Beauty of

VEDIC SPEED MATHEMATICS

(Journey from Limited Intelligence to Human Bio-Calculator)

Sample Pages of Book

HIGHLY USEFUL FOR Standard/Grade **3rd** to **Ph.D** Students; Parents, Mathematics Teachers, Math Lovers, Placement & Job Interviews; All Entrance & Competitive Exams.

Contents

Contents

Unit 1: MULTIPLICATION

Multiplication means times or repeated addition.
Ex.1: 13×3=39 (or 13+13+13=39)
Ex.2: 24×4=96 (or 24+24+24+24=96)

1.1 Multiplication using Base Method

1. Sutra Used is: 2. Nikhilam Navataścaramam Daśatah (निखिलं नवतश्चरमं दशतः) **Meaning:** All from 9 and the last from 10.
2. Bases are any positive numbers ending with 0's (zeroes).
   Ex: 70, 80, 90, 100, 140, 1300, 5600 etc.
3. Working (or functional) Base is always power of 10.
   Ex: 10 (10^1), 100 (10^2), 1000 (10^3), 10000 (10^4) etc.
4. **Complement** = Number – Base
5. **Surplus** = Number – Base
6. **Note:** In multiplication, Base method is preferred if given numbers are nearer (closer) to Working Bases. Otherwise Criss Cross method is preferred.

Number	Base	Complement
8	10	-2
93	100	-7
974	1000	-26
845	1000	-155
57	60	-3
1846	1900	-54

Number	Base	Surplus
12	10	+2
107	100	+7
1145	1000	+145
12364	10000	+2364
57	50	+7
1846	1800	+46

In Vedic Speed Mathematics we get answers quickly if we choose Working Bases. So Prefer Working Bases over Bases. Ex. For 93 Base is both 90 and 100. Choose 100 over 90 because 100 is Working Base.

Abbreviations used	**D:** Digit	**B:** Base	**C:** Complement
	S: Surplus	**BM:** Base Multiple	**BR:** Base Ratio

Case 1: When both numbers (multiplicand and multiplier) are less than the working base:

Working Procedure:

1. Write multiplicand and multiplier one below the other.
2. Write complements of multiplicand and multiplier to its right side with signs.
3. Answer consists of two parts. Left and Right.
4. Left Part: Evaluating any of the cross values.
5. Right Part: Product of both complements (right side values).
6. **Caution:** Total number of digits in the Right Part should be equal to total number of zeroes in the base. If lesser, add required number of zeroes before the right part. If greater then pass the carry (left most excess digits of right part) to left part.

Ex.1: 7×8 Base = 10 7 -3 8 -2 ------ 5 ¦ 6 **56**	**Ex. 1:** Here we need to multiply 7 and 8. We choose base as 10, as both the numbers (7 and 8) are nearer to 10. Numbers 7 and 8 are written one below the other. Their complements are -3 and -2 respectively and they are written at right side. Left Part is 5 {7+(-2) or 8+(-3)}. Right Part is product of complements i.e. -3×-2=6. So final answer is: 56.

Ex.2: 6×7 B = 10 6 -4 7 -3 -------- 3 ¦ 12 3+1 ¦ 2 4 ¦ 2 **42**	**Ex. 2:** 6×7; complements are -4 and -3. Left Part is 3 (6-3 or 7-4) and Right Part is 12 (-4×-3). Here base is having only one zero, so right part should be of single digit. Pass 1 (leftmost excess bit of right part) as carry to Left Part. Left Part: 3+1=4 Right Part: 2. So final answer is: 42.

Ex.3: 94×96 B = 100 94 -6 96 -4 --------- 90 ¦ 24 **9024**	**Ex.3:** 94×96; Base is 100 as both the given numbers (94 and 96) are closer to 100; complements are -6 and -4. Left Part is 90 (94-4 or 96-6). Right Part is 24 (-6*-4). So final answer is: 9024.

Ex.4: 90×89 B = 100 90 -10 89 -11 ---------- 79 ¦ 110 79+1 ¦ 10 80 ¦ 10 **8010**	**Ex.4:** 90×89; complements are -10 and -11. Left Part is 79 (90-11 or 89-10). Right Part: 110 (-10*-11). Here base is having two zeroes, so right part should be of two digits. But, Right Part is of three digits. So 1 is passed as carry to Left Part. So Left Part becomes 79+1=80 and Right Part becomes 10. So final answer is: 8010**.**

Ex.5: 997×993 B = 1000 997 -3 993 -7 ---------- 990 ¦ 021 **990021**	**Ex.5:** 997×993; Base is 1000. Complements are -3 and -7. Left Part is 990 (997-7 or 993-3). Right Part is 021 (-3*-7). So final answer is: 99021. **Note:** Result of product of complements is 21. But we need to add one ZERO before 21. Because base is 1000 & having THREE zeroes.

Ex.6: 950×930 B = 1000 950 -50 930 -70 ---------------- 880 ¦ 3500 880+3 ¦ 500 883 ¦ 500 **883500**	**Ex.6:** 950×930: complements are -50 and -70. Left Part is 880 (950-70 or 930-50). Right Part is 3500 (-50*-70). Here base is 1000 (Three Zeroes) and Right Part is of 4 digits. So 3 is passed as carry to Left Part. So Left Part becomes 880+3=883 and Right Part becomes 500. So final answer is: 883500.

Ex.7: 81×92? (Base = 100) 81 -19 92 -8 ------------------ 73 ¦ 152 73+1 ¦ 52 **74 ¦ 52** **7452**	**Ex.8: 76×95?** (Base = 100) 76 -24 95 -5 ------------------- 71 ¦ 120 71+1 ¦ 20 **72 ¦ 20** **7220**	**Ex.9: 985×960?** (Base = 1000) 985 -15 960 -40 ------------------ **945 ¦ 600** **945600**

Ex.10: 9800×9784? (Base = 10000) 9800 -200 9784 -216 ------------------- 9584 ¦ 43200 9584+4 ¦ 3200 **9588 ¦ 3200** **95883200**	**Ex. 11: 84×94?** (Base = 100) 84 -16 94 -6 ------------------ **78 ¦ 96** **7896**	**Ex.12: 996×975?** (Base = 1000) 996 -4 975 -25 ------------------ **971 ¦ 100** **971100**

"Success is not final; failure is not fatal: It is the courage to continue that counts."

