Risk Sharing, International Comovement, and the Goldilocks Trade Elasticity*

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Abstract

In international business cycle models, lower trade elasticities magnify the international transmission of shocks, helping to match the observation that countries that trade more have more correlated business cycles. Low trade elasticities also increase the correlation between relative consumption and the real exchange rate, while the empirical correlation is negative. Matching both patterns – a strong trade-comovement relationship and negative Backus-Smith correlations – requires a trade elasticity that is "just right." We propose a simple two-country business cycle model that matches these patterns when the calibrated (short-run) trade elasticity is around 0.6.

Note: This is a very preliminary draft, and will be updated before Ryan's presentation in January. Please do not circulate. Statements marked with an $*$ are preliminary.

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1 Introduction

Countries with relatively high consumption typically experience appreciated real exchange rates (Backus and Smith, 1993). Meanwhile, countries that trade more tend to have more correlated business cycles (Kose and Yi, 2006). Reproducing either one of these patterns, however, is difficult in a standard open economy model. Many authors have proposed solutions to one of these "puzzles" or the other, but the literature has few theories that can explain both patterns at once. This paper proposes one possibility.

Central to providing a joint explanation of these two empirical patterns is a deeper understanding of how different elasticities of substitution can influence the channels studied. The literature has typically assumed constant elasticity of substitution utility (equivalently production) functions, typically in a nested form. In the inner nest, an elasticity ϕ governs the elasticity between traded goods produced at home and those produced abroad, typically with some degree of home bias in the share of home versus foreign traded goods. The home consumption basket then combines the aggregate traded good with a domestically produced nontraded good with elasticity, η. Debates about the appropriate values of these elasticities are legion, and most resolutions to the puzzles cited above require particular assumptions about one elasticity or the other, or a particular relation between the two.

The challenge is that solutions to one problem often make the other worse. Consider, for example, Benigno and Thoenissen (2008)'s solution to the Backus-Smith problem. In their model, a rise in home traded-good productivity can cause such a large increase in home's nontraded good price that it swamps the terms-of-trade depreciation caused by higher home traded good supply. This leads to an overall increase in the domestic price level and an empirically realistic negative Backus-Smith correlation. To operate, however, this channel requires that substitution between traded and nontraded goods (η) be quite low, while substitution between home and foreign traded goods (ϕ) remains rather high. A high ϕ, however, implies that an expansion of production in the home traded sector leads to a contraction of traded production in the foreign economy, failing the trade-comovement test.[1]

[1]Several recent international papers, notably Itskhoki and Mukhin (2021), have also attempted to resolve a set of exchange rate puzzles, including the Backus and Smith (1993) challenge, using models of partially

Papers focused on the trade-comovement problem face the same problems in reverse. For example, Drozd et al. (2021) argue that a model with complete asset markets and a very low (short-run) ϕ can help to explain the trade-comovement puzzle. Because these authors assume complete asset markets, it runs afoul of the Backus and Smith (1993) puzzle almost by construction. Even with incomplete asset markets, however, lower trade elasticities imply a very strong terms-of-trade depreciation when home country productivity rises. The large terms-of-trade depreciation implies a real exchange rate depreciation even as home consumption rises, meaning such models cannot explain the Backus-Smith patterns.

Our summary thus far suggests that high trade elasticities are useful to explain the Backus-Smith fact, while low trade elasticities are useful to explain the trade-comovement pattern. An important paper by Corsetti et al. (2008) seems to contradict this. Those authors find that, in the context of an incomplete asset market economy, an extremely low trade elasticity can reverse the typical relationship between home productivity and terms of trade. Intuitively, one might expect that flooding the market with greater quantities of a good that is inelastically demanded should lead to a large fall in the good's price. Yet, Corsetti et al. (2008) find that, with home-biased consumption baskets and sufficiently low elasticities, an increase in home productivity can lead to an appreciation of the home terms of trade, and therefore, an appreciation of the home real exchange rate. Hence, in their context, low elasticities are a *solution* to the Backus-Smith puzzle, rather than an impediment to resolving it.[2]

We begin the paper by reexamining this result from Corsetti et al. (2008). In a series of three propositions, we argue that this finding is a symptom of equilibrium multiplicity and is probably not indicative of the sort of effects we should expect from productivity shocks when trade elasticities are low. To understand why, consider a risk-free world with two ex ante symmetric economies, home and foreign, extremely low trade elasticities, and consumption

segmented asset markets. Those theories are impressively successful in matching several important international facts, but they also tend to assume extremely low levels of real international integration among countries. These theories focus less on international comovement. Nevertheless, they could not match the challenge raised by Kose and Yi (2006) without losing their desirability properties vis-à-vis the exchange rate.

[2]Drozd et al. (2021) point out that, when this sign flip occurs, the implications for comovement are counterfactual.

baskets that feature home-bias. Starting from a relative price of unity, conjecture a small appreciation in the home terms of trade. This appreciation makes the home country richer, so that the world consumption basket is tilted towards the home good. Such a shift in the world-wide consumption basket will lead to a further rise in the price of the home good. With low enough elasticities, this self-reinforcing logic leads to a steady-state in which the home currency is appreciated and the world consumption basket is tilted strongly towards the home good. Given symmetry, there is always a mirror-image steady-state with a strongly depreciated home currency and the world consumption basket tilted to the foreign good.

Our propositions show that, for sufficiently-low trade elasticities, Corsetti et al. (2008)'s simplest two country model admits three steady-state equilibria. The three equilibria consist of the two asymmetric equilibria described above and a symmetric one, with a terms of trade equal to unity. Corsetti et al. (2008) and subsequent authors have typically linearized around the latter, symmetric, steady state. But, we show, this symmetric equilibrium is not stable under the "tatonnement" concept of Samuelson (1947). Local to this unstable equilibrium, the comparative statics follow the patterns described by Corsetti et al. (2008).

By contrast, the two asymmetric equilibria described above are always stable in the Tatonnement sense. And, they have the conventional comparative statics: local to either of these asymmetric equilibria, an increase in home productivity causes a depreciation of the home terms of trade. We argue that this finding reinstates the intuition described earlier: strong complementarity between home and foreign output goods is generally favorable to international comovement, but not to resolving the Backus-Smith puzzle. Whenever home production increases, this raises the relative price of the foreign good and, ceteris paribus, the incentive to produce in the foreign country. But, if low elasticities are good for international comovement in production, they also make the Backus-Smith puzzle more pronounced. For, if the home country experiences an increase in consumption, it should also experience a large real exchange rate depreciation as its productivity rises. In other words, a Backus-Smith correlation near one rather than the negative correlation that appears in the data.

Low elasticities address one puzzle, high elasticities the other: can one find a happy middle ground that addresses both puzzles at once? In most production-based international business cycle models, the answer is “no”: endogenous labor supply and wage effects mean that either the Backus-Smith puzzle or trade-comovement puzzle persists across calibration. Oftentimes, both puzzles appear at the same time.

We propose to resolve this tension in a standard two-country business cycle model, with the addition of a cross-country transmission mechanism built on the ideas of Guerrieri et al. (2022). In that paper, sticky prices and sufficiently low price elasticities mean that productivity shocks in one sector transmit as demand shocks in another. Introducing their idea into a two-country economy, we show that productivity shocks in the home country can transmit as powerful demand shocks in the foreign country. At given wages and with low (but not too low!) elasticities, an increase in home productivity raises the labor demanded in both countries. We assume a sticky real wage, so that labor in each country is entirely demand-determined, and an outward shift in the labor demand curve translates directly into an increase in equilibrium labor. This combination of cross-country demand transmission and appropriately low trade elasticities is enough to resolve both the Backus-Smith and the trade comovement puzzles simultaneously.

