MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT AND ENGINEERING

(Affiliated to NMIMS Deemed to be University, Mumbai)

Project on "Applications of Torque and Rotational Inertia"

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Torque

What is torque?

Torque is the measure of the force that can cause an object to rotate about an axis. Force is what causes an object to accelerate in linear kinematics. Similarly, torque is what causes an angular acceleration. Hence, torque can be defined as the rotational equivalent of linear force. The point where the object rotates is called the axis of rotation. In physics, torque is simply the tendency of a force to turn or twist. Different terminologies such as moment or moment of force are interchangeably used to describe torque.

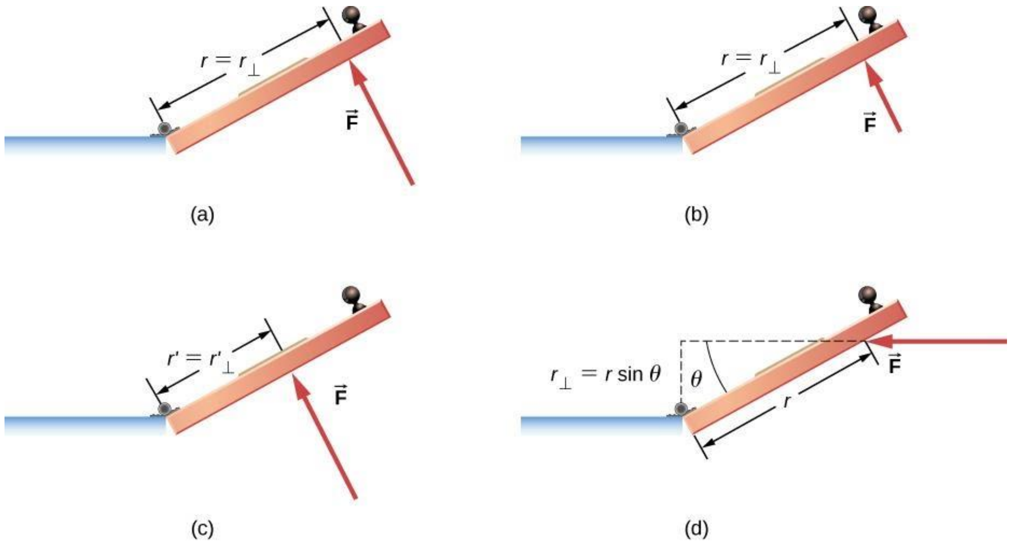


Figure 10.31 Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) A counterclockwise torque is produced by a force $\vec{F}$ acting at a distance r from the hinges (the pivot point). (b) A smaller counterclockwise torque is produced when a smaller force $\vec{F}'$ acts at the same distance r from the hinges. (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) A smaller counterclockwise torque is produced by the same magnitude force as (a) acting at the same distance as (a) but at an angle θ that is less than 90°.

Types of Torque

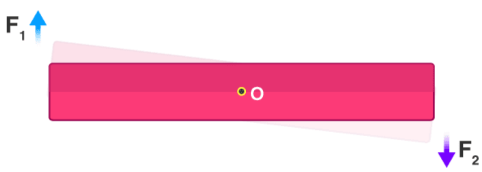
Torque can be either be static or dynamic. Static torque is a torque that does not produce an angular acceleration. A few examples of static torque are as follows:

• A person pushing a closed-door is applying a static door because the door isn't rotating despite the force applied.

• Pedalling a cycle at a constant speed is also an example of static torque as there is no acceleration.

The drive shaft in a racing car accelerating from the start line exhibits dynamic torque because it must be producing an angular acceleration of the wheels, given that the car is accelerating along the track.

To explain torque in detail let us consider the figure.



- We can see that the net force on the body is zero.

- Hence, the body is in translational equilibrium.
- But the rod tends to rotate, thus the turning effect produced by force is known as moment of force or torque.

Now we will consider the example of a door and try to formulate the equation for torque.

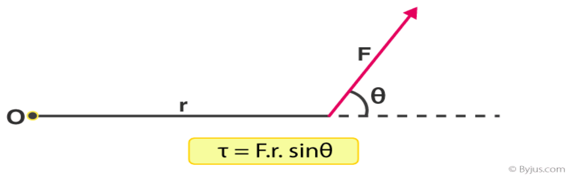
- If we apply force closer to the hinge, then a larger force is required to rotate the door.
- Also, it depends on the direction in which the force is being applied.
- If it is perpendicular to the line joining the hinge and the point of application of force then a smaller force is required.

How is Torque Calculated?

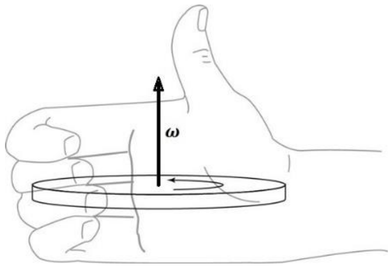
A simple way to calculate the magnitude of the torque is to first determine the lever arm and then multiply it times the applied force.

Now, from the above observation, we conclude that torque produced depends on the magnitude of the force and the perpendicular distance between the point about which torque is calculated and the point of application of force. So, mathematically torque is represented as:

$\tau = F.r. \sin\theta$



The direction of the torque vector is found by convention using the right-hand grip rule. If a hand is curled around the axis of rotation with the fingers pointing in the direction of the force, then the torque vector points in the direction of the thumb.



Measurement of Torque

The unit of torque is Newton–meter (N-m). The above equation can be represented as the vector product of force and position vector.

$\tau = r \times F$

So, as it is a vector product hence torque also must be a vector. Using vector product notations we can find the direction of torque. We will consider an example to see how to calculate torque.

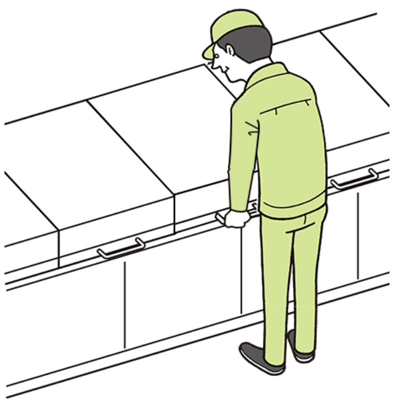
Common symbols	**τ, M**
SI unit	**N·m**
In SI base units	**$kg \cdot m^2 \cdot s^{-2}$**
Dimension	**$M L^2 T^{-2}$**
Other units	**pound-force-feet, lbf·inch**

Application 1: Torque Hinges

What are torque hinges?

- A torque hinge provides continuous resistance to the pivoting motion of the hinge and thus makes the product suitable for holding lids, doors, panels, or display devices at specific angles for extended periods of time.
- It has the ability to retain its angle or position even when a force is applied to it.
- Torque hinges are also called free-stop hinges, friction hinges or constant torque friction hinges.
- Some models of torque hinges like the ones with soft-close prevent door, cabinet doors, lids are used for closing whereas some other types of torque hinges are used to open a heavy cover or door with little effort.

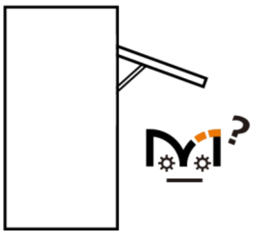
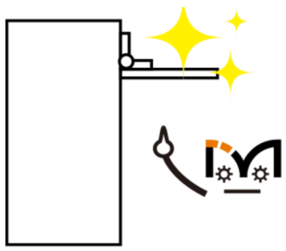
Why are Torque Hinges important?

