

Rise of the SNARKs

an introduction to zero-knowledge proofs, zkSNARKs, and **libsnark**

Howard Wu

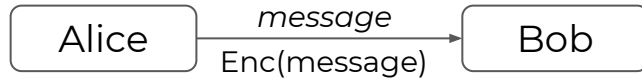
Blockchain at Berkeley Developers Decal

Spring 2018

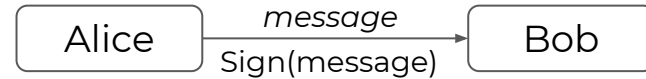
Cryptography is a powerful tool for building secure systems.

Much of the cryptography used today
offers security properties for **data**.

Confidentiality



Authenticity



What about security properties for **computation**?

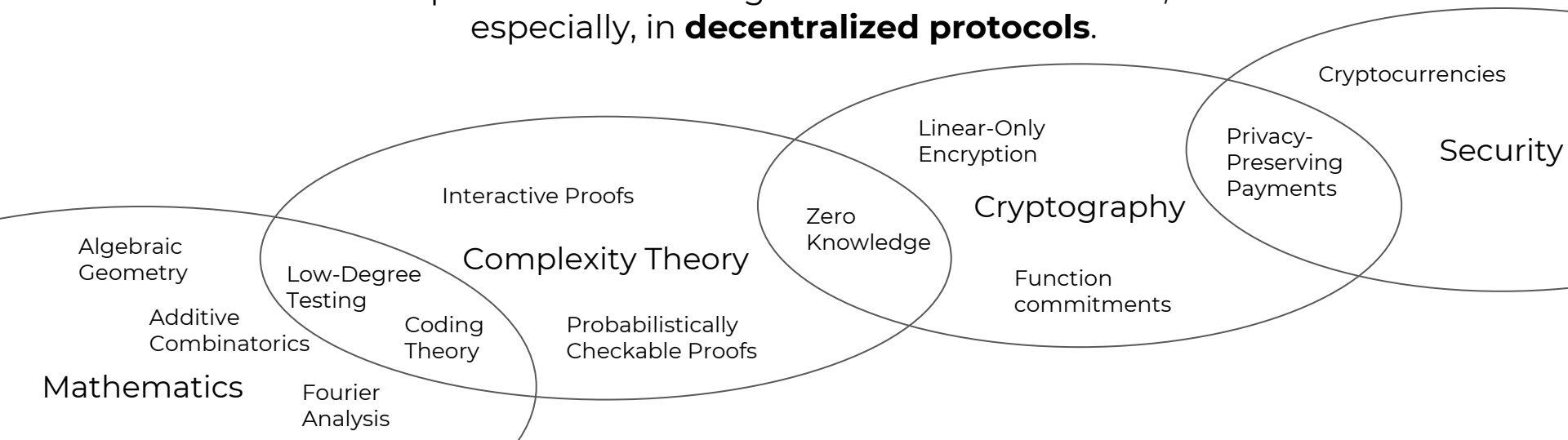
Cryptographic proofs offer
privacy-preserving integrity for computation.

Cryptographic Proofs

A powerful defense against malicious behavior,
especially, in **decentralized protocols**.

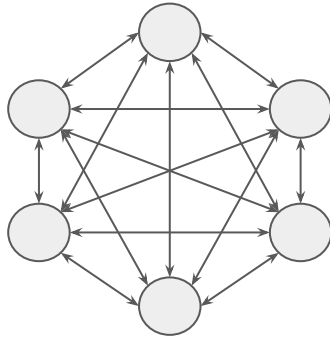
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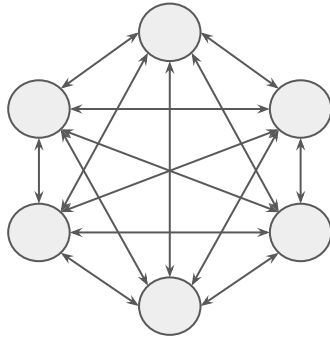
Cryptographic Proofs

1980s Securely compute $y = F(x_1, \dots, x_n)$ via a multi-party protocol



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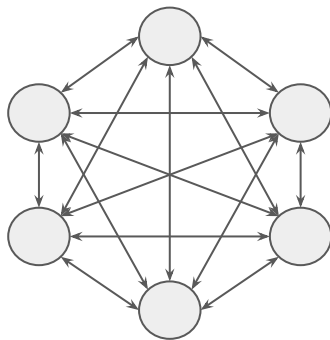


Assumptions

- Closed network
- All nodes are online
- Small number of nodes

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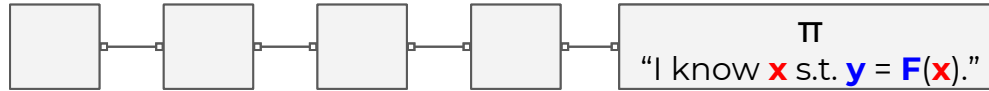
Assumptions

- Closed network
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Zero-knowledge proof that
every message is sent according to the protocol
(consistent with the input and random tape).