"It is better to fail in originality than to succeed in imitation."

"The road to success and the road to failure are almost exactly the same."

"Success usually comes to those who are too busy to be looking for it."

Case 2: When both numbers are greater than the working base:

Working Procedure:

1. Write multiplicand and multiplier one below the other.
2. Write surpluses of multiplicand and multiplier to its right side with signs.
3. Left Part: Adding any of the cross values.
4. Right Part: Product of both surpluses (right side values).
5. **Caution:** Total number of digits in the Right Part should be equal to total number of zeroes in the base. If lesser, add required number of zeroes before the right part. If greater then pass the carry (left most excess digits of right part) to left part.

Ex.1:12×14 B = 10 12 +2 14 +4 ------- 16 ¦ 8 **168**	**Ex.1:** 12×14; here we need to multiply 12 and 14. Numbers 12 and 14 are written one below the other. Their surplus +2 and +4 respectively and they are written at right side. Left Part is 16 (12+4 or 14+2). Right Part is product of surplus i.e. 2×4=8. Here base is 10 (Single Zero). Right part is of single digit. So final answer is 168.

Ex.2:16×17 B = 10 16 +6 17 +7 -------- 23 ¦ 42 23+4¦2 27 ¦ 2 **272**	**Ex. 2:** 16×17; surplus: +6 and +7. Left Part is 23 (16+7 or 17+6). Right Part is 42 (6×7). Base is 10 (Single Zero). But Right part is having two digits. Leftmost digit of right part (here it is 4) is taken to Left part as carry. So Left part becomes 27 (23+4) and Right part becomes 2. So final answer is 272.

It always seems impossible until it's done --Nelson Mandela

Ex.3:109×111 B = 100 109 +9 111 +11 ---------- **120 ¦ 99**	**Ex. 3:** 109×111; surplus: +9 and +11. Left Part is 120 (109+11 or 111+9). Right Part is 99 (9×**11).** Here base is 100 (Two Zeroes). Right part is having two digits. So no any further calculations are required. The final answer is 12099.

Ex.4:117×110 B = 100 117 +17 110 +10 ---------- 127¦170 127+1¦70 **128 ¦70**	**Ex. 4:** 117×110; surplus: +17 and +10. Left Part is 127 (117+10 or 110+17). Right Part is 170 (17×**10).** Here base is 100 (Two Zeroes). But Right part is having three digits. Leftmost digit of right part (here it is 1) is taken to Left part as carry. So Left part becomes 128 (127+1) and Right part becomes 70. So final answer is 12870.

Ex. 5: 1020×1033 B = 1000 1020 +20 1033 +33 ---------- 1053 ¦ 660 **1053660**	**Ex. 5:** 1020×1033; surplus: +20 and +33. Left Part is 1053 (1020+33 or 1033+20). Right Part is 660 (20×33). Here base is 1000 (Three Zeroes). Right part is having three digits. So no any further calculations are required. The final answer is 1053660.

Ex.6: 1050×1030 B = 1000 1050 +50 1030 +30 ---------- 1080¦1500 1080+1¦500 1081 ¦ 500 **1081500**	**Ex. 6:** 1050×1030; surplus: +50 and +30. Left Part is 1080 (1050+30 or 1030+50). Right Part is 1500 (50×30). Here base is 1000 (Three Zeroes). But Right part is having four digits. Leftmost digit of right part (here it is 1) is taken to Left part as carry. So Left part becomes 1081 (1080+1) and Right part becomes 500. So final answer is 1081500.

Ex.7: 112×128?	**Ex.8: 126×104?**	**Ex.9: 1048×1040?**
(Base = 100)	(Base = 100)	(Base = 1000)
112 +12	126 +26	1048 +48
128 +28	104 +4	1040 +40
-------------------	-------------------	-------------------
140 ¦ 336	130 ¦ 104	1088 ¦ 1920
140+3 ¦ 36	130+1 ¦ 04	1088+1 ¦ 920
143 ¦ 36	131 ¦ 04	1089 ¦ 920
14336	**13104**	**1089920**

Ex.10: 12745×10200?	**Ex.11: 1024×1006?**	**Ex.12: 113×107**
(Base = 10000)	(Base = 1000)	(Base = 100)
12745 +2745	1024 +24	113 +13
10200 +200	1006 +6	107 +7
---------------------	-----------------	------------------
12945 ¦ 549000	1030 ¦ 144	120 ¦ 91
12945+54 ¦ 9000	**1030144**	**12091**
12999 ¦ 9000		
129999000		

Case 3: When one number is lesser and other is greater than the working base:

Working Procedure:

1. Write multiplicand and multiplier one below the other.
2. Write complement / surplus of multiplicand and multiplier to its right side with signs.
3. Left Part: Evaluating any of the cross values as per the sign (adding or subtracting).
4. Right Part: Product of both complement and surplus (right side values).
5. **Additional Step:** In this case, in the Right Part we always get negaive value. Let 'n' be the total number of zeroes in the base. To get 'n' digit positive number in the Right Part, Add ‘x’ times

of Base to the Right Part and Parallely Subtract 'x' from Left Part.

Ex.1: 8×13 B = 10 08 -2 13 +3 --------- 11 ¦ -6 11-1¦ -6+10 10 ¦ 4 =**104**	**Ex.1:** 8×13; Complement of 8 is -2 and Surplus of 13 is +3. Left Part is 11 (8+3 or 13-2). Right Part is product of complement and surplus. i.e. -2×3=-6. Here base is 10 & there is only one zero in the base. So, in the Right Part we should have one digit postitive number but having negative value. To get one digit positive number, we need to add **ONE** time of base to Right Part. Parallely we need to subtract 1 from Left Part. Right Part is 4 (∵-6+10=4) and Left Part is 10 (∵11-1=10). So final answer is 104.