We introduce our idea as a minimal deviation from Corsetti et al. (2008) and first demonstrate that it can qualitatively match both patterns at once. Along the way we characterize a “Goldilocks”' range of import elasticities that are consistent with resolving the puzzle. The key to success is strong demand transmission: the Backus-Smith puzzle is resolved when a productivity shock in the home country causes a sufficiently large increase in foreign demand that consumption in the foreign country rises by more than in the home country. When this is true, an increase in home (traded) productivity is accompanied by a depreciation of the real exchange rate, an increase in home consumption, but an even larger increase in foreign consumption. In this way, the economy delivers a realistically negative Backus-Smith correlation even as it delivers strong international comovement across integrated countries.

After establishing that our simple model can qualitatively match the two key patterns in the data, we go on to quantify exactly the Goldilocks elasticity in a more quantitative realistic economy. We do this by searching for a set of model parameters, and specifically elasticities ϕ and η that best match these facts along with several other key moments in the data. The model matches these moments well, and we find that the model best matches data when ϕ is close to 0.6. This value is clearly below the most common estimates in the trade literature, but higher than the values used by Corsetti et al. (2008). We conclude with a discussion of how the elasticity that we estimate should best be mapped to the data.

The paper proceeds as follows. In Section 2, we revisit and reinterpret certain findings of Corsetti et al. (2008) in the context of financial autarky. In Section 3, we introduce our baseline model, which adds endogenous labor demand and wage stickiness to the Corsetti et al. (2008) benchmark. Here, we explore how the model can match the qualitative facts that motivate this paper, and explore why an alternative model without supply-to-demand transmission fails to do so. In Section 4, we add additional realistic features, including a limited set of financial assets, nontraded goods, and a parameter governing steady-state trade frictions. We then calibrate the model to best match quantitative facts and perform counterfactual exercises to better understand the parameter conditions under which the model is consistent with the data.

2 The problem with extremely low ϕ

Intuitively, lower trade elasticities should imply that a relative expansion in home output should cause a relatively larger fall in the home price. Corsetti et al. (2008) find, however, that this intuitive comparative static is reversed when elasticities become extremely low. Instead, when ϕ is below some critical threshold, $\bar{\phi}$, they find that an increase in home productivity actually causes the home terms of trade to appreciate: higher production of the good makes it more expensive. In this section, we revisit that result and conclude that it relies on an implicit and somewhat implausible equilibrium selection. A more plausible equilibrium selection would always imply that increased home productivity leads to a home depreciation.

The Corsetti et al. (2008) Model

We begin by summarizing the simplest version of the model studied in Corsetti et al. (2008). The model economy consists of two symmetric countries, h and f. Each country receives an exogenous endowment of goods, Y_{ht} and Y_{ft}. The home country enjoys a consumption basket that aggregates the home and foreign endowment goods according to

$$C_t = \left(a_h^{1/\phi} C_{ht}^{\frac{\phi-1}{\phi}} + a_f^{1/\phi} C_{ft}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}. \tag{1}$$

In the above equation, the share parameter $a_h \in (0,1)$ implies home bias in consumption baskets, $a_h > a_f$ with $a_h + a_f = 1$, the law of one price holds across for both goods, and the prices of the home and foreign goods are P_{ht} and P_{ft}, respectively.

Corsetti et al. (2008) consider an extreme version of incomplete markets in which countries engage in balanced trade each period. This implies that the home household budget constraint is just

$$P_t C_t = P_{ht} Y_{ht}, \tag{2}$$

where P_t is the home consumption basket price index

$$P_t \equiv \left(a_h P_{ht}^{1-\phi} + a_f P_{ft}^{1-\phi} \right)^{\frac{1}{1-\phi}}. \tag{3}$$

With these preferences home demand for the foreign good is given by

$$C_{ft} = a_f \left(\frac{P_{ft}}{P_t} \right)^{-\phi} C_t. \tag{4}$$

Analogous expressions govern the foreign economy. Specifically, we have

$$C_t^* = \left(a_h^{1/\phi} C_{ft}^{*\,\frac{\phi-1}{\phi}} + a_f^{1/\phi} C_{ht}^{*\,\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}} \tag{5}$$

$$P_t^* C_t^* = P_{ft} Y_{ft} \tag{6}$$

$$P_t^* = \left(a_h P_{ft}^{1-\phi} + a_f P_{ht}^{1-\phi} \right)^{\frac{1}{1-\phi}} \tag{7}$$

$$C_{ht}^* = a_f \left(\frac{P_{ht}}{P_t^*} \right)^{-\phi} C_t^*. \tag{8}$$

The final condition for equilibrium is the trade balance condition. Since they assume financial autarky, the value of imports of the home country must be equal in value to the imports of the foreign country, i.e.,

$$P_{ft}C_{ft} = P_{ht}C^*_{ht}. \tag{9}$$

Since prices are flexible, only relative prices matter. We therefore define the terms of trade $\tau_t \equiv P_{ft}/P_{ht}$, such that a high τ_t corresponds to a depreciation of the home terms of trade. Using this definition, equation (9) can be written as

$$\tau_t = \frac{C^*_{ht}}{C_{ft}}. \tag{10}$$

Equation (10) is useful because it provides a relationship between the equilibrium import demands in each country (which depend only on exogenous endowments and the relative price τ) on the right-hand side and the implied τ, on the left, that would clear the trade balance condition given those demands. Dropping time subscripts and combining demand functions with price index definitions and simplifying, we can rewrite the market-clearing terms of trade in (10) as a "best response" to country-specific import demands:

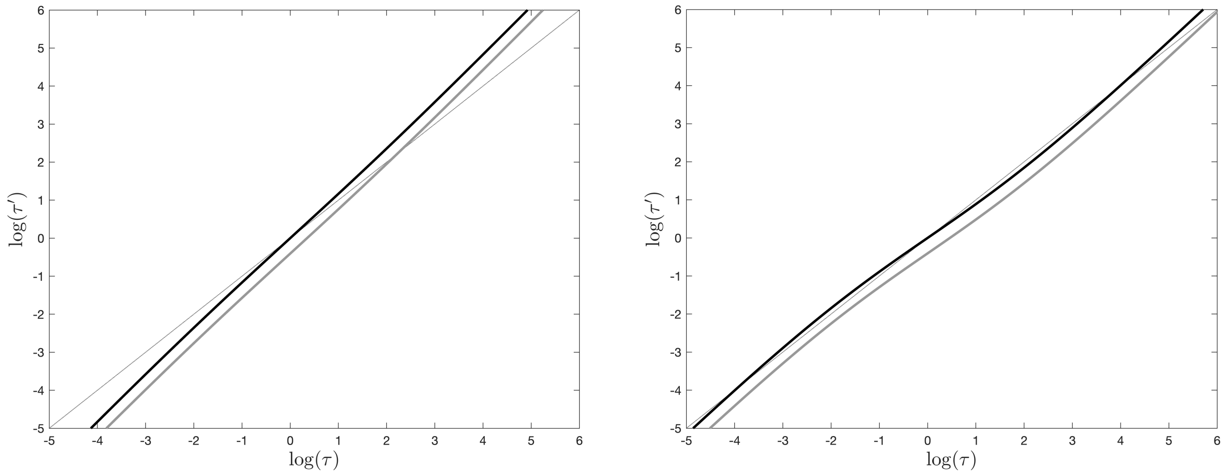
$$\tau' = \mathcal{T}(\tau) \equiv \tau^{1+\phi}\frac{a_h + a_f\tau^{1-\phi}}{a_f + a_h\tau^{1-\phi}}\frac{Y_f}{Y_h}. \tag{11}$$

A steady state of the economy corresponds to a fixed point $\tau = \mathcal{T}(\tau)$ whenever $Y_{ht} = Y_{ft} = \bar{Y}$ are both equal to their steady-state values. Inspecting (11), it is immediately evident that $\tau = 1$ is *a* steady-state equilibrium. This constitutes our first proposition:

Proposition 1. *The model always has a unique symmetric steady-state equilibrium in which* $\tau = 1$*.*

Fluctuations around this symmetric steady state have been the focus of most analysis in the literature. The approach of Corsetti et al. (2008) is to linearize the model around the symmetric equilibrium described in Proposition 1 and to describe how it responds locally to small shocks to country endowments.

