PHY324 - Hall Effect Report

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Abstract

This lab report summarizes the results of the Hall Effect experiment in which we pass a current through Silver and Chromium strips using an external voltage. Placing the strips in an external magnetic field we measure the Hall- Voltage (HV) and Hall-Current (HC) to calculate the Hall Coefficient R_H of each sample. For the Chromium sample we found $R_H^{Cr} = (3.54 \pm 0.03) \cdot 10^{-8} \frac{m^3}{C}$ and for the Silver sample we found $R_H^{Ag} = (-1.9 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$. Moreover, using the four-wire method to measure the resistance of the samples, we calculated the charge carrier density $n_{\mathrm{Cr}} = (1.78 \pm 0.02) \cdot 10^{26} m^{-3}$ and $n_{\mathrm{Ag}} = (3.3 \pm 0.3) \cdot 10^{29} m^{-3}$, drift velocity $v_{\mathrm{Cr}} = (3.3 \pm 0.3) \cdot 10^{-2} \frac{m}{s}$ and $v_{\mathrm{Ag}} = (7 \pm 1) \cdot 10^{-4} \frac{m}{s}$ and conductive mobility $\mu_{\mathrm{Cr}} = (0.049 \pm 0.005) \frac{m^2}{C\Omega}$, and $\mu_{\mathrm{Ag}} = (4.2 \pm 0.9) \cdot 10^{-3} \frac{m^2}{C\Omega}$. The standard values of the Hall Coefficient for Silver and Chromium are $R_H^{\mathrm{Ag}} = -8.9 \times 10^{-11}$ m^3/C and $R_H^{\mathrm{Cr}} = 3.8 \times 10^{-10}$ m^3/C respectively. [1][2]

1 Introduction

The Hall Effect experiment involves passing charged particles through a conductor placed in an external perpendicular magnetic field $\vec{B}$. The Lorentz force acting on the moving charge carriers is perpendicular to the direction of the currnet flow. Hence, positive and negative charges accumulate on the transverse section of the conductor. This charge accumulation creates a voltage difference across the transverse section of the conductor perpendicular to both $\vec{B}$ and direction of current. This creates the Hall electric field opposes further charge accumulation. After a while, this voltage stabilizes and this phenomenon is called the Hall effect.

The experiment setup is seen in Figure 2 and the Silver/Chromium sample circuit is seen in Figure 1. The external voltage for the sample is supplied from (EPS) connected via the C (black) wire. Multimeter (MM2) measures the Hall-Current as it connected to the EPS through a limiting 500Ω resistor and to the sample via the D (red) wire. The A_1 and A_2 wires (blue and brown) are connected to the potentiometer P, which is used to zero the Hall-Voltage in Earth magnetic field. P is connected to a multimeter MM1 which measures the Hall-Voltage across the sample, connected via the B (green) wire. The entire sample (S) is placed in an external magnetic field created by Helmholtz Coils (HC) which is supplied by a power supply with variable current MPS.

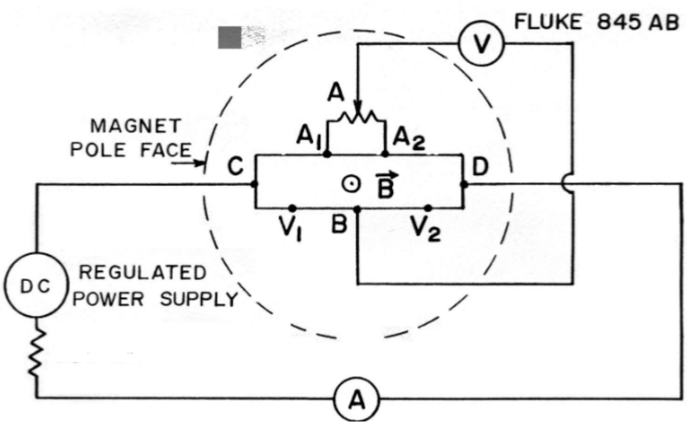


Figure 1: Circuit diagram of 2 along with a schematic of the sample. Wires C, D connect to MM2 via EPS and measure the Hall-Current. A_1, A_2 and B connect to MM1 via the potentiometer P. Magnetic field $\vec{B}$ is created via HC. Note that wires V_1, V_2 are unused in the Hall-Effect experiment, and are used in the 4-wires method to measure the resistance of the sample. Image courtesy of PHY324 Hall-Effect Lab Manual.

In the experiment, we use Silver and Chromium samples. Since these samples have some thickness t, we need the current density given by,

$$J = \frac{I}{wt} \tag{1}$$

[1]Silver Hall Coefficient found at http://www.hep.fsu.edu/ wahl/phy4822/expinfo/hall/HE_silver.pdf
[2]Chromium Hall Coefficient found at https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7865485/

where w is the width of the sample and I is the measured Hall-Current. Moreover, the thickness t can be calculated using the interferometric photographs of each sample using,

$$t = \frac{\text{Fringe Step}}{\text{Fringe Separation}} \cdot 2945 \times 10^{-10} \text{ m} \tag{2}$$

Since the Hall-Voltage is the potential difference across the sample of width w, we have,

$$V_H = -\int_0^w E_y dy = -E_y w \tag{3}$$

where E_y is the Hall vertical electric field and is given by,

$$E_y = v_{d_x} B \tag{4}$$

where B is the external magnetic field and v_{d_x} the horizontal drift velocity. We further define the Hall-Coefficient to be,

$$R_H = \frac{E_y}{J_x B} = \frac{1}{ne} \tag{5}$$

where J_x is the horizontal component of the current density, B is the external magnetic field, n is the charge density of the sample and e is the charge of the electron. Another useful quantity we can define is the electric mobility given by,

$$\mu = \sigma R_H \tag{6}$$

where $\sigma = \frac{1}{\rho}$ is the conductivity and ρ is the resistivity, which can be measured using the 4-wire method using the formula

$$\rho = \frac{R \cdot wt}{l} \tag{7}$$

where R is the measured resistance. We can also use these quantities to calculate the drift velocity of the charge carriers as:

$$v_d = \mu\sqrt{E_x^2 + E_y^2} \tag{8}$$

where $E_x = \text{External Voltage} \cdot l$ is the electric field in the horizontal direction and $E_y = V_H w$ is the electric field in the vertical direction.

2 Procedure

2.1 Materials & Apparatus

The experiment requires a Chromium and Silver sample with appropriate wire connections which is placed in an external magnetic field created by Helmholtz Coils. The resulting Hall Voltage and Hall Current can be measured using two multimeters and a potentiometer connected according to the circuit seen in 1. Experiment setup is seen in Figure 2 along with a potentiometer and the two required power supplies for the Helmholtz Coils and for the sample. To measure the thickness of the sample, we use Equation 2 which requires a microscope and/or Vernier Calipers to measure the fringe separation and step.

2.2 Method

First, we built the circuit seen in Figure 1 connected to the appropriate power supplies and multimeters as seen in Figure 2. Since we want the Hall Effect in a purely external magnetic field which is perpendicular to the sample, we zero the effect induced by Earth's magnetic field by placing the sample in the Coils, setting the current in the coils (MPS) to be 0 A and using the potentiometer to zero the Hall Voltage reading. (Ensure that the external voltage from EPS is at some non-zero value).

