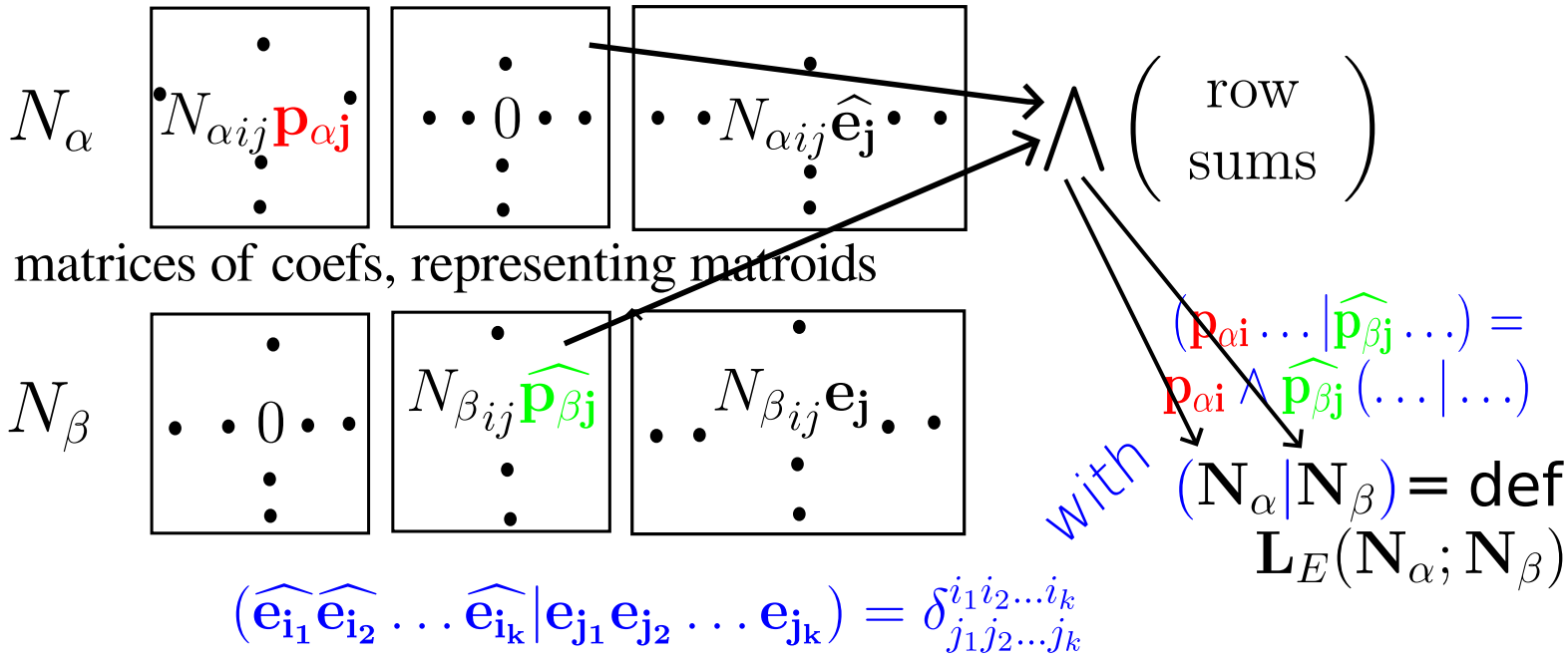
$\mathbf{p}_{\alpha\mathbf{j}}, \mathbf{p}_{\beta\mathbf{j}}, \mathbf{e}_\mathbf{j}$: free generators, space basis elements



$$(\widehat{\mathbf{e}_{\mathbf{i}_1}}\widehat{\mathbf{e}_{\mathbf{i}_2}} \dots \widehat{\mathbf{e}_{\mathbf{i}_k}} | \mathbf{e}_{\mathbf{j}_1}\mathbf{e}_{\mathbf{j}_2} \dots \mathbf{e}_{\mathbf{j}_k}) = \delta^{i_1 i_2 \dots i_k}_{j_1 j_2 \dots j_k}$$

$$\textcircled{1} = \mathbf{L}_{E\setminus e}(\mathbf{N}_\alpha \setminus \mathbf{e}; \mathbf{N}_\beta \setminus \mathbf{e}) + \mathbf{L}_{E\setminus e}(\mathbf{N}_\alpha/\mathbf{e}; \mathbf{N}_\beta/\mathbf{e})$$
$$\in \bigwedge P_\alpha \cup \widehat{P_\beta} : \text{ Exterior Algebra (anti-comm!)}$$

$$\textcircled{2} = \sum_{F\subseteq E}([\mathbf{N}_\alpha/F|P_\alpha] \quad | \quad [\mathbf{N}_\beta/F|\widehat{P_\beta}])$$
$$= \sum_{F\subseteq E}[\mathbf{N}_\alpha/F|P_\alpha] \wedge [\mathbf{N}_\beta/F|\widehat{P_\beta}]$$

Like $|\text{Graph Laplacian}| = |I\ I^t| = \sum_{F\subseteq E} |I(F)|^2$
$= \sum_{T:\text{span. tree}} 1$ (Cauchy-Binet expansion)