

Abstract

Michael Levin is a developmental biology and regenerative medicine scientist who discovered that various kinds of living material can be induced, solely via bioelectric perturbations, to develop persistent forms or behaviors never occurring in nature. He proposed a general viewpoint, “beyond teleology”, where mysterious so-called “emergent” complex patterns or forms are, for scientists, more productively hypothesized to be caused by their pre-existence rather than being outcomes of detailed step-by-step processes. Simpler objects select attractors from a huge, non-physical, rationally and hierarchically structured “latent Platonic space”. Levin thus extends, to evolution, biology and cognitive sciences, the idea that mathematics is discovered, not invented. Among Levin’s labs’ extensive research is that recent simulations produced life-like behavior in biology motivated extensions of game theory’s prisoner’s dilemma. I will survey some of his ideas and results, and suggest that they might lead us to helpful preliminary questions in mathematics.

Surveying Michael Levin's Latent Platonic Spaces

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Mathematical Objects from Biology?

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 <https://chaikens.github.io>

Main sources

 Levin's Lab <https://www.drmichaellevin.org/>

 "personal, broader complement" blog
<https://thoughtforms.life/>

November 18, 2025

Spaces in Mathematics (my remarks); Structure and Properties

Structures

Open sets \mathcal{O} of topological space (U, \mathcal{O}) , with **defining axioms**.

ODE System, with smoothness requirements on the functions.

Vector field — a geometric interpretation of this structure.

Vector field on a manifold — structure on a structure.

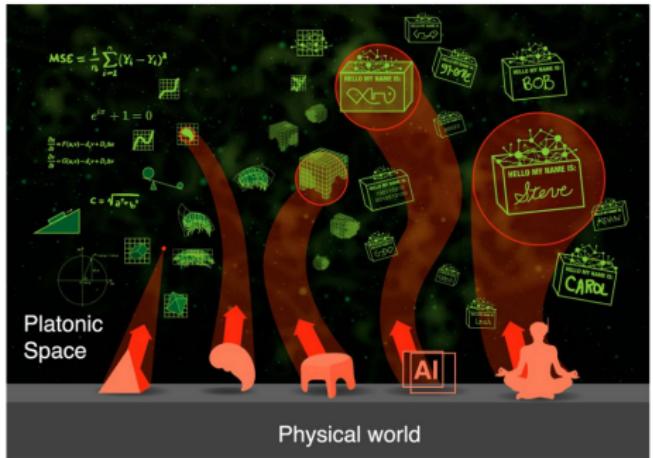
Trained Neural Net \mathfrak{N} — (given its architecture).

Properties

- ▶ **from axioms** Existence and Uniqueness Theorems.
- ▶ **of an instance** — Each critical point's orbit is a stable attractor.
 - \mathfrak{N} 's behavior func: Input \mapsto computed distribution

Care, Rigor. We often say “category” not “kind of space”

Distinguish **types of spaces** eg. category of topological spaces Top, from **spaces** $S \in \text{Obj}(\text{Top})$.



Michael Levin proposes that forms or patterns of even artificial living things **pre-exist** in “latent Platonic space.” A physical thing is an “affordance” or “pointer” to a form, so life was **caused** by more than physics, chemistry and evolution. Platonic spaces are “rationally structured”; science explores / nature exploits their structure and contents.

Although (critics say) physics, chemistry, etc. might explain observations, Levin declares \$ empirical (**not philosophical**) benefits: this “conceptual framework” was crucial for his surprising discoveries. I suggest Levin’s intriguing view of his (≈ 30 year) and other biologists’ work may lead to preliminary **mathematical questions** by perhaps, noticing **life-like** mathematical phenomena.

[picture: [Ingressing Minds: Causal Patterns Beyond Genetics and Environment in Natural, Synthetic, and Hybrid Embodiments, 2025](#)]

Levin's 22 min talk–Selections

Latent Space*

unobserved regularities that underlie observations

The talk motivates:

“ordered structured space that contains not only mathematical truths but also high order patterns that we recognize as kinds of minds”

 Bioelectricity: a bridge between physics and cognition, by way of biology” by Michael Levin Oct 7, 2025

*LevinBot’s definition, cites machine learning papers

“Latent space refers to a conceptual or mathematical space that describes or hypothesizes underlying regularities that are not directly observable in the data stream but need to be inferred. For example, in object perception, the visible input stream (e.g., stimulation of the retina) can be considered a function of latent causes, such as the identity of the object and its location or pose.”

"Cognition all the way down" (2:00) explained well

 (Levin and Dennett, 2020, Aeon) explains how anthropomorphic thinking really helped scientists and engineers, after 1940-1950s cybernetics theories. Also *Bacteria to Bach ...*, Dennett (2017).

Keywords

cognition, (scaled) goals, execution, abstract mental goals, intentions, mind-matter interaction, transduction, interface, embodied minds, remapping memories, agents and agental material, computational matter, agenda, avoiding micromanagement, intelligence, space navigation, anatomical possibilities, perception-action loop, layering, multi-scale problem solving competency in environments, regeneration of forms/patterns, goal seeking (stopping), global plan, intents, fixing error, physiological software vs genetically specified hardware, programmability and reprogramming, computational networks, pattern memories, synthetic life forms, behavior and genetic expression patterns, latent Platonic space, ingestion of patterns.

Levin's speculative hypotheses (my take)

1 Agent: Something with goals; can solve problems in multiple ways upon sensing an environment.

Scientists would be wise to consider **all** objects in our 4×10^9 year evolution to have features and behavior characteristic of **goal-seeking agents with intelligent cognitive abilities**. (Degrees of ability? “physical \oplus cognitive (or care) [possible goals] light-cones”.)

2 This enables objects to navigate **latent Platonic spaces**, taking advantage of existing space structure and properties including “free lunches.”

Levin's approach (paraphrased):

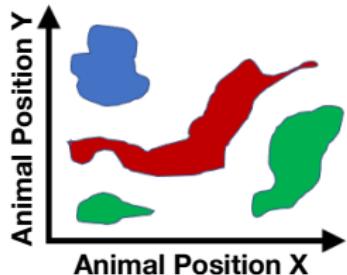
Do experiments and observe trajectory changes — **to discover space structure and how trajectory control machinery works**.

The trajectories are **not** merely and mysteriously just “emergent”— There is something to discover.

Space Examples: Need additional relevant structures?

