CS-301 High Performance Computing

Assignment 2

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1 Hardware Details

Parameter	Value
CPU model	Intel Core i5-4590 3.30 GHz
Number of Cores	4
L1d Cache	$32~\mathrm{kB}$
L1i Cache	$32~\mathrm{kB}$
L2 Cache	$256~\mathrm{kB}$
L3 Cache	6144 kB
Compiler	${ m gcc}$

2 Problem Statement

Write and analyze serial code and parallel code for the following using OpenMP.

- 1. Integration of a function using trapezoidal rule. Verify your code by using it to calculate the value of π .
- 2. Calculation of π using series.
- 3. Summation of two vectors

3 Solution

3.1 Integration using trapezoidal rule

3.1.1 Theory

We use the following formula to compute the value of π :

$$\int_0^1 \frac{4}{1+x^2} dx$$
 (1)

3.1.2 Approach

For the serial code, we use simple numerical integration over 0 to 1. For parallel implementation, we simply run numerical integration over 4 equal ranges of length 0.25 between 0 and 1 over 4 parallel threads. We add each partial sum to the global sum, taking care that the global addition takes place under the **critical** section.

3.1.3 Results and Observations

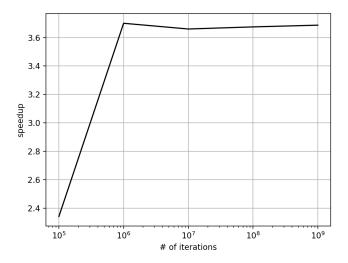


Figure 1: speedup vs number of iterations

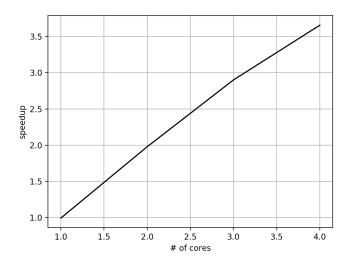


Figure 2: speedup vs number of cores

The above plots show speedup as a function of number of iterations (the problem size) and the number of cores respectively. As the computing unit used during the experiment had 4 cores, the problem could be broken down into 4 parallel processes each having an equal problem size. Hence roughly the speedup should have been at most 4. This is in agreement to our observations which give us a maximum speedup of 3.73.

In figure 2 we see speedup increasing as the number of cores increase. This is in agreement with **Amdahl's law** which gives us a rough upper limit on speedup in terms of parallelization and also gives us speedup as a function of ratio of the parallel code to serial code and the number of cores in a machine. For a piece of code having a ratio s as the serial part and p as a parallel part running on a machine having n cores, the maximum theoretical speedup which can be received is given by:

$$speedup = \frac{1}{s + \frac{p}{n}} \tag{2}$$

From figure 1 we see the benefits of parallelization for large input sizes. For small input sizes of the order 10^5 the parallel code is about 2.4 times faster than its serial counterpart. When the input size grows to 10^7 , the speedup increases by nearly 50%.

4 Coding strategies

Implement some of the important optimization strategies discussed during the lecture today and measure difference in improvement while using the best strategy.

1. Case A

```
for (int i = 0; i < N; i ++)
  if( A [ i ] > 100) {
    flag = true;
    break;
}
```

Case B

```
for (int i = 0; i < N; i ++)
  if( A [ i ] > 100) {
    flag = true;
}
```

Observation: When $N = 1 \times 10^9$ in the above cases, and we search in an already instantiated array, the flag is turned true when the value of A[i] is greater than 100 which in this case was kept at i = 500000000. Theoretically, case B does all the iterations while case A has a break condition if the if condition returns true. Case A takes 0.989066 seconds while case B takes 1.956732 seconds.

Optimization strategy: Trying to perform the least number of operations if the output remains same.

2. Case A

```
A = B**2.0;
```

Case B

```
A = B*B
```

Observation: For large floating point numbers of the order 1×10^{25} , Case A takes 0.000019 seconds while case B takes 0.000000000001 seconds.

Optimization strategy: Try and avoid more resource consuming operations. In the first case, power does multiplication $O(\log(n))$ times while in the second case, multiplication is done only one time.

3. Case A

```
int s = 1 , r =2;
for (int i = 0; i < N ; i ++) {
    A [ i ] = A [ i ] + s + r * sin (0.523598776) ;
}</pre>
```

Case B

```
int s = 1 , r =2;
double tmp = s + r * sin (0.523598776) ;
for (int i = 0; i < N ; i ++) {
    A [ i ] = A [ i ] + tmp ;
}</pre>
```

Observation: As we can see in the code, the operation s + rsin(x) takes place 10^8 times in case A while it takes place only once in case B which makes it faster. Case A takes 3.584542 seconds, Case B takes 2.890220 seconds.

Optimization strategy: Reduce the number of registers used to store data, the number of comparisons made or the number of redundant operations

4. Case A

Branched matrix multiplication where N = 1x10000

Case B

Unbranched matrix multiplication where N = 1x10000

Observation: Case A takes 0.401576 seconds while case B takes 0.194602 seconds. The chances of a branch miss is higher in A then in B because of more number of branches in A than in B. This increases the latency and hence takes more time.

 ${\bf Optimization\ strategy:}\ {\it Avoid\ branching\ wherever\ possible}.$