The key parameter for Corsetti et al. (2008) is the elasticity parameter ϕ that governs the degree to which households will change their consumption basket as relative prices shift. Figure 1 depicts, in log-scale, the best responses function $\mathcal{T}(\tau)$ for two different values of



(a) High elasticity, $\phi > \bar{\phi}$

(b) Low elasticity, $\phi < \bar{\phi}$

Figure 1: Market-clearing notional "best-response" functions for different values of demand elasticity ϕ. Solid line plots the function in steady-state (symmetric endowments.). Light gray line plots with function with high home endowment, i.e., $Y_{ht} > Y_{ft}$.

the elasticity parameter. In panel (a), the ϕ is set to a relatively high number above the threshold $\bar{\phi} \equiv 1 - \frac{1}{2a_h}$ identified by those authors. The figures shows that the response exhibits a single crossing of the 45^o line — in this case "from below" — indicating that the symmetric steady state described by Proposition 1 is the only steady state of the economy.

By contrast, panel (b) plots the best response when ϕ is below the threshold $\bar{\phi}$. In this instance, the Figure shows the best response function exhibits three crossings of the 45^o line, suggesting that for elasticities below this threshold $\bar{\phi}$ the economy may have more than one equilibrium. Indeed, $\phi < \bar{\phi}$ turns out to be both necessary and sufficient for the model economy to have multiple equilibria. Proposition 2 summarizes the existence and stability properties of these additional equilibria.

Proposition 2. *When $\phi < \bar{\phi}$, the economy has exactly two additional asymmetric steady-state equilibria, one which implies $\tau > 1$ and one with $\tau < 1$. Moreover, when $\phi < \bar{\phi}$, only the asymmetric equilibria are Tatonnement stable in the sense of Samuelson (1947).*

Proof. Proved in the Appendix. □

The proposition establishes that the additional asymmetric equilibria appear at exactly the same threshold that determined the terms of trade sign switch emphasized by Corsetti et al. (2008). Moreover, when they exist, an analysis of the excess demand functions in the economy show that *only* the asymmetric equilibria are Tatonnement stable. This finding complicates the interpretation of Corsetti et al. (2008)'s finding of a sign reversal around $\bar{\phi}$. In particular, given that the symmetric equilibrium is not Tatonnement stable, one might wonder if the unexpected result is somehow a peculiarity associated with considering local perturbations around an unstable equilibrium. This indeed appears to be the case.[3]

Proposition 3. *Local to any Tatonnement-stable equilibrium in the Corsetti et al. (2008) model, an increase in home productivity causes a depreciation in the home terms of trade.*

Proof. Proved in the Appendix. □

When we consider local dynamics, it is only close to the unstable steady state that the economy exhibits the terms of trade responses highlighted by Corsetti et al. (2008). The light gray lines in Figure 1 provide graphical intuition for this. Inspecting again equation (11) it is clear that an increase in the home endowment shifts the best response line downwards. That is, for any fixed conjecture of τ, higher home endowment causes the home country to increase its imports while the foreign country does not change its own. Since home country imports appear in the denominator, $\mathcal{T}(\tau)$ shifts downwards. The direction of this shift does not depend on ϕ.

What does depend on ϕ – and on the particular steady state being analyzed – is whether the function $\mathcal{T}(\tau)$ crosses the 45^o line from above or below. If the function crosses from below, then local dynamics will follow a "standard" intuition: more output, low price. Only in the case of the unstable symmetric equilibrium does the function cross from above, leading to the surprising finding that more home output could raise its price.

Panel (b) of the figure also demonstrates a second important point: the unusual appreciation dynamics depend crucially on using a *local* approximation. As depicted in the Figure, the increase in home endowments causes a large enough shift that the "middle" equilibrium disappears: starting from a symmetric steady state, a large enough shock implies that the

[3]To the extent that one follows Corsetti et al. (2008) selecting the symmetric steady state for linearization, an appreciation of higher home productivity would typically cause a fall in labor demand and therefore output in the second country.

only potential equilibrium response is depreciation. The closer the elasticity is to $\bar{\phi}$ from below, the smaller the shock required to invalidate the local approximation, meaning that an appreciation response to the shock is not a possible equilibrium in the model. Hence, even if one is unconcerned with selecting a tatonnement-stable steady state, one still might be concerned that local analysis when $\phi < \bar{\phi}$ may give misleading results.

[Could probably show that $\partial \log(\tau)/\partial Y_{ht}$ is continuous and monotonic regardless of which stable equilibrium one selects.]

3 Towards a Joint Resolution

Section 2 emphasizes an important intuition: other things being equal, low elasticities make it harder for open economy models to explain negative Backus and Smith (1993) correlations. On the other hand, in production economies, elasticities that are low enough to imply complementarity between home and foreign output are favorable for cross-country propagation of shocks. Complementarity, after all, means that an increase in the availability of the home good should lead to a large increase in the relative price of the foreign good, and therefore higher incentives to produce, ceteris paribus – in the foreign country.

This tension clearly creates a challenge for matching these two puzzles. One way to relax this tension somewhat is the introduction of nontraded goods, which creates a wedge between the terms of trade and the ratio of domestic and foreign aggregate prices indexes. If high traded productivity at home increases the price of nontraded goods, it becomes possible for the real exchange rate to appreciate even as the terms of trade depreciate. For this to work, however, a successful calibration requires a relatively high ϕ (so that the terms of trade movements are muted) and relatively low η (so that nontraded price movements are large). Benigno and Thoenissen (2008), for example, calibrates a $\phi = 2$ and $\eta = 0.44$. With such a high trade elasticity, however, the introduction of endogenous production to the model will imply weak and, in most models, negative transmission across countries.

In order to jointly match strong international transmission and negative Backus-Smith correlations, we propose a different modification of the basic model outlined in Section 2.

Our proposal borrows from Guerrieri et al. (2022). That paper shows how, when elasticities of substitution are low, a supply shock in one sector can be transmitted as a demand shock to others. We show that the same mechanism can transmit supply shocks from the home country as demand shocks in the foreign country, thereby matching both of the puzzles that motivate this paper.

