Restricted or Ported Tutte Decomposion and Analogs of All-Minors Laplacian Expansions

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Abstract

The all-minors matrix tree theorem's expansions of a graph's Laplacian minors together comprise one special case of an exterior algebra valued Tutte-type function. For graphs it is constructed after gluing a pointed 1-dim star whose edges (called ("ports") identify the original vertices. The function's value has all the Laplacian's minors for its Plücker coordinates. Each coordinate satisfies Tutte-Grothendieck equations when deletion/contraction is restricted to non-port elements (therefore, so do the exterior algebra values.) How the setup, including homogeneous element weights $(g_e : r_e)$ for each non-port element e, applies to Kirchhoff's electrical networks, gain graphs studied by Reff, Zaslavsky and others, linear matroids and Laplacians of simplicial complexes is described. It also extends to a single exterior algebra expansion that encapsulates the Cauchy-Binet expansions of all the minors of any non-symmetric weighted Laplacian-type matrix $A\,\mathrm{diag}(g_e)B^t$. Thus it applies to digraph directed spanning forest enumeration and solving for the behavior of linear electronic circuits with ideal operational amplifiers.