- **We start with two matrices, N-Alpha and Beta. The matroids they represent have the p & e symbols, each exclusively hatted or not hatted, for their ground set elements. ... For now, think of the row spaces.**
- **We make two exterior algebra elements, boldface N-Alpha and Beta to represent the row spaces.**
- **We construct function L of those N-Alpha and Beta, ... by a bilinear pairing that performs composition ... N-Alpha bar N-Beta.**
- **This bilinear operation has exterior algebra, not field or commutative ring values.**
- **Result 1 is that L obeys Tutte's deletion and contraction identity: BUT only for e type elements, not the p's.**
- **There is also a direct sum identity ... L of a direct sum is an exterior product, not a commutative ring product.**
- **We have TO CAREFULLY DEFINE the exterior algebra operations for deletion & contraction & direct sum so the signs in the L–s we combine are consistent.**
- **As pure, ... or indecomposable anti-symmetric tensors, ... that is, products of vectors, the boldface N–s represent linear subspaces of a big space generated by matroid ground set elements and their hatted versions.**
- **So with these designated bases, related to ground sets, we get duals of basis vectors. We make sure to label the N matrices so hatting is consistent with dualizing. Using duals will be detailed later.**
- **N-Alpha & Beta represent points in the Grassmannian, and have Plucker coordinates. The Plucker coordinates are the maximal minors of the matrices. So, the matroid bases are encoded by which Plucker coordinates are non-zero.**
- **To construct an extensor from a matrix, I multiply each column's boldface symbol with its entries. Boldface N is the exterior product of the row sums.**
- **Finally L equals the bilinear pairing which expresses an linear endomorphism of the exterior algebra generated by the p–s.**
- **I find it interesting that, ... in order to get a Tutte function out of this, it seems require two special things.**
- **One is to make the Tutte function relative.**
- **We need to hold back some elements from deletion or contraction because otherwise they will all disappear and the result will in a trivial algebra.**

- **The distinguished elements we don't delete or contract I like to call PORTS.**
- **In applications, ports relate to variables used to specify inputs or parameters, ... like how much you push on nodes with forces or electric currents, and responses, ... like how the nodes move, or change their voltages.**
- **But when you use L-s value, you can interchange inputs and responses ANY WAY that the relevant matroids encode is feasible and well-posed. Matroids encode which among all variables are independent. Electrical engineers would call L a MULTIPORT LINEAR DEVICE model.**
- **Two, it seems this exterior algebra Tutte function needs to be constructed on two arguments, labelled α & β.**
- **We recover the basis enumerator when the two are equal and P is empty. It is the sum of squared determinants though, not always ones.**
- **Now for the final step.**
- **To define the final bilinear pairing function we distinguish four kinds of generators:
vector e's, vector p's and dual vector $\widehat{\mathbf{e}}$ hats and $\widehat{\mathbf{p}}$ hats.**
- **It's defined here *WITH* these rules for algebra basis monomials. Dual vectors in the left evaluate on vectors in the right, but reverse that ... and they behave like anticommutative coefficients.**
- **The N–s mix vectors and dual vectors ... they represent linear mappings. The hatted and unhatted status of p–s and e–s are interchanged. N_β is like the adjoint of N_α. So ... the bilinear pairing is composition of mappings ... like matrix multiplication.**
- **Therefore ... I call this the Cauchy-Binet form. We get result 2. The common independent set expansion here hides many common basis expansions. One of them is the famous matrix tree theorem.**
- **I finish with some take-homes and morals, and my name, Seth Chaiken of Albany, NY. Three punchlines: One, ports are IM-PORT-ANT. Two, let's do matroid recursion on matrices in exterior algebra. Three, we find a Tutte function there. Thank you!**