CS B553 Assignment 1: Probability fundamentals

Spring 2013
Due: Thursday Jan 24, 11:59PM. (You may submit up to 48 hours late for a 10% penalty.)

Solve the following problems. You must show your work (derivations, calculations, etc.) in order to receive credit. Submit your solutions electronically, via OnCourse. To do this, either type up your answers and all your work in a plain text or PDF format, or neatly write your solutions by hand on paper and scan them into a PDF file.

You may complete this assignment either individually or with a partner. If you work with a partner, submit only a single copy of your assignment work.

Academic integrity guidelines. You may discuss this assignment with the professor and AI, but not with other people except for your partner. You may consult printed and/or online references, including books, tutorials, etc., but you must cite these materials and explicitly state how the materials helped you to solve the problem. Any code you submit must be your own work, which you personally designed and wrote. You may not share written solutions or code with any students other than your partner, nor may you possess solutions or code written by another student, either in whole or in part, regardless of format.

1. Eight parakeets, four green and four blue, land on a telephone wire in random order. What is the probability that the no two adjacent parakeets are the same color?

2. A certain CPU is designed with 8 computing cores. There's a 30% probability that any given computing core will have a manufacturing defect that prevents it from functioning correctly. The defects occur randomly and independent of each other.

   (a) What's the probability that a given CPU will have 8 functioning compute cores?

   (b) To prevent losing money from so many defective CPUs, the manufacturer comes up with an ingenious plan. They introduce a family of 3 CPU models: the Great model, the Advanced model, and the Extreme model. In reality, the Great and Advanced models are simply defective versions of the Extreme model. That is, the only difference between these models is the number of functioning compute cores; Great is guaranteed to have at least 1 functioning core, Advanced has at least 4 functioning cores, and the Extreme has 8 functioning cores. If the company tries to make one thousand 8-core CPUs, how many of each model (Great, Advanced, and Extreme) can it expect to make?

   (c) If the Great model costs \$50, the Advanced model costs \$100, and the Extreme model costs \$1000, when the company sells all of the thousand CPUs, what's their expected revenue?

3. Suppose that in some judicial system, an accused person is tried by a 3-judge panel. Suppose that when the accused person is, in fact, guilty, each judge will independently vote guilty with probability 0.7, whereas when the defendant is, in fact, innocent, this probability drops to 0.2. Suppose that 70% of accused people are actually guilty.

   (a) Suppose Judge 1 has voted guilty. What's the probability that the accused person is in fact guilty?

(b) Suppose all three judges vote guilty. Now what's the probability that the accused person is in fact guilty?

(c) Suppose Judge 1 and Judge 2 have voted innocent. What's the probability that Judge 3 votes guilty?

4. A certain fast food restaurant, MacBurger's, offers a special meal for children called the Cheery Meal. Each Cheery Meal comes with one of the following four action figurines: Suzie Sodium, Charlie Cholesterol, Freddie Fat, or Allie Angina. Assume that the restaurant has an infinite supply of the figurines. Each time a customer orders a Cheery Meal, the restaurant chooses a figurine uniformly at random and gives it to the customer.

   (a) Suppose you already have 3 of the figurines (Suzie, Charlie, and Freddie). How many more Cheery Meals would you expect to have to buy before getting Allie?

   (b) A new customer walks into MacBurger's for the first time. How many Cheery Meals should he or she expect to buy before owning all 4 figurines?

5. As you may know, the U.S. tax system is extremely complicated, and a variety of ideas have been proposed recently to try to simplify it. Here's one additional proposal. The U.S. treasury prints up special red dollar bills in addition to the green bills that it normally prints. Every U.S. citizen is given a special account. Each of these accounts initially contains two bills: a single red bill and a single green bill. Every day, for each account, the government chooses a random bill from the account. If the bill is green, it puts the green bill back in the account, but also prints *another* green bill to add to the account. If the bill is red, it puts the red bill back in the account, but also prints *another* red bill and adds it to the account. So while there are 2 bills in each account on day 0, there are $2 + n$ bills in each account at the end of day n. The government continues to do this for 1 year (365 days). At the end of the year, each citizen counts up the number of red (R) and green (G) bills in their account. If they have more green bills than red bills, they get to keep $G - R$ dollars from the government. If they have more red bills than green bills, they *owe* the government $R - G$ dollars in taxes.

   (a) Suppose you have an account. How much do you expect to receive from the government, $E[G - R]$? What is the variance on this quantity, i.e. $V[G - R]$?

   (b) What's the probability that you will owe at least \$300 to the government at the end of the year?

   (c) What are the answers for (a) - (b) if each account is started with a single green bill and two red bills (as opposed to a single green bill and a single red bill as described above)?

   Hint: If you can't figure out how to solve this problem analytically, then write a program to simulate the above process and compute the expectations, variances, and probabilities empirically. If you do this, submit your source code with your assignment submission.