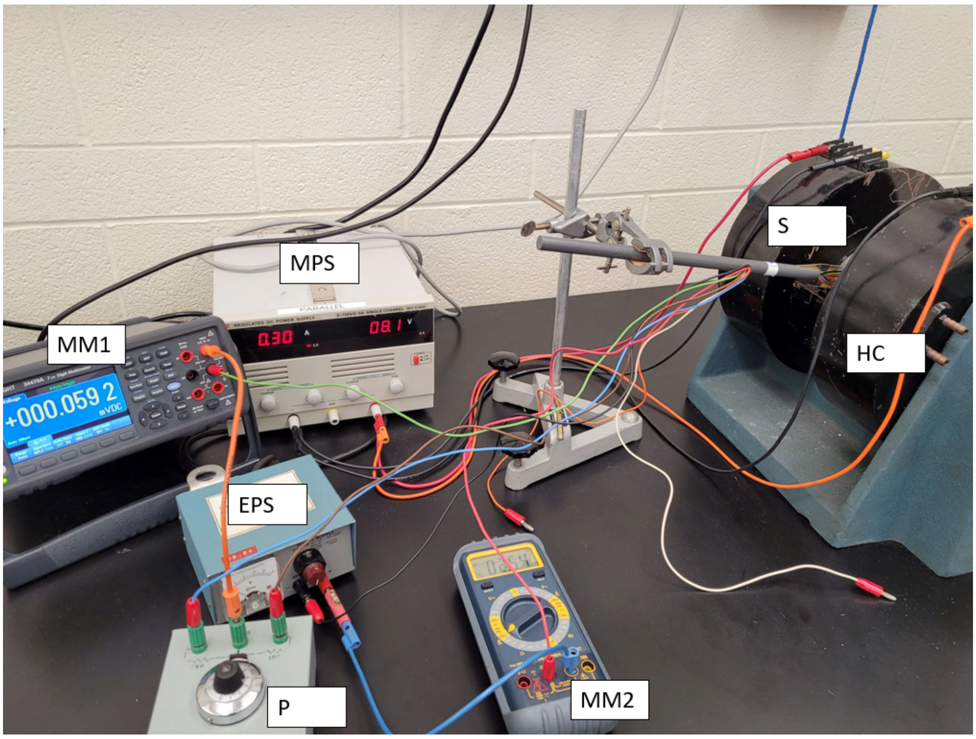


Figure 2: General setup of the Hall-Effect. The sample S is placed in an external magnetic field created by HC and supplied by MPS. An external voltage is passed through the sample via EPS. MM1 measured the Hall-Voltage and can be zeroed via the potentiometer P. MM2 measures the Hall-Current in the sample.

Record the external voltage and increase the coil current and measure the external magnetic field using a probe. Place the sample between the coils and measure the Hall Voltage and Hall Current. During these measurements we found that the Hall Voltage reading would fluctuate a significant amount for the Silver sample while less so for the Chromium sample.

We then set the coil current to 0 A and increase the external voltage and zero the Hall Voltage generated by Earth's magnetic field[3]. We now increase the coil current to its previous amount and measure the Hall-Voltage and Hall- Current. To ensure that the Hall-Current did not exceed 30 mA, we varied the external voltage between 4.0 ± 0.1 V and 16.0 ± 0.1 V in increments of 4 V. Following the same process, we measure the Hall Effect for different magnetic field strengths by varying the coil current between 1.0 ± 0.1 A to 2.5 ± 0.1A for the Silver sample in increments for 0.5A and between 50 ± 1 mA and 250 ± 1 mA in increments on 50mA.

To measure the resistivity, we employ the 4-wire method we requires us to setup the sample according to figure 5 in appendix. We passed high current through A_1 and V_1 (White, Red) and low current through A_2 and V_2 (Orange, Black). Setting the MM1 to the 4-wire resistance mode we measured the resistance of both samples. Finally to use Equation 2 we need to calculate the thickness of the sample. Using a microscope and the provided interferometric photographs, we measure the fringe step and fringe separation, by recording the initial horizontal position and final horizontal position.

[3]Since the Hall Voltage in the sample depends on the external voltage, we re-zero the Hall-Voltage in Earth's magnetic field every time we change the external voltage.

3 Results & Discussion

Using the hall voltage and hall current data collected for both chromium and silver, we plotted $\frac{E_y}{B}$ vs J_x at each magnetic field strength corresponding to different currents in the magnets. E_y was calculated from equation 3 and J_x was calculated from equation 1. For the chromium sample, an example of a $\frac{E_y}{B}$ vs J_x plot at $B = (0.040 \pm 0.001)$ T along with the corresponding residual plot is shown in figures 3a and 3b. For this magnetic field, the Hall Coefficient was calculated from the slope of the line of best fit to be $R_H = (2.96 \pm 0.08) \cdot 10^{-8} \frac{m^3}{C}$. The reduced chi-squared value for this fit was $\chi^2_{red} = 0.43$, which although shows overfitting. However, it is reasonable as the raw data shows a perfect linear trend. The chi-squared probability is 0.6, which shows that our fit is a good one. Further, the residual plot shows that the data points are equally scattered across $y = 0$ axis and are within one error bar of 0.

Accounting all 4 experimental runs, the average hall coefficient was $R_H^{\mathrm{Cr}} = (3.54 \pm 0.03) \cdot 10^{-8} \frac{m^3}{C}$, about 100 times larger than the standard accepted value $3.8 \cdot 10^{-10} \frac{m^3}{C}$ [4]. This excessive discrepancy is inferred to be due to non-linear relationship between the magnetic field strength (B) at the centre of the Helmholtz coils and the current (I) flowing through them. This non-linearity can be seen in figure 6 in Appendix where the B vs I relationship looks fits a quadratic better (with a calculated $\chi^2 = 0.01$ for the quadratic fit). This would mean that the magnetic field varies quite a bit at the centre of the Helmholtz coils, which implies that sample doesn't experience a constant magnetic field providing an inaccurate results. Furthermore, a large systematic error can be contributed by the resistance of the leads that were soldered to the Chromium sample. Nevertheless, R_H^{Cr} being positive means the charge carriers are in Chromium are not electrons but positive ions.

Thus, using eq. 5, the density of charge carriers was calculated as $n_{\mathrm{Cr}} = (1.78 \pm 0.02) \cdot 10^{26} m^{-3}$. Using equation 7, the resistivity was calculated as $\rho_{\mathrm{Cr}} = (7.2 \pm 0.7) \cdot 10^{-7} \Omega m$ which is about 7 times higher than the accepted value of $1.2 \cdot 10^{-07} \Omega m$. We believe this might be a result of impurities in the sample which would have developed over excessive usage This seems to be provide about an order of magnitude error in electric mobility which using eq. 6 yielded the average electric mobility in Chromium as $\mu_{\mathrm{Cr}} = (4.9 \pm 0.5) \cdot 10^{-2} \frac{m^2}{C\Omega}$. Finally, since according to eq. 8 the drift velocity depends on E_x, which changes as we make each measurement in each trial run, we here quote the average across all E_x, E_y: $v_{\mathrm{Cr}} = (3.3 \pm 0.3) \cdot 10^{-2} \frac{m}{s}$.

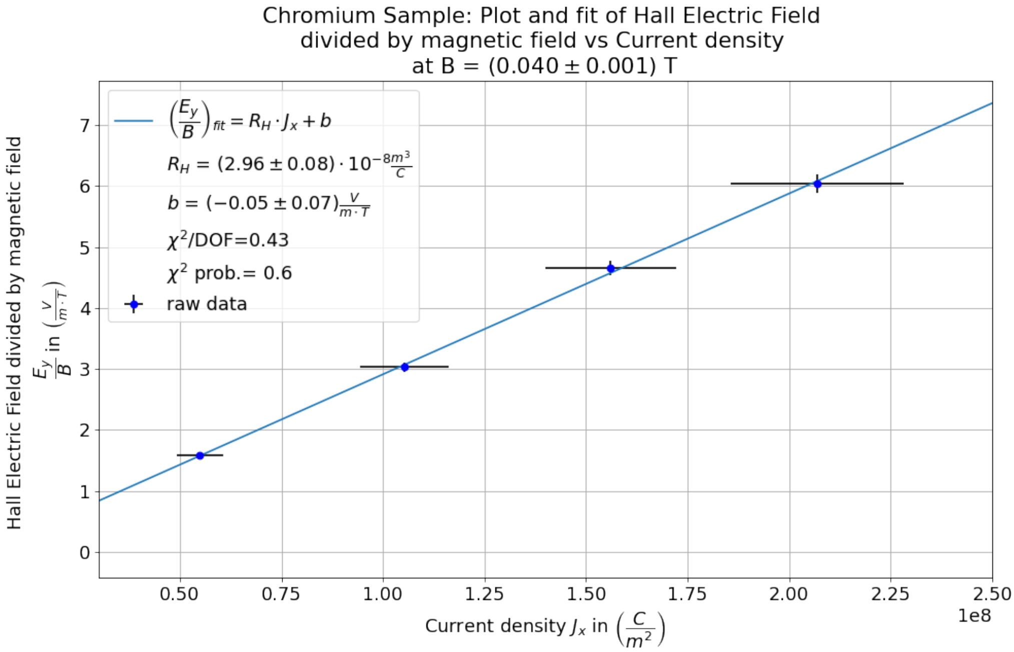
Similar plots were plotted for silver can be seen in exemplar figures 4a and 4b at $B = (0.785 \pm 0.001)$ T. Corresponding plots at other B values are shown in the appendix, whose clear non-linear spread of data points, except at $B = (0.655 \pm 0.001)$ T, imply that only two data sets are good enough. Using these two data sets, the average hall coefficient was found as $R_H^{\mathrm{Ag}}(-1.9 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$, while the standard accepted value of the hall coeffecient of Silver is $8.9 \cdot 10^{-11} \frac{m^3}{C}$. Thus, our results are of the right order of magnitude. Since the hall coeffecient is negative, this implies that the charge carriers in metallic silver are electrons as we would expect in a highly conductive metal. Correspondingly, the average magnitude of charge carrier density was $n_{\mathrm{Ag}} = (3.3 \pm 0.3) \cdot 10^{29} m^{-3}$, resistivity was $\rho_{\mathrm{Ag}} = (4.4 \pm -0.8) \cdot 10^{-9} \Omega m$, average electric mobility was found as $\mu_{\mathrm{Cr}} = (4.2 \pm 0.9) \cdot 10^{-3} \frac{m^2}{C\Omega}$, and the average drift velocity was calculated as $v_{\mathrm{Ag}} = (7 \pm 1) \cdot 10^{-4} \frac{m}{s}$ which is at the same order of magnitude as the accepted drift velocity of electrons in silver which although depends on the area of cross section, is around $10^{-4} \frac{m}{s}$. Thus, this value makes sense.

