SC-374
Computational and Numerical Methods

Assignment 4

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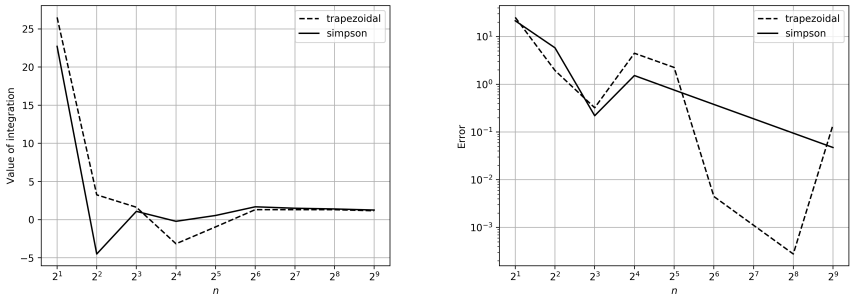
1 Numerical Integration using Trapezoidal and Simpson methods

1.1 $\int_0^\pi e^x \cos(4x)dx$



(a) Value of integration vs n

(b) Error vs n

Figure 1: A comparison of the Trapezoidal and the Simpson Methods

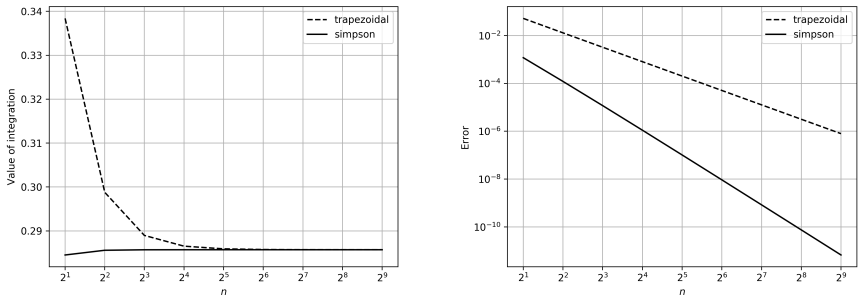
The analytical solution is:

$$\frac{e^\pi - 1}{17} = 1.3024$$

Notice that in this particular case, for certain values of n, the Trapezoidal method gives lesser error than the Simpson method. This is because of the oscillations in the integrand.

n	**Trapezoidal**	**Simpson**	**Trapezoidal Error**	**Simpson Error**
2	26.5163358571	22.7150773715	25.2139421728	21.4126836872
4	3.24905049448	-4.50671129305	1.9466568102	5.80910497733
8	1.62452524724	1.08301683149	0.322131562961	0.219376852786
16	-3.16794185573	-0.221766516671	4.47033554001	1.52416020095
32	-0.951520308268	0.544564269784	2.25391399255	0.757829414498
64	1.3068478855	1.68099858564	0.00445420121631	0.378604901356
128	1.3035056585	1.49171093172	0.00111197421592	0.18931724744
256	1.30267157946	1.39705322755	0.000277895174647	0.094659543273
512	1.16047364026	1.25506383886	0.14192004402	0.0473298454176

1.2 $\int_0^1 x^{\frac{5}{2}} dx$



(a) Value of integration vs n (b) Error vs n

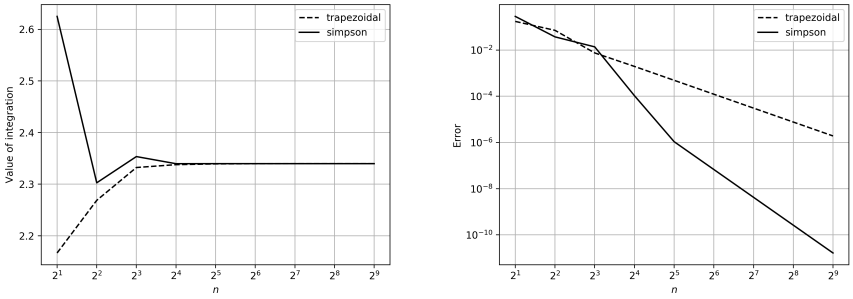
Figure 2: A comparison of the Trapezoidal and the Simpson Methods

The analytical solution is $\frac{2}{7}$ (2.8571).

