Some considerations on the dipole edge effects
31-03-11

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Preamble

The goal of this note is to analyze, in the foot step of K. Brown, the dipole vertical focusing terms present in the edge and fringe field components. The main motivation is driven by the strong effective component in the Thom-X dipole case where the radius of curvature is very short.

First, the main focusing terms induced by the edge pole face rotation is first derived as well as the corresponding transfer map. To end, the correction terms induced by the fringe field effect is also revisited.

Conclusion

The well know relation of the vertical focusing effect in presence of a pole face rotation at the exit of a dipole is derived and is opposite in sign of the horizontal case. This vertical effect is only due to the longitudinal fringe field B_s experienced by the particle together with its horizontal speed or (x') at the exit of the pole. A simple relation is then derived in the kick approximation for the pole face effect and is in good accordance regarding the chromaticities as compared to BETA and MADX code. This kick transfer may restore the TRACY case.

Finally, the vertical correction term induced by the fringe field effect seems to be 4 times smaller than the commonly estimations done by K. Brown K1 dimensionless integral. The difference cames from how the horizontal orbit is considered in the dipole.

Pole face rotation component

The X, Z and S directions are for the horizontal, vertical and longitudinal normal to the pole face. The reference trajectory is composed of arc in the dipole and straight out of the dipole and fix the geometry of the machine. We assume an infinitely large dipole magnet with no horizontal B_x field components.

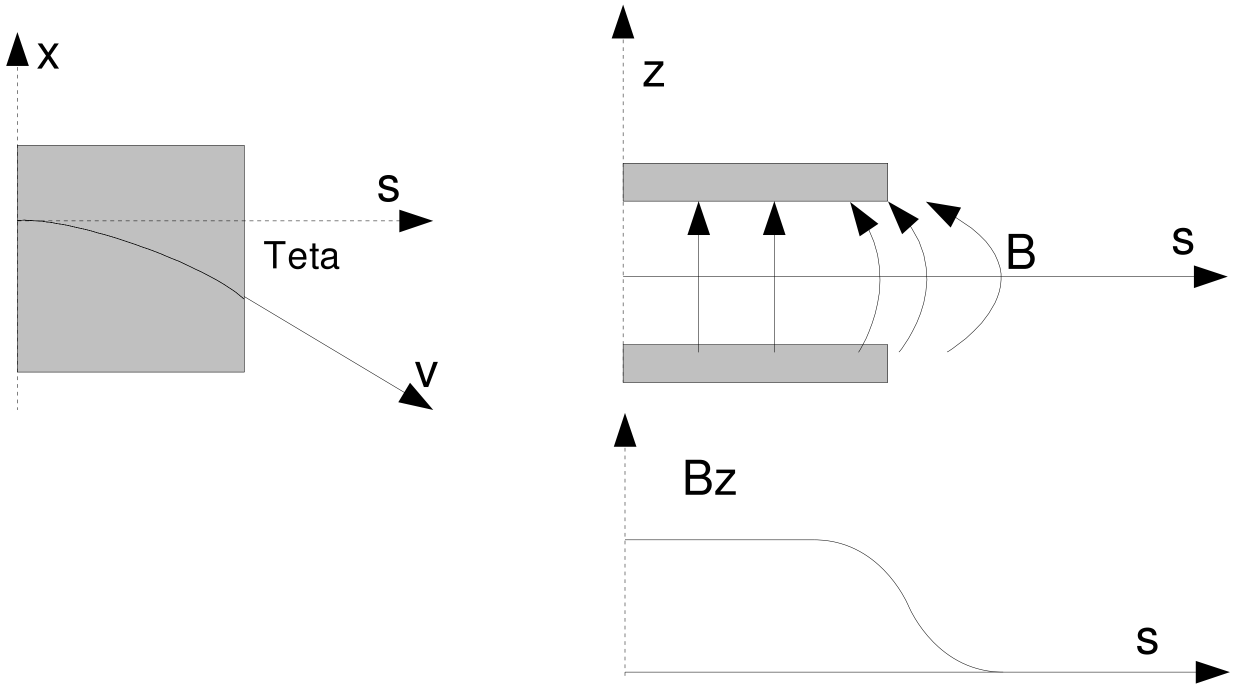


Figure 1 : Sketch of dipole magnet

From the Maxwell equation Rot (B) = 0, the relation between B_s and B_z is :

$\frac{\partial B_s}{\partial z} = \frac{\partial B_z}{\partial s}$ and gives for the longitudinal field $B_s = \frac{\partial B_z}{\partial s} z$ to the first order.

The standard dynamic relations of a moving particle in magnetic field (constant energy) are :

$$\vec{F} = \gamma m \frac{d\vec{V}}{dt} = \begin{pmatrix} V_z B_s - V_s B_z \\ -V_x B_s \\ V_x B_z \end{pmatrix}$$

The vertical force is the given by the term $-V_x B_s$. Neglecting the horizontal orbit perturbation of the fringe field extension, V_x can be replaced by $V_x = \vec{V} \tan(\theta)$. Using the standard relations

$ds = V\,dt$, $z' = \frac{V_z}{V}$ and $B\rho = \frac{\vec{P}}{q}$ we can express the vertical focusing along the fringe field exit of the pole as :

$$\frac{dz'}{ds} = \frac{-\tan(\theta)}{B\rho} \frac{\partial B_z}{\partial s} z = K_z z$$

K_z being the usual notation for the normalized quadrupole strength.

