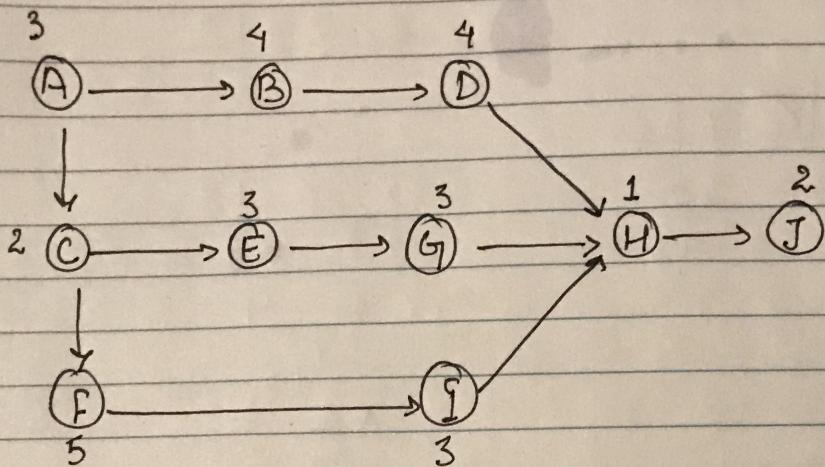


ACTIVITY - 4

(1)

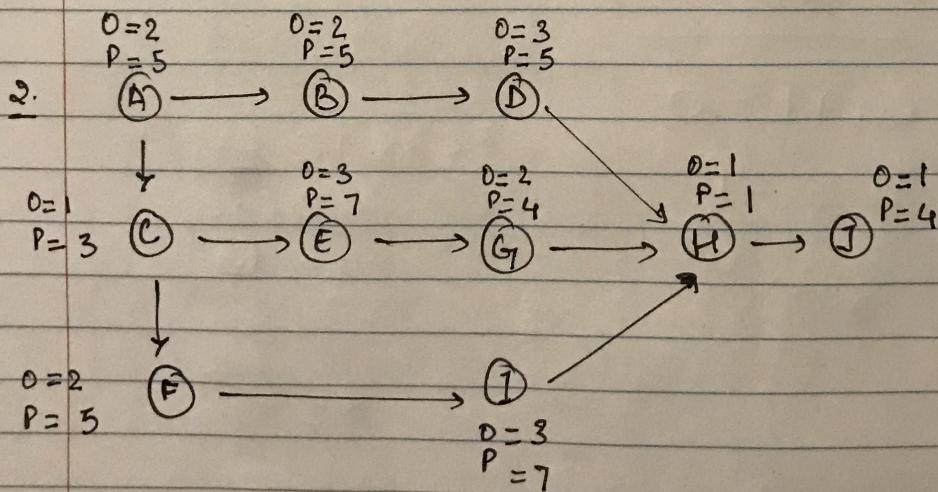
Project networks.



1. A critical path is the longest path in the project network.
we are using estimated duration, considering everything goes fine -

Path	length
$A \rightarrow B \rightarrow D \rightarrow H \rightarrow J$	$3 + 4 + 4 + 1 + 2 = 14 \text{ days}$
$A \rightarrow C \rightarrow E \rightarrow G \rightarrow H \rightarrow J$	$3 + 2 + 3 + 3 + 1 + 2 = 14 \text{ days}$
$A \rightarrow C \rightarrow F \rightarrow I \rightarrow H \rightarrow J$	$3 + 2 + 5 + 3 + 1 + 2 = 16 \text{ days}$

Thus critical path is $A-C-F-I-H-J \rightarrow$ finish
estimated duration = 16 days.



