Expectation Response Functions in Dynamic Linear Economies*

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Abstract

Macroeconomic disturbances affect both current fundamentals and expectations of future fundamentals, but most analyses report only the total of these effects. The expectation response function (ERF) isolates the role of expected future fundamentals in a theory. Defined as the response today to a change in expected fundamentals at each future horizon, the ERF does not depend on the fundamentals' laws of motion, the information held by agents, or the assumption of rational expectations. In applications, we show that (i) the new-Keynesian model implies modest expectational effects of technology and monetary shocks, while (ii) markup shocks in a medium-scale DSGE model have far larger expectational impacts than the "puzzling" effects of forward guidance.

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1 Introduction

Economists have long sought to isolate the macroeconomic effects of beliefs, both in data and in theory. From the perspective of theory, this task is challenging because virtually all model-based assessments of the importance of expectations make assumptions about the processes for fundamentals, the information available to people in the economy, and the manner in which people form their expectations about economic fundamentals. These auxiliary assumptions may preclude significant effects of expectations even when the economic environment requires agents to be extremely forward looking.

To isolate the effects of expectations in a particular theoretical model, it is useful to compute the "expectation response function", or ERF. The ERF isolates the theoretical effects of changes in expectations, without making further assumptions about the source of expectation changes. It describes how a time-t endogenous variable responds to an anticipated change in the fundamental that will occur at time $t+h$. The primary benefit of using the ERF to study expectational effects, particularly compared to an impulse response function, is that it can be computed independently of assumptions regarding how agents make their forecasts or assumptions regarding the nature of the exogenous process for the fundamental. Moreover, the ERF isolates the effects of expectations at different horizons, so researchers can assess more precisely how the timing of expectational changes matters for their present effects.

In this paper, we first define and illustrate the ERF in the context of a simple univariate example. Then we show how to compute it for any linear model. The approach follows a QZ-decomposition strategy similar to the techniques now broadly employed for solving linear models. In contrast to how these tools are usually applied, however, the ERF can be computed without assumptions about the laws of motion of the underlying exogenous processes. Hence, the ERF does not describe a full rational expectations equilibrium, but rather an equilibrium in which agents anticipate that all model equations describing endogenous variables are satisfied but take (their own) expectations of aggregate fundamentals as exogenous.

The ERF appears in prior literature, though it has not been named and has rarely been employed as a direct object of interest.[1] Our contribution is to demonstrate the

[1]The penultimate step of Blanchard and Kahn (1980) is written in terms of the ERF. The ERF appears in Woodford (2003) as an intermediate step in model solution; in a few instances he uses it

ERF's usefulness in isolating the effects of expectations within a given theory, and to show in applications how examining it can influence the conclusions that researchers draw.

A natural question for theorists is the degree to which variables depend on expectations, relative to past states. We present several approaches for answering this question. Like impulse response functions (IRFs), important intuition can be gained by visual inspection of ERFs. Treating ERF coefficients as the coefficients of a notional process, we propose several measures that can be interpreted as a variance decomposition. For example, ERF coefficients can be used as weights to compute a notional "average horizon" of expectations effects. We also propose using the spectral density of the notional process to measure the contribution of medium or long-run expectations.

We use the ERF to diagnose the effects of expectations changes in several models that have been used by applied macroeconomists. In our first application, we compare the expectational effects of future total factor productivity (TFP) in a range of models that have been used to study the empirical importance of TFP news. After reproducing the classic news-comovement problem in the real business cycle (RBC) model, we show that the comovement "fixes" of Jaimovich and Rebelo (2009) solve the qualitative problem but not the quantitative one: expected TFP improvements have small effects on all variables relative to the effects of changes in current TFP. We then show that the effects of anticipated changes in TFP are also modest in both the three equation new-Keynesian model and a medium scale dynamic stochastic general equilibrium (DSGE) model with a full set of frictions. In both models, the size of anticipation effects hinges largely on how aggressively monetary policy fights inflation, especially at longer horizons of anticipation.

In our second application, we review the evidence about the effects of forward guidance on interest rates in the same two versions of the new-Keynesian model. Our results emphasize a point that has already appeared occasionally in the discussion around that literature:[2] the appearance of a "puzzle" relies almost entirely on pre-

to assess the importance of forecasts in setting optimal policy. A closely related object also appears in the literature on adaptive learning (Preston, 2005; Eusepi and Preston, 2011). The non-structural approaches to evaluating policy counterfactuals proposed by McKay and Wolf (2023) and Barnichon and Mesters (2023) presume that the ERF can be econometrically identified.

[2]For an example, see Susanto Basu's 2015 discussion of McKay et al. (2016) at the *Workshop on Forward Guidance and Expectations* at the New York Federal Reserve.

venting the central bank from responding to endogenous variables with interest rate policy. A forward guidance puzzle appears in nominal models when they are calibrated to be close to the region of indeterminacy. The results of this exercise suggest that the questions of equilibrium determinacy and the effects of forward guidance are too closely linked to be studied independently.