Integrating K_z over the fringe field extension (from inside the magnet $B(s_1)=B_0$ to the end of the field $B(s_2)=0$) leads to the total focusing effect (1/fz) in the vertical plane :

$$\frac{1}{f_z}=\int_{s1}^{s2} K_z\, ds=\frac{-\tan(\theta)}{B_0\rho}\int_{s1}^{s2}\frac{\partial B_z}{\partial s}ds=\frac{-\tan(\theta)}{B_0\rho}\int_0^{B_0} dB_z=\frac{-\tan(\theta)}{\rho}$$

It is the well know relation of the vertical focusing effect in presence of a pole face rotation at the exit of a dipole and is opposite in sign of the horizontal case. This vertical effect is only due to the fringe field B_s experienced by the particle together with its horizontal speed or (x') at the exit of the pole. From this statement, we can easily generalize including the horizontal slope component x' as well as energy deviation delta at the exit of the pole by :

$$\Delta z'=\frac{z}{f_z}=\frac{-\tan(\theta\pm x')}{\rho(1+\delta)}z$$

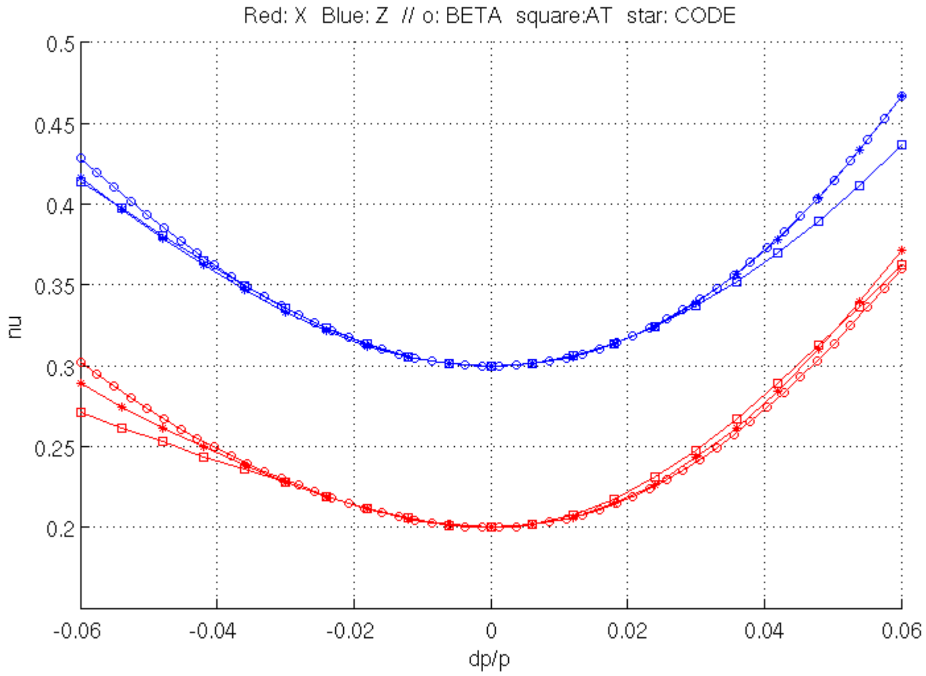
The ± sign stands for entrance and exit of the dipole. These kick formulation is symplectic by construction and is in good accordance with the codes BETA and MADX concerning the chromaticities (large and compact rings). To complete this first derivation, there is a pending effect in the horizontal plane coming from the term $-V_zB_s$. Following the same scheme, the horizontal quadrupole kicks has also to be implemented with z' components :

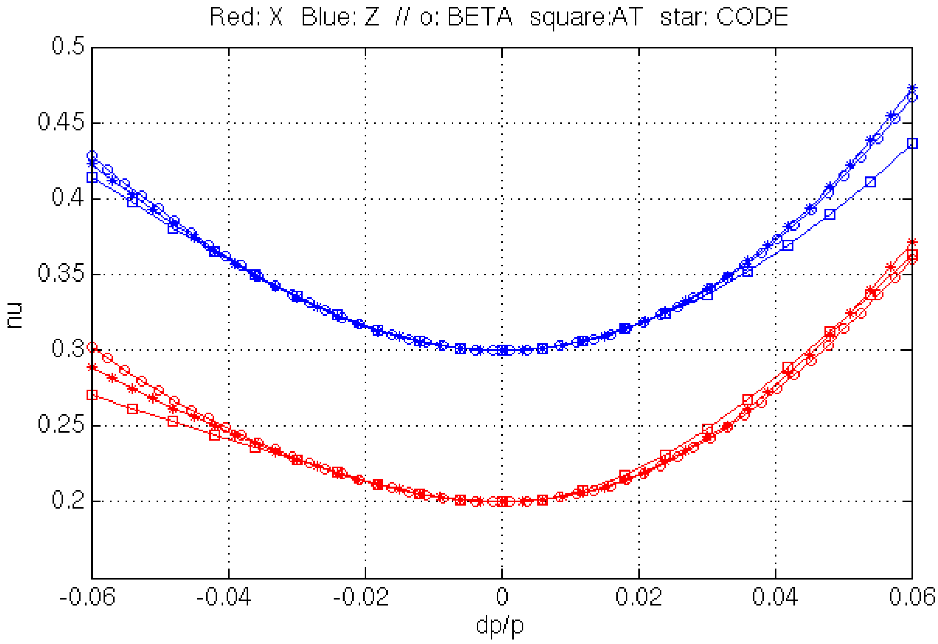
$$\Delta x'=\frac{z}{f_z}=\frac{\tan(\theta\pm z')}{\rho(1+\delta)}z$$