Life Has Embodiment Outside of Familiar 3D space:

3D Space (behavior)

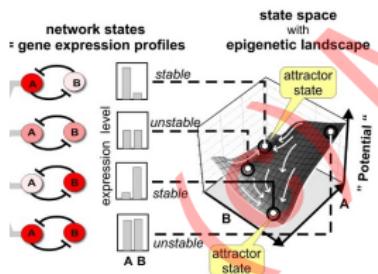


perception-action loop
can happen in other
spaces!

-> unconventional
embodiment for
AI's

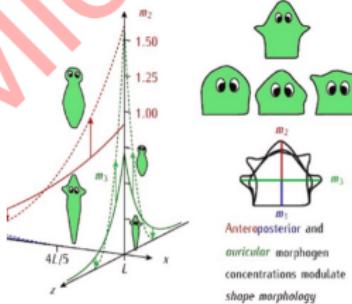
Transcriptional Space

Huang, S.; Emberg, I.; Kaufman, S., *Semin Cell Dev Biol* 2009, 20, (7), 869-76.



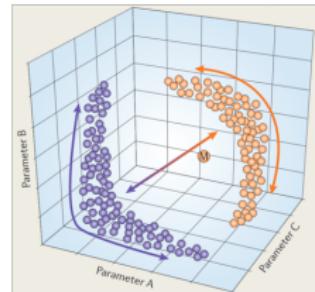
Morphospace

Cervera, J., Levin, M., and Mafe, S., (2021), *BioSystems*, 209:104511



Physiological Space

Marder, E., & Goaillard, J. M. (2006). Variability, compensation and homeostasis in neuron and network function. *Nat Rev Neurosci*, 7(7), 563-574.



Levin's Free Lunch Examples

First: Triangle most fit with 60° angles. 2nd:

1. Voltage gated ion channels “embody” Boolean logic.
2. So the universal property of the NAND gate did not have to be evolved.

Computer Sci (Math) Question

Identify properties (probabilistic?) of observed boolean (partial?) functions and circuits. (Much is known!?)

More generally...

I think we would say the same thing as about the wide range of biological control and reaction networks and fields: They embody differential equations/dynamical systems; a **huge** collection of mathematical properties is known.

Prospecting for Mathematical Gold

What is the mathematical nature of the “affordances or interfaces”? Look at biology results and conceptualize the difference between a space of transversals from its ambient space.

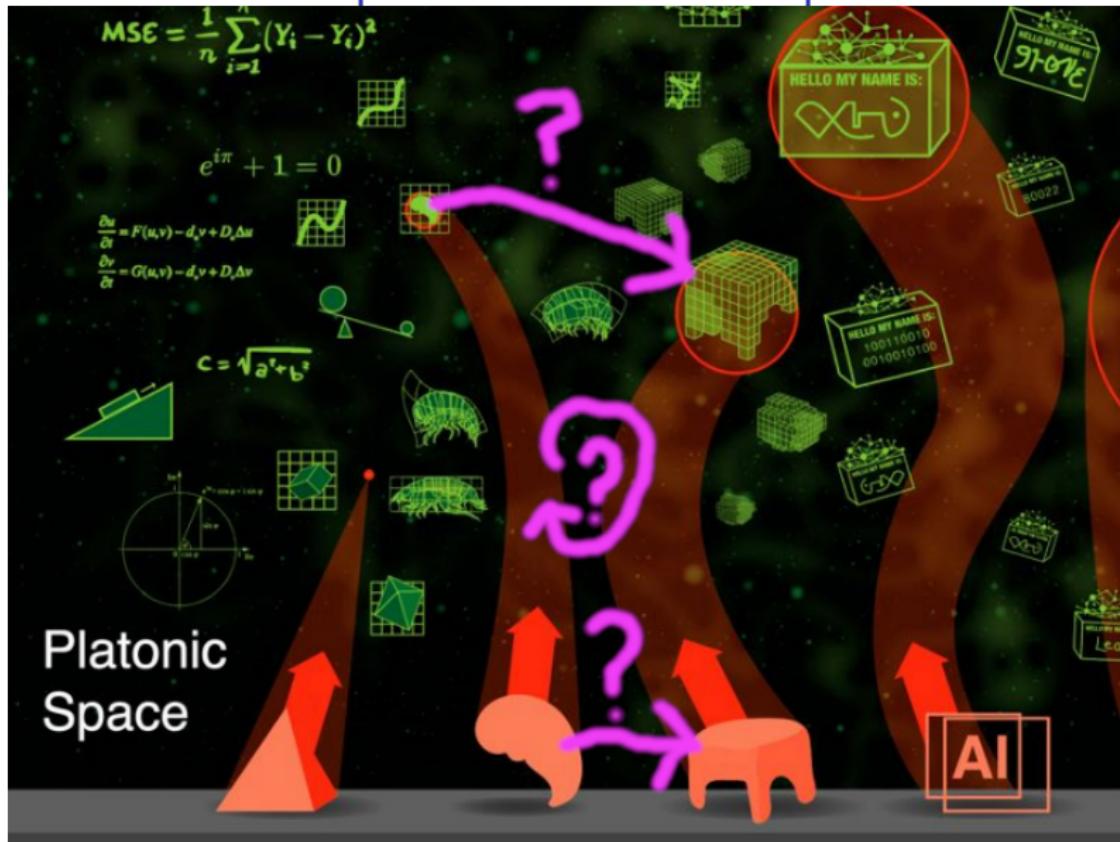
My modest proposal

Ask sharp mathematical questions! Let's try to discover something by conjecturing rigorous, 20th century math models for Levin's Platonic spaces view of some biologists' working concept details.

Higher Category Theory

Like definitions of homotopy composition, morphism composition (modeling some kind of transformation, or environment change) might not be strictly associative, but associative up to a 2-morphism of 1-morphisms (or higher).

Put the Functorial pattern into Levin's picture



Physical world

Lauren Ross described it well.

“How do we clarify distinction between explanation ... [and] description, classification, prediction”

“with causal explanation, main goal is control”

(🔗 citation: Lecture (8) Explanation in Biology: principles and pragmatics - Copied from thoughtforms.life, Oct. 2025)

(🔗 Lauren Ross, Logic and Philosophy of Science Department University of California, Irvine)

“causal explanation” in Math Language!

Understand two categories Afford, Plat and $F : \text{Afford} \rightarrow \text{Plat}$ well enough to prove F is a functor.

