PHY324 - Franck Hertz Report

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Abstract

This lab report summarizes the results of the Franck Hertz experiment in which we used accelerated electrons colliding with mercury vapours to calculate the energy difference between the ground state and the first 5 excited energy levels of Mercury atoms which are all separated by an average of $\Delta E = 4.92 \pm 0.04$eV. The theoretically accepted value is 4.9eV, and so the calculated value is within one error bar of the actual value. This was concluded by observing that the electrons lost discrete amounts of their kinetic energy as their kinetic energy was increased when travelling through the mercury vapours indicating the quantization of energy levels of Mercury atoms.

1 Introduction

The Franck Hertz experiment involves accelerating a beam of electrons through a series of accelerating, collimating, and stopping voltages through a glass tube filled with mercury vapour. These electrons go through inelastic collisions with the Mercury atoms to excite the atoms to energy states above the ground state by completely depositing their energy to the atom if it matches with the energy difference between ground state and a higher energy state. Franck-Hertz used mercury vapour instead of Hydrogen vapour because Mercury atoms have a lower ionisation energy / potential (about 10.44 eV) [1] which is lower than that of Hydrogen (about 13.6 eV) [2]. This means that one can excite mercury atoms with lower kinetic energy electrons which would be better for the experiment as they won't cause other effects which might intrude with the proper current measurements.

For the experiment, as can be seen in the figure 1, the electrons are generated from a tungsten filament connected to f - f_k via thermionic emission produced by filament voltage E_1. These electrons pass through a grid mesh g_1 of positive voltage E_2 and are accelerated by an accelerating voltage E_3 between g_1 and g_2. Moreover, g_2 is a mesh with stopping voltage E_4 that stops electrons with lower kinetic energy before they enter the electrometer for the current to be measured.

In the experiment, we fixed the filament voltage $E_1 = 6.3 \pm 0.1$ V and stopping voltage $E_4 = 1.5 \pm 0.1$ V [4]. For each set of measurements, we fixed the g_1 grid voltage E_2 and increased the accelerating voltage E_3 from $0 - 30$ V and measured the corresponding current via the electrometer. These were plotted and fitted to find the accelerating voltages at which there were dips in the electrometer current. The kinetic energy of the electrons at the particular accelerating voltage is given by:

$$K = e \cdot E_3 \tag{1}$$

If the kinetic energy of the electrons matches the energy difference between the ground state of a Mercury atom and one of its excited states, then the electrons with that particular kinetic energies are completely absorbed and there's a resulting dip in the measured current. So, for 2 current dips at 2 values E_3 and E_3' found via finding dips in

[1] Atomic Data for Mercury (Mercury). (2023). Retrieved 1 March 2023, from https://physics.nist.gov/PhysRefData/Handbook/Tables/mercurytable1.htm

[2] Ionization energy - Definition & Facts. (2023). Retrieved 1 March 2023, from https://www.britannica.com/science/ionization-energy

[3] Schematic courtesy of PHY324 Franck Hertz Experiment Lab Manual. https://www.physics.utoronto.ca/ phy224_324/experiments/franck-hertz/THE-FRANCK-HERTZ.pdf

[4] For E_1 and E_4 the uncertainties were assumed from the significant figures given in the provied values in the setup