Clearly, the clear linear trend in data points for higher magnetic field strengths for Silver hall sample hints that our hall coefficient and other values would been more accurate if higher magnetic fields had been applied to the sample. Further, the inaccuracy in our measurements for Chromium sample could also be related to our observations that when we turned off the external magnetic field, the hall voltage did not collapse to 0 V immediately but decreased very gradually, indicating presence of stray magnetic field in the sample. Thus, this seems to be due to impurities in the sample.

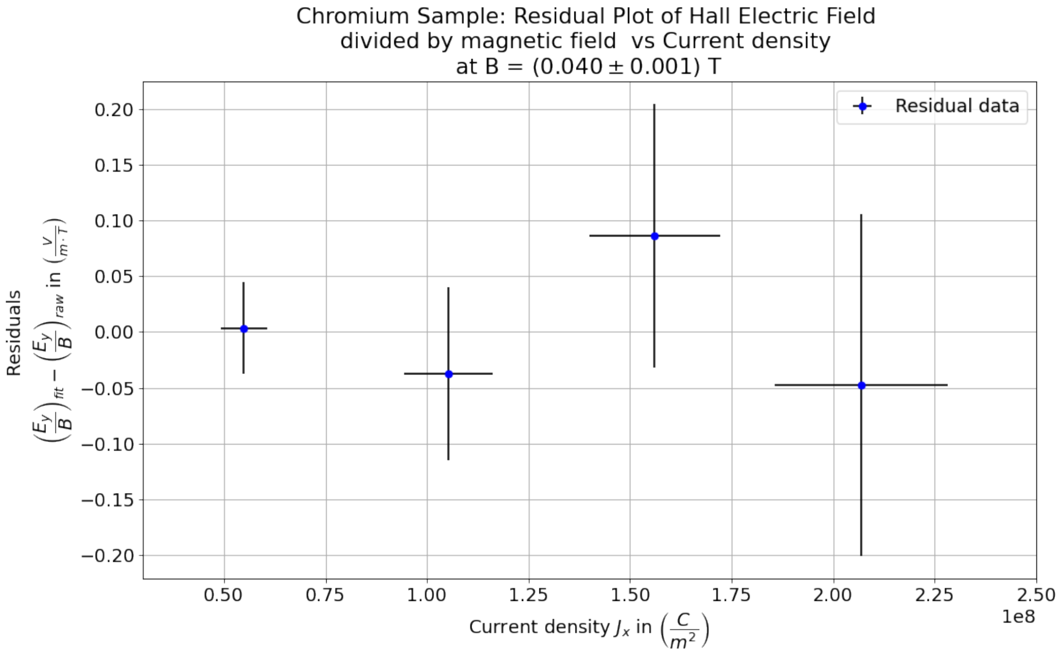
Also, the hall coefficient of Chromium $R_H^{Cr} = (3.54 \pm 0.03) \cdot 10^{-8} \frac{m^3}{C}$ being higher than that of Silver $R_H^{Ag} = (-1.9 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$ implies that Chromium exhibits a much stronger response to external magnetic fields. On the other hand, the resistivity of Silver $\rho_{\mathrm{Ag}} = (4.4 \pm -0.8) \cdot 10^{-9} \Omega m$ is much lower than that of Chromium $\rho_{\mathrm{Cr}} = (7.2 \pm 0.7) \cdot 10^{-7} \Omega m$, and hence Silver has a higher

[4]Entler, S., Soban, Z., Duran, I., Kovarik, K., Vyborny, K., & Sebek, J. et al. (2021). Ceramic-Chromium Hall Sensors for Environments with High Temperatures and Neutron Radiation. Sensors, 21(3), 721. doi: 10.3390/s21030721

conductance σ than Chromium because it is clearly a much better conductor than Chromium. Moreover, it makes sense that the charge carrier density in silver is higher than chromium as there would be more electrons than positive ions in a metal and the charge carriers in chromium are ions while that in silver are electrons.

