Trade Finance and the Durability of the Dollar*

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Abstract

We propose a model in which the emergence of a single dominant currency is driven by the need to finance international trade. The model generates multiple stable steady states, each characterized by a different dominant asset, consistent with the historical durability of real-world currency regimes. The persistence of regimes is caused by a positive interaction between the returns to saving in an asset and the use of that asset for financing trade. A calibrated version of the model shows that the welfare gains of dominance are substantial, but accrue primarily during the transition to dominance. We perform several counterfactual experiments to assess potential threats to the dollar's continued dominance.

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1 Introduction

Historically, the international financial system has been characterized by long-lasting periods in which one "dominant" asset facilitates the majority of international trade and financial flows. Since the 1950s, this role has been played by the US dollar and safe US assets, while the British pound enjoyed dominance before that, and the Dutch Guilder was the dominant asset throughout the 18th century. This pattern suggests that the international financial system favors the emergence of extended periods of currency dominance. The potential for such dominance has been of great interest to prior literature, but the question of why currency regimes often prove so durable has received comparatively little attention.

This paper provides a theory of stable dominant currency regimes. The theory relies on two frictions in international trade, imperfect contract enforcement across borders, which creates a need for collateral guarantees in trade, and a financial friction in obtaining the needed collateral. Trading firms seek to borrow collateral via frictional trade finance markets, generating a positive feedback between the use of an asset to guarantee trade and households' incentives to save in that asset. Firms also benefit from operating with the same collateral as their trade counter-parties, which reinforces the household-firm complementarity. The interaction of these forces gives rise to multiple steady states, each characterized by a different dominant asset and surrounded by a region of unique, stable equilibrium dynamics.

Our model economy is composed of three regions: the United State, the Eurozone, and a continuum of small, open, rest-of-world economies. In each country, there is a continuum of trading firms seeking to engage in profitable transactions with traders from other countries. Imperfect contract enforcement across borders requires the trading firms to collateralize their transactions with safe assets that serve as performance guarantees. To obtain collateral, firms seek an intra-period loan of either US or Eurozone bonds in domestic bond-specific search and matching markets. On the other side of these credit markets are households, who form optimal portfolios and offer intra-period loans of their assets for a fee.

Other things equal, search frictions encourage firms to look for trade financing in the credit market that is less tight, and thus apply for a loan of whichever asset is relatively plentiful in the portfolio of their domestic households. Conversely, households know that an asset that is actively used by traders is more likely to be loaned in the credit markets and earn the associated fee. Thus, the incentives of households and trading firms reinforce each other: high use of an asset in trade finance encourages households to save in that asset, while higher holdings of the asset encourage firms to seek it for financing their trade activities.

We begin our analysis by analytically characterizing the key forces driving dominance and stability within a simplified model. First, we show that the feedback between households and trading firms leads to steady-state multiplicity, including a dollar-dominant steady state in which US safe assets are both the dominant saving vehicle of rest-of-world households and the dominant means of trade finance. The other steady states are a mirror-image euro-dominant steady state, and a "multi-polar" steady state in which portfolios and trade finance use are split equally across the two assets.

While the firm-household interaction can generate dominant-currency steady states, these outcomes may not be stable. Intuitively, when one asset is dominant, the market for trade finance loans of that asset is relatively congested. Hence, an off-equilibrium shift in the portfolio composition of households can drive firms to shift their finance use away from the dominant asset. To ensure stability, we introduce a currency mismatch cost that is incurred by trading pairs who use different types of collateral. This cost can be micro-founded as the expected cost of default by one of the transaction counterparties, since mismatched collateral means the two promises may not be equivalent in all states of the world.[1]

Collateral mismatch costs introduce a second strategic complementarity, now in the type of financing sought by trading firms in different countries. On its own, this complementarity would give rise to sunspot equilibria, since trade finance choices would depend only on what firms' expect their foreign counterparties to choose. Frictions in funding markets limit this indeterminacy, however, because firms' funding choices are also then influenced by the relative availability of different types of trade financing.

Crucially, the interaction between the two complementarity mechanisms – one between households and firms, the other among firms – can make dominant-asset steady states locally stable. Intuitively, wide holdings of an asset make it easier for firms to source, giving it an initial advantage in firms' currency choices. This advantage is then reinforced by firms' cross-country coordination incentives, uniquely anchoring their currency choice on the widely-held asset. Importantly, neither mechanism can achieve this on its own: Without both sources of complementarity, equilibrium is either locally unstable or subject to sunspot indeterminacy.

Having illustrated the key mechanism, we embed it within a rich general equilibrium model, and explore its quantitative implications in and out of steady state. We calibrate the model to match target moments on the size of government debt, trade, currency denomination of trade finance, and import markups. The model exactly matches the target moments and a number of additional, non-targeted moments are also closely aligned with the data.

Using the calibrated model, we first show that dominant-currency steady states in this richer, quantitative framework are indeed *dynamically* stable, and lie within large regions

[1] Amiti et al. (2018) find evidence of such coordination incentives in firm-level trade data from Belgium.

of the state space that uniquely converge to their respective dominant-asset steady state. Within those regions, the equilibrium paths of the economy are determinate (i.e. not subject to sunspot shocks) and the currency regime is uniquely determined by initial conditions. Essentially, the dynamic model tethers the mix of assets used for trade finance to an endogenous state variable, bond holdings, so that the model generates the co-existence of separate dollar- and euro-dominant steady states, each of which has its own unique region of attraction. By contrast, we show that the multi-polar steady state is *unstable*. Hence, the model delivers durable dominant-currency regimes just like in the historical record.

Computing the welfare implications of the model leads to several new insights that showcase the importance of dynamic general equilibrium analysis of currency dominance. The steady-state gain of the dominant country, relative to the other large country, is small: only 0.03% of permanent consumption. At first, this result seems surprising, since the equilibrium interest rate of the dominant asset is roughly one percent lower than that of the other safe asset, reflecting the so-called "exorbitant privilege" associated with dominance.[2] This tension is resolved, however, by observing that in our model the central country necessarily has a significant (and realistic) *negative* net foreign asset position, because dominance requires wide holdings of the central asset by foreigners. Thus, a long run asset imbalance largely offsets the benefits of paying lower interest.

Though dominance has a small effect on long-run consumption, factoring in the transition to dominance dramatically changes welfare conclusions. For example, in the case of a transition from the (unstable) symmetric steady state, the eventual dominant country gains the equivalent of 0.75% of permanent consumption. This happens because, along the transition path, increasing external demand for the assets of the dominant country helps it to fund a temporary boom in consumption. This finding emphasizes the importance of using a dynamic model to evaluate the positive consequences of currency dominance.

We conclude the paper with three counter-factual experiments. First, we analyze the formation of the Eurozone, one of the most important recent developments in the international financial system. Consistent with historical experience, the model shows that starting from a point where the US is the only country furnishing safe assets on a global scale (and thus the dollar-dominant steady state is unique), the introduction of an ex-ante equivalent Eurozone asset is not sufficient to precipitate a currency regime change: While the Eurozone forma-

[2]While our perfect foresight model does not differentiate between covered and uncovered interest parity (UIP), the spirit of our mechanism is most consistent with a violation of UIP. Trade collateral is required to guarantee *future* contract performance, thus the resulting convenience yield is earned by an asset's ability to insure a promised future dollar value, meaning synthetic dollar bonds and US dollar bonds are equivalent to one another, but different from euro bonds.

tion does create a second stable dominant-asset steady state, the world is already dollarized at the euro's introduction, and nothing about the change pushes the world economy away from its dollar-dominated starting point. More broadly, the model implies that we live in a dollar-dominant world today due to the combination of path dependence and the stability of dominance, but the dollar's dominant position is not guaranteed since future shocks could push the economy into the attraction region of the now-stable euro steady state.

In our second experiment, we investigate the implications of a rest-of-world that has grown dramatically vis-a-vis the US. Holding bond supplies constant, we model an increase in the size of the rest-of-world from 45% to 60% of the world economy, while the US falls from 35% to 20%. According to the model, US dollar dominance is the (globally) unique equilibrium when the US is substantially larger than the competing Eurozone block. As the US shrinks, a stable Euro-dominant steady state emerges and thus US dominance is again no longer guaranteed. However, the transition has little impact on the observable US dominance and privilege it earns, as the economy remains in the neighborhood of the dollar-dominant steady state. Looking forward, these first two experiments predict that a new potential challenger, the Chinese renminbi, could threaten dollar dominance if the Chinese economy grows substantially larger than the US *and* the Chinese capital account is sufficiently liberalized (allowing for wider global holdings of renminbi assets).

In our third counterfactual experiment, we consider the consequences of the trade policy of the central country. We find that a trade war in which the US raises tariffs on all imports by 15%, and its trading partners retaliate in kind, disproportionately hurts the US and could be enough to threaten the dominance of the dollar, but does not eliminate it for certain. We find that a potential switch in the dominant-asset in this case would lower US welfare by 1.5% of permanent consumption. Our analysis indicates that US policies favoring free trade have been quite helpful in establishing its preeminent role in the world financial system.

Relation to existing literature

The structure of our model is motivated by both historical accounts of the origins of dollar dominance, which emphasize the role of trade finance (Eichengreen and Flandreau, 2012), and empirical studies showing that the majority of international trade transactions require external financing (Auboin, 2016). Moreover, BIS (2014) documents that outside of the US and the Eurozone, the majority of trade finance is locally sourced via domestic banks, and that trade finance contracts are heavily dollarized, showing that trade is not only denominated in dollars (Gopinath, 2015), but also financed via dollar debt. Lastly, there is also substantial evidence to suggest that trade financing is scarce: Di Capria et al. (2016) estimate an unmet trade finance need of \$1.6 trillion, while a related literature documents how disruptions to trade finance availability cause significant reductions in international trade (e.g. Amiti and Weinstein, 2011; Ahn, 2014; Antras and Foley, 2015; Niepmann and Schmidt-Eisenlohr, 2017; Bruno and Shin, 2019).

The theoretical and empirical literature on currency regimes, which Gourinchas et al. (2019) surveys quite nicely, has explored many distinct properties of dominant international currencies and assets. While this work varies in style and emphasis, much of it can be tied back to one of the three traditional roles of money as a store of value, unit of account, or medium of exchange.

Among these strands, the literature exploring the safety or "reserve" role of dominant assets is comparatively large. Some of this work focuses on asset prices, rationalizing the low returns on safe dollar assets with the US's large role in global consumption risk (Hassan, 2013; Richmond, 2019). Other work aims to explain the high use of dollars in financial markets – e.g. Bocola and Lorenzoni (2017) provide a framework where financial dollarization occurs because of the dollar's unique risk profile, while Brunnermeier and Huang (2018) and Bianchi et al. (2018) explore the role reserve assets play in emerging market crises. Other authors link the special status of the dollar with the US's superior capacity to issue safe assets (Caballero et al., 2008; He et al., 2016), differences in financial development (Mendoza et al., 2009; Maggiori, 2017), or risk-aversion (Gourinchas et al., 2017).

Papers in the unit of account or "currency anchor" line focus on the role of dominant currencies in trade invoicing (Goldberg, 2011; Gopinath, 2015). This literature emphasizes the interaction of nominal price stickiness with pricing complementarities or the denomination of firms' borrowing (e.g. Engel, 2006; Gopinath et al., 2010; Goldberg and Tille, 2016; Mukhin et al., 2018; Eren and Malamud, 2018), but usually does not explain global asset or return imbalances. Authors including Ilzetzki et al. (2019) have also used this mechanism to explain why emerging market central banks "anchor" their currencies to the dollar.

Finally, the literature on medium-of-exchange or "global currencies" centers on search-based theories of money. These works examine the micro-foundations of different trading and payment structures, focusing on the implications for co-existence of multiple currencies (e.g. Matsuyama et al., 1993; Zhou, 1997; Wright and Trejos, 2001; Rey, 2001; Kannan, 2009; Devereux and Shi, 2013; Zhang, 2014; Doepke and Schneider, 2017). Liu et al. (2019) studies instead the co-existence of money and trade finance credit, linking financial development with trade currency choices. Also related are Vayanos and Weill (2008) and Weill (2008), who use search frictions to explain liquidity premia for over-the-counter asset markets.

In our paper, dominance results from the interaction of endogenous liquidity premia and the demand for store of value assets. This is a unique feature of our paper, and links two important aspects of international financial dominance, the instruments of savings and trade finance (not necessarily invoicing or settlement currency, although all three are closely related in the data).

Concurrent work by Gopinath and Stein (2020), which blends the unit of account and safety roles of dominant assets, is perhaps the most closely related to our own. Their model can also generate asymmetric equilibria even when alternative candidate currencies are ex-ante identical in their fundamentals. Nevertheless, the two theories have important conceptual differences, and some key implications are diametrically opposed.

In their model, dollar invoicing of imports makes households across the world prefer saving in dollars. Since Treasuries are scarce offshore (by assumption), local banks have incentives to create locally-sourced dollar assets by offering cheap dollar financing to firms. Attractive dollar financing, in turn, encourages firms to price exports in dollars, closing the feedback loop. By contrast, the complementarity in our theory comes from the interaction between an asset's store of value and liquidity properties: firms prefer to fund their transactions with a more widely-held asset, which generates a liquidity premium on the asset and gives savers a reason to hold it.

Our alternative mechanisms have different implications for what sort of asset is likely to become dominant. Gopinath and Stein (2020) rely on the relative "scarcity" of safe dollar assets to encourage local banks to create dollar-denominated deposits, implying that a high supply of US treasury bills makes the dollar *less likely* to become dominant. By contrast, our model implies that the safe asset in higher supply is the one with a larger region of attraction, and is thus more likely to be dominant. In Section 3.4, we discuss evidence that asset availability is indeed likely to help, rather than hinder, asset dominance.

More broadly, our paper differs from the literature in at least two important respects. First, our model is dynamic, and we emphasize its ability to generate both multiple long-run outcomes (e.g. dollar and euro steady states) and stable equilibrium dynamics. Using our dynamic model, we can thus explain why dominant currency regimes appear so stable in the data, and demonstrate why accounting for dynamics is crucial for evaluating welfare.

Second, we implement our idea in a quantitative, general equilibrium model. Most closely related papers, including Farhi and Maggiori (2016) and Gopinath and Stein (2020), use stylized, two-period models in which some key prices or quantities are fixed. General equilibrium is central to our model's ability to simultaneously match the evidence on US return differentials, net foreign asset positions, and international financial adjustment documented by

Gourinchas and Rey (2007a,b), each of which have important welfare implications.

2 Simplified Model

In this section, we present a simplified version of our model that focuses on the two essential ingredients of our mechanism: the savings decisions of households and the financing decisions of firms. The simplified model allows us to characterize the key forces analytically, and in Section 3, we explore quantitative implications in a rich general equilibrium setting.

The world consists of two symmetric big countries, the United States (US) and the Eurozone (EZ), of equal size $\mu_{us} = \mu_{ez}$, and a continuum of small open economies making up the rest of the world (RW) with total mass μ_{rw}. There are two ex-ante identical assets – a bond issued by the US government and a bond issued by the Eurozone government – each recognized as safe and available in exogenous supply, $\bar{B}$. Both assets serve as saving vehicles and, potentially, as collateral guaranteeing international transactions.

Countries are indexed by $j \in \{us, ez, [0, \mu_{rw}]\}$ and within each country j, there is representative consumer and a continuum of risk-neutral international trading firms. In the simplified model, the only decision of the household is how to allocate its savings between the two available assets and the only decision of a trading firm is which type of asset to seek for financing its trade. We describe the decisions of the two agents types in turn.

Households

Households solve a standard consumption-savings problem, allocating their endowment income between consumption and the two assets. In the simplified model, we assume there is a single consumption good and both assets promise one unit of that good, and we generalize this later. Hence, here ex-antes bonds only differ because of their issuer, yet they may serve different trade financing roles in equilibrium and, therefore, earn different interest rates.

Households in each country $j \in \{us, ez, [0, \mu_{rw}]\}$ solve

$$\max_{C_{jt}, B^{\$}_{jt}, B^{€}_{jt}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma} \qquad \text{subject to}$$
$$C_{jt} + Q^{\$}_t B^{\$}_{jt} + Q^{€}_t B^{€}_{jt} = B^{\$}_{jt-1} + B^{€}_{jt-1} + \Delta^{\$}_{jt} B^{\$}_{jt} + \Delta^{€}_{jt} B^{€}_{jt} + Y_{jt},$$

and a non-negativity constraint on bond-holdings. In the above, Q^c_t is the price of a bond denominated in $c \in \{\$, €\}$, B^c_{jt} is the amount of that bond held by the household, and Y_{jt} is an exogenous endowment of consumption goods. Besides the interest rate, each asset earns an additional return, or "liquidity premium", from the collateral-use fees that firms pay to bond owners. We denote these premia by Δ_j^c, and describe how they are determined below.

