Inference on Bayes nets

CS B553
Spring 2013

Announcements

- Assignment 1 due tonight
- Assignment 2 posted soon!

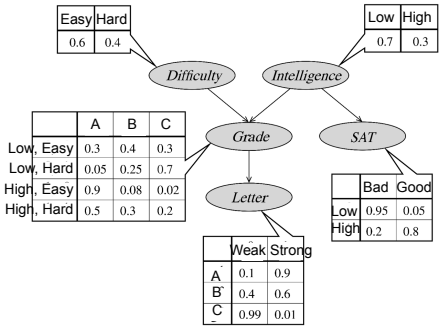
Variable elimination algorithm

1. Sort the non-query variables in an arbitrary order, Z1, Z2, … Zn
2. Initialize set of *factors* **F** to be the conditional probability distributions, P(Zi | Pa(Zi)).
3. For each i=1..n,
   a. Identify subset of factors **F'** involving Zi; these factors have some subset of variables **V** as parameters
   b. Take product of factors **F'**, parameterized by **V**
   c. Sum this product over all values of Zi, producing a new factor f parameterized by **V**-{Zi}
   d. Remove **F'** from **F**, then add f to **F**

Handling evidence

- Suppose we want to compute P(**Y** | **E**=**e**)
  - Set variables in **E** to their known values
  - Eliminate all remaining variables except for **Y**, yielding P(**Y**, **E**=**e**)
  - Then marginalize over **Y** to compute P(**E**=**e**)
  - Finally, calculate P(**Y** | **E**=**e**) = P(**Y**, **E**=**e**) / P(**E**=**e**)

Conditional probability distributions



Running time of Variable Elimination

- Notice that VE is not necessarily efficient

  - Say we want to compute P(E)
  - A smart elimination order: A, B, C, D
  - A less smart order: D, C, B, A

Reasoning about running time

- Define the *induced graph* to be an undirected graph over the nodes of the Bayes net
  - Edge between nodes u and v iff some factor generated during VE involved both u and v
  - Note that this depends on the ordering of VE
- Theorem: Every maximal clique in the induced graph is the scope of some intermediate factor in VE
  - And every intermediate factor corresponds to a clique
  - Running time is exponential in the size of the max clique
- So we'd like to find the "best" elimination ordering for a given Bayes net – i.e. one with the smallest possible maximal clique in the induced graph
  - Unfortunately, this problem is NP hard!

Special cases: Chains and polytrees

- For chains, we can always find an elimination ordering that takes time linear in the number of nodes
  - Start at the beginning of the chain and eliminate variables in node order
- A *polytree* is a dag such that there is at most one trail between every pair of nodes
  - In a polytree, it is always possible to find an elimination ordering that takes time linear in the *size of the conditional probability distributions*
  - E.g. start at leaves of tree and work upwards towards root(s)