MinAvgTwoSlice

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August 27, 2016

Problem. See `https://codility.com/programmers/task/min_avg_two_slice/`

Martin Kysel claimed that we only need to check slices of size 2 and 3 as the min average should be in the sub-slices.[1] In this note, I just want to give a formal proof for this observation.

Without loss of generality, consider a non-empty array $A = A_1, A_2, \ldots, A_N$ and an interger $1 < i < N$. We consider the average of A and its two sub-arrays $A_1, \ldots, A_i$ and $A_{i+1}, \ldots, A_N$. Let

$$B = \frac{A_1 + \ldots + A_N}{N} \qquad C = \frac{A_1 + \ldots + A_i}{i} \qquad D = \frac{A_{i+1} + \ldots + A_N}{N - i}.$$

We claim that either $C \leq B$ or $D \leq B$ $(\star)$. Indeed, assume the opposite, that is, $B < C$ and $B < D$. We have that:

$$\begin{aligned}
B < C \quad &\Leftrightarrow \quad \frac{A_1 + \ldots + A_N}{N} < \frac{A_1 + \ldots + A_i}{i} \\
&\Leftrightarrow \quad i \times (A_1 + \ldots + A_N) < N \times (A_1 + \ldots + A_i) \\
&\Leftrightarrow \quad i \times (A_{i+1} + \ldots + A_N) < (N - i) \times (A_1 + \ldots + A_i) \\
&\Leftrightarrow \quad \frac{A_{i+1} + \ldots + A_N}{N - i} < \frac{A_1 + \ldots + A_i}{i} \\
&\Leftrightarrow \quad D < C
\end{aligned}$$

Similarly, $B < D \Leftrightarrow C < D$.

Then $B < C$ and $B < D$ means $D < C$ and $C < D$, a contradiction! Therefore, claim $(\star)$ holds. What follows is simple, since a slice of size bigger than 3 is composed of sub-slices of size 2 or 3.

[1] `https://www.martinkysel.com/codility-minavgtwoslice-solution/`