The optimal choice of asset holdings $B_{jt}^{\$}$ and $B_{jt}^{€}$ implies the Euler equations

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{1}{Q_t^{\$} - \Delta_{jt}^{\$}} \right] = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{1}{Q_t^{€} - \Delta_{jt}^{€}} \right]. \tag{1}$$

In steady-state, returns are equalized across assets and countries, so that

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta_j^{\$}} = \frac{1}{Q^{€} - \Delta_j^{€}}. \tag{2}$$

Since our analytical results regard steady-states, we suppress t-subscripts until Section 3.

Trading Firms

Trading firms in each country have the opportunity to make a profitable transaction with a randomly-matched foreign partner. If executed, the transaction generates a joint surplus of 2π that is split evenly between the two counterparties. For now, we treat π as exogenous; we endogenize it as the equilibrium profit from international trade in Section 3.

Because of imperfect contract enforcement, each firm must post collateral to guarantee their side of the transaction before executing the deal and realizing this profit.[3] Both the US and the Eurozone safe assets can serve as collateral and the firms' choice of which collateral to seek is central to our mechanism. To obtain this collateral, firms seek an intra-period loan of one of the assets in domestic bond-specific search and matching credit markets. On the other side of these credit markets are domestic households, who make their holdings of safe assets available for loan. We assume that trading firms look for a fixed amount of funding, which we normalize to one, and that they make a binary choice, seeking either dollar or euro collateral (i.e. a US or an Eurozone safe asset). This framework captures the two key empirical features of trade finance outlined in the introduction, that financing is essential for trade and that it is largely supplied domestically.[4]

The probability that a country-j firm seeking to borrow a US asset is successful is given by $p_j^{\$}$, while the probability of sucessfully borrowing an Eurozone asset is $p_j^{€}$. If a firm

[3]Evidence for such frictions is widely documented, e.g. Antras and Foley (2015) and Hoefele et al. (2016).

[4]While stylized, this setup closely resembled the key features of the popular letter of credit form of trade finance, in which two banks guarantee each side of an international contract, and also serves as a tractable abstraction for a wider range of empirically relevant trade finance arrangements (e.g. Schmidt-Eisenlohr, 2013).

successfully borrows a unit of collateral, it pays a fee, $r^{\$}$ or $r^{€}$ respectively, to the household for the use of the asset and proceeds to trade in the international market. If the firm is not successful in these credit markets, it continues on to trade using a "backup" funding plan that still provides its chosen collateral, but absorbs all surplus from the transaction.[5]

The only equilibrium requirement for the funding fees $r^{\$}$ and $r^{€}$ is that they leave firms with a positive ex-post surplus. In parallel with the labor search literature, these prices can be treated as parameters or they could be determined via some bargaining paradigm, which can then be parameterized itself. For simplicity, we follow the first of these paths and fix the funding prices to a common value, $r^{\$} = r^{€} = r < \pi$.[6]

Once successfully funded, the firm is randomly matched with a trading counterparty from another country $j' \neq j$. Upon matching, the pair transacts using their collateral to clear any payments needed and splits the gross transaction surplus of 2π. In the event that the two counterparties' collateral is mismatched — i.e. that one side of the match uses US assets and the other side Eurozone assets as collateral — the transaction's surplus is reduced by a collateral mismatch cost of 2κ. Throughout, we assume that $\kappa < \pi - r$, so that the transaction is profitable even in the event of a mismatch. This "collateral mismatch" cost can be micro-founded as an expected cost in case of default, since having collateral denominated in different currencies means two promises may not be equivalent in all states of the world. In any case, our key results require that κ is not too big, which would explain why it is not hedged away, given reasonable hedging costs.

Putting everything together, a country-j firm chooses to apply for dollar trade financing if the expected payoffs of seeking dollar financing is higher than seeking euro financing. The differential payoff of seeking a US asset over a Eurozone asset is given by

$$V_j^{\$} = p_j^{\$}\left[\pi - r_j^{\$} - \kappa(1-\bar{X})\right] - p_j^{€}\left[\pi - r_j^{€} - \kappa\bar{X}\right] > 0, \tag{3}$$

where $\bar{X}$ is the fraction of all trading firms in the world that use dollar funding and, hence, the probability an individual firm matches with a counterparty that uses dollar financing.

Since our primary aim is to explain third-party use of a dominant currency, we assume that firms in the US and Eurozone always seek to be funded via their respective domestic assets, and solve for the optimal currency choice of the rest-of-world trading firms.[7] Under

[5]This assumption simplifies derivations; in our quantitative model, unfunded firms exit without trading, so that trade finance constitutes a real constraint on equilibrium trade flows.

[6]Nash bargaining with an appropriate choice of bargaining parameter gives identical results.

[7]An earlier version of the paper, Chahrour and Valchev (2017), considered endogenous funding choices for the US and the Eurozone and arrives at very similar conclusions, at the cost of complicating the exposition.

these assumptions, the average use of dollar trade financing around the world is

$$\bar{X} \equiv \mu_{us} X_{us} + \mu_{ez} X_{ez} + \int_{j \in \mu_{rw}} X_j dj = \mu_{us} + \int_{j \in \mu_{rw}} X_j dj,$$

where X_j is the fraction of firms in country $j \in [0, \mu_{rw}]$ that apply for dollar financing.

We assume that the number of matches in a given country-asset credit market is governed by the constant returns to scale den Haan et al. (2000) matching function

$$M^F(B, X) = \frac{BX}{B + X},$$

where B represents units of the asset on offer and X, the number of trading firms demanding that asset. For example, a country-j firm searching for dollar funding succeeds with probability

$$p_j^{\$} = \frac{M^F(B_j^{\$}, X_j)}{X_j} = \frac{B_j^{\$}}{B_j^{\$} + X_j}.$$

Substituting expressions for the funding probabilities into equation (3) yields

$$V_j^{\$} = \frac{B_j^{\$}}{B_j^{\$} + X_j} \left[\pi - r - \kappa(1 - \bar{X})\right] - \frac{B_j^{€}}{B_j^{€} + 1 - X_j} \left[\pi - r - \kappa \bar{X}\right]. \tag{4}$$

Equation (4) reveals the individual firm's three strategic incentives, two with respect to other trading firms and one with respect to the domestic household. First, with respect to other domestic firms, collateral choices are strategic *substitutes*: when a larger share of the other country-j traders apply for dollar funding (higher X_j), the local dollar funding market becomes more congested, lowering the probability a given trader's dollar loan is approved, thus lowering the relative payoff of seeking this type of funding.

Second, funding choices with respect to foreign trading firms ($\bar{X}$), are strategic complements due to collateral mismatch costs. This complementarity can lead to a standard type of sunspot multiplicity, in which both dollar or euro use can be sustained as equilibria, depending on the conjecture of $\bar{X}$.

The third strategic interaction occurs between between the collateral choice of trading firms and the savings choices of households. Equation (4) captures this interaction via the presence of $B_j^{\$}$ and $B_j^{€}$ in the funding probability terms. For example, a trading firm's expected payoff of seeking dollar financing increases with the household's holdings of US bonds ($B_j^{\$}$), as larger household US asset holdings increase the firm's probability of successfully obtaining dollar funding.

This final strategic interaction works against sunspot equilibria: the funding friction makes households' bond positions a *coordination device*, anchoring firms' expectations of others' choices to credit market conditions. To analyze this formally, we define a quasi-equilibrium in trade finance use, in analogy to Mas-Colell et al. (1995, p. 551). We focus on symmetric equilibria in which $X_j = X_{rw}$, $B^{\$}_j = B^{\$}_{rw}$, and $B^{€}_j = B^{€}_{rw}$ for all $j \in [0, \mu_{rw}]$.

Definition 1 (Quasi-equilibrium). *Given household asset holdings in rest-of-world countries $\{B^{\$}_{rw}, B^{€}_{rw}\}$, a symmetric quasi-equilibrium in funding choice is a rest-of-world traders' funding choice X_{rw} such that no trader has an incentive to change its trade finance choice.*

Quasi-equilibrium describes the equilibrium funding choice among trading firms as a function of the asset holdings of the households and we denote the set of quasi-equilibria by the correspondence $X(B^{\$}_{rw}, B^{€}_{rw})$. Using equation (4), the set of quasi-equilibria can be characterized by the condition

$$V^{\$} X_{rw}(1 - X_{rw}) = 0, \tag{5}$$

with $V^{\$} > 0$ only if $X_{rw} = 1$ and $V^{\$} < 0$ only if $X_{rw} = 0$. If there is a unique value of X_{rw} satisfying (5), then $X(B^{\$}_{rw}, B^{€}_{rw})$ becomes a function.

Lemma 1 characterizes some key properties of the quasi-equilibria in our economy:

Lemma 1. *Given household portfolio holdings, the currency quasi-equilibrium is unique for any feasible bond allocation if and only if*

$$\kappa < \kappa^{sunspot} \equiv \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}}.$$

In that case, $X(B^{\$}_{rw}, B^{€}_{rw}) \in \left[\max\{B^{\$}_{rw}, B^{€}_{rw}\}/(B^{\$}_{rw} + B^{€}_{rw}), 1\right]$.

Proof. Proved in Appendix A. ■

Lemma 1 shows that the quasi-equilibrium funding choice can be unique even when there are strategic complementarities across countries (i.e. $\kappa > 0$). This happens because credit market conditions affect the accessibility of funding for firms' counterparties and the chief determinant of that accessibility is the household portfolio position. Hence, bond holdings can become a coordination device that synchronizes trade finance choices on the asset that forms a higher proportion of rest-of-world portfolios.

Naturally, if we reduce the effective financial friction, for example by raising both $B^{\$}_{rw}$ and $B^{€}_{rw}$, the scope for indeterminacy increases and the threshold $\kappa^{sunspot}$ falls. This happens because increasing both asset holdings makes both types of funding easier to obtain, which reduces the congestion effects in credit markets. In fact, if asset holdings of both types become arbitrarily large, potential counterparties' choices become the only payoff-relevant factor in the funding choice and the quasi-equilibrium is not unique for any level of κ:

Corollary 1. *In the limit of $B^{\$}_{rw} \to \infty$ and $B^{€}_{rw} \to \infty$, for any $\kappa > 0$*

$$X(B^{\$}_{rw}, B^{€}_{rw}) \to \{0, 1/2, 1\}.$$

We refer to indeterminacy in the funding choice quasi-equilibrium as "sunspot" multiplicity, because in that case the funding choices are solely determined by the beliefs of what other trading firms would do, i.e. a sunspot shock in the expectations of others' actions.

Next, we characterize the effects of firm funding decisions on household savings choices. The holding premium earned by a bond is equal to the expected intra-period loan fees that the bond earns. These fees are just the probability that the country j household successfully lends this kind of bond times the funding fee r that it receives when it does so. Hence,

$$\Delta^{\$}_j = \frac{M^F(B^{\$}_j, X_j)}{B^{\$}_j} \times r = \frac{X_j}{B^{\$}_j + X_j} r, \tag{6}$$

$$\Delta^{€}_j = \frac{M^F(B^{€}_j, 1 - X_j)}{B^{€}_j} \times r = \frac{(1 - X_j)}{B^{€}_j + (1 - X_j)} r. \tag{7}$$

Since the Euler equation (2) holds for all households j, the liquidity premia earned by each asset are equalized across countries: $\Delta^c_j = \Delta^c$ for all $j \in \{us, ez, [0, \mu_{rw}]\}$. Using this observation, the Euler equations, and the market clearing conditions in bond markets ($\mu_{us} B^c_{us} + \mu_{ez} B^c_{ez} + \int B^c_j dj = \bar{B}$), we can derive Lemma 2.

Lemma 2. *Equilibrium household portfolios, as a function of traders' currency choices, are:*

$$B^{\$}_j = \bar{B} \frac{X_j}{\int_{\mu_{rw}} X_j dj + \mu_{us}} \tag{8}$$

$$B^{€}_j = \bar{B} \frac{1 - X_j}{\int_{\mu_{rw}} (1 - X_j) dj + \mu_{ez}}. \tag{9}$$

Proof. Proved in Appendix A. ∎

Lemma 2 describes the determination of household portfolios, as a function of the mix of assets used for trade financing. This relationship is upward sloping: higher X_j implies higher $B^{\$}_j$. Intuitively, an asset that is heavily used for funding international transactions will deliver a higher liquidity premium ceteris paribus, increasing households' incentive to hold that bond. Since bond premia are equalized everywhere, countries with a higher dollar usage (higher X_j) must also have higher portfolio holdings of US bonds ($B_j^{\$}$).

Together, Lemmas 1 and 2 summarize the strategic firm-household interaction that is both novel and central our mechanism: Higher holdings of a given asset by *rest-of-world* households tilt the quasi-equilibrium in funding choices towards that asset, while use of the asset in funding markets reinforces the rest-of-world households' decision to save in it.

Steady-state equilibria

Having characterized both the financing choices of firms and the savings choices of households, we now analyze steady-state equilibria. We consider symmetric steady states in which the strategies of the ex-ante identical rest-of-world agents are the same.

Definition 2. *A steady-state equilibrium is a rest-of-world currency usage X_{rw}, a set of asset holdings $\{B_{rw}^{\$}, B_{rw}^{€}, B_{us}^{\$}, B_{us}^{€}, B_{ez}^{\$}, B_{ez}^{€}\}$, bond prices $\{Q^{\$}, Q^{€}\}$ and premia $\{\Delta^{\$}, \Delta^{€}\}$ such that*

1. *There is a quasi-equilibrium in currency choice.*

2. *The optimality conditions of household bond holdings are satisfied.*

3. *Bond markets clear:*

$$\bar{B} = \mu_{rw} B_{rw}^c + \mu_{us} B_{us}^c + \mu_{ez} B_{ez}^c, \text{ for } c \in \{\$, €\}.$$

4. *The bond liquidity premia equal fees paid by firms as per equations (6) and (7).*

Proposition 1 summarize the characteristics of the emerging steady-state equilibria.

Proposition 1. *For any $\kappa \geq 0$, the economy has three steady-state equilibria:*

(i) *a dollar-dominant steady state with $X_{rw} = 1$ and $B_{rw}^{€} = 0$;*

(ii) *a euro-dominant steady state with $X_{rw} = 0$ and $B_{rw}^{\$} = 0$; and*

(iii) *a multipolar steady state with $X_{rw} = 1/2$ and $B_{rw}^{\$} = B_{rw}^{€}$.*

Outside of the knife-edge case where $\kappa = \frac{\pi - r}{\bar{B}+1}$, these are also the only steady states.

Proof. Proved in Appendix A. ■

Crucially, each steady-state is jointly characterized by a specific rest-of-world savings composition and a corresponding trade finance choice. For example, at the dollar-dominant steady state, the rest-of-world holds large amounts of the US asset, making dollar financing the most convenient for their firms, reinforcing their trade finance choices. The rest-of-world households, in turn, are happy to concentrate their savings in US assets because the demand for dollar funding supports a liquidity premium on US assets.

This logic also explains why steady-state multiplicity is robust to generalizing many of the details in our model. For example, any strategy that traders may take to avoid paying currency mismatch costs — such as directing their search to counterparties holding a particular type of collateral or renegotiating the settlement currency ex-post — will not eliminate the interaction between households and firms. Similarly, the result is robust to allowing households to lend their assets as collateral in foreign markets, since the optimal allocation of assets across credit markets in each country j must still satisfy (8) and (9).[8]

Stability of steady states

One of our main objectives is to understand why dominant-asset regimes appear to be so *stable* in the data. We have already shown that, when $\kappa < \kappa^{sunspot}$, sunspot shocks cannot change the equilibrium in trade finance markets, *given* household asset positions. We now derive the conditions under which the interaction between households and firms results in locally stable dominant-asset steady states, in the sense that the best response functions of both agent types jointly define a local contraction map. Intuitively, we want to ensure that deviations in saving or trade finance choices would not unravel the dominant asset equilibrium.

Proposition 2 shows that the model with intermediate values of κ can generate dominant-asset steady states that are both locally stable and not subject to sunspot shocks.

Proposition 2. *For $\kappa > \bar{\kappa}$ the dominant-currency steady states are locally stable, where*

$$\bar{\kappa} \equiv \frac{\pi - r}{\bar{B} + 1} < \kappa^{sunspot}.$$

Proof. Proved in Appendix A. ■

The result is due to the interaction among the incentives trading firms face in their financing choice, the differential availability of financing due to the feedback between households

[8] This would be akin to global banks being active in many trade finance markets through country-specific bureaus. While the model is robust to this, our benchmark assumption that trade finance is domestically sourced in rest-of-world countries is consistent with the BIS (2014) data for emerging markets.

and firms, and the cross-country coordination incentive among trading firms.

