Double Auctions in Markets for Multiple Kinds of Goods

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Two-Sided Markets

Both buyers and sellers have private valuations:

Image credit:
Wikipedia “Wireless network”

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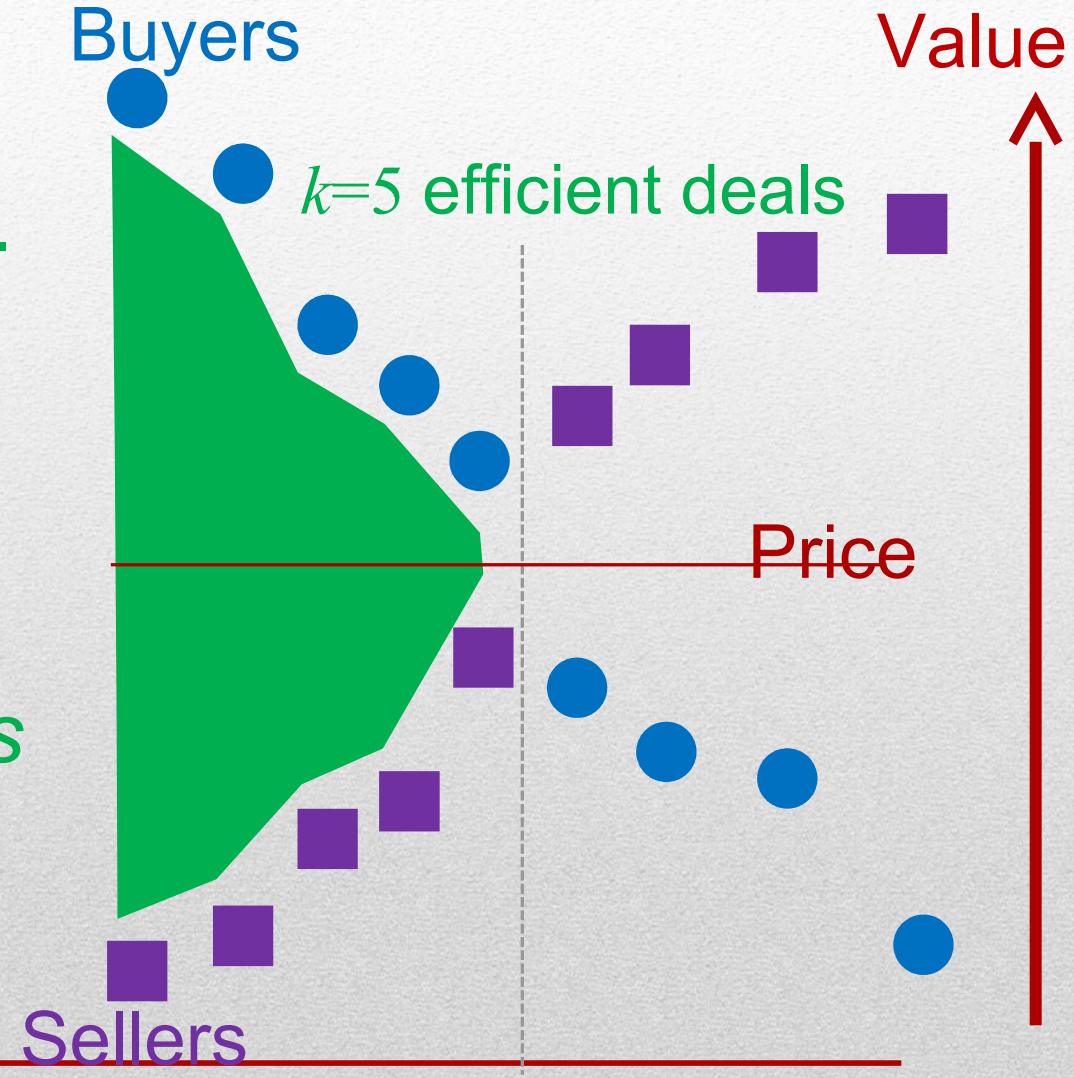
Wikipedia “Haggling”