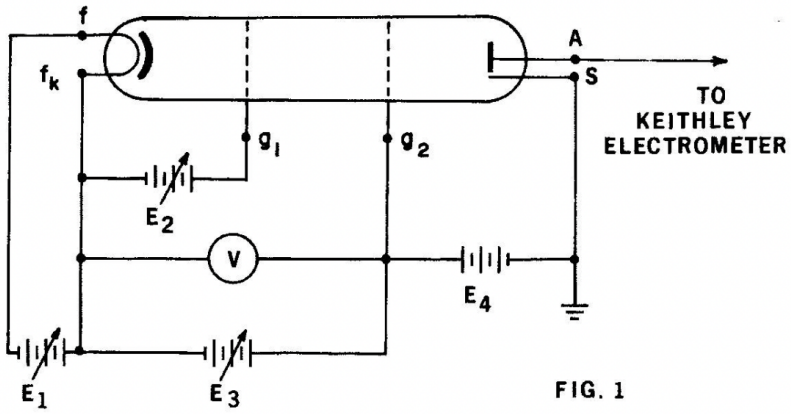

Figure 1: General setup of the Franck Hertz Experiment. The mercury vapours are contained in cylindrical glass tube. The electron beam is generated by applying a voltage E_1 to the tungsten filament connected to f and f_k from which the electrons are attracted to the grid mesh at g_1 by positive voltage E_2. These electrons are then accelerated through the central region via E_3. Lower energy electrons are collected at g_2 and the rest go to an electrometer for current measurement. Image courtesy of PHY324 faculty [3].

the current vs E_3 plot, the energy difference corresponding energy states of the Mercury atom can be calculated via

$$\Delta E = |E_3 - E_3'| \tag{2}$$

Moreover, we can calculate the wavelength of the photon released when the excited electron drops back to its ground state by,

$$\lambda = \frac{hc}{\Delta E} \tag{3}$$

2 Procedure

2.1 Materials & Apparatus

The experiment required a glass tube that only contained a small amount of liquid mercury in addition to the tungsten filament, 2 grid meshes, and various connections as in figure 1. The setup included a large Franck-Hertz system consisting of knobs and voltage meters for E_1 (fixed at 6.3 ± 0.1 V, least count 0.5 V), E_2 (least count 0.1 V) , E_3 (least count 10^{-9} V), E_4 (fixed at 1.5 ± 0.1 V, no scale), and a sweeping circuit to change E_3 automatically with run-reset buttons. To measure current, we used a 600B Keithley electrometer, an oven to vaporize the liquid mercury, a digital thermometer connected to the oven, and a lab view application on a computer connected to the apparatus.

2.2 Method

First, we built the circuit portrayed in Figure 1 by connecting the appropriate voltage terminals to the main Franck-Hertz system via the various components as can be seen in the Figure 2. Since we wanted to record dips in the electron current with different E_3 voltages, we connected one of the terminals of E_3 voltage source to X-port and the output port of the Keithley electrometer to the Y-port. However, the Franck-Hertz labview application plots a voltage instead of a current vs the E_3 accelerating voltage which acts as a proxy to the measured current. Also, the plots are flipped so a peak in the graph corresponds to an actual dip in the measured current.