Lastly, we compare ERFs of the main business cycles variables to six commonly-used shocks in the context of standard medium-scale DSGE model. For several shocks, we again find a strong link between the size of their expectational effects and the responsiveness of the Taylor rule to deviations from target inflation. The exceptions to this pattern, however, are price and wage markup shocks, which have large expectational effects regardless of the Taylor rule. We argue that understanding the cause or causes of these strong expectational effects is an important step in advancing the research agenda on expectational shocks.

2 The Expectation Response Function

This section first defines and discusses the expectation response function in the context of a simple, one-dimensional, purely forward-looking model, and then moves to the more general setting.

2.1 Simple model

The endogenous variable of interest is c_t and the exogenous fundamental is x_t. Since there are no endogenous states, equilibrium depends only on current and future expected states, so that the equilibrium value of c_t can be represented as

$$c_t = \sum_{j=0}^{\infty} \alpha_j E_t[x_{t+j}]. \tag{1}$$

Consider first the impulse response of c_t to some structural shock affecting the fundamental x_t. To compute an impulse response, we must first specify the exogenous process for x_t, for example by assuming x_t follows an invertible one-sided moving average process,

$$x_t = \sum_{l=0}^{\infty} \beta_l \epsilon_{t-l}. \tag{2}$$

While explicit micro-foundations usually underly the coefficients α_j in (1), standard practice usually make restrictive assumptions about the process for x_t based on non-theoretic statistical norms, for example, by assuming that (2) corresponds to an AR(1) process.

Using equation (1), and assuming the full information rational expectations implied by knowledge of the disturbances in (2), the impact impulse response to the shock is

$$\begin{aligned} \mathrm{IRF}(0) &\equiv \frac{\partial c_t}{\partial \epsilon_t} \\ &= \sum_{j=0}^{\infty} \alpha_j \frac{\partial E_t[x_{t+j}]}{\partial \epsilon_t} \\ &= \sum_{j=0}^{\infty} \alpha_j \beta_j. \end{aligned} \tag{3}$$

Similarly, the impulse response after one period is

$$\mathrm{IRF}(1) = \sum_{j=0}^{\infty} \alpha_j \beta_{j+1},$$

and so on for additional horizons.

While impulse responses contain valuable information regarding the total effects of shocks, equation (3) demonstrates the main argument of this paper: the impulse response function conflates the effects of expectations—the α_j—with the sources of expectations changes, here summarized by the coefficients β_j. Not only are the α_j and β_j commingled via multiplication, but they are then summed up across periods, obscuring information about the horizon at which expectations are driving the endogenous variable. Intuitively, if x_t represents productivity, the impact impulse response does not tell us if consumption is moving today because today's productivity has changed or because agents are responding to the shock's implications for future productivity.

Our goal is to provide a tool to examine the effects of expectations implied by the economic model (1), without making commitments regarding formation of the $E_t[x_{t+j}]$ terms. For this, we propose the expectation response function, defined by

$$\mathrm{ERF}(j) \equiv \frac{\partial c_t}{\partial E_t[x_{t+j}]}. \tag{4}$$

Applied to equation (1), this definition implies that $\text{ERF}(j) = \alpha_j$; i.e., the expectation response function is simply the sequence of weights on future expectations.

In the example, the ERF has the advantage that it can be computed without ever committing to the parameters β_j of the exogenous process. Or, indeed, without committing to a particular form of the process or to the full information assumption, provided that the series on the right side of equation (1) converges. In the following section, we show how the simple idea can be applied to generic linear environments, including those with endogenous states.

2.2 More general setting

The simple example above can be generalized in several dimensions: by allowing for many variables and shocks, by admitting endogenous state variables, and by relaxing the assumption of rational (or model consistent) expectations. Here, we present a canonical model form for which the ERF can be solved using standard techniques.[3] Our discussion below explains how each of these generalizations is introduced.

We consider a general linearized model that takes a recursive form:

$$A_k \hat{E}_t[k_{t+1}] + A_y \hat{E}_t[y_{t+1}] = B_k k_t + B_y y_t + A_x \hat{E}_t[x_{t+1}] + B_x x_t \tag{5}$$

In the above equation (5), the variables k_t consist of endogenous states pre-determined at time $t-1$, the variables y_t consist of endogenous jump variables, and x_t the exogenous states of the system. The matrices A_k, A_y and A_x have dimension $(n_{eq} \times n_k)$, $(n_{eq} \times n_y)$ and $(n_{eq} \times n_x)$ respectively, with the same dimensionality for their time t counterparts B_k, B_y, and B_x. Finally, the expectations operator $\hat{E}_t[\cdot]$ encompasses – but is not limited to – rational expectations; all that is needed is that $\hat{E}_t[\cdot]$ satisfies the law of iterated expectations.[4]

We pause here to emphasize that, since our approach does not require assuming any process for or information about x_t, a well-defined model will have $n_{eq} = n_k + n_y$. Standard approaches used in the literature typically require including the process for exogenous variables as an input to solving the model, hence, that $n_{eq} = n_k + n_x + n_y$. Our treatment of the model is distinct because we treat (expectations of) exogenous fundamentals as variables that are determined outside of the economic model.

[3] Our computational approach here shares elements with Klein (2000) and Sims (2001).

[4] Woodford (2013) provides some useful results for representing equilibrium in the form of (5) when expectations are non-rational.