http://youtube.com/watch?v=7TxLKH0ortw&t=13s

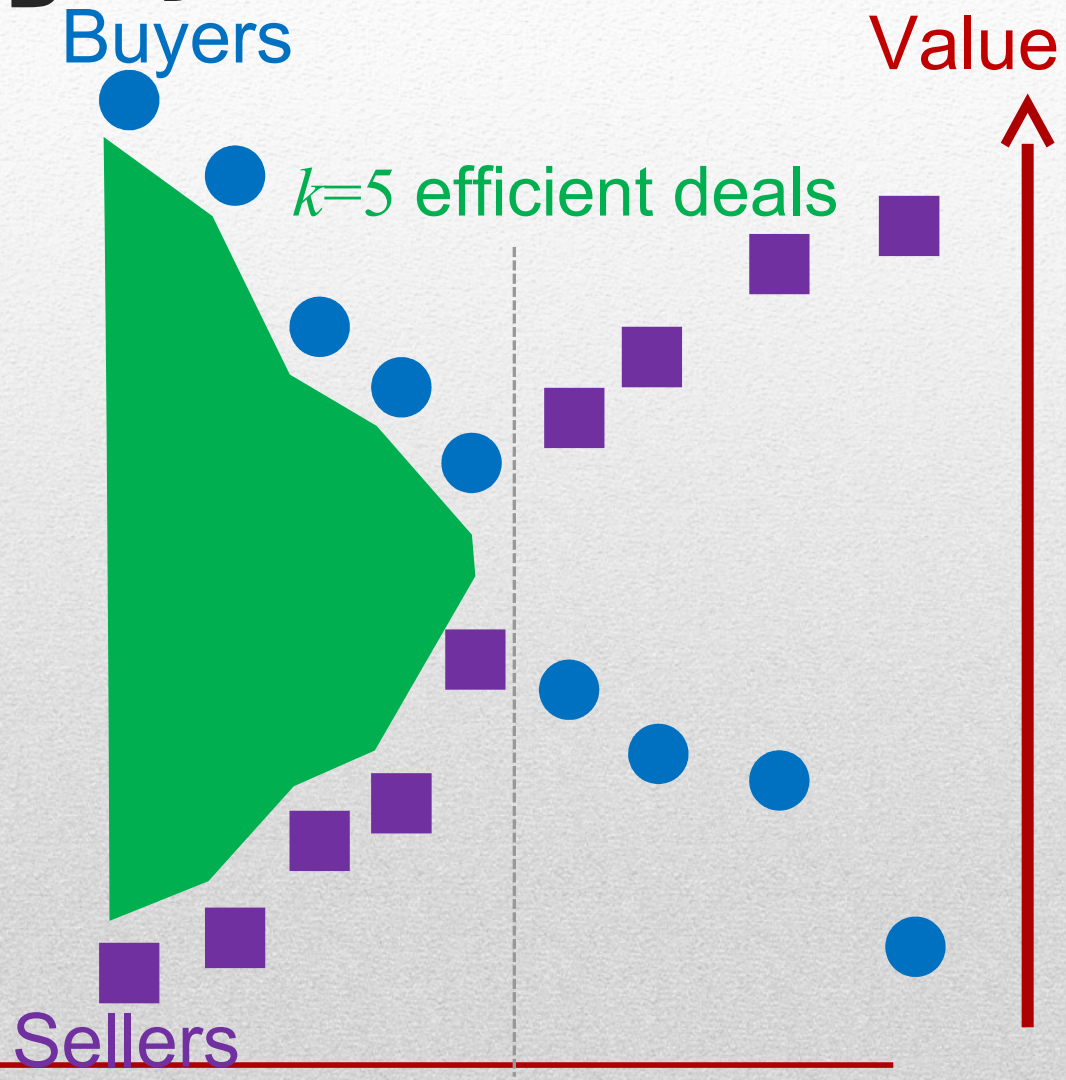
Price-Equilibrium (Walras 1896)

- ✓ Maximum gain-from-trade (GFT).
- ✓ Exists in markets for multiple kinds of goods if all valuations are *Gross-Substitutes*
  (= no complementarities)
- ✗ Not truthful



Impossibility (Myerson & Satterthwaite 1983)

In a two-sided market, a **truthful** mechanism that attains the **maximum gain-from-trade** must run at a deficit (= requires subsidy).



Market Design Goals

1. Truthful;
2. Budget-balanced (no deficit);
3. *Asymptotically*-maximal GFT (approaches maximum when market size approaches ∞);
4. Prior-free.

Some Previous Mechanisms

	Truth-ful	GFT / Max-GFT	Multiple kinds of goods?
Price-Equilib. (Walras, 1896)	No	1 (1st welf. thm)	Yes, gross-substitute vals.
McAfee (JET 1992)	Yes	$1 - 1/k$ (asymp. opt.)	No – only 1-kind 1-unit.
Colini-Baldeschi et al (EC '17)	Yes	$1/6$	Yes – buyers only.
Gonen & Egri (AAMAS '17)	Yes	$1/n$	Yes – buyers and sellers.
MUDA (AAAI '18)	Yes	$1-O(\sqrt{\frac{\ln k}{k}})$ (asymp. opt.)	Many units but 1 kind.

