Solving ...

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0.1 ? Constraint and Generator Duality

0.2 What to solve

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(variables) [2]
  (?? set some variables to zero)
  (set some variables to "inputs")
  (solve for selected variables)
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0.3 Matrix Operations

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(develop with matrix operations and determinants)
(must relate to Schur complement!)
(must relate to Moore-Penrose inverse!)
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0.4 Dependencies

0.5 Extensor Operations

Tie up the stuff about exterior algebra and solving

- Solve for x(B) given Nx = 0 and x(A), and when $H: x(A) \to x(B)$ is a linear map, express det(H) in terms of minors of N.
- Solve for x(B) given $x = (k^r)N^{\perp}$ and x(A), and when $H: x(A) \rightarrow x(B)$ is a linear map, express det(H) in terms of minors of N.

Explain as two dual forms of Cramer's rule, etc.

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(develop same solution with exterior algebra) (Can we relate this solution to BBR [1]?
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0.6 Jacobi's Determinant Theorem

(Various theorems in BBR [1])

0.7 Combinatorial Term Cancellation

(my All-Minors paper [3])
(Kirchhoff [4], Maurer [5], ???)
(with extensors...my 1999-2000 work)

References

- [1] Marilena Barnabei, Andrea Brini, and Gian-Carlo Rota. On the exterior calculus of invariant theory. *Journal of Algebra*, 96:120–160, 1985.
- [2] A. Björner, Michel Las Vergnas, B. Sturmfels, N. White, and G. Ziegler. Oriented Matroids, volume 46 of Encyc. Math. and its Appl. Cambridge Univ. Press, Cambridge, 2nd edition, 1999.
- [3] Seth Chaiken. A combinatorial proof of the all minors matrix tree theorem. SIAM J. Alg. Disc. Meth., 3:319–329, 1982.
- [4] G. Kirchhoff. Über die auflösung der gleichungen, auf welshe man bei der untersuchung der linearen verteilung galvanischer ströme gefuhrt wird. *Ann. Physik Chemie*, 72:497–508, 1847. On the solution of the equations obtained from the investigation of the linear distribution of Galvanic currents, (J. B. O'Toole, tr.) *IRE Trans. Circuit Theory*, 5, 1958, pp. 238–249.
- [5] S. B. Maurer. Matrix generalizations of some theorems on trees, cycles and cocycles in graphs. SIAM J. Appl. Math., 30:143–148, 1976.