It turns out that a qualitative resolution to these puzzles only requires two modifications to the simple Corsetti et al. (2008) model described earlier. First, we introduce a competitive firm sector that hires labor to produce traded output. Home technology shocks therefore shift home labor demand in a standard neoclassical manner, increasing labor demand when productivity is high. Second, we introduce rigid (real) wages so that labor is entirely demand-determined.[4]

Specifically, we assume that firms are competitive and hire labor to produce output. The representative firm production function is given by $Y_{ht} = A_{ht}N_{ht}^{\alpha}$, and the firm solves the simple static optimization problem

$$\max_{N_{ht}} P_{ht}A_{ht}N_{ht}^{\alpha} - W_t N_{ht}. \tag{12}$$

Firm optimality therefore implies

$$W_t = P_{ht}A_{ht}\alpha N_{ht}^{\alpha-1}. \tag{13}$$

An analogous equation applies to foreign firms. We assume that a fixed stock $\bar{N}$ of labor is supplied inelastically by households in each country and that real wages are fixed at a symmetric exogenous level in both countries. Maintaining symmetry, this means

$$W_t/P_t = W_t^*/P_t = \omega. \tag{14}$$

By exploring dynamics local to a steady-state with unemployment, the specific value of $\bar{N}$ becomes immaterial. We refer to this extension of the simple Corsetti et al. (2008) model as

[4] Guerrieri et al. (2022) impose a downward nominal wage rigidity, but the qualitative implications are similar.

our baseline model.

Because we continue to assume financial autarky, both households and firms solve static problems. Combining equilibrium conditions, we can once again derive a "best response" expression for the market clearing terms of trade,

$$\tau' = \tilde{\mathcal{T}}(\tau) \equiv \tau^{1+\phi+\alpha/(1-\alpha)} \left(\frac{a_h + a_f \tau^{1-\phi}}{a_f + a_h \tau^{1-\phi}}\right)^{\Gamma} \left(\frac{A_f}{A_h}\right)^{\frac{1}{1-\alpha}}, \tag{15}$$

where $\Gamma \equiv \frac{1-\phi(1-\alpha)}{1-\alpha-\phi(1-\alpha)}$. Using (15), it is therefore straightforward to show that a steady-state symmetric equilibrium with $\tau = 1$ continues to always exist.

Moreover, the condition for equilibrium uniqueness in this version of the model is

$$\phi > \underline{\phi} \equiv \frac{1}{1-\alpha}\left(1 - \frac{1+\alpha}{2a_h}\right). \tag{16}$$

Comparing this condition with the threshold $\bar{\phi}$ from the model in Section 2, it can be shown that $\underline{\phi} < \bar{\phi}$. In other words, the condition for uniqueness (and for the economy to deliver standard terms-of-trade comparative statics) is relaxed.[5]

To explore the model's implications for the two puzzles' motivating this paper, we log-linearize the fixed point version of equation (15) around the symmetric steady-state. Defining $\psi \equiv \phi + \alpha/(1-\alpha)$, we find an expression for the effects of home productivity on the terms of trade

$$\hat{\tau}_t = \frac{1}{1-\alpha} \frac{1}{\psi + \Gamma(a_f - a_h)(1-\phi)} \hat{A}_{ht}, \tag{17}$$

in which $\hat{x}_t$ denotes the log deviations from the steady-state of variable x_t.

Tracing the effects of $\hat{\tau}_t$ and $\hat{A}_{ht}$ through the remaining model equations, we derive the following proposition:

Proposition 4. *For any $\phi > \underline{\phi}$, an increase in home traded-good productivity leads to a depreciation of the real exchange rate. Moreover, for any $\phi \in (\underline{\phi}, \tilde{\phi})$, home traded-good productivity leads to a joint increase in home consumption, home labor, foreign consumption, and foreign labor, with C_{ft} increasing by more than C_{ht}.*

[5]Indeed, it is possible that $\underline{\phi} < 0$, in which case the model has a unique steady state and standard terms of trade effects for all parameterizations.

Proof. Proved in the appendix. $\square$

In other words, for $\phi \in (\underset{\sim}{\phi}, \tilde{\phi})$, the model can explain strong international comovement of labor and consumption, along with negative Backus-Smith correlations. The threshold range is given by

$$\underset{\sim}{\phi} \equiv \frac{a_h - \alpha}{2a_h(1-\alpha)} \quad \text{and} \quad \tilde{\phi} \equiv \frac{1}{2a_h}. \tag{18}$$

Slightly tedious algebra establishes that $\underline{\phi} < \underset{\sim}{\phi} < \tilde{\phi}$, so that the range of parameters that solve both problems is both non-empty and consistent with a unique equilibrium. Moreover, the distance $\tilde{\phi} - \underset{\sim}{\phi}$ grows as a_h shrinks, implying that the range of parameters that solve the joint problem becomes larger as the economy becomes more open.

The nonempty range $(\underset{\sim}{\phi}, \tilde{\phi})$ constitutes the set of "Goldilocks trade elasticities" mentioned in the title of the paper. This range depends on two parameters, the labor share parameter α and the home bias parameters a_h. While we quantify the model in the next section more rigorously, these parameters already have fairly standard values: a labor share of $\alpha = 0.65$ is common, and a home bias parameter around $a_h = 0.7$ is also common value. In this case, the Goldilocks range spans values of ϕ between roughly 0.10 and 0.71. These remain quite low elasticities relative to what has been found in the much of trade literature, and we discuss this implication in the next section.

A key observation is that this model addresses the Backus-Smith puzzle in a qualitatively different manner than other proposals in the literature. As discussed above, the simple Corsetti et al. (2008) model solves the puzzle by reversing the terms of trade implications of traded productivity. In their model, high home productivity causes home consumption to rise and foreign consumption to fall. This means, while the model captures the "standard" positive correlation between home productivity and home consumption, it is inconsistent with the empirical observation that consumption levels are positively correlated across countries.

By contrast, Benigno and Thoenissen (2008) solve the Backus-Smith puzzle by creating a wedge between the real exchange rate and the terms of trade, again while maintaining the implication that home consumption rises more than foreign. This solution avoids the counterfactual implication of negatively correlated cross-country consumption, but it also implies a negative correlation between the real exchange rate and the terms of trade, which is not something we see in the data.

Our resolution works differently than either of those mentioned above. It maintains the "standard" positive correlation between the real exchange rates and terms of trade, and also between home and foreign consumption. Instead, it solves the Backus-Smith problem by implying that foreign consumption responds *more strongly* than home consumption to home productivity shocks. Increased productivity isn't "bad" for either country, but it boosts foreign consumption by more. Though we do not yet pursue a quantitative match between model and data, intuitively, this finding seems to fit the "US sneezes and the world catches cold" narrative that has grown in importance in recent research and popular media accounts.

We conclude this section by observing that demand-like transmission of productivity shocks is crucial for these implications. If labor markets cleared with a standard disutility of working, then foreign labor would fall in response to the shock. This would imply a smaller foreign consumption response. In that case, the model would fail to match either strong international comovement or negative Backus-Smith correlations. Proposition 5 formalizes this claim.

Proposition 5. **Suppose that the disutility of labor is given by* $v(N) = \gamma \frac{N^{1+\mu}}{1+\mu}$ *and the labor market clears with flexible prices. Then, for any parameterization,* $\frac{\partial N_h}{\partial A_h} < 0$ *and* $corr\left(C_{ht}/C_{ft}, RER_t\right) > 0$.

4 Quantifying the Channels

We now expand the stylized model of Section 3 with several features in order to allow the model to capture several key features of the data. The key modifications are (i) intertemporal preferences with CRRA utility; (ii) the introduction of international savings via trade in a risk-free home-denominated bond; (iii) the introduction of a nontraded goods sector; and (iv) allowing for fixed trade costs. Items (i)-(ii) are standard in most modern business cycle models and allow the model to capture patterns in net exports that provide important target moments for such theories. Item (iii) allows for an empirically realistic share of trade and for imperfect comovement and differing volatilities between the terms of trade and the real exchange rate, which we do see in the data. Finally, item (iv) allows us to include a realistic degree of costs to trade and (eventually) to perform experiments to see how tariff shocks might influence the economy.