They might need to be Higher Categories

if morphism composition is not strictly associative.

Rest of Talk

1. 2025 Levin Lab grid of player Prisoner's Dilemma simulation.
 2. 2025 Life-like patterns from sorting algorithms.
 3. A take-away.
 4. Speculative ideas for future dreaming (work?).
 5. Pointers to more information.
- (? 2025 a simulation of programming morphogenesis.)

Extending Iterated, Spatialized Prisoner's Dilemma to Understand Multicellularity: Game Theory With Self-Scaling Players, Shreesha, Pigozzi, Goldstein and Levin, IEEE Transactions On Molecular, Biological, And Multi-Scale Communications, June 2025.

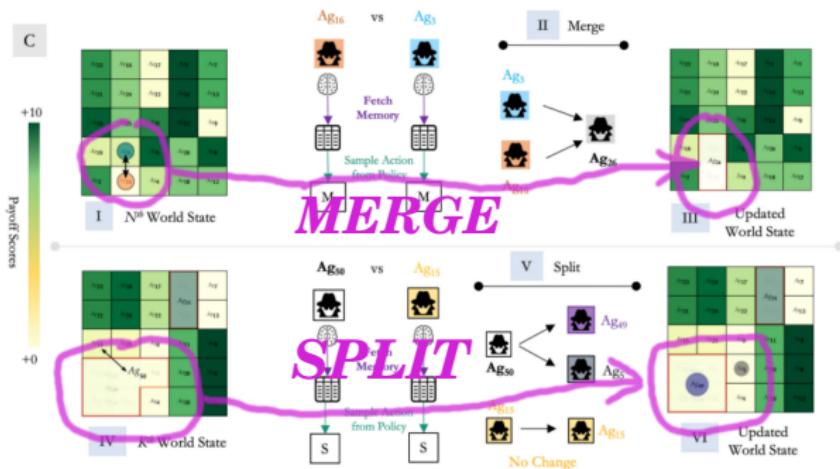
Simulations

- ▶ Prisoner's dilemma game extensions and reinforcement learning of strategy
- ▶ Model cell differentiation in 2-d tissues

Life-like Results

- ▶ Intermediate patterns like in morphogenesis
- ▶ $1 < |\text{memory}| \leq 4 \implies$ convergence to single cell type
- ▶ Punctuated equilibrium dynamics
- ▶ “Smaller sized individuals around larger sized agents”
emergence of “skin” was **not programmed**

Extensions: Agent grid, Adj. M/S, fitness selection



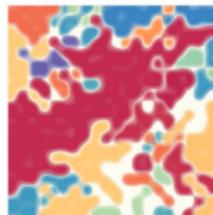
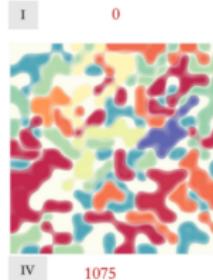
Player X \ Player Y	Cooperate [C]	Defect [D]	Merge [M]	Split [S]
Cooperate [C]	8.0	0.0	8.0	0.0
Defect [D]	10.0	5.0	10.0	0.0
Merge [M]	8.0	0.0	0.0	0.0
Split [S]	0.0	0.0	0.0	0.0

Prisoner's dilemma Payoff Table

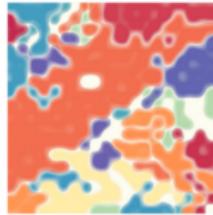
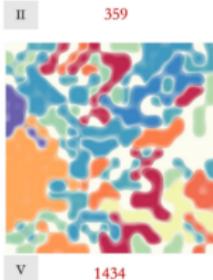
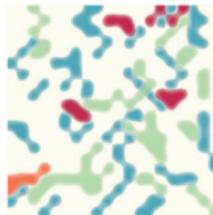
Merge[M] and Split[S] extensions

Result: Patterns like Life's Morphogenesis

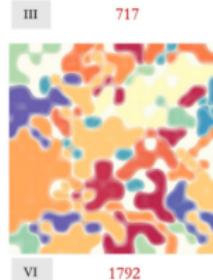
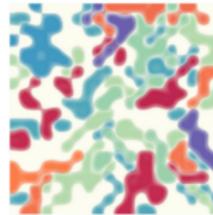
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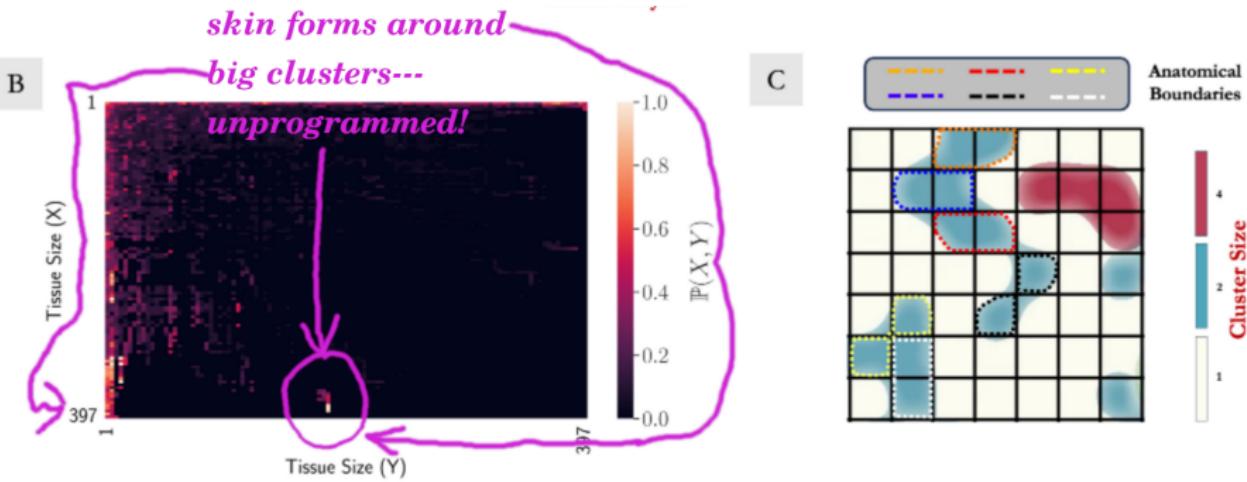


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Result: Another Life-like Pattern



Prisoner's Dilemma summary

- ▶ The cooperative common behavior of cells in an organ, and also biological form, emerges when simulating a natural biology motivated extension in game theory where agents can split and merge.
- ▶ Merely rewarding long-run cooperation, supporting limited memory based strategies, and some fitness selection \implies life-like development patterns and results (organs with boundary membranes), **without anything to explicitly program or reward them in the simulation rules.**

Complex Process from very simple local interaction

 Zhang, Goldstein and Levin (2025)

Classical sorting algorithms as a model of morphogenesis:
Self-sorting arrays reveal unexpected competencies in a minimal
model of basal intelligence.

