Submission Deadline: October 04, 2024, 22:00 hrs

1. Consider the expectation $I = E[\exp(\sqrt{U})]$ where, $U \sim U(0,1)$. Use the following antithetic method to approximate I:

$$\widehat{I}_M = \frac{1}{M} \sum_{i=1}^M \widehat{Y}_i$$
, where $\widehat{Y}_i = \frac{\exp(\sqrt{U_i}) + \exp(\sqrt{1 - U_i})}{2}$ with $U_i \sim U(0, 1)$.

Take the values of M to be $\frac{10^2}{2}$, 10^2 , $\frac{10^3}{2}$, 10^3 , $\frac{10^4}{2}$, 10^4 , $\frac{10^5}{2}$ and 10^5 . Determine the 95% confidence interval for \widehat{I}_M for all the values of M that you have taken.

2. Present the results that you have obtained in Question 1 of Lab 06 and Question 1 of Lab 07 in a tabular form. Your table must consist of the values of estimates (using the two methods), 95% confidence intervals for I (from two methods), and the ratio of widths of both the intervals. How do the estimates of I using the two methods compare with the actual value of I?