Ex.2: 106×76 B = 100 106 +6 76 -24 ---------- 82 ¦ -144 82-2¦-144+200 80 ¦ 56 = **8056**	Surplus of 106 is +6 and Complement of 76 is -24. Left Part is 82 (106-26 or 76+6). Right Part is product of surplus and complement. i.e. 6×-24=-144. Here base is 100 & there are two zeroes in the base. So, in the Right Part we should have two digit postitive number but having negative value. To get two digit positive number, we need to add **TWO** times of base to Right Part. Parallely we need to subtract 2 from
Left Part. Right Part is 56 (∵-144+200=56) and Left Part is 80 (∵82-2=80). Final answer is 104. **Note:** If we add one time of base to Right Part; we get -44 (∵-144+100=-44). We don't want negative value in the Rigt Part. If we add three times of base; we get 156 (∵-144+300=156). We don't want three digit number in the Right Part as our base is 100 and having two zeroes. That's why we choose two times of base. After chosing we get required two digit positive number in the Right Part.	

Ex.3: 109×94 B = 100 109 +9 94 -6 ------------ 103 ¦ -54 103-1¦-54+100 102 ¦ 46 =**10246**	Surplus of 109 is +9 and Complement of 94 is -6. Left Part is 103 (109-6 or 94+9). Right Part is product of surplus and complement. i.e. 9×-6=-54. Here base is 100 & there are two zeroes in the base. So, in the Right Part we should have two digit postitive number but having negative value. To get two digit positive number, we need to add **ONE** time of base to Right Part. Parallely we need to subtract 1 from Left Part. Right Part is 46 (∵-54+100=46) and Left Part is 102 (∵103-1=102). Final answer is 10246.

Ex.4: 97×124 B = 100 97 -3 124 +24 ______________ 121 ¦ -72 121-1 ¦ -72+100 120 ¦ 28 =**12028**	Complement of 97 is -3 and Surplus of 124 is +24. Left Part is 121 (97+24 or 124-3). Right Part is product of complement and surplus. i.e. -3×24=-72. Here base is 100 & there are two zeroes in the base. So, in the Right Part we should have two digit postitive number but having negative value. To get two digit positive number, we need to add **ONE** time of base to Right Part. Parallely we need to subtract 1 from Left Part. Right Part is 28 (∵-72+100=28) and Left Part is 120 (∵121-1=120). Final answer is 12028.

Note: There is an alternative for additional step. Multiply Left Part with base. Add Right Part to it. We will get answer. For example:

Ex.2: Left Part is 82. Base is 100. Multiply both. Product is 8200. Add Right Part (-144) to it. So final answer is 8200 + (-144) = 8200 – 144 = 8056.

Ex.4: Left Part is 121. Base is 100. Multiply both. Product is 12100. Add Right Part (-72) to it. So final answer is 12100 + (-72) = 12100 – 72 = 12028.

Ex.5: 1020×989 B = 1000 1020 +20 989 -11 ------------------ 1009 ¦ -220 1009-1 ¦ -220+1000 1008 ¦ 780 =**1008780**	Surplus of 1020 is +20 and Complement of 989 is -11. Left Part is 1009 (1020-11 or 989+20). Right Part is product of surplus and complement. i.e. 20×-11=-220. Here base is 1000 & there are three zeroes in the base. So, in the Right Part we should have three digit postitive number but having negative value. To get three digit positive number, we need to add **ONE** time of base to Right Part. Parallely we need to subtract 1 from Left Part. Right Part is 780 (∵-220+1000=780) and Left Part is 1008 (∵1009-1=1008). Final answer is 1008780.

Ex.6: 1250×975 Base = **1000** 1250 +250 975 -25 ------------------ 1225 ¦ -6250 1225-7 ¦ -6250+7000 1218 ¦ 750 **=1218750**	Surplus of 1250 is +250 and Complement of 975 is -25. Left Part is 1225 (1250-25 or 975+250). Right Part is product of surplus and complement. i.e. 250×-25=-6250. Here base is 1000 & there are three zeroes in the base. So, in the Right Part we should have three digit postitive number but having negative value. To get three digit positive number, we need to add **SEVEN** times of base to Right Part. Parallely we need to subtract 7 from Left Part. Right Part is 750 (∵-6250+7000=750) and Left Part is 1218 (∵1225-7=1218). Final answer is 1218750

***G**ratitude is heaven itself - William Blake*

*"**H**ave the courage to follow your heart and intuition. They somehow already know what you truly want to become. Everything else is secondary." Steve Jobs*

Ex.7: **89×112?**	**Ex.8:** **92×116?**	**Ex.9:** **976×1030?**	**Ex.10:** **870×1026?**
(Base = 100) 89 -11 112 +12 ---------------- 101 ¦ -132 101-2 ¦ -132+200 99 ¦ 68 **9968**	(Base = 100) 92 -8 116 +16 ------------------- 108 ¦ -128 108-2 ¦ -128+200 106 ¦ 72 **10672**	(Base = 1000) 976 -24 1030 +30 -------------------- 1006 ¦ -720 1006-1 ¦ -720+1000 1005 ¦ 280 **1005280**	(Base = 1000) 870 -130 1026 +26 ---------------- 896 ¦ -3380 896-4 ¦ -3380+4000 892 ¦ 620 **892620**

1.2 Multiplication using Criss Cross Method

Sutra Used: 3. Ūrdhva – tiryagbhyām (ऊर्ध्वतिर्यग्भ्याम्)

Meaning: Vertically & Crosswise

How to Remember? Here you no need to remember any formulas, just you need to understand pattern. Go through graphical representation of various cases and understand pattern. The first part will be multiplication of respective first digits of both multiplier and multiplicand. Last Part will be multiplication of respective last digits of both multiplier and multiplicand. Second Part will be applying criss cross on first two digits of both multiplier and multiplicand. Second last Part will be applying criss cross on second last digits of both multiplier and multiplicand AND SO ON…

Case 1: Two Digit Numbers (2D×2D and 2D×1D) {D: Digit}

Answer consists of three parts.