A coupling components is then present by the fringe field of the dipole.

Codes comparison

SOLEIL case

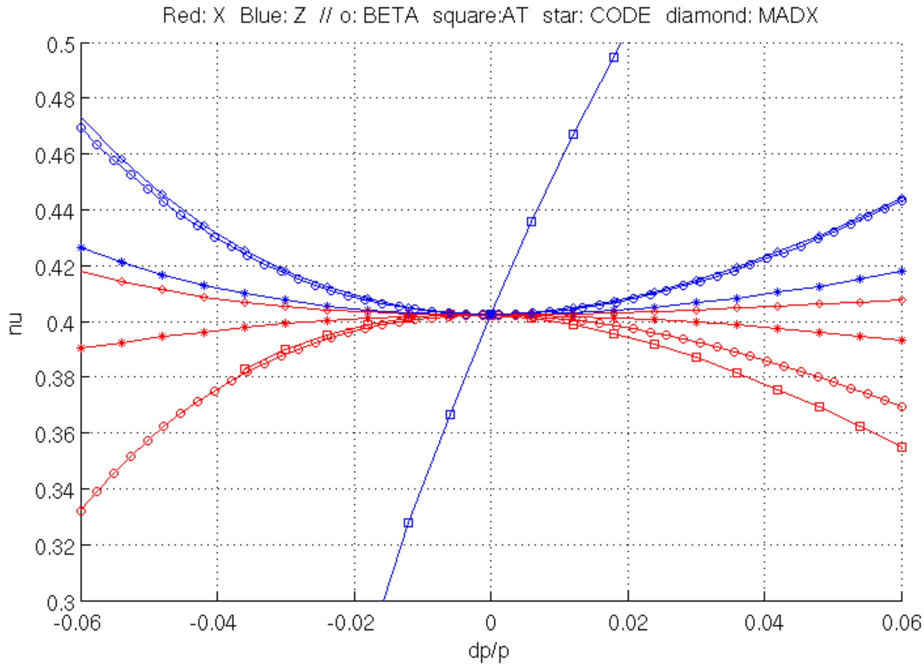


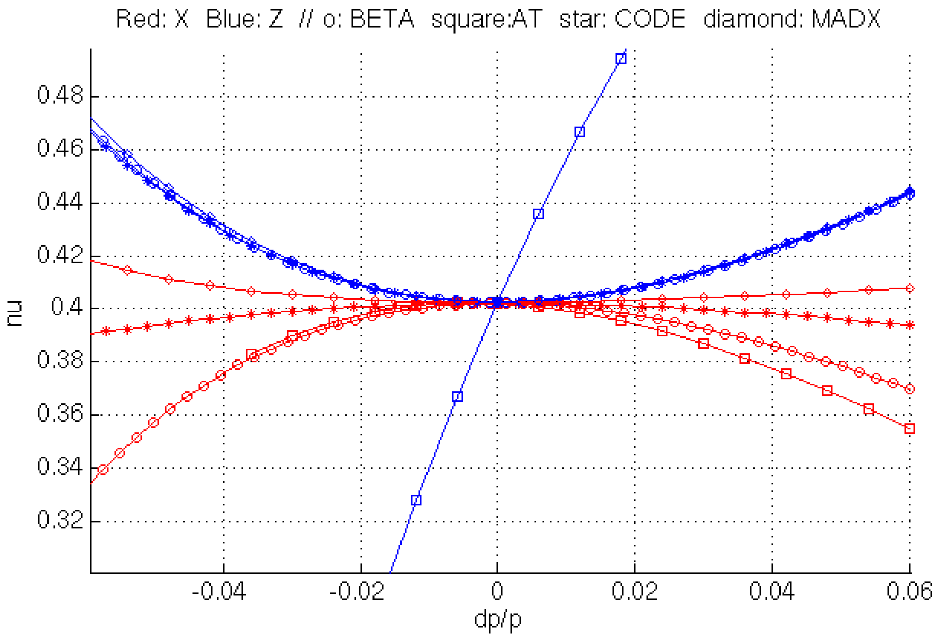


Horizontal (red) and vertical chromaticities tracking from different codes on the SOLEIL lattice. In The second plots, the vertical previous kick edge relation is included in CODE.