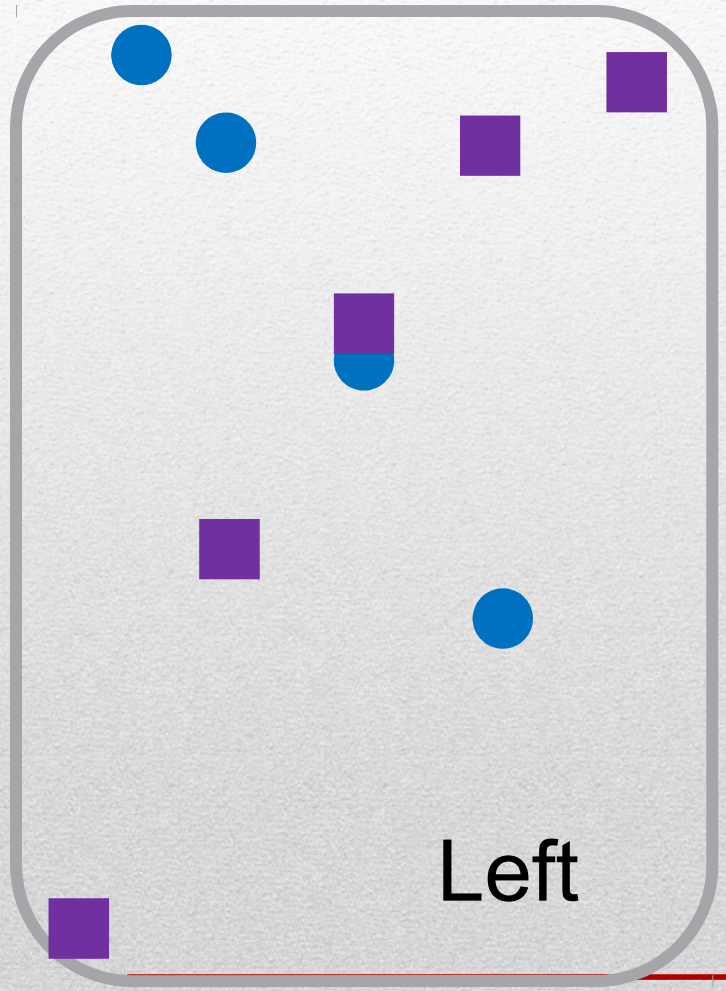
MUDA (AAAI 2018)

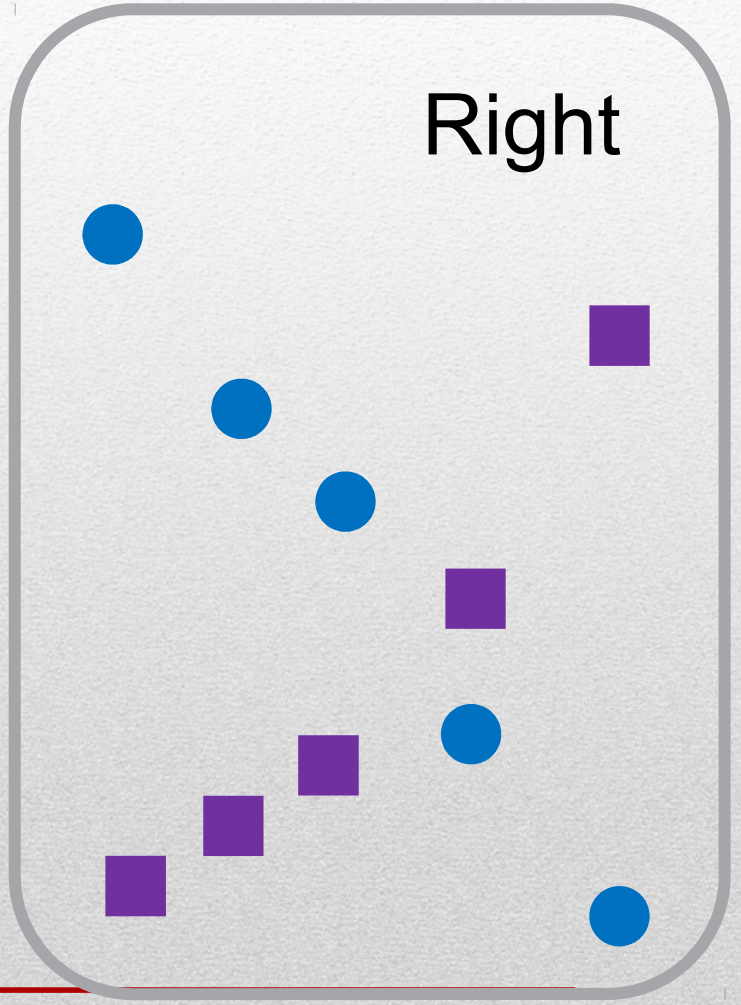
Split traders randomly to “left” and “right”. in each side:

1. Calculate Walrasian equilibrium price (price in which demand=supply).

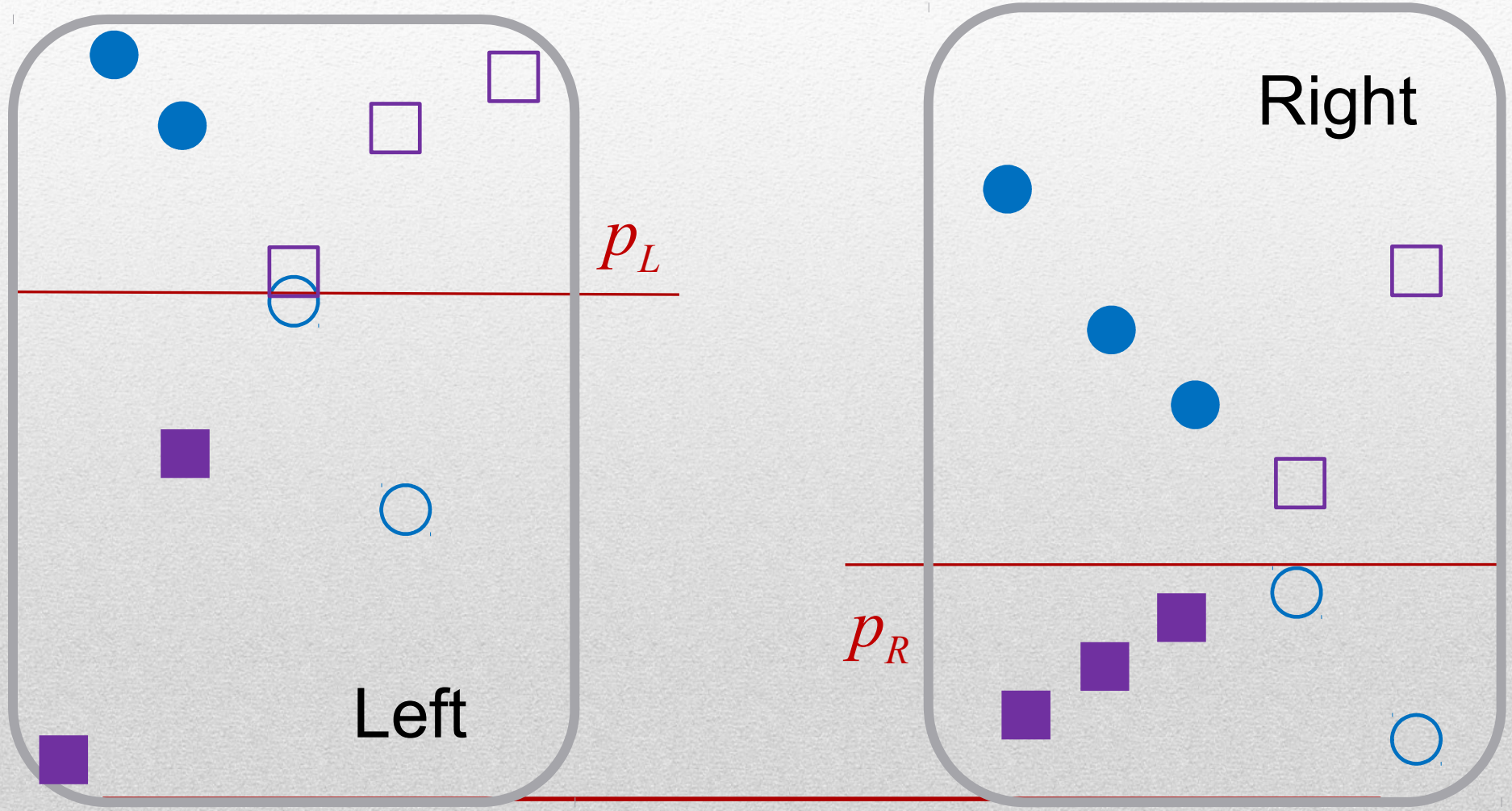
2. Let traders trade at price from other side (guarantees truthfulness).

