New TutteEx Paper–for Math

Theme:Electrical Networks and the Matrix Tree Theorem are the main historical and current context of this paper.

Electrical networks have a long history of (giving, providing, donating, ???) motivations for graph and combinatorial theory. A graphs Laplacian matrix L appears in the equation $L\langle\backslash\text{phi}\rangle = I$ that can be solved for the electrical node potential $\langle\backslash\text{phi}\rangle$ in terms of the electrical currents I flowing into the nodes. [Matrix Tree Theorem and things like Maxwell's rule]

Suppose we fix the potential $\phi(\gamma) = 0$ at node γ; impose unit current $I(\langle\backslash\text{alpha}\rangle) = 1$ into node $\langle\backslash\text{alpha}\rangle$ and insulate the other nodes so $I(\langle\backslash\text{nu}\rangle) = 0$ for all $\langle\backslash\text{nu}\rangle\langle\backslash\text{not}\backslash\text{in}\rangle\langle\backslash\{\rangle\langle\backslash\text{gamma}\rangle, \langle\backslash\text{alpha}\rangle\langle\backslash\}\rangle$. Let us assume $L\phi = I$ can be solved with these conditions. The resulting difference of potential $\langle\backslash\text{phi}\rangle(\langle\backslash\text{alpha}\rangle) - \langle\backslash\text{phi}\rangle(\langle\backslash\text{gamma}\rangle) = \langle\backslash\text{phi}\rangle(\langle\backslash\text{alpha}\rangle)$ is called the equivalent resistance between $\langle\backslash\text{alpha}\rangle$ and $\langle\backslash\textit{gamma}\rangle$. A routine application of Cramer's rule to a submatrix of L together with the matrix tree theorem appied to the network N and to N with $\langle\backslash\text{alpha}\rangle$ and $\langle\backslash\text{gamma}\rangle$ identified will prove Maxwell's Rule (for two nodes):

Maxwells Rule: The equivalent resistance equals

$$\frac{\sum_{F \text{ is } a \text{ spanning tree in } N/(\langle\backslash\text{alpha}\rangle, \langle\backslash\text{gamma}\rangle)} g_F}{\sum_{T \text{ is } a \text{ spanning tree in } N} g_T}$$

Here, $g_F = \prod g_e$ for edges $e\langle\backslash\text{in}\rangle F$. Maxwell's rule in general expresses the potential difference between any two nodes $\langle\backslash\textit{beta}\rangle$, $\langle\backslash\textit{delta}\rangle$ under these conditions. The numerator is a sum of terms representing spanning forests in N with exactly two trees, where $\langle\backslash\text{alpha}\rangle$ and $\langle\backslash\textit{gamma}\rangle$ are in separate trees and also $\langle\backslash\text{beta}\rangle$ and $\langle\backslash\textit{delta}\rangle$ are in separate trees. So, there are two cases: (+) $\langle\backslash\{\rangle\langle\backslash\text{alpha}\rangle, \langle\backslash\text{beta}\rangle\langle\backslash\}\rangle$ are in one tree and $\langle\backslash\{\backslash\textit{gamma}\rangle,\langle\backslash\textit{delta}\rangle\langle\backslash\}\rangle$ in the other tree and (-) $\langle\backslash\{\rangle\langle\backslash\textit{alpha},\backslash\textit{delta}\rangle\langle\backslash\}\rangle$ are in one tree and $\langle\backslash\{\rangle\langle\backslash\textit{beta}\rangle,\langle\backslash\textit{gamma}\rangle\langle\backslash\}\rangle$ are in the other tree. Terms of case (+) appear with positive sign and terms of case (-) apear with negative sign. The denominator is the spanning tree enumerator as before.

Proofs have been given ......

Our contribution is to characterize the signs in Maxwell's rule within the theory of oriented matroids. This requires reformulation of the problem which might be of intependant interest. An outcome is a generalization of Maxwell's rule ....

Theme: My discovery is interesting because it domonstrates a new relationship between two interesting topics.

The spanning tree enumerator is of course the special case for graphic matroids of the basis enumerator, which is one member of the important family of Tutte matroid invariants.

The [SUPER] consequence of our result is that it is not a coincidence that electrical network solutions are ratios of minors of Laplacians and that each minor satisfies the Tutte equations. We have found that when the solution set to a suitably formulated system of electrical network equations is expressed as an extensor, the function that maps the network's graphic matroid to this extensor satisfies the Tutte equations. Of course, this requires that those equations [Tutte?] be extended [rewritten? formulated?] from commutative rings to exterior algebra. This is because the [wedge] exterior product, or wedge, is anti-commutative. This paper presents the technicalities that seem to be necessary [to make this program work?] for this generaliztion to work.

Theme: Understanding my discovery requires that each topic be looked at in a somewhat unfamiliar way to most readers of this journal.

Theme: What is the definition of $M_E(N)$ with the minimum of theory?

Theme: What is the main point of the TutteEx paper, informed by the refs report?

We can formuate a [Laplace/Dirichlet problem, CHECK] so the [solution, kernel, Green's function CHECK] satisfies the [anticommutive, exterior algebra?? these are new to this paper] Tutte equations.

When we express the [??] of the [??] problem in exterior algebra, the [??] satisfies the Tutte equations. Our formulation requires that some elements are distinguished as ports: The ports determine the coordinate system for the [solution??] There is one instance of the Tutte eq. for each of the remaining elements: For each e not a port, $M(N) = r_e M(N$⟨\setminus⟩$e) + g_e M(N/e)$. We believe that this [??] underscores the importance of distinguishing port elements [the ports] [to obtain results of this kind?]

Our other line of work investingates the [consequences of] distinguishing of port elements within matroid theory. There is less novelty. This idea has been studied before, but not in the electrical network context [by mathematicians, but by EEs] We present the fairly straightforward generalizations of Tutte function theory. We can then see how our algebraic result fits into the discrete theory.