AT and beam dynamics course

Tuesday, March 27, 2012

ESRF
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Morning session
Objectives

- Getting set up: download and compile code, set path
- Components of a storage ring and how they are represented. Track a particle through an element. Read in ESRF lattice.
- Linear dynamics. Transfer matrix. Symplecticity. Lattice functions. Tunes.
- Non-linear dynamics. Dynamic aperture.

- If time: add an insertion device via a kick map. Compute tune shift.

Afternoon Session

- Equilibrium emittance and OhmiEnvelope
- Load model machines for different days with different coupling corrections.
- Compute vertical emittance
- Plot coupling angle

Getting AT software ready

- Log in to rnice
- log in to oar cluster
- oarsub -I -l nodes=1/core=3 -p "mem>4000 and cpu_vendor='INTEL '"
- start Matlab
- Set proxy settings in .subversion/servers file:
- [global]
- http-proxy-exceptions = *.esrf.fr, localhost
- http-proxy-host = proxy2.esrf.fr
- http-proxy-port = 3128

  Download software:
- **svn co https://atcollab.svn.sourceforge.net/svnroot/atcollab atcollab**

Setting up in Matlab

Go to Set Path under File menu

Select: Add with Subfolders:

choose:

~/atcollab/trunk

In Matlab, type:

- atmexall

This should compile all the c files into mex files that can be called from Matlab.

That should be it!

Additional files for this course

- create a folder called atcourse

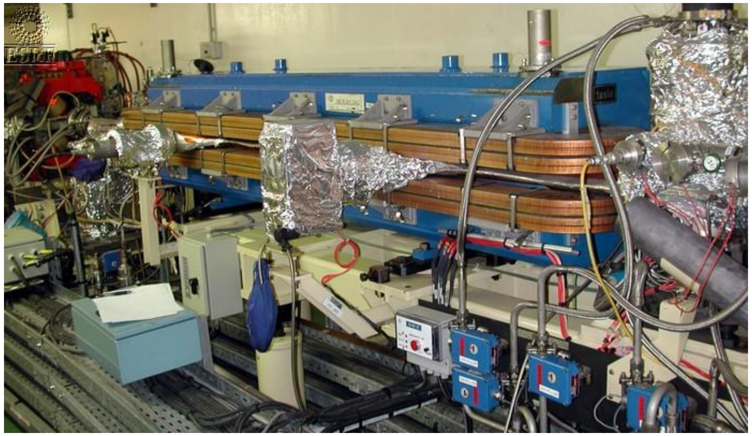
- add it to your path

- copy files from ~nash/atcourse to your atcourse folder.

How to store a high energy electron

- Accelerate to high energy (E=6.04 GeV for ESRF) in linac and booster, then inject into ring.
- Use dipole magnets to create circular trajectory.
- Use quadrupoles to confine the beam transversely.
- Use sextupoles to fix chromatic aberration caused by the quadrupoles.
- Use an RF cavity to replenish energy and confine longitudinally.