To see how these forces interact, consider a situation in which the economy begins at the dollar-dominant steady state, with rest-of-world household portfolios concentrated in US assets and rest-of-world firms using only US safe assets for financing their trade. Suppose now that households shift their savings a little towards Eurozone assets. The increased availability of euro financing gives firms an incentive to shift towards funding via Eurozone assets. However, as long as $\kappa > \bar{\kappa}$, firm choices will not change much: the small shift in portfolios still leaves dollars the most widely available in world markets and, hence, firms still coordinate strongly on dollars. Thus, for a local shift in household portfolios, the cross-country coordination of firms prevents the currency quasi-equilibrium (X) from changing significantly, which in turn means that the conjectured shift in household portfolios is suboptimal.

Proposition 2 implies there is a range of parameter values, $\kappa \in (\bar{\kappa}, \kappa^{sunspot})$, over which dominant asset steady states are both locally stable and not subject to sunspot shocks. The existence of this region depends crucially on the interaction of our two complementarity forces: Currency sunspots require coordination incentive across firms to be so strong that a unilateral deviation in trade finance choice can be sustained, even given fixed portfolios. By contrast, local stability requires weaker coordination incentives, such that firms will only deviate from a dominant asset equilibrium if households dramatically change their portfolios. Hence, necessarily, $\bar{\kappa} < \kappa^{sunspot}$. By the same logic, neither the asset availability mechanism nor cross-country complementarity can achieve such stability on their own: If $\kappa = 0$, dominant-asset steady states are unstable, while if $\kappa > 0$ and bond supplies are infinite (so credit frictions are non-existent), the firms' funding choices are subject to sunspot shocks. We illustrate this argument graphically in Appendix B.

3 Dynamic General Equilibrium Model

Having illustrated the key intuition of our mechanism, we now embed it in a rich dynamic general equilibrium model. We calibrate the model, and show it can match both targeted and untargeted moments, quantify the welfare effects of dominance, and perform counterfactuals to better understand the conditions under which dominant currencies can fall.

3.1 Setup

Like our stylized model, our general environment consists of households and firms in the US, Eurozone, and a continuum of rest-of-world small open economies, but each of those agents now has several margins of choice. Households choose a basket of consumption, as well as their optimal savings patterns. Trading firms make optimal choices about whether to operate and how/where to trade, as well as the choice of how to finance their activities. Finally, all prices and quantities are determined in general equilibrium.

3.1.1 Households

In each country $j \in \{us, ez, [0, \mu_{rw}]\}$, a representative household seeks to maximize the present discounted value of utility of consumption, $E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$. In contrast to Section 2, there are country-specific differentiated goods, and the consumption basket C_{jt} is a Cobb-Douglas aggregate of all domestic and foreign goods. The consumption share of the domestic good is $a_h \in (0,1)$, and the consumption shares for foreign goods are proportional to the size of their origin country. For example, the US consumption basked is

$$C_{us,t} = (C_{us,t}^{us})^{a_h} (C_{us,t}^{ez})^{\frac{(1-a_h)\mu_{ez}}{\mu_{ez}+\mu_{rw}}} (C_{us,t}^{rw})^{\frac{(1-a_h)\mu_{rw}}{\mu_{ez}+\mu_{rw}}}. \tag{10}$$

In the above, C_{jt}^i denotes consumption of good i in country j and $C_{jt}^{rw} \equiv (\int_0^{\mu_{rw}} (C_{jt}^i)^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$ aggregates goods from the small open economies.[9] The consumption baskets of the Eurozone and rest-of-world economies are analogous and presented in the Online Appendix.

Because of the frictions in international trade, the law of one price does not hold in our economy and goods have different equilibrium prices in different locations, with a markup on imports $P_{j,t}^i > P_{j,t}^j$. This markup relative to the origin-country price is endogenous and depends on the equilibrium patterns of trade, as we describe below. Nevertheless, since our economy is real, all prices can be expressed in terms of a numeraire good, which we take as the (identical) domestic price of the small open economy goods, i.e. $P_{rw,t}^{rw} \equiv 1$.

In addition to consumption, households choose how much to save and how to allocate savings among US and Eurozone bonds, each of which yields a risk-free unit of their respective domestic good. The household in country j faces the budget constraint:

$$\begin{aligned} P_{jt}C_{jt} + (1-\Delta_{jt}^{\$})P_{us,t}^{us}Q_t^{\$}B_{jt}^{\$} + (1-\Delta_{jt}^{€})P_{ez,t}^{ez}Q_t^{€}B_{jt}^{€} + \text{adj. costs}_t \\ = P_{us,t}^{us}B_{jt-1}^{\$} + P_{ez,t}^{ez}B_{jt-1}^{€} + P_{jt}^{j}Y_{jt} + \Pi_{jt}^{T} + T_{jt}, \end{aligned} \tag{11}$$

where $Q_t^{\$}$ and $Q_t^{€}$ are the prices of the US and the Eurozone bonds, Y_{jt} is the household's endowment of its domestic good, Π_{jt}^T is the total profit of country j's import/export firms

[9] The price index corresponding to (10) is $P_{us,t} = K^{-1}(P_{us,t}^{us})^{a_h}(P_{us,t}^{ez})^{\frac{(1-a_h)\mu_{ez}}{\mu_{ez}+\mu_{rw}}}(P_{us,t}^{rw})^{\frac{(1-a_h)\mu_{rw}}{\mu_{ez}+\mu_{rw}}}$, where K is a proportionality constant and $P_{us,t}^j$ is the price of country j's differentiated good in the US.

(described below), and T_{jt} are lump-sum taxes. As in our simple model, a household's bond holdings earn an endogenous liquidity premium, given by the intra-period cash flows $\Delta^{\$}_{jt}$ and $\Delta^{€}_{jt}$. We focus on a perfect foresight, symmetric model and assume that all endowments are constant through time and equal, $Y_{jt} = \bar{Y}$ for all j.

Households are also subject to external portfolio adjustment costs, given by

$$\text{adj. costs}_t \equiv P^{us}_{us,t} Q^{\$}_t \tau(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1}) + P^{ez}_{ez,t} Q^{€}_t \tau(B^{€}_{jt}, \underline{B}^{€}_{j,t-1}).$$

These costs are parameterized by the function $\tau(B, \underline{B}) \equiv \frac{\tau}{2}\left(\frac{B-\underline{B}}{\underline{B}}\right)^2 \underline{B}$, which is quadratic in terms of percent deviations from the country-wide bond holdings entering the period, $\underline{B}^{\$}_{j,t-1}$ and $\underline{B}^{€}_{j,t-1}$. These adjustment costs are zero at (any) steady state, and thus have no effect on steady states, but serve to limit the volatility of capital flows outside of steady state.

Intertemporal optimality implies the following household Euler equation for dollar bonds

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P^{us}_{us,t+1}}{P^{us}_{us,t}} \frac{1}{Q^{\$}_t \left(1 - \Delta^{\$}_{jt} + \tau'(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1})\right)} \right]. \tag{12}$$

This equation is same as (1), except that it now reflects the consequences of relative price differences among goods and across time, as well as influence of adjustment costs on effective bond returns. An equation that is analogous to (12) holds for euro bond holdings.

3.1.2 The Import-Export Sector

International goods trade is subject to search and matching frictions as emphasized by the recent trade literature (e.g. Antras and Costinot, 2011). International trade flows through specialized import/export firms, who organize to sell country-j's differentiated good in country i, via a match between a country-j export firm with a country-i import firm.

Once matched, the exporting firm buys goods at the prevailing domestic market price and sells them to the matched foreign importer, who then resells the good to the country-i household at the prevailing market price in that location. Firms optimally choose the intensity with which they search for different types of trade partners (e.g. import from US vs import from the Eurozone), and the resulting matching patterns determine the size and direction of equilibrium trade flows. The import/export firms operate within the period, return profits to households, and disband.[10]

[10] This market structure aligns well with the empirical evidence of significant churn in the firm-to-firm trade relationships (Eaton et al., 2016).

As before, international trade is subject to a financial friction, which implies a need for trade finance. Firms look for a fixed amount of funding, which we normalize to one unit of the numeraire, and firms again make the binary choice of either seeking US or Eurozone safe assets. Both of these assumptions can be relaxed.

A trading firm's choices occur in two stages. In the first, the firm chooses whether or not to pay a fixed cost and become operational in a given period and the likelihood that, if operational, it will pursue an import or an export opportunity and with which partner country. Second, the firm chooses the type of trade financing to apply for. We consider each stage in detail before characterizing equilibrium in the model.

Entry and trading pattern choice

Trading firms pay a fixed cost ϕ in units of their domestic good to enter the international trade market. Thus, a firm enters only if the expected profits from trading net of this cost are positive, and the ones that do enter make a probabilistic choice regarding the direction in which they will trade. Specifically, an active country-j firm chooses the probabilities with which it will become an importer from country i or an exporter to country i, for all i, which probabilities we denote p^{im}_{jit} and p^{ex}_{jit} respectively. In equilibrium, the pattern is such that firms are indifferent between operating as an importer or exporter in any direction. We provide a detailed description of the firm's entry decision in the Online Appendix.

Funding Choice

As in the analytical model, trading firms must arrive to international trade markets with either US or Eurozone safe asset collateral, which they borrow from their domestic households through bond-specific search and matching markets.

Relative to the analytical model in Section 2, we enrich our model of trade finance supply in a two ways. First, we tie the potential liquidity service of an asset to the total *market value* of the household's holdings of that asset. Second, because we will calibrate our model to annual data while the typical trade finance arrangement is much shorter, we introduce a parameter ν that corresponds to the number of times a given bond could be used to intermediate trade within one model period. Thus, the total value of trade flows that the country j holdings of US and Eurozone safe assets can intermediate is given by $\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t$ and $\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t$ respectively, the market value of holdings scaled up by ν.

With these assumptions, the probability of success faced by a country-j trading firm seeking US financing is

$$p^{\$}_{jt} = \frac{M^F\left(m_{jt}X_{jt}, \nu P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_t\right)}{m_{jt}X_{jt}} = M^F\left(1, \frac{\nu P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_t}{m_{jt}X_{jt}}\right), \tag{13}$$

where m_{jt} is the total mass of operational country-j firms, as determined by the zero-profit entry condition, and thus $m_{jt}X_{jt}$ is the mass of country-j firms applying for dollar funding. Note that we also use the general form of the den Haan et al. (2000) matching function $M^F(u,v) = \frac{uv}{(u^{\frac{1}{\xi_F}}+v^{\frac{1}{\xi_F}})^{\xi_F}}$, which allows for an elasticity parameter ε_F that we calibrate to the data. The probability a country-j trading firm seeking Eurozone bonds finds a credit match, $p^{€}_{jt}$, is given by an analogous expression.

In sum, the market tightness of each trade finance market is given by ratio of supply to demand in each: $\frac{\nu P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_t}{m_{jt}X_{jt}}$ and $\frac{\nu P^{ez}_{ez,t}B^{€}_{jt}Q^{€}_t}{m_{jt}(1-X_{jt})}$. We note here that tying the effective trade finance supply to the market value of assets introduces a new channel that generally reinforces the emergence of a dominant asset: Since a dominant asset carries a high equilibrium price, an asset's ability to facilitate trade increases as it becomes dominant.

As in Section 2, we fix the collateral use in the big countries exogenously, though now we calibrate $X_{us} = 1 - X_{ez}$ to the domestic-currency trade finance usage in the US and Eurozone data, which is high but not exactly 100%. We continue to assume that the US and Eurozone firms face the same financing frictions as small open economies, so equation (13) and its euro analgoue apply without modification for these countries as well.

In making their funding choice, the rest-of-world firms compare the respective expected profits of seeking dollar and euro financing. Upon a successful funding match, the expected profit of a country-j trading firm using US safe assets as a collateral guarantee is given by

$$\tilde{\Pi}^{\$}_{jt} = \sum_{i\neq j} p^{im}_{jit}\pi^{\$,im}_{jit} + \sum_{i\neq j} p^{ex}_{jit}\pi^{\$,ex}_{jit},$$

where $\pi^{\$,im}_{jit}$ is the expected profit of a firm importing from i to j that is financed via US bonds, and $\pi^{\$,ex}_{jit}$ is the analogous values for a country-j exporter looking to match with a country-i importer. The corresponding expected profits of a country-j firm funded with Eurozone assets, $\tilde{\Pi}^{€}_{jt}$ is analogous. We describe the determinants of trading profits below.

In return for the intra-period use of the household's bonds, the firm pays a fee r. Thus, the expected net payoff to a country-j firm of seeking dollar funding is then given by

$$\Pi^{\$}_{jt} = p^{\$}_{jt}(\tilde{\Pi}^{\$}_{jt} - r), \tag{14}$$

which is simply the probability of obtaining dollar funding, $p^{\$}_{jt}$, times the expected profit net of the dollar funding costs. The expected payoff to seeking euro funding is $\Pi^{€}_{jt} = p^{€}_{jt}(\tilde{\Pi}^{€}_{jt} - r^{€})$.

Lastly, to match the empirical fact that, despite its dominance, the dollar is not the *only* currency used to finance global trade, we introduce an i.i.d. additive idiosyncratic preference for the type of trade financing $\theta^{(l)}_{jt} \sim N(0, \sigma^2_\theta)$. This generates some idiosyncratic heterogeneity across firms and thus results in an interior equilibrium value for the currency mix X_{jt}, which we can then calibrate to the data.

Combining the probabilities of obtaining each type of funding, the expressions for profits, $\Pi^{\$}_{jt}$ and $\Pi^{€}_{jt}$, and the disturbance $\theta^{(l)}_{it}$, we can compute an individual firm's net benefit of seeking financing via US assets:

$$V^{\$,(l)}_{jt} = \frac{1}{\left[1 + \left(\frac{m_{jt} X_{jt}}{\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t}\right)^{\frac{1}{\xi_F}}\right]^{\xi_F}} (\tilde{\Pi}^{\$}_{jt} - r) - \frac{1}{\left[1 + \left(\frac{m_{jt}(1 - X_{jt})}{\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t}\right)^{\frac{1}{\xi_F}}\right]^{\xi_F}} (\tilde{\Pi}^{€}_{jt} - r) + \theta^{(l)}_{it}.$$

Firm l in country j will then choose to seek dollar funding if and only if $V^{\$,(l)}_{jt} > 0$. Given that the expected payoff of seeking dollar funding is increasing in $\theta^{(l)}_{jt}$, we can express the optimal choice in terms of a threshold strategy, where the firm seeks dollar funding if and only if their idiosyncratic shock exceeds a country-specific threshold $\bar{\theta}_{jt}$. Thus, the fraction of country-j trading firms using US safe assets is

$$X_{jt} = \int_0^1 \mathbb{1}(\theta^{(l)}_{jt} >= \bar{\theta}_{jt}) dl = 1 - \Phi\left(\frac{\bar{\theta}_{jt}}{\sigma_\theta}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

In equilibrium, the cutoff $\bar{\theta}_{jt}$ is defined by $V^{\$,(l)}_{jt} = 0$, the value of the idiosyncratic preference shock that leaves a country-j trader indifferent between choosing one asset or the other. We focus on symmetric equilibria where all ex-ante identical rest-of-world countries have the same equilibrium allocations, hence $\bar{\theta}_{jt} = \bar{\theta}_t$ for all $j \in [0, \mu_{rw}]$.

Exchange of Goods

The remaining steps in our model of trade unfold without further decisions on the part of trading firms. Firms that are successful in obtaining financing search for a foreign trading counterpart. Country-j exporters match with country-i importers according to the technology $M^T(u, v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}} + v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$, which is of the same functional form as the matching function in credit markets, but allows for a different elasticity parameter ε_T.

The probability of a country-j exporter matching with a country-i importer is

$$p^{ei}_{jit} = \frac{M^T\left(\tilde{m}^{ex}_{jit}, \tilde{m}^{im}_{ijt}\right)}{\tilde{m}^{ex}_{jit}} = \left(1 + \left(\tilde{m}^{ex}_{jit}/\tilde{m}^{im}_{ijt}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T}.$$

where $\tilde{m}^{im}_{ijt} = p^{im}_{ijt} m_{it}(p^{\$}_{it}X_{it} + p^{€}_{it}(1 - X_{it}))$ is the mass of *funded* importing firms in country i (given by the term $m_{it}(p^{\$}_{it}X_{it} + p^{€}_{it}(1 - X_{it})))$ seeking trade with country-j firms that are looking to export to i, which are themselves of mass $\tilde{m}^{ex}_{jit} = p^{ex}_{jit} m_{it}(p^{\$}_{it}X_{it} + p^{€}_{it}(1 - X_{it}))$. Using analogous derivations, the probability of a country-j importer matching with a country-i exporter is $p^{ie}_{jit} = \left(1 + \left(\tilde{m}^{im}_{jit}/\tilde{m}^{ex}_{ijt}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T}$.