Despite these additions, the model remains stylized in important dimensions. Most important, perhaps, is the lack of a more detailed input-output microstructure. The recent literature (e.g., Imbs and Mejean, 2015) has emphasized how a detailed understanding of these interactions is crucial for understanding the connection between micro-level and aggregate trade elasticities. Moreover, the literature has also emphasized the distinction between short-run and long-run trade elasticities (Drozd et al., 2021, [Add citation]). For the moment, we leave these important discussions aside. The implication is that the elasticity we estimate to match business cycles statistics should be considered a reduced-form, aggregate, and short-run elasticity. Expanding the model to consider these additional dimensions appears a natural way to build on the exercises presented here.

We now turn to summarizing the model.

Households

The consumption basket consists of three types of goods, nontraded, traded home goods, and traded foreign goods. The two traded goods are aggregated according to the same CES function in 1 above.

$$C_{Tt} = \left(a_h^{1/\phi} C_{ht}^{\frac{\phi-1}{\phi}} + a_f^{1/\phi} C_{ft}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}} \tag{19}$$

This consumption basket is then combined with domestically-produced nontraded goods according to

$$C_t = \left(a_T^{1/\eta} C_{Tt}^{\frac{\eta-1}{\eta}} + a_n^{1/\eta} C_{Nt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{20}$$

In (20), η measures the elasticity of substitution between the traded bundle and the nontraded good. Household maximize the present value of utility,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right] \tag{21}$$

by choosing their demand for the aggregate consumption bundle C_t and for non-state contingent bonds B_{ht+1} denominated in the home-produced traded good. Household supply a fixed amount of labor that, due to wage rigidities, may or may not be employed at any moment. The household budget constraint is

$$P_t C_t + Q_t P_{ht} B_{h,t+1} = P_{ht} B_{ht} + W_t N_t + T_t. \tag{22}$$

In the above, the home aggregate price index is

$$P_t = (a_T P_{Tt}^{1-\eta} + a_N P_{Nt}^{1-\eta})^{\frac{1}{1-\eta}} \tag{23}$$

where $P_{Tt} = \left(a_h P_{ht}^{1-\phi} + a_f (P_{ft}(1+f))^{1-\phi}\right)^{\frac{1}{1-\phi}}$ now reflects the additional cost, f, of a fixed tariff imposed on the household. Tariffs are rebated in lump-sum, T_t, to the household. Analogous expressions hold for the foreign country.

Firms

As before, firms are competitive and therefore earn zero profits. Labor moves freely across the N and h sectors. The N sector solves

$$\max_{L_{Nt}} P_{Nt} A_{Nt} N_{Nt}^{\alpha} - W_t N_{Nt} \tag{24}$$

and the h sector solves

$$\max_{L_{ht}} P_{ht} A_{ht} N_{ht}^{\alpha} - W_t N_{ht}. \tag{25}$$

Wages

We continue to assume that wages are sticky in real terms,

$$W_t / P_t = \zeta \omega. \tag{26}$$

We've experimented with allowing gradual adjustment of wages and this appears to improve the empirical performance of the model only slightly.

Table 1: Model and data moments.

Moment	**Data**	**Model**
$std(\Delta\log(GDP_t))$	2.00	2.00
$corr(\Delta\log(GDP_t), \Delta\log(GDP_{t-1}))$	0.20	0.20
$std(\log(RER_t))/std(\log(GDP_t))$	3.90	1.14
$std(\log(P_N/P_h))/std(\log(GDP_t))$	0.86	1.01
$std(\log(N_t)/std(\log(GDP_t))$	1.19	1.04
$corr(\log(RER_t), \log(C_t/C_t^*))$	-0.71	-0.66
$corr(\log(RER_t), NX_t)$	0.60	0.53
$corr(\log(RER_t), \log(TOT_t))$	0.52	0.58
$corr(\log(GDP_t), \log(GDP_t^*))$	0.68	0.67
$corr(\log(C_t), \log(C_t^*))$	0.60	0.49
$corr(\log(N_t), \log(N_t^*))$	0.54	0.60

Note: All moments based on annual data for the US relative to the OECD. Moments for ΔGDP based on US data from 1955 - 2019. Remaining moments from Corsetti et al. (2008), using HP-filtered log-levels for all variables.

Equilibrium

Equilibrium is defined in the standard way. The Appendix provides a detailed list of all the equilibrium variables and conditions.

4.1 Calibration

We calibrate the model to match a set of empirical targets describing annual comovements between the US (home) and OECD (foreign) economies. We ask the model to match 11 empirical moments, which are reported in Table 1. To discipline the overall variability and persistence in the model, we target the standard deviation of GDP growth and the autocorrelation of GDP growth.

The remaining moments include several of the key relative volatilities and cross-correlations that jointly pose a challenge for standard open economy models. The volatility moments include the standard deviations of the real exchange rate, the relative price of nontradables, and the terms of trade, all relative to the standard deviation of GDP. The first set of correlations focuses on the Backus-Smith patterns, and includes correlation of the real exchange rate with, respectively, relative country consumption, the terms of trade, and net exports. A second set of correlations connects to the puzzle of international transmission, and includes

Table 2: Model calibration.

Description	Parameter	Value
Home traded share	a_h	0.500
Nontraded share	a_n	0.700
Discount factor	β	0.960
Steady-state trade costs	f	0.200
Inverse IES	σ	1.000
Labor share	α	0.720
T-NT elasticity	η	0.264
Trade elasticity	ϕ	0.613
AR of traded tech	ρ_t	0.594
AR of nontraded tech	ρ_n	0.000
SD of traded tech	σ_t	1.038
SD of nontraded tech	σ_n	0.890

the cross-country correlations of GDP, consumption, and total employment. All of these moments are based on HP-filtered data and copied from Table 3 of Corsetti et al. (2008).

We calibrate a set of parameters to standard values for annual frequency. We set the discount rate $\beta = 0.96$, and the intertemporal elasticity $\frac{1}{\sigma} = 1$. The preference parameters are $a_n = 0.7$, and $a_h = 0.5$. This corresponds to a roughly 70% share of nontradable goods in consumption. Notice that we do not assume any home-bias in preferences. For realism, we introduce a steady-state trade cost (tariff) rate of 20%, to capture the roughly 20% average markup on foreign good sold at home. This implies a very small home bias in average traded consumption shares, but it has a negligible impact on any results: no home bias is necessary for the model to match any of the facts we present.

We select the remaining seven parameters to best match our 11 target moments. These include the α, the labor share in the production function, and ϕ and η, the two key elasticities of substitution. The remaining parameters concern the exogenous process for the four productivity parameters in the model. We assume that productivity in all sectors is mutually orthogonal and AR(1) in growth rates. The parameters of the non-traded productivity are ρ_N and σ_N, and ρ_T and σ_T for the traded sector productivity.

The second column of Table 1 reports the model moments corresponding to our empirical targets, while Table 2 reports the parameters required. Despite the stylized nature of the model, most of the parameters fall in the range generally considered reasonable in the literature. For example, the value $\alpha = 0.72$ is quite close to standard values used in the literature. The value $\eta = 0.26$ is rather low, but is important for capturing the substantial volatility of the real exchange rate that cannot be attributed to the terms of trade.

