Particle methods

CS B553
Spring 2013

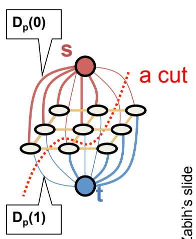
Announcements

- A3 posted
  - Due Friday March 8, 11:59PM

Basic graph cut construction

- One non-terminal vertex per pixel
  - Each pixel has edge to s,t, and neighbors
  - Edge p-s has weight Dp(0), edge p-t has weight Dp(1)
  - Edge (p,q) has weight $V_{pq}(0,1)$
- Run graph cuts to find a min cut
  - Label pixel p 0 if connected to t, and 1 if connected to s
- Cost of cut is the cost of the entire MRF labeling
  - So min cut means we've found min-cost labeling!

$$E(x_1, \ldots, x_n) = \sum_p D_p(x_p) + \sum_{p,q} V_{p,q}(x_p, x_q)$$



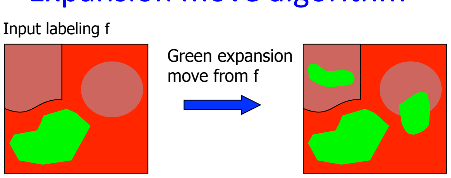
Adapted from R. Zabih's slide

Can this be generalized for multi-label problems?

- Not easily.
  - NP-hard for even the Potts model [K/BVZ 01]
- Two main approaches
  1. Exact solution [Ishikawa 03]
     - Large graph, convex *V* (arbitrary *D*)
  2. Approximate solutions [BVZ 01]
     - Solve a binary labeling problem, repeatedly
     - Expansion move algorithm

4

Expansion move algorithm

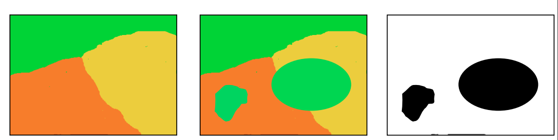


- Make green expansion move that most decreases cost
  - Then make the best blue expansion move, etc
  - Done when no α-expansion move decreases the energy, for any label α
  - See [BVZ 01] for details

5

Adapted from R. Zabih's slide

Binary sub-problem



Input labeling | Expansion move | Binary image

6

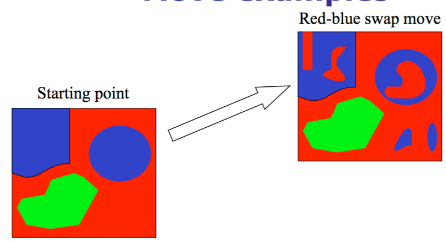
Adapted from R. Zabih's slide

The expansion move algorithm

1. Start with an arbitrary labeling
2. Cycle through every label *A* in some order
   - 2.1 Find the lowest cost labeling that involves an A-expansion move – this is a binary subproblem!
   - 2.2 Make the move if its cost is lower than current labeling
3. If cost did not decrease in the cycle, we're done
   Otherwise, go to step 2

Adapted from R. Zabih's slide

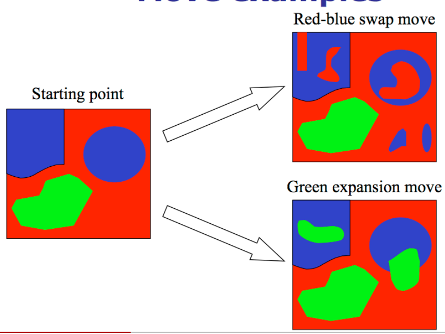
Move examples



8

Adapted from R. Zabih's slide

Move examples



9

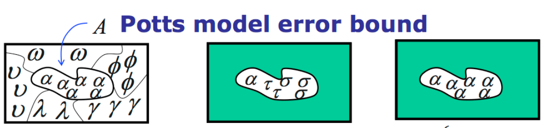
Adapted from R. Zabih's slide

The swap move algorithm

1. Start with an arbitrary labeling
2. Cycle through every label pair *(A,B)* in some order
   - 2.1 Find the lowest cost labeling within a single *AB*-swap
   - 2.2 Go there if its cost is lower than the current labeling
3. If cost did not decrease in the cycle, we're done
   Otherwise, go to step 2