In a successful match between a country-j exporter and a country-i importer, the exporter buys the j good at its domestic market price P^j_{jt} and the importer then sells it to the country-i household at the prevailing market price in that location P^j_{it}. The transaction thus generates a gross surplus of $P^j_{it} - P^j_{jt}$, which is then also subject to a collateral mismatch cost κ.

The importer and exporter in a trading match split the surplus of their transaction via Nash bargaining, with the exporter having a Nash bargaining share of α. The effective "wholesale" price at which a country-j exporter sells to a country-i importer is thus $P^{whol}_{jit} = P^j_{jt} + \alpha(P^j_{it} - P^j_{jt})$. The expected profit of a firm looking to export from country j to i is

$$\pi^{\$,ex}_{jit} = p^{ei}_{jit} \frac{\alpha}{P^{whol}_{jit}} \left[P^j_{it} - P^j_{jt} - \kappa P^{whol}_{jit}(1 - \tilde{X}_{it})\right]. \tag{15}$$

The term in square brackets is the net expected surplus per unit of goods traded, which is given by the gross markup on the imported good, net of the expected currency mismatch cost $\kappa P^{whol}_{ji,t}(1 - \tilde{X}_{it})$. In this expression,

$$\tilde{X}_{it} \equiv \frac{p^{\$}_{it}X_{it}}{p^{\$}_{it}X_{it} + p^{€}_{it}(1 - X_{it})}$$

is the average use of dollar trade financing among the funded country-i firms (which are thus actively searching for trade counterparts), hence $1 - \tilde{X}_{i,t}$ is the probability of matching with a EUR-funded country-i importer, and thus having to incur the expected default cost κ.[11]

Lastly, the financing friction limits the overall value of the transaction to the value of the attached safe collateral. Since we assume each firm borrows one unit of safe assets, to obtain

[11] Note that since a potential trading partner's funding is uncertain, it would be costly to first match with a counterparty, agree on a particular type of collateral, and only then seek that financing. If the financing falls through, the firm looses the opportunity to earn π, which is simply not worth the risk for potentially saving the small mismatch cost κ.

the net expected profit from the view point of a country-j exporter (who earns α fraction of the total surplus), the expected per-unit profit is then scaled by $\frac{\alpha}{P^{whol}_{jit}}$.

Government

We assume that government purchases are zero, and thus governments play a role only in the large countries $j \in \{us, ez\}$, where they issue bonds in fixed supply $\bar{B} = B^{\$} = B^{€}$ and set the level of lump-sum taxes so as to keep their stock of debt constant, so that $\bar{B} = T_{jt} + Q^j_t \bar{B}$. The small rest-of-world countries $j \in [0, \mu_{rw}]$ do not issue debt and set $T_{jt} = 0$.

Equilibrium

In equilibrium, the liquidity premia a country-j household can earn on lending US and Eurozone bonds respectively are equal to the frequency with which the household successfully lends the asset in its respective credit market multiplied by the funding fee r:

$$\Delta^{\$}_{jt} = \frac{\nu m_{jt} X_{jt}}{\left[(m_{jt} X_{jt})^{1/\xi_F} + (\nu P^{us}_{us,t} B^{\$}_{jt} Q^{\$}_t)^{1/\xi_F} \right]^{\xi_F}} r \tag{16}$$

$$\Delta^{€}_{jt} = \frac{\nu m_{jt} (1 - X_{jt})}{\left[(m_{jt}(1 - X_{jt}))^{1/\xi_F} + (\nu P^{ez}_{ez,t} B^{€}_{jt} Q^{€}_t)^{1/\xi_F} \right]^{\xi_F}} r \tag{17}$$

Given those expressions, the rest of the equilibrium is determined by the household and firms' optimal decisions, and market clearing in real goods and bond markets. We focus on the class of symmetric equilibria where the strategies of the ex-ante identical rest-of-world trading firms and households are the same, e.g. $X_{jt} = X_{rw,t}$ for all $j \in [0, \mu_{rw}]$. A complete definition of equilibrium is provided in the Online Appendix.

3.2 Calibration

We fix a set of parameters to standard values, then use the remaining parameters to target several moments, which the model is able to replicate exactly. Table 1 lists the exogenously-fixed parameters. Specifically, we set $\mu_{us} = \mu_{ez} = 0.2$, consistent with the sizes of the US and the Eurozone in world GDP. One model period represents a year, hence we set $\beta = 0.96$; we also assume log preferences ($\sigma = 1$). Next, we minimize the role of the search friction between exporters and importers by using a low value for the elasticity of the trade matching function, $\varepsilon_T = 0.01$, which ensures that firms on the less crowded side of the

Parameter	Concept	Value
β	Time preference	0.960
$\mu_{us} = \mu_{ez}$	Big country measure	0.200
κ	Mismatch cost	0.010
r	Funding fee	0.005
υ	Exog. velocity	8.000
X_{us}	US dollar share	0.900
X_{ez}	EZ dollar share	0.100
α	Exporters bargaining parameter	0.500
σ	Risk aversion	1.000
ε_T	Elasticity of trade matching function	0.010
σ_θ^2	Variance of idio. shock	1e-06
τ	Portfolio adj. costs	0.040

Table 1: Exogenously Fixed Parameters

market are virtually guaranteed a match. We fix $\alpha = 0.5$, thus importers and exporters have equal bargaining power, and set the currency use in the big countries (X_{us} and X_{ez}) so 90% of their firms use their domestic asset, to match the evidence on domestic currency usage in trade finance for the US and the Eurozone (BIS, 2014).

To match the observed maturity of a typical letter of credit contract in the data (45 days), we set $\nu = 8$, implying that an asset can be used up to 8 times per year for trade finance operations. To match the typical cost of letters of credit – which include a substantial fixed component, on average 40 basis points of the principal, plus a spread on top of the LIBOR – we set $r = 0.005$.[12]

Next, we set the collateral mismatch cost $\kappa = 0.01$ to be just 1% of the value of the transaction, which is in fact smaller than typical exchange rate hedging costs.[13] As it turns out, this value of mismatch costs is also both sufficient to ensure local stability of dominant-asset steady states, based on the eigenvalues of linearized solutions around those steady states, and to prevent sunspot multiplicity in the collateral choice quasi-equilibrium.

We fix two additional parameters in order to ensure numerical stability, but minimize their substantive impact. First, we set $\tau = 0.04$ just large enough to prevent large instantaneous jumps in the composition of bond portfolios (e.g. a shift from a portfolio concentrated in US asset, to one concentrated in Eurozone assets within a period) that could lead to equilibrium multiplicity. This value implies that a 10% change in bond positions incurs a cost of just

[12] See guidance by the US Commerce Dept.: acetool.commerce.gov/cost-risk-topic/trade-financing-costs.

[13] See for example Figure 1 in Bonetti (2018) for the historical evolution of USD-EUR hedging costs.

Concept	Data	Model
Gross debt/GDP	0.60	0.60
ROW trade/GDP	0.55	0.55
ROW USD usage	0.80	0.80
Import markup	1.10	1.10

(a) Calibration Targets

Parameter	Concept	Value
$\bar{B}$	US/EZ asset supply	0.613
a_j^h	Home bias	0.718
ε^f	Funding match. elas.	0.294
ϕ	Fixed cost of entry	0.038

(b) Implied Parameter Values

Table 2: Calibration Strategy

2 basis points on the portfolio. We also make the currency preference shocks as small as possible (σ_θ^2 =1e-06), while still ensuring numerically reliable interior solutions for X_{rw}.

We calibrate the remaining parameters to match a set of target steady-state moments. Like our analytical model, the calibrated model has three co-existing steady states – dollar and euro dominant ones, and a symmetric one. Since in our data sample (1984-2017) the dollar has been the dominant currency, we match the empirical moments to those at the dollar-dominant steady state of the model. Panel (a) of Table 2 summarizes the targeted moments: (1) government debt of 60% of GDP, consistent with the US average; (2) rest-of-world trade share ($\frac{\text{Imports + Exports}}{\text{GDP}}$) of 55%, consistent with trade data for non-US and non-Eurozone countries from the World Bank; (3) dollar share in trade financing used by rest-of-world trading firms of 80%, consistent with the evidence on the fraction of letters of credit and trade finance loans denominated in dollars BIS (2014); and (4) import markups of 10%, consistent with micro-level estimates on import markups in Coşar et al. (2018).

We target these four moments with the four remaining free parameters $\{\bar{B}, a^h, \varepsilon^F, \phi\}$. These parameters are: (1) the supply of government debt $\bar{B}$; (2) the home bias parameter in consumption preferences a_h which determines the trade share; (3) the elasticity of the funding matching function ε^F which helps determine the equilibrium level of currency coordination; and (4) the fixed cost of entry in the trading sector ϕ which helps determine import markups. We find the model can exactly match the targeted moments, with the implied parameter values given in Panel (b) of Table 2.

3.3 Quantitative Results

We now consider the model's quantitative implications for non-targeted moments in steady-state and then proceed to use global techniques to solve for the model's (perfect-foresight) transition dynamics. We then explore the model's implications for three counter-factual scenarios. In the first, we examine what could have happened to dollar dominance had the Eurozone continued it expansion beyond its current size. In the second, we explore the implications of the decreasing relative size of the US compared to the world economy. Finally, we explore potential consequences of trade wars initiated by the United States.

Steady State

Table 3 summarizes several key steady state moments in the calibrated economy, and shows that the (empirically-relevant) dollar-dominant steady state matches a number of untargeted phenomena. First, since the rest-of-world countries primarily use dollars for trade finance ($X_{rw} = 0.8$), the US bond earns a higher equilibrium liquidity premium: $\Delta^{\$} > \Delta^{€}$, which in turn results in an interest parity violation: Interest rates on Eurozone bonds must exceed rates on US bonds in order to offset the lower liquidity return. Specifically, we find

$$\frac{1}{Q^{€}} - \frac{1}{Q^{\$}} = \frac{\Delta^{\$} - \Delta^{€}}{\beta} = 1.07\%,$$

which implies the US earns a significant "exorbitant privilege". This size of this excess return is consistent with the Gourinchas and Rey (2007a) evidence on exorbitant privilege and the US Treasury convenience yield estimated by Jiang et al. (2020).

The third line of the table (seignorage) provides a common and simple estimate of the net benefit the US receives from the "privilege" of this interest differential. This number is computed as the counter-factual additional debt servicing payments the US would face if it actually paid an interest rate equal to the inverse of the time discount, holding asset positions constant. Essentially, this is the seignorage the US earns from the liquidity premium on its asset and, at 0.88% of GDP, our model estimates this to be substantial.

Though similar calculations have often been used to estimate the benefits of exorbitant privilege, our model implies that this is an *incomplete* and potentially misleading measure of privilege because it takes asset positions as given. A key insight of our theory is that widespread foreign holdings of a country's assets are necessary to support its dominant status. But such strong external demand leads to a negative steady-state net foreign asset position for the central country, and hence the seignorage benefits of being dominant are at least partially offset by the need to service the resulting negative net foreign asset position.

Indeed, the fourth line in the table shows that the dominant country (i.e. US) has a significant negative net foreign asset position equal to -42% of GDP, while the other big country (i.e. Eurozone) has a much better net foreign asset position of -26% of GDP. This is a manifestation of the fact that rest-of-world households concentrate their savings in US

Moments	USD Dominant US	EZ	RW	Multipolar US	EZ	RW	EUR Dominant US	EZ	RW
	Panel A: Benchmark model								
USD share trade fin. (X_j)	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100\times(i^{\$}-i^{€})$	1.07	-	-	0.00	- 0	-	-	-1.07	-
100×Seignorage/GDP	0.88	0.23	-	0.56	0.56	-	0.23	0.88	-
NFA/GDP	-0.42	-0.26	0.18	-0.38	-0.38	0.19	-0.26	-0.42	0.18
Gross Foreign Assets/GDP	0.04	0.02	0.18	0.02	0.02	0.19	0.02	0.04	0.18
100×Trade bal./GDP	0.87	0.86	-0.45	1.01	1.01	-0.52	0.86	0.87	-0.45
	Panel B: Rest-of-World asset $(\bar{B}_{rw}/(P_{rw}\bar{Y})=0.40)$								
$100\times(i^{\$}-i^{€})$	1.03	-	-	0.00	-	-	-	-1.03	-
NFA/GDP	-0.14	0.01	0.03	-0.10	-0.10	0.05	0.01	-0.14	0.03
Gross Foreign Assets/GDP	0.31	0.29	0.18	0.30	0.30	0.20	0.29	0.31	0.18
100×Trade bal./GDP	-0.25	-0.28	0.14	-0.12	-0.12	0.07	-0.28	-0.25	0.14

Table 3: Steady-state values for baseline model.

assets, which is the second sense in which those assets are dominant. In particular, the model implies that two-thirds of rest-of-world portfolios are invested in US bonds, amounting to a long position in US assets equal to 12% of rest-of-world GDP, consistent with the evidence in Caballero et al. (2008). Overall, at our calibration the US trade balance is only slightly better than that of the Eurozone, despite the excess return the US earns, suggesting that the net welfare effect of dominance is small (which we quantify precisely later) since its position as an external net debtor largely offsets the benefits of exorbitant privilege.

Lastly, we emphasize that while the US trade balance is positive in the benchmark model, our mechanism can indeed generate a negative net foreign asset position *and* a trade deficit at the same time. To illustrate this, Panel B of Table 3 considers a version of the model with a richer asset structure, in which each of the small economies also issues government debt equal to 40% of their GDP. The assets from each small country are measure zero, and hence do not finance international trade, but households in all countries may hold the basket of rest-of-world bonds for investment purposes.[14]

This version of the model shows the US holding a significantly larger *gross* position in

[14]We provide the details on this version of the model in Online Appendix E.

foreign assets – 31% of GDP versus just 4% – as now there is a foreign asset which does not have a high liquidity value to foreigners, and thus offers high returns to Americans. This modification results in a steady-state trade deficit for the US, even as the US net foreign asset position remains significantly negative, because the US is now able to leverage its exorbitant privilege by investing in a high yielding foreign asset.[15] This is an important success of our framework, and something a large class of other models of the exorbitant privilege cannot generate (e.g. Caballero et al., 2008). Still, we abstract from this in our benchmark analysis, in order to minimize the number of state variables and facilitate the global solution of the model dynamics.

Dynamics

We now consider the out-of-steady-state dynamics of the calibrated model, with a particular focus on determining the stability properties of different steady states. Figure 1 plots the respective attraction regions of the models' three steady states. We compute these regions by defining a fine grid on the state space of the model, which is depicted in the axes of the figure.[16] We treat each grid point as a possible initial condition, and compute all perfect foresight equilibrium paths originating from that point and converging to one of the three steady states. Thus, for each grid point we run three separate attempts to compute an equilibrium path – one that ends at the dollar-dominant steady state, one that ends at the multipolar steady state, and one that ends at the euro-dominant steady state. For each point in the blue region, we find that there is only a single possible equilibrium path, which converges to the dollar-dominant steady state. Conversely, for points in the orange region, the only feasible outcome is the euro-dominant steady state. Finally, the purple region corresponds to points where we found perfect foresight paths that arrive at both coordinated steady states – this is a region of dynamic indeterminacy.

A key result of Figure 1 is that only the dominant-currency steady states are dynamically stable, i.e. dynamic paths that are initialized away from the symmetric steady state never converge there.[17] Moreover, each dominant-asset steady state is contained within a large region for which it is the only long-run outcome. For example, whenever rest-of-world

[15] This compositional effect is highly empirically relevant, as discussed by Gourinchas and Rey (2007a).

[16] The model has four state variables: rest-of-world holding of US and Eurozone bonds, US holdings of US bonds, and Eurozone holdings of Eurozone bonds (US and Eurozone foreign holdings are determined by market clearing.) To display Figure 1 in two dimensions (for illustration purposes), we initialize US and Eurozone portfolios shares at their symmetric steady-state level.

[17] We have also confirmed via linearization that the dominant equilibria are locally stable but the symmetric steady state is not.

