Markov net inference

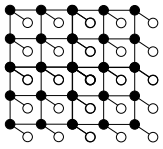
CS B553
Spring 2013

Announcements

- Assignment 2
  - Due next week

Pairwise Markov networks

- In a pairwise Markov network (aka pairwise Markov Random Field or MRF), the max clique size is 2
  - Grid graphs are an especially popular special case



MRFs for images

- MRFs have revolutionized computer vision and image processing over the last decade
- A sample application: Image reconstruction
  - Given a noisy image, estimate the original noise-free image

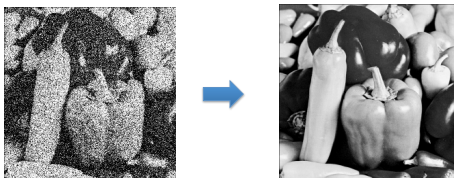


Image reconstruction

- In probabilistic terms, we're given a noisy image N and we want to estimate the original image O,

$$P(O|N) \propto P(N|O)P(O)$$

  - E.g. estimate the most likely original image,

$$O^* = \arg\max_O P(N|O)P(O)$$

- We might assume that noise at each pixel is independent given original pixel data,

$$P(N|O) = \prod_x \prod_y P(N_{x,y}|O_{x,y})$$

What about the prior, P(O)?

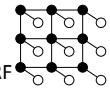
$$O^* = \arg\max_O P(N|O)P(O)$$

- Prior P(O) very difficult to define
  - For a 1 megapixel camera, there are $2^{24000000}$ possible images, so the joint distribution has this many entries!
- Intuitively, captures the fact that some images are much more likely than others

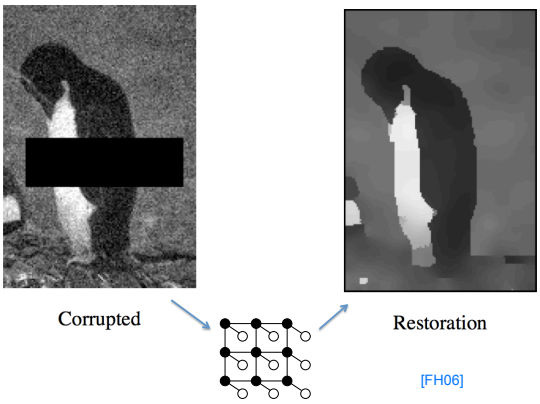
P() > P()

- Simple prior: Encourage "smoothness"
  - Neighboring pixels should generally have similar intensities

Application: Image reconstruction

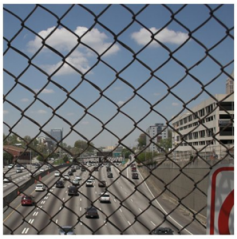


- Given a noisy image, infer original image
- Express problem naturally in terms of an MRF
  - Image is stored as a sampled function on a grid
  - We can observe noisy pixel values, and we'd like to estimate the original, clean pixel values
  - Important constraint: In images of real-world scenes, one pixel's color is correlated with that of its neighbors
  - The pairwise factors model this constraint
- Problem can be solved by doing inference on the Markov network



[FH06]

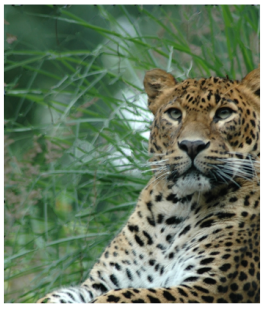
Image restoration



[Liu08]

9

Image restoration



[Liu08]

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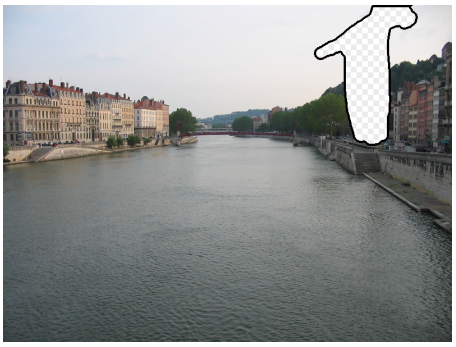
Image restoration

[Liu08]

11

Hays and Efros, SIGGRAPH 2007



Hays and Efros, SIGGRAPH 2007

14

Hays and Efros, SIGGRAPH 2007

Hays and Efros, SIGGRAPH 2007

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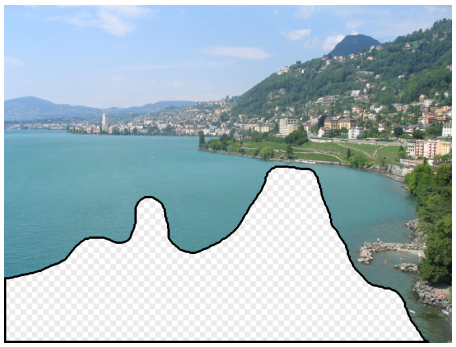
17