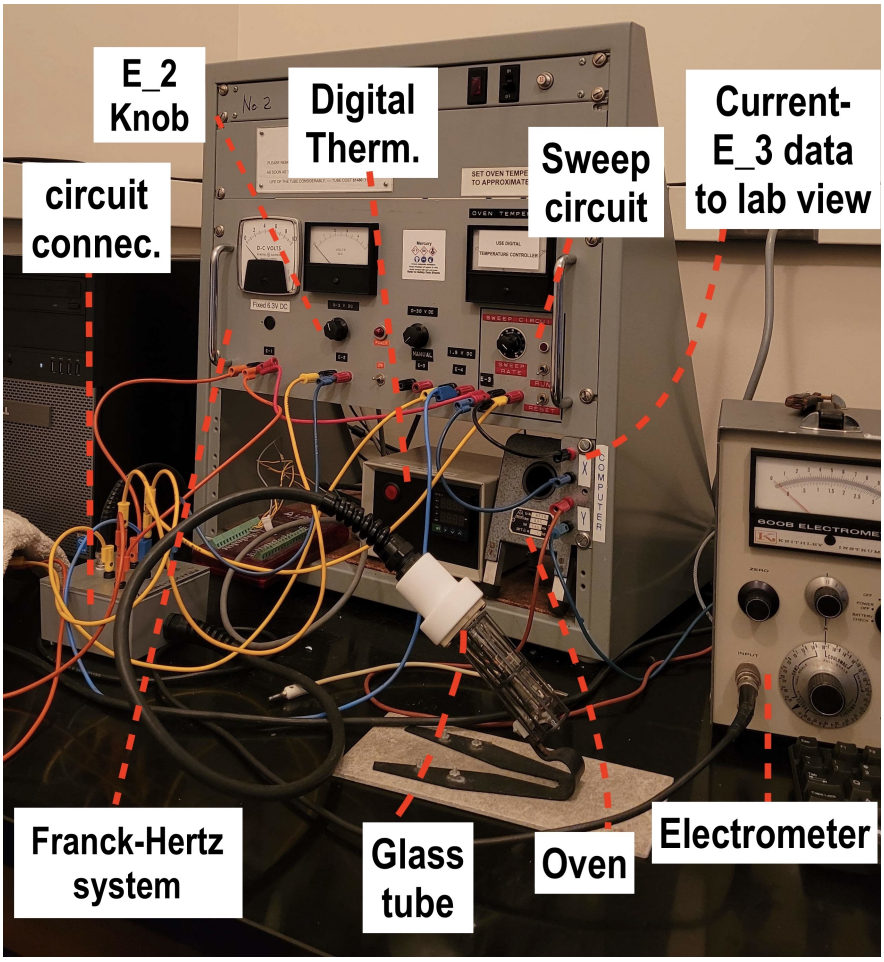

Figure 2: Experimental setup of the Franck Hertz Experiment. The glass tube containing liquid Mercury is to be kept in the oven which needs to be heated to at least $170 \pm 1°$ C before starting the experimental run. The data collection begins by setting E_2 voltage via a knob and running the sweep circuit which automatically increases the accelerating voltage from $0 - 30$ eV at a rate determined by the sweeping rate adjustable through a knob.

Then, we turned on all the components in the shown apparatus. We kept the glass tube in the oven and allowed it to heat up to approximately $170 \pm 1°$ C before beginning a trial. Once heated, we set the E_2 voltage starting at 1.5 ± 0.1 V to 3.4 ± 0.1 V in the increments of 0.1 V because the knob couldn't increase E_2 over 3.4 V and we couldn't observe the dips in the current below 1.5 V. This is because the electrons' kinetic energy was lower than the energy difference between the first energy level and ground state at that voltage.

Once we fixed the E_2 voltage, we reset the sweep circuit in the Franck-Hertz system so that the E_3 accelerating voltage goes to 0 V. We set the sweep rate knob to 100 (from $0 - 100$) in the sweep circuit which determines how quickly the E_3 voltage goes from $0 - 30$ V. It was set at maximum so that the electrons can get the maximum possible kinetic energy as fast as possible because they already travel very quickly through the glass tube and this way we would be able to observe all the dips in the current more quickly. Then, we turned on the Franck-Hertz labview system on the connected computer and clicked the 'Collect Data' button so that it was ready to record and plot the data. Then, we flicked the run toggle switch in the sweep circuit which would sweep the E_3 voltage through $0 - 30$ V at the preset sweep rate. After it's done sweeping, we can observe the peaks and dips in the electron current, see figure 3 for an example. Sometimes in a data run, there were spikes in the current (see figure 8 in Appendix) which would be caused by electrons

missed by the stopping grid g_2 (refer to figure 1). Since this hindered in analyzing the data computationally by mistaking them as peaks in current, we ignored such collected data and redid the trial. After saving the collected data as a text file, we repeated the same procedure by incrementing the E_2 voltage by 0.1 V. The collected data sets were then used to find the E_3 voltages at which there are peaks in the voltage, which here is a proxy to inverse of measured current. This means the peaks actually correspond to dips in the electron current and the difference between them was used to find the kinetic energy of the electrons deposited to the Mercury atoms which is equal to energy level gaps.

3 Results & Discussion

During the experiment we varied E_2 from 1.5 ± 0.1V to 3.4 ± 0.1V in increments of 0.1 V while collecting more than 1 trial for some of the values of E_2 for a total of 28 sets of data. In this section we summarize the analysis for the trial where the voltage $E_2 = 1.8 \pm 0.1$V.