- ▶ Runs of randomized parallel sorting algorithms
- ▶ Keys' code was faulty
- ▶ Multiple variants of keys' code

Observations

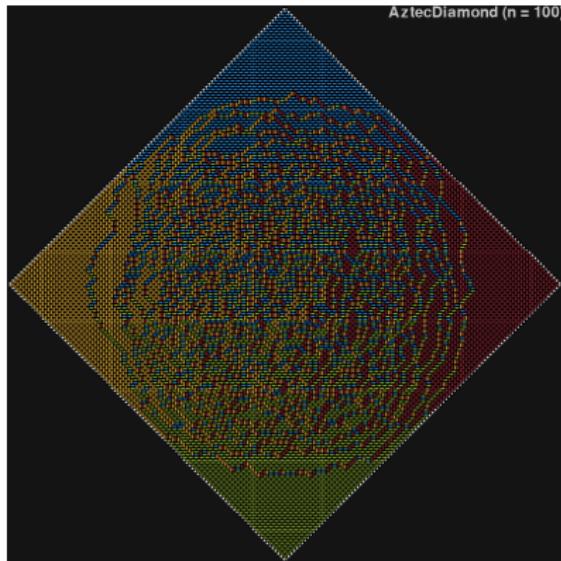
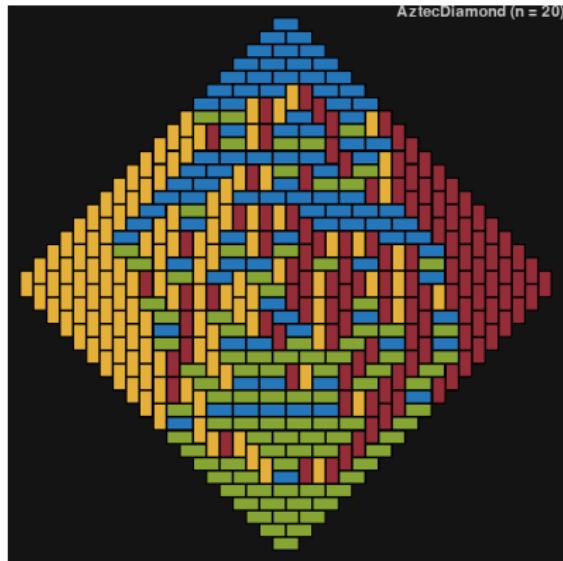
Problem solving & pattern formation behavior characteristic of life.

Conclusion

Do seek phenomena from behavioral, information & bio. sciences
in simpler systems

Similar pictures in statistical & dynamical combinatorics of James
Propp, Jessica Striker, & others in Dynamical Algebraic
Combinatorics!

Tilings of Aztec Diamond with Dominoes



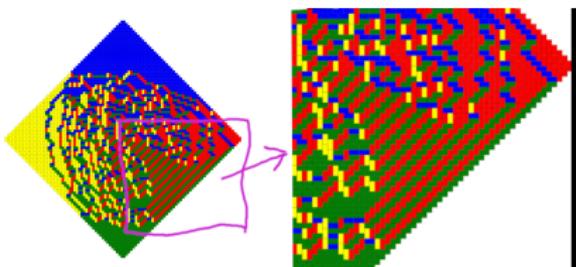
Theorem (Jockusch, Propp and Shor 1995 4 KINDS?!)

(the Arctic Circle Theorem): Fix $\epsilon > 0$. Then for all sufficiently large n , all but an ϵ fraction of the domino tilings of the Aztec diamond of order n will have a temperate zone whose boundary stays uniformly within distance ϵn of the inscribed circle.

Recent

- ▶ Bufetov, Petrov, Zografos. Domino Tilings of the Aztec Diamond in Random Environment & Schur Generating Functions (2025)
arXiv:2507.08560

(Visualizations)



- ▶ Tomas Berggren. “Domino tilings of the Aztec diamond with doubly periodic weightings.” Ann. Probab. 49 (4) 2021

1970

T. BERGGREN

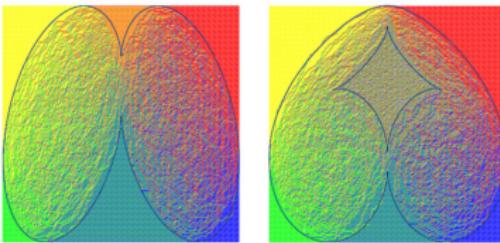


FIG. 6. Two examples of periodic weighting of the Aztec diamond which is covered by Theorem 1.1 but not studied in the rest of the paper.

Pessimism

Maybe all this stuff is what today's biological, bioinformatics and data scientists do anyway, but it's interesting.

Take-Away (I think)

- ▶ Local, similar interactions between elements in a planar lattice result in interesting dynamical patterns, quite generally.
- ▶ The hard forward problem is predicting pattern properties without simulating.
- ▶ The even harder backward problem is to find parameters or boundary conditions for a given pattern.
- ▶ Biology, including theorizing about agents/goals, may lead us to new, interesting examples: Both of **rules** and **dynamic patterns**.
 - ▶ **Why?** Evolution made use of the Platonic Space subsets that made **us possible**, not at all obvious how, so it's a good bet that its tractable structures will have interesting mathematics.
 - ▶ perturbation tolerance, programmability, mix of randomness and predictables (e.g. fingerprints), etc.

Speculative Questions

- ▶ Might goals/purposes (now accepted in biology) force any solution to the problem of achieving them to have certain structure and properties? For example,
 - ▶ The problems of communication, community cohesion, culture maintenance (teaching), etc. solved by language during human evolution can only be solved by systems whose structure has properties identified by Chomsky and later cognitive scientists?
 - ▶ Geoffry Hinton (2024) recounted that feature based learning systems are far more effective for producing intelligence than reasoning systems. Could it be that the problems solved by certain kinds of intelligence require that every solution has the multi-level feature recognition and learning structure embodied by natural and artificial neural nets?