Define $A \equiv [A_k \;\; A_y]$ and $B \equiv [B_k \;\; B_y]$. Using a QZ decomposition, we can find matrices $\{Q, Z, S, T\}$ that satisfy $QAZ = S$ and $QBZ = T$, with S and T block upper triangular and Z unitary. The matrices S, T, and Z can be partitioned into the blocks

$$S = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix}, T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix}, \text{ and } Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix},$$

where S_{11} and $(n_k \times n_k)$, S_{12} is $(n_k \times n_y)$, and S_{22} is $(n_y \times n_y)$ and similarly for T and Z. Letting $\Gamma \equiv T_{22}^{-1} S_{22}$ and $M \equiv \left[T_{22}^{-1} Q_2 A_x + \Gamma T_{22}^{-1} Q_2 B_x\right]$, we show in the appendix that the equilibrium value of y_t is given by

$$y_t = Z_{21} Z_{11}^{-1} k_t + \sum_{j=0}^{\infty} \text{ERF}(j) \hat{E}_t[x_{t+j}], \tag{6}$$

in which the expectation response function coefficients correspond to

$$\text{ERF}(0) \equiv -(Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}) T_{22}^{-1} Q_2 B_x \tag{7}$$

for horizon zero, and by

$$\text{ERF}(j) \equiv -(Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}) \Gamma^{j-1} M \tag{8}$$

for any horizon $j > 0$.

In this algorithm, the usual conditions for existence and local uniqueness of equilibrium are assumed to hold; namely, that Z_{11} is invertible.

2.3 Summary statistics of forward-looking behavior

One barrier to interpreting the ERF is the sheer number of coefficients described by (7) - (8). Because the ERF is an infinite dimensional object, it can be difficult to select particular coefficients for analysis. In this section, we propose several approaches to summarize and visualize the information contained in the ERF.

Our approach to summarizing the ERF starts with defining a notional infinite-order moving average process using the ERF coefficients. Let

$$\tilde{y}_t \equiv \sum_{j=0}^{\infty} \text{ERF}(j) u_{t-j}, \tag{9}$$

where the u_t is a vector of i.i.d. white noise disturbances. While $\{\tilde{y}_t\}_{t=0}^{\infty}$ is a purely notional process, and (9) does not correspond to the dynamics in an equilibrium of the model, a high degree of persistence in $\tilde{y}_t$ corresponds to strongly forward-looking behavior in y_t. While one could invoke any standard approach to measuring the persistence of $\tilde{y}_t$, we propose a few specific statistics that have not appeared before in the literature.[5]

The first measure is the "share of future weights." It captures the fraction of (squared) coefficient weights that are applied to terms other than the current fundamental. The share of future weights of the ERF for variable k in response to fundamental l is defined as

$$\mathrm{SFW}_{kl} \equiv \frac{\sum_{j=1}^{\infty}(\mathrm{ERF}_{kl}(j))^2}{\sum_{j=0}^{\infty}(\mathrm{ERF}_{kl}(j))^2}.$$

This statistic can be interpreted as the share of variance of $\tilde{y}_t$ that is explained by disturbances not occurring at t.

The share of future weights is a good measure of the relative importance of future versus current fundamentals, but it does not distinguish weights on relatively near-future expectations from weights applied to expectations of the distant future. The "mean horizon of expectation" better captures the horizon of expectations, and is defined as

$$\mathrm{MHE}_{kl} \equiv \frac{\sum_{j=0}^{\infty}(\mathrm{ERF}_{kl}(j))^2 \times j}{\sum_{j=0}^{\infty}(\mathrm{ERF}_{kl}(j))^2}.$$

Essentially, the MHE_{kj} corresponds to a weighted average of horizons j, with weights that are proportional to the variance contribution of the notional disturbances at each horizon.[6]

Alternatively, one may recast $\tilde{y}_t$ using a Fourier transform and employ techniques from frequency-domain analysis to measure the degree of forward looking behavior in the model. The frequency response function of $\tilde{y}_t$ is

$$\varphi(\lambda) \equiv \sum_{j=0}^{\infty} \mathrm{ERF}(j) e^{-i\lambda j}.$$

[5]One obvious candidate, the autocorrelation coefficient, can be quite difficult to interpret when $\tilde{y}_t$ does not closely resemble an AR(1) process.

[6]Woodford (2003) uses a similar measure to capture the forward-lookingness of the optimal target variable in optimal monetary policy problems. His analogue to the MHE excludes the $j = 0$ horizon and uses a simple sum of coefficients, rather than their square.

One natural measure is the low-pass filtered variance share,

$$\text{LPF}_k(m) \equiv \frac{\int_0^{2\pi/m} \varphi_k(\lambda)\varphi_k(\lambda)' d\lambda}{\int_0^{\pi} \varphi_k(\lambda)\varphi_k(\lambda)' d\lambda},$$

where m is the periodicity of the highest frequency terms considered. Of course, different values of m can be considered; in our analysis we consider $m = 32$, which corresponds to periodicities beyond the standard business cycle range (more than eight years when periods are quarters), and $m = 100$, which corresponds to much longer-run fluctuations (more than 25 years).