From the perspective of our motivation in this paper, the key parameter is the trade elasticity. For this, we estimate $\phi = 0.62$. Compared to some estimates in the trade literature, this is a low value. But note that this low elasticity must stand in for the many reasons that substituting across import and home-traded goods is difficult in the short run. Among these, the literature has emphasized supply chain linkages, for example di Giovanni and Levchenko (2010) or Johnson (2014).

4.2 Inspecting the Mechanism

We next examine how the quantitative model matches the two patterns in the data, and explore how it depends on parameters. To do this, we focus on the response to a home-traded productivity shock, both because traded productivity is the main shock driving the qualitative data, and this is also the shock for which we derive propositions in Section 3.

Figure 2 depicts some key patterns for different values of the elasticity ϕ and for different degrees of international openness, a_h. The top-panel shows the impact response of the real exchange to an increase in A_{ht}. Darker lines correspond to a more integrated economy, with the darkest line corresponding to our baseline calibrations of the economy. The top-left panel shows the intuitive pattern that, as the trade elasticity rises, the real exchange rate becomes less volatile, confirming our intuition from the introduction of the paper. On the other hand, the size of the impact real exchange rate response actually shrinks as the economies become more integrated. The top right panel shows that the terms of trade follows the same patterns, although the size of the response is somewhat less that the real exchange rate, also consistent with the unconditional moments.

The bottom-left panel shows the impact response of relative consumption in response to the same shock. For relatively low values of ϕ, this response is negative: foreign consumption

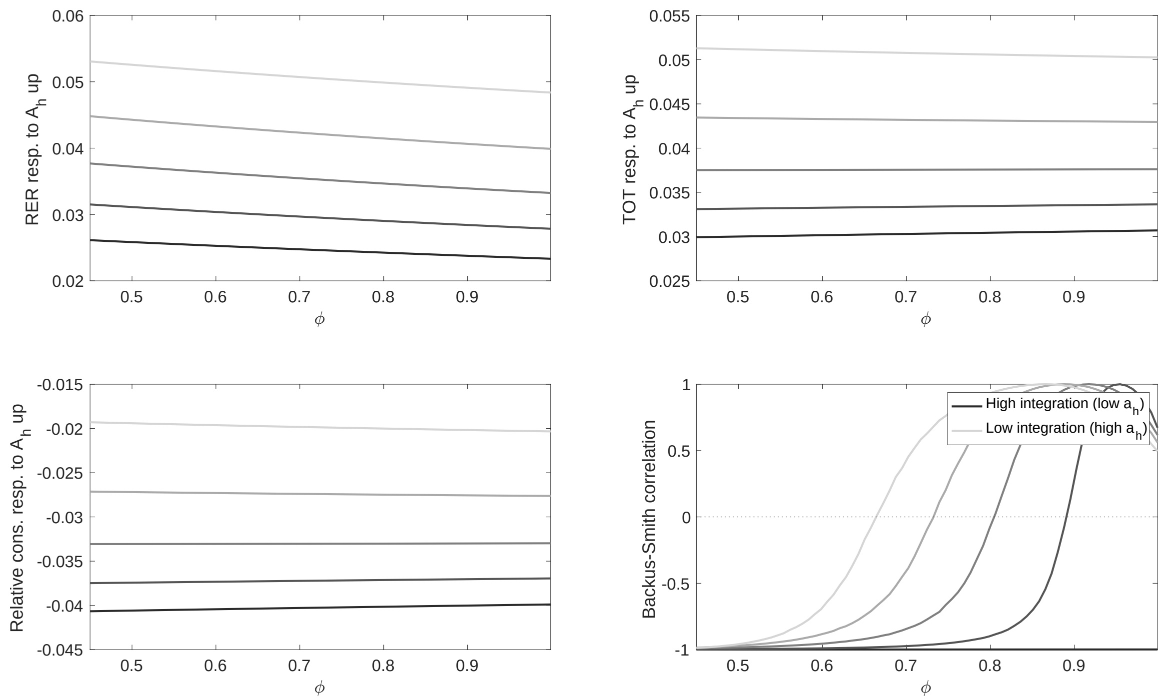


Figure 2: Impact responses and Backus-Smith correlations conditional on shocks to A_h, for different values of a_h and ϕ. The first three panels depict impact responses to the shock. The last panel depicts the correlation of HP-filtered real exchange rate and relative consumption. All other parameters are held fixed at their calibrated values.

rises by more than home consumption in response to a home productivity shock. Again, this confirms the pattern described in Proposition 4 of Section 3: this model solves the Backus-Smith problem by reversing the typical implication of productivity for relative consumption, rather than change the implication for the real exchange rate or terms of trade.

Finally, the bottom-right panel of Figure 2 depicts the correlation, conditional on only the home trade productivity shock, of the real exchange rate and relative consumption. This value is negative for lower values of ϕ, consistent with data. Interestingly, this correlation tends to be less negative (or more positive) for economies that are less integrated.

Figure 3 depicts several model implications that are key for understanding its implications for international comovement. The top-left panel shows that, in this version of the model, aggregate home labor can actually rise or fall in response to home productivity shocks. This is a manifestation of extremely strong price effects on both home trade and home nontraded goods. Since these relative prices fall, and wages cannot fall commensurately, labor actually

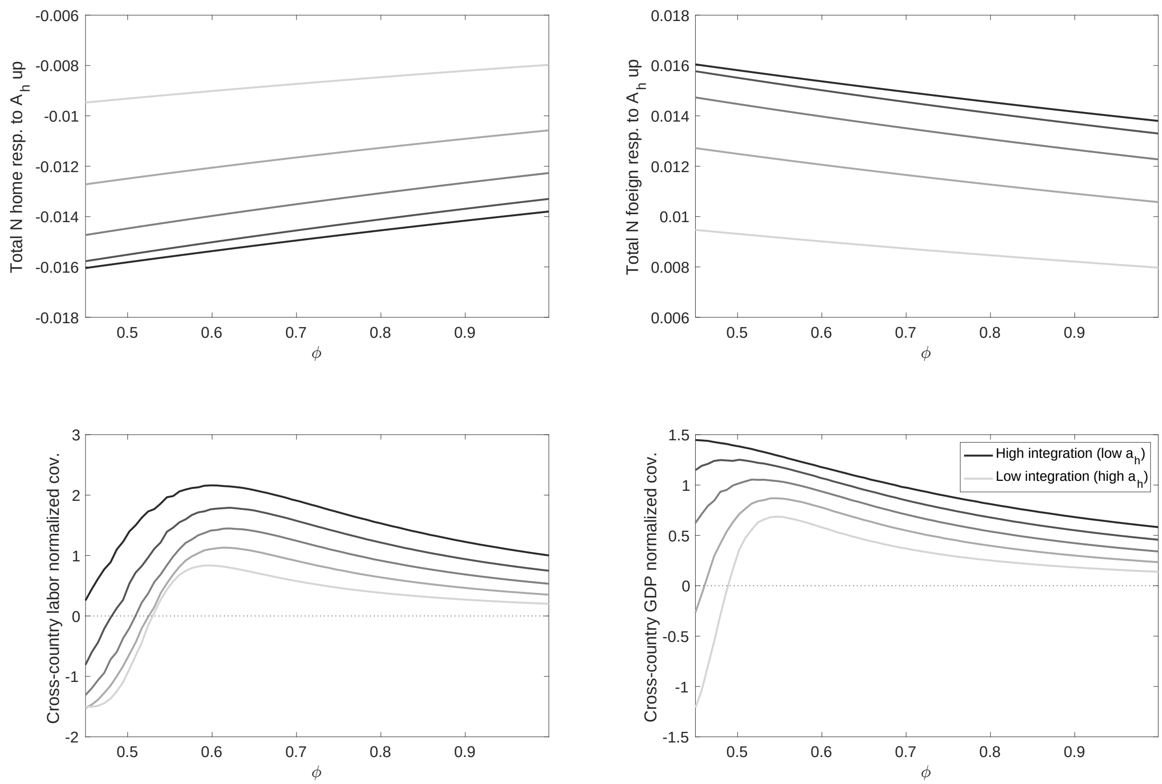


Figure 3: Impact responses and cross-country correlations conditional on shocks to A_h, for different values of a_h and ϕ. The first two panels depict impact responses to the shock. The third panel depicts the covariance of cross-country labor, divided by the home variance: numbers larger than 1 indicate a relatively larger foreign response. The last panel depicts the same object for output.

falls at these parameterizations.