Notice that the Simpson method converges to the solution much faster than the Trapezoid method. Also notice that for a given n, the Simpson method has a much lower error.

n	**Trapezoidal**	**Simpson**	**Trapeoidal Error**	**Simpson Error**
2	0.338388347648	0.284517796864	0.052674061934	0.00119648884986
4	0.298791496231	0.285592545759	0.0130772105171	0.000121739955264
8	0.28897473967	0.285702487483	0.00326045395586	1.17982312106e-05
16	0.286528567896	0.285713177305	0.000814282181752	1.10840961681e-06
32	0.285917779699	0.285714183633	0.000203493984448	1.02081319453e-07
64	0.28576515225	0.285714276434	5.08665361767e-05	9.27991378186e-09
128	0.285727001721	0.285714284878	1.27160068126e-05	8.3630879999e-10
256	0.28571746466	0.285714285639	3.17894550972e-06	7.4924566551e-11
512	0.285715080446	0.285714285708	7.9473136344e-07	6.68542998739e-12

1.3 $\int_0^5 \frac{1}{1+(x-\pi)^2} dx$



(a) Value of integration vs n (b) Error vs n

Figure 3: A comparison of the Trapezoidal and the Simpson Methods

The analytical solution is :

$$\arctan(5) + \arctan(5 - \pi) = 2.3397$$

Notice that the Simpson method converges to the solution almost at the same rate as Trapezoid method initially. Also notice that for a given large n, the Simpson method has a much lower error.

n	**Trapezoidal**	**Simpson**	**Trapeoidal Error**	**Simpson Error**
2	2.16665484358	2.62509546184	0.173111440085	0.285329178167
4	2.26866764398	2.30267191077	0.0710986396921	0.0370943728946
8	2.33227046587	2.3534714065	0.00749581779951	0.0137051228314
16	2.33781288101	2.33966035273	0.00195340265534	0.000105930940619
32	2.33927712301	2.33976520368	0.000489160655167	1.07998844268e-06
64	2.33964394293	2.33976621624	0.000122340738076	6.7432379236e-08
128	2.33973569532	2.33976627945	3.0588347208e-05	4.21691925823e-09
256	2.33975863638	2.3397662834	7.64728449809e-06	2.6359270322e-10
512	2.33976437183	2.33976628365	1.91183347953e-06	1.6474821507e-11

1.4 $\int_0^{10} e^{-x^2} dx$

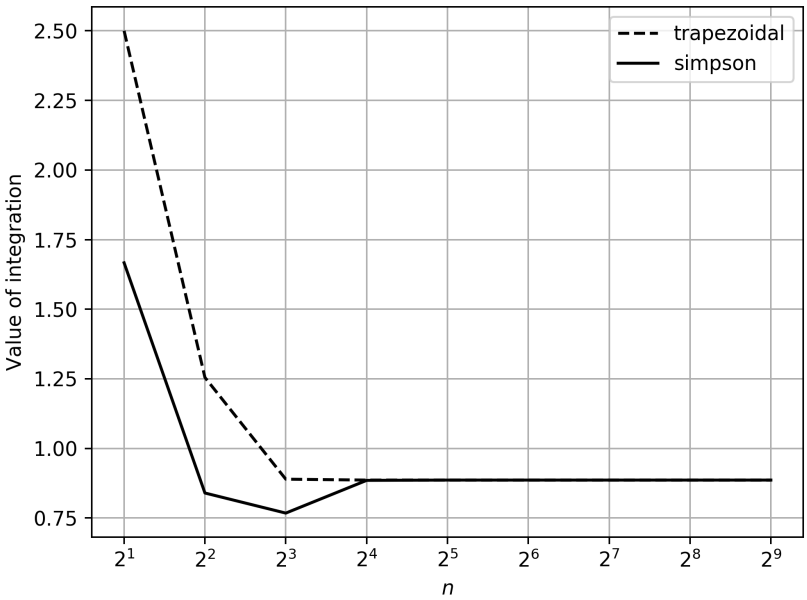


Figure 4: A comparison of the Trapezoidal and the Simpson Methods

n	**Trapezoidal**	**Simpson**
2	2.50000000007	1.66666666676
4	1.25482613538	0.839768180477
8	0.889428278063	0.767628992292
16	0.886226925472	0.885159807941
32	0.886226925453	0.886226925446
64	0.886226925453	0.886226925453
128	0.886226925453	0.886226925453
256	0.886226925453	0.886226925453
512	0.886226925453	0.886226925453