Hays and Efros, SIGGRAPH 2007

[Hays07]

18



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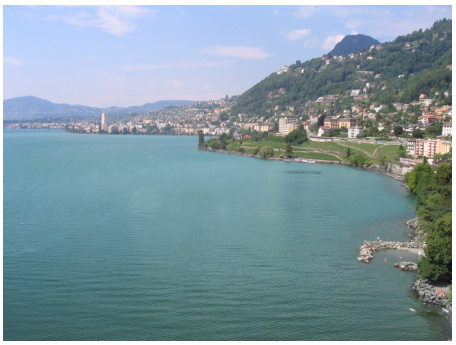
Hays and Efros, SIGGRAPH 2007



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Hays and Efros, SIGGRAPH 2007

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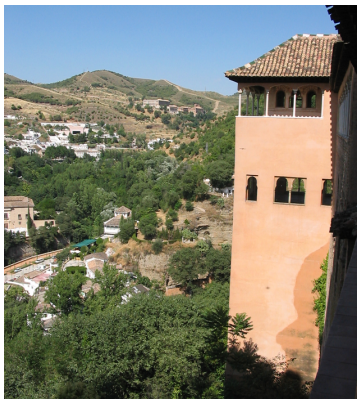
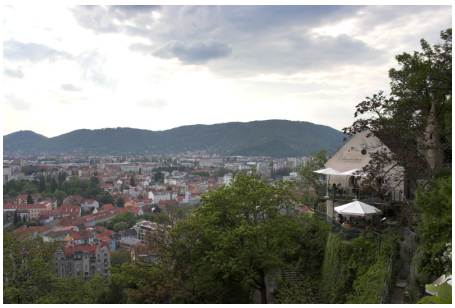
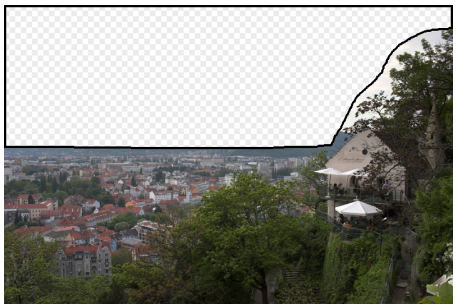
Hays and Efros, SIGGRAPH 2007



24

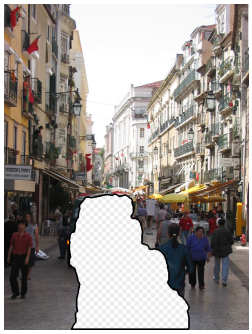
Hays and Efros, SIGGRAPH 2007





Failures

Failures



Hays and Efros, SIGGRAPH 2007

Failures

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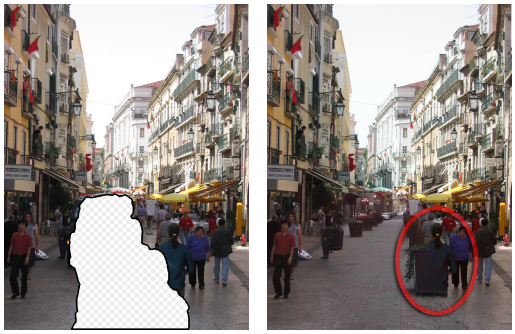
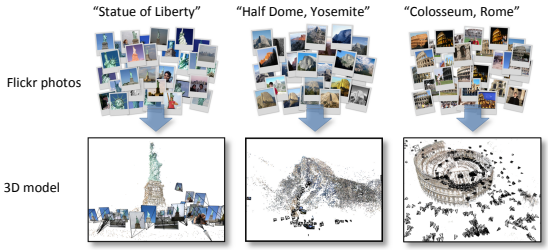
Failures



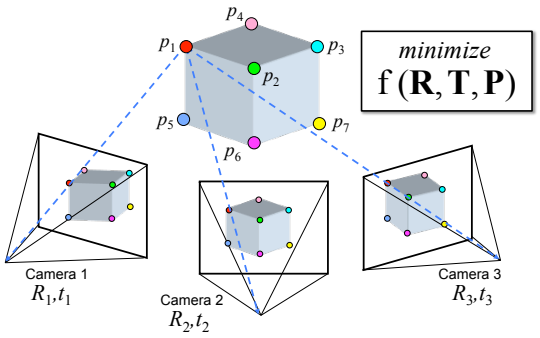
Hays and Efros, SIGGRAPH 2007

Automatic 3-D reconstruction

- From Internet photo collections [Snavely06]