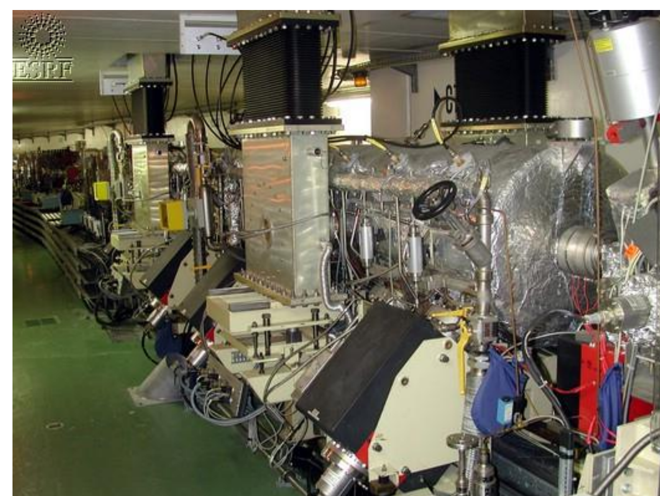
Components needed to store electrons



dipole

quadrupole

sextupole



RF cavity

Load the ESRF lattice

>> load esrf.mat

This is a cell structure with 1648 elements

```
>> esrf{1}

ans =

      FamName: 'SDHI'
       Length: 3.0526
   PassMethod: 'DriftPass'
     BetaCode: 'SD'
       Energy: 6.0400e+09
```

```
>> esrf{10}

ans =

        FamName: 'S6'
         Length: 0.4000
       PolynomB: [0 0 -4.1054]
       PolynomA: [0 0 0]
       MaxOrder: 2
    NumIntSteps: 10
     PassMethod: 'StrMPoleSymplectic4Pass'
       BetaCode: 'SX'
         Energy: 6.0400e+09
```

```
>>  findcells(esrf,'FamName','S6')
ans=…
```

Electrons move through elements

$$\vec{z} = \begin{pmatrix} x \\ x'(1+\delta) \\ y \\ y'(1+\delta) \\ \delta \\ cT \end{pmatrix}$$

6-D phase space for
Electron or ‘ray’ entering
Plane at element

$$\delta = \frac{E - E_0}{E_0}$$

Integrators

- AT has the following integrators:

ApertuePass.c
BendLinearPass.c
BndMPoleSymplectic4E2Pass.c
BndMPoleSymplectic4E2RadPass.c
BndMPoleSymplectic4Pass.c
BndMPoleSymplectic4RadPass.c
CavityPass.c
CorrectorPass.c
DriftPass.c
EAperturePass.c
IdTablePass.c
IdentityPass.c
Matrix66Pass.c
QuadLinearPass.c
SolenoidLinearPass.c
StrMPoleSymplectic4Pass.c
WiggLinearPass.c

element

In-coming phase-space point

Calling syntax:

>>StrMPoleSymplectic4Pass(QF2,[.001 0 0 0 0 0]')

Drift space

- The simplest example is the electron just passing through empty space.

$$\Delta\vec{Z} = \begin{pmatrix} x'L \\ 0 \\ y'L \\ 0 \\ 0 \\ \frac{L}{2}(x'^2 + y'^2) \end{pmatrix}$$

path length effect
note this is non-linear!

Bend

L, theta, theta_i, theta_f

rho=L/theta

hard bend

```
>> esrf{14}

ans =

        FamName: 'B1H'
         Length: 2.1573
   BendingAngle: 0.0923
  EntranceAngle: 0.0491
      ExitAngle: 0.0432
     PassMethod: 'BndMPoleSymplectic4E2Pass'
              K: 0
       PolynomA: [0 0 0]
       PolynomB: [0 0 0]
       MaxOrder: 2
    NumIntSteps: 10
       BetaCode: 'DI'
      FringeInt: 0
        FullGap: 0
         Energy: 6.0400e+09
```

soft bend

```
>> esrf{15}

ans =

        FamName: 'B1S'
         Length: 0.2927
   BendingAngle: 0.0059
  EntranceAngle: -0.0432
      ExitAngle: 0.0491
     PassMethod: 'BndMPoleSymplectic4E2Pass'
              K: 0
       PolynomA: [0 0 0]
       PolynomB: [0 0 0]
       MaxOrder: 2
    NumIntSteps: 10
       BetaCode: 'DI'
      FringeInt: 0
        FullGap: 0
         Energy: 6.0400e+09
```

