1) a. The impulse response of the system $y(t) = x(t) + \alpha y(t-T)$ is simply:

$$h(t) = \sum_{k=0}^{\infty} \alpha^k \delta(t - kT)$$

b. The system is BIBO stable if $\int_{-\infty}^{\infty} |h(t)|dt$ is bound. In our case we get that this integral is equal:

$$\int_{-\infty}^{\infty} \left| \sum_{k=0}^{\infty} \alpha^k \delta(t-kT) \right| dt$$

If $\alpha$ is positive we get that:

$$\sum_{k=0}^{\infty} \int_{0}^{\infty} \alpha^k \delta(t-kT)dt = \sum_{k=0}^{\infty} \alpha^k$$

This is bound if $\alpha < 1$ and unbound otherwise. Hence, if $0 \leq \alpha < 1$ the system is BIBO stable, and it is unstable otherwise.

c.

$$h(t) = \sum_{k=0}^{\infty} \alpha^k \delta(t - kT)$$

can also be inverted with the LTI system

$$\boxed{h_1(t) = \delta(t) - \alpha\delta(t-T)}$$

We can show this by looking at the properties of the convolution:

$$\begin{aligned} y(t) &= x(t) * h(t) \\ h_1(t) * y(t) &= h_1(t) * \Big(x(t) * h(t)\Big) \\ &= \Big(h_1(t) * h(t)\Big) * x(t) \end{aligned}$$

Therefore, if $(h_1(t) * h(t)) = \delta(t)$ (which is trivial) we get that

$$h_1(t) * y(t) = x(t)$$

2) a. Starting with the problem

$$\ddot{y}(t) + 300\dot{y}(t) + 2 \times 10^4 y(t) = 10^3 \dot{x}(t)$$

we integrate it over time twice to yield

$$\begin{aligned} y(t) + 300 \int y(t)dt + 2 \times 10^4 \iint y(t)dt^2 &= 10^3 \int x(t)dt \\ y(t) &= 10^3 \int x(t)dt - 300 \int y(t)dt - 2 \times 10^4 \iint y(t)dt^2 \end{aligned}$$

For the diagram, see the attached.

b.

$$\ddot{y}(t) + 300\dot{y}(t) + 2 \times 10^4 y(t) = 10^3 \dot{x}(t)$$

Since $e^{jwt}$ is an eigenfunction for the above LTI system, we get that

$$\begin{aligned}
y(t) &= A_w e^{jwt} \\
\dot{y}(t) &= A_w (jw) e^{jwt} \\
\ddot{y}(t) &= A_w (wj)^2 e^{jwt} \\
\dot{x}(t) &= (jw) e^{jwt}
\end{aligned}$$

Substituting all of these into the equation yields:

$$\begin{aligned}
A_w (jw)^2 e^{jwt} + A_w 300(jw) e^{jwt} + A_w 2 \times 10^4 e^{jwt} &= 10^3 e^{jwt} \\
e^{jwt} A_w \Big( (jw)^2 + 300(jw) + 10^4 \Big) &= e^{jwt} 10^3
\end{aligned}$$

$$\begin{aligned}
A_w &= \frac{10^3}{-w^2 + 10^4 + 300wj} \\
&= \boxed{\frac{10^3(-w^2 + 10^4 - 300wj)}{(-w^2 + 10^4)^2 + (300w)^2}}
\end{aligned}$$

3) a. For the diagram, see attached.

b.

$$y[n] + 20y[n-1] + 1700y[n-2] = x[n] + 20x[n-1]$$

Since $e^{jwn}$ is an eigenfunction for the above LTI system, we get that

$$\begin{aligned}
y[n] &= A_w e^{jwt} \\
y[n-1] &= A_w e^{jwn} e^{-jw} \\
y[n-2] &= A_w e^{jwn} e^{-2jw} \\
x[n] &= e^{jwn}
\end{aligned}$$

Substituting all of these into the equation yields:

$$\begin{aligned}
A_w e^{jwt} + A_w e^{jwt} 20 e^{-jw} + A_w e^{jwt} 1700 e{-}2jw &= e^{jwt} + e^{jwt} 20 e^{-jw} \\
A_w e^{jwt} (1 + 20e^{-jw} + 1700e{-}2jw) &= e^{jwt} (1 + 20e^{-jw})
\end{aligned}$$

$$\boxed{A_w = \frac{(1 + 20e^{-jw})}{(1 + 20e^{-jw} + 1700e{-}2jw)}}$$

4) a.