The plot of the inverted electron-current intensity as a function of the accelerating voltage is shown in Figure 3 because the voltage on y-axis plotted by the labview application is just a proxy for inverted electron current intesnsity in some arbitary units (AU). Notice that the peaks are "smooth" rather than sharp saw-tooth as one would expect.

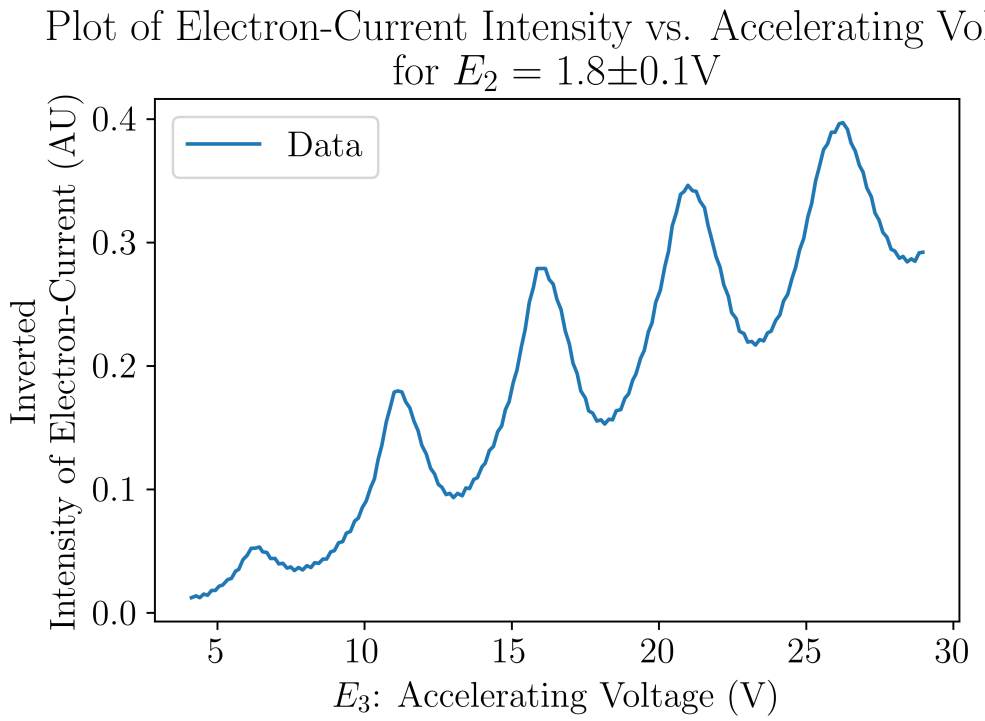

Figure 3: Electron-current vs accelerating voltage E_3 plot for $E_2 = 1.8 \pm 0.1$V for Franck-Hertz experiment trial run. The accelerating voltage ranges from 0 to 30 V. Here we plot accelerating voltage from 4 eV since the beginning is random signal noise. We also truncate the end and only plot till 29 eV, since the end is also random noise. Here, the peaks correspond to the dips in actual electron current measured by the electrometer.

This is due to multiple factors, one of the major being: as the electrons travel through the Mercury vapor, they collide with the gas which cause the electrons to lose some of their energy randomly (energy lost to random collisions is not the same for each collision). As

a result, the electrons which are measured at the anode do not all have exactly the same energy, leading to a broadening of the dips. This affect the results as we no longer have a discrete value for the peaks, rather we have smooth curves.

Notice that the 5 peaks[5]in Figure 3 can be approximated as Gaussian curves around the peak location. This suggests that we can find the peaks by fitting 5 Gaussian curves and finding the mean for the peak location and the standard uncertainty in the peak location by the standard deviation of the Gaussian. However, since the curve is not horizontal, we first find the baseline and subtract that away.

