

- The bottom reviews how the Cauchy-Binet theorem proves a graph's Laplacian determinant counts spanning trees.
- In first order stiffness theory there are matrices like the Laplacian, ... and Poisson and Dirichlet type problems.
- I'd be very happy to meet collaborators to help go from my electrical network thinking with its one-dimensional voltages and currents to the higher dimensional world of this workshop, and see what problems are analogous to electrical ones.
- Briefly, (1) solution matrices can be encoded as pure exterior algebra elements, (2) As such, they package Plucker coordinate values, and (3) the solution is a Tutte function of a class of problems.
- Entries in solution matrices for nice electrical network analyses ... and also larger minors in those matrices ... turn out to be ratios of those Plucker coordinates.
- I conjecture that's true and might be interesting for stiffness problems.
- Now some details.
- We start with two matrices, N -Alpha and Beta. A column label, identified a matroid element, is either hatted or not, ... but the same letter never appears both hatted and not over the same matrix. We can therefore consider the unhatted letters to belong to a special basis in a big space generated by all matroid elements, and consider each hatted version to be the corresponding dual vector.
- We make two pure exterior algebra elements, boldface N -Alpha and Beta to represent the row spaces. From now on, every pure, that is, indecomposable element, a product of vectors and or dual vectors, we will call an "extensor".
- We make them by formally multiplying the matrix rows sums after plugging in hatted or unhatted versions of unique symbols naming the columns, and using the anticommutative exterior product. The coefficient of a given monomial is therefor a Plucker coordinate, the determinant of the submatrix with the given column labels.
- We construct function L by a bilinear pairing so L 's extensor value represents a linear endomorphism on the exterior algebra generated by free p type symbols. For this, each hatted symbol e -hat or p -hat functions as the dual of the vector symbolized e or p .
- Result 1 is that L obeys Tutte's deletion and contraction identity: BUT only for e type elements, not the p 's.

- There is also a direct sum identity ... L of a direct sum is an exterior product, not a commutative ring product.
- We have TO CAREFULLY DEFINE the extensor operations for deletion & contraction & direct sum so the signs in the L -s we combine are consistent.
- I find it interesting that, ... in order to get a Tutte function out of this, it seems require two special things.
- One is to make the Tutte function relative, ... to the set of ports P .
- We need this because the e—s disappear when they are contracted or deleted.
- I like to call the distinguished elements ports. ... We never delete or contract them.
- In applications, ports relate to variables used to specify inputs or parameters, ... like how much you push on nodes with forces or electric currents, and responses, ... like how the nodes move, or change their voltages.
- But this setup, you can interchange inputs and responses ANY WAY that the relevant matroids encode is feasible and well-posed. Electrical engineers would call L a MULTIPORT LINEAR DEVICE model.
- Two, it seems this extensor Tutte function needs to be constructed on two arguments, labelled α & β .
- Result 2 is a Cauchy-Binet kind of expansion. The bracketed minors denote sums of terms because they are extensor representations of minors without any non-port elements. The notation hides common basis expansions of many different minors our two matroids.
- I finish with some take-homes and morals, and my name, Seth Chaiken of Albany, NY.
- Three punchlines: One, ports are IM-PORT-ANT. Two, let's do matroid recursion on matrices in exterior algebra. Three, we find a Tutte function there. Thank you!