More

- ▶ Levin asks to find simple systems with agental behavior, as in his bubble-sort project. Are there some behaviors impossible in simpler systems, versus some that are possible? Computer scientists, beginning with Godel and Turing have studying this more or less abstractly for well over fifty years, starting with Godel and Turing. It's called Computational Complexity. But today's problems are different.

For example, (1) to-date, the only successful AI neural networks are huge. (2) Formalizing math well-enough for really assisting research (Mathematical Components library, etc. in Rocq (COQ), Lean, etc.) uses a large, intricate, hard to understand body of code. Meanwhile, human math learning seems to be an easy (for some!) step-by-step experience, but it relies on the unquestionably large human brain.

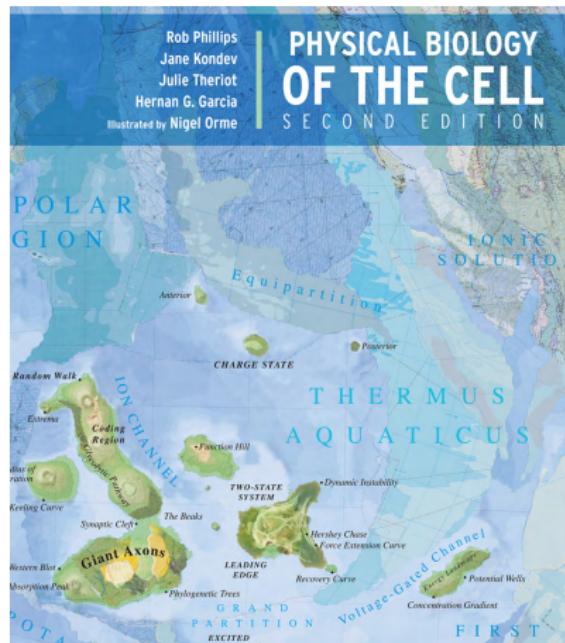
How I started

- 💻 Sept 21, 2023 Michael Levin “Emergent Selves and Unconventional Intelligences: where philosophy and engineering meet” at [🔗 Topos Institute Colloquium.](#)
- 🔗 I proposed group discussion of Levin’s Fall 2025 Levin’s asynchronous [🔗 Symposium On The Platonic Space.](#)
[🔗 thoughtforms.life/symposium-on-the-platonic-space/](#)

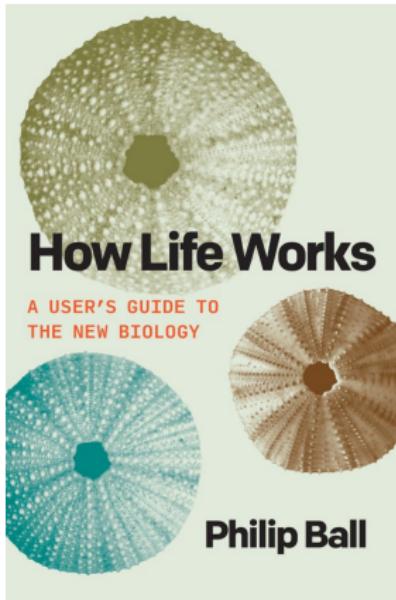
Good start: Its kickoff talk, Aug 28, 2025 by Levin. And then:

- 💻 Patterns of Form and Behavior Beyond Emergence
- 💻 Levin, Platonic Space: brief argument and research agenda Oct 29, 2025 (linked from Symposium discussion)
- 📄 Ingressing Minds: Causal Patterns Beyond Genetics and Environment in Natural, Synthetic, and Hybrid Embodiments, 55pp, 2025
- 🔗 Levin’s **Forms of life...** blog entry on *Platonic Space* . . .

Other Cool Writings



(2013) Rob Phillips et al.
Cell biology organized by
physics principles
not cell components



(2023) a favorite of Levin
30 year physical sciences editor for
Nature now prolific freelance
popular science writer

How Life Works, Philip Ball (2023)

Gene Regulation

- ▶ ... first challenge ... genes don't relate to traits in any straightforward way.
- ▶ ... complexity lies not with genes but how they are regulated.
- ▶ ... not governed ... precisely defined ... network.
- ▶ ... rules ... different levels of organization that don't relate [obviously] to those ... above and below.

proteins “disordered, not globular” [“floppy” ... [lots function as adapters or messengers]

Necessity of Disorder, Redundancy

- ▶ Some ... error is necessary for evolution ...
- ▶ ... evolution requires ... the right amount ...
- ▶ On progression ... prokaryotic to eukaryotic ... unicellularity to multicellularity ... vertebrates ... mammals.
- ▶ locus of causation ... to higher organizational levels ...
- ▶ ... new ways of handling information ... cognition

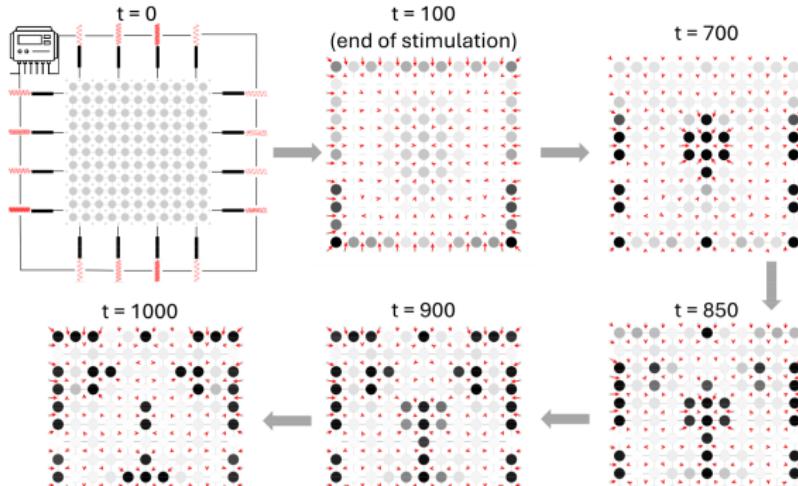
Quote from Ball

"If the first challenge to the conventional story of our own biology comes with the recognition that genes don't relate to traits in any straightforward way, and the second comes from the discovery that our organismal complexity lies not with the genes themselves but with how they are regulated, the third major shift in thinking is that this regulation is not governed by simple switching processes in a precisely delineated interactome network. The reality is startlingly different, and there is plenty about it that we still don't understand. What we do know, though, is astonishingly, gratifyingly ingenious. It reiterates the point that life has rules at many different levels of organization that don't relate in any obvious way to those of the levels above and below. None is more fundamental than any other."