Without Torque Hinge	With Torque Hinge
	

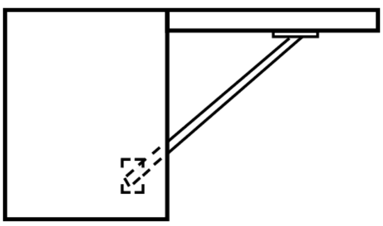
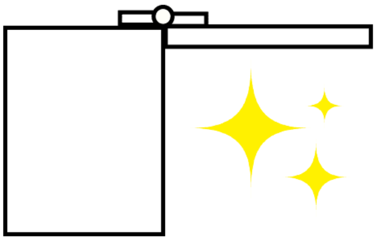
As we can see in the figure above, torque hinges can increase worker efficiency and maintenance simplicity. By holding flaps and lids in place, workers can focus on other tasks and keep their hands free. Torque hinges reduce the risk of flaps and lids slamming shut, reducing the change of injury.

There are 3 main advantages of torque hinges:

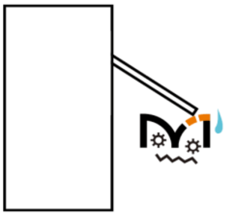
1. Efficiency

Without	With
 Applications need some type of stopper to hold in place	 Torque hinges allow the lid or flap to Hold at any angle without any additional components

2. Design

Without	With
 Multiple components are required.	 Simplified design without the need for a lid support.

3. Safety

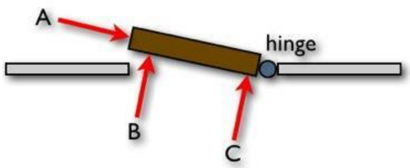
Without	With
 May accidentally or unintentionally close, increasing the risk of injury.	 Holds in whatever position it is set at.

Science behind Torque Hinges

When you push on a door it cannot freely translate because it is confined (or pinned) by the hinges. It does, however, rotate on the hinges. When it comes to hinges, torque is the amount of energy or effort needed to open or close a door or panel. The energy required depends on the door's weight; the travel distance or arc of motion from closed to open; and whether the hinge must hold the door or panel partially open or the door will always go from fully open to completely closed.

The opening of a door and its hinges is caused by torque, and the hinges are the pivot point. If you try to open a door by pushing on the door near its hinges, it most likely will not open because there is not enough torque to force it to do so. The length of the moment arm when the door is pushed near its hinges is not large enough to supply enough torque to open the door. In order to open the door, you have to push on the side of the door opposite from the hinges to provide a substantial moment arm which allows for an increased torque to open the door. The rotation itself depends on where you apply the force. As you get closer to the hinge, you must apply a larger force to make the door swing. As you get farther from the hinge, you can apply a smaller force to make the door swing.

The product of the force and the perpendicular distance to a pivot (or hinge) is called the torque or the moment.



Torque Hinge strength Variation

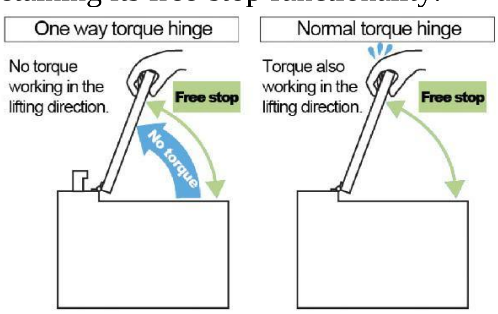
Depending on the type, format and internal mechanisms, there can be large variations in torque strength. Further, all torque hinges have a certain amount of variability. All torque hinges operate within an expected range, this is why you will almost always see a ± symbol beside a torque hinge. Even if you do not see this symbol, there is variability. This is not unique to any manufacturer but is an inevitable by-product of the process of creating torque hinges. This is why it is important to accurately calculate the use case in order to ensure that the hinge will work properly, even with variation.

Choosing a product that is below but close to the lower limit is important. If it is above the lower limit, the hinge is not guaranteed to provide free stop (undercompensating). If is too far below the lower limit, the lid/flap/panel/ or other movable component will become very rigid and difficult to move.

Different types of torque hinges based on their functions:

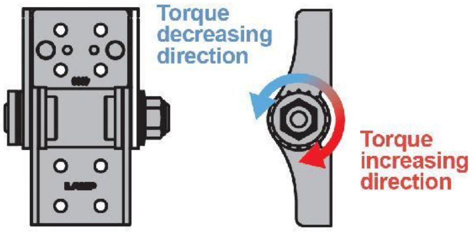
1. One way torques hinges

These hinges have friction which only activates in one direction. This feature makes opening and lifting easy while still retaining its free-stop functionality.



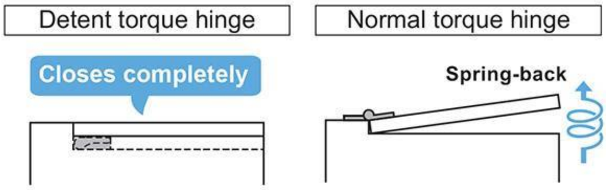
2. *Adjustable Torque Hinges*

Adjustable torque hinges contain the functionality to adjust the level of torque exerted in the hinge. This can be very useful when fine tuning torque hinges to make operational feeling as ideal as possible.



3. Detent Torque Hinge

Detent torque hinges are able to hold torque hinges completely closed. This protects against "spring back", an inherent property of torque hinges without a decent. "Spring back" is a phenomenon that occurs due to the presence of torque. This causes hinges to be unable to close completely.

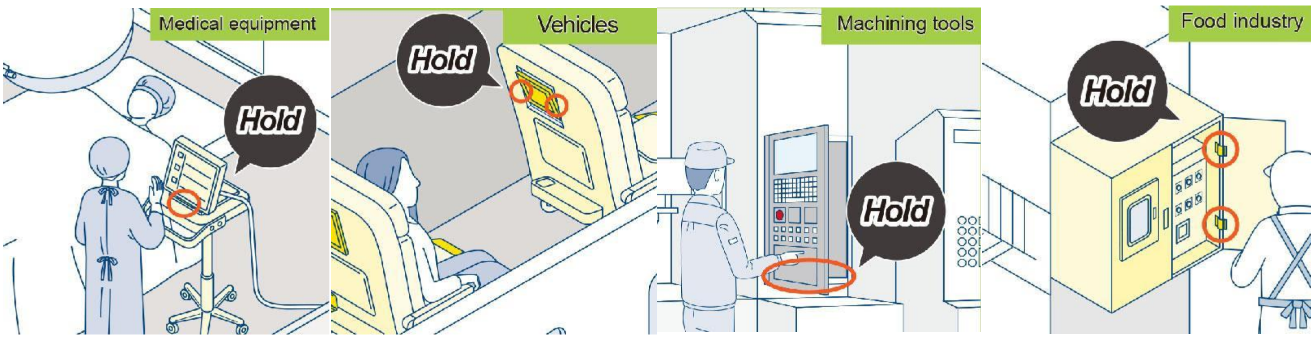


Applications:

Most people use torque hinges daily without ever giving them a thought. Generally, if something opens, closes, tilts or rotates, and you'd like to be able to adjust it and have it remain in the chosen position, a torque hinge is the answer. Torque hinges are used for holding laptop screens, shop cabinets, and commercial doors open and at the user's desired angle. If a traditional hinge were used, lids, doors, and equipment panels wouldn't stay open or upright, but instead would slam shut because no friction or resistance would be present. While regular hinges have been used for hundreds of years, the innovation of torque hinges has been a welcome addition for manufacturers, contractors, and homeowners.