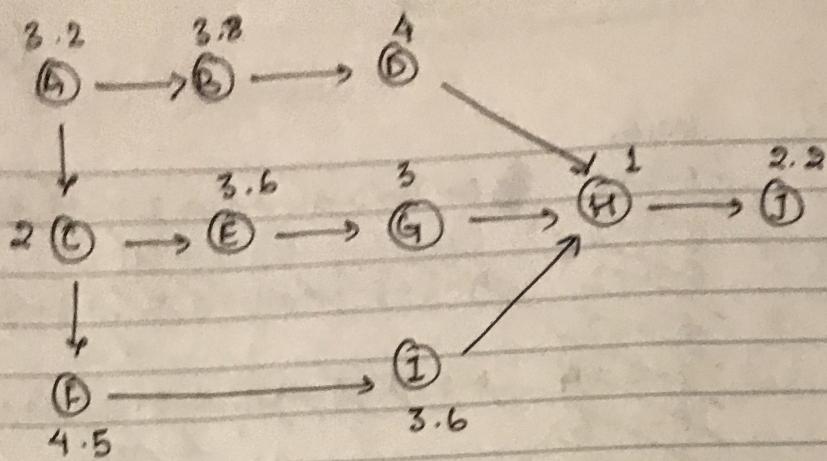
$$\text{Slack} = \text{Pessimistic duration} - \text{Optimistic duration}$$

(2)

	slack	
A	$5-2=3$	$A-B-D-H-I = 3+3+2+4+3 = 11$
B	$5-2=3$	
C	$3-1=2$	$A-C-E-G-H-J = 3+2+4+3 = 12$
D	$5-3=2$	
E	$7-3=4$	$A-C-F-X-H-J = 3+2+3+4 = 12$
F	$5-2=3$	
G	$4-2=2$	
H	$1-1=0$	
I	$7-3=4$	
J	$4-1=3$	

1. expected time = $\frac{O+4M+P}{6}$

	expected time
A	$2+(4 \times 3)+5/6 = 3.2$
B	$2+(4 \times 4)+5/6 = 3.8$
C	$1+(4 \times 2)+3/6 = 2$
D	$3+(4 \times 4)+5/6 = 4$
E	$3+(4 \times 3)+7/6 = 3.6$
F	4.5
G	3
H	1
I	3.6
J	2.2



(3)

A critical path is the longest path in the Project Network.

Path	Length -
A → B → D → H → J	$3.2 + 3.8 + 4 + 1 + 2.2 = 14.2$
A → C → E → G → H → J	$3.2 + 2 + 3.6 + 3 + 1 + 2.2 = 15$
A → C → F → I → H → J	$3.2 + 2 + 4.5 + 3.6 + 1 + 2.2 = 16.2$

The critical path is A → C → F → I → H → J
Duration : 16.2 days

2. To find the slack time we need to calculate the Early start, Early finish or latest start and latest finish.

ES for first activity is always 1.

	ES	EF	LS	LF	slack (LS-ES)
A	1	3.2	0.7	2.9	0.3
B	4.2	7	6.2	9	2
C	4.2	5.2	3.9	4.9	0.3
D	8	11	10	13	2
E	6.2	8.8	7.4	10	1.2
F	6.2	9.7	5.9	9.4	0.3
G	9.8	11.8	11	13	1.2
H	14.3	14.3	14	14	0.3
I	10.7	13.3	10.4	13	0.3
J	13.8	16.5	15	16.2	0.3

$$EF \text{ of } A = ES \text{ of } A + \text{duration} - 1$$
$$= 1 + 3.2 - 1 = 3.2$$

(4)

$$ES \text{ of } B = EF \text{ of } A + 1$$
$$= 4.2$$

$$EF \text{ of } B = ES \text{ of } B + \text{duration} - 1$$
$$= 4.2 + 3.8 - 1$$
$$= 7$$

$$ES \text{ of } C = ES \text{ of } A + 1$$
$$= 3.2 + 1$$
$$= 4.2$$

using expected time did not give us ~~the~~ slack time as 0 for any activities

$$EF \text{ of } C = ES \text{ of } C + \text{duration} - 1$$
$$= 4.2 + 2 - 1$$
$$= 5.2$$