Extra Material

Manicka & Levin Oct 2025 Field-mediated [morphogenesis simulation]

- ▶ Levin **had** proved patterns of frog faces, etc. occur **first** in bioelectric fields.
- ▶ Tissue morphogenesis, by bioelectrical interactions, modeled by a non-linear dynamical system
- ▶ Variables have 11×11 array structure.
- ▶ Electric field \vec{e} at cell adjacencies.
- ▶ Stimulation: Initial time series applied around the boundary.



Gradient descent was used to find 2 stimulations to produce a face pattern.

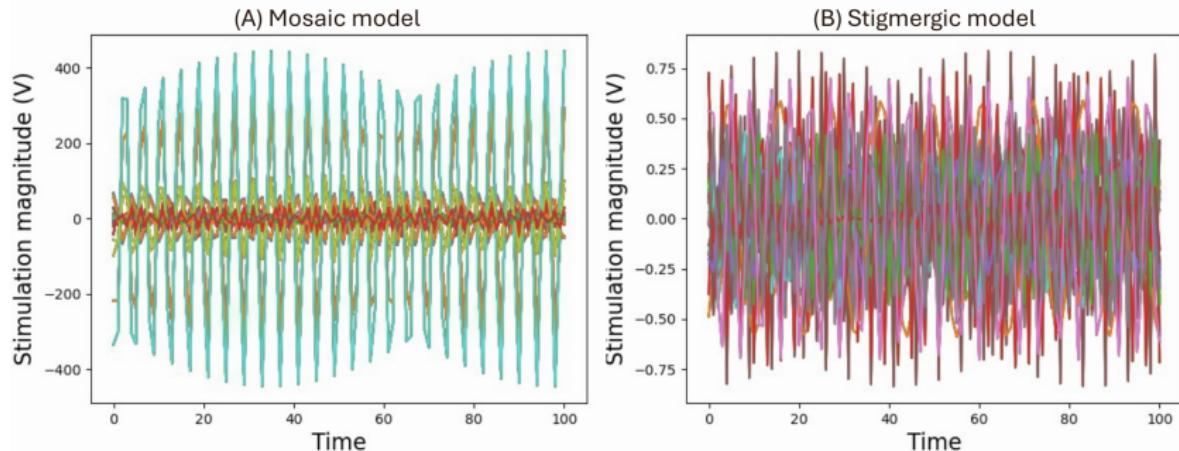
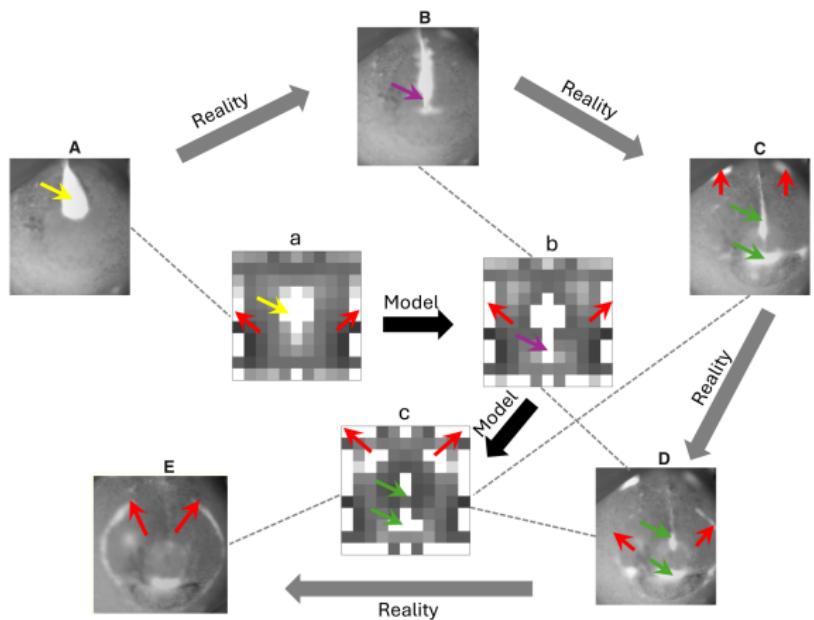
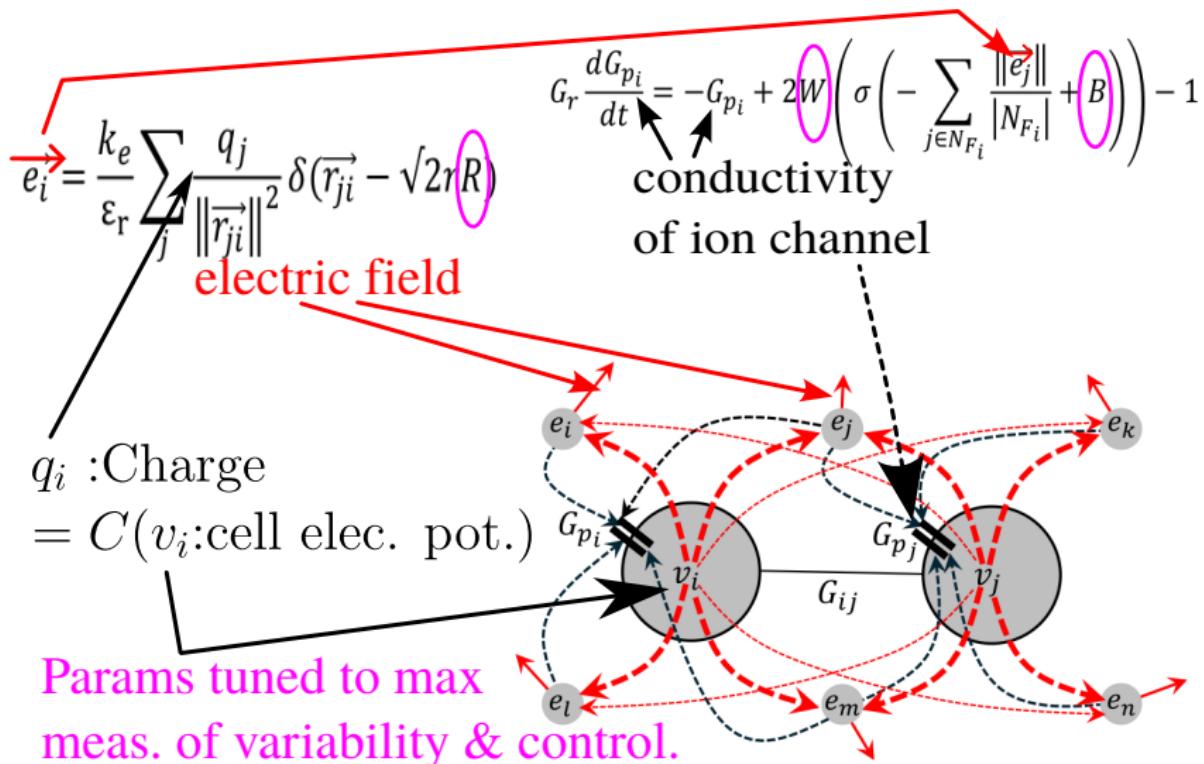


