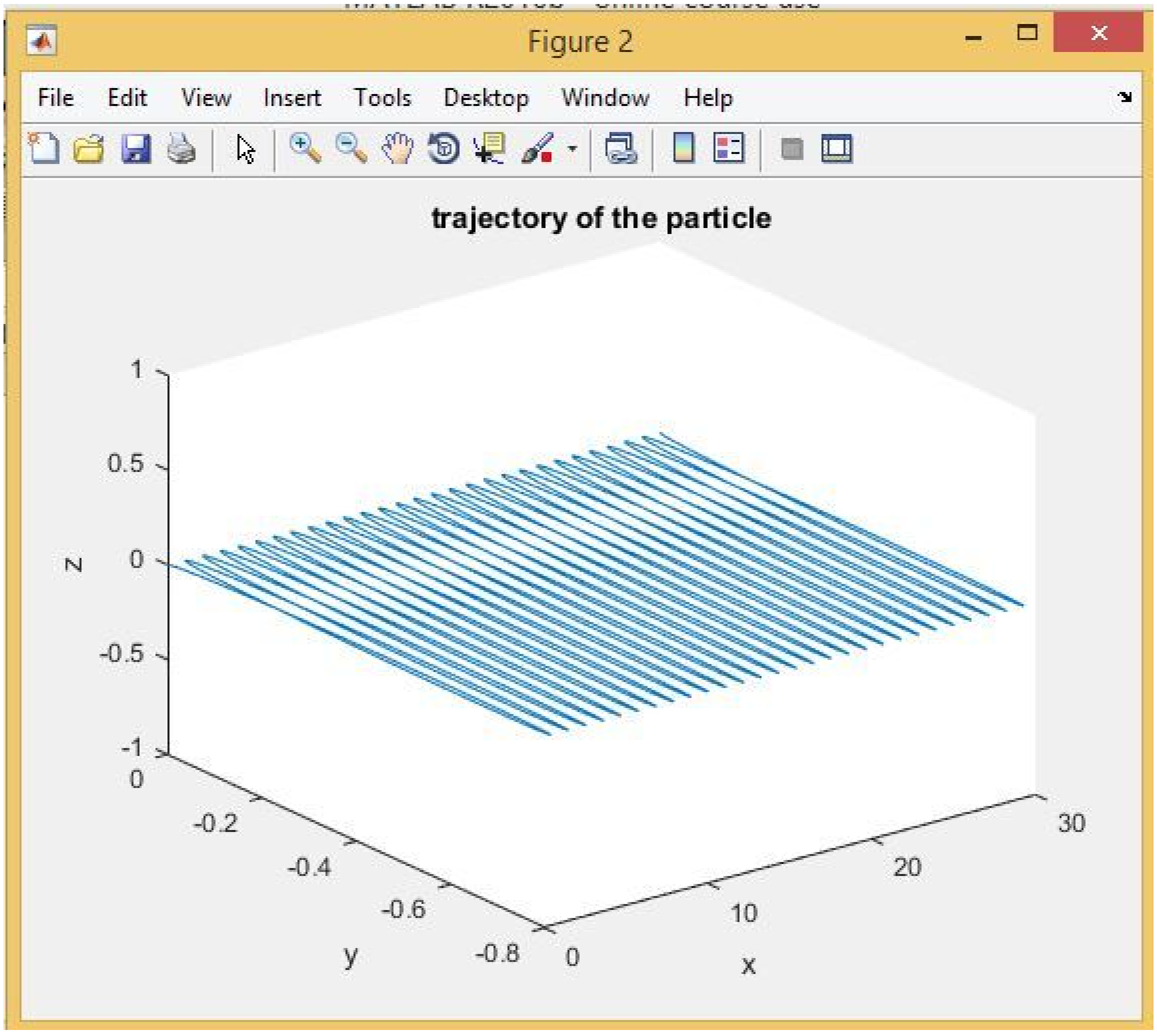
Computational Approach:

Assumption: Charge to mass ratio (q/m) for a charged particle is take in order of 10^0 to 10^4 Because our ODE45 solver works only for the order of 10^6.So our parameters (acceleration and velocity) should be in range of 10^6