All the codes seems to be OK ... weak dipole (radius=5.36m)

Thom-X case





Horizontal (red) and vertical chromaticities tracking from different codes on the SOLEIL lattice. In The second plots, the vertical previous kick edge relation is included in CODE.

In the second plot, all the codes are in goods accordance in the vertical plane except TRACY. In the horizontal plane, they all disagree : H strong dipole (radius=0.352m) approach may differ.

Fringe field term component

At this stage we do not take into account the horizontal orbit depression in the fringe fields area. These term is known as the correction phi (K. Brown, Transport Code Notice) to be added to the vertical focusing quadrupole edge kick.

In figure 2, an example of dipole Bz profile as well as the relative X orbit depression are plotted from Thom-X ring model. The presence of fringe field induced a large orbit depression of the order of -1.4 mm. The total deviation with the fringe field extension is kept constant. In other words, the dipole bore length as well as the fringe field extension as been designed to fit the hard edge model deviation (reference orbit).

Removing the pole face rotation term (teta) to simplify, the vertical focusing along the fringe field exit of the pole as induced by this horizontal orbit perturbation x is also expressed by the relation :

$$\frac{1}{f_z}=\int_{s1}^{s2} K_z\, ds=\frac{-1}{B_0\rho}\int_{s1}^{s2} \tan(x')\frac{\partial B_z}{\partial s}\, ds\approx\frac{-1}{B_0\rho}\int_{0}^{B_0} x'\frac{\partial B_z}{\partial s}\, ds$$

with x'=Vx/V is the horizontal slope considered to be small. The K. Brown formulation is expressed as follow for arbitrary field profile :

$$\frac{1}{f_z}=\int_{s1}^{s2} K_z\, ds\approx -K_1\frac{g}{\rho^2} \qquad K_1=\frac{1}{g B_0^2}\int_{s1}^{s2} B_z(s)(B_{z0}-B_z(s))\, ds$$

with g the pole gap and B0 the field in the flat region well inside the magnet. There no easily available source were can be found the demonstration this relation, but an other derivation, in the specific case of a linear drop off off the field can be found (H. Wiedemann, Particles Accelerators Physics Books p. 131-134). In both cases, for a linear drop over dipole gap length, the K_1 integral is equal 1/6.