Additional possible applications for torque hinges include:

- Adjustable shields or operator's screens on machinery
- Doors that need to remain open and not close accidentally for safety or convenience such as emergency exits, air locks, or during loading and unloading
- Cabinets, lockers, electrical boxes, or control panels
- Windows, doors, or hatches that need to remain partially open for ventilation



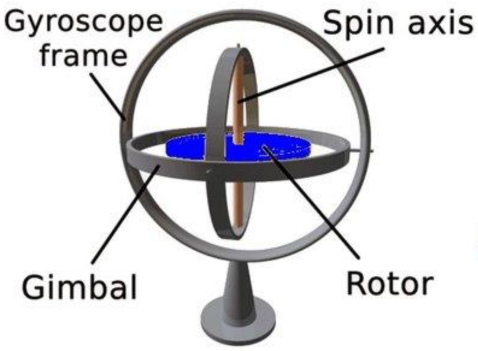
Application 2: Gyroscope

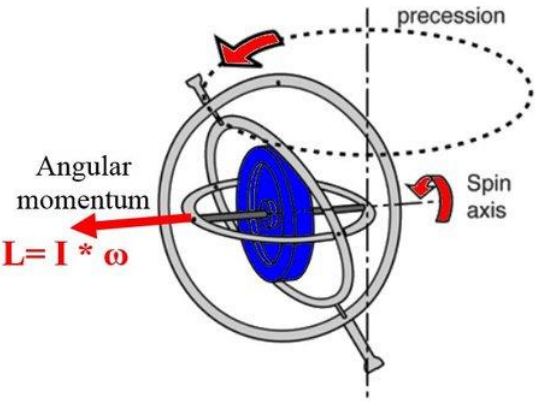
What is it?

A gyroscope is a device used for measuring or maintaining orientation and angular velocity. It is a spinning wheel or disc in which the axis of rotation is free to assume any orientation by itself.

It is a device consisting of a wheel that turns very quickly inside a frame and does not change position when the frame is moved.

A gyroscope consists of a wheel-like disk, called a flywheel, mounted on an axle, which in turn is mounted on a larger ring perpendicular to the plane of the wheel itself. An outer circle on the same plane as the flywheel provides structural stability, and indeed, the gyroscope may include several such concentric rings. Its focal point, however, is the flywheel and the axle. One end of the axle is typically attached to some outside object, while the other end is left free to float.





Construction

Once the flywheel is set spinning, gravity has a tendency to pull the unattached end of the axle downward, rotating it on an axis perpendicular to that of the flywheel. This should cause the gyroscope to fall over, but instead it begins to spin a third axis, a horizontal axis perpendicular both to the plane of the flywheel and to the direction of gravity. Thus, it is spinning on three axes, and as a result becomes very stable—that is, very resistant toward outside attempts to upset its balance.

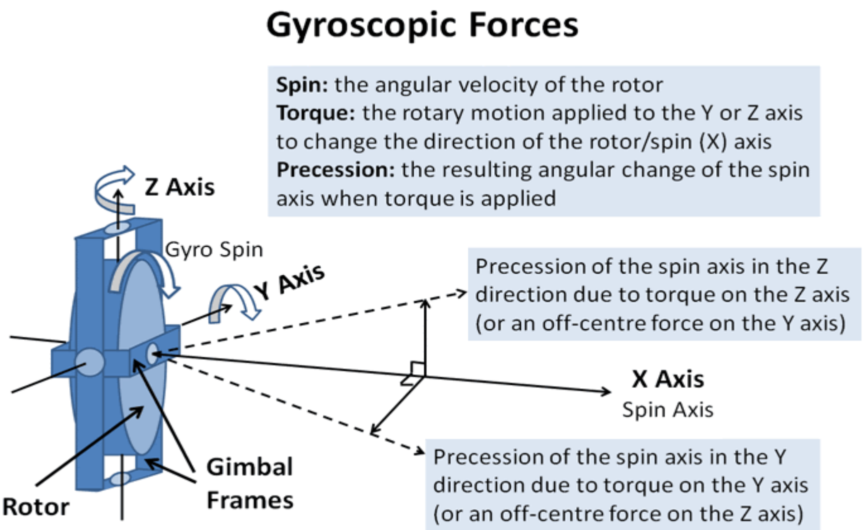
This in turn makes the gyroscope a valued instrument for navigation: due to its high degree of gyroscopic inertia, it resists changes in orientation, and thus can guide a ship toward its destination. Gyroscopes, rather than magnets, are often the key element in a compass. A magnet will point to magnetic north, some distance from "true north" (that is, the North Pole.) But with a gyroscope whose axle has been aligned with true north before the flywheel is set spinning, it is possible to possess a much more accurate directional indicator. For this reason, gyroscopes are used on airplanes—particularly those flying over the poles—as well as submarines and even the Space Shuttle.

Working

Torque, along with angular momentum, is the leading factor dictating the motion of a gyroscope. Think of angular momentum as the momentum (mass multiplied by velocity) that a turning object acquires. Due to a principle known as the conservation of angular momentum, a spinning object has a tendency to reach a constant level of angular momentum, and in order to do this, the sum of the external torques acting on the system must be reduced to zero. Thus, angular momentum "wants" or "needs" to cancel out torque.

The "right-hand rule" can help you to understand the torque in a system such as the gyroscope. If you extend your right hand, palm downward, your fingers are analogous to the moment arm. Now if you curl your fingers downward, toward the ground, then your fingertips point in the direction of g —that is, gravitational force. At that point, your thumb (involuntarily, due to the bone structure of the hand) points in the direction of the torque vector.

When the gyroscope starts to spin, the vectors of angular momentum and torque are at odds with one another. Were this situation to persist, it would destabilize the gyroscope; instead, however, the two come into alignment. Using the right-hand rule, the torque vector on a gyroscope is horizontal in direction, and the vector of angular momentum eventually aligns with it. To achieve this, the gyroscope experiences what is known as gyroscopic precession, pivoting along its support post in an effort to bring angular momentum into alignment with torque. Once this happens, there is no net torque on the system, and the conservation of angular momentum is in effect.



The gyroscope's axis, like that of a top, will tend to remain fixed in space, but if it is perturbed by an external force it will move or precess at a right angle to the force exerted. The resistance to precession is directly proportional to the gyroscope's angular momentum which is the product of its mass and its rate of rotation.

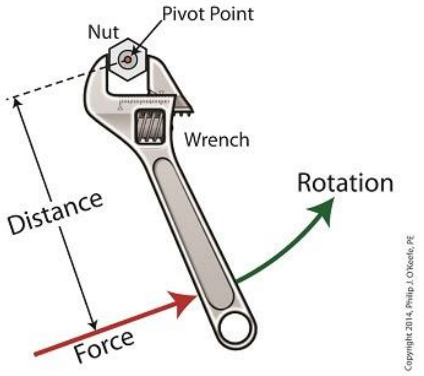
Uses

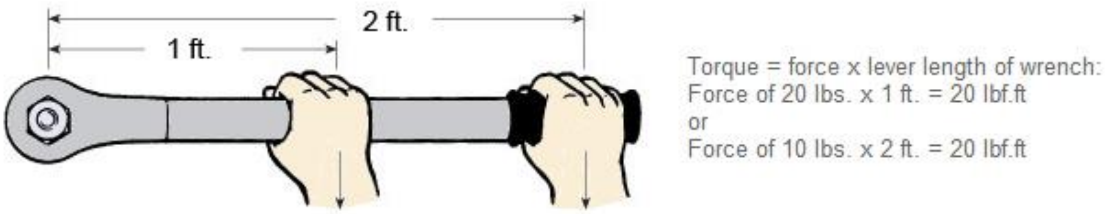
Gyroscopes are used in compasses and automatic pilots on ships and aircraft, in the steering mechanisms of torpedoes, and in the inertial guidance systems installed in space launch vehicles, ballistic missiles, and orbiting satellites.

Application 3: Sea-Saw and Wrenches

The balance-point on a see-saw is the pivot point which is the centre point of any object experiencing torque. While in the wrench-and-lug nut combination this point is lug nut itself. Now, this is the area around which all the forces are directed. In both cases, there is a place where force is being applied. For the seesaw, it is the seats, each holding a child of differing weight.

