The Forward Signalling Channel of Inflation

Ryan Chahrour   Gaetano Gaballo

Inflation expectations reflect more than just past experiences with inflation. Aggregate and idiosyncratic economic developments, as well as central bank forward guidance, are all reflected in inflation expectations and therefore in realized inflation. This paper models the information aggregation properties of inflation. Peoples' private economic foresight makes inflation appear to be "backwards looking", even when past fundamental disturbances have no effect on current inflation. More public forward guidance weakens the connection between shorter- and longer-term inflation expectations, consistent with the patterns highlighted by Goldstein and Gorodnichenko (2022). The monetary policy rule changes the forward information people glean from inflation itself, with a higher degree of inflation stabilization causing inflation to depend more on potential developments far in the future.

1 Introduction

Inflation expectations reflect more than just past experiences with inflation. Yet, a recent literature concerned with *measured* inflation expectations, as reflected in a variety of surveys, tries to capture inflation expectations using models in which inflation is a backwards looking AR(1) process and people have (only) noisy signals about past inflation. Goldstein and Gorodnichenko (2022) point out a set of facts about survey inflation expectations that are inconsistent with these theories of inflation expectations, and push for a perspective that puts forward information – information that could not be gleaned from past experiences of inflation – at the center of a theory of inflation expectations.

In this paper, we seek to build a theory designed to match the facts highlighted by Goldstein and Gorodnichenko (2022). A the heart of our theory is a joint determination between the foresight reflected in inlation expectations and inflation itself. Rather than assume an exogenous process for inflation, we show how private foresight about inflation, as well as forward guidance by the central bank, work together to determine the equilibrium process for inflation, which itself is a crucial input back into inflation expectations.

Our model of forward inflation expectations reveals some surprising features. We show that equilibrium inflation, taken alone, may appear to be a backward look process even as inflation itself is actually entirely determined by expected future macroeconomic developments. Private information about these potential developments is aggregated and then revealed to people in their observations of inflation itself.

Central bank policy influences inflation expectations (and therefore the equilibrium process that inflation follows) through at least two key channels. First, forward guidance changes the profile of forward information available to agents, leading to independent variation in people's forecasts of inflation at different horizons, consistent with the findings of Goldstein and Gorodnichenko (2022), but inconsistent with other leading models of inflation expectations. Second, the monetary policy rule itself influences the information about future developments revealed by inflation, generating a new, indirect channel through which monetary policy influences macroeconomic outcome.

This note outlines our approach to modeling these phenomena and summarizes some of our preliminary findings.

2 Facts about inflation expectations

Below is brief summary of facts about survey expectations from Goldstein and Gorodnichenko (2022) that we think a good theory of inflation expectations should speak to. The basic estimating equation from their paper is

$$x^i_{t+h|t} = \beta^h x^i_{t+h-1|t} + \epsilon_{i,t} \tag{1}$$

where $x^i_{t+h|t}$ is the forecast of agent i at time t about the realization of x in h periods ahead. This regression estimates the average autocorrelation in individual forecasts at different horizons.

Fact 1:

The regression coefficient β^h in (1) rises for longer horizon forecasts h.

Fact 2:

The r-squared in the regression in (1) rises with horizon h.

Fact 3:

Both the coefficient and the r-squared in (1) fall from 1980 to the 2000's.

Fact 4:

Estimating (1) at the individual level reveals the same patterns: they are not a composition effect of who's in the survey

Fact 5:

Adding fed information "controls" to equation (1) makes facts 1 and 2 less pronounced: β^h rises, and rises more low h than it does for high h; similarly for r-squared.

Fact 6:

Define

$$FI^i_{t+h|t} \equiv x_{t+h|t} - (\hat{c}_t + \hat{\beta}^h_t x_{t+h-1|t})$$

as the individual-level forward information. We need some trick to estimate these parameters appropriately. We find that the average $\bar{FI}_{t+h|t}$ is a good predictor of future inflation, beyond past realized inflation, and the contribution of this term falls as h increases.

