Internal Resistance Lab

Chaitanya Kumar Mahajan Alex He-Mo

October 20 2022

1 Introduction

1.1 Abstract

When calculating resistance, voltage or current in a theoretical setting, we assume that the measurement devices are ideal, i.e. the power supply/cell battery have no internal resistance and that multimeters are ideal. However when making actual measurements these facts must be taken into account as they systematically shift the measurements (Power-supply/cell battery have small internal resistance, ammeter have small resistance and voltmeter allows some current to pass through.) Thus the goal of this lab is to determine the internal resistance of a DC power-supply and a cell battery and to estimate the resistance of the ammeter and estimate the current flow through a voltmeter.

1.2 Background Information

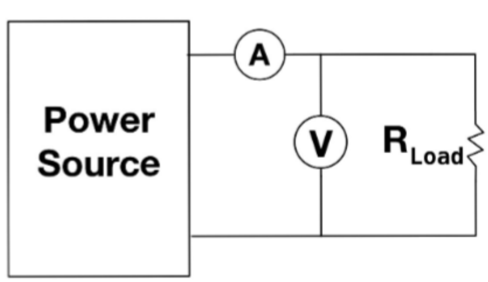
Ohm's Law asserts a relationship between voltage, resistance, and current of a circuit in the form of $V = IR$. This can be extended to each individual component of a circuit having its own resistance and voltage drop, while sharing the value of current with other components in series. One of these components is the power supply (or battery) itself.

There is inherent resistance within the power supply, which will produce voltage drop, leading to an effective voltage difference (terminal voltage) from the battery which is different from the voltage given by the power supply (open-circuit voltage). As the internal resistance can be treated as a resistor, Ohm's Law shows that the relationship between the open-circuit voltage (V_∞) and the terminal voltage V can be represented by

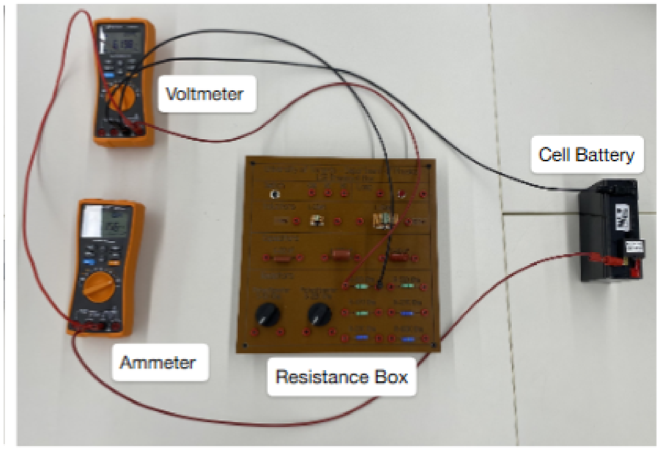
$$V = V_\infty - R_{int} I$$

where the internal resistance is R_{int} and the current is I. This is an inverse linear relationship between effective voltage and the current, where the slope is the internal resistance.

1.3 Procedure and Apparatus



(a) Circuit Diagram of experiment (figure from lab instructions)



(b) Apparatus setup (figure from lab instructions)

Q1 - We believed that option B would be a better experiment to conduct given that the multimeters are not ideal. With non-ideal multimeter, the voltage drop across an ammeter is not 0 and thus measuring the voltage drop across the ammeter and resistor would give a smaller voltage measurement (since non ideal ammeter add to overall resistance of the circuit.)

We outline the procedure below,

1. Setup the circuit board as shown in the diagram (as in (b)) using a cell-battery of known voltage (ammeter in series and voltmeter in parallel and across the resistor). Note the range of the voltmeter and ammeter.
2. Change the resistor used by adding new resistors in series and/or using higher Ohm resistors to measure the current and voltage drop.
3. Change the cell-battery to a power-supply and set the voltage to a desired value (note: depending on the voltage, change to the connector to max 10A in the ammeter to not blow the fuse.)
4. Using a variety of resistors, measure the current and voltage drop.
5. Repeat 3-4 with at least two different voltage setting on the power-supply.

