Investigation of the Electron Charge-to-Mass Ratio

Chaitanya Kumar Mahajan Alex He-Mo

October 31 2022

1 Introduction

1.1 Abstract

The aim of this investigation is to find the electron charge mass ratio by passing an electron beam through a magnetic field created by Helmholtz Coils to measure the radius of curvature. The electron beam is created from an electron gun which is passed through an accelerating voltage. Varying the accelerating voltage and the current in the Helmholtz Coils, we can measure variety of trajectories. From this we can calculate the electron charge mass ratio using the equations mentioned below.

1.2 Background Information

A particle with some charge e and velocity v will experience a force while moving in a magnetic field with strength B which is equal to $F_{\text{magnetic field}} = ev \times B$. When the particle moves in a direction perpendicular to the direction of the magnetic field, the force will always be perpendicular to the direction of movement, leading to a circular orbit and centripetal motion. This centripetal force would be

$$ev \times B = \frac{mv^2}{r}. \tag{1}$$

When the particle is accelerated using a voltage potential difference V, its kinetic energy is

$$\frac{1}{2}mv^2 = eV. \tag{2}$$

We can combine these two equations by manipulating $\frac{1}{2}mv^2 = eV$ into $v = \sqrt{\frac{2eV}{m}}$ and then substituting this equation for v in $ev \times B = \frac{mv^2}{r}$. We arrive at

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}} \tag{3}$$

Thus, as mass and charge of the particle constant, we can manipulate the strength of the magnetic field and the voltage difference for accelerating the particle to find a relationship between them and the radius of the trajectory in the magnetic field, potentially being able to find $\sqrt{\frac{e}{2m}}$ and thus the charge-mass ratio of the particle. The strength B of the magnetic field is represented by

$$B = \frac{\mu_0 I R^2}{2\sqrt{R^2+z^2}^{3/2}} \tag{4}$$

where R is the coil separation distance, μ_0 is the permittivity of free space, z is the distance from the coil, and I is the current through the coil. Using this equation, the strength of the magnetic field from the coil is,

$$B_c = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n I}{R} \tag{5}$$

If we take $k = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}$, we can manipulate B through the relationship $B = I \cdot k$. However, the strength of the magnetic field is also affected by the external magnetic field from outside the experiment B_e, where $B = B_c + B_e$, where B_c is the field from the coil. Thus, we can combine this idea with equation 3 to arrive at

$$\frac{1}{r} = \sqrt{\frac{e}{m}} k \frac{I + \frac{1}{\sqrt{2}} I_0}{\sqrt{V}} \tag{6}$$

where I_0 is defined as B_e/k.

1.3 Materials and Apparatus

1. Electron Gun
2. Glass Bulb (with Helium)
3. Helmholtz Coils
4. Power Supplies (for Coils and Electron Gun/Anode)
5. Rheostat
6. Two Multimeters
7. Self-Illuminating Measurement Scale

1.4 Procedure

1. Connected the apparatus in accordance with the Figure (1).
2. Turned on the electron gun power supply and allowed it to heat up before proceeding.
3. Turned on the anode power supply and the coils power supply.
4. Rotated the glass bulb until the visible electron trajectory was in the form of a circle, see Figure (5) in appendix.
5. Used the illumination scale to measure the diameter of the electron trajectory at the different current values for some constant accelerating voltage, as reported in Part I of Results.
6. Used the illumination scale to measure the diameter of the electron trajectory at the different accelerating voltage values, keeping coil current constant as reported in Part II of Results.