1.5 $\int_0^2 \arctan(1+x^2)dx$

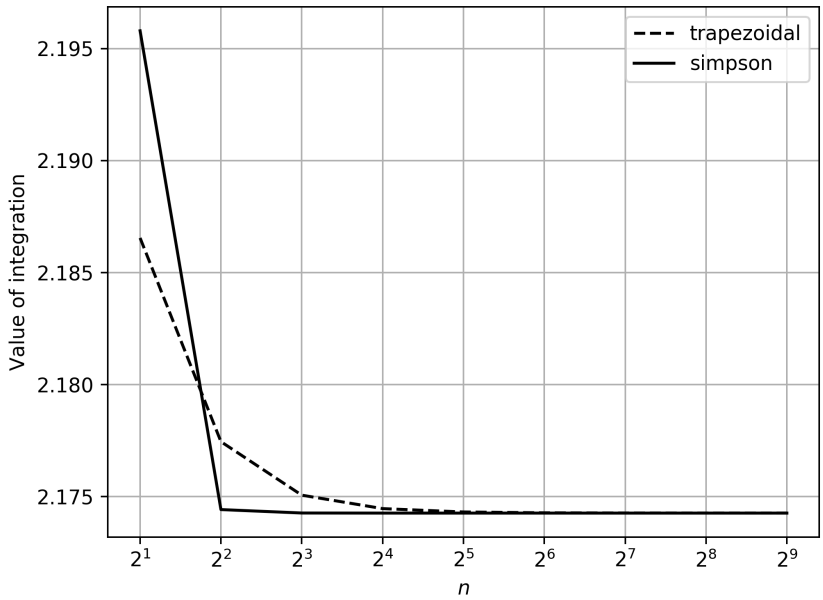


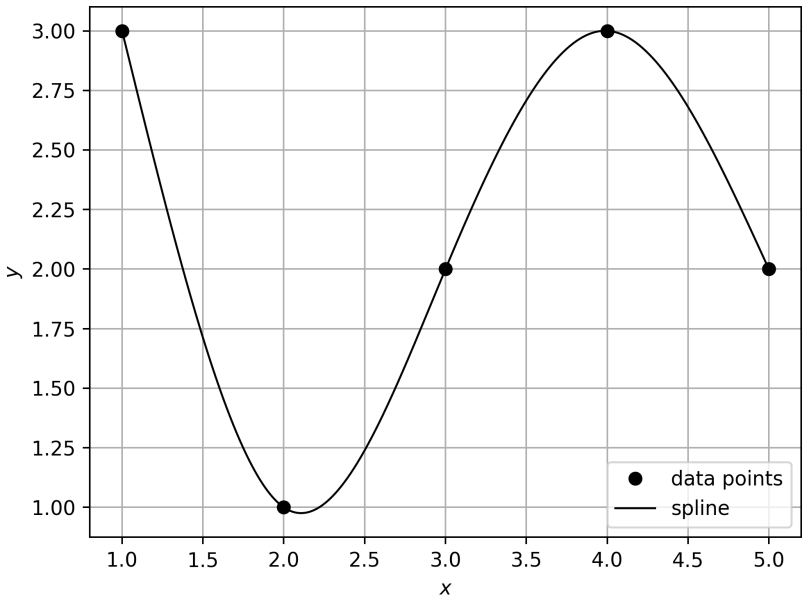
Figure 5: A comparison of the Trapezoidal and the Simpson Methods

n	**Trapezoidal**	**Simpson**
2	2.18654818297	2.19579793384
4	2.17745048137	2.17441791418
8	2.17506135648	2.17426498151
16	2.17446133091	2.17426132239
32	2.174311149	2.17426108836
64	2.17427359256	2.17426107374
128	2.17426420276	2.17426107283
256	2.17426185527	2.17426107277
512	2.17426126839	2.17426107277