2 Results and Analysis

2.1 Experiment with the Battery Supply

Table 1: Voltage and Current in the Circuit with Varying Resistance and a 6.5V Battery

Resistor (±5% Ω)	**Current (mA)**	**Uncertainty (±A)**	**Voltage (V)**	**Uncertainty (±V)**
100	62.5	0.2	6.25	0.02
220	28.9	0.1	6.31	0.02
320	19.93	0.09	6.34	0.02
470	13.79	0.07	6.34	0.02
690	9.37	0.07	6.35	0.02
2700	2.35	0.01	6.36	0.02
3170	2.02	0.01	6.36	0.02
127000	0.05	0.01	6.36	0.02

We noticed the later resistor heated up.

Here the voltage and current uncertainty is calculated by[1],

$$u(V) = (0.05\% \times \text{ measured value } + 2 \text{ counts of last significant digit})$$
$$u(I) = (0.3\% \times \text{measured value} + 10 \text{ counts of last significant digit})$$

[1]Uncertainty values from U1272A millimeter manual

$$u(V) = (6.25 \times 0.0005 + 2 \times 0.001) \approx 0.02$$
$$u(I) = (62.5 \times 0.003 + 10 \times 0.001) \approx 0.2$$

Use correct count values based on U1272A manual. For current that are larger than 30A range and for values smaller than 3A.

Q2 - We chose resistance values incrementally based on the resistors that were available to us. We made sure that the resistance was not too low so we would not approach I_{max} which may lead us away from linear relationship needed to accurately measuring the internal resistance.

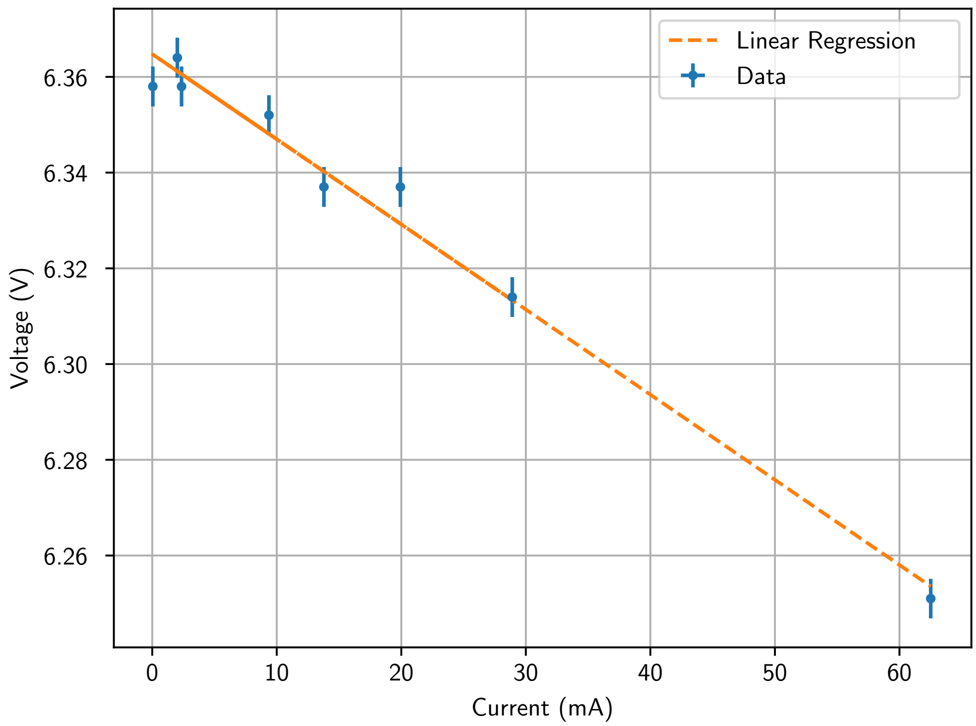


Figure 2: Voltage drop over the variable resistor as a function of the current in the circuit. *Current error bars are present, however they are* ≈ 100 *times smaller*. Data points are from Table 1, Section 2.1.

