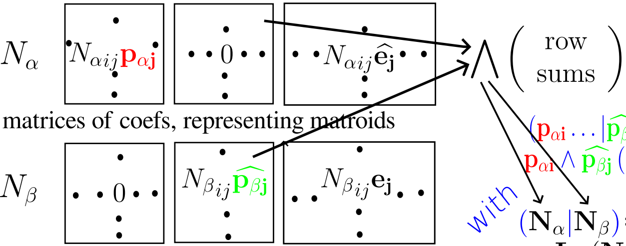
$\mathbf{p}_{\alpha \mathbf{j}}, \mathbf{p}_{\beta \mathbf{j}}, \mathbf{e}_{\mathbf{j}}$: free generators, space basis elements



$(\mathbf{p}_{\alpha \mathbf{i}} \ldots | \widehat{\mathbf{p}_{\beta \mathbf{j}}} \ldots) = \mathbf{p}_{\alpha \mathbf{i}} \wedge \widehat{\mathbf{p}_{\beta \mathbf{j}}} (\ldots | \ldots)$

$(\mathbf{N}_\alpha | \mathbf{N}_\beta) =$ def $\mathbf{L}_E(\mathbf{N}_\alpha; \mathbf{N}_\beta)$

$(\widehat{\mathbf{e}_{\mathbf{i_1}}} \widehat{\mathbf{e}_{\mathbf{i_2}}} \ldots \widehat{\mathbf{e}_{\mathbf{i_k}}} | \mathbf{e}_{\mathbf{j_1}} \mathbf{e}_{\mathbf{j_2}} \ldots \mathbf{e}_{\mathbf{j_k}}) = \delta^{i_1 i_2 \ldots i_k}_{j_1 j_2 \ldots j_k}$

1 $= \mathbf{L}_{E \setminus e}(\mathbf{N}_\alpha \setminus \mathbf{e}; \mathbf{N}_\beta \setminus \mathbf{e}) + \mathbf{L}_{E \setminus e}(\mathbf{N}_\alpha / \mathbf{e}; \mathbf{N}_\beta / \mathbf{e})$

$\in \bigwedge P_\alpha \cup \widehat{P_\beta}$: Exterior Algebra (anti-comm!)

2 $= \sum_{F \subseteq E} ([\mathbf{N}_\alpha / F | P_\alpha] \quad | \quad [\mathbf{N}_\beta / F | \widehat{P_\beta}])$

$= \sum_{F \subseteq E} [\mathbf{N}_\alpha / F | P_\alpha] \wedge [\mathbf{N}_\beta / F | \widehat{P_\beta}]$

Like |Graph Laplacian| $= |I \; I^t| = \sum_{F \subseteq E} |I(F)|^2$

$= \sum_{T:\text{span. tree}} 1$ (Cauchy-Binet expansion)

1. Distinguish matroid element "ports" associated with electric or elastic system parameter and solution variables of interest. (All vars are paired: (voltage, current), (force, displacement), etc. One gets a pair of submodels with dual matroids in elementary situations; not duals otherwise.)
2. Exterior algebra forms of deletion and contraction of a non-port yield a pair of simpler systems.
3. Cancelling non-port elements with a kind of bilinear pairing yields the parameter/solution variables of interest relation, in the form of an **exterior algebra valued function** of systems, that **is a Tutte function** (when the minor and direct sum operations are sign-consistent).

With the suitable incidence matrix form, we get the all-minors matrix tree theorem; but all the minors are packed into **one exterior algebra object** that is a Tutte function of graphs.
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