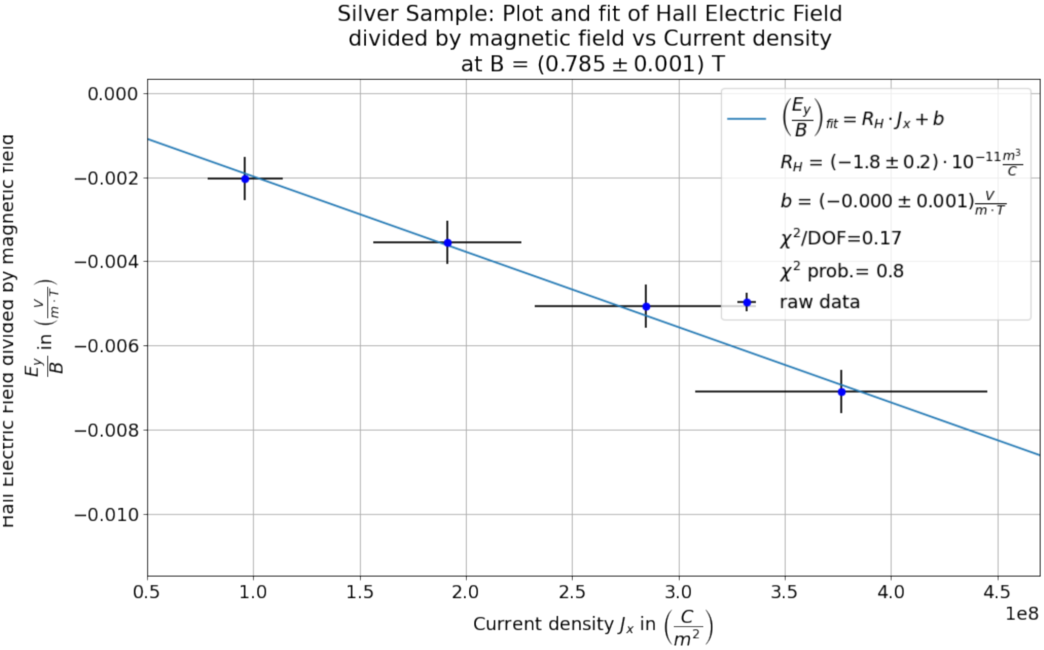


(a) Plot of $\frac{E_y}{B}$ vs J_x

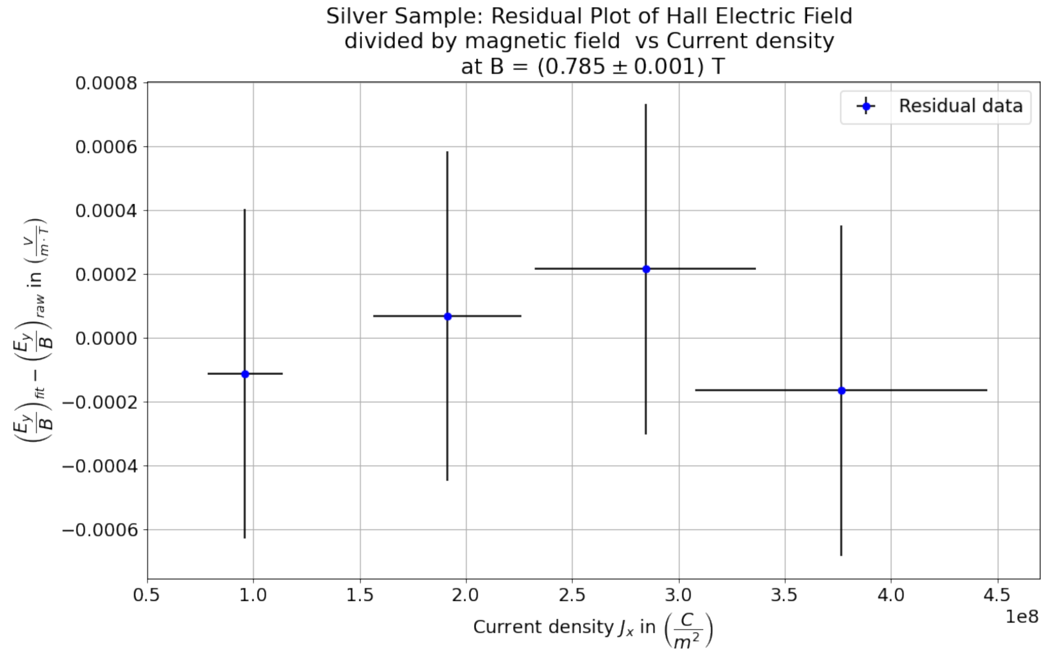


(b) Residual Plot: difference between $\frac{E_y}{B}$ and the line of best fit $\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$, where $R_H = (2.96 \pm 0.08) \cdot 10^{-8} \frac{m^3}{C}$ and $b = (-0.05 \pm 0.07) \frac{V}{m \cdot T}$.

Figure 3: Plot, fit, and residuals of raw data $\frac{E_y}{B}$ vs J_x at $B = (0.040 \pm 0.001)$ T for Chromium Sample in Hall Effect Experiment. The hall coefficient for this setup was found $R_H = (2.96 \pm 0.08) \cdot 10^{-8} \frac{m^3}{C}$, about 100 times larger than the standard accepted value $3.8 \cdot 10^{-10} \frac{m^3}{C}$. The reduced χ^2 value is $0.43 < 1$ which shows slight overfitting (but is expected given that the fit is linear). In (b), all the data points pass through the line $y = 0$ within their corresponding vertical uncertainty. This shows that the line of best fit passes through all the points within their uncertainty ranges.



(a) Plot of $\frac{E_y}{B}$ vs J_x



(b) Residual Plot: difference between $\frac{E_y}{B}$ and the line of best fit $\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$, where $R_H = (-1.8 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$ and $b = (0.000 \pm 0.001) \frac{V}{m \cdot T}$.

Figure 4: Plot, fit, and residuals of raw data $\frac{E_y}{B}$ vs J_x at $B = (0.785 \pm 0.001)$ T for Silver Sample in Hall Effect Experiment. The hall coefficient for this setup was found $R_H = (-1.8 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$. The reduced χ^2 value is $0.17 < 1$ which is because of comparatively large uncertainty in $\frac{E_y}{B}$. In (b), all the data points pass through the line $y = 0$ within their corresponding vertical uncertainty. This shows that the line of best fit passes through all the points within their uncertainty ranges.

4 Conclusion

This lab report summarizes the results of the Hall Effect experiment in which we pass a current through Silver and Chromium strips using an external voltage. Placing the strips in an external magnetic field we measure the Hall- Voltage (HV) and Hall-Current (HC) to calculate the Hall Coefficient R_H of each sample. For the Chromium sample we found $R_H^{Cr} = (3.54 \pm 0.03) \cdot 10^{-8} \frac{m^3}{C}$ and for the Silver sample we found $R_H^{Ag} = (-1.9 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$. Moreover, using the four-wire method to measure the resistance of the samples, we calculated the charge carrier density, the drift velocity and conductive mobility. The standard values of the Hall Coefficient for Silver and Chromium are $R_H^{\text{Ag}} = -8.9 \times 10^{-11}$ m^3/C and $R_H^{\text{Cr}} = 3.8 \times 10^{-10}$ m^3/C respectively. These discrepancies were inferred to be from using lower magnetic field strength (since the higher coil current data-sets resulted in data which fit a linear line better.) Moreover from the sign of the R_H we see that positive charges were moving in the Chromium sample which we believe is cased due to impurities in the sample.

Appendix

Setup Images

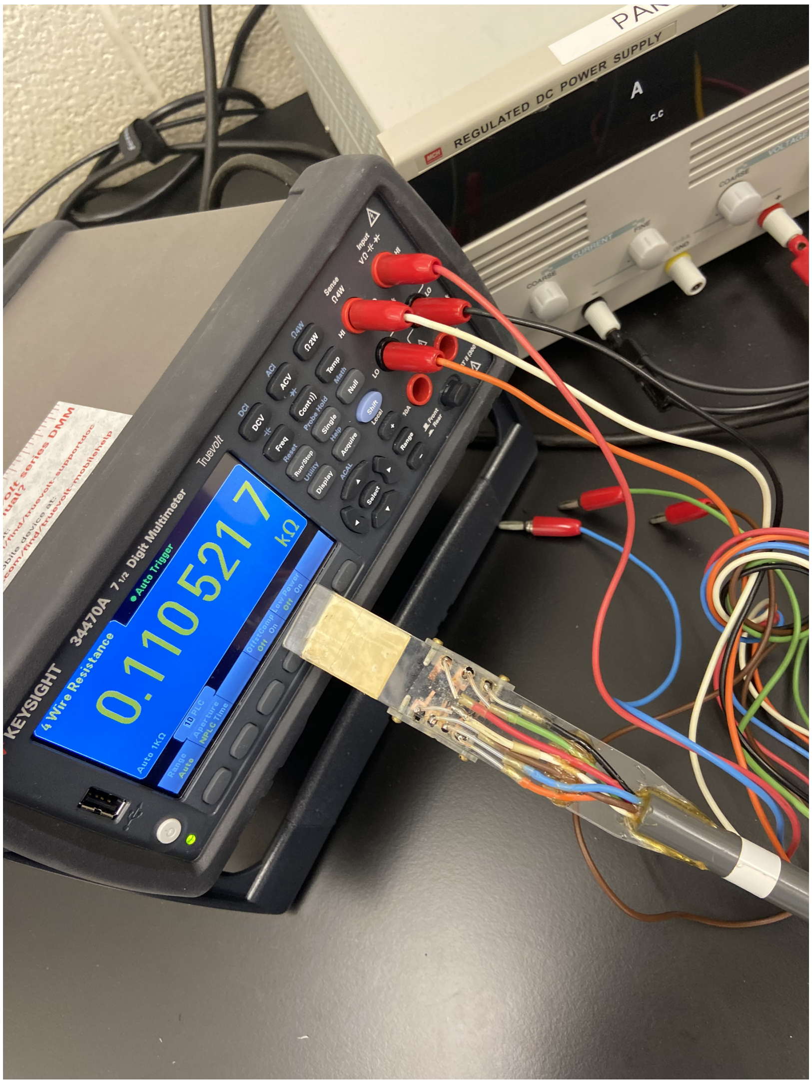


Figure 5: Circuit connections for the 4-Wire method of measuring resistance. We pass a test high current through the sample via the Red and White wires and a low sense current through the sample via the Black and Orange wires. This gives us the desired resistance of the sample across which the wires are connected. It is important to use wires of the same length to ensure they all have the same internal resistance.