Adapted from R. Zabih's slide

Potts model error bound



optimal solution f^* local minimum $\hat f$

$$f^\alpha = \begin{cases} \alpha & p \in A \\ \hat f_p & p \notin A \end{cases}$$

$$\hat f \mapsto f^\alpha \quad \Rightarrow \quad E(\hat f) \le E(f^\alpha)$$

$$E_A(\hat f) \le E_A(f^\alpha) \le E_A(f^*)$$

Summing up over all labels:

$$E(\hat f) \le E(f^*) + E_\partial(f^*) \le 2E(f^*)$$

11

Adapted from R. Zabih's slide

Multi-label graph cuts

- The approximate algorithm works for:
  - D of any form
  - V must satisfy a (generalized) submodularity constraint:

$$V(\alpha,\alpha) \;+\; V(f(p),f(q)) \quad \le$$
$$V(f(p),\alpha) \;+\; V(\alpha,f(q))$$

12

Adapted from R. Zabih's slide

Graph cuts properties

- Binary graph cuts is key step of inner loop
- In each iteration of graph cuts, the total cost can't increase
  - Converges to a solution in O(n) steps
  - In practice, typically converges in just a few steps
- At convergence, the solution is a local minimum
  - And, we can prove an *approximation bound:* The cost of the graph cuts solution is within a factor of 2 of the cost of the exact solution!

Why does graph cuts work so well?

- It's an iterative, hill-climbing approach, but one in which every step is searching over a *huge* space
  - Every step searches over $O(2^n)$ labelings!
  - Starting from an arbitrary labeling, you can get to the optimal labeling in just *k* of these steps
- Compare this to other, more obvious hill-climbing techniques, e.g. change a single pixel at a time
  - Every step searches over just O(1) labelings
  - Generally yields a weak local minimum

Graph cuts vs BP

(a) Tsukuba Image (b) Graph Cuts (c) Synchronous BP (d) Accelerated BP

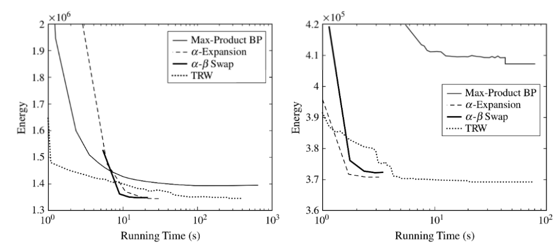
Figure 3. Results produced by the three algorithms on the Tsukuba image. The parameters used to generate this field were $s = 50$, $T = 4$, $P = 2$. Again, Graph Cuts produces a much smoother solution. Belief Propagation does maintain some structures that are lost in the Graph Cuts solution, such as the camera and the face in the foreground.

Tappen 2003

Adapted from R. Zabih's slide

Comparing techniques on stereo

- Compare techniques on cost of best solution ("energy") versus time



Ground truth vs Graph cuts vs BP

Image	Energy of MRF Labelling Returned ($\times 10^3$) Ground-Truth	Graph Cuts	Synchronous Belief Prop	% Energy from Occluded Matching Costs
Map	757	383	442	61%
Sawtooth	6591	1652	1713	79%
Tsukuba	1852	663	775	61%
Venus	5739	1442	1501	76%

Figure 2. Field Energies for the MRF labelled using ground-truth data compared to the energies for the fields labelled using Graph Cuts and Belief Propagation. Notice that the solutions returned by the algorithms consistently have a much lower energy than the labellings produced from the ground-truth, showing a mismatch between the MRF formulation and the ground-truth. The final column contains the percentage of each ground-truth solution's energy that comes from matching costs of occluded pixels.