Notice that the valleys of Figure 3, appear to have a power-law fit. Fitting the equation,

$$y = b(x - a)^n$$

to the valleys of the data (found using `scipy.signal.find_peaks`) and using `scipy.optimize` we plot Figure 4. We then subtract the baseline to get a "normalized" curve. Fitting a curve made of 5 Gaussians we can find the peaks and the standard deviations, as seen in Figure 5. For this trial, the peaks were found to be,

$$p_1 = 6.1\pm0.8\text{eV}, \quad p_2 = 11.2\pm0.7\text{eV}, \quad p_3 = 16.0\pm0.8\text{eV}, \quad p_4 = 21.0\pm0.9\text{eV}, \quad p_5 = 26.1\pm0.9\text{eV}$$

where the uncertainty is the standard deviation of the respective Gaussian (see Equation 7 in Appendix for full fit function).

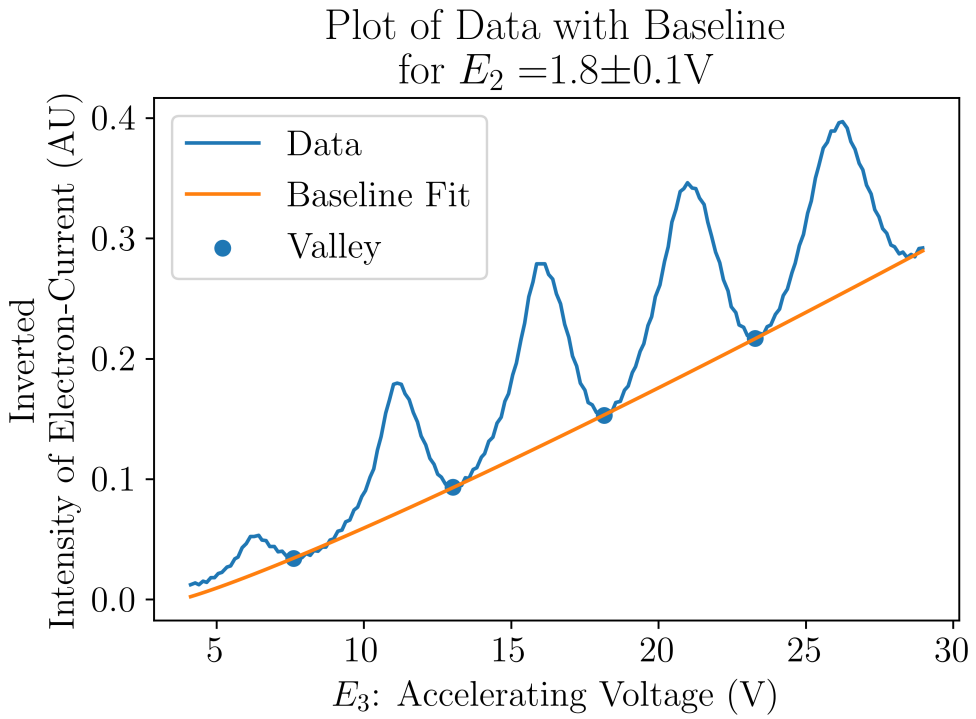

Figure 4: Electron-current plot for $E_2 = 1.8 \pm 0.1$V along with the fitted baseline and the valleys of the current. The power-law fit for this trial is $y = (3.8\pm0.1)(x-(0.007\pm0.002))^{1.14\pm0.03}$ eV

[5]The peaks are a result of the Franck-Hertz labview plotting software which flips the plot. The peaks correspond to drops in the electron-current.