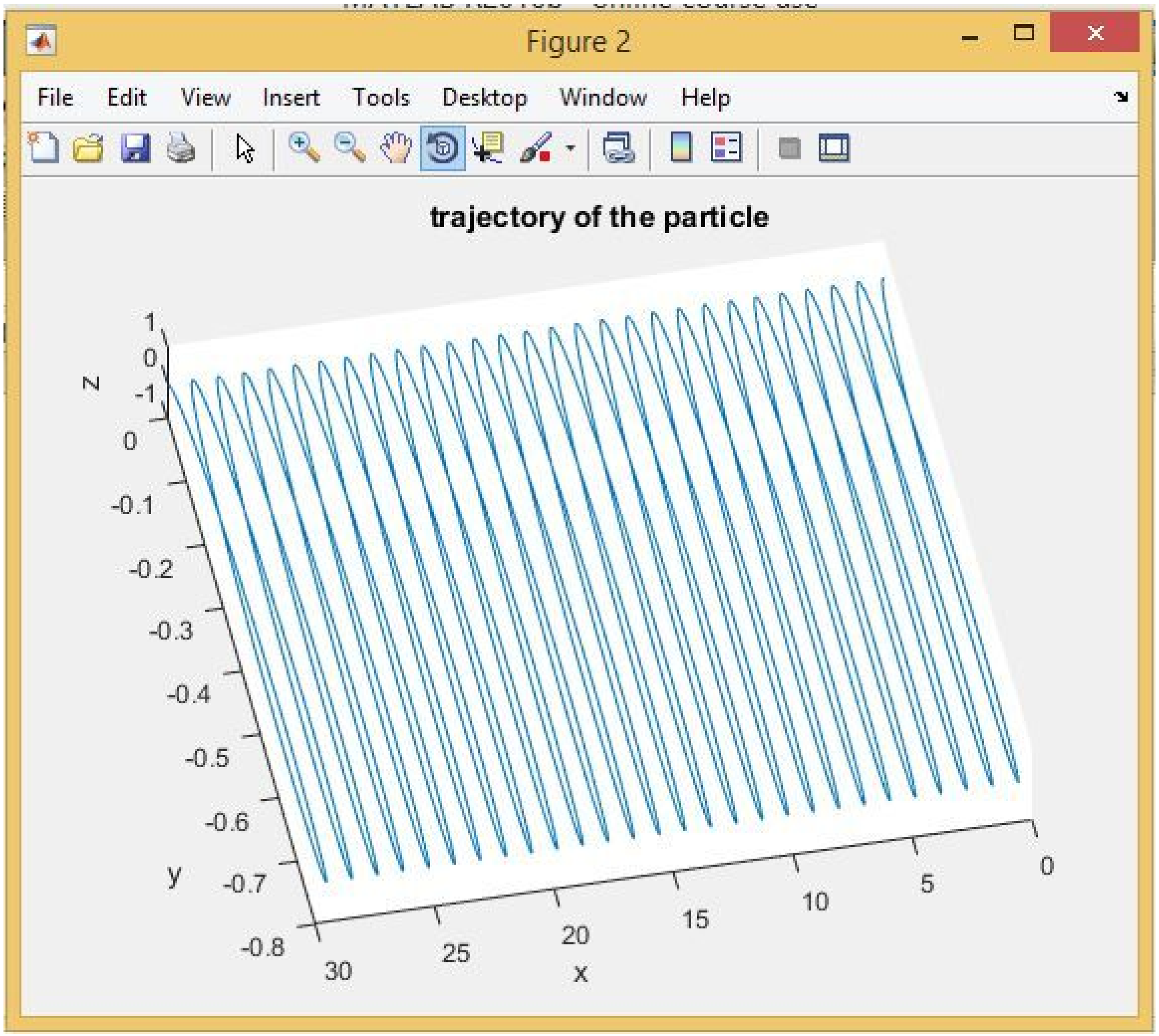
```
f = zeros(6,1);
q = 1.6e-19;
m = 1e-19;


f(1) = cmr*(u(2)*B(3)-B(2)*u(3));
f(2) =  (-cmr*(u(1)*B(3)-B(1)*u(3)))-g;
f(3) =  cmr*(u(1)*B(2)-B(1)*u(2));
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

Plots:

Trajectory of particle in 3-D:



Drift in 2-D due to gravitational force:-



Conclusion:

Here presence of magnetic field cause circular motion in plane perpendicular to it(X-Y plane) and presence of gravitational force(Y direction) will cause drift in motion in X direction.

If we change the value of mass , drift velocity changes accordingly**.**

2. i) . Compute with your matlab code, the cyclotron frequency and the cyclotron radius for – an electron in the Earth's ionosphere at 300 km altitude, where the magnetic flux density B~.00005 Tesla, considering that the electron moves at the thermal velocity (kT/m), with T=1000 K, where "k" is Boltzmann's constant. Plot a graph to show the motion/results and compare your results with analytical calculations.