The moment arm is the distance from the pivot point to the vector on which force is being applied, it is always perpendicular to the direction of force.
In wrenches operating on a lug nut:
-The nut is the pivot point
-the moment arm is the distance from the lug nut to the place where the person operates the wrench has applied force
-Thus, the torque (here, lug nut) is the product of the moment arm multiplied by force at the pivot point. The greater the amount of torque, the greater the tendency of the object to be put into rotation.



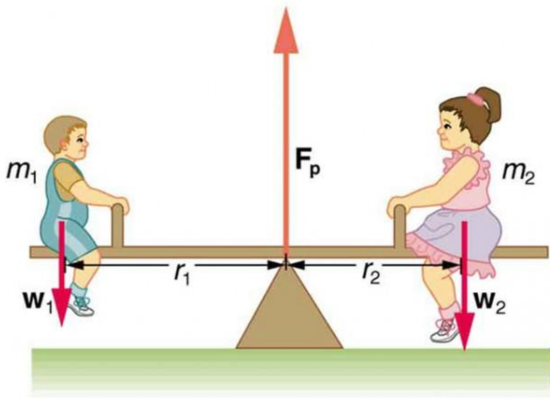


In the case of a seesaw, the fact that it sits on the ground means that its board can never undergo anywhere near 360° rotation; Nonetheless, the board does rotate within relatively narrow parameters. Imagine the clockwise rotational behaviour of a seesaw viewed from the side, with a child sitting on the left and a teenager on the right which will allow us to see the effects of torque (right being heavier than left). We see that the torque will be imbalanced, the side holding the teenager will rotate clockwise, toward the ground, causing the child's side to also rotate clockwise—off the ground.

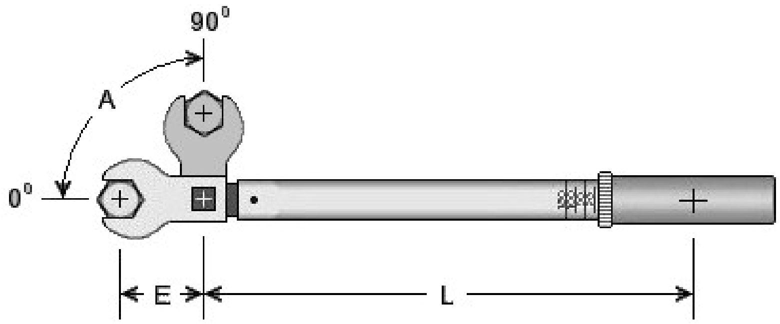
For the two to balance one another perfectly, the torque on each side has to be adjusted.

1. Change in position of moment arm. If the teenager weighs exactly twice as much as the child, the moment arm on the child's side must be exactly twice as long as that on the teenagers.

2. Change in weight



Suppose one wants to disengage a lug nut, and after applying all your force, it still would not come loose. So, to rotate in wrench increase the moment arm either by grasping the wrench further from the pivot point, or by using a longer wrench.



Application 4: Complex Machines

- Torque is a factor in several complex machines such as the electric motor that runs most household appliances.
- Torque comes from the engine, but it has to be supplied to the transmission. In an automatic transmission, there are two principal components: the automatic gearbox and the torque converter.
- The torque converter helps to transmit power from the flywheel of the engine to the gearbox, and it has to do so as smoothly as possible.
- The torque converter consists of three elements:
  - An impeller, which is turned by the engine flywheel;
  - A reactor that passes this motion on to a turbine;
  - The turbine itself, which turns the input shaft on the automatic gearbox.
- An infusion of oil to the converter assists the impeller and turbine in synchronizing movement, and this alignment of elements in the torque converter creates a smooth relationship between engine and gearbox.
- Torque in the context of electricity involves reference to current, conduction, magnetic field, and other topics relevant to electromagnetic force.

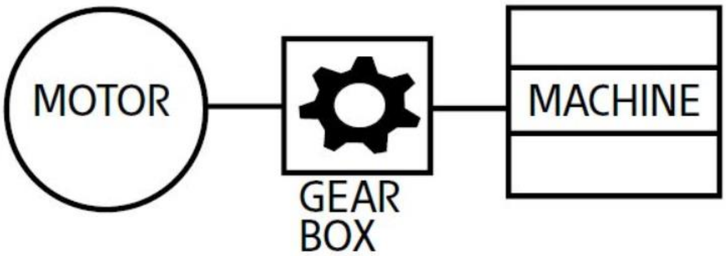


Figure 1: - Schematic diagram of a motor-gearbox arrangement to drive a machine

Now let's take an example of a car: -

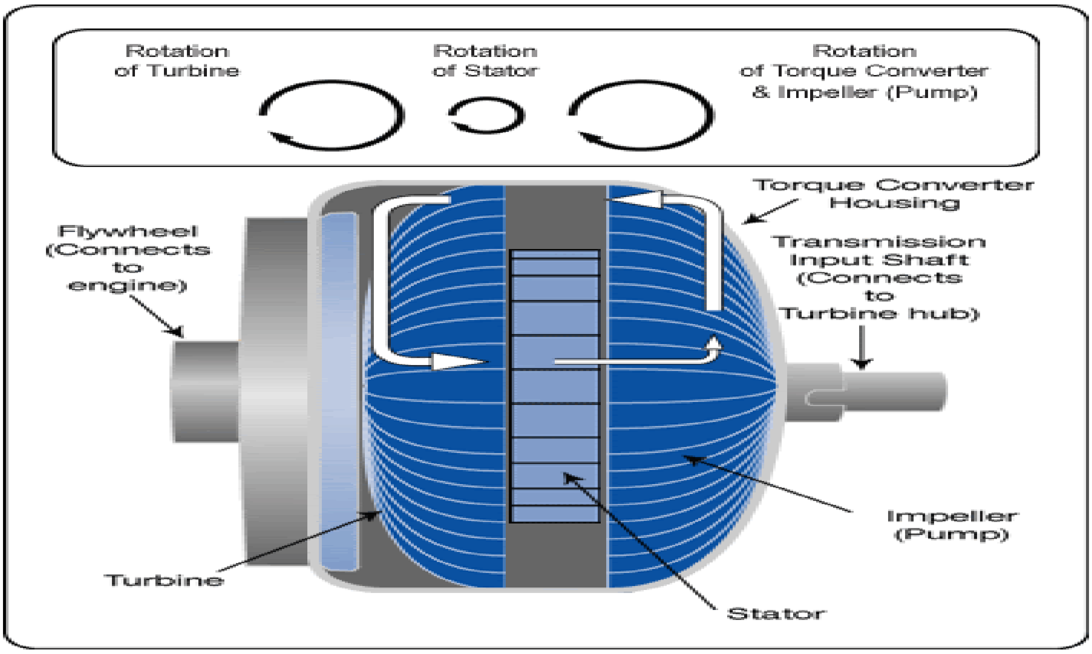
- Torque is a crucial part of generating power from a car's engine, as it represents the load an engine can handle to generate a certain amount of power to rotate the engine on its axis. The force is measured in pounds (lb) per foot (ft) of rotation around one point. Multiply this torque force (in lb-ft.)

Working of a torque converter: -

A torque converter is a type of fluid coupling which is used to transfer rotating power from the engine of a vehicle to the transmission. It takes place of mechanical clutch in an automatic transmission. The main function of it is to allow the load to be isolated from the main power source. It sits in between the engine and transmission. It has the same function as the clutch in manual transmission. As the clutch separates the engine from the load when it stops, in the same way it also isolates the engine from load and keep engine running

when vehicle stops its main functions are:
1. It transfers the power from engine to the transmission input shaft.
2. It drives the front pump of the transmission.
3. It isolates the engine from the load when the vehicle is stationary.