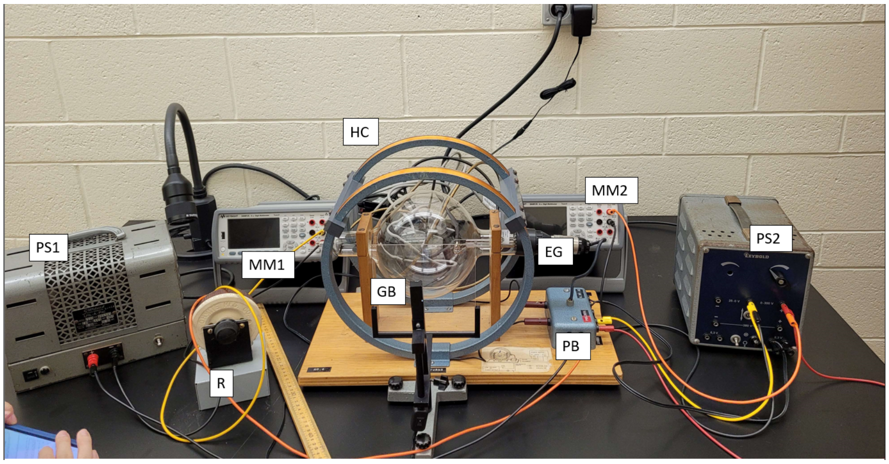

Figure 1: Apparatus setup. Legend: EG - Electron Gun, GB - Glass Bulb, HC - Helmholtz Coils, MM1 - Coil Ammeter, MM2 - Anode Voltmeter, PB - Power Box, PS1 - Coils Power Supply, PS2 - Anode and Filament Power Supply, R - Rheostat.

2 Results and Analysis

Extra Measurements on the Helmholtz Coils:

- Radius of Coils - 16.1 ± 0.1 cm
- Distance between Coils - 15.9 ± 0.1 cm
- Number of turns of the coil - 130

2.1 Part I

It was noticed that both with high current and low accelerating voltage, the electron arc would become very faint and barely visible when observed with the self-illuminated scale. This introduced a massive error because often the illuminated scale will have to be turned off in order to clearly see the trajectory. A solution to this was to introduce some sort of physical marker which would appear in the same place from the perspective of looking at the arc through the self-illuminated scale, so that it would be a substitute which could be measured to an error of within one millimeter.

We also ensured that parallax was not an error by ensuring that the self-illuminated scale and reflector were perfectly horizontal and by making two separate measurements by different people. By averaging the two different measurements we get an accurate result, not impacted by parallax.

In terms of the effects of other ferromagnetic materials on the experiment, we found that their effects were negligible, and at most within error of the illuminated scale. Our electronics (cell phone, laptop) could only visibly affect the electron trajectory when almost between the two Helmholtz coils, and otherwise were kept at least a meter away from the glass bulb.

Table 1: Coil Current and Radius of the Electron Arc when Voltage is $V = 200.2 \pm 0.5V$

Current (A)	**Current Uncertainty (± A)**	**Radius of Arc (m)**	**Uncertainty (± m)**
1.715	0.001	0.032	0.001
1.665	0.001	0.034	0.001
1.603	0.001	0.035	0.001
1.525	0.001	0.037	0.001
1.426	0.001	0.039	0.001
1.324	0.001	0.042	0.001
1.232	0.001	0.045	0.001
1.473	0.001	0.037	0.001
1.380	0.001	0.040	0.001
1.261	0.001	0.043	0.001

To correct for the consideration that the magnetic field is weaker farther away from the centre of the coils, we found that the electron trajectory began approximately at a distance of 5cm from the center of the coils (approximately 3cm from the edge), which was used in our calculations to adjust the perceived strength of the magnetic field.