Magnetic Field Plots

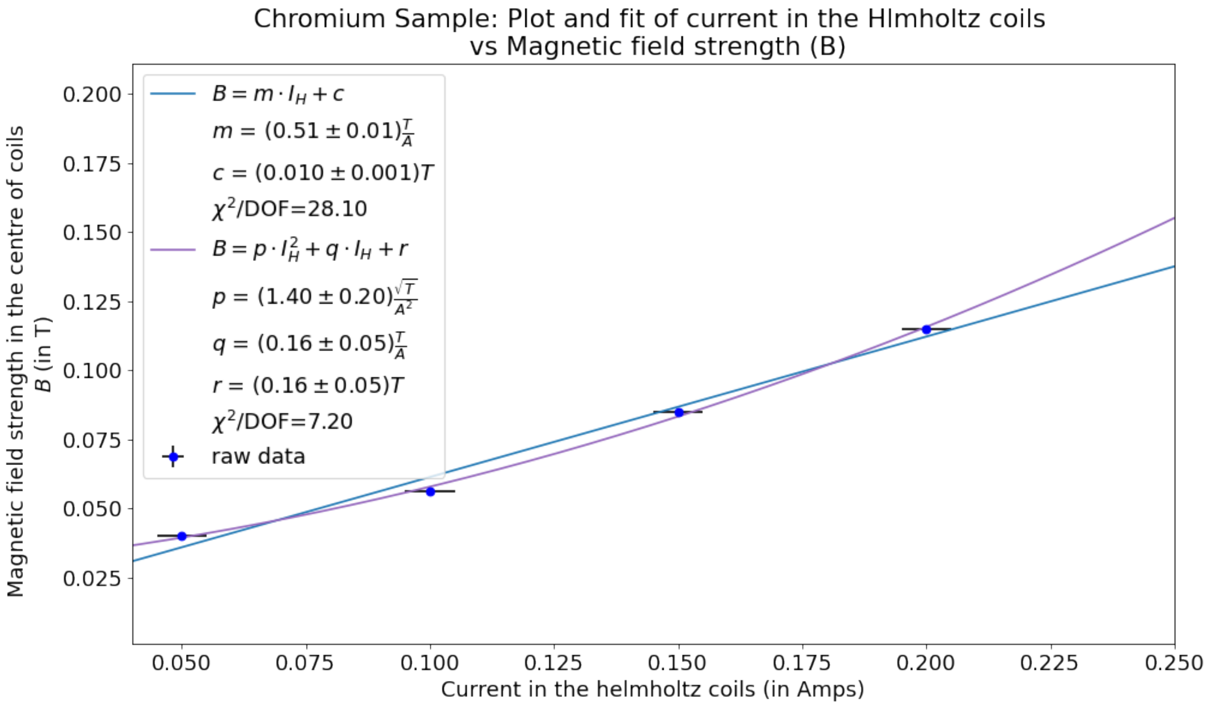


Figure 6: Magnetic field vs current in coils for Chromium

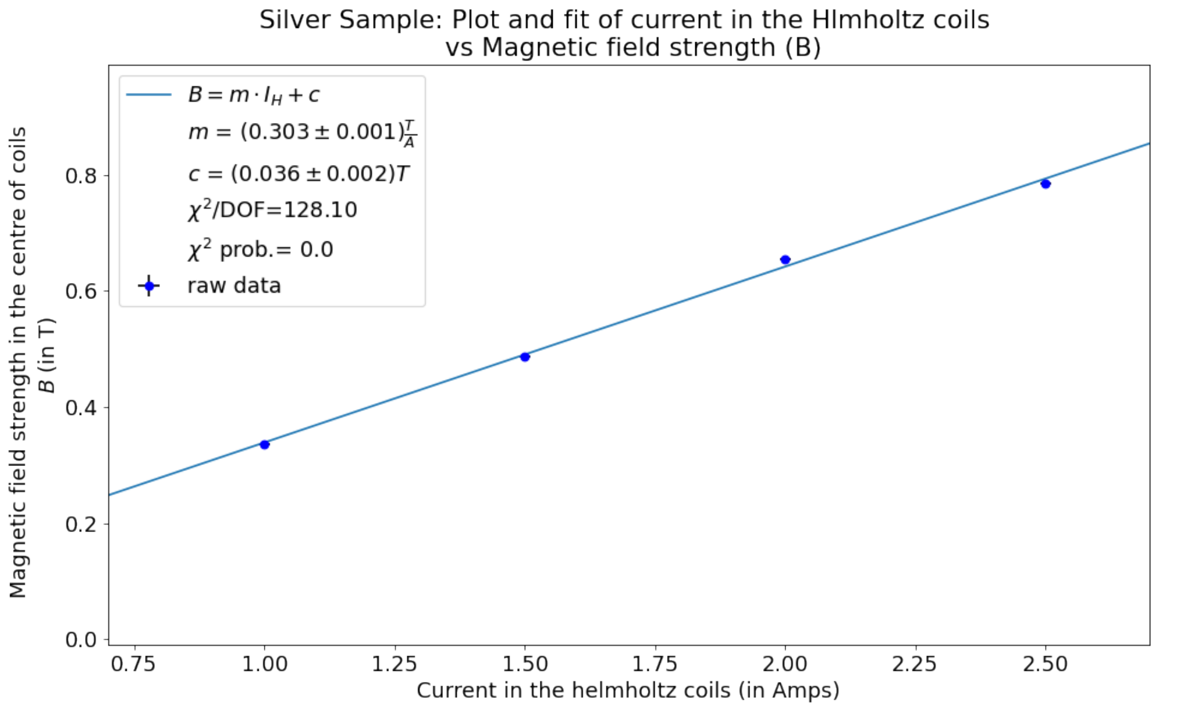
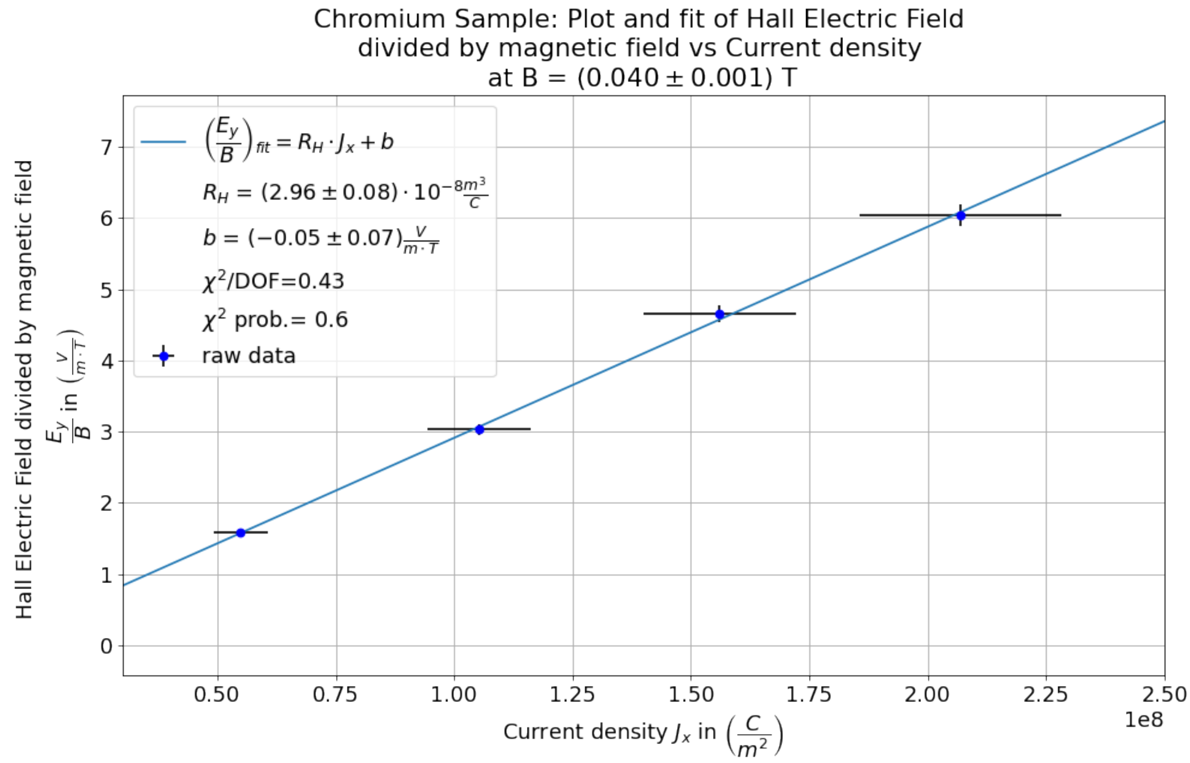