3 Applications

In this section we use the ERF to diagnose the effects of expectations changes in three different models that have been used by applied macroeconomists.

3.1 Effects of Anticipated Productivity

Much of the literature on anticipated economic shocks has focused on the potential importance of anticipated productivity. Here, we revisit the implications of a set of theories in which such news shocks have been considered, with emphasis on the features of the economic environments (rather than information structure) that are necessary for anticipation of TFP to play an important role in generating business cycle fluctuations. We provide more details on model construction and calibration in the Appendix.

As a baseline and to fix ideas, we first consider the expectation response of aggregate variables to productivity in the standard RBC model. The comovement properties of surprise TFP shocks are known to depend on the parameterization of the TFP process, particularly on the persistence parameter in AR(1) specifications for TFP. Meanwhile, the effects of anticipated productivity are known to give counterfactual implications for business cycle comovements.

The black line in Figure 1 provides a new perspective on these two related facts. The figure shows that contemporaneous changes in productivity, captured by the ERF at horizon zero, deliver a strong comovement of output, consumption, hours, and investment. By contrast, anticipated changes in productivity, captured by the

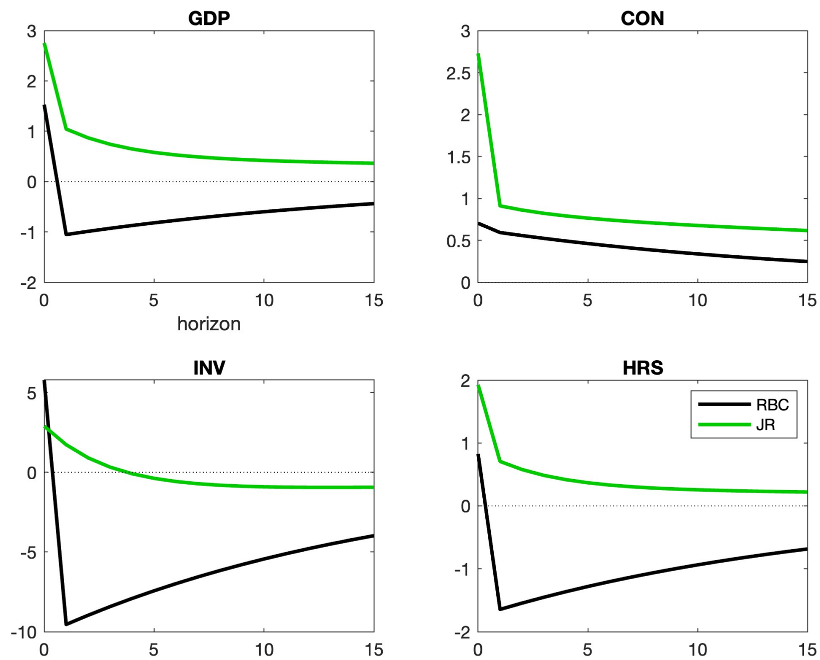

Figure 1: Effects of anticipated productivity growth in real business cycle models.

ERF at horizon one and beyond, always deliver counterfactual comovements, in line with the predictions of Barro and King (1984).

In light of the RBC results in Figure 1, it is possible to reinterpret the common observation that the comovement associated with contemporaneous productivity shocks depend on AR(1) parameters: Since a shock to an AR(1) productivity process implies changes in both contemporaneous and future productivity, the impact effects of the shock can be read as a weighted sum of expectation response functions, where higher AR(1) parameters imply more weight on expected future changes relative to current changes. Evidently, a fundamental shock that moves future productivity enough relative to current will no longer deliver the comovements required by the data.

Jaimovich and Rebelo (2009) introduce a set of modifications to the basic RBC model with the goal of resolving the comovement problems associated with anticipated shocks. The green lines in the figure plot the ERF associated with neutral productivity shocks in their model using their baseline calibration. Indeed, an expected change in productivity can now produce comovement, but the picture also reveals a limitation of their approach: anticipated changes must be sufficiently close in the future—within four quarters—for them to lead to positive investment comovement with other variables.

	Output				Consumption			
	SFW	MHE	LPF(32)	LPF(100)	SFW	MHE	LPF(32)	LPF(100)
RBC	0.80	6.82	0.55	0.34	0.86	7.30	0.79	0.49
JR	0.53	11.71	0.58	0.46	0.71	20.02	0.76	0.66
3-eq NK (stand.)	0.49	0.96	0.31	0.09	0.49	0.96	0.31	0.09
Med. NK (stand.)	0.50	1.36	0.36	0.16	0.69	3.90	0.52	0.33
3-eq NK (accom.)	0.97	66.53	0.95	0.87	0.97	66.53	0.95	0.87
Med. NK (accom.)	0.97	94.69	0.97	0.93	0.99	96.42	0.98	0.94

Table 1: ERF summary statistics for technology: output and consumption

The figure also reveals a more subtle point about the potential for the model to deliver an important expectational component in business cycles.[7] This limitation on the potential of expectations is a consequence of the restriction that, under rational expectations, beliefs about a variable can never be more volatile than the variable itself. Because of this restriction, it is not possible to "activate" expectation effects without also adding additional variance due to the impact effects pictured in the ERF. Since the ERF in the case of the Jaimovich and Rebelo (2009) model is strongly downward sloping, this implies that we cannot induce much variance via expectations without *also* inducing even larger variance from the shock realizations themselves: the ratio of purely expectations-driven fluctuations to those driven by the fundamental realizations themselves is bound to be small. This pattern echoes the finding of Chahrour and Jurado (2018) in a related version of the Jaimovich and Rebelo (2009) model estimated by Schmitt-Grohé and Uribe (2012) that the contribution purely due to expectations is small even when most shocks are anticipated in advance.