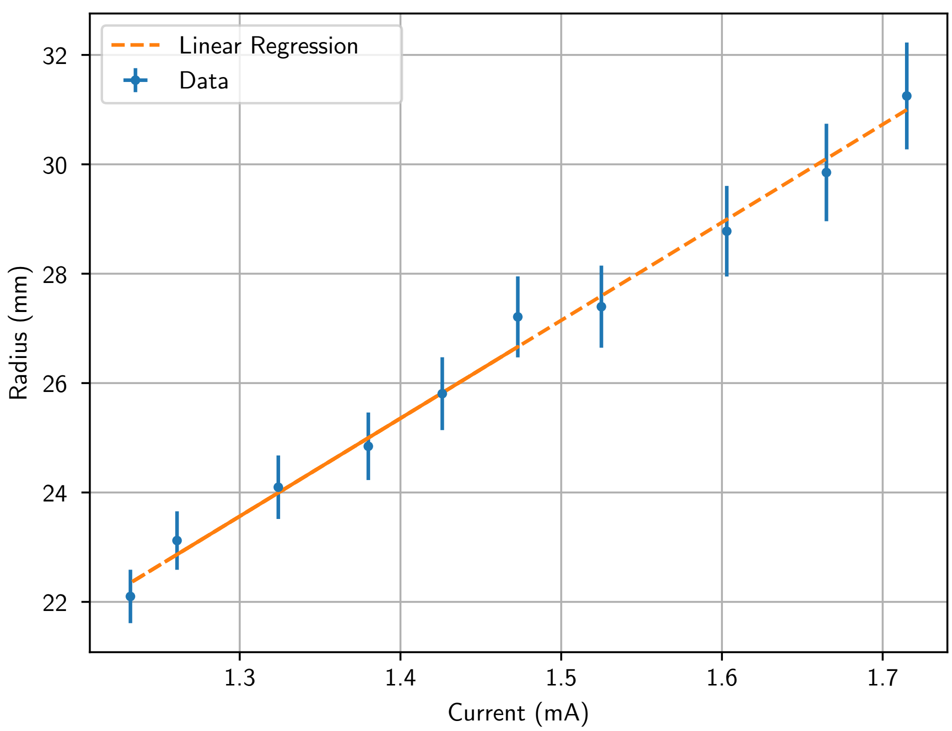

Figure 2: Change in electron trajectory radius as a function of the current in the coils. *Current error bars are present, however they are* ≈ 100 *times smaller*. Data points are from Table 1 above.

Using our Python script, we found that the suggested relationship between $\dfrac{1}{r}$ and I is $\dfrac{1}{r} = 17.909I +$

0.2829, which is a linear relation between $\frac{1}{r}$ and I of 17.909. The slope is 17.909 ± 1.421 Using equation 6, we can see that $17.909 = \sqrt{\frac{e}{m}} k \frac{1}{\sqrt{V}}$ meters^{-1}. Substituting $k = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n}{R}$ and $V = 200.2$, we can find that $\frac{e}{m}$ is approximately equal to 243630115819.46136, which is around $2.4 \cdot 10^{11}$ C/kg. We found the uncertainty to be around $\pm 0.4367 \cdot 10^{11}$, which means that our empirical charge-to-mass ratio is $2.4 \pm 0.4 \cdot 10^{11}$ C/kg.
Using our Python script, we found that $\frac{1}{r} = 17.909I + 0.2829$, which suggests that I_0 is the y-intercept, 0.2829 ± 0.9942. Using equation 6, we know that the y-intercept would be equal to $\sqrt{\frac{e}{2m}} \frac{k}{\sqrt{V}} \frac{1}{\sqrt{2}} I_0$, for which we can substitute our aforementioned values, to get $I_0 = 0.2829 \cdot \sqrt{2}/17.909 = 0.022$. To find $B_e = k \cdot I_0$, we can use the values to get $B_e = 0.00051 \cdot I_0 = 11 \cdot 10^{-6} T$, which is slightly lower than expected.
Note that there was a massive error ($> \pm 100\%$) when using `scipy.optimize.curvefit` with the y-intercept, and thus the true intercept may be very different. The error was found to be around $\pm 32 \cdot 10^{-6}$ T.