MUDA 0: Random Halving

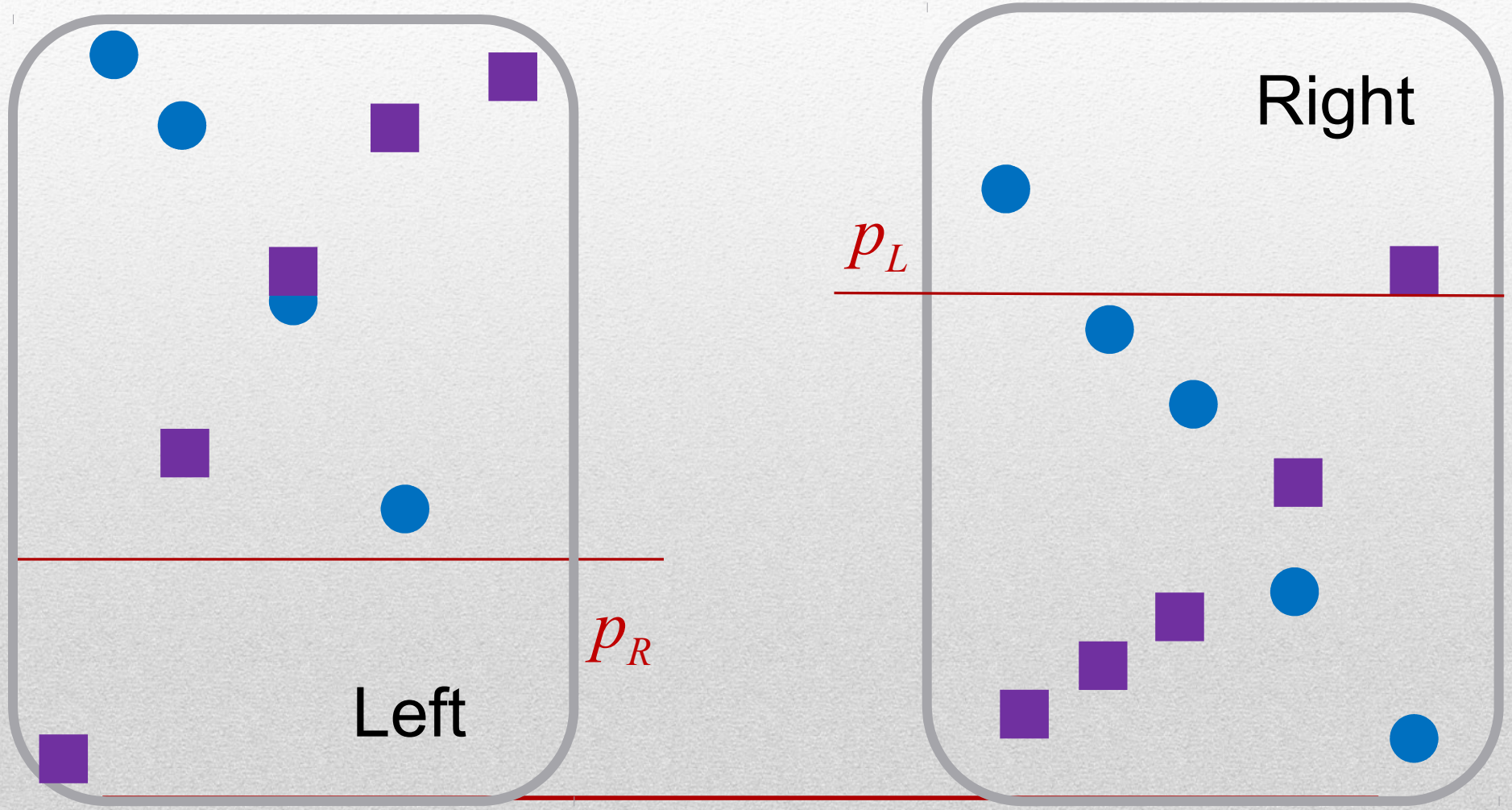




MUDA 1: Equilibrium Calculation

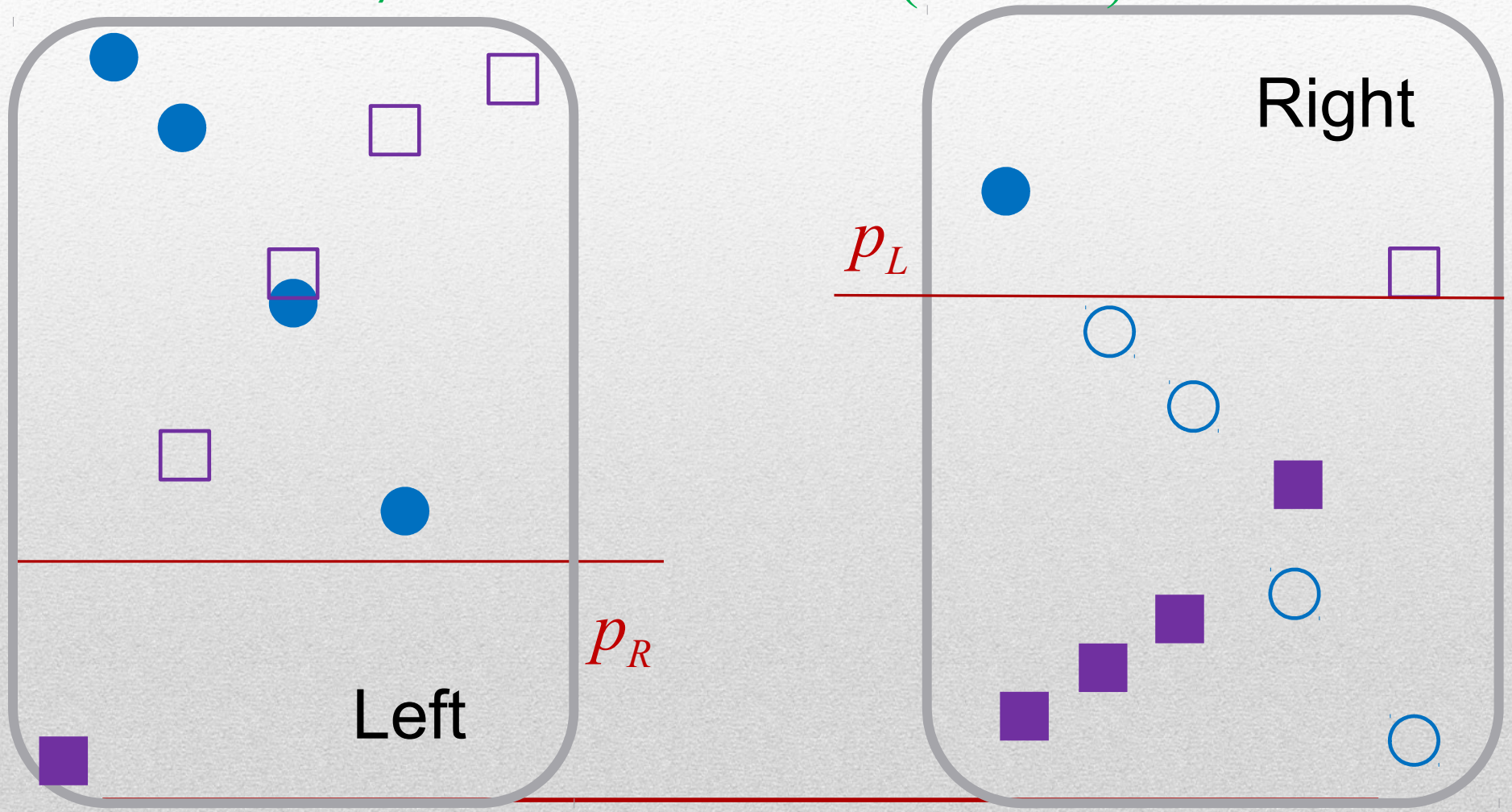


