1

a. b. c. d. e.

2

a. b. c. d. e. f.

3

a. $\delta_1(t) = \lim_{a\to\infty} \frac{a}{2} e^{-a|t|}$
Property #1
$\delta_1(t-t_0) = \lim_{a\to\infty} \frac{a}{2} e^{-a|t-t_0|}$

Let's explore two cases $|t-t_0| = 0$ and $|t-t_0| = m \neq 0$, in case 1, the limit takes the value of $\lim_{a\to\infty} \frac{a}{2} e^0 = \lim_{a\to\infty} \frac{a}{2}$ which is clearly non-zero.

When $|t-t_0| = m \neq 0$ the limit is:

$$\lim_{a\to\infty} \frac{a}{2} e^{-am} = \lim_{a\to\infty} \frac{\frac{a}{2}}{e^{am}}$$

using L'Hopital's rule we get that:

$$= \lim_{a\to\infty} \frac{\frac{1}{2}}{me^{am}} = 0$$

Property #2

$$\lim_{a\to\infty} \int_{-\infty}^{\infty} \frac{a}{2} e^{-a|t|}\, dt = \lim_{a\to\infty} \frac{a}{2} 2 \int_0^{\infty} e^{-at}\, dt$$
$$= \lim_{a\to\infty} \frac{1}{a} e^{-at}|_0^{\infty} = \lim_{a\to\infty} \frac{a}{a}$$
$$= 1$$

b. $\delta_2(t) = \lim_{a\to 0} \frac{1}{2a} \Pi(\frac{t-a}{2a})$
Property #1

$$\delta_2(t-t_0) = \lim_{a\to 0} \frac{1}{2a} \Pi(\frac{(t-t_0)-a}{2a})$$

By using standard shifting and scaling we can write $\Pi(\frac{(t-t_0)-a}{2a})$ piecewise:

$$\Pi(\frac{(t-t_0)-a}{2a}) = \begin{cases} 1 & : t \in [t_0, t_0+2a] \\ 0 & : t \notin [t_0, t_0+2a] \end{cases}$$

As the $a \to 0$ the only value that stays consistantly non-0 is $t = t_0$ all the other values become zero.
Property #2

$$\lim_{a\to 0} \int_{-\infty}^{\infty} \frac{1}{2a} \Pi(\frac{t-a}{2a}) dt = \lim_{a\to 0} \frac{1}{2a} \int_0^{2a} dt = \lim_{a\to 0} \frac{2a}{2a}$$
$$= 1$$

4

a.

$$z \times z^* = (re^{j\theta})(re^{-j\theta}) = r^2 e^{j(\theta-\theta)=r^2}$$

b.

$$\frac{z}{z^*} = \frac{re^{j\theta}}{re^{-j\theta}} = \frac{r}{r}e^{j(\theta+\theta)} = e^{2j\theta}$$

c.

$$(z_1 z_2)^* = (r_1 e^{j\theta_1} r_2 e^{j\theta_2})^* = (r_1 r_2 e^{-j(\theta_1+\theta_2)})^* = r_1 r_2 e^{-j(\theta_1+\theta_2)}$$

$$z_1^* z_2^* = (r_1 e^{j\theta_1})^* (r_2 e^{j\theta_2})^* = (r_1 e^{-j\theta_1})(r_2 e^{-j\theta_2}) = r_1 r_2 e^{-j(\theta_1+\theta_2)}$$

$$\rightarrow z_1^* z_2^* = (z_1 z_2)^*$$

d.

$$(\frac{z_1}{z_2})^* = (\frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}})^* = (\frac{r_1}{r_2} e^{j(\theta_1-\theta_2)})^* = \frac{r_1}{r_2} e^{j(\theta_2-\theta_1)}$$

$$\frac{z_1^*}{z_2^*} = \frac{r_1 e^{-j\theta_1}}{r_2 e^{-j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_2-\theta_1)}$$

$$\rightarrow \frac{z_1^*}{z_2^*} = (\frac{z_1}{z_2})^*$$

5

a. $u(t)$

$$Odd = \frac{u(t) - u(-t)}{2}$$

$$Evem = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$$

b. $Cos(2\pi t)u(t)$

$$Odd = \frac{Cos(2\pi t)u(t) - Cos(-2\pi t)u(-t)}{2} = \frac{Cos(2\pi t)}{2}(u(t) - u(-t))$$

$$Even = \frac{Cos(2\pi t)u(t) + Cos(-2\pi t)u(-t)}{2} = \frac{Cos(2\pi t)}{2}(u(t) + u(-t))$$

$$= \frac{Cos(2\pi t)}{2}$$

6

a. $h(t) = e^t u(t)$ is not *BIBO* stable system, an example of a bounded input resulting in an unbound output is $x(t) = u(t)$ which results in the output

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} u(\tau) e^{t-\tau} u(t-\tau)\, d\tau$$

$$= \int_0^t e^{t-\tau}\, d\tau = e^t \int_0^t e^{-\tau}\, d\tau =$$

$$= \boxed{e^t(1 - e^{-t})}$$

which is an unbound function.