First Part:	**Second Part:**	**Third Part:**
a b c d	a b c d	a b c d
(a×c)	**(a×d) + (b×c)**	**(b×d)**

Ex. 1: 42 × 57	**Ex. 2: 84 × 36**
(4×5) ¦ (4×7 + 2×5) ¦ (2×7)	(8×3) ¦ (8×6 + 4×3) ¦ (4×6)
20 ¦ 28+10 ¦ 14	24 ¦ 48+12 ¦ 24
20 ¦ 38 ¦ 14	24 ¦ 60 ¦ 24
20 ¦ 38+1 ¦ 4	24 ¦ 60+2 ¦ 4
20 ¦ 39 ¦ 4	24 ¦ 62 ¦ 4
20+3 ¦ 9 ¦ 4	24+6 ¦ 2 ¦ 4
23 ¦ 9 ¦ 4	30 ¦ 2 ¦ 4
2394	**3024**

Ex.1: Put values as per formula. Evaluate all parts. All Parts except first one should contain only one digit. Start observation from Right to Left. If you find more than one digit, then pass excess digits (leftmost) to its immediate left part.

Ex. 3: 67 × 89	**Ex. 4: 76 × 59**
(6×8) ¦ (6×9 + 7×8) ¦ (7×9)	(7×5) ¦ (7×9 + 6×5) ¦ (6×9)
48 ¦ 54+56 ¦ 63	35 ¦ 63+30 ¦ 54
48 ¦ 110 ¦ 63	35 ¦ 93 ¦ 54
48 ¦ 110+6 ¦ 3	35 ¦ 93+5 ¦ 4
48 ¦ 116 ¦ 3	35 ¦ 98 ¦ 4
48+11 ¦ 6 ¦ 3	35+9 ¦ 8 ¦ 4
59 ¦ 6 ¦ 3	44 ¦ 8 ¦ 4
5963	**4484**

Ex. 5: 78 × 08	**Ex. 6: 83 × 07**
(7×0) ¦ (7×8 + 8×0) ¦ (8×8)	(8×0) ¦ (8×7 + 2×0) ¦ (3×7)
0 ¦ 56+0 ¦ 64	0 ¦ 56+0 ¦ 21
0 ¦ 56 ¦ 64	0 ¦ 56 ¦ 21
0 ¦ 56+6 ¦ 4	0 ¦ 56+2 ¦ 1
0 ¦ 62 ¦ 4	0 ¦ 58 ¦ 1
0+6 ¦ 2 ¦ 4	0+5 ¦ 8 ¦ 1
6 ¦ 2 ¦ 4	5 ¦ 8 ¦ 1
624	**581**

Unit 2: DIVISION

Division undoes Multiplication.
Ex. 39÷13=3(Q); 0(R)
Dividend = 39; Divisor = 13; Quotient (Q) = 3; Remainder (R) = 0
39÷3=13
∴ 13×3=39 (∵ Division undoes Multiplication.)

2.1 Division Using Base Method

Sutra Used is: 2. Nikhilam Navataścaramam Daśatah
(निखिलं नवतश्चरमं दशतः)

Meaning: All from 9 and Last from 10.
Note: This Formula is preferred when divisor is below the Working base (9, 8, 7, 74, 88, 649, 8463, 9874 etc.).
Keywords: Divisor, Dividend, Quotient, Remainder, Division, Complement, Left Part, Right Part, Vertical Line (¦).
In Ex. 1: Divisor (9), Dividend (12), Quotient (1), Remainder (3), Division (Operation), Complement (1), Left Part (1), Right Part (2), Vertical Line (¦).

Working Procedure:

1. **First Line:** Split the dividend into two parts (Left and Right) using vertical line (¦). Total number of digits in the Right Part should be equal to total number of zeroes in the Base.
2. **Second Line:** Left Part = Blank; Right Part = (p*C); where 'p' is Left Part of First Line and '**C**' is Complement of Divisor.
   Note: Here, Ignore Negative Sign of Complement.
3. **Third Line or Answer Line:** Left Part: Fetch Left Part Value of First Line to Third Line as it is; Right Part: Add Right Part Values of First and Second Lines. **Left Part is Quotient and Right Part is Remainder.**

4. **Note:** If Remainder is greater than divisor, then divide Remainder by same divisor using above process. For Quotient: Add Quotient Parts of all Iterations and for Remainder just consider Remainder Part of Last Iteration.

Ex.1:12÷9 B:10; C:1 9) 1 ¦ 2 ¦ 1 --------- 1 ¦ 3 Q:**1;** R:**3**	**Ex.1:** Here divisor is 9 and Dividend is 12. As divisor is of one digit, right part contains only one digit. So left part is 1 and right part is 2. In the second line, left part is blank and right part is 1×1=1 (Left Part is 1 and complement is 1). In the third line we add left and right parts. Left part becomes 1 (1+0) and right part becomes 3 (2+1). Left part is quotient and right part is Remainder. So 1 is quotient and 3 is Remainder.

Ex.2: 21÷8	**Ex.3: 12÷7**	**Ex.4: 10÷6**	**Ex.5: 11÷6**
B: 10; C: 2	B: 10; C: 3	B: 10; C: 4	B: 10; C: 4
8) 2 ¦ 1 ¦ 4 --------- 2 ¦ 5 Q: **2;** R: **5**	7) 1 ¦ 2 ¦ 3 --------- 1 ¦ 5 Q: **1;** R: **5**	6) 1 ¦ 0 ¦ 4 --------- 1 ¦ 4 Q: **1;** R: **4**	6) 1 ¦ 1 ¦ 4 --------- 1 ¦ 5 Q:**1;** R:**5**

Ex.6: 74÷9 (B: 10; C: 1)		
9) 7 ¦ 4 ¦ 7 --------- 7 ¦ 11 ----- **(a)**	9) 1 ¦ 1 ¦ 1 ------------ 1 ¦ 2 ------ **(b)**	(a+b) ¦ b (7+1) ¦ 2 8 ¦ 2 Q: **8;** R: **2**

2.2 Division using Transpose and Apply

Sutra: 4. Parāvartya Yojayet (परावर्त्य योजयेत)

Meaning: Transpose and Apply

Note: This Sutra is used in division when divisor is both below and above the base.