Plotting the data and using a linear curve-fit from `scipy.optimize` we find that the slope of the line of best fit is, $m = -0.00178$, with a $u(m)$ of 0.00008. We know that the slope of the curve is internal resistance R_b of the cell battery. Thus,

$$R_b = 1.78 \pm 0.08 m\Omega$$

2.2 Experiment with the DC Power Supply

For the DC regulated power supply, we had two trials with three different voltages, 10V, 15V and 20V. Below we give the data,

Table 2: Voltage and Current in the Circuit with Varying Resistance and a 10V DC Regulated Power-Supply

Resistor (±5% Ω)	**Current (mA)**	**Uncertainty (±A)**	**Voltage (V)**	**Uncertainty (±V)**
220	45.5	0.1	9.93	0.02
320	31.3	0.1	9.96	0.02
470	21.67	0.09	9.97	0.02
2700	3.70	0.01	9.99	0.02
3170	3.16	0.01	10.00	0.02

Table 3: Voltage and Current in the Circuit with Varying Resistance and a 15V DC Regulated Power-Supply

Resistor (±5% Ω)	**Current (mA)**	**Uncertainty (±A)**	**Voltage (V)**	**Uncertainty (±V)**
220	68.1	0.2	14.90	0.02
320	46.8	0.1	14.93	0.02
470	32.5	0.1	14.95	0.02
2700	5.55	0.02	14.99	0.02
3170	4.75	0.01	14.99	0.02

Table 4: Voltage and Current in the Circuit with Varying Resistance and a 20V DC Regulated Power-Supply

Resistor (±5% Ω)	**Current (mA)**	**Uncertainty (±A)**	**Voltage (V)**	**Uncertainty (±V)**
220	90.7	0.2	19.87	0.02
320	62.4	0.2	19.91	0.02
470	43.4	0.1	19.94	0.02
2700	7.40	0.06	19.99	0.02
3170	6.33	0.06	19.99	0.02

All uncertainties are calculated as shown above, using appropriate values from U1272A manual.

We plot these three trials below,

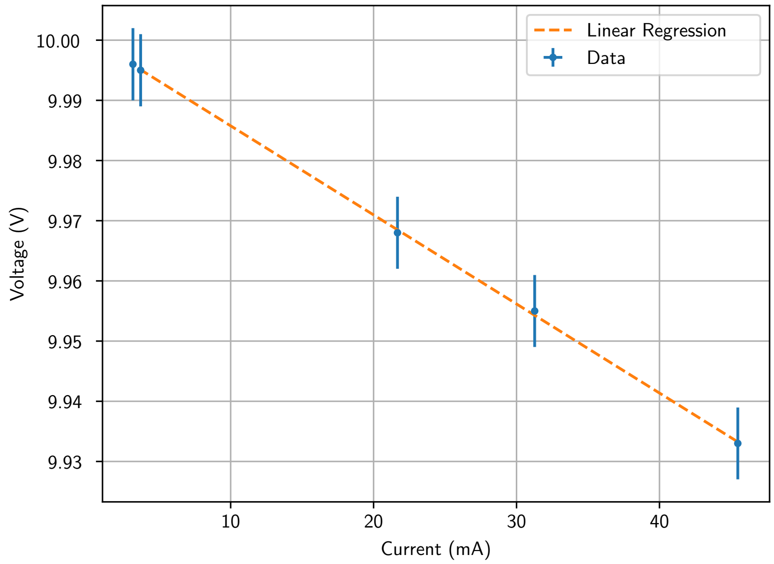


Figure 3: Voltage drop over the variable resistor as a function of the current in the circuit when direct current is supplied by a 10V power supply. *Current error bars are present, however they are* ≈ 100 *times smaller*. Data points are from Table 2, Section 2.2.

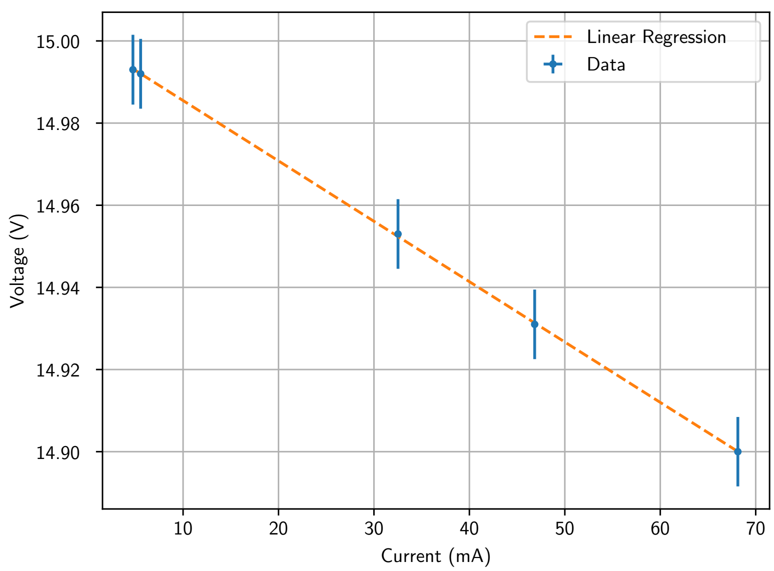


Figure 4: Voltage drop over the variable resistor as a function of the current in the circuit when direct current is supplied by a 10V power supply. *Current error bars are present, however they are* ≈ 100 *times smaller*. Data points are from Table 3, Section 2.2.