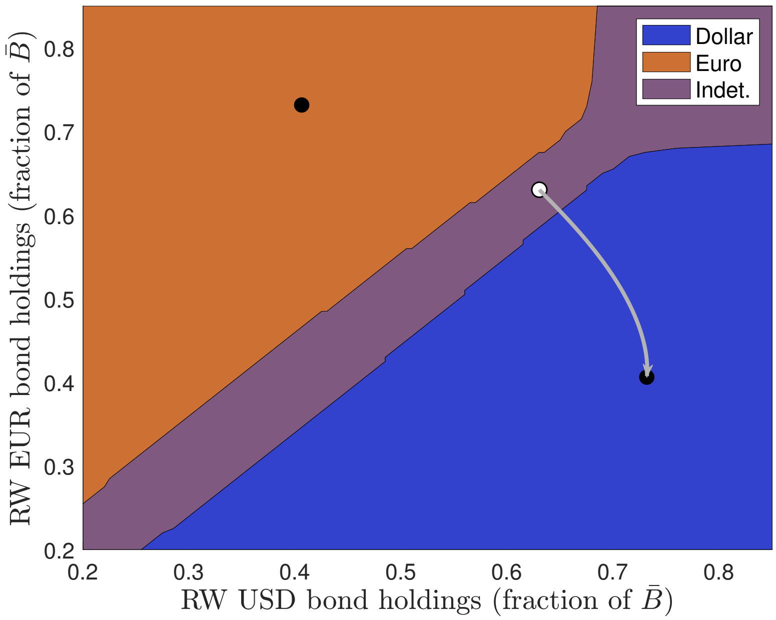


Figure 1: Steady state attraction regions.

households' initial portfolios are sufficiently biased towards US assets (bottom right), the unique equilibrium path converges to the dollar-dominant steady state. This is a manifestation of the interactions we explored in Section 2: When rest-of-world household portfolios are sufficiently biased towards US bonds, firms tilt their actions towards financing international trade with US assets, perpetuating the households' decision to save primarily in US assets.

These large unique attraction regions show that dominant-asset regimes in the model are endogenously persistent and sustainable indefinitely, so long as no large shocks push the economy out of the respective basins of attraction. And even in that case, the model will still converge to one or the other dominant-currency steady states, confirming that a dominant-currency regime is the eventual outcome in any given simulation.

We next explore the transition paths that underly Figure 1 in more detail. As an example, Figure 2 plots the transition of several endogenous variables, when the economy starts at the (unstable) symmetric steady state and converges to the dollar-dominant steady state. The top right panel shows the evolution of the equilibrium mix of collateral used (X_{rw}), which starts close to equally balanced and then gradually converges to dominant dollar usage over the subsequent 15-20 years. Along this transition path, the exorbitant privilege of the US gradually builds as rest-of-world household portfolios (right column, middle plot and also the gray line in Panel (a) of Figure 1) shift towards US assets. The shift towards US assets, in turn, worsens the US net foreign asset position.

The top-left panel of Figure 2 plots the paths of US and Eurozone consumption during

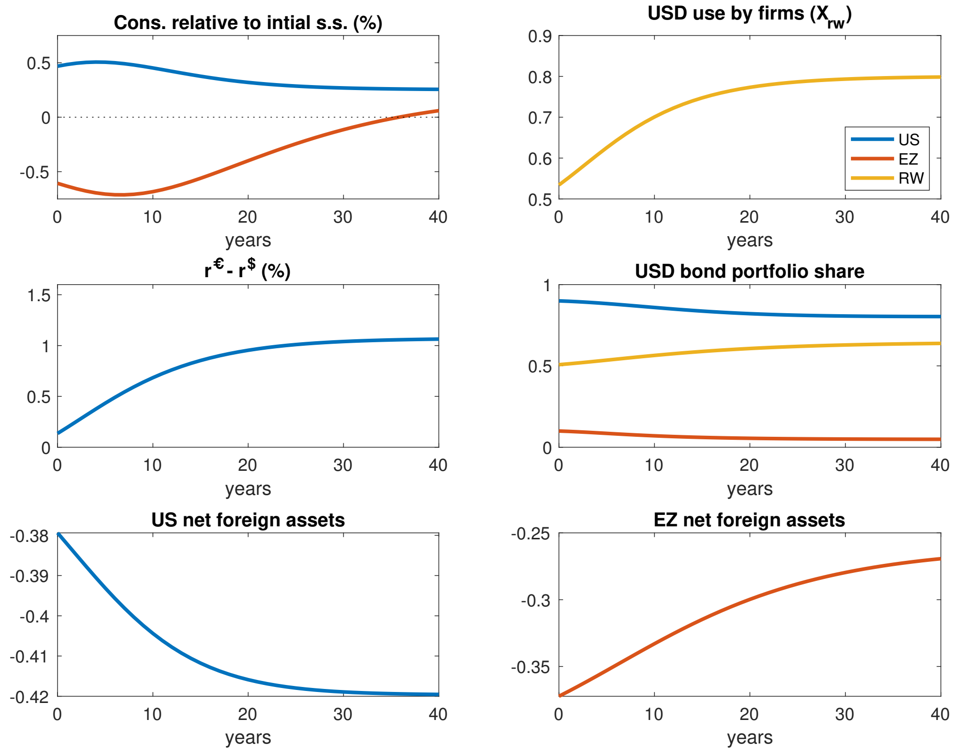


Figure 2: Transition from the symmetric steady state to dollar-dominant steady state.

this transition and shows that US consumption is elevated for an extended period. This is a result of the *increasing* foreign demand for US assets, which allows the US to steadily increase its borrowing from the rest of the world at a low interest rate. Meanwhile, Eurozone consumption is significantly depressed, as the Eurozone increases savings in order to repatriate some of its assets, which were previously held abroad but are now in lower demand externally.

Though US consumption is relatively high throughout the transition period depicted in Figure 1, the steady-state findings in Table 3 show that the eventual consumption levels of the US and the Eurozone are similar, suggesting that transition and long run welfare implications can be very different. Table 4 summarizes the welfare effects of dominance in consumption equivalent units.

While the steady-state welfare calculation (top row) shows essentially no harm of dollar dominance for the Eurozone, incorporating the transition dynamics to the dollar-dominant steady state reverses this conclusion. Taking into account the transition period, in which the US is able to maintain consumption significantly above its eventual steady-state level for an extended period of time, the US permanent consumption equivalent is 0.75% higher than that of the Eurozone. This is an order of magnitude larger than the 0.03% welfare

	US	EZ	RW
steady-state only	0.25%	0.22%	-0.12%
incl. transition	0.37%	-0.37%	-0.00%

Table 4: Welfare gain/loss at dollar-dominant steady state, as percentage of symmetric steady state consumption.

gain implied by steady-state consumption difference alone. Thus, accounting for both out of steady state dynamics and general equilibrium portfolio and asset positions, is crucial for properly assessing the welfare implications of issuing the dominant currency.

3.4 The emergence of dollar dominance

The model rationalizes the historical experience of prolonged dominant-asset regimes – e.g. the British pound was dominant before the 1940s, and the US dollar since – because only dominant-asset steady states are dynamically stable. But why is the dollar, specifically, playing the dominant role currently? Our model suggests that both an initial advantage in the US's ability to supply safe assets and path dependence play important roles.

Regarding the dollar's initial advantage, our model suggests two features were particularly important. First, after WWII the US had a unique ability to credibly supply large quantities of safe assets. Second, under the Bretton Woods agreement, the US was the *only* country with virtually no capital controls, which facilitated international access to its (abundant) supply of safe assets.[18] In such an asymmetric world, our model implies that there is only one steady state, with coordination on the single abundant asset the unique outcome globally, which explains how the dollar established its initial post-war dominance.

Since the dollar gained dominance in world markets, however, the US has become less unique both in terms of its ability to supply large quantities of a safe asset on a global scale, and in terms of its economic size and central role in world trade. However, since historical experience suggests (occasional) transitions are possible, we next explore how empirically relevant changes to the global economy might affect the dollar's position and dominance.

[18] See Ghosh and Qureshi (2016) for a detailed description of the evidence on capital controls. They also note that the US Treasury secretary at the time was well aware of the benefit of an open US capital account, having stated in his 1948 testimony to Congress that controlling capital inflows would require exchange controls that "would do maximum violence to our position as a world financial center."

Introduction of the Euro

Many academics and policy makers once speculated that the formation of the Eurozone might precipitate a shift in the equilibrium (e.g. Chinn and Frankel, 2007). In our model, the emergence of a Eurozone block similar to the US creates a second stable steady state, but path dependence prevents the economy from transiting to it absent some other precipitating shock. To illustrate this point, we simulate two alternative, counter-factual scenarios of the euro introduction. In both cases, we take the starting point (i.e. the world pre-1999) as one in which the supply of the alternative asset is 60% of the size of the US safe asset supply, $\bar{B}^{€} = 0.6\bar{B}^{\$}$, consistent with the supply of German bonds at the time. We then model the formation of the Eurozone as an increase in the supply of the alternative safe asset over time, reflecting the increased fiscal capacity of the Eurozone.[19]

Before exploring dynamics, we first look at the model's steady-state implications before the Eurozone is introduced. When the supply of the alternative asset is only 60% of that of the US asset, we find the model has a unique, dollar-dominant steady state: The initial asymmetry in the supply of the two assets is strong enough to guarantee world coordination on the dollar. Initializing the economy at this steady state, we then consider the resulting transition path(s) as the supply of Eurozone assets grows with the introduction of the euro.

In the first scenario, plotted in Figure 3 with a blue line, we assume that the total supply of Eurozone safe assets converges to the same level as that of the US asset over a 10 year period (i.e. to our benchmark calibration). The results of this exercises are displayed by the blue line. We find that there is a unique transition path out of the dollar-dominant, pre-Eurozone steady state, a path which converges to the new dollar-dominant steady state. Along the transition path, dollar use, the interest rate differential, and the US net foreign asset position are essentially unchanged, consistent with the continued dominance of the dollar in the data. Thus, our model agrees that path dependence is too strong for the introduction of the euro, by itself, to change the currency regime. The euro introduction creates a new, euro-based steady state, but without further shocks, the world would not converge to it.

In the second scenario, we consider the counter-factual possibility that the Eurozone eventually grows significantly larger than the US, to the point that the euro-dominant steady state becomes unique. Making the euro-dominant steady state unique requires the supply of Eurozone assets to exceed the supply of US assets by 30%: i.e. $\frac{\bar{B}^{€}}{\bar{B}^{\$}} > 1.3$.

[19] We model a gradual transition because interest rates on euro-area sovereigns took several years to converge to those of the German Bund, suggesting that markets only gradually accepted euro bonds as a homogeneous safe asset.

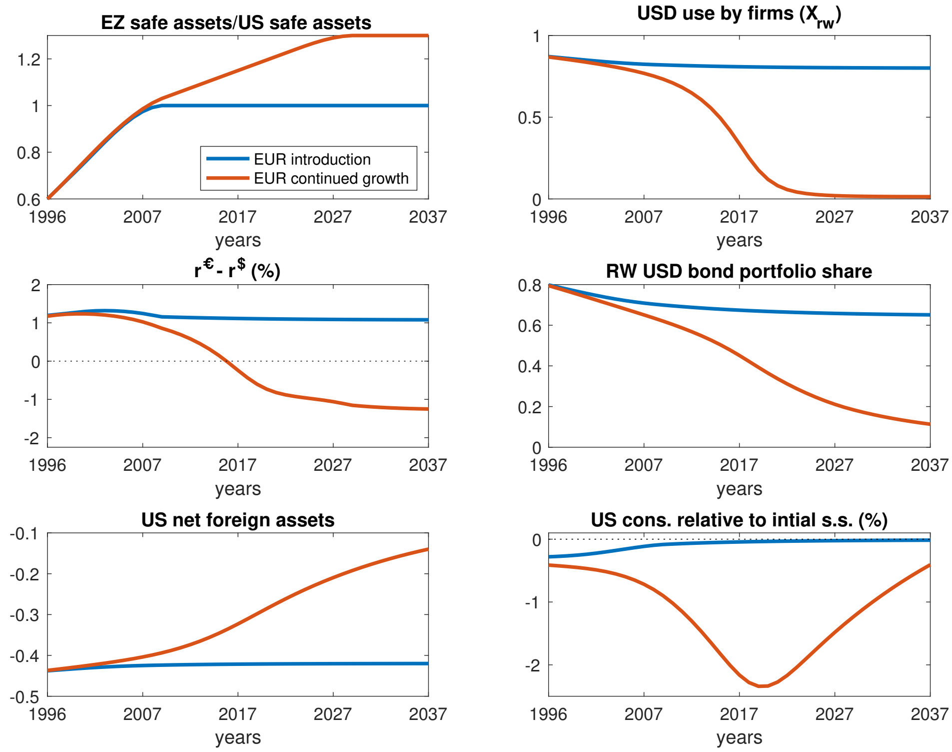


Figure 3: Introduction of euro

We plot this scenario with a red line in Figure 3. Along the path to the new, uniquely stable euro-dominant steady state, the US net foreign asset position shrinks towards zero, and the US's exorbitant privilege benefits disappears as the US safe assets largely leave international markets and return to US portfolios. The welfare impact is significant: the welfare swing between the US and the Eurozone is equivalent to 1.4% of permanent consumption.

The figure also highlights an insight that we can only derive thanks to the model's dynamic nature. Namely, there are significant anticipation effects, as much of the shift in portfolios and international currency usage occurs *before* the supply of Eurozone assets has actually converged to its new, high steady state. This means that a switch away from the dollar-dominant regime could have occurred upon the creation of the Eurozone, if investors around the world were convinced that additional countries such as the UK and Sweden would eventually join, *even if this consolidation never actually happens.* Just the expectation of further expansion, if held long enough for rest-of-world portfolios to rebalance towards Eurozone assets, could have precipitated a change in currency regime.

Overall, these counterfactuals show that merely introducing a fundamentally *equivalent* asset is not enough to shift the currency regime; to become dominant, the alternative asset needs to offer a substantially bigger base than the dollar, now or in the future. Yet, the 30%

threshold implied by the model is not so extreme, and it could realistically be surpassed by either the Eurozone or China in the future.

Growth of the Rest-of-world

The US share of world output has shrunk by roughly 15 percentage points since 1960, with a corresponding rise in the output share of developing (non-G7) countries. A natural question is whether this fall in the relative size of the US imperiled the dominance of the US dollar or if, by contrast, it actually increased the benefits of issuing the dominant currency since the dollar asset now intermediates a larger quantity of rest-of-world trade.

To explore this question, we consider a counter-factual experiment in which the US starts as the largest economy and gradually shrinks (from $\mu_{us} = 0.35$ to 0.20, our benchmark calibration), while the rest-of-world reciprocally grows (from $\mu_{rw} = 0.45$ to 0.60). To isolate the effects of size, we hold constant the aggregate supply of US and euro bonds, $\bar{B}$.[20]

A first insight is that at the starting point where the US is substantially bigger than the Eurozone again there exists only one steady state, at which the dollar is dominant. In this case, the use of dollars in the rest of the world is anchored by the relatively high probability that rest-of-world firms will trade with dollar-intensive US firms. The fall in the size of the US opens *the possibility* of a change in the dominant currency, as our benchmark calibration features two stable steady states. In this sense, our model implies that the dollar's dominance has become more fragile over the last sixty years, both because of the introduction of alternative assets and because of the US's reduced share of the global economy.

To understand whether the changing relative economic size of the US had a material impact on the US's dominance in practice, Figure 4 plots the evolution of several key variables as the economy follows the (unique) transition path, which leads to the dollar-dominant steady state of our symmetric baseline model. The top-right panel of the figure shows that, as the US shrinks, the intensity of the rest-of-world dollar usage ($\bar{X}_{rw}$) falls somewhat but the dollar clearly remains dominant. The intensity of dollar use falls because rest-of-world trading firms now face a substantially smaller chance of encountering US firms, who primarily use dollars in their trading activity. This reduces their incentive to use dollars, a reduction that is reinforced by strategic incentives in general equilibrium.

Even though average use of dollars among rest-of-world firms falls, the *mass* of rest-of-

[20] One subtlety in performing this exercise is that changing country size also changes the relative supply of each country-specific good, which impacts relative prices. To control for this, we adjust the Cobb-Douglas shares in preferences so that (in a frictionless economy) the size changes have no effect on relative prices. Since this is a change in preferences, we refrain from making welfare comparisons.

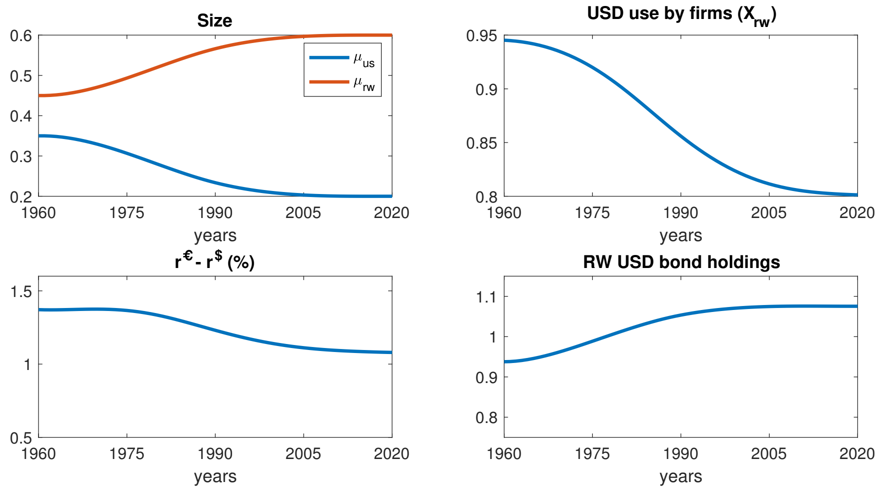


Figure 4: Growth of rest-of-world

world firms that use dollars actually grows due to the increase in the economic size of the rest-of-world countries. This, in turn, increases the total external demand for US assets (due to the increased volume of dollar-denominated trade), as can be seen from the bottom right panel of Figure 4 (B^{rw}). Overall, the US's shrinking size gives rise to offsetting effects on US dominance, which on net leave the US in a similar position in 2020 as it was in 1960 — the clear hegemon, with a somewhat lower exorbitant privilege in terms of the interest rate spread $r^€ - r^\$$ (bottom left panel of the Figure, mainly due to fall in $\bar{X}_{rw}$), but enjoying a higher overall external demand for its assets.