Ex: Below Base (9; 8; 74; 69; 849; 736; 9746; 6478; 71255 etc.)

Ex: Above Base (12; 104; 246; 4264; 24364; 42361, 36431 etc.)

First we will understand about Vinculum Numbers Concept.

2.2.1 Vinculum Numbers

A number that has atleast one vinculum digit is called vinculum number. Notation: Either dotted or dash above the number or Strikethrough.

Ex. 1~~3~~2; 9~~68~~1; 22~~3~~; ~~6~~23~~8~~; ~~8~~4; 2~~3~~; 7~~3~~2~~6~~; ~~8~~; ~~3~~

1~~3~~2 = 100-30+2 = 72

9~~68~~1 = 9000-600-80+1 = 8321

22~~3~~ = 200+20-3 = 217

~~6~~23~~8~~ = -6000+200+30-8 = -5778

~~8~~4 = -80+4 = -76

2~~3~~ = 20-3 = 17

7~~3~~2~~6~~ = 7000-300+20-6 = 6714

~~8~~ = -8

~~3~~ = -3

2.2.2 Division using Transpose & Apply (Above Working Base)

Working Procedure:

1. Split the dividend into two parts (left and right) using vertical line (| or ¦). Total number of digits in right part should be equal to total number of zeroes in Base. Write divisor to the left of dividend and Kiles just below the divisor.
2. Bring down the first digit of left part.

3. Multiply first digit with each digit of Kiles and go on placing the product from second spot of second line onwards.
4. Calculate the value of column (addition or subtraction, as per signs) and write the result on third line (answer line).
5. Again multiply the next number of answer line with each digit of Kiles and go on placing the product from third spot and so on until you reach an end.

About Kiles:

1. Kiles are negation of complements/surpluses separted by semicolons.
2. Total number of Kiles should be Equivalent to total number of Zeroes in the Base.
3. If lesser, add required number of Zeroes before Kiles.
4. If Greater, multiply/divide the Divisor by suitable number to get new Divisor (which should be nearer to Working Base). To get final Quotient multiply/divide the intermediate Quotient Part by the same number. Remainder is Constant.

Note:

1. Left Part is Quotient and Right Part is Remainder.
2. If Right Part (Remainder) is negative then add divisor to Right Part and Parallely Subtract Quotient by 1.
3. If you encounter vinculum numbers, convert them to regular numbers.

Ex.1: 123÷11; Here Base=10 and Surplus=1; Kiles=-1

Step 1:	**Step 2:**	**Step 3:**	**Step 4:**	**Step 5:**
11) 1 2 | 3 -1	11) 1 2 | 3 -1 ------------ 1 |	11) 1 2 | 3 -1 -1 ------------ 1 |	11) 1 2 | 3 -1 -1 | ------------ 1 1|	11) 1 2 | 3 -1 -1 | -1 ------------- 1 1 | 2

So, Answer is = 11 (Q); 2 (R)

Ex.2: 1793÷163 B: 100; S: 63	**Ex.3: 147÷12** B: 10; S: 2	**Ex.4: 1232÷114** B: 100; S: 14
163) 1 7 ¦ 9 3 -6;-3 -6 ¦ -3 ¦ -6 -3 ------------------ 1 1 ¦ 0 0 11 ¦ 0 Q: **11**; R: **0**	12) 1 4 ¦ 7 -2 -2 ¦ -4 ------------- 1 2 ¦ 3 **Q: 12; R: 3**	114) 1 2 ¦ 3 2 -1;-4 -1 ¦ -4 ¦ -1 -4 --------------- 1 1 ¦ $\bar{2}$ $\bar{2}$ 1 1 ¦ -20-2 1 1 ¦ -22 (11-1) ¦ (-22+114) 10 ¦ 92 **Q: 10; R: 92**

In Ex.4: In the right part, after converting Vinculum number to regular number, we get -22. We got negative value in the right part. So add divisor (114) to -22. Parallely subtract 1 from left part. In **Ex.** 5 to 10: Vinculum numbers are generated. Convert them to regular numbers.

Ex.5: 248÷16 B: 10; S: 6	**Ex.6: 241÷11** B: 10; S: 1	**Ex.7: 1179÷123** B: 100; S: 23
16) 2 4 ¦ 8 -6 -12 ¦ 48 --------------- 2 -8 ¦ 56 (20-8) ¦ 56 12 ¦ 56 12+3 ¦ 56-48 **Q: 15; R: 8** **(∵16*3=48)**	11) 2 4 ¦ 1 -1 -2 ¦ -2 ------------- 2 2 ¦ $\bar{1}$ 22 ¦ -1 (22-1) ¦ (11-1) 21 ¦ 10 **Q: 21; R: 10**	123) 1 1 ¦ 7 9 -2;-3 -2 ¦ -3 ¦ 2 3 ----------1------ 1 $\bar{1}$ ¦ 7 2 (10-1) ¦ 72 9 ¦ 72 **Q: 9; R: 72**

Unit 5: SQUARES

What is Square: a square is the result of multiplying a number by itself.
For example Square of 3 is 9 (3×3)
Square of 12 is 144 (12×12)
Square of -12 is 144 (-12×-12)
Square of -45 is 2025 (-45×-45).

5.1 Square Using One More than the Previous One

Sutra: 1. Ekādhikena Pūrvena
एकाधिकेन पूर्वेण
Meaning: One More than the Previous One

Note: This sutra is used to obtain squre of given number which ends with digit 5 (Ex. 15, 125, 345, 4585, 6485, 9745 etc.).

Working Procedure:

1. Split the given number into two parts (left and right) using vertical line (¦) or using any other symbol. Right part is last digit i.e 5 and Left part is remaining digits.
2. Multiply left part with its next number in the number line. Right part is 25 (Square of 5).
3. Remove vertical line, the obtained number is required square of given number.