2.2 Part II

Table 2: Accelerating Voltage and Radius of Electron Arc when Coil Current is $I = 1.510 \pm 0.001A$

Voltage (V)	Uncertainty (V)	Radius (mm)	Uncertainty (mm)
151.5	0.5	32.0	0.5
160.5	0.5	33.3	0.5
170.8	0.5	34.3	0.5
181.2	0.5	35.5	0.5
191.2	0.5	37.5	0.5
204.8	0.5	38.5	0.5
220.5	0.5	39.8	0.5
232.5	0.5	41.8	0.5
241.5	0.5	42.3	0.5
251.6	0.5	43.0	0.5
262.6	0.5	43.5	0.5
271.5	0.5	44.0	0.5
284.2	0.5	45.3	0.5
291.2	0.5	46.0	0.5
301.4	0.5	46.8	0.5

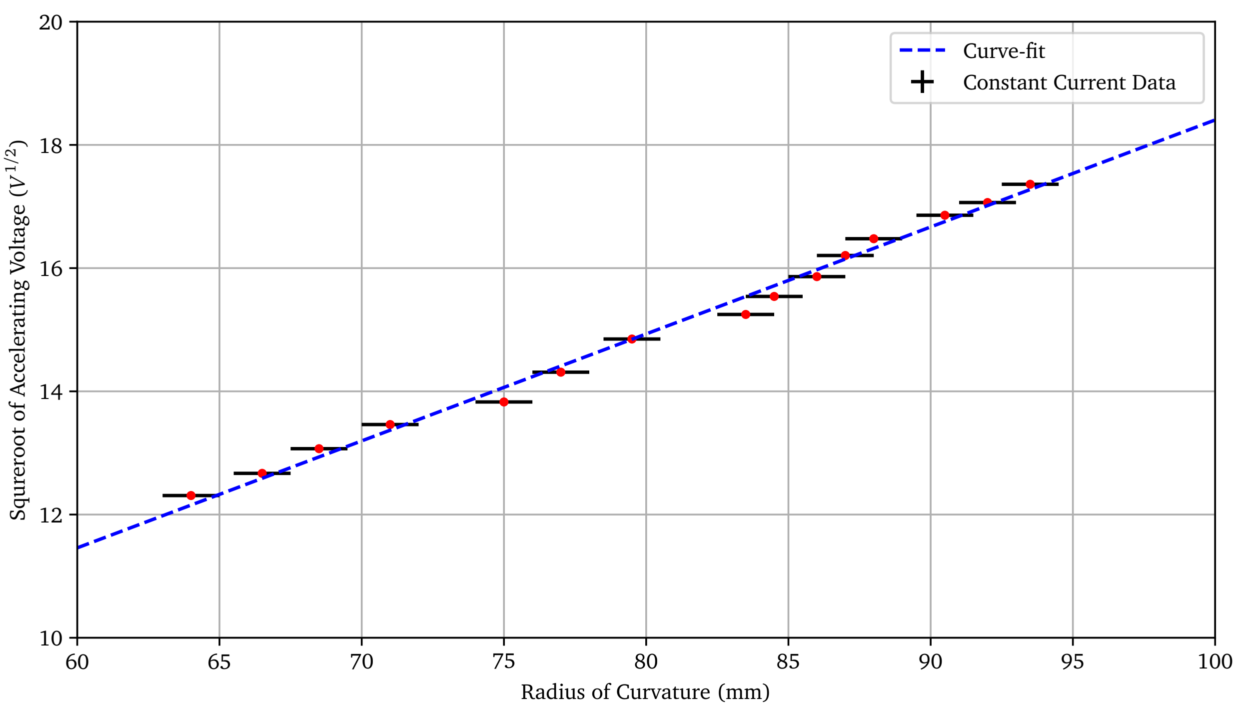

Figure 3: Radius of curvature of electron beam as accelerating voltage was increased while keeping coil current constant. Curve-fit line from `scipy.optimize`.