THIS is calculated using expected time

$$ES \text{ of } D = EF \text{ of } B + 1$$
$$= 7 + 1$$
$$= 8$$

$$EF \text{ of } D = ES \text{ of } D + \text{duration} - 1$$
$$= 8 + 4 - 1$$
$$= 11$$

$$ES \text{ of } E = EF \text{ of } C + 1$$
$$= 5.2 + 1 = 6.2$$

$$EF \text{ of } E = ES \text{ of } E + \text{duration} - 1$$
$$= 6.2 + 3.6 - 1 = 8.8$$

$$ES \text{ of } F = ES \text{ of } C + 1$$

$$= 5.2 + 1 = 6.2$$

(3)

$$EF \text{ of } F = ES \text{ of } F + \text{duration} - 1$$

$$= 6.2 + 4.5 - 1$$

$$= 9.7$$

$$ES \text{ of } G = EF \text{ of } E + 1$$

$$= 8.8 + 1 = 9.8$$

$$EF \text{ of } G = ES \text{ of } G + \text{duration} - 1$$

$$= 9.8 + 3 - 1$$

$$= 11.8$$

$$EF \text{ of } I + 1 = 14.3$$

$$ES \text{ of } H + d - 1 = 14.3$$

$$+ 1 - 1$$

$$= 14.3$$

$$ES \text{ of } H = EF \text{ of } G + 1$$

$$= 11.8 + 1 = 12.8$$

$$= EF \text{ of } D + 1$$

$$= 11 + 1 = 12$$

$$EF \text{ of } H = ES \text{ of } H + \text{duration} - 1$$

$$= 12.8 + 1 - 1$$

$$= 12$$

$$ES \text{ of } I = EF \text{ of } F + 1$$

$$= 9.7 + 1 = 10.7$$

$$EF \text{ of } I = ES \text{ of } I + \text{duration} - 1$$

$$= 10.7 + 3.6 - 1$$

$$= 13.3$$

$$14.3 + 1 = 15.3$$

$$ES \text{ of } J = EF \text{ of } H + 1 \longrightarrow$$

$$= 12.8 + 1$$

$$= 13.8$$

$$= \cancel{12} + \cancel{1}$$

$$= 13$$

$$EF \text{ of } J = ES \text{ of } J + \text{duration} - 1$$

$$= 13.8 + 2.2 - 1 = 15 \longrightarrow 15.3 + 2.2 - 1 = 16.5$$

or 14.2.

Late start = late finish - activity duration + 1 (6)

Late finish = late start of successor activity - 1

$$LF \text{ of } J = 16.2 \text{ days}$$

$$LS \text{ of } J = 16.2 - 2.2 + 1 \\ = 15$$

$$LF \text{ of } H = 15 - 1 = 14$$

$$LS \text{ of } H = LF \text{ of } H - \text{duration} + 1 \\ = 14 - 1 + 1 \\ = 14$$

$$LF \text{ of } D = LS \text{ of } H - 1 \\ = 14 - 1 = 13$$

$$LS \text{ of } D = LF \text{ of } D - \text{duration} + 1 \\ = 13 - 4 + 1 \\ = 10$$

$$LF \text{ of } G = LS \text{ of } H - 1 \\ = 14 - 1 = 13$$

$$LS \text{ of } G = LF \text{ of } G - \text{duration} + 1 \\ = 13 - 3 + 1 \\ = 11$$

$$LF \text{ of } I = LS \text{ of } H - 1 \\ = 14 - 1 = 13$$

$$LS \text{ of } I = LF \text{ of } I - \text{duration} + 1 \\ = 13 - 3.6 + 1 \\ = 10.4$$

$$LF \text{ of } E = LS \text{ of } G - 1 \\ = 11 - 1 = 10$$

$$LS \text{ of } E = LF \text{ of } E - \text{duration} + 1 \\ = 10 - 3.6 + 1 \\ = 7.4$$