Ex.1:15^2	**Ex.2:25^2**	**Ex.3:75^2**	**Ex.4:95^2**	**Ex.5:115^2**
1 ¦ 5	2 ¦ 5	7 ¦ 5	9 ¦ 5	11 ¦ 5
1×2 ¦ 25	2×3 ¦ 25	7×8 ¦ 25	9×10 ¦ 25	11×12 ¦ 25
2 ¦ 25	6 ¦ 25	56 ¦ 25	90 ¦ 25	132 ¦ 25
225	**625**	**5625**	**9025**	**13225**

Ex.6:-145^2	Ex.7:-205^2	Ex.8: 795^2	Ex.9: 1015^2	10:7995^2
14 ¦ 5	20 ¦ 5	79 ¦ 5	101 ¦ 5	799 ¦ 5
14×15 ¦ 25	20×21 ¦ 25	79×80 ¦ 25	101×102 ¦25	799×800¦25
210 ¦ 25	420 ¦ 25	6320 ¦ 25	10302 ¦ 25	39200¦25
21025	**42025**	**632025**	**1030225**	**3920025**

Ex.3: Left part is 7 and right part is 5. Multiply 7 with its next number in the nuber line (8). It gives 56. Right part is 25 (square of 5). After removing vertical line we get 5625, which is square of 75.

Ex.8: Left part is 79 and right part is 5. Multiply 79 with its next number in the number line (80). It gives 6320. Right part is 25 (square of 5). After removing vertical line we get 632025, which is square of 795.
Note: For negative numbers; ignore sign.

5.2 Square Using Complements/Surpluses
Sub Sutra 7: Yāvadūnam Tāvadūnīkrtya Vargaňca Yojayet
Meaning: Lessen by the Deficiency and set up the square of that deficiency.

Note: This sutra is used to obtain squre of given numbers which are nearer to working (functional) base (Ex 87, 76, 112, 980, 1021 etc).

Case 1: When Number is below the Working Base.
Working Procedure:
1. Note down given number, its Base and Complement.
2. Answer consists of Two Parts (Left Part and Right Part)
3. Right Part is square of Complement.
4. Left Part = (Given Number + Complement).

5. **Note:** Total number of digits in the Right Part should be same as total number of zeroes in the base. If lesser add required number of zeroes, if greater pass the carry (leftmost excess digits of right part) to left part.

Ex.1: 94^2	**Ex.2: 97^2**	**Ex.3: 87^2**
Base: 100	Base: 100	B:100; C: -13
Complement: -06	Complement: -03	87-13 ¦ -13^2
94-6 ¦ -6^2	97-3 ¦ -3^2	74 ¦ 169
88 ¦ 36	94 ¦ 09	74+1 ¦ 69
8836	**9409**	75 ¦ 69
		7569

Ex.4: 893^2	**Ex.5: 9790^2**	**Ex.6: 98930^2**
B:1000; C:-107	B:10000; C:-210	B:100000; C: -1070
893-107 ¦ -107^2	9790-210 ¦ -210^2	98930-1070 ¦-1070^2
786 ¦ 11449	9580 ¦ 44100	97860 ¦1144900
786+11 ¦ 449	9580+4 ¦ 4100	97860+11 ¦ 44900
797 ¦ 449	9584 ¦ 4100	97871 ¦ 44900
797449	**95844100**	**9787144900**

Case 2: When Number is above the Working Base.

Working Procedure:

1. Note down given number, its Base and Surplus.
2. Answer consists of Two Parts (Left Part and Right Part)
3. Right Part is Square of Surplus.
4. Left Part = (Given Number + Surplus).
5. **Note:** Total number of digits in the Right Part should be same as total number of zeroes in the base. If lesser add required number of zeroes, if greater pass the carry (leftmost excess digits of right part) to left part.

Ex.1: 108^2	**Ex.2: 103^2**	**Ex.3: 1104^2**	**Ex.4: 1250^2**
Base: 100	Base: 100	B:1000; S:+104	B:1000; S:+250
Surplus:+08	Surplus: +03	1104+104 ¦ 104^2	1250+250¦250^2
108+8 ¦ 8^2	103+3 ¦ 3^2	1208 ¦ 10816	1500 ¦ 62500
116 ¦ 64	106 ¦ 09	1208+10 ¦ 816	1500+62 ¦ 500
11664	**10609**	1218 ¦ 816	1562 ¦ 500
		1218816	**1562500**

Ex.5: 1205^2	**Ex.6: 1301^2**	**Ex.7: 11320^2**
B:1000; S:+205	B:1000; S:+301	B:10000; S:+1320
1205+205 ¦ 205^2	1301+301 ¦ 301^2	11320+1320¦1320^2
1410¦42025	1602 ¦ 90601	12640¦1742400
1410+42 ¦ 025	1602+90 ¦ 601	12640+174 ¦ 2400
1452 ¦ 025	1692 ¦ 601	12814 ¦ 2400
1452025	**1692601**	**128142400**

Unit 12: POLYNOMIALS

Polynomials: Polynomial is addition /subtraction /multiplication /division of constants (coefficients), variables and exponents, but

1. Division by variable is not allowed (but division by constant is allowed).
2. Variable's exponents can only be whole numbers (0,1,2,3,..).
3. Number of terms should be finite.

- Constants: 14, 36, -74, -963 etc.
- Variables: x, y, z, a, b, c, p, q, r, s etc.
- Exponents: x^2, x^3 etc.
- If p(x) is a polynomial in x, the highest power of x is called degree of polynomial.
- Polynomials with 1 term is called monomial, 2:binomial; 3:Trinomials

Ex.1: $x^2+7x+12$ (Degree: **2**)
Ex.2: $x^3-13x^2+2x-87$ (Degree: **3**);
Ex.3: x^4-8x^2+12x (Degree:**4**); etc.

Types of Polynomials:

- A polynomial of degree 1 is called linear polynomial.
- A polynomial of degree 2 is called quadratic polynomial.
- A polynomial of degree 3 is called cubic polynomial.
- A polynomial of degree 4 is called biquadratic (or quartic) polynomial.