Fact 7:

$FI^i_{t+h|t}$ accounts for a large fraction of the cross sectional heterogeneity in inflation forecasts.

Interpretation

Facts 1 and 2 are consistent with the idea that at longer horizons, further out forecasts are a simple extrapolation of earlier forecasts, i.e there is less predicted future information in forward forecasts. Fact 3 says that agents got more future information from 1980 to 2000. Fact 4 says that this force acts at the individual level. Fact 5 points out to fed information embodying this common component of forward information. Fact 6 checks whether residual of the regression are indeed predictor of future inflation. Fact 7 says that such residuals are responsible for most of observed cross sectional heterogeneity.

3 A model of dispersed foresight and public news

3.1 Basic Model

To start, we consider a dispersed information version of the Phillips curve.

$$\pi_t = \beta \int E[\pi_{t+1}|\Omega_{i,t}]\, di \;+\; \theta_t$$

where the fundamental

$$\theta_t \;=\; \theta_{s,t} \;+\; \theta_{n,t},$$

with $\theta_{\cdot,t} \sim N(0,\sigma)$ is composed by i.i.d. surprise and a news components, and the individual information set

$$\Omega_{i,t} = \{\pi_t, \pi_{t+1} + \eta_{i,t}, \theta_{n,t}, \theta_{n,t+1}\}$$

includes the t and $t+1$ public news.

To solve for the equilibrium, let us first write down the public expectation

$$E[\pi_{t+1}|\Omega_t] \equiv E[\pi_{t+1}|\pi_t, \theta_{n,t}, \theta_{n,t+1}]$$

and define:

$$\hat{\pi}_t \equiv \pi_t - \theta_{n,t}$$

so that

$$\hat{\pi}_{t+1} = \pi_{t+1} - \theta_{n,t+1}$$

is the statistics on which agents would optimally receive forward information (the optimality of forward information in this form follows under the working hypothesis that π_{t+1} only contains $\pi_{t+\tau>t+1}$, whcih will be verified later).

Agents will form expectations according to:

$$E[\hat{\pi}_{t+1}|\Omega_{i,t}] = (1-k)\, E[\hat{\pi}_{t+1}|\Omega_t] \;+\; k\,(\hat{\pi}_{t+1} + \eta_{i,t})$$

so that:

$$E[\pi_{t+1}|\Omega_{i,t}] = E[\hat{\pi}_{t+1}|\Omega_{i,t}] + \theta_{n,t+1},$$

where k represents a rational degree of attention to information about the future, that could be well taken as a primitive. Notice for k to be optimal it has to be equal to the ration between the precision of the private signal and total precision.

Conjecture now

$$\hat{\pi}_t = b\,\hat{\pi}_{t+1} + \; d_n\,\beta\,\theta_{n,t+1} \; + d_s\,\theta_{s,t} \tag{**}$$

implying

$$\hat{\pi}_{t+1} = \sum_{\tau=0}^{\infty} b^{\tau}(d_n\,\beta\,\theta_{n,t+2+\tau} \;+\; d_s\theta_{s,t+1+\tau}\,),$$

with $|b| < 1$. Thus, we have

$$V_{\hat{\pi}} = \frac{1}{1-b^2}\left(d_n^2\beta^2 V_{\theta_n} + d_s^2 V_{\theta_s}\right).$$

Note that, since $\theta_{n,t+1}$ is known, then using $\hat{\pi}_t$ and $\theta_{n,t+1}$ to infer $\hat{\pi}_{t+1}$, leads to

$$E[\hat{\pi}_{t+1}|\Omega_t] = \frac{Cov(\hat{\pi}_{t+1}, \frac{\hat{\pi}_t - d_n\beta\theta_{n,t+1}}{b})}{Var(\frac{\hat{\pi}_t - d_n\beta\theta_{n,t+1}}{b})}\left(\frac{\hat{\pi}_t - d_n\beta\theta_{n,t+1}}{b}\right) =$$