These implications track the historical experience. The US has indeed remained dominant through the last sixty years, although that dominance is somewhat less pronounced given the small but rising share of "other" currencies used internationally as compared to sixty years ago. Still, our model implies that the US is managing to extract a similarly large effective seignorage, thanks to the increased volume of international trade.

Comparison to Gopinath and Stein (2020)

The predictions explored in the above experiments contrast with other models. For example, according to the model of Gopinath and Stein (2020), increasing the supply of an asset (our first experiment) should decrease the likelihood it become dominant whereas our model implies the reverse.[21] Similarly, in their model, increasing the quantity of world trade while keeping asset supplies constant should have significantly boosted dollar dominance.

[21] For example, they highlight this implication in their discussion of proposals for a "Eurobond" in Section 5.3, noting their model implies such a development should only harm the Euro's international status.

While these alternative mechanisms need not be mutually exclusive, we think historical experience favors the sort of relationship implied by our model. First, a strong empirical pattern is that the hegemon country carries a substantial negative net foreign asset position, as exemplified by the persistent US imbalance. This observation is explained endogenously by our model – an asset that is dominant must be widely held abroad – but seems in tension with their model in which the special asset is the scarce one.

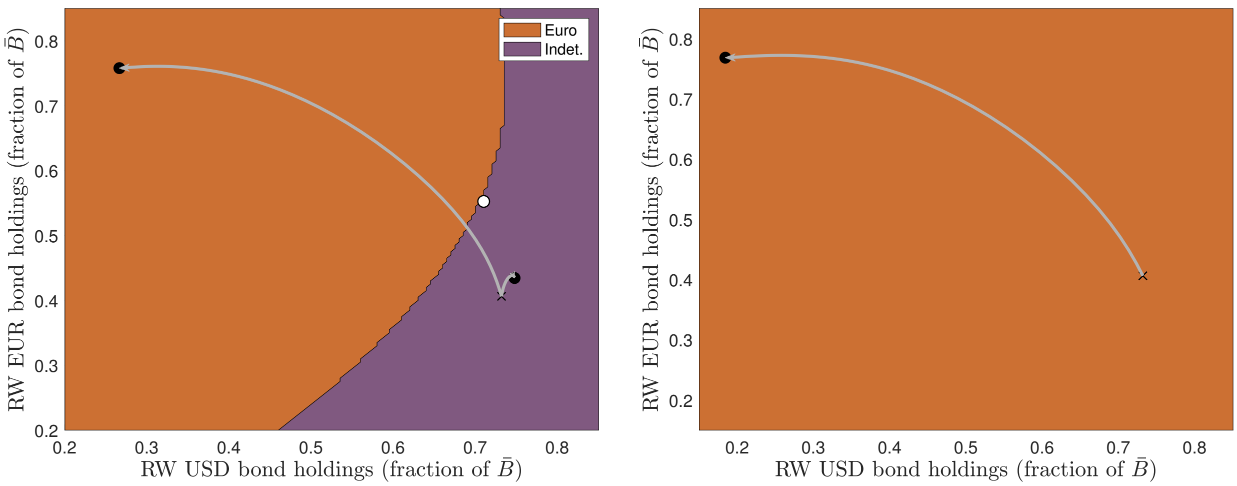
Second, historical accounts of the emergence of the dollar tends to emphasize the importance of dollar "availability" rather than "scarcity." For example, Eichengreen and Flandreau (2012) documents the crucial role played by the Federal Reserve Act of 1913, which significantly increased the availability of offshore dollar trade financing, in jump-starting the US dollar's international role. More recently, Bahaj and Reis (2020) document a similar effect in terms of the growing importance of the Chinese renminbi and the increased number of swap agreements the Chinese Central Bank has signed with rest-of-world counterparts.

As some final supporting evidence evidence, our online Appendix C documents two stylized patterns in cross-sectional data that support the hypothesis that asset availability favors an asset's dominant status. In that appendix, we relate a country's holdings of US Treasuries with the use of dollars in invoicing and the issuance of dollar-denominated bank liabilities.

In the first case, we find that larger holdings of Treasuries in country i are associated with higher dollar invoicing. In the second case, we find that larger holdings of Treasuries in country i are associated with greater issuance of dollar bank liabilities in that country.[22] Both results seem to contrast with the mechanism in Gopinath and Stein (2020), which relies on the offshore scarcity of the dominant asset, but are consistent with our model which relies instead on wide-spread holdings of an asset to drive dominance. Since these are correlations, not causal relationships, we view the results as only suggestive evidence.

As a final observation, we note there is no tension between our theoretical findings and the empirical evidence of e.g. Krishnanurthy and Vissing-Jorgensen (2011) that asset supply and convenience yields are negatively correlated. In our model, the positive association between supply and liquidity premia appears only when comparing steady states. By contrast, near a dominant-currency steady state, local increases in an asset's supply will still imply a fall in its liquidity premium, just as in the data.

[22]We use invoicing because the data on currency composition of trade finance in the cross-section is sparser. However, studies by BIS (2014) and Friberg and Wilander (2008) suggest that these choices are highly correlated in the data.



(a) Attraction regions, moderate trade war

(b) Attraction regions, acute trade war

Figure 5: Attraction regions under trade ware scenarios.

3.5 Trade Policy and Barriers

Recent trade conflicts between the US and other countries have raised the possibility that trade barriers might affect the currency regime. To shed some light on this, we consider two "trade war" scenarios in which the US introduces a proportional tariff γ on all imports and the Eurozone and rest-of-world countries respond by levying the same tariffs on US imports.

Tariffs are implemented as taxes paid by consumers, so while US importers receive the price of $P^j_{us,t}$ for each imported type j good, US consumers pay the effective price

$$\tilde{P}^j_{us,t} = (1+\gamma)P^j_{us,t}$$

Similarly, Eurozone and rest-of-world consumers pay import taxes on US imports, and hence face the prices $\tilde{P}^{us}_{jt} = (1+\gamma)P^{us}_{jt}$. In all countries, tariff revenues are reimbursed lump-sum to households. Even though they are refunded, the tariffs lead to expenditure switching on the consumer side, and thus also shift the equilibrium patterns of trade flows. We consider two scenarios, a "moderate" trade war where $\gamma = 0.15$ and an "acute" one where $\gamma = 0.30$.

Moderate Trade War

Figure 5, Panel (a) depicts the consequences of a permanent trade war, with tariffs set to $\gamma = 0.15$. The tariffs change the position of the steady states and also their respective attraction regions (old steady states are marked with $\times$; the new steady states with a dot).

	US	EZ	RW
Dollar remains dominant	-1.34%	0.07%	0.09%
Euro becomes dominant	-2.13%	0.79%	0.10%

Table 5: Gain/loss as percentage of dollar-dominant steady state consumption.

As the figure shows, the region of unique attraction to the dollar-dominant steady state is eliminated, and both the old and new dollar-dominant steady states lie within the region of equilibrium indeterminacy. Moreover, the region of unique attraction to the euro-dominant steady state is significantly increased. Hence, even a moderate trade war could potentially endanger the position of the dollar, as it makes the dollar-dominant regime unstable.

The trade war weakens dollar dominance for two reasons. First, it diverts rest-of-world trade away from the US towards the Eurozone. Since Eurozone firms are far more likely to use Eurozone assets in their trade, rest-of-world firms become more likely to encounter euro-funded trading partners and, hence, to prefer euros. Second, the overall world trade level falls, decreasing trade quantities relative to total asset supply. As trade financing become less scarce, the equilibrium anchoring effects of the financial friction become weaker, increasing the indeterminacy region.

The first row of Table 5 reports the welfare implications of the moderate trade war, *assuming the dollar remains dominant.* The US is disproportionately hurt by the trade war for two reasons. First, there is the standard effect from the fact that the distortions created by the tariffs hit all of its exports, while the Eurozone and the rest-of-world face tariffs only on their direct trade with the US. Second, as world trade levels fall, the US' seignorage revenue decreases as both fewer firms require liquidity and a smaller portion choose dollars.

Since the starting position of the economy (i.e. the dollar-dominant steady state before the tariffs) is now inside the indeterminate region, however, this is not the only possible outcome. In the second row of Table 5, we report the welfare implications of the same trade war scenario, but assuming the alternative long-run outcome that is now possible – that the dollar looses dominance and the economy transitions to the euro steady state.

In this second case, the US is significantly worse off and Eurozone welfare is substantially improved. On the one hand, at the new steady state the US loses all of its seignorage, as the world currency use and the resulting exorbitant privilege shift to the Eurozone. On the other hand, the transition to this new steady state itself, depicted by the long gray line in Panel (a) of Figure 5, is particularly painful because during the transition the US runs significant trade surpluses as external demand for dollar assets dries up.

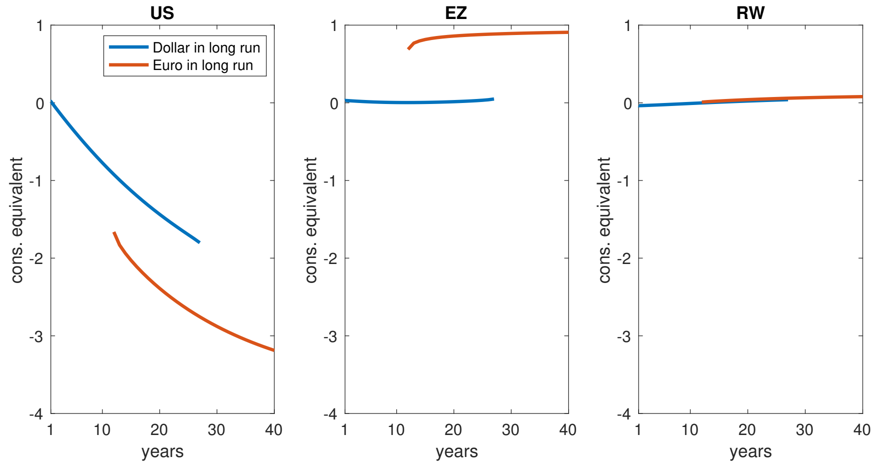


Figure 6: Welfare costs of an acute trade war as a function of duration.

The welfare difference between the US and Eurozone when the euro becomes dominant is 2.9% of permanent consumption. Comparing to the first line of the table, where the US-EZ welfare differential is 1.4%, we conclude that the loss of currency dominance is worth 1.5% of permanent consumption. However, we find that the transition dynamics generate two-thirds of the effect, hence a steady-state-only model would understate the loss by a factor of three.

Acute Trade War

Figure 5, Panel (b) depicts the implications of a permanent 30% tariff between the US and Eurozone/rest-of-world, a scenario we call an "acute" trade war. In this case, the effects of the trade barriers are strong enough to eliminate the symmetric and the dollar-dominant steady states, thereby guaranteeing a transition to the now-unique euro steady state.

This is a strong implication, but a *permanent* 30% tariff on all imports is (we hope) implausible. Hence, we also consider the effects of a temporary trade war of the same magnitude. Figure 6 depicts the welfare cost for different possible durations, ranging from one to 40 years. The figure shows that for trade wars lasting under 10 years, the economy cannot transition to the euro steady state absent other shocks. Within this range, longer trade wars are worse, but not discretely so, and showcases the general stability of the model – even 10 years of a very acute trade war is not enough to shift the currency regime.

For trade wars lasting more than 10 years, however, transition to the euro-dominant steady state becomes possible as in this case we enter the indeterminacy region. If a transition occurs, the US is discretely worse off, and suffers an additional loss of roughly 1% of permanent consumption. If the trade war lasts longer than 28 years, the *unique* outcome is a transition to the euro-dominant steady state, despite the fact that tariffs return to zero in the long-run. Thus, certain temporary shocks, could have *permanent* effects.

4 Conclusions

This paper presents a new theory describing the emergence of dominant international assets. Our model is quantitatively realistic and tractable enough to use for standard macroeconomic analysis. Throughout, we have abstracted from risk: both the potential for short run shocks that perturb the economy around a given steady state and possible longer-run stochastic transitions between currency regimes. Both of these extensions are straightforward: Business-cycle analysis can be conducted using policy functions approximated locally around a given steady state, or via global solution techniques, such as the ones suggested by Richter et al. (2013). Such extensions could help the model address the observation of Gourinchas et al. (2017) that an "exorbitant duty" coincides with the privilege of being the dominant currency. We leave exploration of this issue to future work.

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Appendix

A Proofs

Proof of Lemma 1. First, we prove uniqueness. We aim to show that, under the condition in the Lemma, the quasi-equilibrium correspondence $X(B^{\$}_{rw}, B^{€}_{rw})$ is a scalar valued function for all possible asset holdings. Note that, due to the fact that the total supply of each asset is $\bar{B}$, then market clearing ($\bar{B} = \mu_{rw}B^c_{rw} + \mu_{us}B^c_{us} + \mu_{ez}B^c_{ez}$) puts an upper bound on the feasible rest-of-world asset holdings:

$$B^{\$}_{rw} < \frac{\bar{B}}{\mu_{rw}}$$

$$B^{€}_{rw} < \frac{\bar{B}}{\mu_{rw}}$$

If either $B^{\$}_{rw} = 0$ or $B^{€}_{rw} = 0$, it is trivially true that the quasi-equilibrium is unique. For example, if $B^{€}_{rw} = 0$ then

$$V^{\$}(X_{rw}) = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + X_{rw}} \left(\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})\right) > 0$$

since $\kappa < \pi - r$. Thus, the only quasi-equilibrium in funding is $X_{rw} = 1$. Similarly, if $B^{\$}_{rw} = 0$, the only quasi-equilibrium is $X_{rw} = 0$.

Next, we show that for any pair of bond holdings $\{B^{\$}_{rw}, B^{€}_{rw}\}$, such that $B^{\$} \in (0, \frac{\bar{B}}{\mu_{rw}})$ and $B^{€} \in (0, \frac{\bar{B}}{\mu_{rw}})$, the net payoff of using dollars crosses zero exactly once, thus the quasi-equilibrium is unique. Using $\mu_{us} = \mu_{eu} = \frac{1-\mu_{rw}}{2}$, and evaluating equation (4) at $X_{rw} = 1$:

$$V^{\$}(1) = -\frac{\pi - r}{B^{\$}_{rw} + 1} + \kappa \frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1}$$

This is strictly negative (and thus $X_{rw} = 1$ is not a quasi-equilibrium) if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}.$$

A similar argument implies that $V^{\$}(0) > 0$ if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{€}_{rw} + \frac{1}{2}) + \frac{1}{2}}.$$

But since $B^{\$}_{rw} < \frac{\bar{B}}{\mu_{rw}}$,

$$\frac{\pi - r}{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}} > \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}} = \kappa^{sunspot}$$

Thus, when $\kappa < \kappa^{sunspot}$, $X_{rw} = 1$ is not a quasi-equilibrium for any feasible allocation of asset holdings when $B^{\$} > 0$ and $B^{€} > 0$. Similar argument, shows that $X_{rw} = 0$ is not a quasi-equilibrium either in this case. Hence, all existing quasi-equilibria must be interior and thus solve $V^{\$}(X_{rw}) = 0$.

Evaluating equation (4) gives:

$$V^{\$}(X_{rw}) = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + X_{rw}} \left[\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})\right] - \frac{B^{€}_{rw}}{B^{€}_{rw} + (1 - X_{rw})} \left[\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{ez})\right].$$

Setting the above expression equal to zero and multiplying by $\frac{1}{\kappa}(B^{\$}_{rw} + X_{rw})(B^{€}_{rw} + (1 - X_{rw}))$ results in a quadratic equation in X_{rw}. Simplifying further, and dividing through by κ, allows us to express the resulting quadratic polynomial as $P(X_{rw})$:

$$\begin{aligned} P(X_{rw}) &= (B^{€}_{rw} - B^{\$}_{rw})\mu_{rw} X^2_{rw} \\ &+ \left(B^{\$}_{rw} \left(\frac{1}{2} + \frac{3}{2}\mu_{rw} - 2\frac{\pi - r}{\kappa} \right) + B^{€}_{rw} \left(\frac{1}{2} - \frac{\mu_{rw}}{2} + 2B^{\$}_{rw}\mu_{rw} - \frac{\pi - r}{\kappa} \right) \right) X_{rw} \\ &+ B^{\$}_{rw} \left(\frac{\pi - r}{\kappa} - \left(\frac{1}{2} + \mu_{rw}(B^{€}_{rw} + 1) \right) \right) \end{aligned}$$

Since the quadratic polynomial $P(X_{rw})$ has different signs at $P(0)$ and $P(1)$, it can only have a single crossing in the range $X \in (0, 1)$ and thus there is a unique quasi-equilibrium.