Cavity

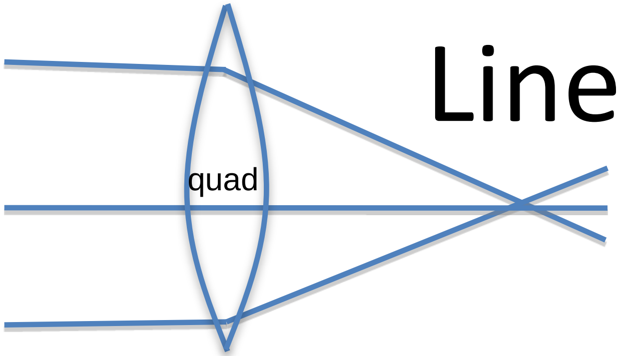
```
>> esrf{end}

ans =

      FamName: 'CAV'
       Length: 0
   PassMethod: 'IdentityPass'
      Voltage: 562500
    Frequency: 3.5220e+08
   HarmNumber: 992
     BetaCode: 'CA'
       Energy: 6.0400e+09
```

cavity turned off in model by default

cavity in long straights: distributed model. Note reduced RF voltage to give total correct value.

```
>> findcells(esrf,'FamName','CAV')

ans =

  Columns 1 through 9

       103        206        309        412
515        618        721        824        927

  Columns 10 through 16

      1030       1133       1236       1339
1442       1545       1648
```

Linear Optics: Quadrupole



$$x'' + k_x(s)x = 0$$

$$z'' + k_z(s)z = 0$$

(Hill's equation)

$$k_x = \frac{1}{\rho^2} - \frac{B_1}{B\rho} \qquad k_y = \frac{B_1}{B\rho} \qquad B_1 = \frac{\partial B_y}{\partial x}$$

$$B\rho[Tm] = 3.3357\, p[GeV/c]$$

=20.15 for 6.04 GeV, ESRF

```
>>QF2=esrf{6}
QF2
    FamName: 'QF2'
      Length: 0.9434
          K: 0.3910
    PassMethod:'StrMPoleSymplectic4Pass'
      PolynomA: [0 0 0]
      PolynomB: [0 0.3910 0]
      MaxOrder: 2
   NumIntSteps: 10
```