These are calculated using ~~estimated~~ ~~expected~~ time

$$LF \text{ of } B = LS \text{ of } D - 1 \\ = 10 - 1 = 9$$

(1)

$$LS \text{ of } B = LF \text{ of } B - \text{duration} + 1 \\ = 9 - 3.8 + 1 \\ = 6.2$$

$$LF \text{ of } F = LS \text{ of } I - 1 \\ = 10.4 - 1 = 9.4$$

$$LS \text{ of } F = LF \text{ of } F - \text{duration} + 1 \\ = 9.4 - 4.5 + 1 \\ = 5.9$$

$$LF \text{ of } C = LS \text{ of } F - 1 \\ = 5.9 - 1 = 4.9$$

(we select the one with
least start date between
F and E)

$$LS \text{ of } C = LF \text{ of } C - \text{duration} + 1 \\ = 4.9 - 2 + 1 \\ = 3.9$$

$$LF \text{ of } A = LS \text{ of } C - 1 \\ = 3.9 - 1 = 2.9$$

$$LS \text{ of } A = LF \text{ of } A - \text{duration} + 1 \\ = 2.9 - 3.2 + 1 \\ = 0.7$$

Path
A - B - D - H - J

These are
calculated
using expected time
length slack

Path
A - B - D - H - J

$$0.3 + 2 + 2 + 0.3 + 0.3 = 4.9$$

A - C - E - G - H - J

$$0.3 + 0.3 + 1.2 + 1.2 + 0.3 + 0.3 = 3.6$$

A - C - F - I - H - J

$$0.3 + 0.3 + 0.3 + 0.3 + 0.3 + 0.3 \\ = 1.8$$

Path A - B - D - H - J has the most slack time of 4.9

2.

$$\begin{aligned} \text{EF of A} &= \text{ES of A} + \text{duration - 1} \\ &= 1 + 3 - 1 = 3 \end{aligned}$$

(8)

$$\begin{aligned} \text{ES of B} &= \text{EF of A} + 1 \\ &= 3 + 1 = 4 \end{aligned}$$

$$\begin{aligned} \text{EF of B} &= \text{ES of B} + \text{duration - 1} \\ &= 4 + 4 - 1 = 7 \end{aligned}$$

$$\begin{aligned} \text{ES of C} &= \text{EF of A} + 1 \\ &= 3 + 1 = 4 \end{aligned}$$

$$\begin{aligned} \text{EF of C} &= \text{ES of C} + \text{duration - 1} \\ &= 4 + 2 - 1 = 5 \end{aligned}$$

$$\begin{aligned} \text{ES of D} &= \text{EF of B} + 1 \\ &= 7 + 1 = 8 \end{aligned}$$

$$\begin{aligned} \text{EF of D} &= \text{ES of D} + \text{duration - 1} \\ &= 8 + 4 - 1 = 11 \end{aligned}$$

$$\begin{aligned} \text{ES of E} &= \text{EF of C} + 1 \\ &= 5 + 1 = 6 \end{aligned}$$

$$\text{EF of E} = 6 + 3 - 1 = 8$$

$$\begin{aligned} \text{ES of F} &= \text{EF of C} + 1 \\ &= 5 + 1 = 6 \end{aligned}$$

$$\begin{aligned} \text{EF of F} &= 6 + 5 - 1 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{ES of G} &= \text{EF of E} + 1 \\ &= 8 + 1 = 9 \end{aligned}$$

$$\begin{aligned} \text{EF of G} &= 9 + 3 - 1 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{ES of I} &= \text{EF of F} + 1 \\ &= 10 + 1 = 11 \end{aligned}$$

$$\begin{aligned} \text{EF of I} &= 11 + 3 - 1 \\ &= 13 \end{aligned}$$

$$\begin{aligned} \text{ES of H} &= \text{EF of I} + 1 \\ &= 13 + 1 = 14 \end{aligned}$$

$$\begin{aligned} \text{EF of H} &= 14 + 1 - 1 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{ES of J} &= \text{EF of H} + 1 \\ &= 14 + 2 = 16 \end{aligned}$$