Assumptions:

Charge to mass ratio (q/m) for a charged particle is take in order of 10^0 to 10^4 Because our ODE45 solver works only for the order of 10^6 .So our parameters (acceleration and velocity) should be in range of 10^6

Initial condition: Initially, charged particle is moving with v_x velocity in x direction.

Analytic Solution :

Force **F**;
Charge q;
Magnetic field **B**;

Centrifugal force can be given by

F = q*v*B;

$\frac{m*v^2}{R}$ = q*v*B;

R = $\frac{m*v}{q*B}$;

v = r* ω;

$m*r*\omega^2 = q*v*B;$

$\omega = q * B / m;$

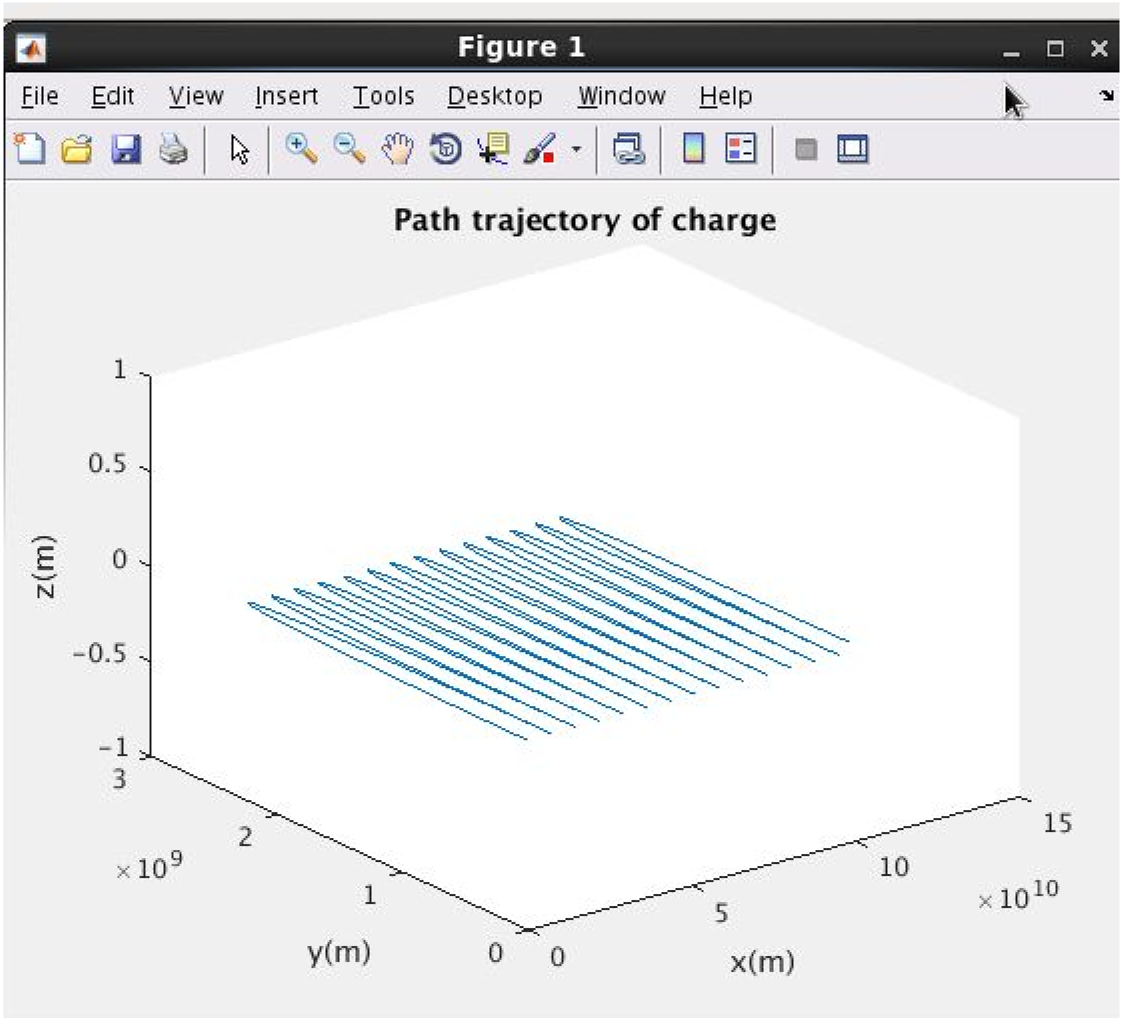
Computational Approach:

For electron charge by mass ratio (q/m) is in order of 10^{12} for which our ODE45 will not work . So here we have taken q/m ratio in order of 10^1 - 10^2 . So now our ODE parameters(acceleration and velocity) are of order of 10^6 .

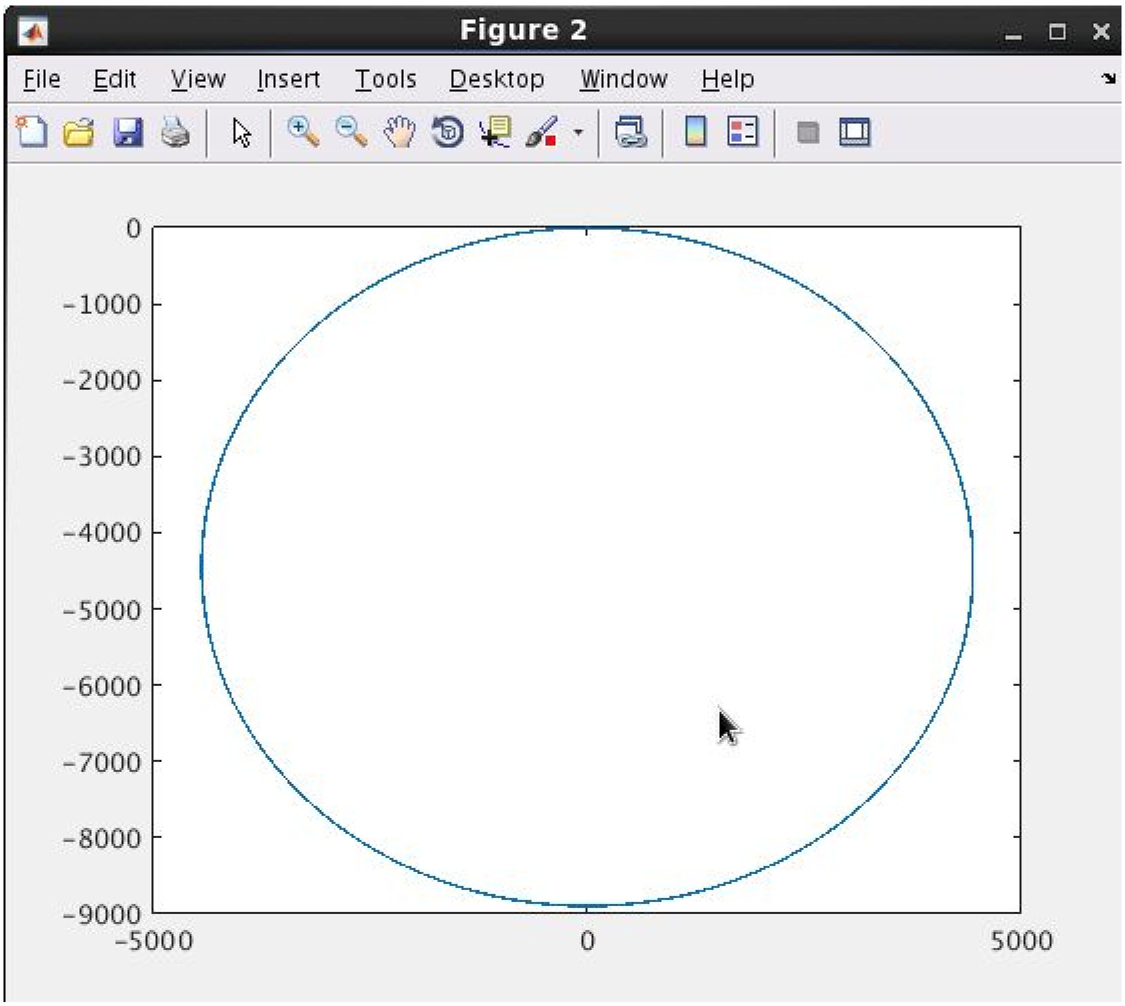
```
f = zeros(6,1);
q = -1.6e-19;
m = 9.1e-20;