2 Spline Interpolation

Problem 1

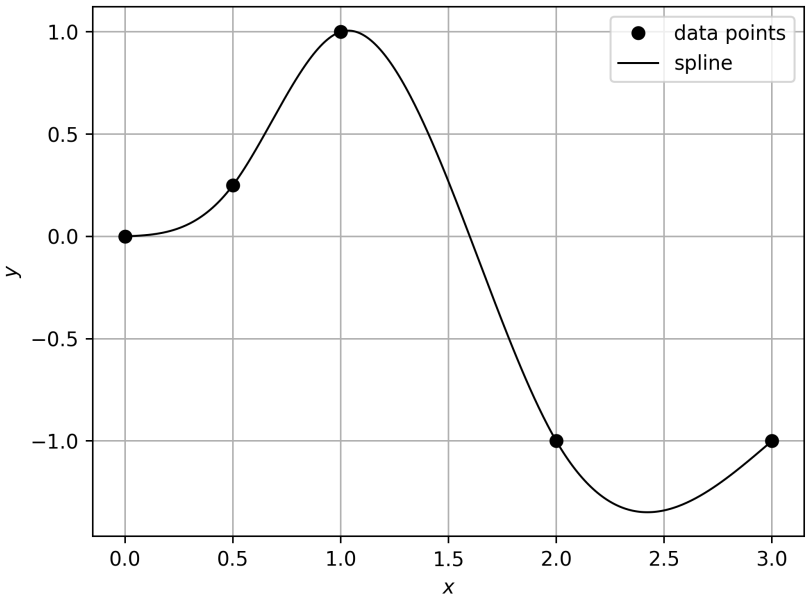
x	1	2	3	4	5
y	3	1	2	3	2



Coefficient	**Value**
M_0	0
M_1	4.6071428571428568
M_2	-0.42857142857142849
M_3	-2.8928571428571432
M_4	0

Problem 2

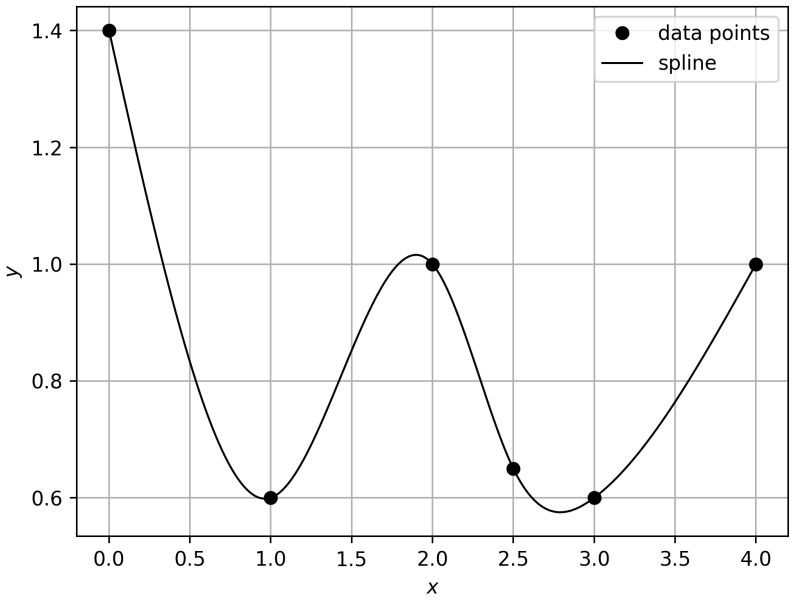
x	0	0.5	1	2	3
y	0	0.25	1	-1	-1



Coefficient	**Value**
M_0	0
M_1	5.4285714285714297
M_2	-9.7142857142857153
M_3	5.4285714285714288
M_4	0

Problem 3

x	0	1	2	2.5	3	4
y	1.4	0.6	1.0	0.65	0.5	1.0



Coefficient	**Value**
M_0	0
M_1	2.6788381742738592
M_2	-3.5153526970954365
M_3	2.5344398340248961
M_4	0.57759336099585101
M_5	0

3 Linear Interpolation

x	0	1	2	3	4	5	6
y	2	2.1592	3.1697	5.4332	9.1411	14.407	21.303