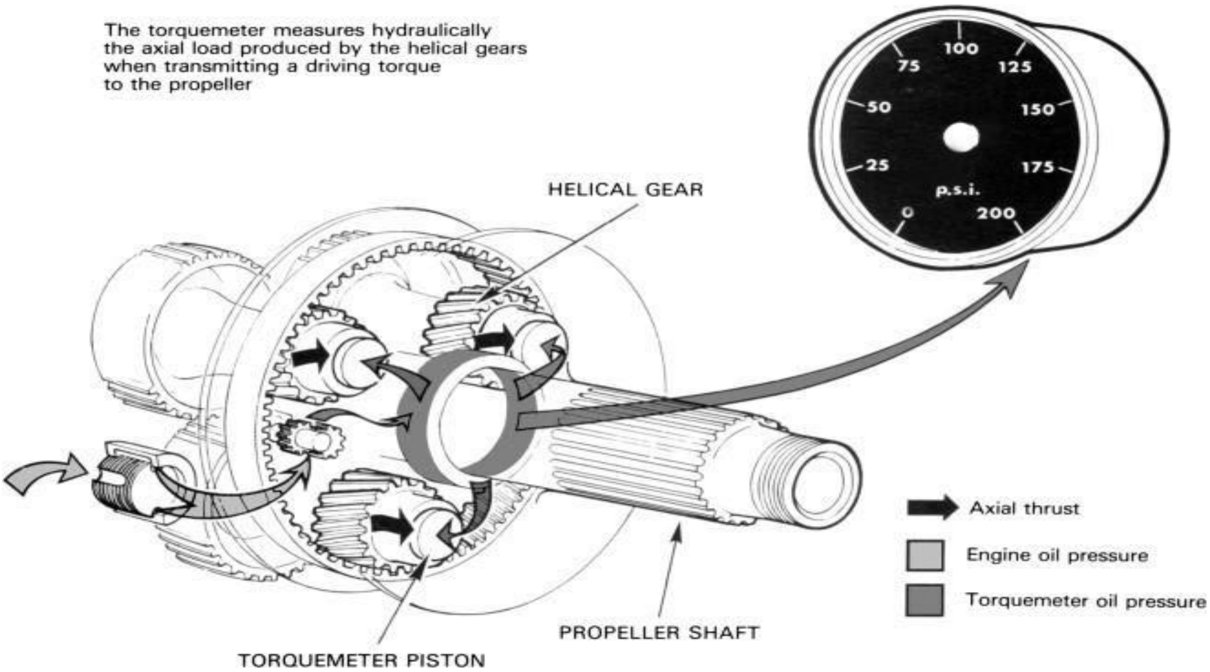
4. It multiplies the torque of the engine and transmits it to the transmission. It almost doubles the output torque.



HOW TORQUE WORKS IN A VEHICLE

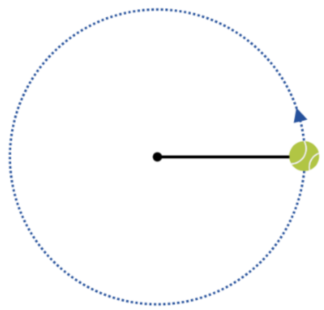
- All engines, whether gasoline or hybrid, generate a certain amount of both horsepower and torque. The two are related to one another, and both express engine output in different ways. Torque is even used in the calculation of an engine's horsepower. Both horsepower and torque are measured to give shoppers a sense of the performance they might expect from their vehicle.
- Engines in mainstream cars and trucks typically generate 100 to 400 lb.-ft of torque. That torque is created by the pistons within an engine as they reciprocate up and down on the engine's crankshaft, causing it to rotate (or twist), continuously. This torque is then transferred to the vehicle's wheels through the transmission and drivetrain.
- Torque output is a result of many variables, including the size of the engine, and how it is designed to operate.
- In simple terms, the more torque an engine has, the better suited it is to hard work such as towing, hauling or climbing steep grades. This is why torque is often the figure of utmost importance when moving something big and heavy, like a truck with a trailer attached.
- Engine Torque is used to indicate the power that is developed by a turbo-propeller engine, and the indicator is known as a torque meter. The engine torque or turning moment is proportional to the horsepower and is transmitted through the propeller reduction gear.
- A torque meter system is shown in below. In this system, the axial thrust produced by the helical gears is opposed by oil pressure acting on a number of pistons; the pressure required to resist the axial thrust is transmitted to the indicator.

The torquemeter measures hydraulically the axial load produced by the helical gears when transmitting a driving torque to the propeller



Rotational Inertia

Rotational inertia is a property of any object which can be rotated. It is a scalar value which tells us how difficult it is to change the rotational velocity of the object around a given rotational axis.
Rotational inertia of an object depends on its mass and the distribution of that mass relative to the axis of rotation. Rotational inertia is given the symbol I. For a single body such of mass m, rotating at radius r from the axis of rotation the rotational inertia is $I = mr^2$.



: A tethered tennis ball rotating about a central point.

Fig: Here single body is a tennis body

Calculating the total rotational inertia for any shape about any axis can be done by summing the rotational inertia of each mass. $I = m_1 r_1^2 + m_2 r_2^2 + \ldots.$
Rotational inertia is commonly known as moment of inertia OR second moment of mass because it depends on the length of the moment arm squared.

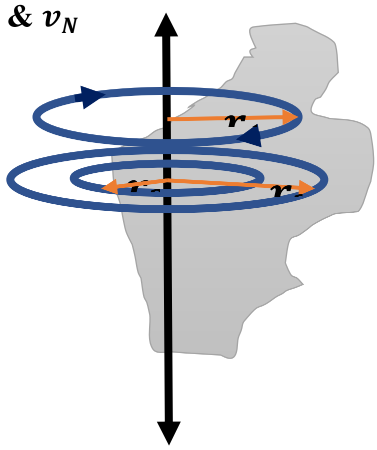
To derive an equation for inertia for system of particles:
Consider N number of point objects of masses $\boldsymbol{m_1, m_2, m_3 \ldots\ldots \;\&\; m_N}$ situated at position vectors $\boldsymbol{r_1, r_2, r_3 \ldots\ldots \;\&\; r_N}$ respectively from origin.
Let the given body performs rotational motion with angular speed "$\boldsymbol{\omega}$", as shown in diagram.
From diagram,

$$\boldsymbol{r_1 = x_1 + y_1 + z_1}$$
$$\boldsymbol{r_2 = x_2 + y_2 + z_2}$$
$$\boldsymbol{r_3 = x_3 + y_3 + z_3}$$

On continuing,

$$\boldsymbol{r_N = x_N + y_N + z_N}$$

Then the linear velocity of particles can be given as $\boldsymbol{v_1, v_2, v_3 \ldots\ldots \;\&\; v_N}$
Therefore,



$$\boldsymbol{v_1 = r_1\omega}$$
$$\boldsymbol{v_2 = r_2\omega}$$
$$\boldsymbol{v_3 = r_3\omega}$$

On continuing,

$$\boldsymbol{v_N = r_N\omega}$$

As, kinetic energy of a particle is given by

$$\boldsymbol{K.E._{Rotational} = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2}$$