$$= \frac{V_{\hat{\pi}}}{V_{\hat{\pi}} + \frac{d_s{}^2}{b^2}V_{\theta_s}}\frac{\hat{\pi}_t - d_n\beta\theta_{n,t+1}}{b} = b\;\underbrace{\frac{\frac{1}{d_s^2}\frac{1}{V_{\theta_s}}}{\frac{b^2}{d_s^2}\frac{1}{V_{\theta_s}} + \frac{1}{V_{\hat{\pi}}}}}_{=v}(\hat{\pi}_t - d_n\beta\theta_{n,t+1}) \tag{*}$$

Given the result above we can write

$$\int E[\hat{\pi}_{t+1}|\Omega_{i,t}]di = (1-k)bv\,(\hat{\pi}_t - d_n\beta\theta_{n,t+1}) \;+\; k\hat{\pi}_{t+1}$$

and

$$\int E[\pi_{t+1}|\Omega_{i,t}]di = \int E[\hat{\pi}_{t+1}|\Omega_{i,t}]di + \theta_{n,t+1}$$

that, once put back into the actual law of motion, gives

$$\pi_t = \beta(1-k)vb\,(\hat{\pi}_t - d_n\beta\theta_{n,t+1}) \;+\; \beta k\hat{\pi}_{t+1} + \beta\theta_{n,t+1} + \theta_{n,t} + \theta_{s,t}$$

and finally

$$\hat{\pi}_t = \underbrace{\frac{\beta k}{1-\beta(1-k)vb}}_{=b}\hat{\pi}_{t+1} + \underbrace{\frac{1-\beta(1-k)vbd_n}{1-\beta(1-k)vb}}_{=d_n}\beta\,\theta_{n,t+1} + \underbrace{\frac{1}{1-\beta(1-k)vb}}_{=d_s}\theta_{s,t}. \tag{2}$$

Here is what we learn from equation (2). First, inflation is a *purely forward-looking process* which depends only on current and future realizations of the two fundamental disturbances. Second, (2) implies that regression of inflation on its past values would deliver a coefficient b: taken in isolation, inflation would appear to be a purely backwards looking process. Third, allowing for forward guidance (news about $\theta_{n,t+1}$) changes the process for inflation, leading to forecasts at short horizons that are not just "discounted" versions of current inflation. Finally, changing the amount of forward guidance changes the value b. That is, changing the forward information released by the central bank changes the persistence of the inflation process as well as the information embedded in current inflation.

3.2 Regression at different horizons

We can compute the model implied coefficients for the regressions given in (1), to see if this simple model of inflation determination is consistent with the facts we started with.

Given that

$$E[\pi_{t+1}|\Omega_{i,t}] = E[\hat{\pi}_{t+1}|\Omega_{i,t}] + \theta_{n,t+1}$$

$$E[\pi_{t+2}|\Omega_{i,t}] = E[\hat{\pi}_{t+2}|\Omega_{i,t}] + E[\theta_{n,t+2}|\Omega_{i,t}]$$

$$E[\pi_{t+2}|\Omega_{i,t}] = (b + \delta_{\theta_{n,t+2}})E[\hat{\pi}_{t+1}|\Omega_{i,t}]$$

$$\delta_{\theta_{n,t+2}} = \frac{\beta V_{\theta_n}}{V_{\hat{\pi}}}$$

for the individual regressions we have

$$\begin{aligned}
\frac{Cov\left(E[\pi_{t+2}|\Omega_{i,t}], E[\pi_{t+1}|\Omega_{i,t}]\right)}{Var(E[\pi_{t+1}|\Omega_{i,t}])} &= (b + \delta_{\theta_{n,t+2}})\frac{V(E[\hat{\pi}_{t+1}|\Omega_{i,t}])}{V(E[\hat{\pi}_{t+1}|\Omega_{i,t}]) + V(\theta_n)} \\
&= \frac{b\, V_{\hat{\pi}} + \beta V_{\theta_n}}{V_{\hat{\pi}}} \frac{\frac{b}{\beta} V_{\hat{\pi}}}{\frac{b}{\beta}\, V_{\hat{\pi}} + V_{\theta_n}} = b \\
\frac{Cov\left(E[\pi_{t+3}|\Omega_{i,t}], E[\pi_{t+2}|\Omega_{i,t}]\right)}{Var(E[\pi_{t+2}|\Omega_{i,t}])} &= b
\end{aligned}$$