The first two rows of Table 1 provide ERF summary statistics for output in the RBC and Jaimovich and Rebelo (2009) models.[8] The share of future weights measure

[7]The question here is whether noise shocks, which are orthogonal to fundamentals, could play an important role. Chahrour and Jurado (2018) provide detailed analysis of the relation between news shocks, which are correlated with later fundamental realizations, and noise shocks, which are not.

[8]Though we do not report them, summary statistics for other variables all give qualitatively similar conclusions in the cases we have looked at.

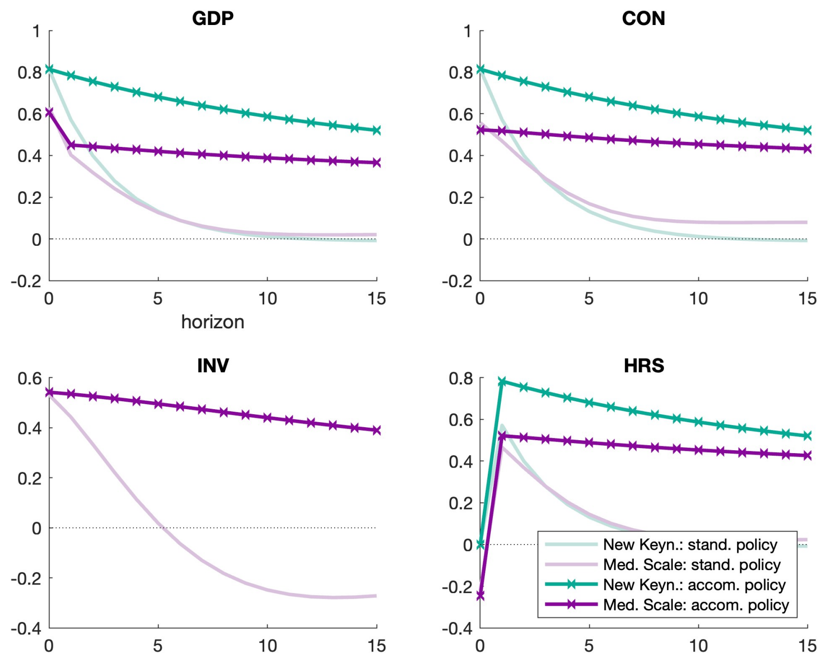

Figure 2: Effects of anticipated productivity in new-Keynesian models under alternative specifications of the monetary policy rule.

shows that both models place more than half of the ERF weights (again, our statistic has an interpretation as a variance share in the notional $\tilde{y}_t$) on expectations. The second column shows the mean horizon of expectations (MHE) for these models. Clearly the SFW and MHE contain different information: the RBC model depends more on expectations overall, but the Jaimovich and Rebelo (2009) depends more on further-dated expectations, meaning the latter has a relatively large MHE statistic. The two LPF statistics tell a story similar to the MHE: the longer the horizon of expectation, the relatively more important are expectations in the Jaimovich and Rebelo (2009) model compared to the RBC model.

We next consider the effects of anticipated productivity shocks in two versions of the new-Keynesian model. The first model we consider is a standard three equation new-Keynesian model. The second model is a quantitative medium scale DSGE model with nominal wage and price frictions, in the spirit of Christiano et al. (2005) and Smets and Wouters (2007). Both models include a version of the Taylor rule for monetary policy,

$$i_t = \rho i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y \hat{y}_t) + \epsilon_t. \tag{10}$$

In (10), i_t the nominal interest rate, ρ_i is the interest rate smoothing parameter, ϕ_π is the weight on inflation and ϕ_y is the weight on the output gap.

Figure 2 depicts the expectation response function to neutral productivity in the two versions of the new-Keynesian model. The lighter lines depict the effects of anticipated productivity under a fairly standard calibration of the Taylor rule, with $\phi_\pi = 1.5, \phi_y = 0.5$, and $\rho_i = 0.5$. We call this specification “standard” monetary policy. In these cases, both models deliver modest effects of anticipated productivity, effects which die out at horizons of anticipation just over a year. As discussed just above, the downward sloping nature of the ERFs indicates that these models, which are calibrated with fairly strong nominal rigidities, are rather unlikely to deliver a large role for pure expectation shocks under rational expectations.

In contrast, the darker lines in the figure depict the effects of the same changes in expectations under a less active Taylor rule with coefficients $\rho_i = 0.5583$ $\phi_\pi = 1.02$, and $\phi_y = 0.005$, which are the values estimated by Blanchard et al. (2013). (Coefficients similar to these have been estimated in various contexts.) We call this specification “accommodative” monetary policy. With less active Taylor rules, both models now deliver strong expectation responses even at horizons of four years or longer, as well as empirically realistic comovements at shorter horizons.