Observations: When collecting data for this part, we noticed the power supply (PS2) was getting noticeably hot and so we decided to turn it off and wait for it to cool down. During this part, we also noticed that the coil current fluctuated between $1.502A$ to $1.512A$, which could have resulted in the fluctuations we see in the plot.

The slope of the plot above is 0.1736 ± 0.001 $\text{V}^{1/2}/\text{mm}$. Notice that the slope would allow us to calculate the desired electron charge-mass ratio if we did not have an external magnetic field. To account for that, we apply the correction mentioned above and plot the magnetic field strength at each current value (in Part 1) as per equation (6) against $1/r$ to get the following,

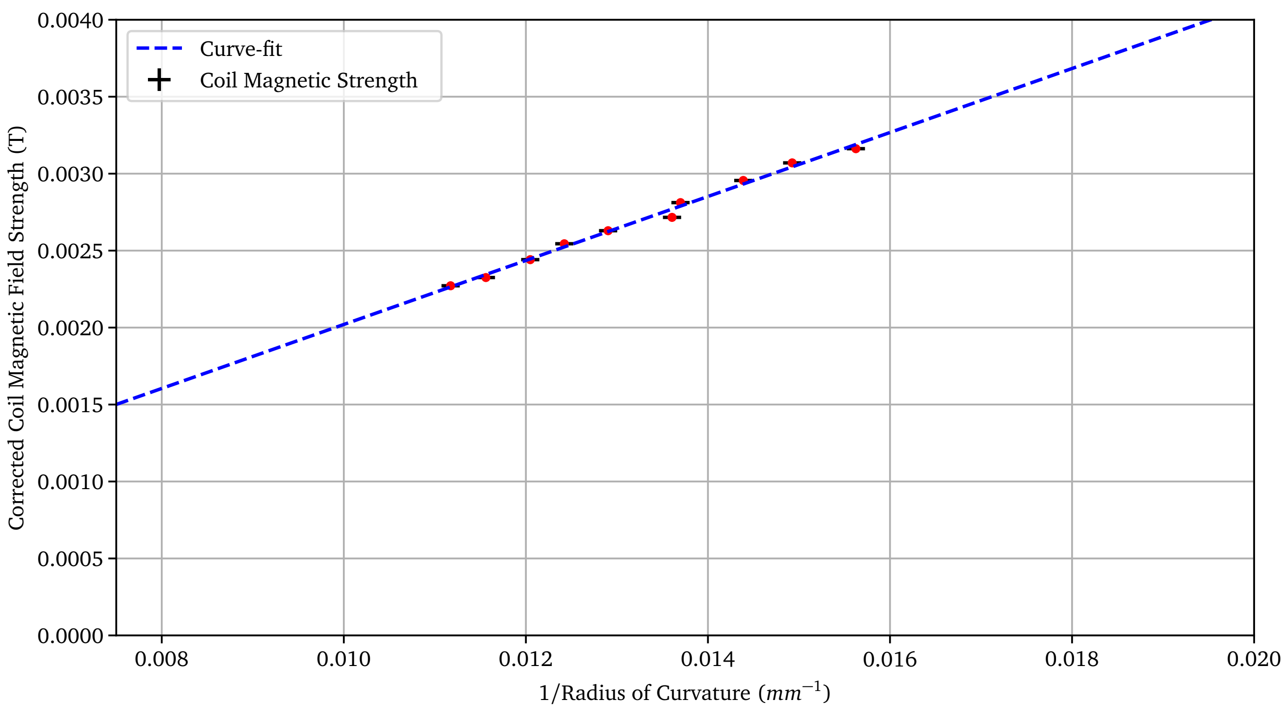

Figure 4: Plot of magnetic field strength against 1/radius of curvature with constant accelerating voltage (Part 1 data). Curve-fit line from `scipy.optimize`.