Let $X^* \in (0, 1)$ be the unique quasi-equilibrium value that satisfies $P(X^*) = 0$ for a given level of bond holdings $\{B^{\$}_{rw}, B^{€}_{rw}\}$. To see that $X^* > \frac{B^{\$}_{rw}}{B^{\$}_{rw}+B^{€}_{rw}}$ whenever $B^{\$}_{rw} > B^{€}_{rw}$, notice that if $B^{\$}_{rw} > B^{€}_{rw}$

$$P(X_{rw} = \frac{B^{\$}_{rw}}{B^{\$}_{rw} + B^{€}_{rw}}) = \frac{B^{\$}_{rw} B^{€}_{rw} \mu_{rw} \left(B^{\$}_{rw}(1 + B^{\$}_{rw}) - B^{€}_{rw}(1 + B^{€}_{rw}) \right)}{(B^{€}_{rw} + B^{\$}_{rw})^2} > 0.$$

Hence since $P(1) < 0$, the zero of the quadratic polynomial $P(X)$ must be at a value $X^* > \frac{B^{\$}_{rw}}{B^{\$}_{rw}+B^{€}_{rw}}$.

To prove that $\kappa < \frac{\pi - r}{\bar{B}^{\$} + \frac{\mu_{rw}}{2} + \frac{1}{2}}$ is a sufficient condition for uniqueness, let $\kappa > \frac{\pi - r}{\bar{B} + \frac{\mu_{rw}}{2} + \frac{1}{2}}$ and note that if $B^{\$}_{rw} = \frac{\bar{B}}{\mu_{rw}}$ and $B^{€}_{rw} = \frac{\bar{B}}{\mu_{rw}}$:

$$V^{\$}(1) = -\frac{\pi - r}{B^{\$}_{rw} + 1} + \kappa \frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1} > 0$$

and

$$V^{\$}(0) = \frac{\pi - r}{B^{\$}_{rw} + 1} - \kappa \frac{\mu_{rw}(B^{\$}_{rw} + \frac{1}{2}) + \frac{1}{2}}{B^{\$}_{rw} + 1} < 0$$

Thus, in this case both 1 and 0 are quasi-equilibria, and hence $X(\frac{\bar{B}}{\mu_{rw}}, \frac{\bar{B}}{\mu_{rw}})$ is set valued, Hence, $X(B^{\$}_{rw}, B^{€}_{rw})$ is not a scalar-valued function across the whole state space, and we conclude the quasi-equilibrium is not always unique.

■

Proof of Lemma 2. Throughout, let $j \in [0, \mu_{rw}]$ be an arbitrary index of one of the small open economies. Accounting explicitly for the inequality constraint in bonds, the steady-state Euler equations (2) for dollars can be written

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta^{\$}_j} + \lambda^{\$}_j = \frac{1}{Q^{\$} - \Delta^{\$}_{us}} + \lambda^{\$}_{us} = \frac{1}{Q^{\$} - \Delta^{\$}_{eu}} + \lambda^{\$}_{eu}, \tag{18}$$

where the weakly positive λ's are appropriately-scaled Lagrange multipliers and the complementarity slackness conditions

$$\lambda_j B^{\$}_j = \lambda_{us} B^{\$}_{us} = \lambda_{eu} B^{\$}_{eu} = 0 \tag{19}$$

must hold for all countries.

We being by proving the following additional Lemma which would be helpful:

Lemma 3. $B^{\$}_j = 0$ *if and only if* $X_j = 0$*. Similarly,* $B^{€}_j = 0$ *if and only if* $X_j = 1$*.*

We prove the statement for dollar bond holdings, the statement for euro holdings follows by a parallel argument.

If: If $X_j = X_{ez} = 0$ the premia $\Delta^{\$}_j = \Delta^{\$}_{eu} = 0$ by equation (6), while $\Delta^{\$}_{us} > 0$. Hence equation (18) reduces to

$$\frac{1}{Q^{\$}} + \lambda_j^{\$} = \frac{1}{Q^{\$}} + \lambda_{ez}^{\$} = \frac{1}{Q^{\$} - \Delta_{us}^{\$}} + \lambda_{us}^{\$}. \tag{20}$$

But this equation shows that $\lambda_j > \lambda_{us}$ and $\lambda_{ez} > \lambda_{us}$. Since $\lambda_{us} \geq 0$, we know that $\lambda_j > 0$ and $\lambda_{ez} > 0$ and, from complementary slackness, that $B_j^{\$} = B_{eu}^{\$} = 0$.

Only If: To find a contradiction, suppose that $X_j > 0$ but $B_j^{\$} = 0$. Since $B_j^{\$} = 0$, there exists some other country j' (it could be the US or it could be another small country) which holds positive bonds and pays premium $\Delta_{j'}^{\$} < r$. In this case, equation (18) implies

$$\frac{1}{Q^{\$} - \Delta_{j'}^{\$}} = \frac{1}{Q^{\$} - r} + \lambda_j^{\$}. \tag{21}$$

which cannot be true since $\lambda_j^{\$} \geq 0$. Hence, $B_j^{\$} > 0$. And the helper Lemma 3 is proved.

An implication of the proof above is that $B_{ez}^{\$} = B_{us}^{€} = 0$. We can now use the equality of the remaining premia along with market clearing conditions to compute the expressions in the text. For example, $\Delta_{us}^{\$} = \Delta_j^{\$}$ implies that

$$\frac{X_j}{B_j^{\$} + X_j} = \frac{X_{us}}{B_{us}^{\$} + X_{us}},$$

which simplifies to

$$B_{us}^{\$} = B_j^{\$} \frac{X_{us}}{X_j}.$$

Using this expression and the fact that $B_{ez} = 0$, $B_{us}^{\$}$ and $B_{ez}^{\$}$ can be eliminated in the market clearing condition for dollar bonds.

$$\bar{B} = \int_{\mu_{rw}} \frac{B_{us}^{\$}}{X_{us}} X_i di + \mu_{us} B_{us}^{\$} \tag{22}$$

$$\bar{B} = \frac{B_j^{\$}}{X_j} \int_{\mu_{rw}} X_i di + \mu_{us} \frac{B_j^{\$}}{X_j}. \tag{23}$$

Where in the second line we pick a specific country j and substitute in $X_{us} = 1$. Solving expression (23) for $B_j^{\$}$ gives equation (8) in the Lemma 2. The same steps for euro bonds imply equation (9).

■

Proof of Proposition 1. To show that the dollar-dominant steady state exists, conjecture that the rest-of-world traders all use dollars and thus $X_{rw} = 1$. Using Lemma 2, the optimal bond holdings of the rest-of-world households are then

$$B_{rw}^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}}$$

$$B_{rw}^{€} = 0.$$

Plugging those expression into the relative payoff of seeking dollar vs euro funding for a rest-of-world trader ($V^{\$}$), we have:

$$V^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}} (\pi - r - \kappa(1 - \mu_{rw} - \mu_{us})) > 0.$$

Thus, using dollars is strictly preferred by any rest-of-world trader, and the dollar dominant steady state where $X_{rw} = 1$ is indeed sustained. Conjecturing $X_{rw} = 0$, instead, and following a similar argument shows that the euro-dominant steady state $X = 0$ also exists.

Lastly, we look for interior steady-state equilibria where $X_{rw} \in (0, 1)$. In that, case the optimal bond holdings for the rest-of-world households are given by:

$$B_{rw}^{\$} = \bar{B}\frac{X_{rw}}{\mu_{rw}X_{rw} + \mu_{us}}$$
$$B_{rw}^{€} = \bar{B}\frac{1 - X_{rw}}{\mu_{rw}(1 - X_{rw}) + \mu_{ez}}$$

Substituting in those expressions for bond holdings in the value of seeking dollar collateral relative to euro collateral for a rest-of-world trader, we have

$$\begin{aligned} V^{\$}(X_{rw}) = & \frac{\bar{B}}{\bar{B} + \mu_{rw}X_{rw} + \mu_{us}} [\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})] - \\ & \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - X_{rw}) + \mu_{ez}} [\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{us})] . \end{aligned} \tag{24}$$

Any interior equilibrium must satisfy $V(X_{rw}) = 0$ – these are the points at which the traders are indifferent between seeking dollar and euro financing. To find the zeros of $V^{\$}(X_{rw})$, we set (24) equal to 0, and multiply through with $(\bar{B} + \mu_{rw}X_{rw} + \mu_{us})(\bar{B} + \mu_{rw}(1 - X_{rw}) + \mu_{ez})$. Then further dividing by $\bar{B}$, gives us the condition

$$(\bar{B}+\mu_{rw}(1-X_{rw})+\mu_{ez}) [\pi - r - \kappa(1 - X_{rw}\mu_{rw} - \mu_{us})]-(\bar{B}+\mu_{rw}X_{rw}+\mu_{us}) [\pi - r - \kappa(X_{rw}\mu_{rw} + \mu_{us})] = 0$$

Using the fact that $\mu_{us} = \mu_{ez}$ and $\mu_{us} + \mu_{eu} + \mu_{rw} = 1$, this equation simplifies to

$$\mu_{rw}(\kappa(\bar{B} + 1) - \pi - r)(2X_{rw} - 1) = 0.$$

This linear equation has the unique solution $X_{rw} = \frac{1}{2}$ when $\kappa \neq \frac{\pi - r}{\bar{B}+1}$, and admits any $X_{rw} \in [0, 1]$ as a solution in the knife edge case $\kappa = \frac{\pi - r}{\bar{B}+1}$. Thus, for any $\kappa \geq 0$ that is different from $\frac{\pi - r}{\bar{B}+1}$ there are three steady states, $X_{rw} \in \{0, \frac{1}{2}, 1\}$, and when $\kappa = \frac{\pi - r}{\bar{B}+1}$ there is a continuum of steady states $X_{rw} \in [0, 1]$. The associated steady-state bond holdings are then immediately implied by Lemma 2. ■

Proof of Proposition 2. To prove local stability of a given steady state, we need to show that the best-response functions define a contraction in the neighborhood of that steady state. Define the vector of best response functions of trading firms and households in country j, given the actions of all other firms, X_{rw}, and households in the rest of the world $B^{\$}_{rw}$ and $B^{€}_{rw}$:

$$\begin{aligned}
\varphi_X(X, B^{\$}, B^{€}) &= \frac{B^{\$}(\pi - \kappa(B^{€}+1) + \kappa(\mu_{rw}X + \mu_{us})(2B^{€}+1))}{(B^{\$}+B^{€})\pi + \kappa((\mu_{rw}X+\mu_{us})(B^{\$}-B^{€}) - B^{\$})} \\
\varphi_{B^{\$}}(X, B^{\$}, B^{€}) &= \bar{B}\frac{X}{\mu_{rw}X + \mu_{us}} \\
\varphi_{B^{€}}(X, B^{\$}, B^{€}) &= \bar{B}\frac{1-X}{\mu_{rw}(1-X) + \mu_{ez}}
\end{aligned}$$

Stacking these in the vector $\Phi \equiv [\varphi_X, \varphi_{B^{\$}}, \varphi_{B^{€}}]$, we want to show that Φ is a local contraction map, which is the case whenever the eigenvalues of the Jacobian $\nabla\Phi$ lie inside the unit circle.

The Jacobian has the form

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\varphi_X}{\partial X} & \frac{\partial\varphi_X}{\partial B^{\$}} & \frac{\partial\varphi_X}{\partial B^{€}} \\ \frac{\partial\varphi_{B^{\$}}}{\partial X} & 0 & 0 \\ \frac{\partial\varphi_{B^{€}}}{\partial X} & 0 & 0 \end{bmatrix}$$

hence its eigenvalues are given by the roots of the characteristic polynomial

$$\lambda\left(\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - \frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} - \frac{\partial\varphi_X}{\partial B^{€}}\frac{\partial\varphi_{B^{€}}}{\partial X}\right) = 0.$$

Clearly, one of the solutions is $\lambda = 0$, so we just need to ensure that the roots of the quadratic expression in the parenthesis are inside the unit circle We proceed to check this condition for each steady state.

Case I: Symmetric Steady State

At the symmetric steady state we have that $\frac{\partial\varphi_X}{\partial B^{\$}} = -\frac{\partial\varphi_X}{\partial B^{€}}$ and $\frac{\partial\varphi_{B^{\$}}}{\partial X} = -\frac{\partial\varphi_{B^{€}}}{\partial X}$. Hence, the relevant condition for the eigenvalues reduces to

$$\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - 2\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} = 0$$

with roots

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial X} \pm \sqrt{\left(\frac{\partial\varphi_X}{\partial X}\right)^2 + 8\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X}}\right).$$

At the symmetric steady state.

$$\frac{\partial \varphi_X}{\partial X} = \frac{(1+2\bar{B})\kappa\mu_{rw}}{2\pi - \kappa} > 0$$

since $\kappa < \pi$. Hence, the bigger root (in absolute value) is

$$\lambda^* = \frac{1}{2}\left(\frac{\partial \varphi_X}{\partial X} + \sqrt{\left(\frac{\partial \varphi_X}{\partial X}\right)^2 + 8\frac{\partial \varphi_X}{\partial B^{\$}}\frac{\partial \varphi_{B^{\$}}}{\partial X}}\right).$$

Lastly, since we also have that

$$\frac{\partial \frac{\partial \varphi_{B^{\$}}}{\partial X}}{\partial \kappa} = \frac{\partial \frac{\partial \varphi_X}{\partial B^{\$}}}{\partial \kappa} = 0,$$

the root is growing in κ. The threshold $\bar{\kappa}$ that ensures the root is within the unit circle solves $\lambda^* = 1$, which after some re-arranging results in:

$$1 - \frac{\partial \varphi_X}{\partial X} - 2\frac{\partial \varphi_X}{\partial B^{\$}}\frac{\partial \varphi_{B^{\$}}}{\partial X} = 0.$$

Solving for the threshold κ, we obtain

$$\bar{\kappa} = \frac{\pi - r}{\bar{B} + 1}$$

Hence, in the neighborhood of the symmetric steady state, the roots of the characteristic polynomial are inside the unit circle so long as $\kappa < \bar{\kappa}$.

Case II: Dollar-dominant Steady State

At the dollar dominant steady state $X_{rw} = 1$ and

$$\frac{\partial \varphi_X}{\partial X} = \frac{\partial \varphi_X}{\partial B^{\$}} = 0.$$

Hence the roots λ are given by

$$\lambda^2 = \frac{\partial \varphi_{B^{€}}}{\partial X}\frac{\partial \varphi_X}{\partial B^{€}}$$

where

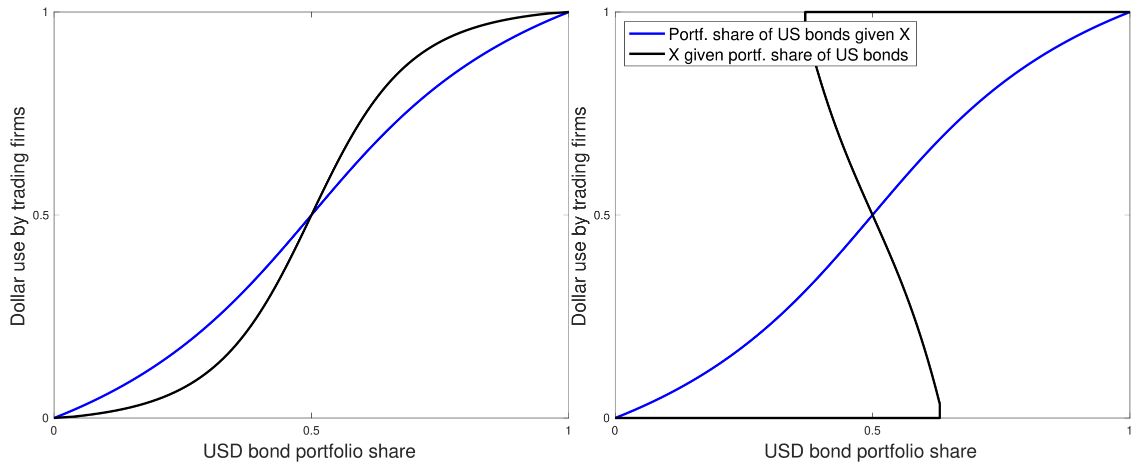
$$\frac{\partial \varphi_{B^{€}}}{\partial X}\frac{\partial \varphi_X}{\partial B^{€}} = \frac{(1+\mu_{rw})(\pi - r) - \kappa(\frac{1}{2} + \mu_{rw}(1 + 2\bar{B} + \frac{\mu_{rw}}{2}))}{(1-\mu_{rw})(\pi - r - \frac{\kappa}{2}(1-\mu_{rw}))}.$$

If $\kappa < \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ then the above expression is positive. In that case, if it is also true that

$$\kappa > \frac{\pi - r}{\bar{B}+1} = \bar{\kappa}$$

(i.e. $\kappa \in (\bar{\kappa}, \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})})$ then $\lambda < 1$.