The case of the medium-scale DSGE model is especially notable because ERFs are so flat: the response to a 10 year ahead change in productivity growth remains more than half as large as the response to a productivity change next quarter. The extremely flat ERF profiles, along with strong comovement at all horizons, make this version of the model especially conducive to finding a large role for expectation shocks.

The last four rows of Table 1 provide the ERF summary statistics for the four different versions of the new-Keynesian models. These statistics clearly show that the importance of expectations of productivity is strongly influenced by the parameters of monetary policy, and much less influenced by the additional structure of the medium-scale model. For example, the LPF(100) statistic is between 9 and 16% for the models when monetary policy is more active, and is over 85% for both models when monetary policy is less active.

3.2 Forward Guidance and Equilibrium Determinacy

We now shift gears to use the ERF to analyze the effect of anticipated changes in monetary policy. Following the tradition of Campbell et al. (2012), we think of forward guidance as anticipated (exogenous) deviations from a monetary policy rule. Hence, implicit in the expectation response function, is that agents anticipate the central bank will follow its endogenous rule for policy at all horizons other than the one in which it is expected to deviate.

Figure 3 reports the expectation response function for monetary deviations. Once again, the lighter line reports the response to anticipated deviations from a Taylor rule with a standard specification. The results are quite similar to the case of the productivity shocks, with anticipated monetary deviations having extremely small real effects if they are anticipated to occur at horizons greater than a year. In short, with a standard calibration of monetary policy, there is no evidence in any of the models of a "forward guidance" puzzle.

By contrast, the darker lines plot the effects of anticipated deviations under the accommodative Taylor rule. Under this calibration, the endogenous response of monetary policy to deviations from inflation and output gap targets is quite muted; i.e. this is calibration very near to the edge of the determinacy region in both models.

In this case, the effects of anticipated monetary deviations are large, and extend far out into the future. Indeed, for the medium scale model, the effects of a one-year anticipated shock exceed those of a surprise shock. Nevertheless, responses for real variables stop growing with the anticipation horizon after this point and all ERFs eventually return to zero. Hence, even in this calibration the "puzzle" is much less extreme than in some formulations of the famous forward guidance experiments conducted in the literature.

Combining the results of Sections 3.1 and 3.2, some themes begin to emerge. First, the effects of anticipated changes in fundamentals depend to a large extent on assumptions about the endogenous response of policy. Second, large expectations responses are more likely to emerge when the Taylor principle is only barely satisfied, i.e. when the economy is near to a calibration of indeterminacy. These results suggest that it may be unwise to study the effects of forward guidance in isolation, without considering the empirical plausibility of responses to other shocks or the issues related to determinacy in new-Keynesian environments.

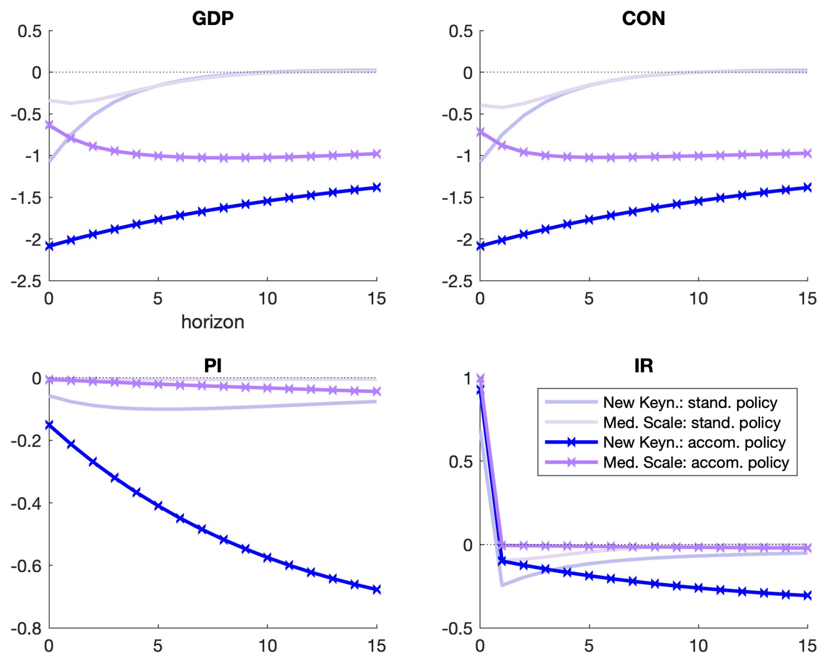

Figure 3: Effects of forward guidance in new-Keynesian models under alternative specifications of the monetary policy rule.

3.3 Comparing Shocks in a Medium Scale DSGE Model

In our final application, we consider a set of six exogenous fundamental drivers within our quantitative medium-scale DSGE model. The exogenous driving forces are government spending, investment-specific technology, neutral technology (TFP), interest rate deviations, price markup shocks, and wage markup shocks.