b. $h(t) = (t-1)^2 e^{1-t} u(t)$ The system is *BIBO* stable since:

$$\begin{aligned} y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} |u(t) e^{1-t} (t-1)^2 \, dt \\ = e \end{aligned}$$

which is bound, hance the integral is bound.

c. $h[n] = u[n-4]$ is not *BIBO* stable system, an example of a bounded input resulting in an unbound output is $x(t) = u(t)$ which results in the output:

$$\begin{aligned} y[t] = h[t] * x[t] = \sum_{k=-\infty}^{\infty} u[k] u[n-k-4] = \sum_{k=0}^{n-4} 1 \\ = n - 4 + 1 = \boxed{n-3} \end{aligned}$$

which is an unbound function. Therefore, the system is not BIBO stable

d. $Cos[2\pi n] u[n]$ is not *BIBO* stable system, an example of a bounded input resulting in an unbound output is $x[n] = u[n]$ which results in the output

$$y[t] = h[t] * x[t] = \sum_{k=-\infty}^{\infty} u[k] u[n-k] = \sum_{k=0}^{n} 1 = \boxed{n+1}$$

which is clearly unbound.

e. $\sum_{n=-\infty}^{\infty} \delta(t-2n)$ is not *BIBO* stable system, an example of a bounded input resulting in an unbound output is $x(t) = u(t)$ which results in the output

$$\begin{aligned} y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} u(t-\tau) \sum_{n=-\infty}^{\infty} \delta(\tau - 2n) \, d\tau \\ = \int_{-\infty}^{t} \sum_{n=-\infty}^{\infty} \delta(\tau - 2n) \, d\tau = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{t} \delta(\tau - 2n) \, d\tau \\ = \sum_{n=-\infty}^{\lfloor \frac{t}{2} \rfloor} 1 = \boxed{\infty} \end{aligned}$$

which is clearly unbound

7

a. $x(t) = \Pi(t-1); h(t) = r(t)$

$$\begin{aligned}
y(t) = x(t) * h(t) &= \int_{-\infty}^{\infty} r(\tau)\Pi(t-\tau-1)\, d\tau \\
&= \int_{(t-1)-\frac{1}{2}}^{(t-1)+\frac{1}{2}} r(\tau)d\tau = \int_{t-\frac{3}{2}}^{t-\frac{1}{2}} r(\tau)d\tau \\
&= \begin{cases} \int_{t-\frac{3}{2}}^{t-\frac{1}{2}} \tau d\tau & : t \geq \frac{3}{2} \\ \int_{0}^{t-\frac{1}{2}} \tau d\tau & : t \in [\frac{1}{2}, \frac{3}{2}) \\ 0 & : t < \frac{1}{2} \end{cases} \\
&= \boxed{\begin{cases} \frac{(t-\frac{1}{2})^2-(t-\frac{3}{2})^2}{2} & : t \geq \frac{3}{2} \\ \frac{(t-\frac{1}{2})^2}{2} & : t \in [\frac{1}{2}, \frac{3}{2}) \\ 0 & : t < \frac{1}{2} \end{cases}}
\end{aligned}$$

b. $x(t) = e^{-t}u(t); h(t) = \Pi(t-\frac{1}{2})$

$$\begin{aligned}
y(t) = x(t) * h(t) &= \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\Pi(t-\tau-\frac{1}{2})\, d\tau \\
&= \int_{(t-\frac{1}{2})-\frac{1}{2}}^{(t-\frac{1}{2})+\frac{1}{2}} e^{\tau}u(\tau)d\tau = \int_{t-1}^{t} e^{\tau}u(\tau)(\tau)d\tau \\
&= \begin{cases} \int_{t}^{t-1} e^{\tau}d\tau & : t \geq 1 \\ \int_{0}^{t} e^{\tau}d\tau & : t \in [0,1) \\ 0 & : t < 0 \end{cases} \\
&= \boxed{\begin{matrix} e^t - e^{t-1} & : t \geq 1 \\ e^t & : t \in [0,1) \\ 0 & : t < 0 \end{matrix}}
\end{aligned}$$

c. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-\frac{1}{2}-n); h(t) = \Pi(t)Sin(2\pi t)$

$$y(t) = x(t) * h(t) = \sum_{n=-\infty}^{\infty} \delta(t-\frac{1}{2}-n) * h(t) = \sum_{n=-\infty}^{\infty} h(t-\frac{1}{2}-n) =$$

$$\sum_{n=-\infty}^{\infty} \Pi(t-\frac{1}{2}-n)Sin(2\pi(t-\frac{1}{2}-n)) = \sum_{n=-\infty}^{\infty} \Pi(t-\frac{1}{2}-n)Sin(2\pi t-\pi-2n\pi) =$$

$$-\sum_{n=-\infty}^{\infty} \Pi(t-\frac{1}{2}-n)Sin(2\pi t) =$$

$$\boxed{Sin(2\pi t)}$$