MUDA 2: Posted Pricing



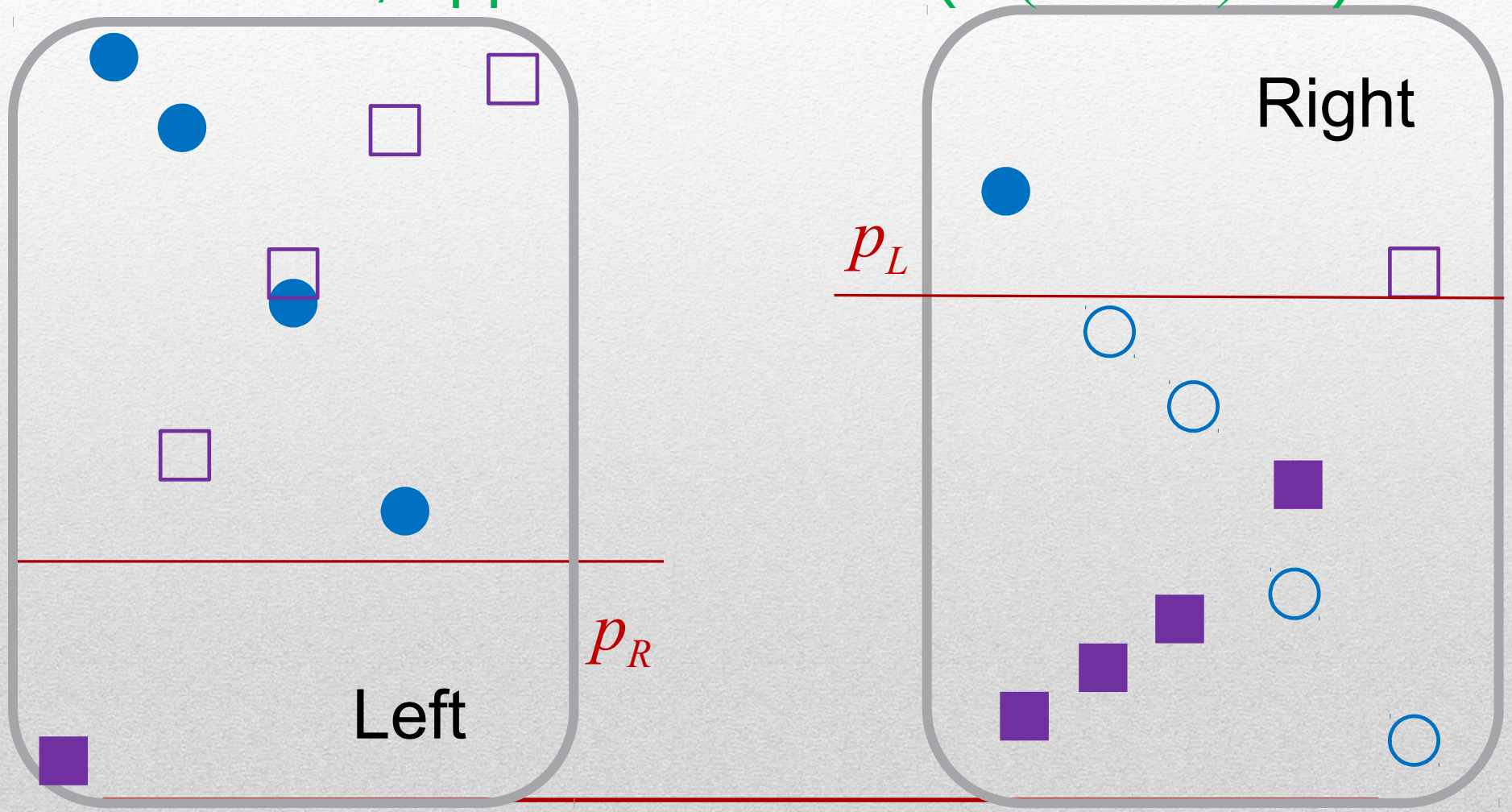
MUDA 2: Trade Loss

With 1 kind, # lost deals is $O(\ln k \sqrt{k})$.



