- **Here are $N_\alpha$ and $\beta$, the 2 matrices we start with. They represent two matroids. The p—s & e—s, unhatted and hatted forms, are ground set elements. But for now, think of the row spaces.**
- **From each matrix we make an exterior algebra element, denoted in boldface, to represent its row space.**
- **We construct a Tutte Function $L$ of those two elements by a kind of bilinear pairing ... $\mathbf{N}_\alpha$ bar $\mathbf{N}_\beta$.**
- **But this bilinear function has exterior algebra, not field or commutative ring values.**
- **Result 1 is that $L$, as a function of $\mathbf{N}_\alpha$ & $\mathbf{N}_\beta$ obeys Tutte's deletion and contraction identity: BUT only for $e$ type elements, not the $p$'s.**
- **I omit the direct sum identity, but its product is exterior, not commutative ring product.**
- **We have TO CAREFULLY DEFINE the exterior algebra operations for deletion & contraction & direct sum so the signs in $L$'s expansion are consistant.**
- **Here's some more background.**
- **As pure, ... or indecomposable anti-symmetric tensors, ... that is products of vectors, the boldface N–s represent linear subspaces of the big space generated by matroid ground set elements and their hatted copies.**
- **Having a basis related to ground set elements, we will in a moment consider the unhatted and hatted versions to be vector duals.**
- **So, $\mathbf{N}_\alpha$ & $\mathbf{N}_\beta$ represent points in the Grassmannian, and have Plucker coordinates.**
- **The Plucker coordinates are the maximal minors of the matrices.**
- **So, the matroid bases are encoded by which Plucker coordinates are non-zero.**
- **I multiply each column's boldface symbol with its entries.**
- **What are the boldface Ns? They are the exterior products of the row sums.**
- **Finally $L$ equals the bilinear pairing on these two.**
- **I find it interesting that, ... in order to get a Tutte function out of this, it seems require two special things.**

- **One is to make the Tutte function relative, ... to the set of ports** $P$.
- **We need that in order to get a non-trivial exterior algebra value because the e—s disappear when they are contracted or deleted.**
- **I like to call the distinguised elements ports. ... We never delete or contract them.**
- **In applications, ports relate to variables used to specify inputs or parameters, ... like how much you push on a node with force or an electric current, and responses, ... like how much every node moves, or changes its voltage.**
- **Two, it seems this exterior algebra Tutte function needs to be constructed using two arguments, labelled** $\alpha$ **&** $\beta$**.**
- **We recover the basis enumerator when the two are equal and** $P$ **is empty. It is the sum of squared determinants though, not always ones.**
- **Now for the final step.**
- **To define the final bilinear pairing function we distinguish four kinds of generators:**<br>**vector e's, vector p's and dual vector** $\widehat{\mathbf{e}}$ **hats and** $\widehat{\mathbf{p}}$ **hats.**
- **It's defined here *WITH* these rules for algebra basis monomials. Covectors in the left evaluate on vectors in the right, but reverse that ... and they behave like anticommutative scalars.**
- **I found this very recently, ... an algebra valued, not scalar valued bilinear function on tensors or their relatives. I'd appreciate any pointers. I also must ask some physicists and try the old subscipts approach.**
- **I call** $L$**-'-s construction the Cauchy-Binet form because of the expansion in result 2.**
- **I used to define** $L$ **from** $\mathbf{N}_\alpha$**, and the DUAL of** $\mathbf{N}_\beta$**.**
- **I took their exterior product and then contracted all of** $E$**...I call it the Laplace form.**
- **I finish with some take-homes and morals, and my name, Seth Chaiken of Albany, NY, which I'll leave for you to read at your own pace. Thank you!**