$$\frac{B_y + iB_x}{B\rho} = \sum_{n=1}^{N+1} (iA_n + B_n)(x + iy)^n$$

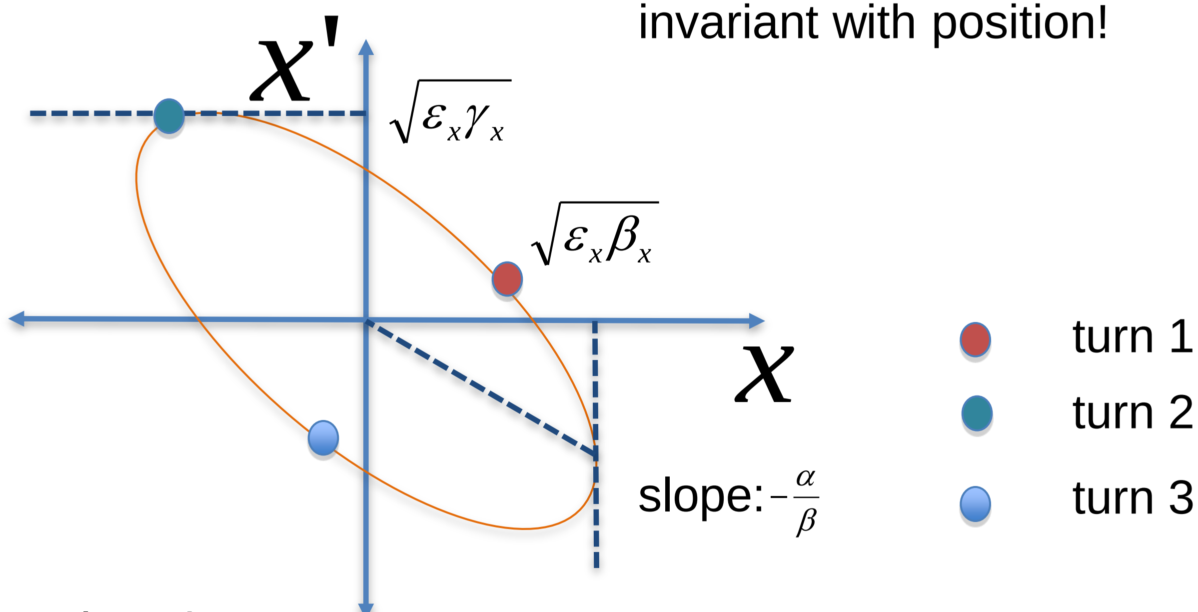
Multipole expansion

Harmonic oscillator in phase space
Twiss Parameters

measuring the position
over time, it will oscillate

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

invariant with position!



tune is defined by
number of oscillations about closed
orbit over 1 turn

This is at one position in the ring.

Linear Dynamics, continued

All this can found from one-turn map matrix.

```
>>M=findm66(esrf)
```

In uncoupled case, beta functions from eigenvectors of M

$$M\vec{v}_j = \lambda_j \vec{v}_j j \qquad \vec{v}_x = \begin{pmatrix} \sqrt{\beta_x} \\ \frac{i-\alpha_x}{\sqrt{\beta_x}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{v}_y = \begin{pmatrix} 0 \\ 0 \\ \sqrt{\beta_x} \\ \frac{i-\alpha_y}{\sqrt{\beta_y}} \\ 0 \\ 0 \end{pmatrix}$$

Tunes from eigenvalues

$$\lambda_j = e^{2\pi i \nu_j}$$

```
>> eigs(M)
```

Find tunes!

This matrix satisfies a property called symplecticity:

$$M^T J M = J$$

Verify this property for ESRF lattice.

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

Compute the lattice functions around the ring: atlinopt function

```
>>[lindata,tunes,xsi]=atlinopt(esrf,0,1:length(esrf)+1);
```

The fields of lindata:

SPos

Dispersion

B

beta

A

mu

alpha

C

gamma

```
>> tunes
tunes =
0.4396    0.3901
```

nx=36,
ny=14

ElemIndex

```
>>xsi
xsi =
2.6599    5.2293
```

M44

Plotting the results of atlinopt

because the result is an array of arrays, one needs to use the cat function to pull out the individual fields.

```
>>beta = cat(1,lindata.beta);
>>betax= beta(:,1);
>>betay=beta(:,2);
>>disp = cat(2,lindata.Dispersion);
>>dispx=disp(1,:);
```

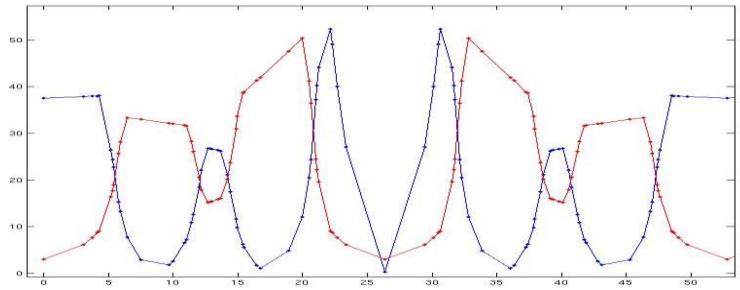
small inconsistency in the array orientation!

use ‘hold on’ so that later plots don’t replace earlier

```
>>spos=cat(1,lindata.SPos)
```

```
>>plot(spos,betax,'.-b',spos,betay,'.-r')
```

Note that there are 32 super-periods, and a total circumference of 844.39 m Each cell is 26.38 m long. Note mirro symmetry of adjacent cells.



Add Radiation

```
>>[esrfrad,radindex,cavindex]=atradon(esrf);
```

What changed?

Check symplecticity condition for esrfrad.