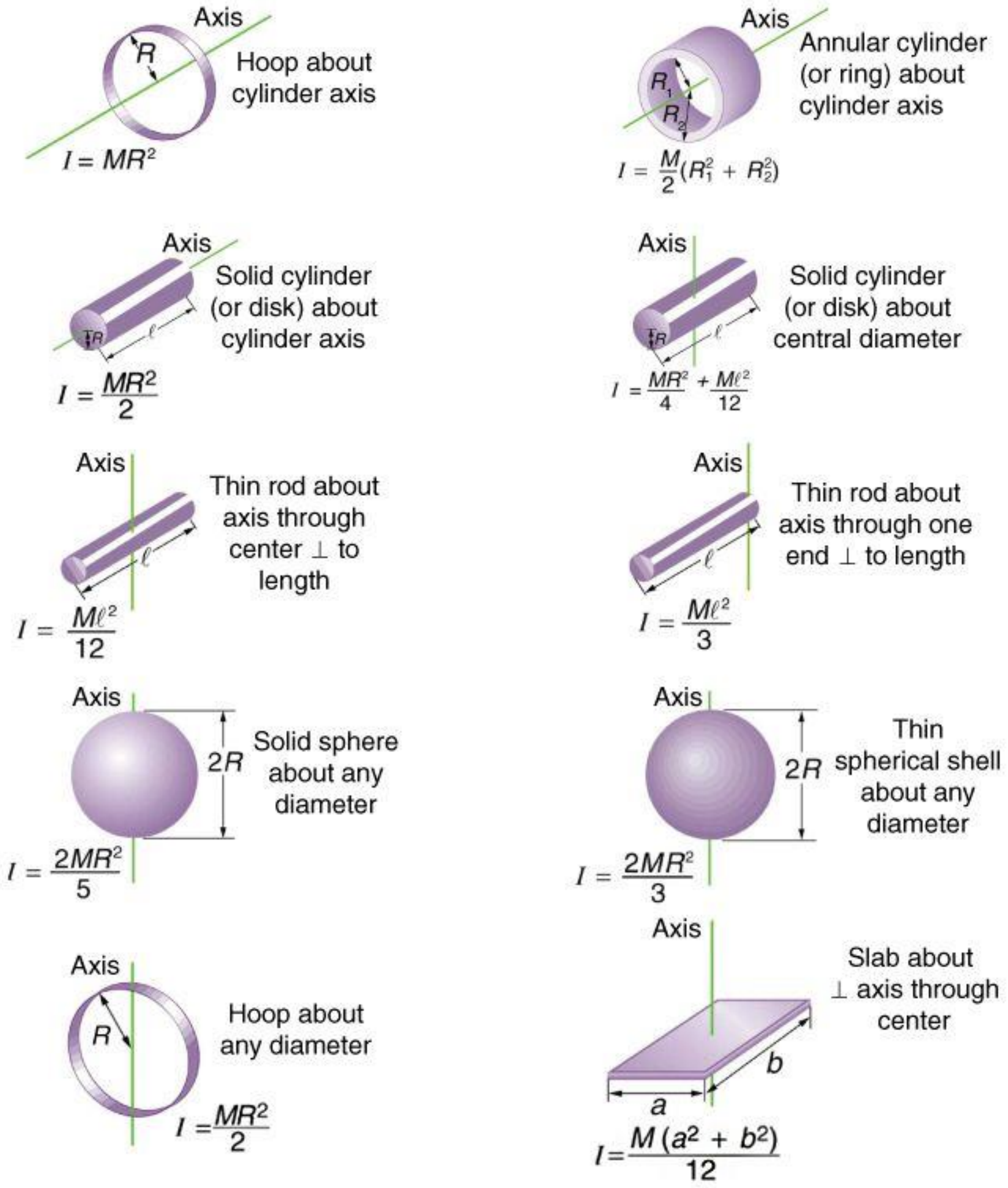
Hence total kinetic energy is given by

$$K.E._{Rotational} = \frac{1}{2}mv_1{}^2 + \frac{1}{2}mv_2{}^2 + \ \frac{1}{2}mv_3{}^2 + \cdots + \frac{1}{2}mv_N{}^2$$

$$K.E._{Rotational} = \frac{1}{2}mr_1{}^2\omega^2 + \frac{1}{2}mr_2{}^2\omega^2 + \frac{1}{2}mr_3{}^2\omega^2 + \cdots + \frac{1}{2}mr_N{}^2\omega^2$$

$$K.E._{Rotational} = \left(\frac{1}{2}mr_1{}^2 + \frac{1}{2}mr_2{}^2 + \frac{1}{2}mr_3{}^2 + \cdots + \frac{1}{2}mr_N{}^2\right)\omega^2 = \frac{1}{2}\left(\sum_{1}^{N} mr_a{}^2\right)\omega^2$$

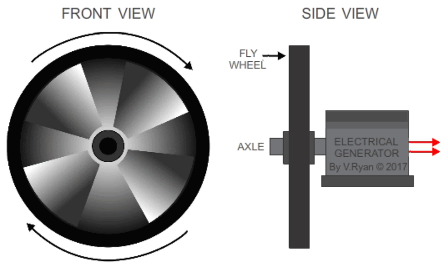
$$K.E._{Rotational} = \frac{1}{2}I\omega^2$$



Application 1: Flywheel

What is a flywheel?

A flywheel is a mechanical device which stores energy in the form of rotational momentum. Torque can be applied to a flywheel to cause it to spin, increasing its rotational momentum. This stored momentum can then be used to apply torque to any rotating object, most commonly machinery or motor vehicles. In the case of motor vehicles and other moving objects, the rotational inertia of the flywheel can have an effect due to gyroscopic motion, resisting a change in the direction of the vehicle. If the mass of the flywheel is significant compared to the overall mass of the vehicle, turning and stopping the vehicle is difficult. This fact means careful design of a flywheel is required for application in moving vehicles.



A flywheel is essentially a very heavy wheel that takes a lot of force to spin around. It might be a large-diameter wheel with spokes and a very heavy metal rim, or it could be a smaller-diameter cylinder made of something like a carbon-fibre composite. Either way, it's the kind of wheel you have to push really hard to set it spinning. Just as a flywheel needs lots of force to start it off, so it needs a lot of force to make it stop. As a result, when it's spinning at high speed, it tends to want to keep on spinning, which means it can store a great deal of kinetic energy. You can think of it as a kind of "mechanical battery," but it's storing energy in the form of movement rather than the energy stored in chemical form inside a traditional, electrical battery.
Flywheels come in all shapes and sizes. The laws of physics tell us that large diameter and heavy wheels store more energy than smaller and lighter wheels, while flywheels that spin faster store much more energy than ones that spin slower.
Modern flywheels are a bit different from the ones that were popular during the Industrial Revolution. Instead of wide and heavy steel wheels with even heavier steel rims, 21st-century flywheels tend to be more compact and made from carbon-fibre or composite materials, sometimes with steel rims, which work out perhaps a quarter as heavy.

The physics behind flywheel
Things moving in a straight line have momentum and kinetic energy because they have mass and velocity. In the same way, rotating objects have kinetic energy because they have what's called a moment of inertia and an angular velocity. Moment of inertia is the equivalent of mass for spinning objects, while angular velocity is like ordinary velocity only going around in a circle.

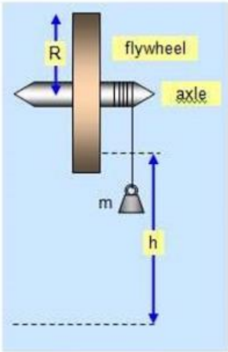


Figure 1

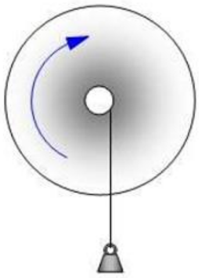


Figure 2

The energy stored inside of a flywheel is related to the speed of its rotation and its moment of inertia. The following equation shows the energy of a flywheel

$E_{rotation} = \frac{I\omega^2}{2}$ where,

- $E_{rotation}$ is the energy stored in the rotational momentum (Joules, J)
- I is the object's moment of inertia (kilogram * meters^2, kgm^2)
- ω is the rotational speed (radians per second, rad/s)

The total stored energy in a flywheel depends on the rotational speed (ω) or the inertia (I) of the flywheel. A typical flywheel consists of a solid cylinder with radius r and mass m. The moment of inertia, I, of this type of flywheel is given by the equation:

$$I = \frac{mr^2}{2}$$

"Moment of inertia" sounds horribly abstract and confusing, but it's much easier to understand than you might think. What it really means is that, from the viewpoint of kinetic energy and momentum, the effective mass of a spinning object depends not just on how much actual mass it has but on where that mass is located in relation to the point it's spinning around. The further from the centre the mass is, the more effect it has on the object's momentum and kinetic energy—and we quantify that by saying the mass has a higher moment of inertia. So, a large diameter, lightweight, spoked flywheel with a very heavy steel rim might have a higher moment of inertia than a much smaller, solid flywheel, because more of its mass is further from the point of rotation.

