# PORTED, RESTRICTED, SET POINTED PARAMETRIZED TUTTE FUNCTIONS

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Abstract.

## Introduction

The Tutte polynomial is a well-known invariant of graphs and matroids. Several related polynomials have applications that involve graphs whose edges each bear one or more weights and/or some of whose edges (or vertices) are distinguished. In this context, the additive parametrized Tutte identity applied to graph $M$ with edge set $S(M)$

$$T(M) = x_e T(M/e) + y_e T(M \setminus e) \text{ if } e \in S(M) \text{ is neither a loop nor a coloop in } M \tag{1}$$

is assumed only when $e$ is *not* a distinguished edge. (A distinguished vertex remains an identifyable element within a set of collapsed original vertices during each contraction of edges.) The applications, questions and results have natural generalizations to matroids. So, for now, we will use matroid language and refer to graphs via their graphic matroids.

The purpose of this paper is to extend, to matroids and graphs a set of distinguished elements $P$, known results and methods pertaining to solutions to 1 together with the multiplicative identities

(2)
$T(M) = X_e T(M/e)$ if $e \in S(M)$ is a coloop$T(M) = Y_e T(M \setminus e)$ if $e \in S(M)$ is a loop$T(M_1 \oplus M_2) = T(M_1)$

when these identities are restricted to apply only when $e \notin P$.

When all the $x_e, y_e$ parameters are 0 and, independently of $e$, each coloop multiplier $X_e = z$ and each loop multiplier $Y_e = t$, and $T(\emptyset) = 1$, the solutions have been characterized by Las Vergnas .... . The theory is ...

Interesting complications arise with other the parameter and multiplier values. We extend known results about this to the case when $P \neq \emptyset$.

When the Tutte identities are parametrized, it is especially important to carefully distinguish between solutions (functions of matroids that satisfy all the relevent identities) and formal polynomials that result from using a subset of the identities to try to calculate $T(M)$ for one $M$. Unlike in the non-parametrized case, even without distinguished elements, different formal polynomials result from different calculations. These formal polynomials are in the parameters, values for $T$ of loops, coloops and the initial value $T(\emptyset)$. Additional polynomial identities in these values must be satisfied in order for the values together with the to have a solution. A solution of course means that $T(M')$ for all the (matroid) minors of $M$ satisfy all the Tutte identities; equivalently, all calculation sequences give the same result. When the parameters are all 1, the value for each loop is $t$, the value for each

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coloop is $z$ and the initial value $T(\emptyset) = 1$, it is well-known that the identities have a unique solution which is a polynomial in $z$ and $t$. That is the famous Tutte polynomial. It is well-known as a universal Tutte invariant $U(M)(t, z)$–all Tutte invariants $F(M)$ are obtained as $F(M) = U(M)(F(U_{0,1}), F(U_{1,1})$, where $U_{0,1}$ is the loop matroid on any ground set $\{e\}$ and $U_{1,1}$ is the coloop matroid. Since additional conditions involving parameters, loop/coloop, and empty matroid values are needed for a solution, we will avoid the term "Tutte polynomial" for now and use the term Tutte function to denote any solution to the parametrized Tutte equations.

Several recent papers have demonstrated the essential equivalences between most different, seemingly incompatible forms for solutions of parametrized Tutte equations (which have been called "Tutte polynomials"). Their conclusions are that, like for Tutte invariants, universal solutions can be given.

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