An Exterior Algebra Valued Tutte Function on Linear Matroid Pairs

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Abstract

Let N be a linear representation (i.e, matrix) of a matroid whose ground set S includes a finite, distinguished subset P. We give function $L(N)$ that, unlike what we know of other Tutte functions and work like the Hopf algebra variants of Krajewski, Moffatt and Tanasa, has values in an *anti-commutative* algebra. Let deletion and contraction be limited to $e \notin P$. Then, the values are in the exterior algebra generated by $P_\alpha \coprod P_\beta$. The construction relies on concrete minor operations to establish consistent signs of the constituent terms so that, with suitable accounting for sequential orderings of set elements, $L(N) = L(N \setminus e) + L(N/e)$ in the exterior algebra. Our construction is derived from the structure of the equilibrium equations for linear electrical networks, and of their generalization to multi-dimensional elastic frameworks. Further, the construction does not require orthogonality for the spaces that generalize spaces of feasible currents and voltages, or of forces and displacements. Hence L will be defined on equal rank pairs (N_α, N_β) (where originally, $N_\alpha = N_\beta = N$). We take the Tutte identities those for Welsh and Kayibi's linking polynomial of matroid pairs. With $N_\alpha \neq N_\beta$, we can derive the digraph all-minors matrix tree theorem by taking P to be the set of vertices. We so get ratio of common basis expansion solutions for linear electrical and other linear systems with multi-terminal amplifiers (where a voltage or force at one place is a multiple of current or displacement at a different place). To incorporate resistance (r_e), conductance (g_e), elasticity coefficient, etc. parameters, we use parametrized Tutte function theory for which $L(N) = r_e L(N \setminus e) + g_e L(N/e)$; the term for common basis B includes $\prod_{e \in B} g_e \prod_{e \notin B \cup P} r_e$.