12.1 Multiplication using Criss Cross Method

Sutra 3: Ūrdhva – tiryagbhyām; Meaning: Vertically & Crosswise

Note: Read Multiplication using Criss Cross Method from Multiplication Unit

Steps:

1. Write coefficients of given polynomials one below the other separated by space or vertical bar.
2. Multiply coefficients using formula (refer formula OR graphical representation).
3. Last part is constant. Go on incrementing powers of variable by 1 from right. Second last is x, then x^2, x^3, x^4, x^5 and so on.
   Note: Write coefficient as zero if any term is absent.

CASE 1: (2×2; 2×1)

First Part:	**Second Part:**	**Third Part:**
a b c d	a b c d	a b c d
(a×c)	**(a×d + b×c)**	**(b×d)**

Ex.1: (x+3) (x+5)	**Ex.2: (x+3) (x-5)**	**Ex.3: (x-3) (x-5)**
1 3 1 5 ------------------ (1×1)|(1×5+1×3)|(3×5) 1 | 8 | 15 x^2+8x+15	1 3 1 -5 ------------------- (1×1)|(1×-5+1×3)|(3×-5) 1 | -2 | -15 x^2-2x-15	1 -3 1 -5 ----------------- (1×1) | (1×-5+1×-3) | (-3×-5) 1 | -8 | 15 x^2-8x+15
x^2+8x+15	**x^2-2x-15**	**x^2-8x+15**

Never think there is anything impossible for the soul-Swami Vivekananda

<table>
<tr><th>Ex.4: (x+3) (x)</th><th>Ex.5: (-x+3) (-x+5)</th><th>Ex.6: (-x-3) (-x-5)</th></tr>
<tr><td>1 3
1 0
(1×1) | (1×0+1×3) | (3×0)
1 | 3 | 0
x^2+3x+0</td><td>-1 3
-1 5
(-1×-1) | (-1×5+-1×3) | (3×5)
1 | -8 | 15
$x^2-8x+15$</td><td>-1 -3
-1 -5
(-1×-1) | (-1×-5+-1×-3) | (-3×-5)
1 | 8 | 15
$x^2+8x+15$</td></tr>
<tr><td>x^2+3x</td><td>$x^2-8x+15$</td><td>$x^2+8x+15$</td></tr>
</table>

CASE 2: (3×3; 3×2; 3×1)

<table>
<tr><th>First Part:</th><th>Second Part:</th><th>Third Part:</th><th>Fourth Part:</th><th>Fifth Part:</th></tr>
<tr><td>a b c
d e f</td><td>a b c
d e f</td><td>a b c
d e f</td><td>a b c
d e f</td><td>a b c
d e f</td></tr>
<tr><td>(a×d)</td><td>(a×e + b×d)</td><td>(a×f + b×e + c×d)</td><td>(b×f + c×e)</td><td>(c×f)</td></tr>
</table>

<table>
<tr><th>Ex.1: $(x^2+5x+1)(3x^2-10x+15)$</th><th>Ex.2: $(2x^2-4x-7)(4x^2+20x-12)$</th></tr>
<tr><td>1 5 1
3 -10 15

(1×3) | (1×-10+3×5) | (1×15+5×-10+1×3) | (5×15+-10×1) | (1×15)

3 | 5 | -32| 65| 15
$3x^4+5x^3-32x^2+65x+15$</td><td>2 -4 -7
4 20 -12

(2×4) | (2×20+4×-4) | (2×-12+-4×20+4×-7) | (-4×-12+20×-7) | (-7×-12)

8 | 24 | -132| -92| 84
$8x^4+24x^3-132x^2-92x+84$</td></tr>
<tr><td>$3x^4+5x^3-32x^2+65x+15$</td><td>$8x^4+24x^3-132x^2-92x+84$</td></tr>
</table>

The greatest sin is to think that you are weak.--Swami Vivekananda

CASE 3: (4×3; 4×3; 4×2; 4×1)

First Part:	Second Part:	Third Part:	Fourth Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h	a b c d e f g h
(a×e)	**(a×f) + (b×e)**	**(a×g) + (b×f) + (c×e)**	**(a×h) + (b×g) + (c×f) + (d×e)**

Fifth Part:	Sixth Part:	Seventh Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h
(b×h) + (c×g) + (d×f)	**(c×h) + (d×g)**	**(d×h)**

Ex.1: $(x^3+5x^2+3x+2)(2x^3-4x^2-7x+3)$
1 5 3 2 2 -4 -7 3 (1×2) | (1×-4)+(5×2) | (1×-7)+(5×-4)+(3×2) | (1×3)+(5×-7)+(3×-4)+(2×2) | (5×3)+(3×-7)+(2×-4) | (3×3)+(2×-7) | (2×3) 2 | 6 | -21 | -40 | -14 | -5 | 6 $2x^6+6x^5-21x^4-40x^3-14x^2-5x+6$
$2x^6+6x^5-21x^4-40x^3-14x^2-5x+6$

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*"**A**s long as one keeps searching, the answers come." - Joan Baez*

*"**W**hatever you can do or dream you can, begin it. Boldness has genius, power and magic in it." - Johann von Goethe*

*"**T**hey say that time changes things, but you actually have to change them yourself." - Andy Warhol*

12.2 Division using Transpose and Apply

Sutra: 4. Parāvartya Yojayet (परावर्त्य योजयेत्)

Meaning: Transpose and Apply

Steps:

1. At the right side of **#**: Write dividend
2. At the left side of #: Write negation of coefficients of all terms of divisor except first one (separate coefficients using **|**).
3. Now write the coefficient of first term of dividend below the dotted lines.
4. Individually go on multiplying left part of # with coefficients (which are present below dotted lines) and go on placing the product below the second term onwards of dividend.
5. Evaluate columns and write the value below the line.
6. Continue Step 4& 5 till end.

Now separate Quotient and Remainder parts using **¦**. Left side is Quotient Part and Right side is Remainder Part. Total number of values in the reminder part is same as that of degree of divisor or the total number of values present at the left of #.

For Final Quotient and Remainder: Last is constant, go on incrementing powers of variable by 1. Second last is x, then x^2, x^3, x^4, x^5 and so on.