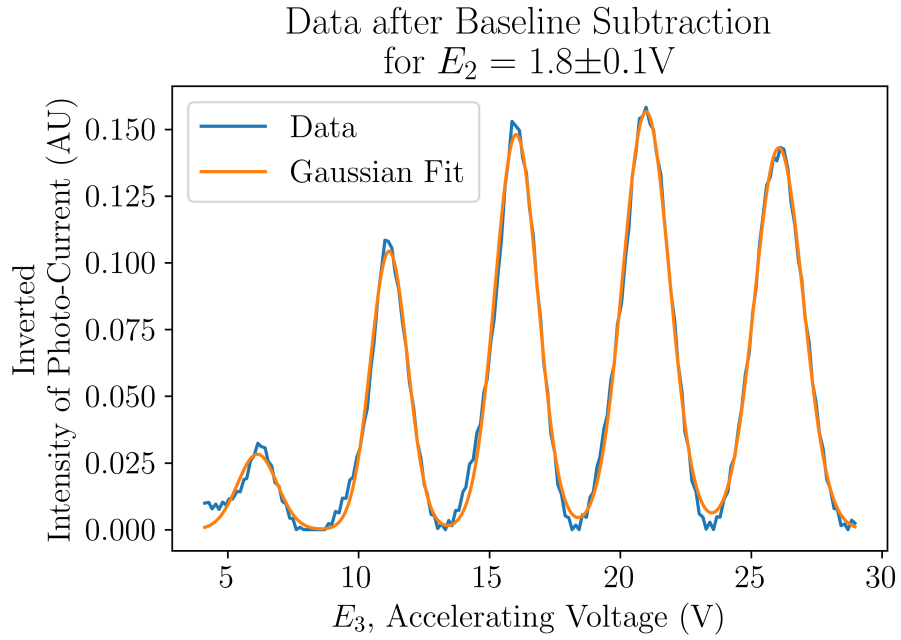

Figure 5: Electron-current vs accelerating voltage E_3 data after baseline subtraction along with the optimized fit (see eq. 7 in Appendix). Notice that the peaks of the fit line up with the peaks of the data in the horizontal direction and deviates in the vertical direction. However, as we are only interested in the accelerating voltage corresponding to the peak, so the fit is appropriate.

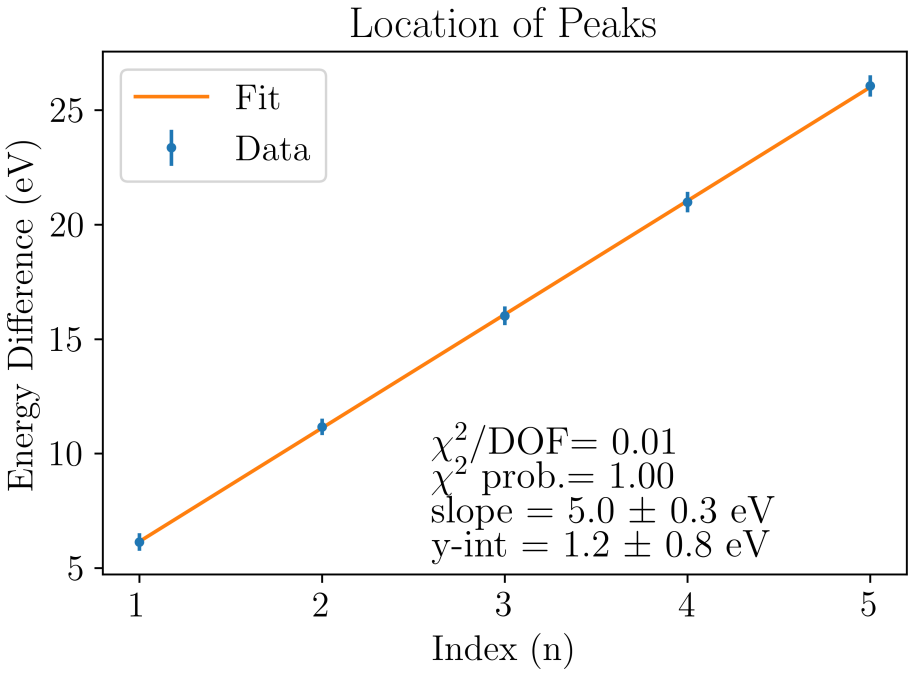

Figure 6: Plot of the relative peaks locations along with an optimized linear fit for $E_2 = 1.8 \pm 0.1$V trial. The slope (which is the energy difference between the peaks) was found to be 5.0 ± 0.3eV. This means that according to this trial, the energy difference between the first few energy levels is constant at about 5.0 ± 0.3 eV.