To change the inertia of the flywheel, the radius or mass of the flywheel must be changed. There are clear restrictions for increases in these two properties. If the mass of the flywheel is significant compared to the overall weight of the vehicle, the gyroscopic effect will make turning the vehicle difficult. Since most flywheels will need to fit inside of another design, increasing the radius of the flywheel is restricted by the overall size of the system it is used in.

Applications of Flywheel

1. The flywheel is also used as a part of the clutch mechanism and fluid drive unit. The outer edge of the flywheel has forged teeth to mesh with the electric cranking motor-driven pinion when the engine is being cranked to start it.

2. Modern flywheel developed by NASA

A modern flywheel developed by NASA for use in space. Note how the silver-coloured centre of the wheel is mostly empty space and spokes, while the mass of the wheel is concentrated around the rim. This gives the wheel what's known as a high moment of inertia and allows it to store more energy.

3. Water Wheel

Water wheels use the simple flywheel principle to keep themselves spinning at a steady speed. This is a model of an undershot water wheel.

Application 2: Tornadoes

Tornadoes are weather phenomena that are examples of physical vortexes.

A tornado is a violent windstorm with a twisting, funnel shape cloud and is usually spawned by thunderstorms when cool air and warm air meet, forcing warm air to rise quickly. Damage from tornadoes are due to high speed winds and flying debris.

When a pocket of air is heated, it expands and the volume increases, thus density decreases. And if the density of the air pocket is less than the density of the surrounding cooler air it will rise. Rising air in the centre meets less resistance because it is surrounded by air that is also rising. Rising air creates a vacuum causing cooler air from the sides to move in to replace the rising air. This causes a wind which further pushes the rising air, which leads to the formation of such storms.

The physics described by a rotating observer it's different from that described on a rotating system. For example, hurricanes are due to the rotating motion of atmosphere and oceans and there is no way to introduce them by means of a rotating observer. So, in this case inertia "produces" the physical effect.

Rotational Momentum

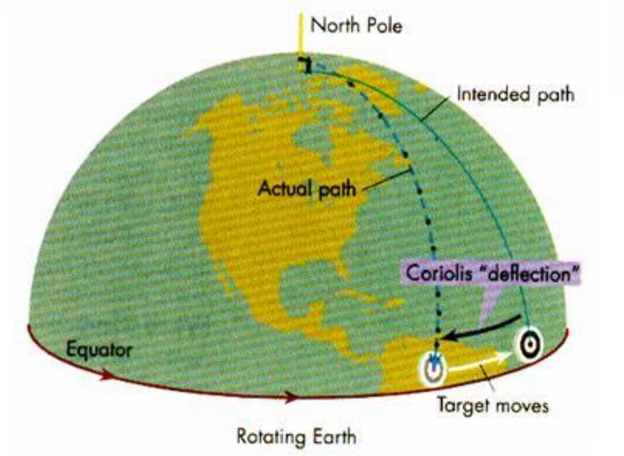
- The rotational momentum of a spinning object depends on both its rotational inertia and its rotational velocity (how fast it is spinning)
- If either the rotational inertia or rotational velocity changes, the other parameter must also change to keep the rotational momentum constant.
    - If the rotational inertia changes the rotational velocity must also change.
    - If the rotational inertia increases, the rotational velocity must decrease.
    - If the rotational inertia decreases, then the rotational velocity must increase.
- Technical term for such a storm is mesocyclone.
- Intense updrafts stretch the mesocyclone vertically.
- As it is stretched upward it gets increasingly narrower.
- As it gets narrower, its rotation speed increases.
- When the radius of the tornado is bigger the rotational inertia will also be bigger.
- When the radius decreases near the ground the tornado will spin faster to keep the rotational momentum constant.

Factors on which tornados depend:

<u>*Coriolis Force*</u>

The Coriolis Force is an inertial force that was described by the French engineer-mathematician Gustave Gaspard Coriolis in 1835.

In a rotating frame of reference, it is an inertial force acting to the right of the direction of movement when rotating counter-clockwise and to the left of the direction of movement when rotating clockwise. It occurs because the Earth rotates eastward and it rotates faster as you approach the Equator and slower at the poles.



The Coriolis force is very, very weak and plays an insignificant role in the spinning of water in a sink or a toilet. The way the water spins is more likely due to the oval shape of the bowl or the off-centre drain. On the scale of large storms and hurricanes, the Coriolis force causes air to rotate around the centre in a cyclonic direction (counter clockwise in the northern hemisphere, clockwise in the southern hemisphere).

Rotational Force

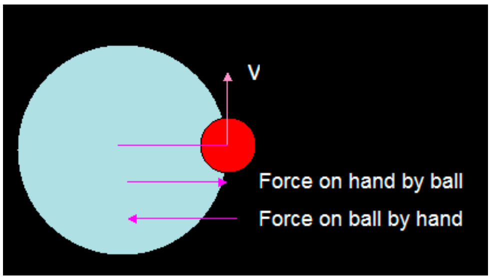
Acceleration is the change of velocity in a short period of time: $a = \Delta v/\Delta t$

When an object is rotating around something, the acceleration force to keep it rotating is:

$F = ma = mv^2/r$

The acceleration causes the direction of velocity to continually change to keep it rotating.

For example: when you swing a ball on a string, you exert a force on the string, causing a tension, which then exerts a force on the ball. At the same time, the ball exerts an equal amount of force in the opposite direction. The ball exerts a force on the string, causing a tension, which then exerts a force on your hand.



Angular Momentum

Ordinary linear momentum is a measure of an object's tendency to move at constant speed along a straight path. Linear momentum depends on speed and mass.

When objects moved in curved paths, we can generalize the idea of linear momentum to something called angular momentum, an object's tendency to spin.

Like linear momentum, the total angular momentum of an isolated object is conserved.

Imagine an object with mass, rotating around an axis. Here, the angular momentum is the product of its mass, velocity, and radius (the distance from the axis point around which the object is spinning around).

$L = mvr$

Note that this applies to an object that has its mass at the distance r from the axis. However, we can generalize this to an object whose mass is distributed all along the distance r and not just at the end of that distance r.

For example, the body of a figure skater has most of its mass near the axis of rotation, and the mass of his or her arms and hands at a farther distance out.

Let us assume all the mass is at the distance r from the axis of rotation.

Conservation of angular momentum then explains why figure skaters or divers spin faster when they bring in their arms or tuck themselves in a roll.

For angular momentum to conserve in a spin, the angular momentum before must equal the angular momentum after. The mass of the figure skater doesn't change. So, if the radius decreases, then the velocity increases.

Similarly, the same can be applied to a tornado.

$$L_{before} = L_{after}$$

$$m\, v_i\, r_i = m\, v_f\, r_f$$

Application 3: Figure Skating

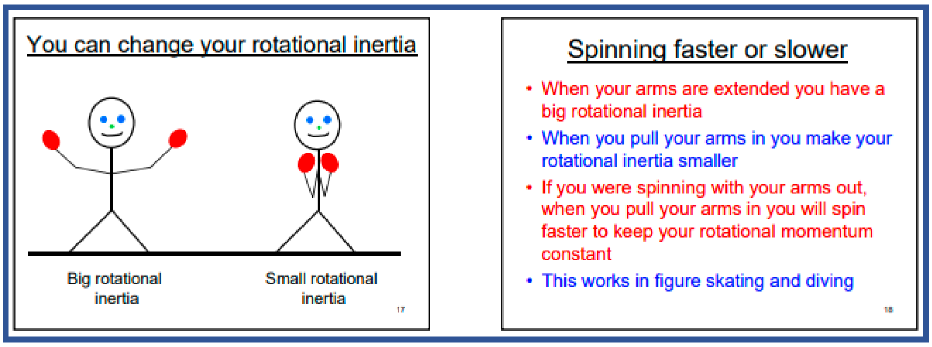
In figure skating, rotational inertia takes place as a skater is spinning or jumping because in order to perform the required amount of revolutions spinning on the ice or in the air, they must decrease their moment of inertia. This can be acquired by pulling their arms in closer so they reduce their average radius, or average distance to the centre.

