- **Here N_α and β, the 2 matrices we start with. They represent two matroids. The p—s & e—s are ground set elements. But think of the row spaces for now.**
- **We make two exterior algebra elements to represent those row spaces: boldface $\mathbf{N}_\alpha$ & $\mathbf{N}_\beta$.**
- **We construct a Tutte Function L of those two things by a kind of bilinear pairing $\mathbf{N}_\alpha$ bar $\mathbf{N}_\beta$:**
- **But this bilinear function has exterior algebraic, not field or ring values.**
- **Result 1 is that L, as a function of pair $\mathbf{N}_\alpha$ & $\mathbf{N}_\beta$ obeys result ONE, Tutte's deletion and contraction identity: BUT only for e type elements, not the p's.**
- **I omit the direct sum identity, but its product is exterior, not commutative ring product.**
- **We have TO CAREFULLY DEFINE the exterior algebra operations for deletion & contraction & direct sum so the signs in L's expansion are consistant.**
- **Here's some more background.**
- **As pure, ... or indecomposabale anti-symmetric tensors, ... that is products of vectors, the boldface Ns represent linear subspaces over the basis comprised of the matroids' ground sets.**
- **The ground elements correspond to the boldface p & e vectors, ... or dual vectors when they have hats.**
- **So, $\mathbf{N}_\alpha$ & $\mathbf{N}_\beta$ represent points in the Grassmannian, and have Plucker coordinates.**
- **The Plucker coordinates are the maximal minors of the matrices.**
- **So, the matroid bases are encoded by which Plucker coordinates are non-zero.**
- **I multiply a column's boldface symbol with every numeric entry.**
- **What are the boldface Ns? They are the exterior products of the row sums.**
- **Finally to get L I use the bilinear pairing on the two Ns.**

- **I find it interesting that, ... in order to get a Tutte function out of this, it seems require two special things.**
- **One is to make the Tutte function relative, ... to the set of ports P.**
- **We need that in order to get a non-trivial exterior algebra for the Tutte function values to live in.**
- **The e—s disappear when they are contracted or deleted.**
- **I like to call the distinguised elements ports. ... We never delete or contract them.**
- **Those port elements in the exterior algebra are vectors, and the hatted ones are covectors, ie., 1-forms or dual vectors.**
- **In applications, ports relate to variables used to specify inputs or parameters, ... like how much you push on a node with force or an electric current, and responses, ... like how much every node moves, or change its voltage.**
- **Two, it seems the Tutte function needs pairs, $\mathbf{N}_\alpha$ ... $\mathbf{N}_\beta$.**
- **We recover the basis enumerator with P empty and $\mathbf{N}_\alpha$ ... $\mathbf{N}_\beta$.**
- **The bilinear pairing in the final step to construct L ... is defined after distinguishing four kinds of generators:**
vector e's, vector p's and dual vector $\widehat{\mathbf{e}}$ hats and $\widehat{\mathbf{p}}$ hats.
- **It's defined on the slide *WITH* these rules for algebra basis monomials.**
- **I found this very recently, ... a pairing that should return algebra values instead of ring values. I'd appreciate pointers if anyone recognizes something like it.**
- **I call L'—s construction the Cauchy-Binet form because of the expansion in result 2.**
- **A while ago I defined L from $\mathbf{N}_\alpha$, and the DUAL of $\mathbf{N}_\beta$.**
- **I took their exterior product and then contracted all of E...and called it the Laplace form.**
- **I'll finish with some take-homes and morals, and my name, Seth Chaiken, which I'll leave for you to read at your own pace. Thank you!**