In Figure 6 we plot the E_3 voltages corresponding to the peaks from figure 5 and fit a linear function to find the average difference in the energies. Since the energy of

an electron which was accelerated through 1V is exactly 1eV. We directly convert the accelerating voltage to electron energy. The line of best fit was found to be,

$$y = (5.0 \pm 0.3)x + (1.2 \pm 0.8) \text{ eV} \tag{4}$$

where, the slope is the energy difference between the peaks, $\Delta E = 5.0 \pm 0.3$ eV. Figure 7 shows the residual plot of the linear fit. Since the residuals are within one error bar of 0, we expect a very small reduced χ^2 which we calculated to be $\chi^2_{red} = 0.01$. This suggests that taking the standard deviation of the Gaussian as the uncertainties was an over-estimate. An appropriate uncertainty would then be half the standard deviation, since we are only interested in region around the peak and not entire Gaussian.

We also calculated the chi squared probability to be $p = 1.00$. This means that the observed data fits the distribution perfectly up to the error bar. This is expected since the fitted function is linear.

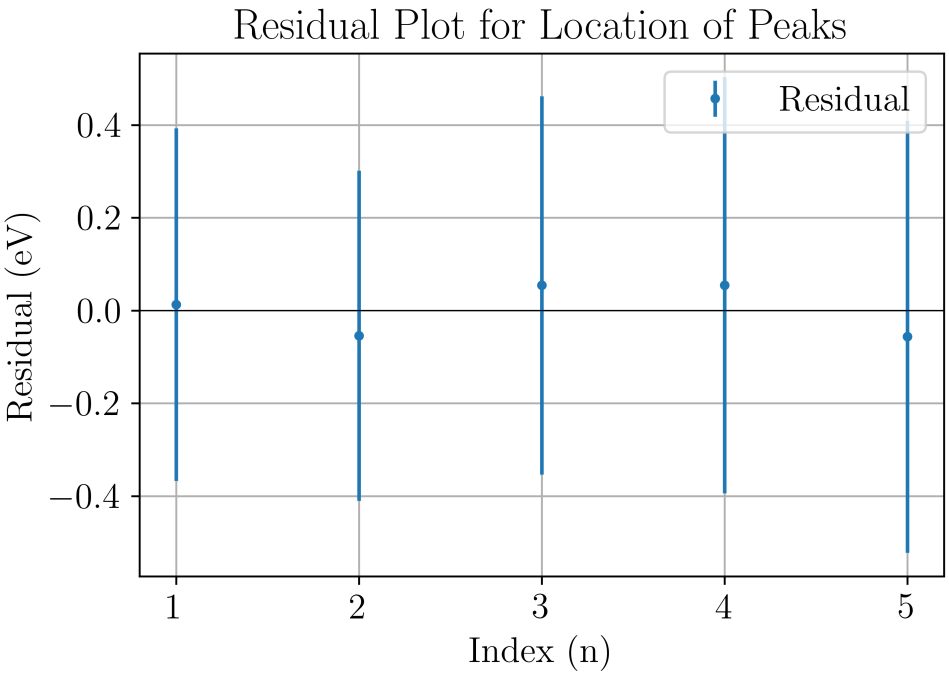

Figure 7: Residue plot or the difference between the best fit line y (eq. 4) and the data points in figrue 6. Since each error bar contains 0, we can conclude that the fit is well.

During the inelastic collision, ΔE was the average energy transferred from the accelerated electron to excite an electron in Mercury's valence shell for $E_2 = 1.8 \pm 0.1$V trial. Similarly for the other data sets, we calculate ΔE for each trial. Taking the average and standard deviation of all the trials we get the energy transferred from an electron to an Mercury atom in an inelastic collision,

$$\boxed{\Delta E_{\text{avg}} = 4.92 \pm 0.04 \text{ eV}} \tag{5}$$

Note that the uncertainty is due to a statistical factor and is not directly minimized from changes in procedure. The other source of uncertainty is from the Labview with gives a precision of 10^{-9} and has no significant impact on the overall calculated uncertainty.