f(1) = cmr*(u(2)*B(3)-B(2)*u(3));
f(2) = cmr*(u(1)*B(3)-B(1)*u(3))-g;
f(3) = cmr*(u(1)*B(2)-B(1)*u(2));
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

Plots:

Trajectory of particle in 3-D:



Trajectory in 2-D:-



Conclusion:

Particle has initial velocity in x direction. So force will be exerted on the particle due to presence of Magnetic field in y direction . Velocity and Field are mutually perpendicular to each other so force will also perpendicular to motion. And this will create circular motion of charged particle in plane perpendicular to field.

Change in value of initial velocity change the radius of circular motion but will not change the frequency of motion.Radius increases as increment in velocity and vice versa.

Change in value of charge and mass will change the value of frequency.

In this case frequency will be very small so to analyze the motion we have to time period in order of 10^6 - 10^7 and as time steps according to it.So we can analyze the motion.

ii)What will be the effect of gravitational drift velocity “vg” in this case?

Assumptions: Here in Spherical coordinates gravitational force is in r direction downwards.change in value of g is negligible.

Initialization: Initially charged particle is moving with v_{x0} velocity in θ direction.

Analytic solution:

Force **F**;
Charge q;
Magnetic field **B**;
Gravitational acceleration g = 9.8 m/s^2;
Gravitational Force F_g = m*g;

a_r = q*($v_\theta * B_\phi - v_\phi * B_\theta$)/m-g ;

a_θ = q*($v_\phi * B_r - v_r * B_\phi$)/m;

a_ϕ = q*($v_\theta * B_r - v_r * B_\theta$)/m;

$v_r = a_r * t;$

$v_\theta = a_\theta * t;$

$v_\phi = a_\phi * t;$

r = $\frac{1}{2} * a_r * t^2$;

θ = $\frac{1}{2} * a_\theta * t^2$;