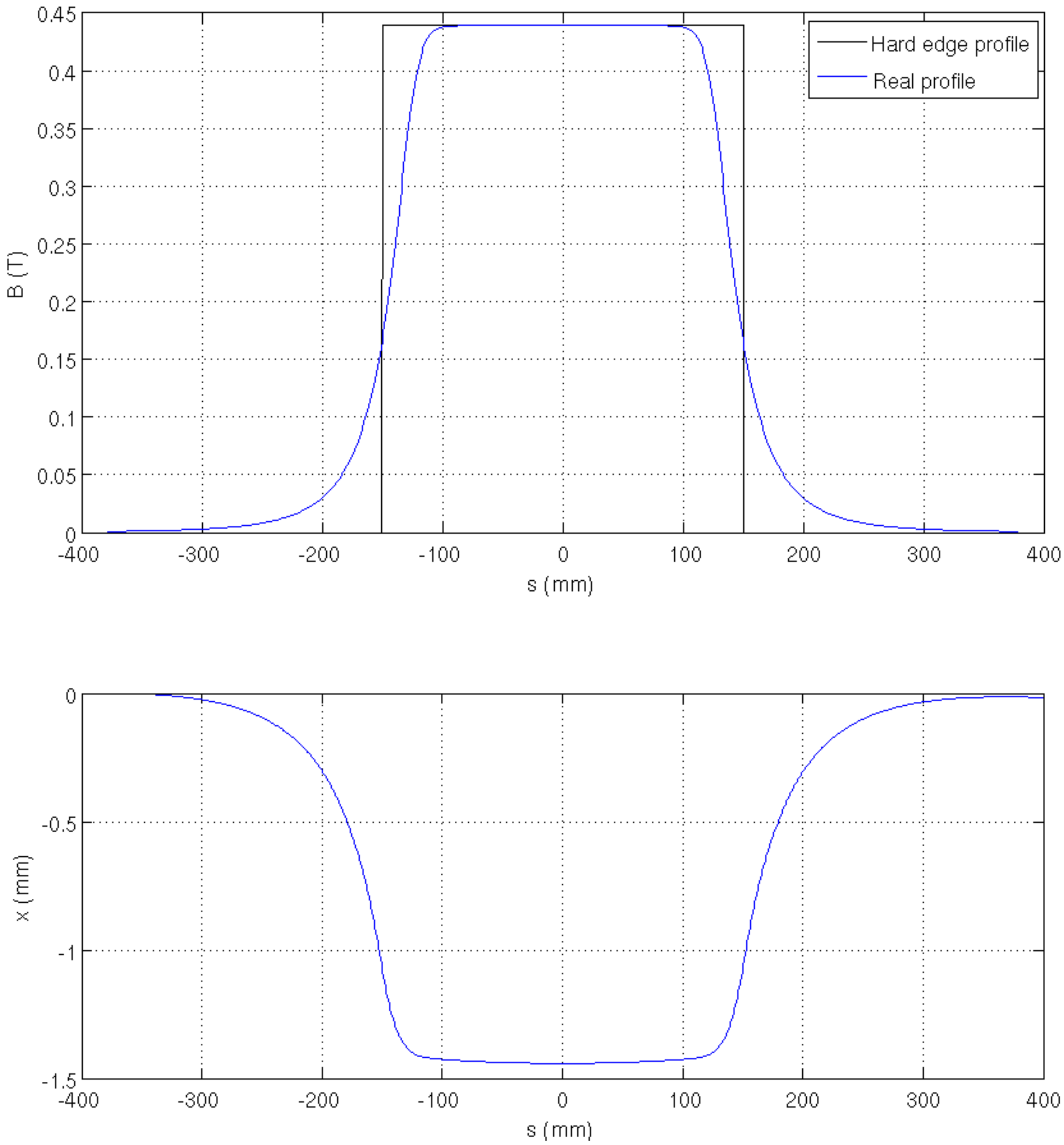


Figure 2 : Bz field profile and relative X orbit for Thom-X ring dipole (45 deg, r=0.352 m, Leff=300 mm, 50 MeV)

Linear fringe field

With the B drop profile expressed as $B(s)=B_0(1-\frac{s}{g})$, the K_1 integral is (Transport page 96) :

$$K_1=\frac{1}{gB_0^2}\int_0^g B_z(s)(B_0-B_z(s))ds=\frac{1}{g}\int_0^g(\frac{s}{g}-\frac{s^2}{g^2})ds=\frac{1}{6} \qquad \frac{1}{f_z}=\frac{g}{6\rho^2}$$

In the H. Wiedemann derivation, using the previous relations, with $B(s)=B_0(1-\frac{s}{g})$ and $dB(s)/ds=-B_0/g$ one get for the horizontal slope x' :

$$x'=\frac{1}{B\rho}\int_0^s \Delta B\, ds=\frac{1}{\rho}\int_0^s (s/g)ds=\frac{s^2}{2g\rho}$$

The focusing term is the given by

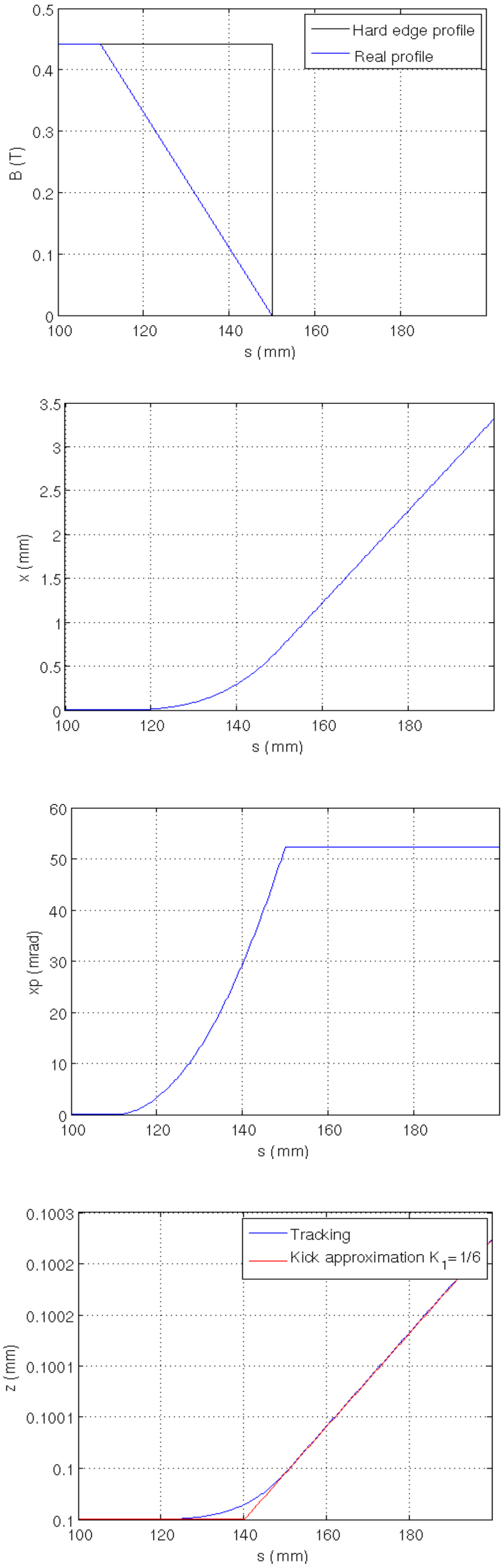
$$\frac{1}{f_z}=\frac{-1}{B_0\rho}\int_0^g x'\frac{\partial B_z}{\partial s}ds=\frac{1}{2\rho^2 g}\int_0^g s^2 ds=\frac{g}{6\rho^2}$$

The two results are in accordance but reveal that the horizontal orbit used is not closed and the slope x' presents a quadratic diverging behavior. The drop off of the field has been set such as the total integrated field is not conserved. The vertical quadrupole kick being strongly correlated to this slope, the kick strength may be really lower in case of closed orbit behavior (x'=0 at the end of the fringe field).