MUDA 2: Trade Loss

With 1 kind, approx. ratio is $O(1-(\ln k \sqrt{k}) / k)$.



Challenges extending to multi-kind

1. Truthfulness challenge: traders may strategize over what kinds to trade, even when prices are fixed.

2. Gain-from-trade challenge: small sampling errors in one kind may have large effects on other kinds.

Current assumptions

1. Each **seller** sells **one kind**.

2. Both buyers and sellers have **gross-substitute valuations**.

MIDA (Multi Item-kind Double Auction)

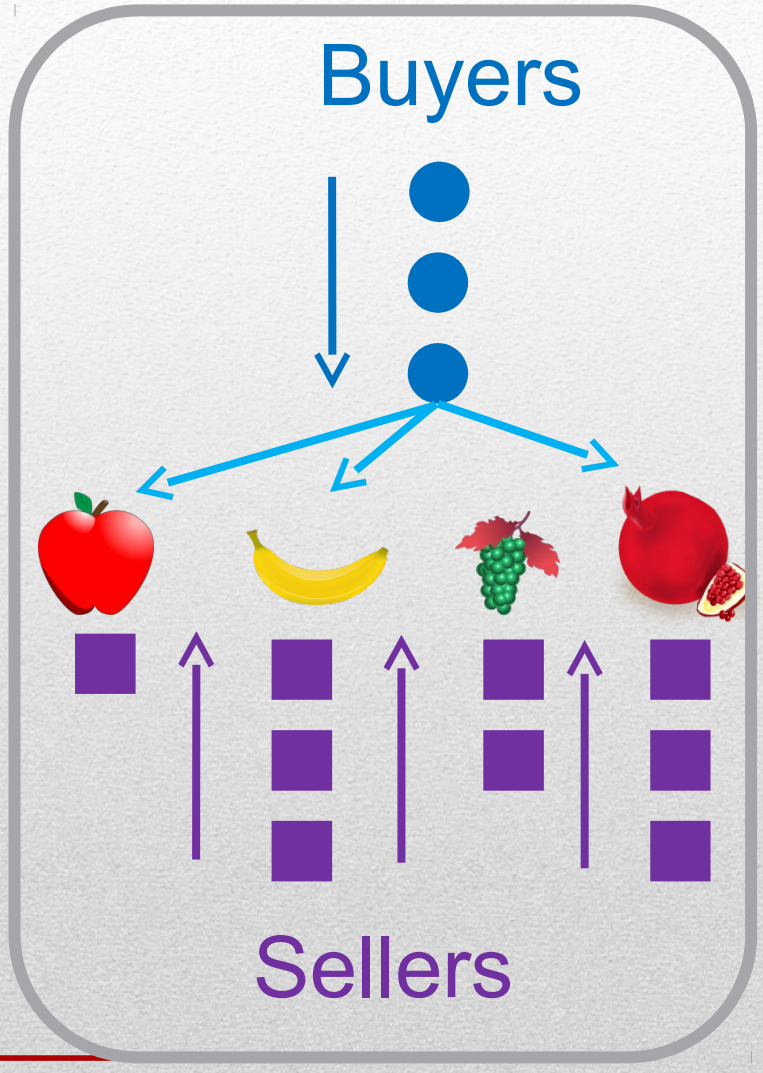
Split traders randomly. In each side:

1. Calculate equilibrium price-**vector**.
2. Traders trade at other price-vector:

- Order sellers of same good randomly.
- Order buyers randomly.
- First buyer buys a best bundle from first sellers and goes home.
- First seller may go home at any time.