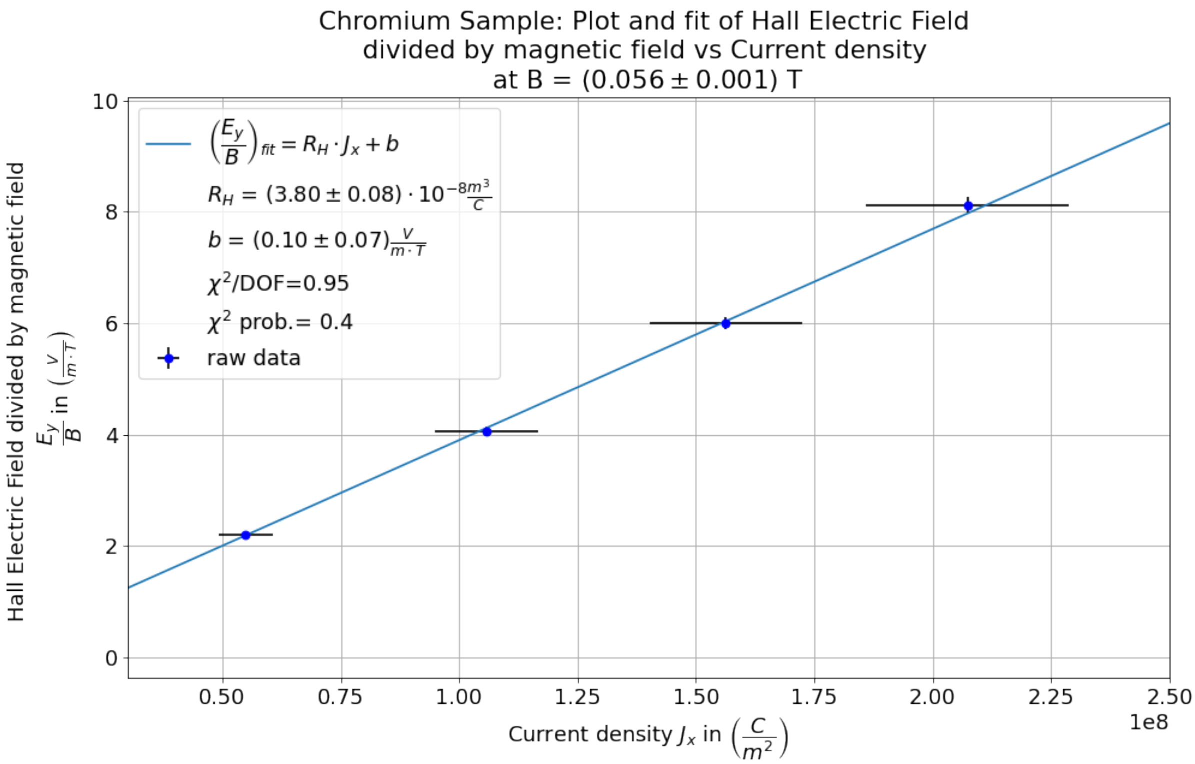
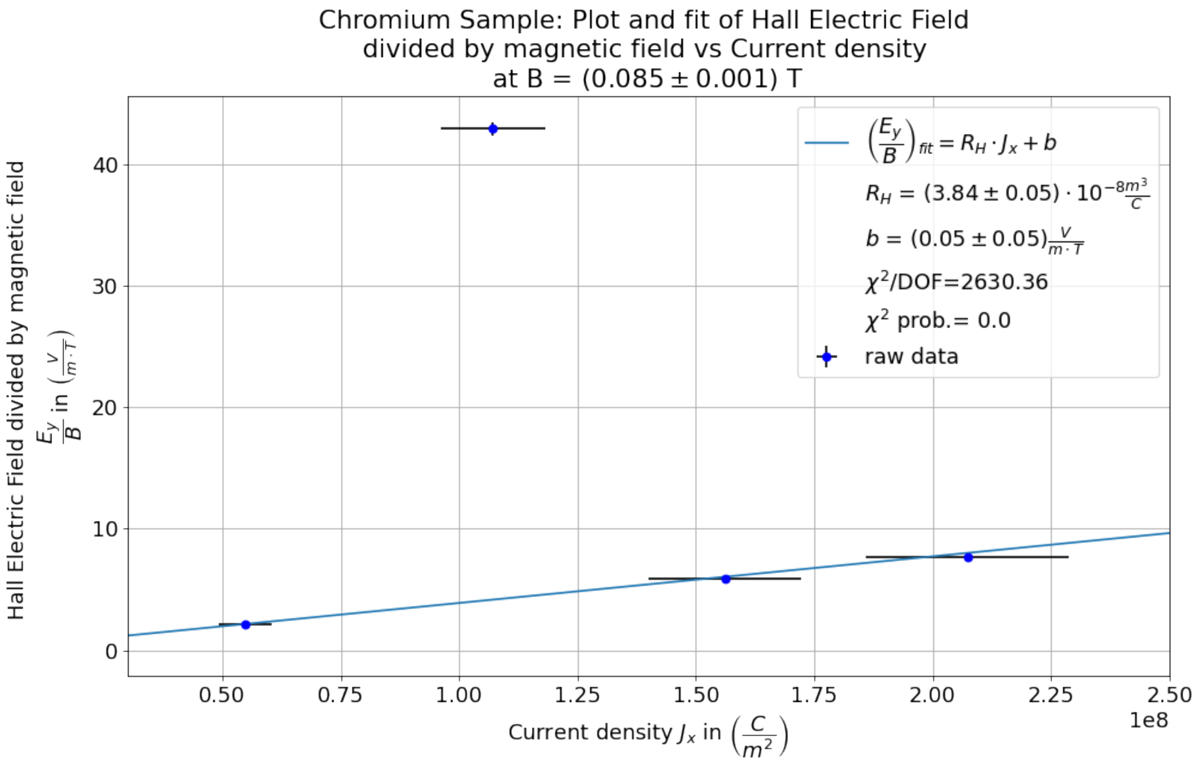


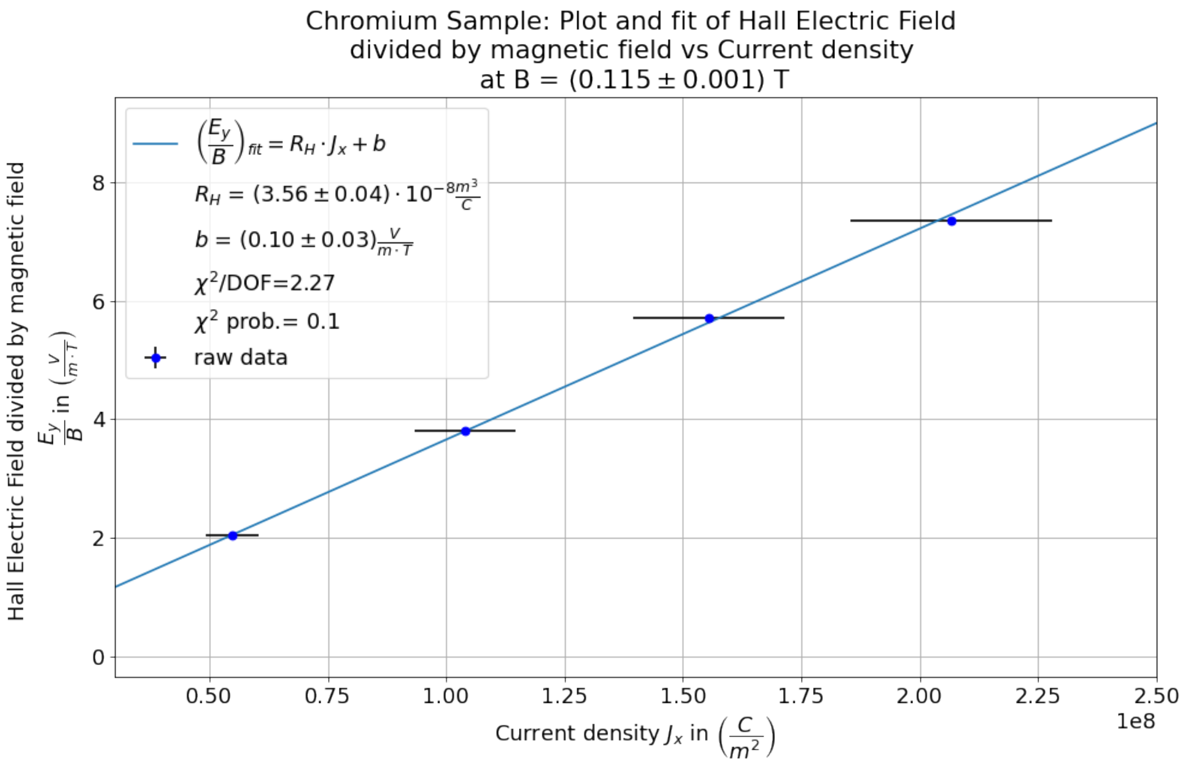
Figure 7: Magnetic field vs current in coils for Silver