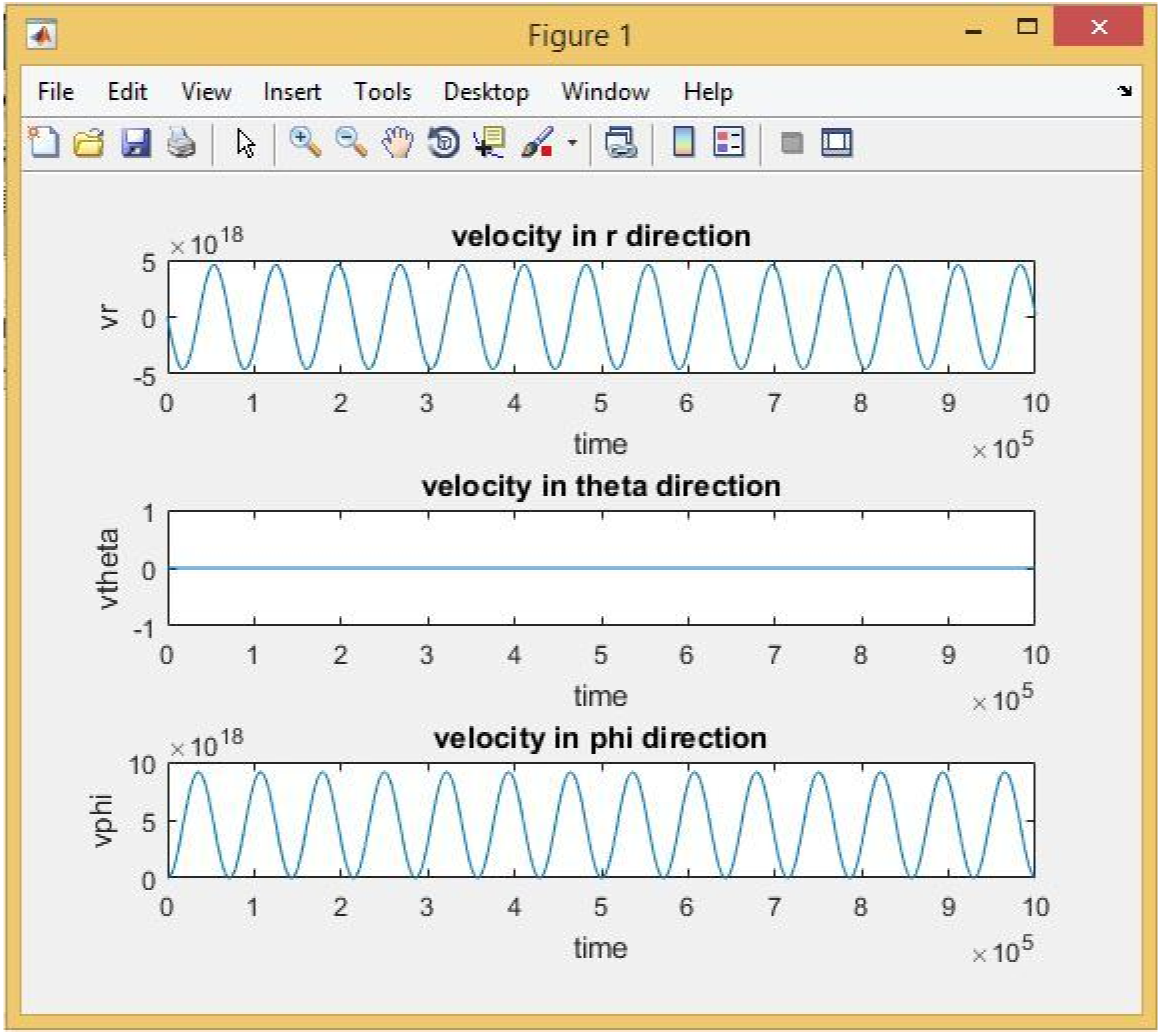
ϕ = $\frac{1}{2} * a_\phi * t^2$;

Computational Approach:

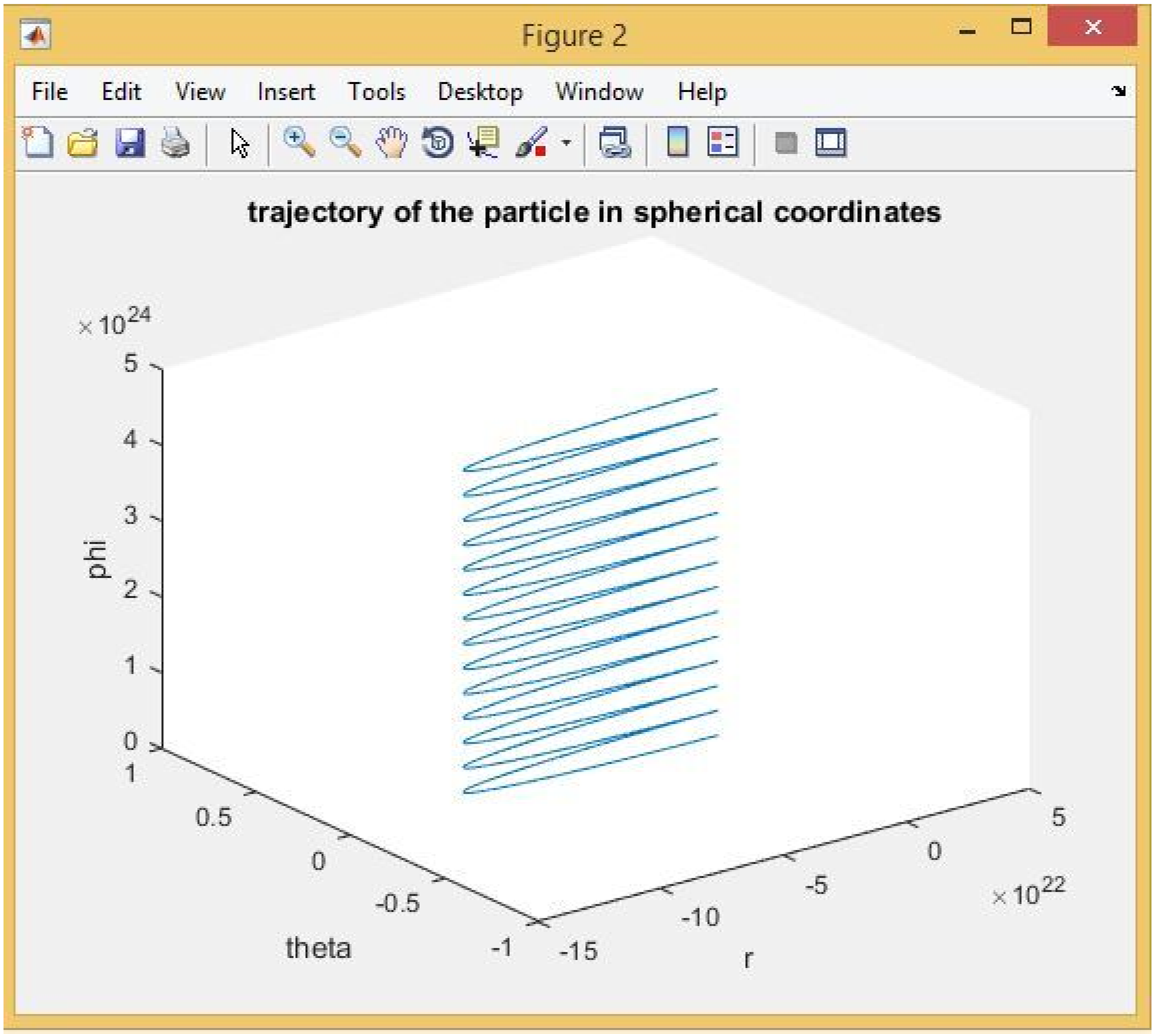
```
f = zeros(6,1);
q = -1.6e-19;
m = 9.1e-20;
g=.988;

f(1) = (q*(u(2)*B(3)-B(2)*u(3))/m)-(g*((R*R)/((R+u(4))*(R+u(4)))));
f(2) =  -q*(u(1)*B(3)-B(1)*u(3))/m;
f(3) =  q*(u(1)*B(2)-B(1)*u(2))/m;
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

Plots:

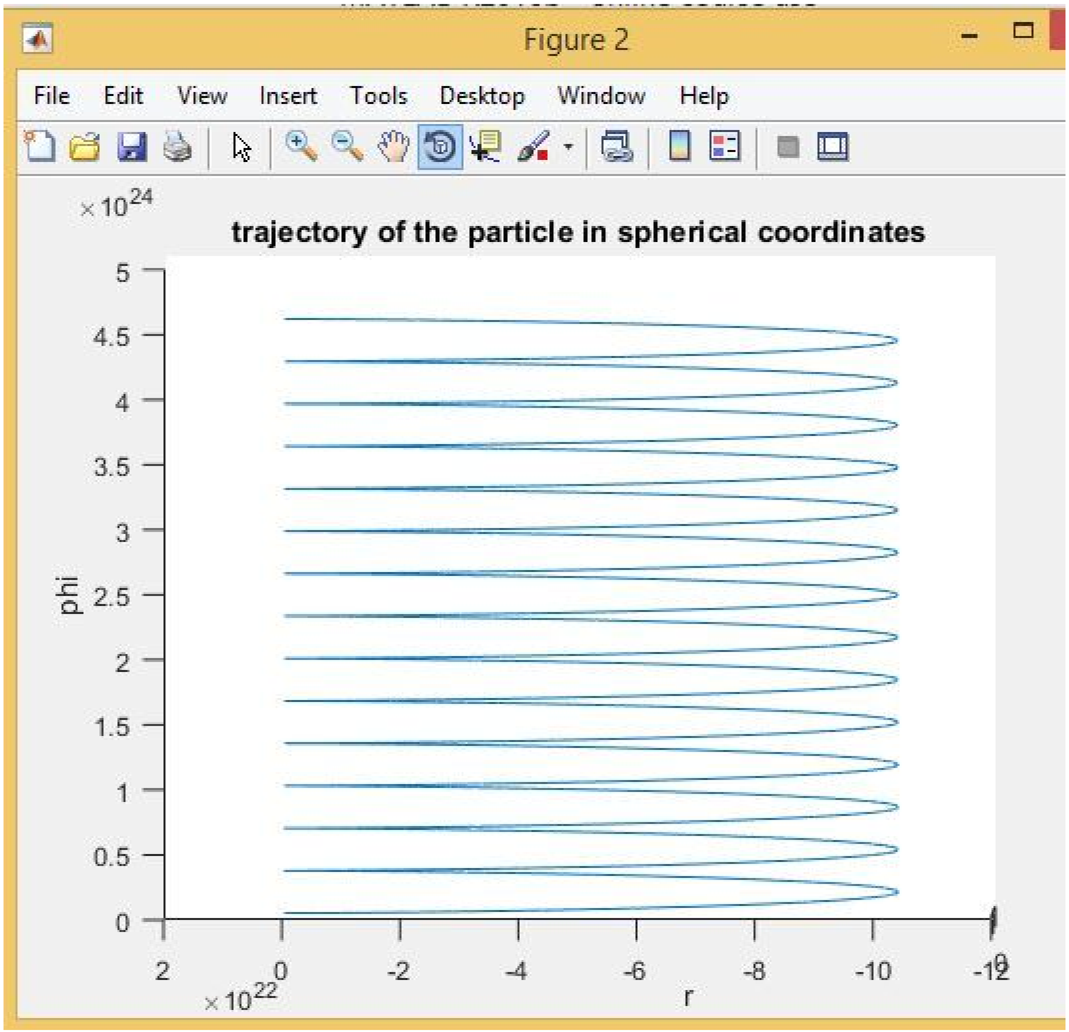
1) velocity in three directions:-



2)trajectory of particle in 3-D:-



3) drift in 2-D due to gravitational force:-



Conclusion:

Here presence of gravity cause drift in the motion in θ direction.the value of drift velocity depend upon the value of g and magnetic field.Also It depends upon the value of charge but doesn't depend on the value of mass. Here again in order to analyze the motion of particle we took time period in order of $10^6 - 10^7$.