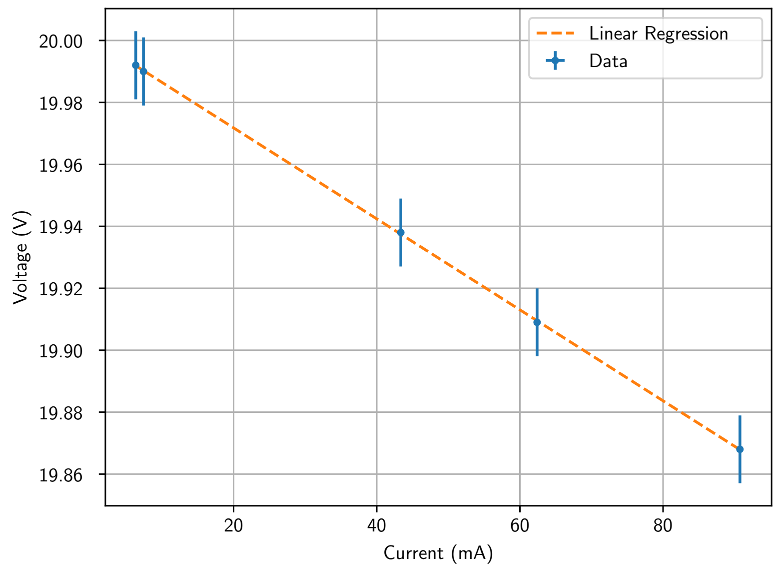


Figure 5: Voltage drop over the variable resistor as a function of the current in the circuit when direct current is supplied by a 10V power supply. *Current error bars are present, however they are* ≈ 100 *times smaller*. Data points are from Table 4, Section 2.2.

Using `scipy.optimize` we find the parameters of linear best fit for the three plots are,

$$10V: \quad V = -0.00148I + 10.00056 \tag{1}$$
$$15V: \quad V = -0.00147I + 15.00019 \tag{2}$$
$$20V: \quad V = -0.00147I + 20.00115 \tag{3}$$

Averaging the slopes (which is the internal resistance of the power supply), we find that the mean average internal resistance is,

$$R_{ps} = 1.5 \pm 0.2 m\Omega$$

where the uncertainty is the square root of `pcov`. *Note: we did not use standard deviation divided by* $\sqrt{3}$ *since this gave an uncertainty of* $\approx 3 \times 10^{-6}\Omega$.

Using the calculated internal resistance of the power-supply,

Q3 - We can approximate the resistance of the ammeter by treating it as an unknown R which would add in series to to overall resistance of the circuit. Then, $R + 220\Omega + 1.5m\Omega = \dfrac{10V}{45.48mA} \implies R = 0.12\Omega$. Note that we could have calculated a better estimate if we had smaller Ohm resistor, since the smaller scale would be impacted more by the resistance of the ammeter. Similarly, we can estimate the current flow through the voltmeter to be ≈ 0.003A.

Q4 - Notice that if we solve for equations (1-3) for when $V = 0$ we find that $I_{\text{intercept}} = 6.757A, 10.204A, 13.606A$ respectively. However, we know that terminal voltage and current have a linear relation only for a small range of currents and this relationship breaks for large current when approaching I_{max}. Since $I_{\text{max}} < I_{\text{intercept}}$ we can estimate that V_∞ has an upper bound of 6.7A, although we would except the actual $I_{\max}$ to be much less. Moreover we know that $I_{\max}$ has to be constant and does not change with V_{∞}.

We know that beyond the regulation regime, the R_{ps} can't handle the demand of the larger current and thus will non-linearly increase until terminal voltage reaches zero. This reflects that I_{max} is realistically less than the intercept if R_{ps} were to continue to linearly increase.