Since the moment of inertia acts as a form of resistance, a skater can increase their angular velocity (rpm) by reducing their average radius to decrease their moment of inertia. This means when skaters have their arms out = larger average radius, resulting in a larger moment of inertia = slower angular velocity. When skaters pull their arms in = smaller average radius, resulting in less rotational inertia = more angular velocity.

A STUDY

A skater's every twist, turn and leap begins with balance. Balance relies on being able to keep your centre of mass directly over the point of contact with the ice. For the human body, the centre of mass tends to be a bit below the navel. Through glides, spins, take-offs and landings, a figure skater has to keep their centre of mass aligned with a foot on the ice—or can tumble.

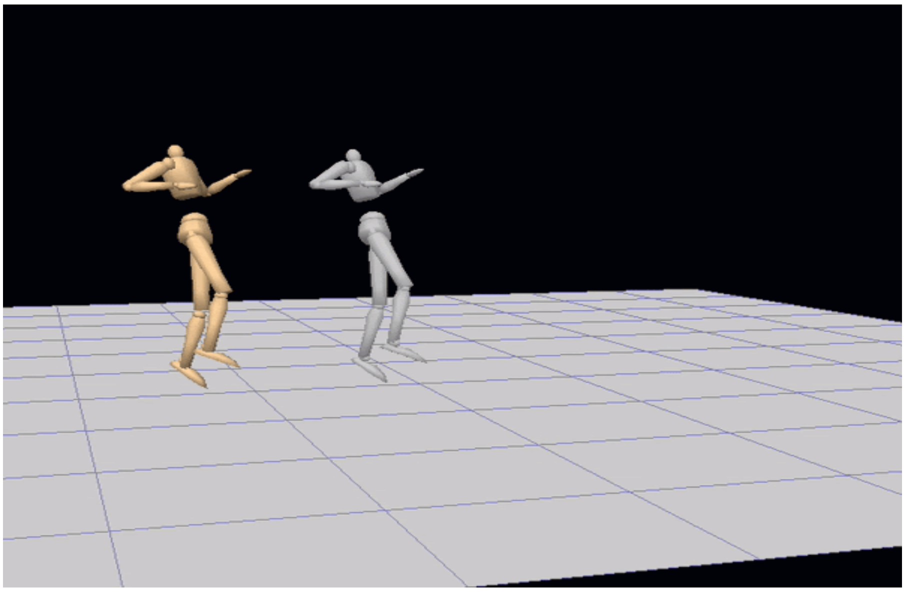
Moment of inertia plays a major role. When a skater performs a spin, they control their rotational speed by pulling their arms in to decrease the moment of inertia and speed up rotation or spreading them out to decrease moment of inertia and slow rotation. This increase is due to a principle called the conservation of angular momentum. A higher moment of inertia corresponds to a lower rotational speed, and a lower moment of inertia corresponds to a higher rotational speed.



“It amounts to three components: how much angular momentum do you leave the ice with, how small can you make your moment of inertia in the air, and how much time you can spend in the air,” says James Richards on improving jump techniques, a professor of kinesiology and applied physiology at the University of Delaware.

At first, the lab had some difficulty translating these findings into advice for skaters. All mathematics were taken into consideration & a simple construct was made; they took high-speed videos of the skaters and transferred that data to an avatar of the skater. Then they would go in and tweak the body position at the point of the jump where the skater had some room to improve. The skater could then see the comparison between what they did and what the jump would look like with some small modifications. Skaters still have to practice getting used to the changes, but the visualization tools help them know what they should be working on.

Moreover, researchers gave skaters small hand weights; so, when the skaters brought their arms in, there was a bigger change in moment of inertia, which gave their rotational speed a boost. For instance, in an office chair, if one starts with books or any weights in your hands, you will speed up even more when you pull your arms in.

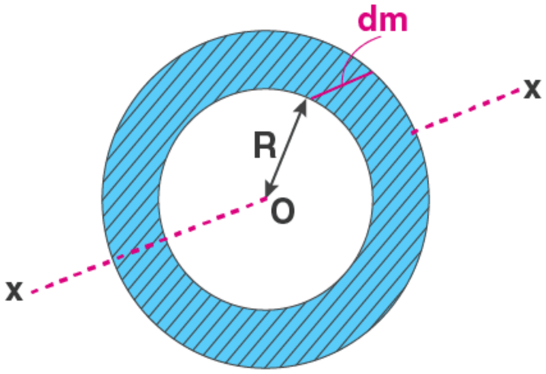


In order to jump higher, skaters might build strength, which could cause them to gain muscle mass. That extra mass could further increase their moment of inertia, slowing them down in the air. “You can lose more from the increase in moment of inertia than you gain from increased time in the air,” Richards says. In other words, achieving balance on the ice takes its own balance.

Researchers gave skaters small hand weights; when the skaters brought their arms in, the increased weight meant there was a bigger change in moment of inertia, which gave their rotational speed a boost.

Application 4: Merry-Go-Round

Moment of Inertia of a Circular Ring about its Axis



The moment of inertia of a circular ring about its axis (I) = MR^2.

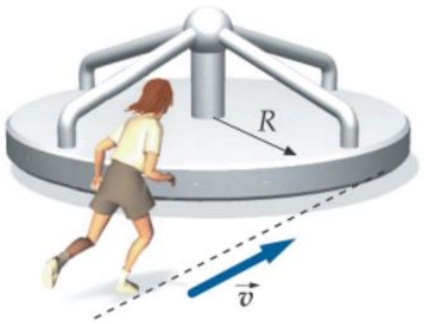
Rotational Inertia in Merry-Go-Round: -

For an extended object (like the merry go round), the angular momentum is (in the scalar form):

$$\mathbf{L_{object} = I\omega}$$

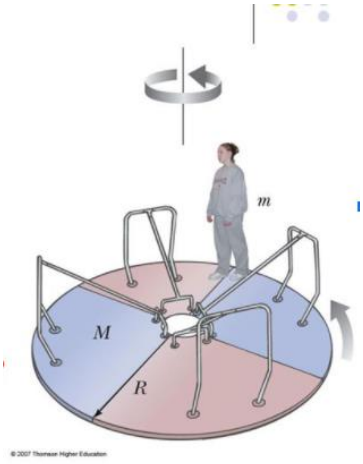
Here, *I* is the moment of inertia for that object (rotational mass). Basically, it depends on the mass of that object, the size, and how the mass is distributed about the axis of rotation. ω is the angular velocity of the object.

A playground merry-go-round has a horizontal circular platform. The radius of the platform is R and the moment of inertia of the entire merry-go-round about the axis of rotation is I. The merry-go-round is initially at rest, but may route on a frictionless axle. The z axis coincides with the axle. A person with mass m, runs toward the edge of the merry-go-round with velocity $v = v_i$ and grabs onto the edge of the merry-go-round, which causes the merry-go-round to rotate.



Conservation of angular Momentum:

- The moment of inertia of the system is the moment of inertia of the platform plus the moment of inertia of the person
- Assume the person can be treated as a particle.
- The rotational inertia of the merry-go-round remains unchanged throughout the whole event. But as the person moves in toward the axis of rotation, the rotational inertia of the person decreases, therefore, in order to maintain a constant angular momentum of the system, the angular velocity of the system has to increase.



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