**Submission Deadline: October 25, 2024, 22:00 hrs**

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**For the background material, refer to Glasserman (Chapter 3, Sections 3.1 & 3.2)**

1. Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion
$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} \cdot Z_{i+1},$$
with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Also estimate $E[W(2.5)]$ and $E[W(5)]$ from the 10 paths that you have generated.

2. Repeat the above exercise with the following Brownian motions ($BM(\mu, \sigma^2)$) discretization
$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$
Take $X(0) = 5$ and the following sets of values: (a) $\mu = 0.09$ and $\sigma = 0.5$; (b) $\mu = 0.9$ and $\sigma = 0.25$; and (c) $\mu = 0.9$ and $\sigma = 0.05$.

3. The Euler approximated recursion with time dependent $\mu$ and $\sigma$ is given by
$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$
Repeat the above exercise by taking
$$Y(0) = 5,\ \mu(t) = 0.0325 - 0.05t,\ \sigma(t) = 0.012 + 0.0138t - 0.00125t^2.$$

4. Let us simulate a geometric Brownian motion (GBM), which is the most fundamental model for the value of a financial asset. As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < \cdots < t_n = T$ as:
$$S(t_{i+1}) = S(t_i) \exp\left(\left[\mu - \frac{1}{2}\sigma^2\right](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}\right),\ i = 0, 1, \ldots, n-1,$$
where $Z_1, Z_2, \ldots, Z_n$ are independent $\mathcal{N}(0, 1)$ variates. In the interval $[0, 5]$, taking both positive and negative values for $\mu$ and for at least two different values of $\sigma^2$, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points, say 5000 values, for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$.

   Now, taking $S(0) = 5, \mu = 0.06, \sigma = 0.3$, first simulate $S(1)$ by taking $n = 50$ steps to reach time 1 (i.e., with time intervals of length 0.02). Then, compute the expectation of $S(1)$ (i.e., $E(S(1))$) by simulating $N$ values of $S(1)$. By varying values for $N$ from 1 to 1000, plot the values of $E(S(1))$ as a curve. Simulate and plot 5 such curves ($N$ in the $x$-axis and $E(S(1))$ in the $y$-axis).

Put all your observations in the report.

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