3) Modify the program to describe the motion of a charged particle in an oscillating electric field. Consider a uniform alternating electric field in x-direction E=Ex0 sin (omega t) Angular freq Omega= 2 (pi) (frequency) Amplitude Ex0 Plot the trajectory of the particle in time.

What happens when the particle has non-zero initial velocity in x or y direction.

Assumptions: There is static and uniform Magnetic field in Z direction. And there is oscillatory Electric field in X direction.Particle has initial velocity in Y direction.We have taken 2 cases,in first we have taken oscillatory frequency high compared to frequency of motion of charged particle . Whereas in second case we have taken same oscillatory frequency as frequency of motion of charged particle .

Initial conditions: Particle has initial velocity in Y direction.

Analytic Approach :

Force **F**;
Charge q;
Magnetic field **B**;

a_x= q*($v_y * B_z - v_z * B_y$)/m -q* $E_0 * sin(\omega * t)$ /m;

a_y= q*($v_z * B_x - v_x * B_z$)/m ;

a_z= q*($v_y * B_x - v_x * B_y$)/m;

$v_x = a_x * t;$

$v_y = a_y * t;$

$v_z = a_z * t;$

x = $\frac{1}{2} * a_x * t^2;$

y = $\frac{1}{2} * a_y * t^2;$

z = $\frac{1}{2} * a_z * t^2;$

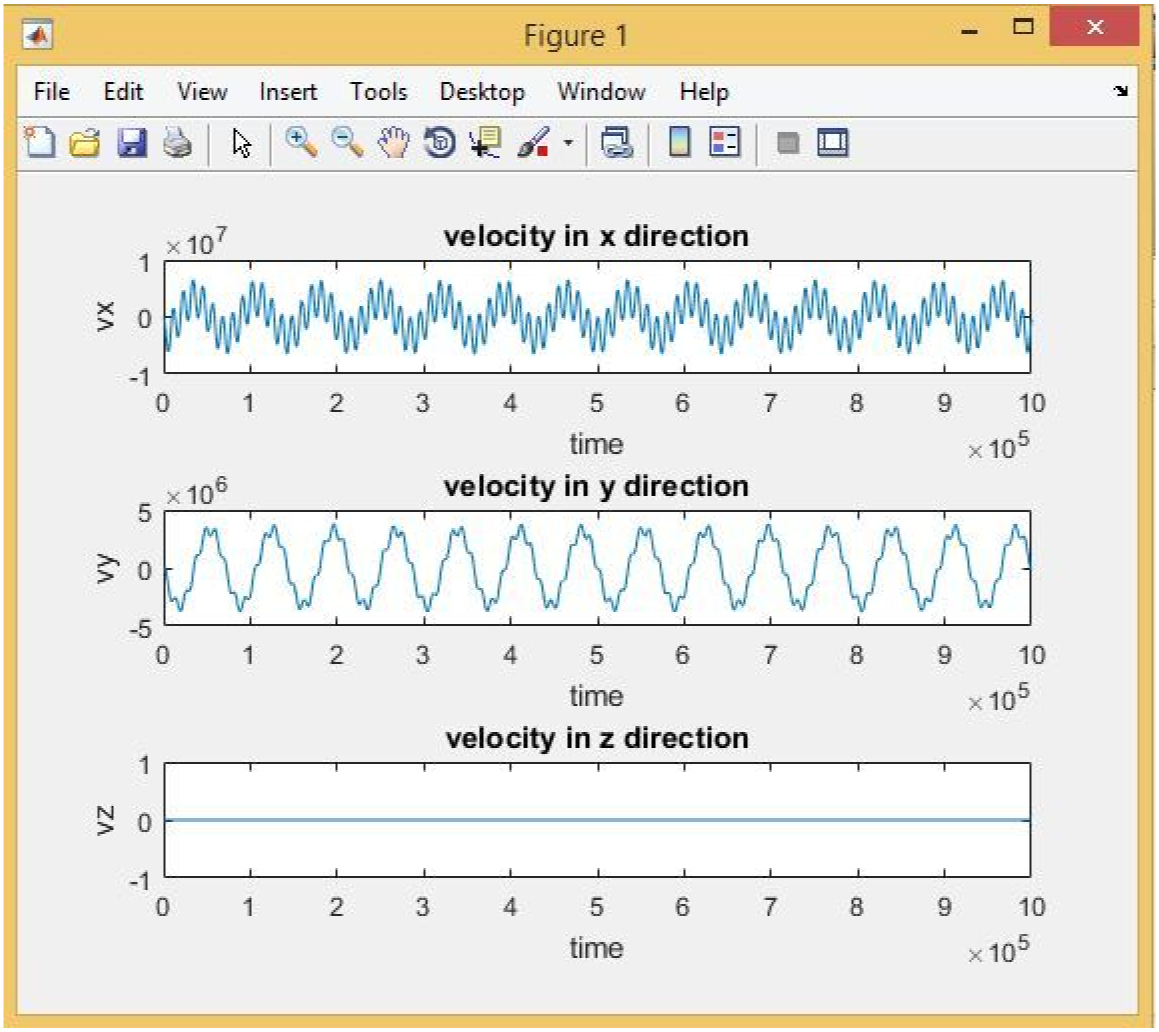
Computational Approach:

RHS :

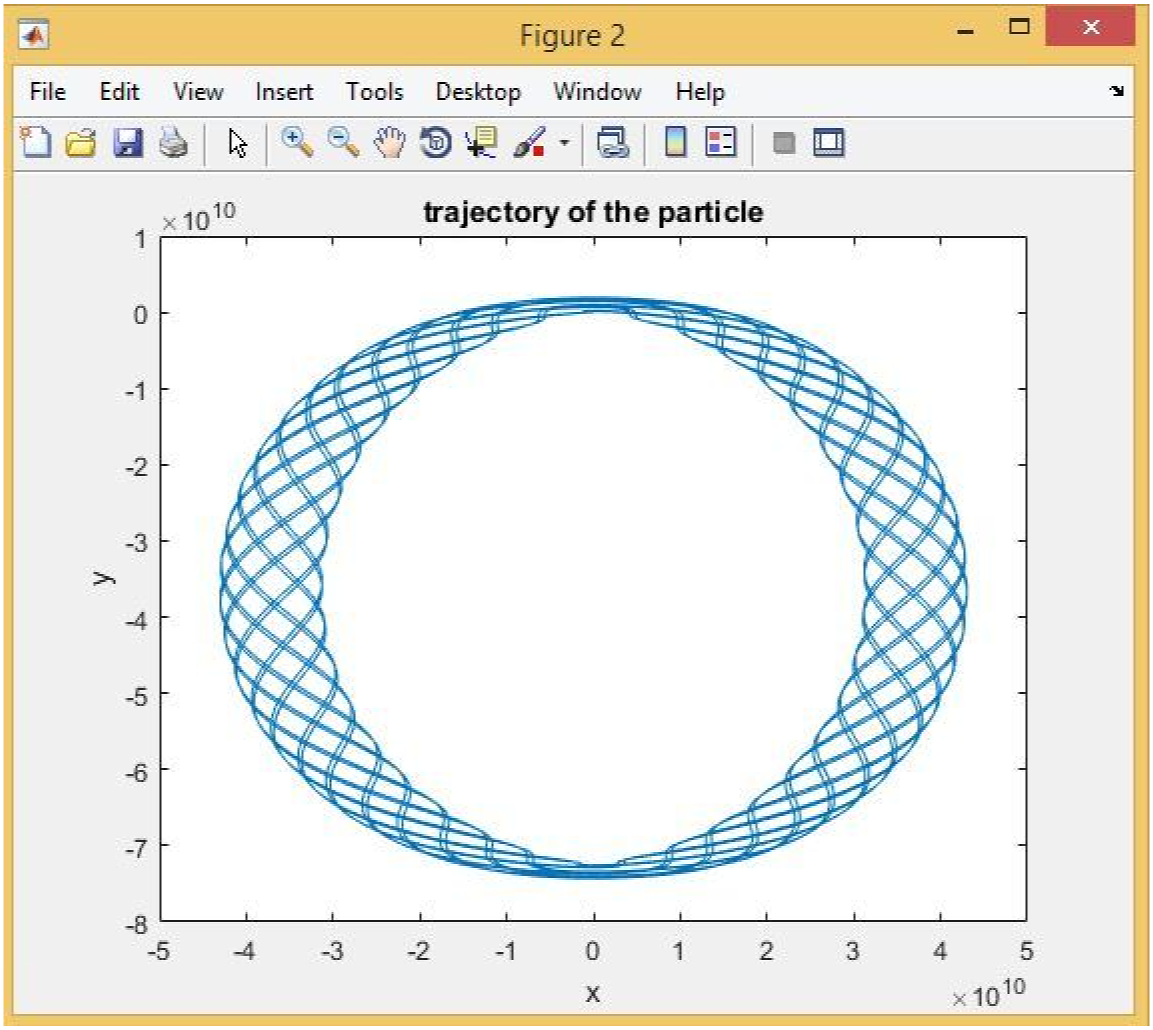
```
fr = q*B(3)/m;
E0 = 1000;

f(1) = cmr*(u(2)*B(3)-B(2)*u(3)-E0*sin(2*pi*fr*t));
f(2) =  -cmr*(u(1)*B(3)-B(1)*u(3));
f(3) =  cmr*(u(1)*B(2)-B(1)*u(2));
f(4)=u(1);
f(5)=u(2);
f(6)=u(3);
```

Plots:

Velocity in x,y,z direction:



Trajectory of the particle:



Conclusion:

When oscillatory frequency is high, drift velocity is insignificant, When oscillatory frequency is equal to frequency of charge particle , drift velocity is considerable.