1 Composition Function

```
→   compose (L, x) :=
      block ([r : x], for e in L do r : subst (e, x, r), r) $;
```

Note that subst(a, b, c) substitutes a for b in c

2 Trivial decompositions; compare with wolframalpha

```
→   f1 : x^8 + 4·x^7 +10·x^6 + 16·x^5 + 29·x^4+ 36·x^3 + 40·x^2 + 24·x + 39;
```

(f1) $x^8+4x^7+10x^6+16x^5+29x^4+36x^3+40x^2+24x+39$

```
→   pl1:polydecomp(f1, x);
```

(pl1) $[x^2+3, x^2+5, \frac{x^2+3}{4}, 2x+1]$

Indeed, we have

```
→   r:x;
```

(r) x

```
→   r: subst(x^2 +3, x, r);
```

(r) x^2+3

```
→   r:subst(x^2+5,x,r);
```

(r) $(x^2+5)^2+3$

```
→   r:subst((x^2+3)/4, x, r);
```

(r) $\left(\frac{(x^2+3)^2}{16}+5\right)^2+3$

```
→   r:expand(subst(2·x+1, x, r));
```

(r) $x^8+4x^7+10x^6+16x^5+29x^4+36x^3+40x^2+24x+39$

2.1 Another one (Cohen p. 179)

```
→   f2 : x^4 + 4·x^3 + 3·x^2 − 2·x + 3;
```

(f2) $x^4+4x^3+3x^2-2x+3$

```
→   pl2:polydecomp(f2, x);
```

(pl2) $[\frac{x^2+11}{4}, 2x^2-3, x+1]$

```
→      pl2a : [x^2+2·x+1, x^2−3·x+5];
```
(pl2a) $[x^2+2\ x+1,x^2-3\ x+5]$

```
→      expand(compose(pl2a,x));
```
(%o3) $x^4-6\ x^3+21\ x^2-36\ x+36$

2.2 Cohen p. 180

```
→      f3: x^4−6·x^3+21·x^2−36·x+36;
```
(f3) $x^4-6\ x^3+21\ x^2-36\ x+36$

```
→      pl3: polydecomp(f3, x);
```
(pl3) $[x^2,\frac{x^2+15}{4},2\ x-3]$

```
→      expand(compose(pl3,x));
```
(%o6) $x^4-6\ x^3+21\ x^2-36\ x+36$

```
→      pl3a : [x^2+2·x+1, x^2−3·x+5];
```
(pl3a) $[x^2+2\ x+1,x^2-3\ x+5]$

```
→      expand(compose(pl3a,x));
```
(%o13) $x^4-6\ x^3+21\ x^2-36\ x+36$

3 *Non-trivial decomposition*

```
→      f4 : x^6 − 2·x^4 − 2·x^3 + x^2 + 2·x −a + 1;
```
(f4) $x^6-2\ x^4-2\ x^3+x^2+2\ x-a+1$

```
→      pl4:polydecomp(f4, x);
```
(pl4) $[x^2-a,x^3-x-1]$

```
→      expand(compose(pl4, x));
```
(%o10) $x^6-2\ x^4-2\ x^3+x^2+2\ x-a+1$

```
→      expand(subst(x^2 − a, x, x^3 − x − 1));
```
(%o11) $x^6-3\ a\ x^4+3\ a^2\ x^2-x^2-a^3+a-1$