and for aggregate regression we have,

$$\begin{aligned}
\frac{Cov\left(\int E[\pi_{t+2}|\Omega_{i,t}], \int E[\pi_{t+1}|\Omega_{i,t}]\right)}{Var(\int E[\pi_{t+1}|\Omega_{i,t}])} &= (b + \delta_{\theta_{n,t+2}})\frac{V(\int E[\hat{\pi}_{t+1}|\Omega_{i,t}])}{V(\int E[\hat{\pi}_{t+1}|\Omega_{i,t}]) + V(\theta_n)} \\
&= \frac{b\, V_{\hat{\pi}} + \beta V_{\theta_n}}{V_{\hat{\pi}}} \frac{\frac{b}{\beta} V_{\hat{\pi}} - k(1 - \frac{b}{\beta})V_{\hat{\pi}}}{\frac{b}{\beta}\, V_{\hat{\pi}} - k(1 - \frac{b}{\beta})V_{\hat{\pi}} + V_{\theta_n}} < b \\
\frac{Cov\left(\int E[\pi_{t+3}|\Omega_{i,t}], \int E[\pi_{t+2}|\Omega_{i,t}]\right)}{Var(\int E[\pi_{t+2}|\Omega_{i,t}])} &= b
\end{aligned}$$

where we used

$$V(\int E[\hat{\pi}_{t+1}|\Omega_{i,t}]) = \underbrace{V(E[\hat{\pi}_{t+1}|\Omega_{i,t}])}_{=\frac{b}{\beta}V_{\hat{\pi}}} - \underbrace{k^2\sigma_\eta^2}_{=\underbrace{\frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{V_{\hat{\pi}}}+\frac{\beta^2k^2}{V_{\theta_s}}}}_{=k}\underbrace{\frac{1}{\frac{1}{\sigma_\eta^2}+\frac{1}{V_{\hat{\pi}}}+\frac{\beta^2k^2}{V_{\theta_s}}}}_{=(1-\frac{b}{\beta})V_{\hat{\pi}}}}$$

4 Model

The next steps is to embed this mechanism in a microfounded model of inflation and the real economy.

4.1 Households

Utility function of agent $i \in (0,1)$ depends on individual consumption of an homogeneous consumption good C_i, and the supply of an homogeneous type of labor N_i :

$$E\left[\sum_{\tau=t}^{\infty}\beta_{i,t}^{\tau-t}\left(\frac{C_{i,\tau}^{1-\sigma}}{1-\sigma}-\frac{N_{i,\tau}^{1+\varphi}}{1+\varphi}\right) \mid \Omega_{i,t}\right]$$

where β_i is an individual and idiosincratic discont factor. Agent i chooses consumption and labor supply subject to

$$C_{i,t}P_t + Q_tB_t = W_tN_{i,t} + B_{t-1}.$$

First order conditions are

$$\begin{aligned} C_{i,\tau}^{-\sigma} &= P_t\lambda_{i,t} \\ N_{i,\tau}^{\varphi} &= W_t\lambda_{i,t} \\ Q_t\lambda_{i,t} &= \beta_{i,t}E\lambda_{i,t+1} \end{aligned}$$

that combined in logs give

$$\begin{aligned} -\sigma c_{i,t} &= p_t - w_t + \varphi n_{i,t} \\ -\sigma c_{i,t} - p_t &= \underbrace{\log Q_t}_{i_t} + \underbrace{\log \beta_{i,t}}_{z_{i,t}} - E\left[\sigma c_{i,t+1} + p_{t+1}|\Omega_{i,t}\right], \end{aligned}$$

giving the individual Euler equation as:

$$c_{i,t} = -\frac{1}{\sigma}\left(i_t - E_{i,t}\left[\pi_{t+1}\right]\right) + E_{i,t}\left[c_{i,t+1}\right] + z_{i,t}.$$