$$\Pi(t/8) * \mathrm{comb}(t/10)$$

By examination of the signal, we get that the period is $T_0 = 10$ and the fundamental frequency is $\boxed{w_0 = \frac{\pi}{5}}$. We can compute $a_k$ by:

$$\begin{aligned}
a_k &= 10 \int_{-4}^{4} e^{-jw_0 kt} dt \\
&= 10 \frac{1}{-jw_0 kt} \left( e^{-jw_0 k4} - e^{jw_0 k4} \right) \\
&= \boxed{\frac{20 \sin(4w_0 k)}{10 w_0 k}}
\end{aligned}$$

b.

$$\Pi(4t) * \text{comb}(t/10)$$

By examination of the signal, we get that the period is $T_0 = 10$ and the fundamental frequency is $\boxed{w_0 = \frac{\pi}{5}}$. We can compute $a_k$ by:

$$\begin{aligned} a_k &= 10 \int_{-\frac{1}{8}}^{\frac{1}{8}} e^{-jw_0kt} dt \\ &= \frac{10}{-jw_0kt} \left( e^{-jw_0k\frac{1}{8}} - e^{jw_0k\frac{1}{8}} \right) \\ &= \boxed{\frac{20\sin(\frac{1}{8}w_0k)}{w_0k}} \end{aligned}$$

c.

$$\Big(\Pi(t-1) * \Pi(t/2)\Big) * \text{comb}(t/10)$$

By examination of the signal, we get that the period is $T_0 = 10$ and the fundamental frequency is $\boxed{w_0 = \frac{\pi}{5}}$. We can compute $a_k$ by:

$$\begin{aligned} a_k &= 10 \int_{-\frac{1}{2}}^{\frac{5}{2}} e^{-jw_0kt} \Big(\Pi(t-1) * \Pi(t/2)\Big) dt \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jw_0kt} \left(\frac{1}{2} + t\right) dt + \int_{\frac{1}{2}}^{\frac{3}{2}} e^{-jw_0kt} dt + \int_{\frac{3}{2}}^{\frac{5}{2}} e^{-jw_0kt} \left(\frac{5}{2} - t\right) dt \end{aligned}$$

$$\boxed{a_k = \frac{4e^{jkw} \sin\left(\frac{kw}{2}\right) \sin(kw)}{k^2w^2}}$$

5. a.

$$a_k = \sum_{n=0}^{8-1} (\delta[n-1] + \delta[n-2]) e^{-jnk\frac{2\pi}{8}}$$

$$\boxed{a_k = e^{-jk\frac{\pi}{4}} + e^{-jk\frac{\pi}{2}}}$$

b.

$$\begin{aligned} a_k &= \sum_{n=0}^{32-1} (-1)^n \ e^{-jnk\frac{2\pi}{32}} \\ &= \sum_{n=0}^{32-1} (-e^{-jk\frac{2\pi}{32}})^n \\ &= \frac{1 + e^{-jk(32)\frac{2\pi}{32}}}{1 + e^{-jk\frac{2\pi}{32}}} \end{aligned}$$

$$\boxed{a_k = \frac{1 + e^{-jk2\pi}}{1 + e^{-jk\frac{\pi}{16}}}}$$

6. a. $x_1(t) = \cos\left(t\frac{2\pi}{(\frac{1}{60})}\right)$ Therefore $T_0 = \frac{1}{60}$ and $w_0 = \frac{2\pi}{(\frac{1}{60})}$, we can use Euler's identity to write $x_1(t)$ as:

$$x_1(t) = \frac{e^{tjw_0} + e^{-tjw_0}}{2}$$

hence the Fourier Series is:

$$x_1(t) = \frac{1}{2}e^{tjw_0} + \frac{1}{2}e^{-tjw_0}$$

And its power is ultimately

$$\boxed{P = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}}$$

b. **[TODO]**

c. **[TODO]**

d. **[TODO]**