In the closed orbit case, one has to integrate over 0 to g/2 and multiply by 2 by symmetry :

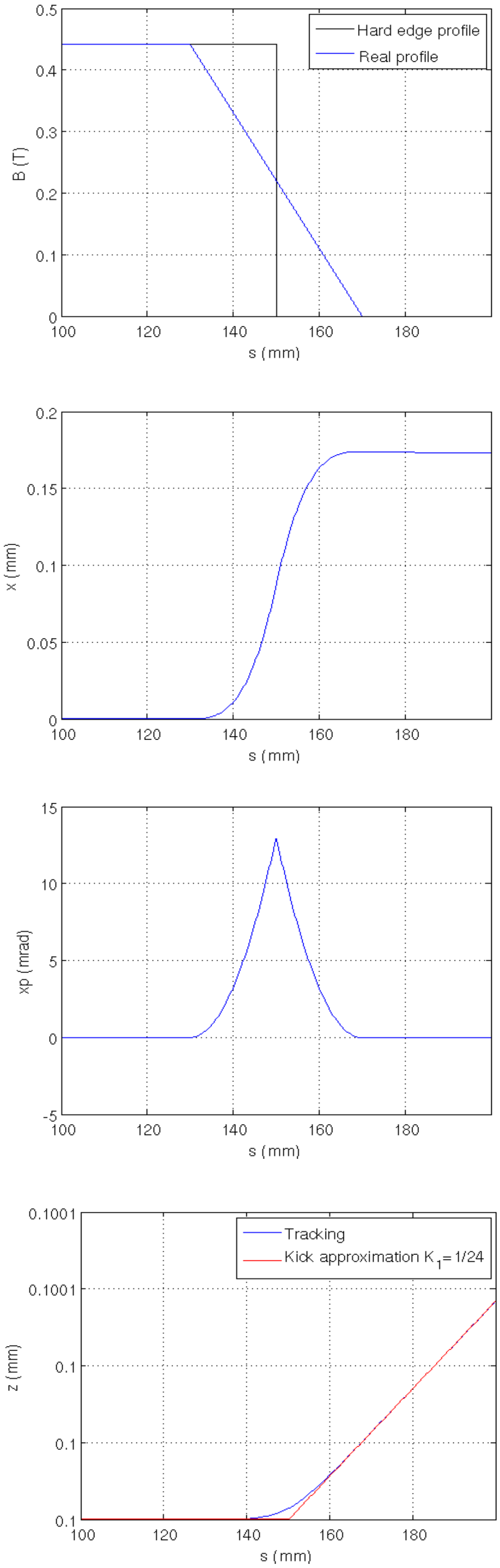
$$\frac{1}{f_z}=\frac{-2}{B_0\rho}\int_0^{g/2} x'\frac{\partial B_z}{\partial s}ds=\frac{1}{\rho^2 g}\int_0^{g/2} s^2 ds=\frac{g}{24\rho^2}$$

The effective strength is then 4 times weaker. To illustrate these differences, a set of plots are sketched in both cases, without closed orbit consideration and with. The plot are the following from top to bottom : B profile, horizontal orbit, horizontal slope x' and comparison between tracking in the field and kick effect in the vertical plane. The previous results are well depicted in this case of linear field drop off over one gap.



Case K. Brown/ H. Wiedemann

with diverging orbit or not closed.



Case with closed orbit

General fringe field

From the dynamic equation we derived the following relation for the vertical focusing effect :

$$\frac{1}{f_z} \approx \frac{-1}{B_0 \rho} \int_{s1}^{s2} x' \frac{\partial B_z}{\partial s} ds$$

The dimensionless K_1 integral, as defined by K. Brown is the given by :

$$K_1 = \frac{-\rho}{B_0 g} \int_{s1}^{s2} x' \frac{\partial B_z}{\partial s} ds$$

Using the well known trick of integration by part :

$$\int_{s1}^{s2} x' \frac{\partial B_z}{\partial s} ds = \int_{s1}^{s2} \frac{\partial (x' B_z)}{\partial s} ds - \int_{s1}^{s2} \frac{\partial x'}{\partial s} B_z ds$$

The first term of left hand side is null and K_1 integral is then given by :

$$K_1 = \frac{\rho}{B_0 g} \int_{s1}^{s2} \frac{\partial x'}{\partial s} B_z ds$$

With lack of deviation or non-closed orbit, the differential horizontal slope is $dx'/ds = (B_0 - B_z)/B_0 \rho$ leading to K. Brown relation :