4.2 Firms

From the firm problem

$$\begin{aligned}
&\max_{P_{j,t}} E\left[\sum_{\tau=t}^{\infty} (\theta)^{\tau-t} Q_{t|t+\tau}\, \Pi_{j,t} \mid \Omega_{j,t}\right] \\
\Pi_{j,t} &= P_{j,t}\left(\frac{P_t}{P_{j,t}}\right)^{\varepsilon} A_{j,t}N_{j,t} - W_t N_{j,t},
\end{aligned}$$

we get the log linear optimal price equation

$$p^*_{i,t} = (1-\beta\theta)\, E_{i,t}\left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t}\ mc_{j,t}\right]$$

where the real marginal cost is

$$mc_{j,t} = w_t - p_t - a_{j,t}.$$

By aggregating the first order condition for labor and the technology constraint

$$\begin{aligned}
w_t - p_t &= \varphi n_t + \sigma c_t \\
c &= y = n_t + a_t
\end{aligned}$$

we can rewrite

$$\begin{aligned}
mc_{j,t} &= \varphi n_t + \sigma c_t - a_{j,t} \\
&= \varphi\left(c_t - a_t\right) + \sigma c_t - a_{j,t}
\end{aligned}$$

and finally get the fowing equations.

4.3 Reduced form

In aggregate we have

$$\begin{aligned}
c_t &= -\frac{1}{\sigma}\left(i_t - \int E_{i,t}\left[\pi_{t+1}\right]\right) + \int E_{i,t}\left[c_{t+1}\right] + \left(\int E_{i,t}\left[c_{i,t+1}\right] - \int E_{i,t}\left[c_{t+1}\right]\right) + z_t \\
\pi_t &= \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\left(\varphi+\sigma\right)c_t - a_{j,t} - \varphi a_t\right) + \beta\int E_{i,t}\left[\pi_{t+1}\right] + \int E_{i,t}\left[p^*_{i,t+1} + p^*_{t+1}\right] \\
i_t &= \phi_\pi \pi_t + \phi_c c_t
\end{aligned}$$

We work under the guess to be verified that

$$\int E_{i,t}\left[p^*_{i,t+1} + p^*_{t+1}\right] = \int E_{i,t}\left[c_{i,t+1}\right] - \int E_{i,t}\left[c_{t+1}\right] = 0.$$

We put the system in matrix form

$$\begin{bmatrix} \frac{1}{\sigma}\phi_\pi & 1+\frac{1}{\sigma}\phi_c \\ 1 & -\kappa\left(\varphi+\sigma\right) \end{bmatrix}\begin{bmatrix} \pi_t \\ c_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma} & 1 \\ \beta & 0 \end{bmatrix}\begin{bmatrix} \int E_{i,t}\left[\pi_{t+1}\right] \\ \int E_{i,t}\left[c_{t+1}\right] \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -\kappa\left(1+\varphi\right) \end{bmatrix}\begin{bmatrix} z_t \\ a_t \end{bmatrix}$$

to get

$$x_t = B \int E_{i,t}\left[x_{t+1}\right] + C\eta_t \tag{3}$$

with

$$\begin{aligned} B &= \frac{1}{\sigma+\phi_c+\kappa\left(\sigma+\varphi\right)\phi_\pi}\begin{bmatrix}\left(\sigma+\phi_c\right)\beta+\kappa\left(\sigma+\varphi\right) & \kappa\sigma\left(\sigma+\varphi\right)\\ 1-\beta\phi_\pi & \sigma\end{bmatrix}, \\ C &= \frac{1}{\sigma+\phi_c+\kappa\left(\sigma+\varphi\right)\phi_\pi}\begin{bmatrix}\kappa\sigma\left(\sigma+\varphi\right) & -\kappa\left(\varphi+1\right)\left(\sigma+\phi_c\right)\\ \sigma & \kappa\phi_\pi\left(\varphi+1\right)\end{bmatrix}. \end{aligned}$$