Figure 4 displays the LPF(100) summary statistic for all six shocks under a variety of policy coefficient ϕ_π, after fixing $\phi_y = 0.5$ and $\rho_i = 0.5$. The figure reveals several interesting patterns. First, the effects of anticipated changes in government spending are extremely small for all variables. This observation is particularly interesting because the literature that seeks to identify the effects of government spending includes many attempts to control for such effects.[9]

Other shocks have relatively large anticipation effects only for certain variables. For example, anticipated changes in the interest rate have relatively large effects on investment, but much smaller anticipation effects on other variables. Qualitatively, a similar story holds for investment specific productivity shocks.

[9] In simulations, however, Chahrour et al. (2012) have found that such controls are not important.

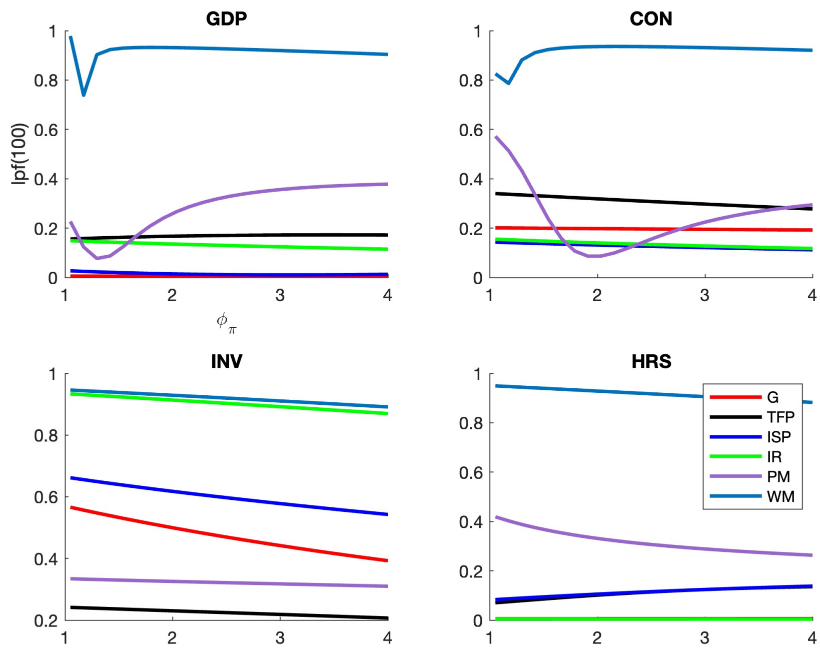

Figure 4: ERF summary statistics for six "standard" shocks under different monetary rules.

Moreover, for most shocks and variables, the relative importance of anticipation effects generally falls modestly as policy become more restrictive. But there are exceptions. For example, the effect of anticipated changes in price markup shocks on GDP actually tend to increase for values of ϕ_π larger than 1.25, and they are strongly non-monotonic for consumption.

The most salient feature of Figure 4, however, are the extremely large values for the LPF(100) statistic for the wage markup shock. In fact, it has the largest value of LPF(100), for all four variables and for every policy variable depicted. This implies that, relative to its impact effect, expectations of future wage markup shocks can have very large effects on the economy. Moreover, the relative size of their LPF(100) statistic (and similarly for other ERF summary statistics) doesn't depend very much on policy. One way to describe the intuition for this result is that the wage markup shock is a particularly "stagflationary" economic disturbance, creating expectations of both recession and increased inflation. The standard monetary policy rule can do relatively little to offset expectations of such a shock, leaving it relatively high latitude to influence endogenous variables.

4 Conclusions

In this paper, we have shown how the expectation response function can be used to isolate the effects of expectations in linear DSGE models. The tool separates assumptions about exogenous processes, information, and expectations formation from the direct effect of expectations themselves. In three applications, we have shown how the tool can be used to diagnose issues related to expectations in some standard models, and to uncover productive new avenues for future research on the effects of expectations in the macroeconomy.

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Appendix

A Computing ERFs

Let, e.g., $k_{t+1|t} \equiv \hat{E}_t[k_{t+1}]$. Then define

$$\begin{bmatrix} k_t^* \\ y_t^* \end{bmatrix} \equiv Z^{-1} \begin{bmatrix} k_t \\ y_t \end{bmatrix}$$

so that system in (5) can we rewritten as

$$AZ \begin{bmatrix} k_{t+1|t}^* \\ y_{t+1|t}^* \end{bmatrix} = BZ \begin{bmatrix} k_t^* \\ y_t^* \end{bmatrix} + A_x x_{t+1|t} + B_x x_t. \tag{11}$$

Premultiply equation (11) by Q to get

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} k_{t+1|t}^* \\ y_{t+1|t}^* \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} k_t^* \\ y_t^* \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} A_x x_{t+1|t} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} B_x x_t \tag{12}$$

Using equation (12), we can solve for

$$y_t^* = T_{22}^{-1} S_{22} y_{t+1|t}^* - T_{22}^{-1} Q_2 A_x x_{t+1|t} - T_{22}^{-1} Q_2 B_x x_t \tag{13}$$

Letting $\Gamma \equiv T_{22}^{-1} S_{22}$ and $M \equiv \left[T_{22}^{-1} Q_2 A_x + \Gamma T_{22}^{-1} Q_2 B_x\right]$, we can solve for

$$y_t^* = -T_{22}^{-1} Q_2 B_x x_t - \Gamma^0 M x_{t+1|t} - \Gamma^1 M x_{t+2|t} + ... \tag{14}$$

Noting that

$$\begin{bmatrix} k_t \\ y_t \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} k_t^* \\ y_t^* \end{bmatrix}$$

we have

$$y_t = Z_{21} Z_{11}^{-1} k_t + (Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}) y_t^*. \tag{15}$$

Combining (14) and (15), we have

$$\frac{\partial y_t}{\partial x_t} = -(Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}) T_{22}^{-1} Q_2 B_x$$

and

$$\frac{\partial y_t}{\partial x_{t+j|t}} = -(Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}) \Gamma^{j-1} M$$

for all $j > 0$.