The top-right panel shows that aggregate foreign labor rise consistently in response to the home trade productivity shock. This increase in labor, however, falls off quickly as the elasticity parameter rises. Moreover, for elasticities roughly larger than 0.45, the cross-border transmission of the shock is weaker when the economies are less integrated.

The bottom two rows of the figures show a measure of the relative transmission from the home to foreign country. The figure depicts, for each parameterization the covariance between the home and foreign quantity (again logged and HP-filtered) relative to the home variance of the quantity, conditional on the home traded productivity shock. Hence, to the degree the number is large than unity, this implies that the shock induces more variation in the foreign country than in the home country.

The bottom-left panel of Figure 3 depicts this statistic for labor. Evidently, this value is larger than one for a wide range of the parameter space, implying again an international transmission that is more than one-for-one. Moreover, note that this transmission shrinks quickly with lower degrees of international integration. Indeed, international transmission is about 1/3 as large by this measure when $a_h = 0.85$ (the highest value contemplated in the figure) than when it is $a_h = 0.5$ (the baseline from our calibration exercise). In other words, the figure shows that the model provides an explanation for the trade-comovement puzzle: as trade shares fall, so does labor comovement.

Finally, the bottom-right panel of Figure 3 shows the same statistic for cross-country GDP transmission. Even those foreign labor responses are more than one-for-one; for most calibrations, this ratio is at or below unity. This indicates that the labor expansion in the foreign country is not large enough to offset the joint effect of productivity expansion and labor demand changes in the home country. Again, the figure shows that this international transmission is weakened as the countries become less integrated.

5 Conclusions

This paper has argued that international business cycle models faced an inherent tension between low trade elasticities, which strengthen international comovement, and higher elasticities, which make it easier to explain Backus-Smith correlations. According to our theoretical results, there are two key ingredients to success: (1) supply shocks in one country should transmit as demand shocks to the other country; and (2) the trade elasticity must be low, but not too low.

Regarding (1), there are likely several other models of demand-like transmission of supply shocks that could be as good or better at matching the empirical facts. Introducing nominal rigidities might help the model match the data better, and it would allow the model to take into account several other exchange rate puzzles, to see if the same channels are helpful – or counterproductive – in addressing those.

Regarding (2), the Goldilocks range of elasticities implied by our simple theory are low.

Yet, as a reduced-form for all the channels that lower short-run elasticities, we believe values in this range are plausible. Future work should examine how frictions in adjusting trade levels could rationalize (or not) our estimate.

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Appendix

A Proofs of Propositions

Proof of Proposition 2-3. We prove the propositions by analyzing the excess demand function following the analysis of Mas-Colell et al. (1995). The excess demand function for the foreign good is

$$X(\tau) \equiv \frac{a_h}{a_h + a_f \tau^{\phi-1}} Y_f + \frac{a_f}{\tau^{\phi}(a_h + a_f \tau^{1-\phi})} Y_h - Y_f \tag{27}$$

An equilibrium is a point τ^* where $X(\tau^*) = 0$, and an equilibrium τ^* is tatonnement stable when $X'(\tau^*) < 0$. Setting $Y_h = Y_f = 1$ without loss of generality, we can compute the derivative

$$X'(\tau) = \frac{a_h}{(a_h + a_f \tau^{\phi-1})^2} a_f (1-\phi)\tau^{\phi-2} - \frac{a_f}{(a_h \tau^{\phi} + a_f \tau)^2} \left(a_h \phi \tau^{\phi-1} + a_f\right). \tag{28}$$

Our strategy is first to prove the existence and properties of the equilibria described in the proposition, then to prove that no other equilibria could exist.

The existence of the symmetric equilibrium $\tau^* = 1$ equilibrium follows from inspection of (27). Evaluating the derivative (28) at $\tau = 1$, we have

$$\begin{aligned}
X'(1) &= \frac{a_h}{(a_h + a_f)^2} a_f (1-\phi) - \frac{a_f}{(a_h + a_f)^2} (a_h \phi + a_f) \\
&= \frac{1}{(a_h + a_f)^2} \left(a_h(1-a_h)(1-\phi) - (1-a_h)(1-a_h+a_h\phi)\right) \\
&= \frac{1-a_h}{(a_h + a_f)^2} \left(a_h(1-\phi) - (1-a_h+a_h\phi)\right) \\
&= (1-a_h)\left(2a_h(1-\phi) - 1\right).
\end{aligned} \tag{29}$$

Expression (29) is negative if and only if $\phi > 1 - \frac{1}{2a_h}$. This establishes that the symmetric equilibrium is tatonnement stable only when $\phi > \bar{\phi}$.

To establish the existence of additional equilibria when $\phi < \bar{\phi}$, first compute the limit

$$\lim_{\tau \to 0} X(\tau) = \infty. \tag{30}$$

But since $X(1) = 0$ and $X'(1) > 0$ when $\phi < \bar{\phi}$, we know that there also exists a δ sufficiently small that $X(1-\delta) < 0$. Since $X(\tau)$ is continuous, we know there exists at least one crossing

$\underline{\tau} \in (0,1)$, such that $X(\underline{\tau}) = 0$ and $X'(\underline{\tau}) < 0$. Similarly, compute $\lim_{\tau\to\infty} X(\tau) = -\infty$. Since there exists a δ such that $X(1+\delta) > 0$, there must be at least one equilibrium $\bar{\tau} \in (0,\infty)$, again wtih with $X'(\bar{\tau}) < 0$.

To see the equilibria have the local properties described in the proposition, take a first order Taylor expansion of $X(\tau)$ around $Y_h = 1$ and $\tau = \tau^*$ for any equilibrium τ^*

$$X(\tau) \approx X'(\tau^*)\hat{\tau} + \mu_{imp}\hat{Y}_h = 0, \tag{31}$$

where $\mu_{imp} \equiv \frac{a_f}{a_h\tau^{*\phi}+a_f\tau^*}$, $\hat{\tau} \equiv \tau - \tau^*$, and $\hat{Y}_h \equiv Y_h - 1$. Solving for $\hat{\tau}$ in terms of $\hat{Y}_h$, we have

$$\hat{\tau} = -X'(\tau^*)\mu_{imp}^{-1}\hat{Y}_h. \tag{32}$$

Since μ_{imp} is positive, this establishes that sign of the excess demand function at the cross is equal to minus the sign of the local terms-of-trade response to an increase in home output, proving proposition 3: all the equilibria described above have the local properties summarized in the proposition.

