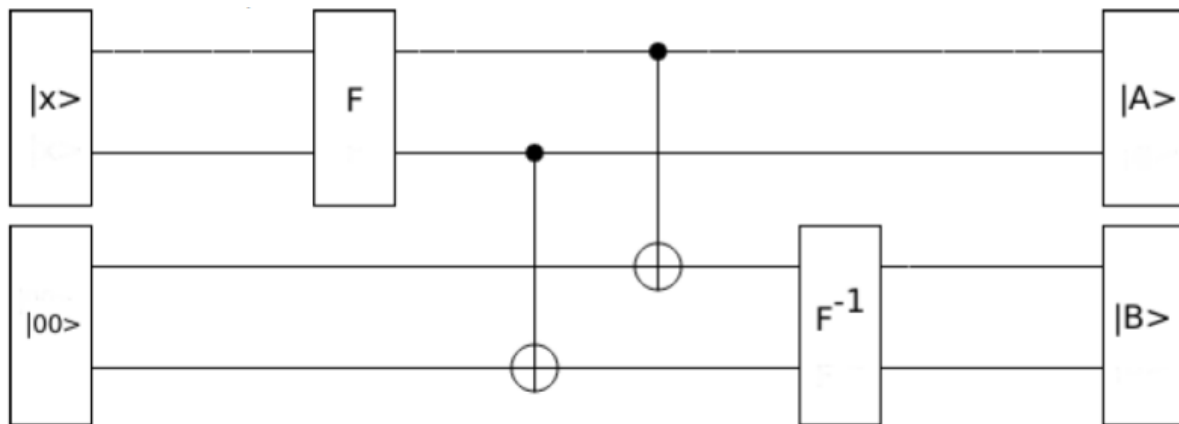


Analysis of the circuit:



For the given circuit let's assume, lower two-qubit gate as $|y\rangle$ which is at ground state $|00\rangle$.

Now we have $|x\rangle$ and $|y\rangle$ as our input two-qubit and $|A\rangle$ and $|B\rangle$ as our output two-qubit quantum registers.

Given circuit has two CNOT gates in the middle of the network. Where, qubits x_1, x_2 from register $|x\rangle$ are controlling y_1, y_2 qubits from register $|y\rangle$ using CNOT gates respectively.

If qubit $x_1=1$, then y_1 will be flipped. If $x_2=1$, then y_2 will be flipped.

We have a gate F operating on $|x\rangle$ which we do not know what it does. Depending upon the properties of the gate F and input $|x\rangle$ the output quantum registers $|A\rangle$ and $|B\rangle$ will change values.

Let's consider some cases.

Case 1:

If F is a Pauli X gate. F^{-1} becomes Pauli X gate.

Thus, for any value of x_1, x_2 in $|x\rangle$ we get,

Qubits in $|x\rangle$ will flip and we will get those in $|A\rangle$.

Whereas $|y\rangle$ ($|00\rangle$) after flipping twice will provide $|B\rangle = |x_1x_2\rangle = |x\rangle$.

Case 2:

If F is a CNOT gate. F^{-1} becomes CNOT.

We find $F|x\rangle$ in $|A\rangle$ and $|x\rangle$ in $|B\rangle$.

$|A\rangle = F|x\rangle$ and $|B\rangle = |x_1x_2\rangle = |x\rangle$.

Case 3:

F is a Hadamard gate, F^{-1} becomes H^{-1} .

Here, we will get incoherence after one operation. But, after repeating the circuit thrice in a series, we will get, $|A\rangle=|00\rangle$ and $|B\rangle=|x_1x_2\rangle=|x\rangle$.

Considering cases 1, 2, and 3, we can say that the given circuit is producing outputs based on some arbitrary F and input $|x\rangle$ from $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. Where, $|A\rangle$ is F operating on $|x\rangle$ and $|B\rangle$ is same as input $|x\rangle$.
 $|A\rangle=F|x\rangle$ and $|B\rangle=|x\rangle$.

2. Determining operation of gate F:

Consider gate F is some arbitrary 4 by 4 matrices as it is operating on two qubits $|x\rangle$. Thus F^{-1} becomes 4 by 4 matrix. Now CNOT gates can have only $|00\rangle$ and $|01\rangle$ states as input $|y\rangle$ is set to ground. That means all the operations will depend upon either first or second column of the CNOT matrix.

Gate F operating on $|x\rangle$ will produce $F|x\rangle$ any value in $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

For output $|B\rangle$, F^{-1} gate will nullify the effect of gate F on the input quantum register $|x\rangle$. Which in turn will give $|B\rangle = |x\rangle$.

Knowing the outputs being in a pattern, we can backtrack and solve matrices to find the operations of gate F.

This could become possible because of the **probabilistic nature of qubits**, i.e., values between and including **0 and 1**. Qubits can be represented continuously, thus we can backtrack and find what gate F is doing.

Whereas, in classical computing nature of bits is discrete, i.e., either **0 or 1**. Classical bits are irreversible, once we pass them through the circuit, we cannot fetch input from output bits by going in reverse.