Sextupole element

```
>>S6 = esrf{10}

S6 =

       FamName: 'S6'
        Length: 0.4000
     PolynomB: [0 0 -4.1054]
     PolynomA: [0 0 0]
     MaxOrder: 2
  NumIntSteps: 10
   PassMethod:
'StrMPoleSymplectic4Pass'
     BetaCode: 'SX'
       Energy: 6.0400e+09
```

Note that the element here comes directly from the PolynomB.

From multipole expansion, work out the form for the magnetic field for the sextupole.

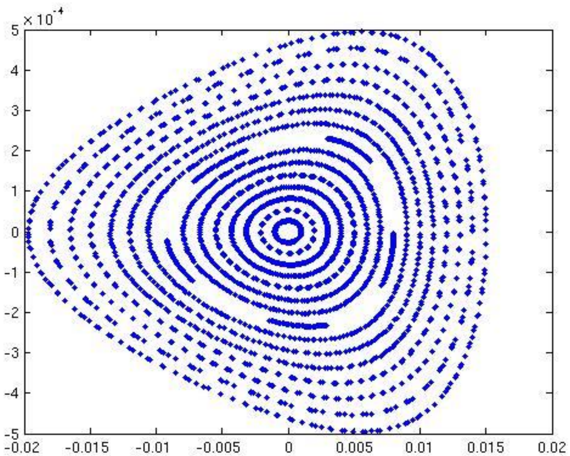
Non-linear dynamics

Sextupoles result in non-linear dynamics. Many new phenomena arise.

Motion can now be unstable at large amplitudes.

Motion is chaotic. Accurate tracking for large number of turns: need to be sophisticated in tracking algorithm. Symplectic integrators.

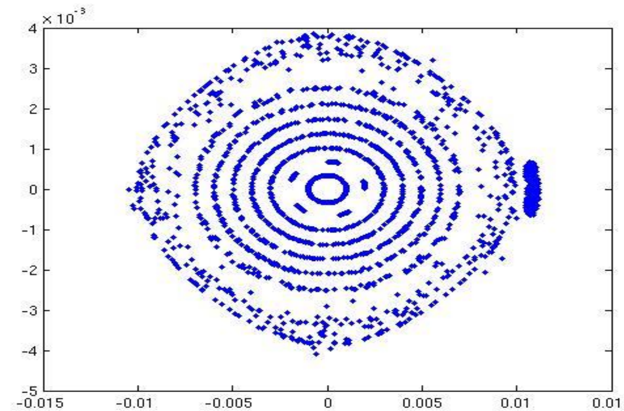
x'(rad)



x(m)

```
>> z0=[0.001 0 0 0 0 0]'*(1:15);
>> zf=ringpass(esrf,z0,200);
>> plot(zf(1,:),zf(2,:),'.b')
```

y'(rad)



y(m)

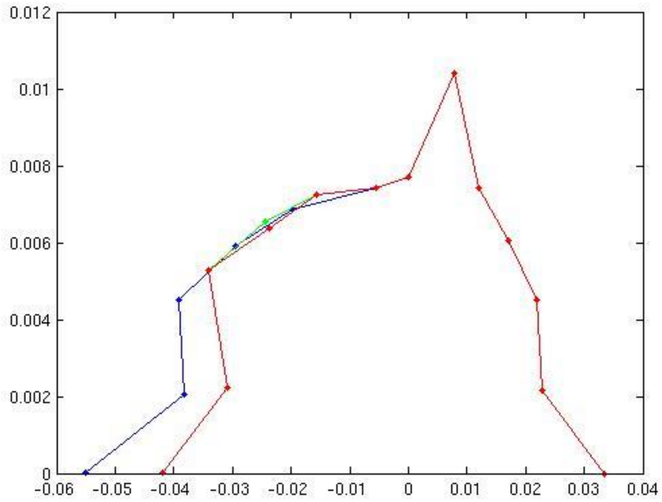
```
>> z0=[0 0 0.001 0 0 0]'*(1:15);
>> zf=ringpass(esrf,z0,100);
>> plot(zf(3,:),zf(4,:),'.b')
```

Non-linear dynamics continued

Compute dynamic aperture for ESRF lattice using atdynap function: Vary the number of tracking turns and see how the dynamic aperture varies.