B Model Summaries

Here we present a brief summary and calibrations details for each of the models used in our applications above. The summaries are necessarily brief, and we provide references for further details.

RBC model

The household maximizes utility by choosing $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize utility, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) - \chi H_t\right),$$

subject to the flow budget constraints

$$C_t + K_{t+1} - (1-\delta)K_t = W_t H_t + R_t K_t.$$

Firms solve a static problem,

$$\max_{\{Y_t, H_t, K_t\}} Y_t - W_t H_t - R_t K_t \;\text{ subject to }\; Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}.$$

We linearize the model around a balanced-growth path in which the potentially non-stationary productivity process A_t has no drift. The model parameters are $\beta = 0.985$, $\alpha = 0.36$, $\delta = 0.0125$, and $\chi = 1$.

Jaimovich and Rebelo (2009) Model

Our version of the Jaimovich and Rebelo (2009) model is identical to the baseline version of their model. Relative to the RBC model above, it (i) adds type of habit to preferences (ii) introduces variable capacity utilization (iii) introduces investment adjustment costs. Like the RBC model, the decentralized equilibrium and planner solutions give the same allocations. The planner chooses $\{C_t, H_t, I_t, Y_t, u_t, X_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize household utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \psi H_t^{\theta} X_t)^{1-\sigma}}{1-\sigma},$$

subject to the following constraints

$$
\begin{aligned}
Y_t &= C_t + I_t \\
X_t &= C_t^{\gamma} X_{t-1}^{1-\gamma} \\
K_{t+1} &= I_t \left[1 - \varphi\left(I_t / I_{t-1}\right)\right] + (1 - \delta(u_t)) K_t \\
Y_t &= A_t (u_t K_t)^{1-\alpha} H_t^{\alpha}
\end{aligned}
$$

We use the same parameters as Jaimovich and Rebelo (2009). For parameters that overlap with the RBC model, including $\sigma = 1$, these are identical to the RBC model described above. For the remaining parameters, Jaimovich and Rebelo (2009) set $\theta = 1.4$, $\gamma = 0.001$. In the linear-approximate solution, we only need to calibrate steady-state values for the second derivative $\phi''(1) = 1.3$, $\delta(1) = 0.0125$, and $\delta''(1)/\delta'(1) = 0.15$.

New-Keynesian Model

The standard new-Keynesian model is derived similarly to the formulation in Woodford (2003). Households choose $\{C_{it}, H_{it}\}_{t=0}^{\infty}$ to maximize household utility,

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{H_{it}^{1-\zeta}}{1-\zeta} \right),
$$

subject to the flow budget constraint

$$
P_t C_{it} + B_{it} = R_{t-1} B_{i,t-1} + W_{it} N_{it} + T_t.
$$

Wages are set in a monopolistically competitive manner, but without any nominal stickiness. The real wage is given by

$$
W_t^R = \frac{\eta_w}{\eta_w - 1} C_t N_t^{\zeta}.
$$

Firms maximize the net present value of profits subject to a CES demand function

$$
Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\eta_p}
$$

a production function

$$
Y_{it} = (A_t N_{it})^{1-\alpha},
$$

and a Calvo probability of adjusting prices θ_p.

For parameters, we the values from Blanchard et al. (2013) where possible (see below). We use $\beta = 0.99$, $\sigma = 1$, $\theta_p = 0.877$, $\alpha = 0.1859$, $\zeta = 2.0871$, and $\eta_p = 4\frac{1}{3}$.

Aggregating equilibrium conditions and linearizing around a non-stochastic steady state yields the three-equation model

$$\pi_t = \kappa \hat{y}_t + \beta E_t[\pi_{t+1}] \tag{16}$$

$$\hat{y}_t = E_t[\hat{y}_{t+1} + \pi_{t+1} - i_t + (1-\alpha)\Delta A_{t+1}] \tag{17}$$

$$i_t = \rho i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y \hat{y}_t) + \epsilon_t \tag{18}$$

where $\kappa = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \frac{1+\zeta}{1+\alpha(\eta_p - 1)}$ and i_t is the log of nominal interest rate.

Medium-scale model

Our medium scale model is that of Blanchard et al. (2013). This model adds several features to the new-Keynesian model from the previous section; namely, habits in consumption, variable capacity utilization, investment adjustment costs, government consumption, and wage stickiness. Aside from parameters of the policy function, all parameters correspond to the point estimates (posterior medians) reported in Blanchard et al. (2013).