Figure S4. Profiles of the exogenous stimulation applied to the vertebrate face patterning models. (A) Timeseries of the oscillatory stimulation applied to the boundaries of the mosaic model during the initial 10% (equal to 100 steps) of the simulation phase, and (B) equivalently for the stigmergic model. The total number of traces in each panel is 44, indicating the total number of field grid points at the boundary of the 11x11 tissue with a 12x12 layout of the field grid points. Due to the bilateral symmetry of the smiley pattern, the total number of distinct traces is only 22, corresponding to one vertical half of the boundary, with the other being its mirror reflection.

Simulated and real bioelectric pattern trajectories compare.



selected details



details if needed

$$C \frac{dv_i}{dt} = -\frac{G_{p_i}(v_i - E_p)}{1 + e^{\frac{x(v_i - v_{th})}{v_r}}} - \frac{G_{d_i}(v_i - E_d)}{1 + e^{\frac{-x(v_i - v_{th})}{v_r}}} + \sum_{j \in N_{F_i}} (v_j - v_i) G_{ij}$$

$$G_{ij} = \frac{2GG_r}{1 + \cosh\left(\frac{v_i - v_j}{v_0}\right)}$$

$$G_r \frac{dG_{p_i}}{dt} = -G_{p_i} + 2W \left(\sigma \left(- \sum_{j \in N_{F_i}} \frac{\|\vec{e}_j\|}{|N_{F_i}|} + B \right) \right) - 1$$

$$\vec{e}_i = \frac{k_e}{\epsilon_r} \sum_j \frac{q_j}{\|\vec{r}_{ji}\|^2} \delta(\vec{r}_{ji} - \sqrt{2}rR)$$

$$q_j = Cv_j$$

Where:

$r = 5\mu m$, the radius of a cell

n_c = total number of cells

N_{F_i} = the set of neighboring cells of cell i

n_f = total number of field grid points

N_{F_i} = the set of neighboring (within perception range) field grid points of cell i

$v_i = V_{mem}$ of cell i (Volts)

\vec{e}_i = electric field vector at field grid point i ; $\|\vec{e}_i\|$ measured in (Volts/m)

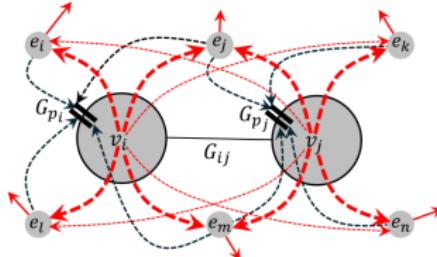
q_j = charge content of cell j (Coulombs)

\vec{r}_{ji} = distance vector from cell j to field grid point i

G_{ij} = the conductance of the gap junction connecting cells i and j (Siemens)

$G_{p_i} = P * G_r; P \in [0.0, 2.0] \in [0.0, 2.0]$,

the conductance of the polarizing ion channel of cell i (Siemens)



$G_{d_i} = 1.5 \text{ nS}$, the conductance of the depolarizing ion channel of cell i (S)

$G_r = 1.0e^{-9} \text{ S}$, the reference value for scaling ion channel and gap junction conductance (Siemens)

$C = 0.1e^{-9} \text{ F}$, the capacitance of the cell membrane (Farads)

$E_p = -55 \text{ mV}$, the equilibrium potential of polarizing channel

$E_d = -5 \text{ mV}$, the equilibrium potential of depolarizing channel

G = Gap junction strength $\in [0.0, 1.0]$ (A.U.)

R = field action range (A.U.)

W = field transduction weight (A.U.)

B = field transduction threshold (Volts/m)

$\sigma(\cdot)$ = sigmoid transformation

$\delta(\vec{r}_{ji} - d) = \text{delta function with value 1 when } \|\vec{r}_{ji}\| \leq d \text{ and 0 otherwise}$

$k_e = 8.987e^9 \text{ Nm}^2 \text{ C}^{-2}$, the Coulomb electric field constant

$\epsilon_r = 10^7$, the presumed relative permittivity of cytoplasm

Possibly Useful Out-takes

Research Program:

- Build new interfaces to observe new ingressing forms - our synthetic morphology work provides tools/vehicles/periscopes for exploration of the space.
- Infer a rigorous mapping between properties of the pointers and the patterns they facilitate
- Quantify the “free lunch” aspects - how much information/influence/evolvability is injected into the physical world? Free compute?
- Are the contents of this space under positive pressure?
- Is the space sparse? Are some attractors “better” than others?
- Are the contents of this space purely passive (eternal, unchanging) or can we define a kind of “chemistry” of how these things interact and live in their own space?
- Are mathematical objects really “low agency”? Can we extend standard behaviorist tests to their native space?
- Why? Where did the Platonic Space and its structure/contents ‘come from’? Could it have been otherwise?

- ▶ Levin studies Developmental Biology and Regenerative Medicine.
- ▶ He first showed that “bioelectric fields” *control morphogenesis* in *Xeopus* (aquatic frog) embryo eyes and in *Planaria* (flat worms) and found ion channel changes produce novel forms.
- ▶ He found that each cut-off piece of his novel two-headed *Planaria* grew back into that two-headed form, indicating the pieces retain a *non-genetic* memory of the whole form.
(Unaltered *Planaria* do this.)
- ▶ The bioelectric fields powered by ion transport across cell membranes are now known to exist in *all cellular tissues*, not just neural (bacterial films too!)

