TOO LONG AT ABT 6 min.

- **On the Mass Pike from Albany, I found a better title: Forest Counting Laplacian Minors Lift to Exterior Algebra Tutte Functions.**
- **The math at the bottom reviews how the Cauchy-Binet theorem proves a graph's Laplacian determinant counts spanning trees.**
- **Applied to minors, it shows that each minor counts forests with constraints on whether some vertices must be in different trees, forcing pairs of vertices to be in the same tree.**
- **At least ... when terms don't cancel.**
- **I see in first order stiffness theory, ... matrices like the Laplacian, ... and Poisson and Dirichlet type problems.**
- **With generic constants for electrical conductance, ... analogous to Hooke's law constants, there's no cancellation.**
- **I'd be very happy to meet collaborators to help go from my electrical network thinking with its one-dimensional voltages and currents to the higher dimensional world of this workshop.**
- **Basically, the exterior algebra Tutte function values package Plucker coordinates.**
- **Entries in solution matrices for nice electrical network analyses ... and also larger minors in those matrices ... turn out to be ratios of Plucker coordinates of our extensor function values.**
- **I conjecture that's true and might be interesting for stiffness problems.**
- **Now some details.**
- **We start with two matrices, N-Alpha and Beta. The resulting matroids have for their elements a mixture of hatted and not-hatted, ... p & e type symbols. For now, think of the row spaces.**
- **We make two exterior algebra elements, boldface N-Alpha and Beta to represent the row spaces.**
- **They and all the other exterior algebra elements we'll see today are pure or indecomposible—that is, products of vectors or dual vectors.**
- **For brevity I'll call them "extensors".**
- **Only extensors represent subspaces, that is, points in Grassmannians. They have Plucker coodinates which satisfy the Grassmann-Plucker relations.**

• I'll remind everyone that the Plucker coordinates are proportional to the maximal minors of full-row rank matrices whose rows generate the subspace. So ... non-zero Plucker coordinates encode which subsets are matroid bases.

• We construct function L of those N-Alpha and Beta, ... by a kind of composition and bilinear pairing ... N-Alpha bar N-Beta.

• This bilinear operation has extensor values, not field or commutative ring values.

• Result 1 is that L obeys Tutte's deletion and contraction identity: BUT only for e type elements, not the p's.

• There is also a direct sum identity ... L of a direct sum is an exterior product, not a commutative ring product.

• We have TO CAREFULLY DEFINE the extensor operations for deletion & contraction & direct sum so the signs in the L–s we combine are consistent.

• The boldface N extensors represent linear subspaces of a big space generated by matroid ground set elements and their hatted versions.

• So the boldface N–s are the exterior products of the row sums after plugging a column label related vector or dual vector multiplying each matrix entry.

• So with these designated ground set related bases, we get duals of basis vectors.

• Finally L equals the bilinear pairing or composition on these two. The status of vector versus dual vector is used in defining this.

• I find it interesting that, ... in order to get a Tutte function out of this, it seems require two special things.

• One is to make the Tutte function relative, ... to the set of ports P.

• We need this because the e—s disappear when they are contracted or deleted.

• I like to call the distinguised elements ports. ... We never delete or contract them.

• In applications, ports relate to variables used to specify inputs or parameters, ... like how much you push on nodes with forces or electric currents, and responses, ... like how the nodes move, or change their voltages.

- **But this setup, you can interchange inputs and responses ANY WAY that the relevant matroids encode is feasible and well-posed. Electrical engineers would call L a MULTIPORT LINEAR DEVICE model.**
- **Two, it seems this extensor Tutte function needs to be constructed on two arguments, labelled α & β.**
- **Now for the final step.**
- **We define the bilinear pairing function after distinguishing four kinds of generators:**
  vector e's, vector p's ... and dual vector $\widehat{\mathbf{e}}$–hats and $\widehat{\mathbf{p}}$–hats.
- **The pairing function is defined *WITH* rules on the slide for basis monomials. Dual vectors in the left evaluate on vectors in the right. But ... when you reverse that ... they behave like anticommutative coefficients.**
- **The N–s mix vectors and dual vectors ... they represent linear mappings. Between Alpha and Bets, the hatted and unhatted status of p–s and e–s are interchanged. $\mathbf{N}_\beta$ is like the adjoint of $\mathbf{N}_\alpha$. So ... the bilinear pairing is composition of mappings ... like matrix multiplication.**
- **Result 2 is a Cauchy-Binet kind of expansion. The bracketed minors denote sums of terms because they are extensor representations of minors without any non-port elements. The notation hides common basis expansions of many different minors our two matroids.**
- **I finish with some take-homes and morals, and my name, Seth Chaiken of Albany, NY.**
- **Three punchlines: One, ports are IM-PORT-ANT. Two, let's do matroid recursion on matrices in exterior algebra. Three, we find a Tutte function there. Thank you!**