Chromium sample Trial Run Plots for all B field values

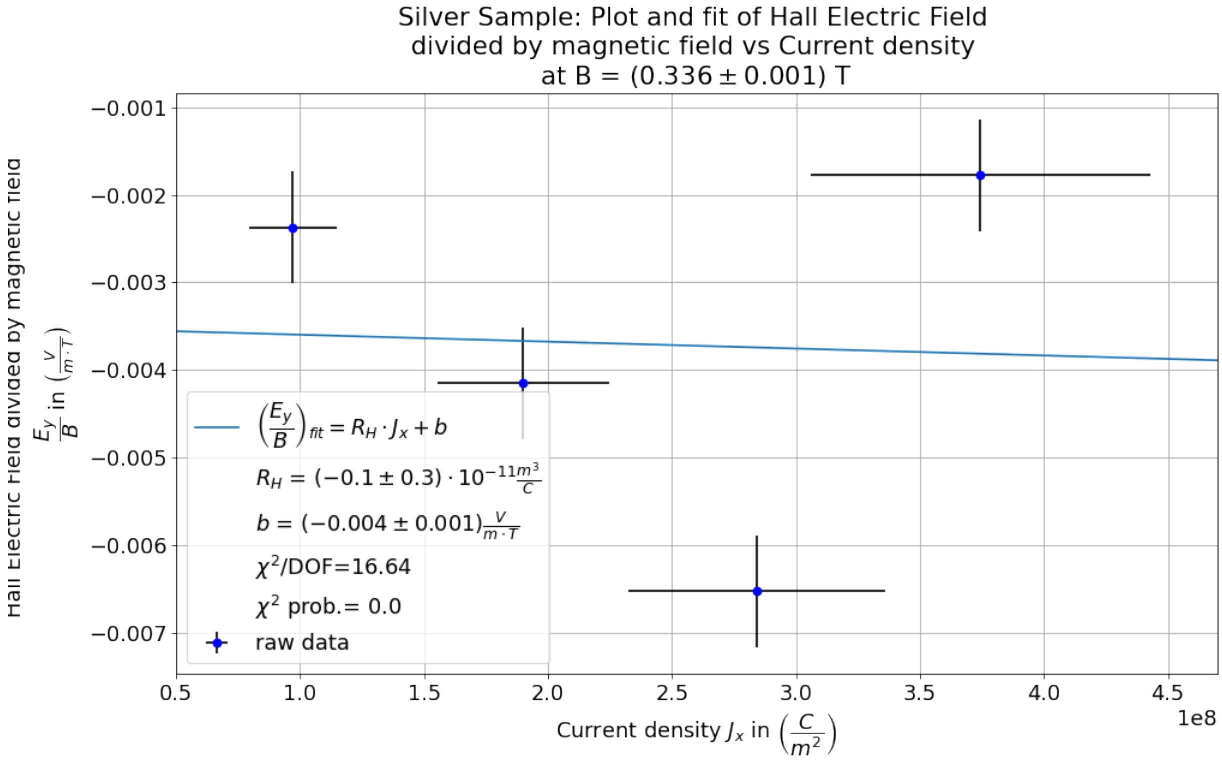


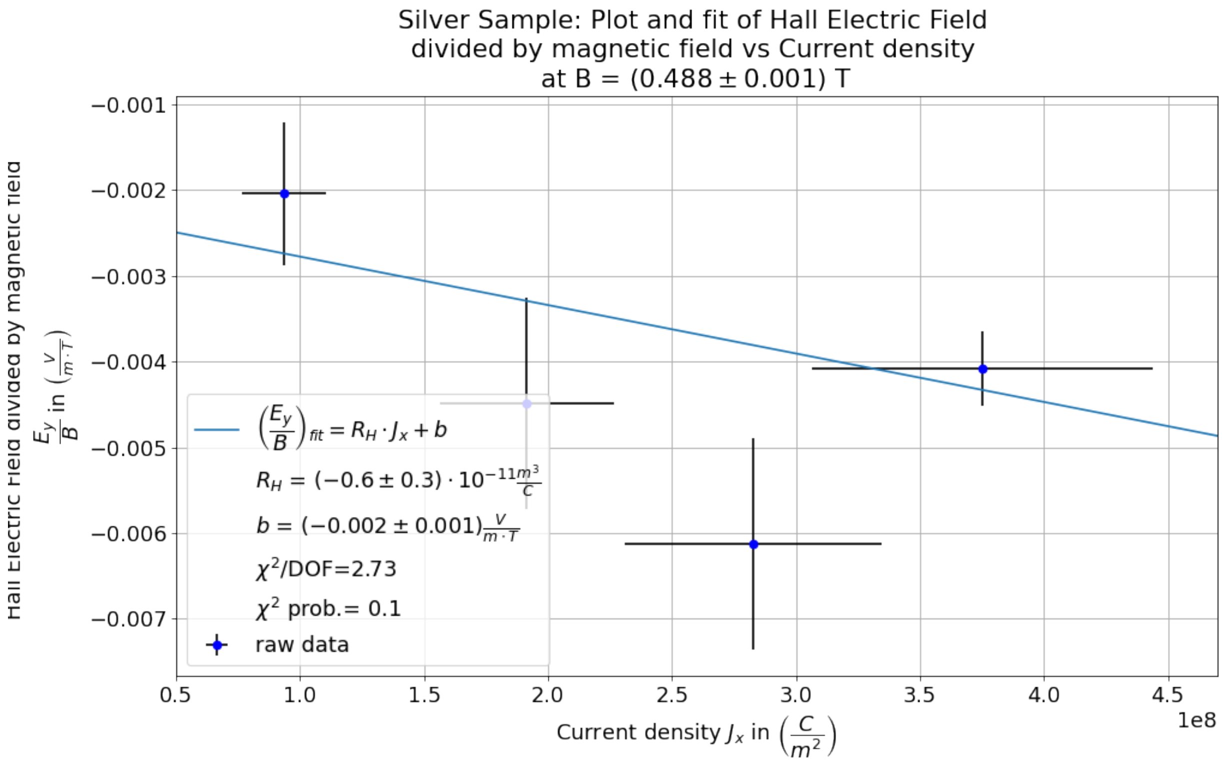


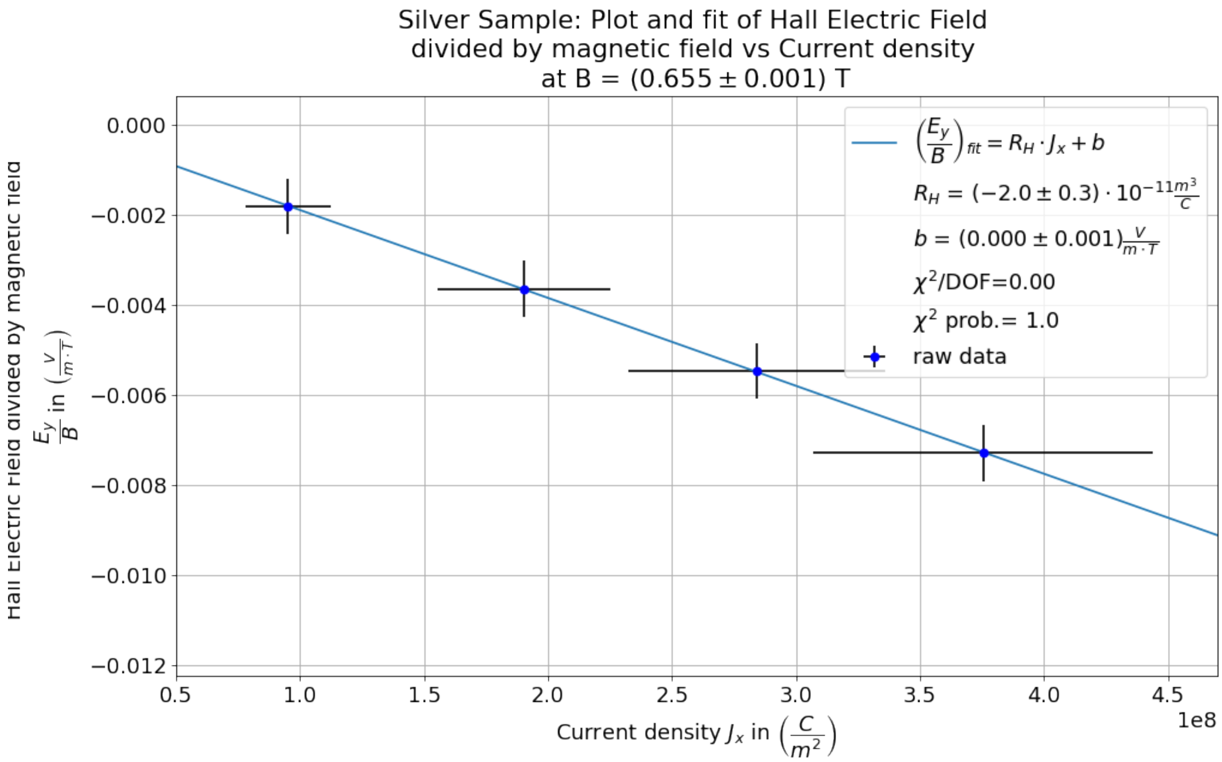

Chromium Sample: Plot and fit of Hall Electric Field
divided by magnetic field vs Current density
at B = (0.085 ± 0.001) T
$\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$
$R_H = (3.84 \pm 0.05) \cdot 10^{-8} \frac{m^3}{C}$
$b = (0.05 \pm 0.05) \frac{V}{m \cdot T}$
χ^2/DOF=2630.36
χ^2 prob.= 0.0
raw data
Hall Electric Field divided by magnetic field
$\frac{E_y}{B}$ in $\left(\frac{V}{m \cdot T}\right)$
Current density J_x in $\left(\frac{C}{m^2}\right)$
1e8

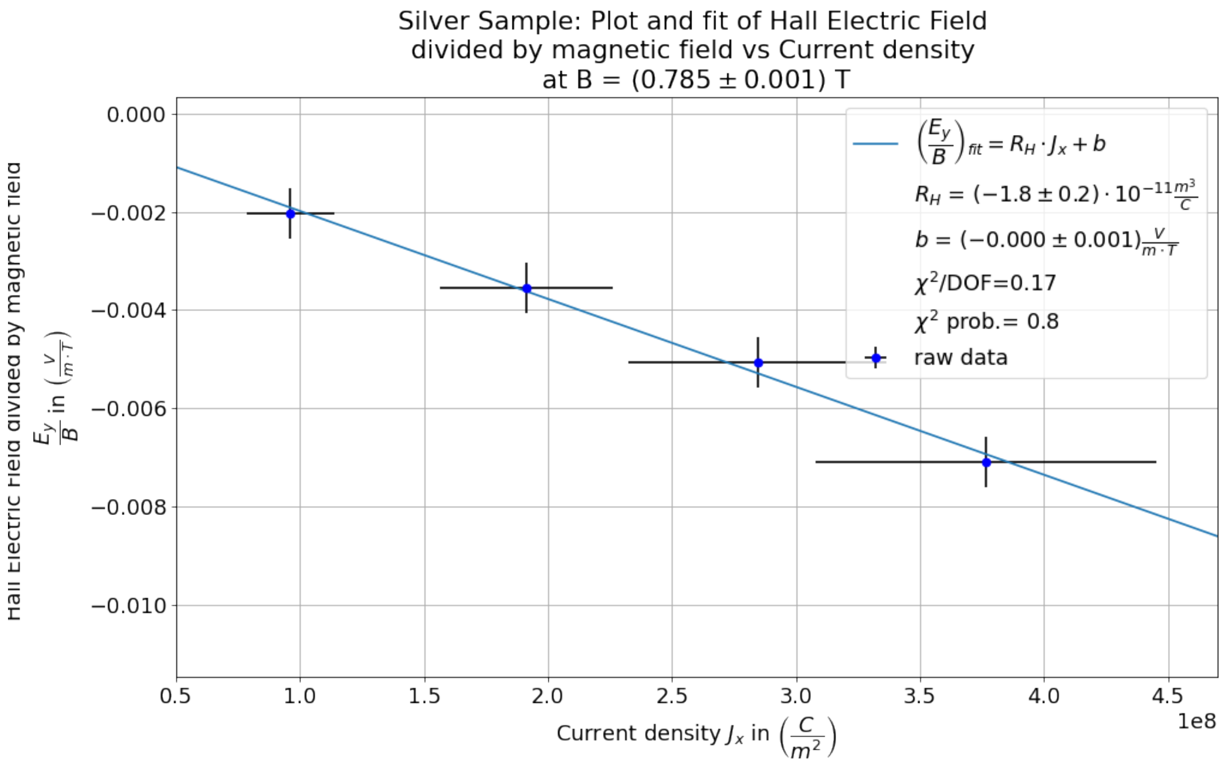

Chromium Sample: Plot and fit of Hall Electric Field
divided by magnetic field vs Current density
at B = (0.115 ± 0.001) T
$\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$
$R_H = (3.56 \pm 0.04) \cdot 10^{-8} \frac{m^3}{C}$
$b = (0.10 \pm 0.03) \frac{V}{m \cdot T}$
χ^2/DOF=2.27
χ^2 prob.= 0.1
raw data
Hall Electric Field divided by magnetic field