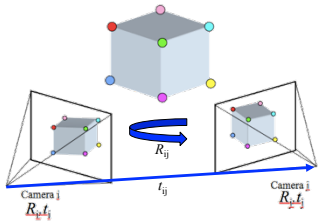


Structure from Motion



Constraints on camera *pairs*

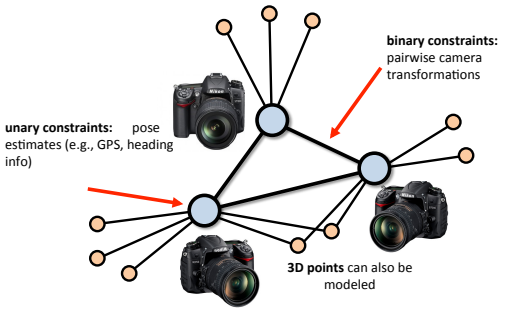
- Estimate relative pose between cameras, $\boldsymbol{R}_{ij}$ and $\boldsymbol{t}_{ij}$



- Want to solve for absolute camera poses, ($\boldsymbol{R}_i$, $\boldsymbol{t}_i$) and ($\boldsymbol{R}_j$, $\boldsymbol{t}_j$)
  - Two key constraints:

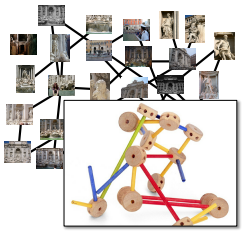
$$\mathbf{R}_{ij} = \mathbf{R}_i^\top \mathbf{R}_j \qquad \lambda_{ij}\mathbf{t}_{ij} = \mathbf{R}_i^\top(\mathbf{t}_j - \mathbf{t}_i)$$

The MRF model



Our approach

- View SfM as inference over a Markov random field, solving for all camera poses at once



- **Vertices** are cameras (or points)
- Both **pairwise** and **unary** constraints
- **Inference problem:** label each image with a camera pose, such that constraints are satisfied

Inference on Markov nets

- Of course, all of these applications (and the thousands of others…) require efficient inference
  - This is the hard part!

Variable elimination on Markov nets

1. Sort the non-query variables in an arbitrary order, Z1, Z2, … Zn
2. Initialize set of *factors* **F** to be the conditional probability distributions, P(Zi | Pa(Zi)).
3. For each i=1..n,
   a. Identify subset of factors **F'** involving Zi; these factors have some subset of variables **V** as parameters
   b. Take product of factors **F'**, producing ψ(**V**)
   c. Sum this product over all values of Zi, producing a new factor τ parameterized by **V**-{Zi}
   d. Remove **F'** from **F**, then add τ to **F**

Variable elimination on Markov nets

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Inference on Markov nets

- What's the running time of Variable Elimination on a Markov network?
- Recall that, in general, we'll have a factor for every maximal clique in the Markov net
  - If the size of the largest clique is *N*, and each variable can take on *s* possible labels, then we'll need to process a factor with $O(s^N)$ possible labels
  - So the running time is at least exponential in the size of the maximal clique

Example

- What is the running time of Variable Elimination on:
  - A tree?
  - A simple cycle?
  - A grid graph?

Running time of VE

- The running time of Variable Elimination is controlled by the size of the scope of the largest factor
  - Including the original factors and the intermediate factors produced by the algorithm
  - Intermediate factors depend on elimination order
  - But it's often impossible to find a "good" elimination order
- In other words, running time is controlled by:
  - The maximal clique size of the original Markov net
  - Something about the structure of the Markov net

Cluster graph

- *Cluster graph:* An undirected graph in which:
  - Each node represents a subset of variables
  - Edge *might* connect nodes **X** and **Y** if $\mathbf{X} \cap \mathbf{Y} \neq \emptyset$
- For a given execution of Variable Elimination, we can draw a cluster graph to "visualize" the process

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Nodes of clique tree are values of V

Clique tree

- A particularly interesting cluster graph is that produced as follows from a run of VE:
  - Every subset **V** produced by VE is a node in the graph
  - Edge connects **X** and **Y** iff VE used $\psi(\mathbf{X})$ when computing $\psi(\mathbf{Y})$
- This graph is always a tree
  - Called a *clique tree* or *junction tree*