4.4 Dispersed Forward Looking Expectations

We first guess a forward looking perceived law of motion

$$x_t = \Phi\, x_{t+1} + D\eta_t,$$

with $\Sigma_p \equiv V(x_t)$. The information set of agents is composed by the observation of the current outcome x_t and a signal about the future

$$s_{i,t} : x_{t+1} + \eta_{i,t}$$

where

$$\eta_{i,t} \sim N(0, \Sigma_i)$$

is iid Gaussian individual noise. We then establish that, given the guess, the optimal expectation has to take the form

$$E_{i,t}\left[x_{t+1}|x_t, s_{i,t}\right] = \left(1-K\right)E_{i,t}\left[x_{t+1}|x_t\right] + K\left(x_{t+1}+\eta_{i,t}\right) \tag{4}$$

where

$$E_{i,t}\left[x_{t+1}|x_t\right] = \frac{Cov(x_{t+1}, x_t)}{Cov(x_t, x_t)}\, x_t = \Sigma_p\Phi'\Sigma_p^{-1}x_t \neq \frac{\Phi'\,\Sigma_p}{\Sigma_p}\, x_t = \Phi'\, x_t,$$

and K directly depend on the rational inattention choice implicit in Σ_i, for whatever D. (Note our previous result that the regression coefficient would be the same if the perceived law of motion was a AR(1) instead!) In particular

$$K = \frac{\Sigma_p}{\Sigma_p + \Sigma_i}.$$

Actually, I think

$$K = \frac{\tilde{\Sigma}_p}{\tilde{\Sigma}_p + \Sigma_i}$$

where

$$\tilde{\Sigma}_p = (I - \Sigma_p\Phi'\Sigma_p^{-1}\Phi)\Sigma_p(I - \Sigma_p\Phi'\Sigma_p^{-1}\Phi)' + \Sigma_p\Phi'\Sigma_p^{-1}DD'(\Sigma_p\Phi'\Sigma_p^{-1})'$$

(probably can simplify).

Therefore we have that

$$\int E_{i,t}\left[x_{t+1}|x_t, s_{i,t}\right]di = \left(I-K\right)\Sigma_p\Phi'\Sigma_p^{-1}\, x_t + K\, x_{t+1} \tag{5}$$

that we can plug in the actual law of motion (3) to get

$$x_t = B((I - K)\,\Sigma_p \Phi' \Sigma_p^{-1}\, x_t + K\, x_{t+1}) + C\eta_t$$

or [this line not yet updated]

$$x_t = \frac{B\,K}{1 - B(1-K)\Phi}\, x_{t+1} + \frac{C}{1 - B(1-K)\Phi}\eta_t,$$

so that

$$\Phi = \frac{B\,K}{I - B((I-K)\,\Sigma_p \Phi' \Sigma_p^{-1})}, \tag{6}$$

identifies the equilibrium condition, which exactly encompasses the one-dimensional case.

5 Conclusions

Though incomplete this note Goldstein and Gorodnichenko (2022) describes a new approach to modeling the joint determination of inflation and inflation expectations. The framework allows us to consider how forward information, including forward guidance, will change the information about the future that people can deduce from inflation itself, as well as how it will change the process for inflation itself. These channels are new in the literature, and deliver a model of inflation that appears to match many of the facts about inflation expectations that have been missed by simple models of backwards looking inflation.

References

Nathan Goldstein and Yuriy Gorodnichenko. Expectations formation and forward information. Working Paper 29711, National Bureau of Economic Research, January 2022. URL `http://www.nber.org/papers/w29711`.