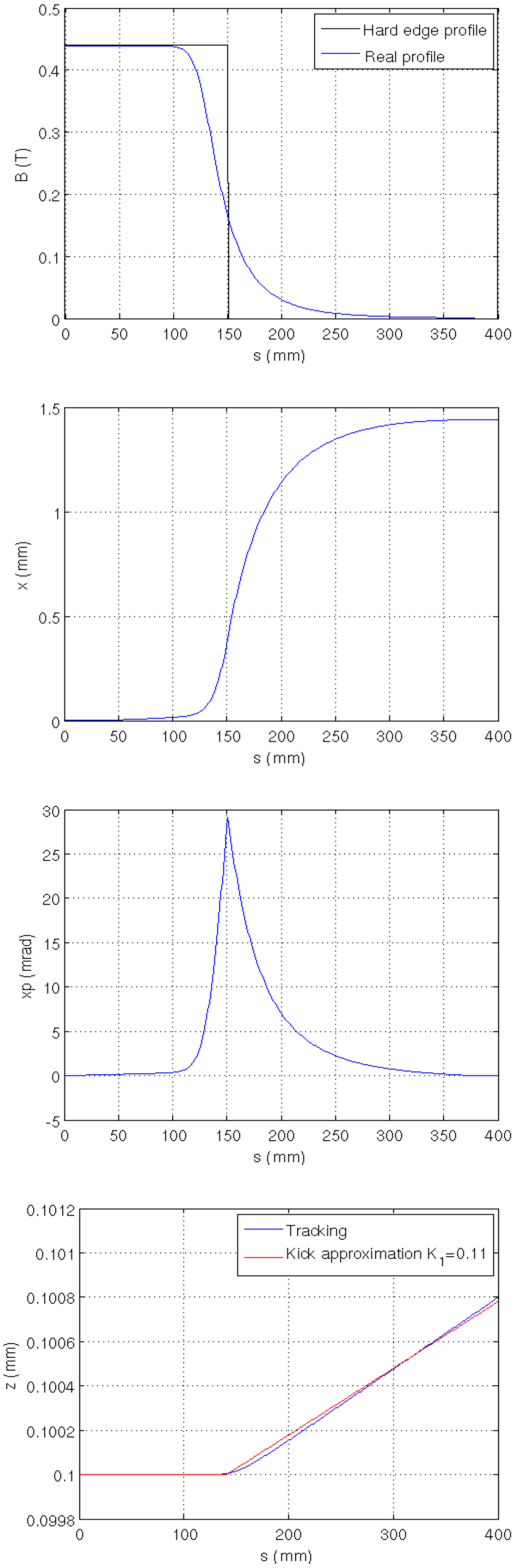
$$K_1 = \frac{1}{B_0^2 g} \int_{s1}^{s2} B_z (B_0 - B_z) ds$$

With closed orbit, the differential horizontal slope is $dx'/ds = (B_0 - B_z)/B_0 \rho$ from s_1 to s_0 (hard edge limit) and to $dx'/ds = -B_z/B_0 \rho$ from s_0 to s_2 leads to the modified K. Brown relation :

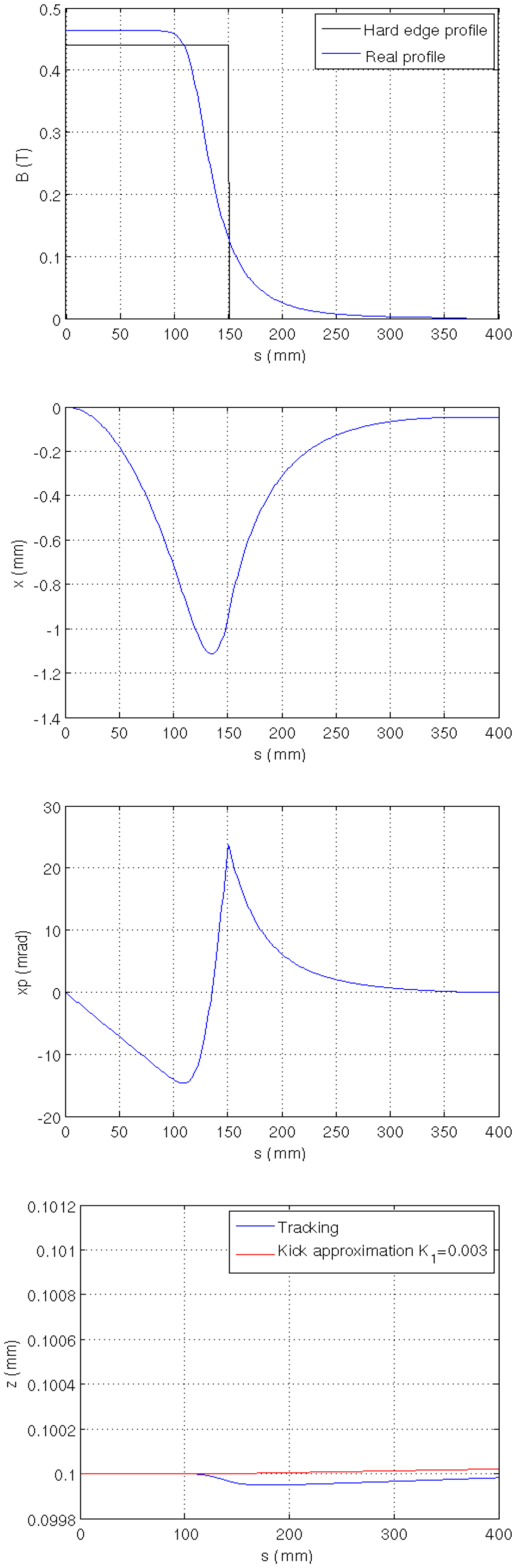
$$K_1 = \frac{1}{B_0^2 g} \left[\int_{s1}^{s0} B_z (B_0 - B_z) ds - \int_{s0}^{s2} B_z^2 ds \right]$$