These values
are calculated using
estimated activity
time

$$LF \text{ of } J = 16$$

$$LS \text{ of } J = 16 - 2 + 1 \\ = 15$$

$$LF \text{ of } H = LS \text{ of } J - 1 \\ = 14$$

$$LS = 14 - 1 + 1 = 14$$

$$LF \text{ of } D = LS \text{ of } H - 1 \\ = 14 - 1 = 13$$

$$LS \text{ of } D = 13 - 4 + 1 \\ = 10$$

$$LF \text{ of } G = LS \text{ of } D - 1 \\ = 13$$

$$LS \text{ of } G = 13 - 3 + 1 \\ = 11$$

$$LF \text{ of } I = LS \text{ of } G - 1 \\ = 13$$

$$LS \text{ of } I = 13 - 3 + 1 \\ = 11$$

$$LF \text{ of } E = LS \text{ of } I - 1 = 11 - 1 = 10 \\ = \cancel{11} - 1 = 10$$

$$LS \text{ of } E = 10 - 3 + 1 = 8$$

$$LF \text{ of } B = LS \text{ of } E - 1 \\ = 10 - 1 = 9$$

$$LS \text{ of } B = 9 - 4 + 1 \\ = 6$$

$$LF \text{ of } F = LS \text{ of } B - 1 \\ = 11 - 1 = 10$$

$$LS = 10 - 5 + 1 = 6$$

$$LF \text{ of } C = LS \text{ of } F - 1 \\ = 6 - 1 = 5$$

$$LS \text{ of } C = 5 - 2 + 1 = 4$$

$$LF \text{ of } A = LS \text{ of } C - 1 \\ = 4 - 1 = 3$$

$$LS \text{ of } A = 3 - 3 + 1 \\ = 1$$

(It has two successors
and we select the
one with least start date)

(10)

ES EF LF LS

	ES	EF	LF	LS	slack time (LS-ES)
A	1	3	3	1	0
B	4	7	9	6	2
C	4	5	5	4	0
D	8	11	13	10	2
E	6	8	10	8	2
F	6	10	10	6	0
G	9	11	13	11	2
H	14	14	14	14	0
I	11	13	13	11	0
J	15	16	16	15	0

Hence the critical path is A-C-F-H-I-J
as it has 0 slack time

Path	slack time
A-B-D-H-J	0+2+2+0+0 = 4
A-C-E-G-H-J	0+0+2+2+0+0 = 4

Both of these above mentioned paths have most slack time.

3 Reducing activity E's time to half number of days would not affect the critical path because , activity E does not lie on critical path

Path	Length
A-B-D-H-J	$3.2 + 3.8 + 4 + 1 + 2.2 = 14.2$
A-C-E-G-H-J	$3.2 + 2 + 1.8 + 3 + 1 + 2.2 = 13.2$
A-C-F-I-H-J	$3.2 + 2 + 4.5 + 3.6 + 1 + 2.2 = 16.2$

(11)

4. Activity F is crashed to half number of days.

Path	Length
A-B-D-H-J	$3.2 + 3.8 + 4 + 1 + 2.2 = 14.2$
A-C-E-G-H-J	$3.2 + 2 + 3.6 + 3 + 1 + 2.2 = 15$
A-C-F-I-H-J	$3.2 + 2 + 2.25 + 3.6 + 1 + 2.2 = 14.25$

Since we have crashed the activity F, A-C-F-I-H-J is no longer the critical path and the new critical path is A-C-E-G-H-J as it has the longest duration of project completion of 15 days.