$\frac{E_y}{B}$ in $\left(\frac{V}{m \cdot T}\right)$
Current density J_x in $\left(\frac{C}{m^2}\right)$
1e8

Silver sample Trial Run Plots for all B field values






Silver Sample: Plot and fit of Hall Electric Field divided by magnetic field vs Current density at B = (0.655 ± 0.001) T
$\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$
$R_H = (-2.0 \pm 0.3) \cdot 10^{-11} \frac{m^3}{C}$
$b = (0.000 \pm 0.001) \frac{V}{m \cdot T}$
χ^2/DOF=0.00
χ^2 prob.= 1.0
raw data
Hall Electric Field divided by magnetic field $\frac{E_y}{B}$ in $\left(\frac{V}{m \cdot T}\right)$
Current density J_x in $\left(\frac{C}{m^2}\right)$
1e8


Silver Sample: Plot and fit of Hall Electric Field divided by magnetic field vs Current density at B = (0.785 ± 0.001) T
$\left(\frac{E_y}{B}\right)_{fit} = R_H \cdot J_x + b$
$R_H = (-1.8 \pm 0.2) \cdot 10^{-11} \frac{m^3}{C}$
$b = (-0.000 \pm 0.001) \frac{V}{m \cdot T}$
χ^2/DOF=0.17
χ^2 prob.= 0.8
raw data
Hall Electric Field divided by magnetic field $\frac{E_y}{B}$ in $\left(\frac{V}{m \cdot T}\right)$
Current density J_x in $\left(\frac{C}{m^2}\right)$
1e8