We know that,

$$B_c = \alpha \frac{1}{r} - B_e, \quad \alpha = \sqrt{\frac{2m}{e}\Delta V} \tag{7}$$

The plot above has a y intercept is $-B_e$ thus by equation (7) $B_e = 58 \times 10^{-6}$ T with an uncertainty of 30×10^{-6} T (found by `scipy.optimize`). Notice that the average Earth magnetic field on surface is anywhere from 40 μT to 60 μT depending on the location.

Using the the value for B_e from above, we use equation (6) and figure (4), to find that the slope is,

$$\text{slope} = 0.1736 = \sqrt{\frac{e}{m}}k\left(I + \frac{I_0}{\sqrt{2}}\right)$$

where k, I_0 is defined in Background Info section. Calculating e/m using $I = 1.510 \pm 0.001\ A$ and $B_e = 58 \pm 30\mu T$ we get,

$$e/m = 0.5 \pm 0.4 \times 10^{11} \text{ C/kg}$$

where the uncertainty is found by adding the square of the partials of the each component with an uncertainty.

3 Conclusion

In this investigation we varied the current in the coils to vary the magnetic field, and we varied the accelerating voltage of the electron beams to create a circular trajectory and measured the radius.

In Part 1 we held the accelerating voltage constant at $V = 200.2 \pm 0.5V$ and varied the current and measured the radius. From this we plotted $\frac{1}{\sqrt{r}}$ against I and using our equations (notably equation 6) from the background information section, found that e/m was $2.4 \pm 0.4 \cdot 10^{11}$ C/kg. B_e was found to be around $11 \cdot 10^{-6}$ T, which is smaller than the literature value of 30 to $60 \cdot 10^{-6}$ T.

In Part 2 we help the coil current constant (with minute variation) at $1.510 \pm 0.001A$ and varied the accelerating voltage and measured the radius of curvature. Plotting $\sqrt{V}$ against r we find the slope of the curve to be 0.01736. However, since there is an external magnetic field, we also plot a curve to find B_e taking into account the correcting factor. Using B_e and the slope we found that the charge mass ratio is $0.5 \pm 0.4 \times 10^{11}$ C/kg, however the theoretical value is 1.71×10^{11} C/kg.

Notice that the theoretical value falls outside the uncertainty range which illuminates that a large systematic/random error exists. Throughout the experiment we noticed minor fluctuations in the measured values and as well as the effect of electronics such as out phones and laptops on the trajectory of the electron beam. It is possible that these events could have contributed to the smaller calculated value. Another source or potential error is the measured radius values were not accurate - since many beams were somewhat thick ($\approx$ 2—3mm) and required guesses for the radius of curvature. These guesses could have had a large impact on the slope of figure 3 and 4, impacting both the calculated value of B_e and hence e/m.

4 Appendix

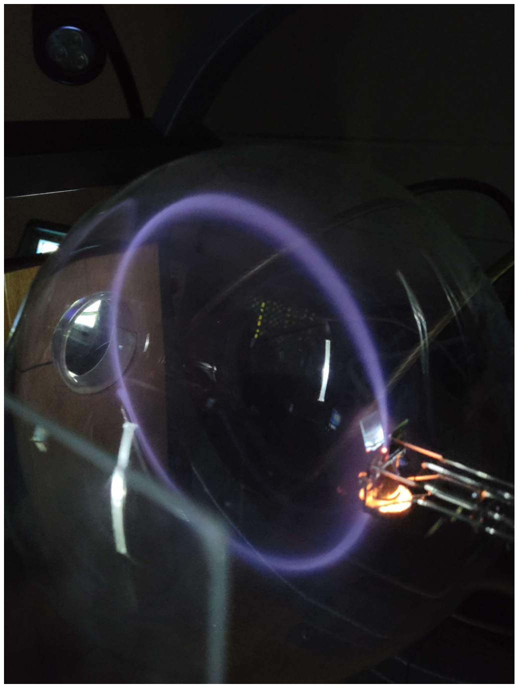

Figure 5: Example electron beam trajectory, where glass bulb has been rotated to create a complete circle.