Change the value of a sextupole setting: esrf{10}.PolynomB=[0 0 -2], and recompute dynamic aperture.

y (m)

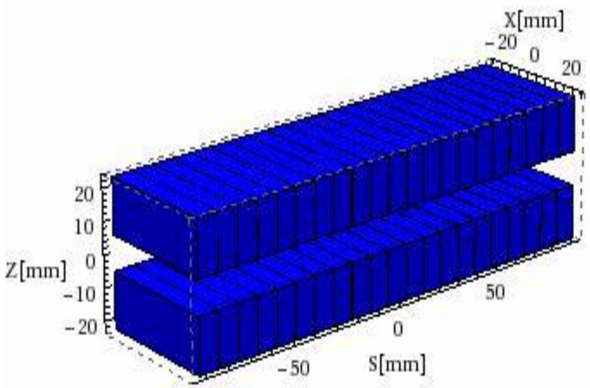


X (m)

```
>>[xm100,ym100]=atdynap(esrf,100)
>>[xm200,ym200]=atdynap(esrf,200)
>>[xm300,ym300]=atdynap(esrf,300)
>>[xm400,ym400]=atdynap(esrf,400)
>>plot(xm100,ym100,'.-b',xm200,ym200,'.-g',xm300,ym300,'.-
r',xm400,ym400,'.-y')
```

Modelling Undulator impact on beam

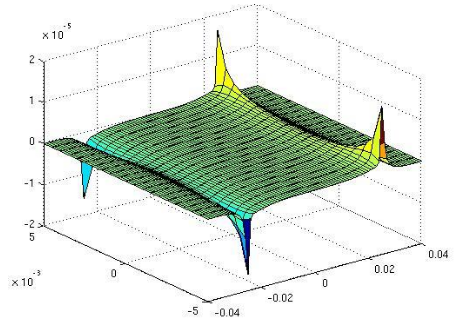
1.6 m undulator B_max= 0.76 T, lam= 35 mm.
Model in Radia. U35.mat contains this kick map in a Matlab variable.



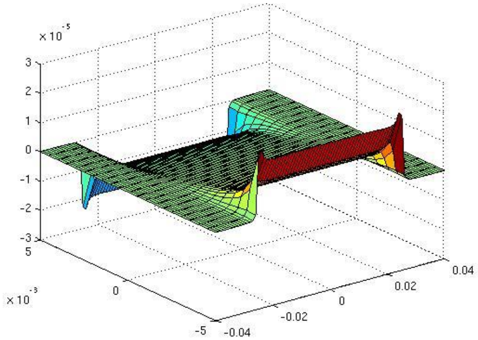

```
>> U35elem=atidtable('U35',1,'U35.mat',6.04,'IDTablePass')

U35elem =

      FamName: 'U35'
       Nslice: 1
     MaxOrder: 3
  NumIntSteps: 10
           R1: [6x6 double]
           R2: [6x6 double]
           T1: [0 0 0 0 0 0]
           T2: [0 0 0 0 0 0]
   PassMethod: 'IDTablePass'
       Length: 1.6000
         NumX: 81
         NumY: 21
       xtable: [1x81 double]
       ytable: [1x21 double]
        xkick: [21x81 double]
        ykick: [21x81 double]
       xkick1: [21x81 double]
       ykick1: [21x81 double]
     PolynomA: [0 0 0 0]
     PolynomB: [0 0 0 0]
```

Plot the kick maps

```
>> xtable=U35elem.xtable;
>> ytable=U35elem.ytable;
>>xkick = U35elem.xkick;
>> surf(xtable,ytable,xkick)
>> ykick = U35elem.ykick;
>> surf(xtable,ytable,xkick)
>> surf(xtable,ytable,ykick)
```



horizontal kick



vertical kick

Find the tune shift and DA impact from this ID

Add the U35 to the ring:

>>esrf_U35=add_ID(esrf,U35elem)

For B=0.76 T

$$\lambda_0 = 3.5cm$$

$$\Delta\nu_y = \frac{\langle \beta_y \rangle L}{8\pi\rho^2}$$

$$k = \frac{B_0[kG]\lambda_0[cm]}{10.7}$$

$$k = 2.49$$

$$\rho = 26.51m$$

Compute the new tunes.
Compute the new dynamic aperture.
Try decreasing the energy when creating the element to give a stronger effect.

Afternoon

Equilibrium Emittance

- We saw that with radiation, the linear motion is not symplectic. It is damped. If you run enough turns, all particles go to 0. Is this the truth?
- There's another effect. The radiated energy has quantum fluccuations. This causes a diffusion effect. Causes a change in second moments of beam size.

$$P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2} \qquad (C_\gamma = 8.85*10^{-5}\ meter - GeV^{-3})$$

Second moments with Radiation

Mathematically, we write:

$$\vec{z}_2 = M\vec{z}_1 + \vec{\Delta}$$

One turn map,
with radiation damping
Deterministic (reversible)

Random fluccuation
From quantum effect.
Average (first moment) is
zero, non-zero second
moment.

Define

$$\Sigma_{ij} = \left\langle z_i z_j \right\rangle$$

Average over ensemble.

Then,

$$\Sigma_2 = M\Sigma_1 M^T + \bar{B}$$

$$\bar{B}_{ij} = \left\langle \Delta_i \Delta_j \right\rangle$$

Equilibrium beam

Equilibrium if:

$$\Sigma = M\Sigma M^T + \overline{B}$$

findm66(esrfrad)

$$\overline{B}_j = \sum_i M_{i\to j}{}^{-1} B_j M_{i\to j}$$

findmpoleraddiffmatrix(B1Hrad,[0 0 0 0 0 0]')
(esrfrad{14})

This is all combined together in
>>[env, rmsdp, rmsbl] = ohmienvelope(esrfrad,radindex,1:length(esrfrad)+1)

energy spread

bunch length

Envelope structure

env =

1x1649 struct array with fields:
    Sigma
    Tilt
    R

```
ENVELOPE is a structure with fields
  Sigma   - [SIGMA(1); SIGMA(2)] - RMS size [m] along
            the principal axis of a tilted ellips
            Assuming normal distribution exp(-
(Z^2)/(2*SIGMA))
  Tilt    - Tilt angle of the XY ellips [rad]
            Positive Tilt corresponds to Corkscrew (right)
            rotatiom of XY plane around s-axis
  R       - 6-by-6 equilibrium envelope matrix R
```

OhmiEnvelope computation included in atx function

```
lindata=atx(esrf,0);
tilt=cat(1,lindata.tilt)*360/(2*pi); //Convert to
degrees.
spos=findspos(esrf,1:length(esrf));
tiltdat=[spos tilt];
plot(spos,tilt)
```

Connection to Emittances

To a good approximation, one can also write the Gaussian beam distribution in the form

$$f(z) = \frac{1}{N} e^{-\frac{g_1}{2\varepsilon_1} - \frac{g_2}{2\varepsilon_2} - \frac{g_3}{2\varepsilon_3}}$$

In the uncoupled case, the g's are the Courant-Snyder invariants.
The eps's are the emittances.

Vertical emittance from coupling

- Try to add a skew quadrupole component

to create non-zero vertical emittance.

Plot beam sizes around ring.

Real errors

- Measure a response matrix.
- Apply corrections.
- AT lattices are a part of this process.
- Find esrf08Nov2011.mat and

esrf27Sept2011.mat

In the atcourse folder.

Compute the vertical emittance, and beam sizes around ring for these two machines. Plot coupling angle and compare for the two.