- ▶ He asserts that cognition in the brain is the same kind of problem solving activity that controls morphogenesis of tissues and organisms.
- ▶ The spectra of kinds of *intelligence* and *agency* that brains, bodies, reaction networks, etc. occupy are broader than most people consider. He characterizes, cybernetically, agency as goal-seeking, and intelligence as the ability to solve problems in multiple ways. (Physics and Math are at the lower end! Levin later asks maybe not.)

“future medicine is going to look a lot more like a kind of semantic psychiatry than it’s going to look like chemistry.”

- ▶ Observation of such spaces are done in living systems and simulations.
- ▶ Levin declared this view is empirically effective. It was necessary to lead him to novel experiments giving unexpected discoveries.
- ▶ Although a discovery once made can be explained by the ordinary causality sciences, he said he could not have made them without his latent Platonic Space approach.

I think it's interesting to ask mathematical questions, beginning with the most elementary, about these Platonic spaces, such as dimension, perhaps fractal. Many have mathematical representations (examples: morphological, gene expression, bioelectric field spaces.)

- ▶ Observation of such spaces are done in living systems and simulations.
- ▶ Levin declared this view is empirically effective. It was necessary to lead him to novel experiments giving unexpected discoveries.
- ▶ Although a discovery once made can be explained by the ordinary causality sciences, he said he could not have made them without his latent Platonic Space approach.

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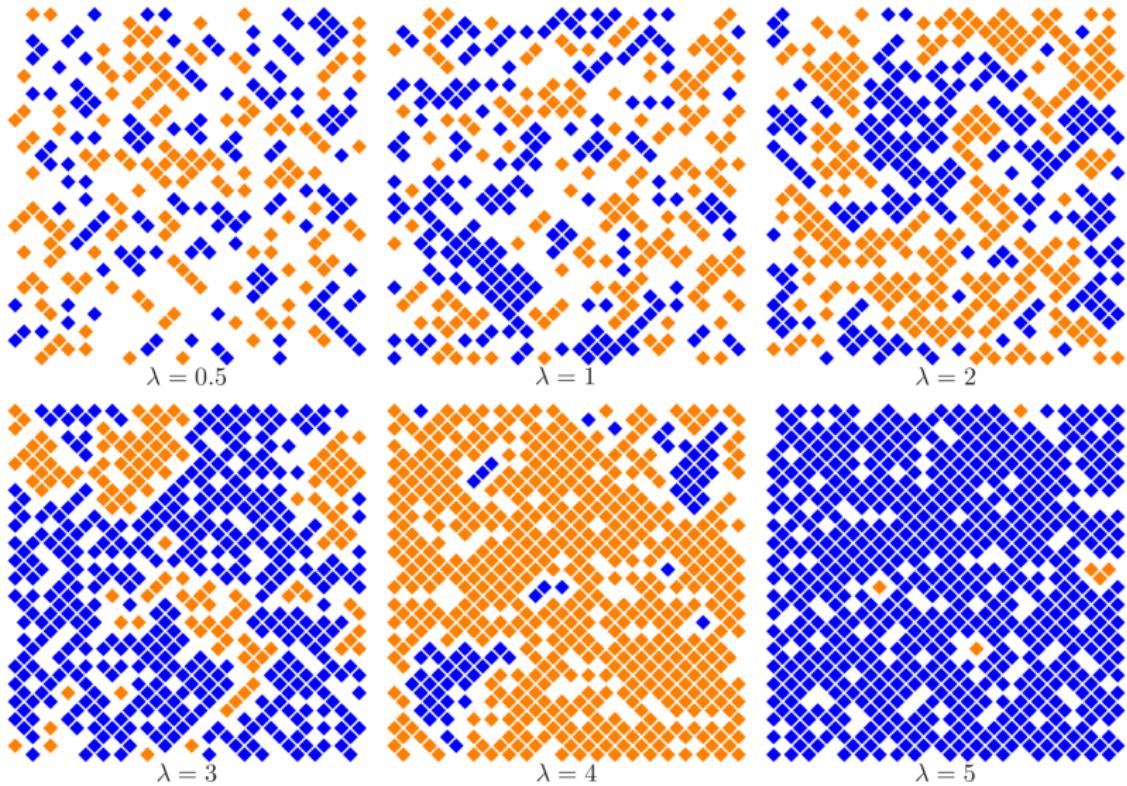
Outtake Research Ideas

- ▶ What examples of transformations on affordances mapping to Latent Platonic Spaces are somehow functorial? Might there be higher category structure (associative law holds up to morphisms between morphisms ...)?
- ▶ Voronoi diagrams seem to appear. [?? Look into devel. bio. refs and Physical Biology of the Cell
What properties do their defining metrics have?
- ▶ Properties of prime residues relate to prime multi-year Circada reproductive cycles.

Tilings and Dynamical Combinatorics

Dynamical Combinatorics

Apply Levin et. als. approach for bubble-sort and ... work on behavior of distributed sorting subject to failures and multiple local algorithms to algorithms on graphs; and tiling pattern statistics and **Dynamical Algebraic Combinatorics**, work of James Propp et. al.



James Propp on Aztec Diamond Tilings

Platonic Spaces

“Mathematicians are already very comfortable ... that there is a non-physical space of truths which we discover, ... and space has a structure that enables exploration. I make the conjecture that this space contains not only ... facts about triangles ..., but also a very wide variety of high-agency patterns”

Minds Everywhere!

that we call kinds of minds. ... physical bodies don't create, or even connect to (and thus have) minds – instead, minds are the patterns, with their ingressions into the physical world enabled by the pointers of natural or synthetic bodies. ... whenever ... is built acts as an interface to numerous patterns

More

- ▶ Connections with fractals, holograms, compressed sensing.
(Hologram fragments all project the original image with loss of resolution)
- ▶ Why do living things have so many different types of parts (e.g., chemical species) compared to non-living things?
- ▶ What ideas from category theory can give us fruitful insight resulting from the hierarchy and repetition of things with common type (latter called “symmetry” by Fields and Levin) found in biological structures?
Category theory develops consequences of two fundamental mathematical phenomena **associativity** and **transitivity**.
- ▶ Same question for hierarchy and processing possibly “embodying” Martin-Loff dependent type theory/functional programming, the logic foundation for today’s very productive **automated proof assistants**: Rocq (COQ), Lean, Agda, etc.