Adapted from R. Zabih's slide

Graph cuts vs BP

- Graph cuts typically finds slightly lower-energy solutions
  - However, lower-energy is not necessarily better…
- More theoretical results are known for graph cuts
  - On 2 label problems, graph cuts gives exact solution
  - On multilabel problems with convex cost functions, GC gives solutions in polynomial time (but not practical in practice)
  - On other multilabel problems, GC has an approx. bound
- BP is more general
  - Works on any graph structure, and any pairwise cost function
  - Can choose MAP inference or compute marginals
  - Easier to implement

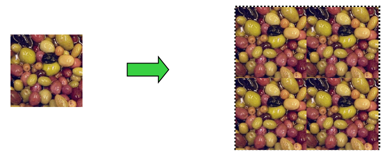
Adapted from R. Zabih's slide

GrabCut

Grabcut [Rother et al., SIGGRAPH 2004]

Adapted from R. Zabih's slide

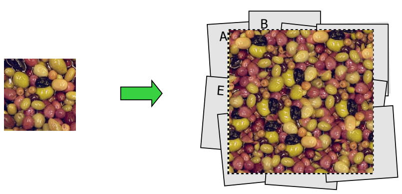
Application: texture synthesis



"Graphcut textures" [Kwatra et al 03]

20

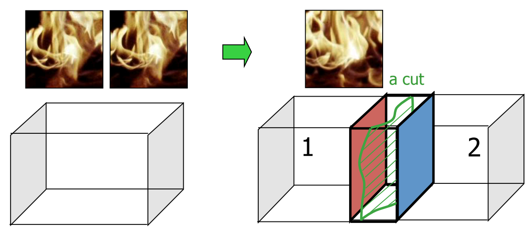
Application: texture synthesis



"Graphcut textures" [Kwatra et al 03]

21

Graphcuts video textures



Short video clip

Long video clip

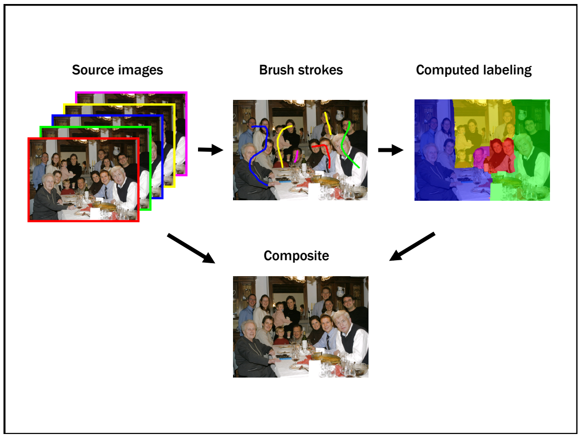
22

Interactive Digital Photomontage

Aseem Agarwala, Mira Dontcheva, Maneesh Agrawala, Steven Drucker, Alex Colburn, Brian Curless, David Salesin, Michael Cohen

University of Washington & Microsoft Research


set of originals
photomontage


Source images
Brush strokes
Computed labeling
Composite

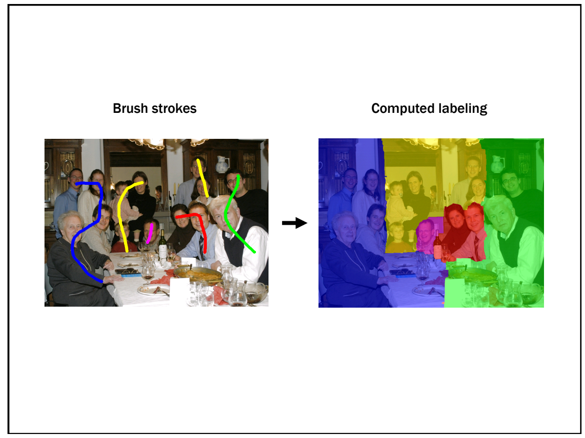

Brush strokes
Computed labeling


Image objective
0 for any label
0 if red
∞ otherwise