[*It remains to show these are the only equilibria possible.] We next establish that the equilibria described above are the only ones. First note that, because of continuity – and regardless of the value of ϕ – equilibria other than those described in the proposition can exist only if there are additional crossings of $X(\tau)$ "from below", i.e. with $X'(\tau)$ upwards sloping at those crossings.

For a contradiction, suppose that such an equilibrium exists, and call the corresponding price $\hat{\tau}^*$. First, note that (27) implies that for any equilibrium $\tau^* \neq 1$, it must be the case that

$$a_h = \frac{(\tau^* - 1)}{(1+\tau^{*\phi})(\tau^{*1-\phi} - 1)} \tag{33}$$

Second, rearrange the derivative (28) to find it is negative if and only if

$$\frac{(1-\phi)a_h\tau^{-1}}{a_h\phi + a_f\tau^{1-\phi}} < \tau^{-2\phi}\frac{(a_h + a_f\tau^{\phi-1})^2}{(a_h + a_f\tau^{1-\phi})^2}. \tag{34}$$

For the case of $\phi < \bar{\phi}$, substitute (33) into (34) and simplify to to rewrite the condition

$$\frac{\tau^{2\phi}(\tau+\tau^{\phi})^2(\tau-\phi\tau-\phi\tau^{2\phi}-\tau\tau^{2\phi}+\phi\tau^2+\phi\tau\tau^{2\phi})}{\tau^3(\tau-\tau^{\phi})(\tau^{\phi}+1)^3} > 0. \tag{35}$$

Take first the case of $\phi > \bar{\phi}$.

To do this, we show that if, $X(\tau) = 0$ and $\tau \neq 1$, then the $X'(\tau)$ is negative. When $\phi > \bar{\phi}$, this implies that

To show that the symmetric $\tau^* = 1$ equilibrium is unique when , we next show that $X(\tau)$ is strictly monotone decreasing when $\phi > \bar{\phi}$. Rearranging (28) and using $a_f = 1 - a_h$, we have that $X'(\tau) < 0$ if an only if

$$\frac{(a_h + a_f\tau^{\phi-1})^2}{(a_h\tau^{\phi} + a_f\tau)^2} > \frac{(1-\phi)a_h\tau^{\phi-2}}{\phi a_h + (1-a_h)\tau^{1-\phi}} \tag{36}$$

We next show that $X(\tau)$ is strictly convex for $\tau < 1$ and that there is exists a $\tau < 1$ for which $X'(\tau) > 0$. This establishes there can be only one crossing point $X(\underline{\tau}) = 0$ for $\underline{\tau} < 1$. Moreover, we know that $X'(\underline{\tau}) < 0$, establishing that $\underline{\tau}$ represents a tatonnement-stable steady-state equilibrium.

□

B Quantitative Model: Summary of Equilibrium Conditions

The following conditions characterize equilibrium in our quantitative model.

$$P_t = (a_T P_{Tt}^{1-\eta} + a_N P_{Nt}^{1-\eta})^{\frac{1}{1-\eta}} \tag{37}$$

$$P_t^* = (a_T {P_{Tt}^*}^{1-\eta} + a_N {P_{Nt}^*}^{1-\eta})^{\frac{1}{1-\eta}} \tag{38}$$

$$C_{Tt} = a_T \left(P_{Tt}/P_t\right)^{-\eta} C_t \tag{39}$$

$$C_{Tt}^* = a_T \left(P_{Tt}^*/P_t^*\right)^{-\eta} C_t^* \tag{40}$$

$$C_{Nt} = a_N \left(P_{Nt}/P_t\right)^{-\eta} C_t \tag{41}$$

$$C_{Nt}^* = a_N \left(P_{Nt}^*/P_t^*\right)^{-\eta} C_t^* \tag{42}$$

$$P_{Tt} = \left(a_h P_{ht}^{1-\phi} + a_f P_{ft}^{1-\phi}\right)^{\frac{1}{1-\phi}} \tag{43}$$

$$P_{Tt}^* = \left(a_h P_{ft}^{1-\phi} + a_f P_{ht}^{1-\phi}\right)^{\frac{1}{1-\phi}} \tag{44}$$

$$C_{ft} = a_f (P_{ft}/P_{Tt})^{-\phi} C_{Tt} \tag{45}$$

$$C_{ht}^* = a_f (P_{ht}/P_{Tt}^*)^{-\phi} C_{Tt}^* \tag{46}$$

$$\frac{C_t^{-\sigma}}{P_t} = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \frac{P_{h,t+1}}{P_{ht}} \frac{1}{Q_t}\right] \tag{47}$$

$$\frac{C_t^{*-\sigma}}{P_t^*} = \beta E_t \left[\frac{{C_{t+1}^*}^{-\sigma}}{P_{t+1}^*} \frac{P_{h,t+1}}{P_{ht}} \frac{1}{Q_t}\right] \tag{48}$$

$$Y_{ht} = C_{ht} + C_{ht}^* \tag{49}$$

$$Y_{ft} = C_{ft} + C_{ft}^* \tag{50}$$

$$Y_{Nt} = C_{Nt} \tag{51}$$

$$Y_{Nt}^* = C_{Nt}^* \tag{52}$$

$$Y_{ht} = A_{ht} N_{ht}^{\alpha} \tag{53}$$

$$Y_{ft} = A_{ft} N_{ft}^{\alpha} \tag{54}$$

$$Y_{Nt} = A_{Nt} N_{Nt}^{\alpha} \tag{55}$$

$$Y_{Nt}^* = A_{Nt}^* {N_{Nt}^*}^{\alpha} \tag{56}$$

$$W_t = P_{Nt} A_{Nt} \alpha N_{Nt}^{\alpha-1} \tag{57}$$

$$W_t = P_{ht} A_{ht} \alpha N_{ht}^{\alpha-1} \tag{58}$$

$$W_t = \omega P_t \tag{59}$$

$$W_t^* = P_{Nt}^* A_{Nt}^* \alpha {N_{Nt}^*}^{\alpha-1} \tag{60}$$

$$W_t^* = P_{ht}^* A_{ht}^* \alpha {N_{ft}}^{\alpha-1} \tag{61}$$

$$W_t^* = \omega P_t^* \tag{62}$$

$$P_{ht} C_{ht}^* = P_{ft} C_{ft} + Q_t P_{ht} B_{h,t+1} - P_{ht} B_{ht} \tag{63}$$

$$P_t C_t = P_{ht} Y_{ht} + P_{Nt} Y_{Nt} + P_{ht} B_{ht} - Q_t P_{ht} B_{h,t+1} \tag{64}$$

$$P_t^* C_t^* = P_{ft} Y_{ft} + P_{Nt}^* Y_{Nt}^* - P_{ht} B_{ht} + Q_t P_{ht} B_{h,t+1} \tag{65}$$

$$P_{ht} = 1 \tag{66}$$

That's 30 equations. List of 19 quantities:

$$[C_t, C_t^*, C_{Nt}, C_{Nt}^*, C_{Tt}, C_{Tt}^*, C_{ht}, C_{ft}, C_{ht}^*, C_{nt}^*, Y_{Nt}, Y_{ht}, Y_{Nt}^*, Y_{ft}, N_{Nt}, N_{ht}, N_{Nt}^*, N_{ft}, B_{h,t+1}] \tag{67}$$

List of 11 prices:

$$[P_t, P_t^*, P_{Nt}, P_{Nt}^*, P_{Tt}, P_{Tt}^*, P_{ht}, P_{ft}, W_t, W_t^*, Q_t] \tag{68}$$