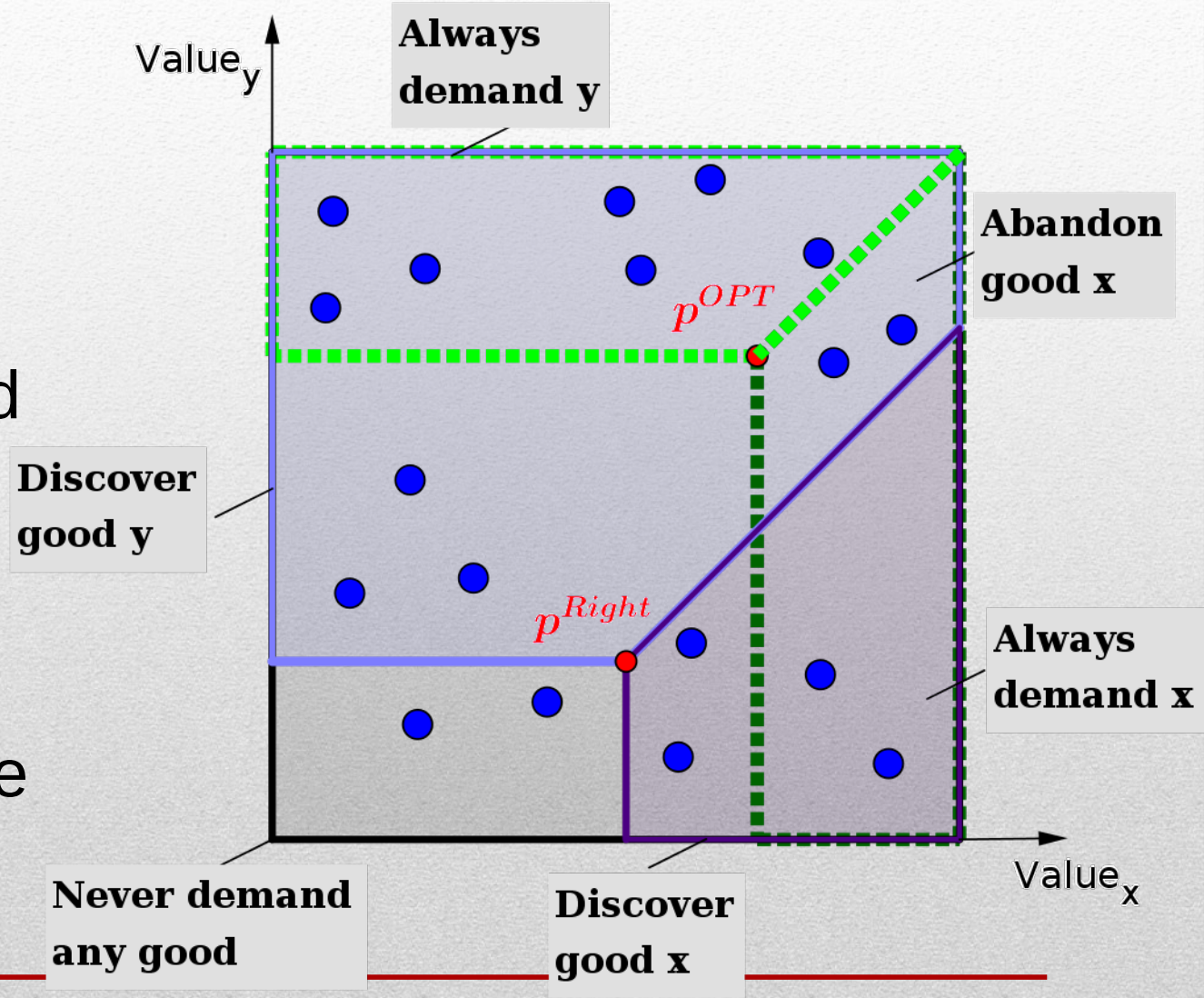
Truthfulness challenge

Theorem 1. If each seller sells multiple units of a *single* kind, and all buyers and sellers have gross-substitute valuations, then MIDA is truthful (ex-ante and ex-post).



Efficiency challenge

Lemma (downward-demand-flow). If a GS agent abandons a good whose price decreased, he must have discovered a good whose price decreased even more.



Efficiency challenge

Theorem 2. If all traders have gross-substitute valuations, then the expected competitive ratio of MIDA is at least:

$$1 - 18 \cdot M^{2G+2} \cdot c \cdot \frac{\ln(10Gk_{\max})}{\sqrt{k_{\max}}} - o(1/k_{\max})$$

G = number of good-kinds;

M = max #units traded by one person;

$k_{max} = \max_x k_x$; k_x = #units of good x traded in optimal situation; $c = k_{\max} / k_{\min}$.

Previous Mechanisms

	Truthful	$\frac{\text{GFT}}{\text{Max-GFT}}$	Multiple kinds of goods?
Price-Equilib. (Walras, 1896)	No	1 (1st welf. thm)	Yes, gross-substitute vals.
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Colini-Baldeschi et al (EC '17)	Yes	1/6	Yes – buyers only.
Gonen & Egri (AAMAS '17)	Yes	~ 0.5 (simulations)	Yes – buyers and sellers.
MIDA (IJCAI '18)	**Yes**	1 - ... (asymp. opt.)	**Yes in one side;** gross-substitute vals.

Open Question

Is there a double-auction mechanism that is:

1) truthful,

2) budget-balanced,

3) attains asymptotically-optimal GFT;

4) allows **both** buyers **and** sellers to trade multiple kinds of goods?

Acknowledments

- Game theory seminar in BIU
- Ron Peretz
- Simcha Haber
- Tom van der Zanden
- Assaf Romm
- Economic theory seminar in HUJI
- Econ.&Comp. seminar in HUJI
- Algorithms seminar in TAU
- Game theory seminar in Technion