

# Solving ...

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## 0.1 ? Constraint and Generator Duality

## 0.2 What to solve

(variables) [2]

(?? set some variables to zero)

(set some variables to “inputs”)

(solve for selected variables)

## 0.3 Matrix Operations

(develop with matrix operations and determinants)

(must relate to Schur complement!)

(must relate to Moore-Penrose inverse!)

## 0.4 Dependencies

## 0.5 Extensor Operations

Tie up the stuff about exterior algebra and solving

- Solve for  $x(B)$  given  $Nx = 0$  and  $x(A)$ , and when  $H : x(A) \rightarrow x(B)$  is a linear map, express  $\det(H)$  in terms of minors of  $N$ .
- Solve for  $x(B)$  given  $x = (k^r)N^\perp$  and  $x(A)$ , and when  $H : x(A) \rightarrow x(B)$  is a linear map, express  $\det(H)$  in terms of minors of  $N$ .

Explain as two dual forms of Cramer’s rule, etc.

(develop same solution with exterior algebra)

(Can we relate this solution to BBR [1]?)

## 0.6 Jacobi’s Determinant Theorem

(Various theorems in BBR [1])

## 0.7 Combinatorial Term Cancellation

(my All-Minors paper [3] )  
(Kirchhoff [4], Maurer [5], ???)  
(with extensors...my 1999-2000 work)

## References

- [1] Marilena Barnabei, Andrea Brini, and Gian-Carlo Rota. On the exterior calculus of invariant theory. *Journal of Algebra*, 96:120–160, 1985.
- [2] A. Björner, Michel Las Vergnas, B. Sturmfels, N. White, and G. Ziegler. *Oriented Matroids*, volume 46 of *Encyc. Math. and its Appl.* Cambridge Univ. Press, Cambridge, 2nd edition, 1999.
- [3] Seth Chaiken. A combinatorial proof of the all minors matrix tree theorem. *SIAM J. Alg. Disc. Meth.*, 3:319–329, 1982.
- [4] G. Kirchhoff. Über die auflösung der gleichungen, auf welshe man bei der untersuchung der linearen verteilung galvanischer ströme gefuhrt wird. *Ann. Physik Chemie*, 72:497–508, 1847. On the solution of the equations obtained from the investigation of the linear distribution of Galvanic currents, (J. B. O’Toole, tr.) *IRE Trans. Circuit Theory*, 5, 1958, pp. 238–249.
- [5] S. B. Maurer. Matrix generalizations of some theorems on trees, cycles and cocycles in graphs. *SIAM J. Appl. Math.*, 30:143–148, 1976.