When the excited electron drop down to the valence shell, it will release a photon of wavelength given in Equation 3. Thus the wavelength associated with photons emitted

by Mercury atoms when decaying from the first excited state to the ground state is,

$$\boxed{\lambda = 254 \pm 2 \text{ nm}} \tag{6}$$

4 Conclusion

By conducting the Franck-Hertz experiment, we found that there were regular dips in the electron current at specific electron kinetic energies travelling through mercury vapour. We found that the average energy difference between the current dip energies to be $\Delta E = 4.92 \pm 0.04$eV. This shows that Mercury atoms have discrete energy levels to which they can be excited to if the kinetic energy of the incoming electron exactly matches the corresponding energy difference. The theoretically accepted value for the Mercury energy level difference is 4.9eV, and so the calculated value is within one error bar of the accepted value. The chi-squared probabilities on average were very close to 1.0 which gives confidence that the results of energy difference was very precise. Also, the wavelength of the photons emitted by the Mercury atom transitioning down by one energy level from an excited state was found to be $\lambda = 254 \pm 2$ nm.

Appendix

A - Data Tables and Images

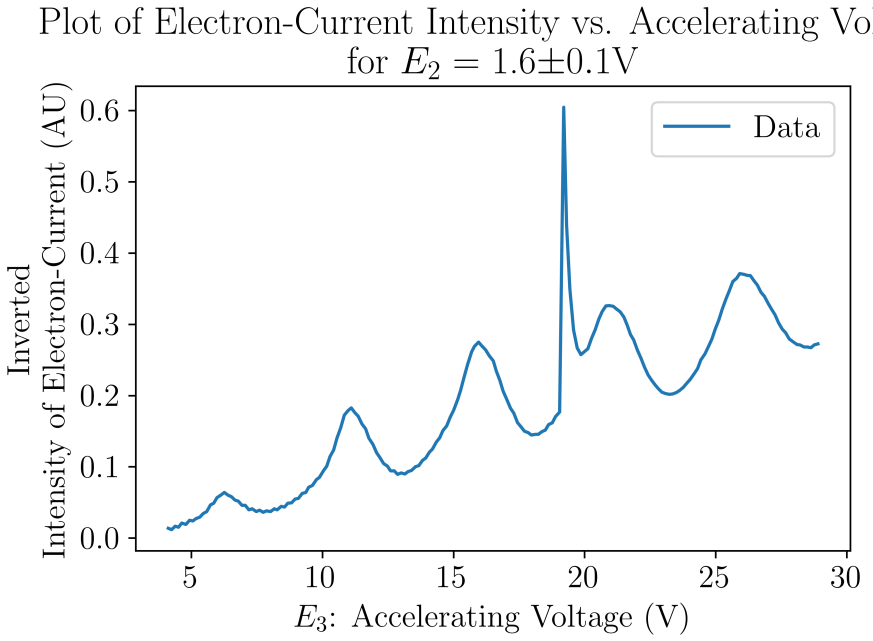

Figure 8: A plot of intensity of electron current vs the accelerating voltage E_3 showing a bad data. The ideal data should not have a large spike as can be seen in the figure. It is a result of a stray electron missing the stopping grid g_2 which is supposed to lower the kinetic energy of electrons. We ignored such bad data sets in our analysis.

Figure 5: Equation of best fit

$$\begin{aligned}\text{fit}(x) =&(0.028 \pm 0.001) \cdot e^{(-x-(6.14\pm0.05))^2/(2(0.76\pm0.05)^2)} + \\ &(0.104 \pm 0.002) \cdot e^{(-x-(11.17\pm0.01))^2/(2(0.71\pm0.01)^2)} + \\ &(0.148 \pm 0.002) \cdot e^{(-x-(16.022\pm0.009))^2/(2(0.815\pm0.009)^2)} + \\ &(0.157 \pm 0.001) \cdot e^{(-x-(20.985\pm0.009))^2/(2(0.896\pm0.009)^2)} + \\ &(0.143 \pm 0.001) \cdot e^{(-x-(26.06\pm0.01))^2/(2(0.93\pm0.01)^2)}\end{aligned} \tag{7}$$