Ex.1: $(x^3+9x^2+20x+12) \div (x+1)$ -1 **#** x^3 $+9x^2$ $+20x$ $+12$ -1 -8 -12 ----------------------------- 1 8 12 ¦ **0** **Q:**$x^2+8x+12$ **R:**0	**Ex.2:** $(3x^4-2x^3+x^2-2x+3) \div (x-3)$ +3 **#** $3x^4$ $-2x^3$ $+x^2$ $-2x$ $+3$ +9 +21 +66 +192 ----------------------------------- 3 +7 +22 +64 ¦ **+195** **Q:** $3x^3+7x^2+22x+64$ **R:**195

3: $(3x^4-2x^3+x^2-2x+3)\div(x^2-2x+6)$ +2|-6 # $3x^4$ $-2x^3$ $+x^2$ $-2x$ $+3$ +6 -18 8 -24 -18 54 ----------------------------------- 3 +4 -9 ¦ **-44** **+57** **Q:** $3x^2+4x-9$ **R:** $-44x+57$	**4:** $(2x^5+2x^4-x^3+x^2-2x+2)\div(x^2+3x-4)$ -3|+4 # $2x^5$ $+2x^4$ $-x^3$ $+x^2$ $-2x$ $+2$ -6 8 12 -16 -57 76 216 -288 --- 2 -4 19 -72 ¦ 290 -286 **Q:** $2x^3-4x^2+19x-72$ **R:** $290x-286$

Exercise:

1. $(2x^3-3x^2+2x+3)\times(x^3-2x^2-3x+4)$
2. $(3x^3+4x^2+2)\times(2x^3+6x^2-7x-2)$
3. $(3x^4-2x^3-2x^2+4)\times(3x^3+2x^2-4x-3)$
4. $(2x^4+3x^3+x^2)\times(2x^3-5x^2-x-7)$
5. $(2x^6+x^4-x^3+x^2-2x-2)\div(x^2-3x+5)$
6. $(x^6+2x^4-3x^3+x^2-2x-4)\div(x^3-2x+6)$
7. $(x^5+2x^4-3x^3-4)\div(x^2+3)$
8. $(3x^6+4x^5-3x^3+x^2-4)\div(2x^3-2x+6)$

Answers:

1. $(2x^6-7x^5+2x^4+16x^3-24x^2-x+12)$
2. $(6x^6+20x^5+3x^4-30x^3+4x^2-14x-4)$
3. $(9x^7-22x^5-5x^4+26x^3+14x^2-16x-12)$
4. $(4x^7-4x^6-15x^5-22x^4-22x^3-7x^2)$
5. Q: $(2x^4+6x^3+9x^2-4x-56)$ **R:** $(-150x+278)$
6. Q: (x^3+4x-9) **R:** $(9x^2-44x+50)$
7. Q: (x^3+2x^2-6x-6) **R:** $(18x+14)$
8. Q: $(3/2.x^3+2x^2+3/2.x-4)$ **R:** $(-8x^2-17x+20)$

We need silence to be able to touch souls. - Mother Teresa

Unit 13: FACTORIZATION

It is the decomposition of a mathematical object (number / polynomial) into a product of other objects (factors) which when multiplied together gives the original.

12 = 3×4 (3 and 4 are the factors of 12)
144 = 16×9 (16 and 9 are the factors of 144)

$x^2+7x+12 = (x+3)(x+4)$
Factors of ($x^2+7x+12$) are (x+3) and (x+4)

$x^2-38x+48 = (x-6)(5x-8)$
Factors of ($x^2-38x+48$) are (x-6) and (5x-8)

13.1 Type I: Factorization of Simple Quadratic Polynomials using "Proportionately" and "The First by the First & Last by the Last"

Sub Sutra 1 and 3

General Form of Quadratic Equation: ax^2+bx+c

Step 1: Spit the middle coefficient (b) into two parts (say i and j) such that
b=i+j and a×c=i×j
Step 2: First Factor: **ax+i (and REDUCE if Possible)**
Step 3: Second Factor: **ax+j (If possible reduce only if you haven't reduced in above Step. Don't reduce here if you have reduced in above step)**

Note: Verification of answer is done using Sub Sutra "The Sum of the Product is equal to the Product of the Sum" {Sub Sutra 13: Gunitasamuccayah Samuccayagunitah (गुणितसमुच्चयः समुच्चयगुणितः) sutra} Refer Introduction Unit to know more.

Read **'F'** as**:** Factor **V** as: Verification of Answers using sub sutra 13.

	Ex.1: $x^2+7x+12$	**Ex.2: $5x^2+24x+27$**
a;b;c	1; 7; 12	5; 24; 27
i & j	3 & 4	15 & 9
∵	7=3+4; 1×12=3×4	24=15+9; 5×27=15×9
1st **F**	**(x+3)**	5x+15 => 5(x+3) => **(x+3)**
2nd **F**	**(x+4)**	**(5x+9)**
Final	(x+3) and (x+4)	x+3 and 5x+9
V	(1+3)(1+4)=(1+7+12) 20=20	(1+3)(5+9)=(5+24+27) 56=56

Ex.2: Here in 1st Factor, we got (5x+15). 5 is common in the both the terms. We remove that common 5 and factor becomes (x+3).

	Ex.3: $5x^2-38x+48$	**Ex.4: $3x^2+18x+15$**
a;b;c	5; -38; 48	3; 18; 15
i & j	-30 & -8	3 & 15
∵	-38=-30-8; 5×48=-30×-8	18=3+15; 3×15=3×15
1st **F**	5x-30 => 5(x-6) => **(x-6)**	3x+3 => 3(x+1) => **(x+1)**
2nd **F**	**(5x-8)**	**(3x+15)**
Final	(x-6) and (5x-8)	(x+1) and (3x+15)
V	(1-6)(5-8)=(5-38+48) 15=15	(1+1)(3+15)=(3+18+15) 36=36

Ex.3: Here in 1st Factor, we got (5x-30). 5 is common in the both the terms. We remove that common 5 and factor becomes (x-6).
Ex.4: Here in 1st Factor, we got (3x+3). 3 is common in the both the terms. We remove that common 3 and factor becomes (x+1). Our 2nd Facor is (3x+15). Here common term is 3, but here we are not reducing as we have reduced in step 2. So keep 2nd factor as it is.