Finally, depending on how the dipole effective length is set relative to the geometric arc and total deviation preservation as well, there is many different issues from this last relation. It can even be canceled with a shorter effective length. Nevertheless, in the standard case, where the dipole effective length is equal to the arc length and deviation preserved, this last dimensionless integral is about 4 times smaller than the usual relation derived by K. Brown as previously shown for the linear drop off case.

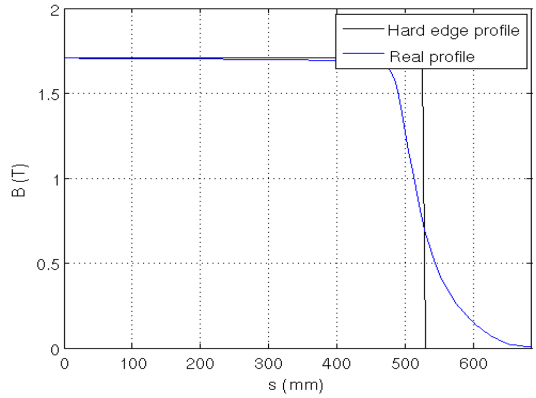
To illustrate these differences, a set of plots are sketched in tow cases for the Thom-X dipole modelized by RADIA code: with the effective length fitted to the arc length (300 mm) and with an effective length shorter in order to cancel the defocusing vertical correction terms. The plot are the following from top to bottom : B profile, horizontal orbit, horizontal slope x' and comparison between tracking in the field and kick effect in the vertical plane. The kick model is based on the modified dimensionless integral.



Case with effective length fitted to the arc length and orbit closed.



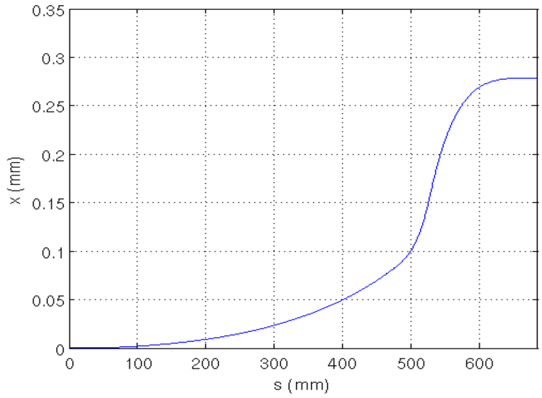
Case with effective length shorter to cancel the vertical kick



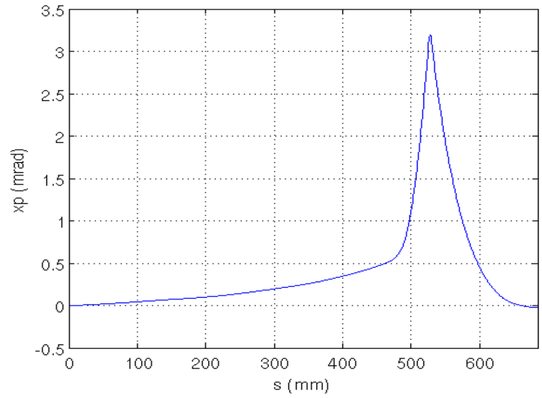
Application to the SOLEIL storage ring dipole :

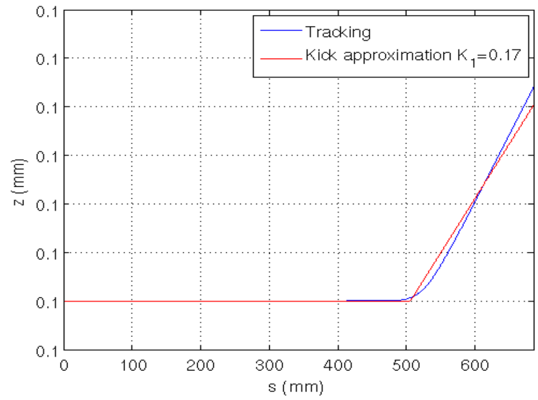
Lff=1.05 m, Bmax=1.71 T

Straigth dipole of pi/16 deviation.



The horizontal orbit depression is about 0.3 mm





The K_1 integral is only 0.17 leading to a fringe field length Lff=6*K1*gap=38 mm.

The present value in used is K_1=0.7, leading to Lff=160 mm.

Here again, a reduced factor by 4 is expected. The impact is small, about 0.015 in vertical tune shift.