On the other hand, if $\kappa > \frac{(\pi-r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ the best response function φ_X hits its upper



(a) $\kappa \in (\bar{\kappa}, \kappa^{sunspot})$

(b) $\kappa > \kappa^{sunspot}$

Figure 7: Best response functions

bound of 1. In particular, in that case $\frac{\partial \varphi_X}{\partial B^{€}} > 0$ in the neighborhood of the dollar-dominant steady state. This implies that starting at the dollar-dominant steady state, a small increase in $B^{€}$ will increase φ_X even further, going over 1. However, $X_{rw} = 1$ is the upper bound on X, and enforcing this, means that for $\kappa > \frac{(\pi - r)(1+\mu_{rw})}{\frac{1}{2}+\mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ effectively $\frac{\partial \varphi_X}{\partial B^{€}} = 0$ and thus $\lambda^2 = 0$. Thus, all eigenvalues of $\nabla\Phi$ are zero, and the system is very stable locally.

Thus, the dollar-dominant steady state is stable for any $\kappa > \bar{\kappa} = \frac{\pi - r}{\bar{B}+1}$.

Case III: Euro-dominant Steady State

Can be proven with identical steps to Case II.

■

B Visual illustration of simplified model

Figure 7 illustrates Proposition 2. First, the black line shows the quasi-equilibrium in trade finance choice X_{rw}, as a function of bond holdings (x-axis). Panel (a) demonstrates local stability, which can be seen from the fact that the black line is relatively flat in the corners, implying that firms do not respond too much to a small change in portfolios. Panel (b) showcases the possibility of sunspots when κ is large, in which case X becomes a correspondence.

Appendix for Online Publication

C US bond holdings and currency use in the data

A key implication of the model is that the holdings of US assets in country j are positively correlated with the intensity of dollar use in the international trade of that country. To test this in the data, we regress the share of dollar invoicing of a country's trade on the share of US bonds in that country's aggregate portfolio.[23] In particular, we estimate the regression

$$X_j = \alpha + \beta_{B_{usd}} \frac{\text{Holdings of US bonds}_j}{\text{Total Foreign Bond Holdings}_j} + \beta_{UStrade} \frac{\text{Trade with US}_j}{\text{Total Trade}_j} + \varepsilon_j$$

where X_j is the share of dollar invoicing in country j's trade (data from Gopinath (2015)), while portfolio data is from the IMF's CPIS database. The estimates are presented in Table 6. In addition to bond holdings and US trade, we also include an euro dummy variable which takes the value of one for a country inside the Eurozone. We include these countries to maximize sample size, but the results remain qualitatively the same if we exclude them from the sample.

As predicted by the model, we find that the portfolio share of US bonds in total foreign bond holdings is highly positively correlated with the share of trade invoiced in dollars (column (1)). This is not simply a proxy for direct trade with the US – controlling for the volume of trade with the US does not change the significance or magnitude of the estimated relationship with US bond holdings (column (3)). In fact, direct US trade is not significantly associated with invoicing once we control for bond holdings (while it is by itself). Lastly, for a subset of the countries, we also have data on the currency composition of their foreign bonds holdings.[24] We find a similarly strong relationship between dollar denominated bond holdings and invoicing (columns (4) and (5)).

Thus, the data indeed strongly supports the key implication of our model that as a country saves more in US assets its firms also trade (internationally) in dollars more.

Our regressions are related, but distinct, from the ones in Gopinath and Stein (2020) who use the share of dollars in the aggregate bank liabilities of country j as the regressor, not the US bonds' share in the country's holdings of *foreign* debt assets. And in fact, our regression can help differentiate the empirical implications vis-a-vis this other mechanism.

Specifically, in Gopinath and Stein (2020) dollar usage is actually driven by a shortage of US Treasury holdings in a given country, which incentivizes local banks to create dollar

[23] As documented by BIS (2014) and Friberg and Wilander (2008), the currency of invoicing is closely related to the currency in which trade is settled and financed, for the countries for which there is data for both invoicing and trade financing. However, the coverage in terms of invoicing data is much better, giving us a significantly larger sample size, thanks to the dataset built by Gopinath (2015).

[24] Maggiori et al. (2018) show that investors have an affinity for dollar-denominated foreign assets, even when the issuer has a different local currency, so the currency composition of portfolios is likely even more highly concentrated in dollars, than in US assets alone.

Table 6: Dollar invoice share and portfolio share of dollar bond holdings

	(1)	(2)	(3)	(4)	(5)
$\frac{\text{Holdings of US bonds}_j}{\text{Total Foreign Bond Holdings}_j}$	0.73***		0.65***		
	(0.18)		(0.22)		
$\frac{\text{Trade with US}_j}{\text{Total Trade}_j}$		0.80***	0.21		1.25**
		(0.30)	(0.34)		(0.58)
$\frac{\text{Holdings of USD bonds}_j}{\text{Total Foreign Bond Holdings}_j}$				0.52***	0.36**
				(0.17)	(0.18)
Euro dummy	Yes	Yes	Yes	Yes	Yes
R^2	0.52	0.41	0.53	0.47	0.56
N	39	39	39	31	31

deposits for local households to save in. It is a story of dollarization of the bank sector due to a relative *shortage* of US safe assets, while in our model, instead, dollar usage is motivated by the relative *abundance* of US-issued bonds in rest-of-world household portfolios.

Thus, our regressions, which use the composition of the *foreign* asset holdings of countries as the regressor not only broadly supports our model, but also helps differentiate with this different mechanism. Given our finding that countries which own a large amount of US Treasuries are indeed associated with a high usage of dollars in their trade activities, we conclude that the usage of dollars in trade is more likely driven by the fact that US Treasuries are abundantly available in global markets, as our model suggests, rather than being relatively scarce.

To further this analysis, we can directly test whether the dollarization of the local banking industry is a associated with a relative lack of US Treasuries at the country level, or rather by a relatively abundance of Treasuries. Specifically, we estimate the regression

$$\frac{\text{Bank Liab. to non-banks}_j}{GDP_j} = \alpha + \gamma_{B_{us}} \frac{\text{Holdings of US bonds}_j}{GDP_j} + \gamma_{UStrade} \frac{\text{US Trade}_j}{GDP_j} + \varepsilon_j$$

The left-hand-side variable is from the BIS database on local bank statistics, and effectively captures the size of bank deposits owned by non-bank entities. The model of Gopinath and Stein (2020) implies that $\gamma_{B_{us}} < 0$ – i.e. holding the volume of trade with the US constant (which generates a demand for dollar savings), the more US Treasuries country j owns, the lower is the need and incentive for locally creating additional, synthetic dollar safe assets via dollar deposits with local banks.

To the contrary, in the data we find a strong positive association between the holdings of US bonds at the country level, and the bank dollarization. The results are presented in Table 7. We consider several variation of the basic regression, alternatively using the holdings of dollar denominated foreign bonds (instead of holdings of US-issued bonds only), and also

Table 7: Dollar bank deposits and US bond holdings

	(1)	(2)	(3)	(4)
$\frac{\text{Holdings of US bonds}_j}{\text{GDP}_j}$	0.68***		0.71***	
	(0.16)		(0.16)	
$\frac{\text{Trade with US}_j}{\text{GDP}_j}$	0.14	0.43		
	(0.11)	(0.30)		
$\frac{\text{Holdings of USD bonds}_j}{\text{GDP}_j}$		0.44***		0.50***
		(0.07)		(0.03)
$\frac{\text{USD invoiced Trade}_j}{\text{GDP}_j}$			0.12**	0.28***
			(0.06)	(0.04)
Euro dummy	Yes	Yes	Yes	Yes
R^2	0.60	0.85	0.67	0.96
N	22	15	21	15

proxying for the demand for dollar savings with the total volume of trade invoices in dollars in country j, rather than just using its trade directly with the US.

In all cases, we find a highly positive $\gamma_{B_{us}}$, which combined with the results in Table 6, suggests that the dollarization of both trade and banking is associated with a relative *abundance* of US bond holdings in a given country, consistent with our liquidity-based mechanism.

D Additional Quantitative Model Details

D.1 Households

For $j \in \{us, ez\}$, foreign imports consist of the good of the other big country and an aggregate of rest-of-the world goods. Hence, the big country consumption aggregator is

$$C_{jt} = (C_{jt}^{j})^{a_h} \left((C_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (C_{jt}^{rw})^{(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}})} \right)^{1-a_h}, \tag{25}$$

where j' is the complement of j and $C_{jt}^{j'}$ is the consumption in country j of the good of country j', and a_h controls the degree of home bias in consumption. Rest-of-world consumption goods are aggregated according to $C_{jt}^{rw} = (\int (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$. The corresponding aggregate consumption price index is

$$P_{jt} = \frac{1}{K} (P_{jt}^{j})^{a_h} \left((P_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}} \right)^{1-a_h}. \tag{26}$$

where $K \equiv a_h^{a_h}(1-a_h)^{1-a_h}\left(\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}\right)^{(1-a_h)\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}}\left(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}\right)^{(1-a_h)\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}}$.

For small countries $j \in [0, \mu_{rw}]$, the consumption basket includes imports from both big countries and all other rest-of-world small countries:

$$C_{jt} = C_{jt}^{j\ a_h}\left((C_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}(C_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}(C_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}\right)^{1-a_h}. \tag{27}$$

The associated price index is

$$P_{jt} = \frac{1}{K_{rw}}(P_{jt}^{j})^{a_h}\left((P_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}(P_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}(P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}}\right)^{1-a_h}. \tag{28}$$

where $K_{rw}$is defined analogously to K above.

D.2 The Import-Export Sector

The following subsection provide additional details on the trading structure of the general equilibrium model.

Trading Round and Profits

Let $c = (j, j')$ be a double index, capturing an arbitrary country pair, and let $\tilde{m}_{ct}^{im}$ be the mass of *funded* importing firms in country j seeking trade with funded exporting firms in country j' at time t. Then the probability of a country j importer matching with a country j' exporter is

$$p_{ct}^{ie} = \frac{\tilde{m}_{c',t}^{ex}}{\left[(\tilde{m}_{c',t}^{ex})^{1/\varepsilon_T} + (\tilde{m}_{ct}^{im})^{1/\varepsilon_T}\right]^{\varepsilon_T}},$$

where $c' \equiv (j', j)$. Using analogous definitions, the probability of a country j exporter matching with a country j' importer is

$$p_{ct}^{ei} = \frac{\tilde{m}_{c',t}^{im}}{\left[(\tilde{m}_{c',t}^{im})^{1/\varepsilon_T} + (\tilde{m}_{ct}^{ex})^{1/\varepsilon_T}\right]^{\varepsilon_T}}.$$

Let $\tilde{X}_{jt}$ be the fraction of funded country j firms who hold dollar collateral, as defined in the main text. Then the expected profits of a country-j importer importing from j' who holds dollars is given by

$$\pi_{c,t}^{\$,im} = p_{c,t}^{ie}\frac{(1-\alpha)}{P_{c',t}^{whol}}\left[P_{j,t}^{j'} - P_{j',t}^{j'} - \kappa P_{c',t}^{whol}(1-\tilde{X}_{jt})\right],$$

while if it hold euros, expected profits are

$$\pi_{c,t}^{€,im} = p_{c,t}^{ie}\frac{(1-\alpha)}{P_{c',t}^{whol}}\left[P_{j,t}^{j'} - P_{j',t}^{j'} - \kappa P_{c',t}^{whol}\tilde{X}_{jt}\right].$$

Similar expressions hold for exporters:

$$\pi_{c,t}^{\$,ex} = p_{c,t}^{ei} \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[P_{j',t}^{j} - P_{j,t}^{j} - \kappa P_{c,t}^{whol} (1 - \tilde{X}_{j',t}) \right]$$
$$\pi_{c,t}^{€,ex} = p_{c,t}^{ei} \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[P_{j',t}^{j} - P_{j,t}^{j} - \kappa P_{c,t}^{whol} \tilde{X}_{j',t} \right].$$

Firm Formation

Equilibrium with interior p_{ct}^{im} and p_{ct}^{ex} requires that, prior to learning their private currency choice, firms are ex post indifferent between importing and exporting to the various countries. Hence, for example in the US, we must have

$$X_{us} \pi_{(us,j),t}^{\$,im} + (1 - X_{us}) \pi_{(us,j),t}^{€,im} = X_{us} \pi_{(us,j'),t}^{\$,im} + (1 - X_{us}) \pi_{(us,j'),t}^{€,im}$$

for all US potential trading partners j and j'. Similarly,

$$X_{us} \pi_{(us,j),t}^{\$,im} + (1 - X_{us}) \pi_{(us,j),t}^{€,im} = X_{us} \pi_{(us,j),t}^{\$,ex} + (1 - X_{us}) \pi_{(us,j),t}^{€,ex}.$$

The above equations are sufficient to pin down the equilibrium probabilities for importing and exporting to and from each country pair.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$W_{jt} = \max_{\{p_{jit}^{im}, p_{jit}^{ex}\}} X_{jt} \Pi_{jt}^{\$} + (1 - X_{jt}) \Pi_{jt}^{€} - \phi P_{jt} = 0 \quad \text{s.t.} \quad \sum_{i \neq j} p_{jit}^{im} + \sum_{i \neq j} p_{jit}^{ex} = 1.$$

is satisfied. Thus, the equilibrium mass of active firms in country j, which we label m_{jt}, is determined by the condition $W_{jt} = 0$, and the optimal trade pattern is such that firms are indifferent between operating as an importer or exporter in any direction.

Equilibrium Defintion

Definition 3 (Equilibrium). *A symmetric equilibrium is a pair of bond prices $\{Q_t^{\$}, Q_t^{€}\}$, and a set of country specific allocations $\{C_{jt}^{us}, C_{jt}^{ez}, C_{jt}^{rw}, B_{jt}^{\$}, B_{j}^{€}, X_{jt}, m_{jt}, p_{jit}^{im}, p_{jit}^{ex}\}$, prices $\{P_{jt}^{us}, P_{jt}^{ez}, P_{jt}^{rw}\}$, and liquidity premia $\{\Delta_{jt}^{\$}, \Delta_{jt}^{€}\}$ for $j \in \{us, ez, rw\}$ such that*

1. *The household optimality conditions are satisfied.*

2. *The trading firms optimality conditions are satisfied.*

3. *Liquidity premia earned by households are given by (16)-(17).*

4. *The mass of successful cross-border trading matches is consistent with consumption of foreign goods $C_{jt}^{j'}$ for all $j \neq j'$.*

E Rest-of-World Asset Supply

Our baseline economy abstracts from the presence of any savings vehicle issued by the rest of the world. This is in part because, absent a liquidity premium term, adding such an asset would create an indeterminacy in long-run wealth levels. The same sort of indeterminacy is pervasive in open economy models with incomplete asset markets.

A simple way to include a rest-of-world asset market is to assume there exists an exogenous liquidity demand z_j for the assets of each country $j \in [0, \mu_{rw}]$. Though we don't model this role explicitly, we assume it is proportional to the measure of firms in the country j economy, so that the liquidity wedge a the rest-of-world asset is given by

$$\Delta_{jt}^{RW} = \frac{M^f \left(m_{jt} z_j, \nu P_{rw,t}^{rw} B_{jt}^j Q_t^j \right)}{\nu P_{rw,t}^{rw} B_{jt}^j Q_t^j} r.$$

In the numerical implementation of this model, we make z_j negligible ($z_j = 0.01$), so that it plays no substantive role except for making asset positions determinate.

Since the small open economies j are identical, the price and liquidity premia of all of these assets are the same, e.g. $Q_t^j = Q_t^{rw}$, hence it is sufficient to treat it as a basket of rest-of-world assets denoted by RW. The household Euler equation for the rest-of-world basket of bonds is

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{rw,t+1}^{rw}}{P_{rw,t}^{rw}} \frac{1}{Q_t^{rw}(1 - \Delta_{jt}^{rw} + \tau'(B_{j,t}^{rw}, B_{j,t-1}^{rw})} \right].$$

A desirable feature of this approach to determining holding of the rest-of-world bond is that the steady state portfolio allocations are independent of a scale shift in the z_j.