SC-374
Computational and Numerical Methods

Assignment 5

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1 Accretion discs

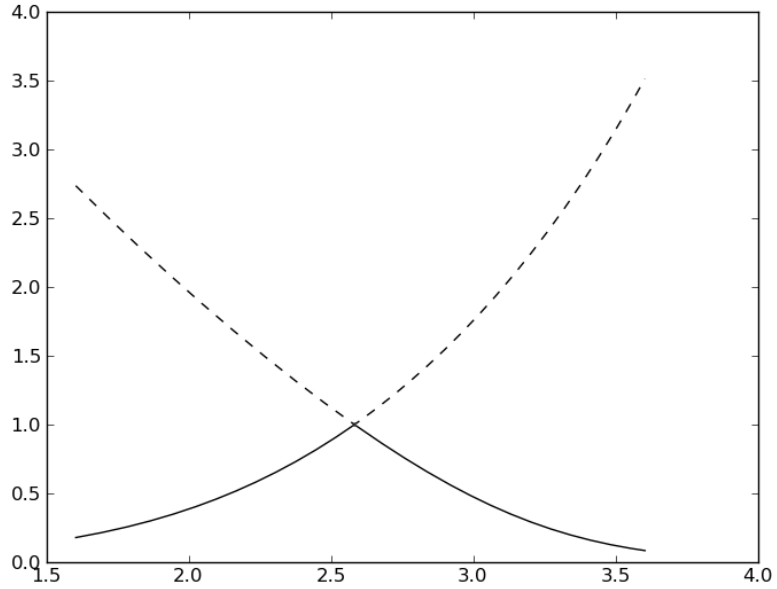


Figure 1: Assignment 2

2 Gaussian Elimination Method

1. Solve system of linear equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + 2x_2 + 3x_3 &= 3 \\ -x_1 - 3x_2 &= 2 \end{aligned}$$

Soln:

x_1	1
x_2	-1
x_3	1

2. Solve system of linear equations:

$$\begin{aligned} 4x_1 + 3x_2 + 2x_3 + x_4 &= 1 \\ 3x_1 + 4x_2 + 3x_3 + 2x_4 &= 1 \\ 2x_1 + 3x_2 + 4x_3 + 3x_4 &= -1 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= -1 \end{aligned}$$

Soln:

x_1	0
x_2	1
x_3	-1
x_4	0

3. Find the inverse of the following matrix:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{bmatrix}$$

Soln:

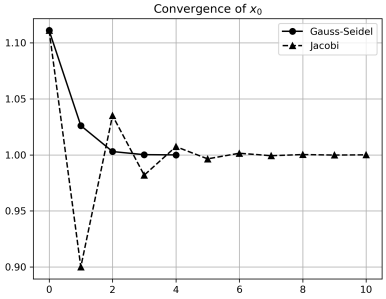
$$\begin{bmatrix} 2.00000000e+00 & -1.00000000e+00 & -2.49800181e-16 \\ 1.50000000e+00 & -5.00000000e-01 & 5.00000000e-01 \\ 2.50000000e+00 & -1.50000000e+00 & 5.00000000e-01 \end{bmatrix}$$

3 Jacobi Iteration and Gauss-Seidel Methods

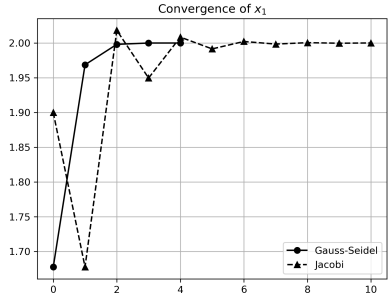
$$\begin{aligned} 9x_1 + x_2 + x_3 &= 10 \\ 2x_1 + 10x_2 + 3x_3 &= 19 \\ 3x_1 + 4x_2 + 11x_3 &= 0 \end{aligned}$$

Initial guess values of $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$ and using a tolerance of 0.001.

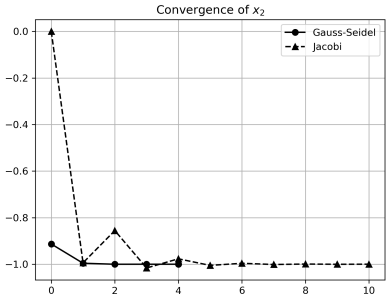
variable	Analytic solution	Jacobi	Gauss-Seidel
x_1	1	1.00000701759	1.00000701759
x_2	2	2.00010775418	2.00001652
x_3	-1	-0.99985948542	-1.00000792175



(a) Convergence of x_1



(b) Convergence of x_2

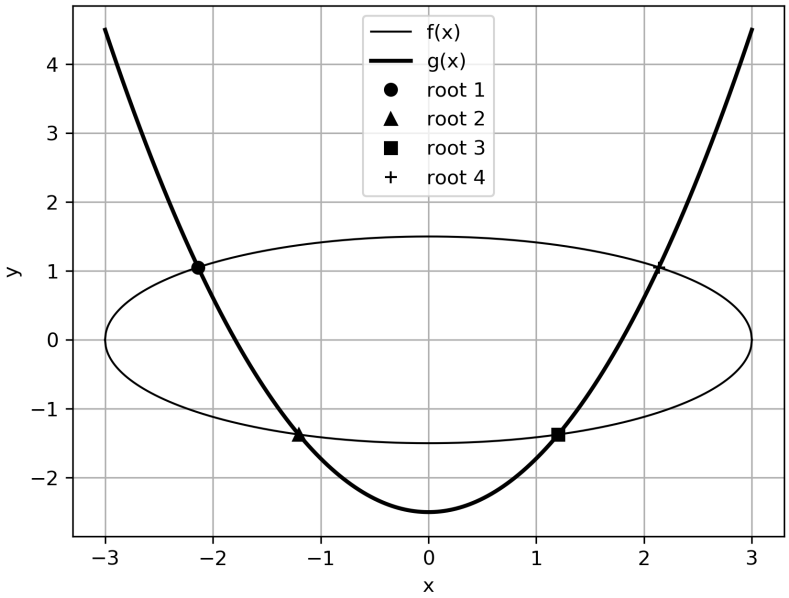


(c) Convergence of x_3

Figure 2: A comparison of the convergence of all roots for Jacobi and Gauss-Seidel methods

4 Non-linear Systems using Netwon Raphson method

$$\begin{aligned} f(x,y) \equiv x^2 + 4y^2 - 9 &= 0 \\ g(x,y) \equiv 18y - 14x^2 + 45 &= 0 \end{aligned}$$



(x_1, y_1)	(-2.13721674 , 1.05265196)
(x_2, y_2)	(-1.20316696 , -1.37408053)
(x_3, y_3)	(1.20316696 , -1.37408053)
(x_4, y_4)	(2.13721674 , 1.05265196)

$(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ took 4, 5, 5, 4 iterations respectively to converge.