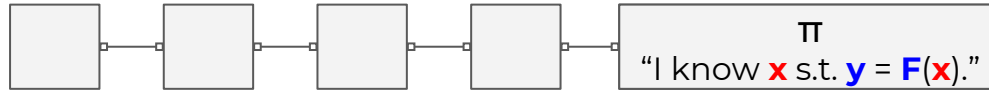
Cryptographic Proofs

2010s Blockchain Technology



Cryptographic Proofs

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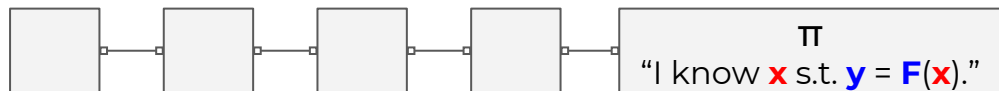


Assumptions

Open network
Nodes can be online or offline
Planetary scale

Cryptographic Proofs

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Desirable Proof Properties

Succinct
Non-Interactive
Publicly Verifiable
Zero Knowledge

Zero-Knowledge Proofs

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computational integrity.

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computational integrity.

*I can't tell you the secret,
but I can prove to you that I know the secret.*

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computational integrity.

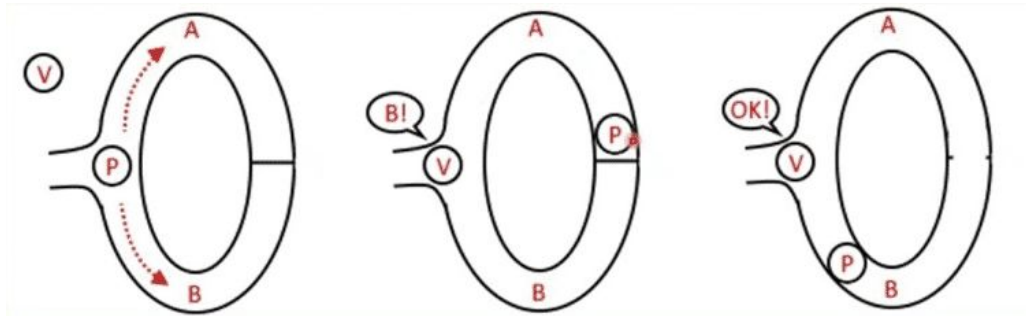


*Can you prove to me where Waldo is,
without saying anything about where he is?*

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computational integrity.

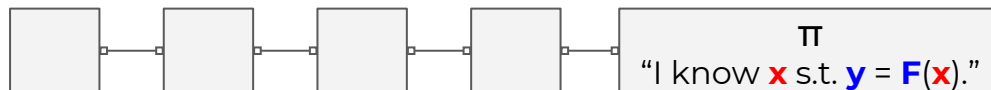
The Ali Baba Cave



from "How to Explain Zero-Knowledge Protocols to Your Children"

Zero-Knowledge Proofs

2010s Blockchain Technology



Assumptions

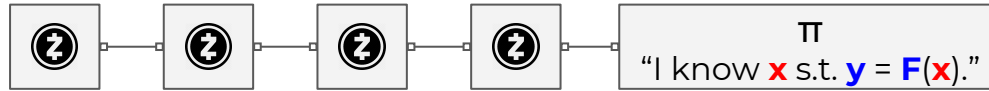
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Proof Properties

Succinct
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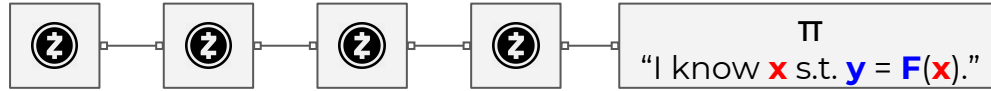
Zero-Knowledge Proofs

2010s Zerocash Protocol (Zcash)



Zero-Knowledge Proofs

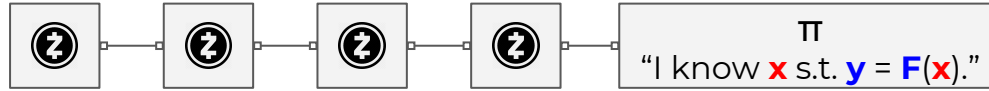
2010s Zerocash Protocol (Zcash)



A cryptographic protocol achieving a digital currency that is:

Zero-Knowledge Proofs

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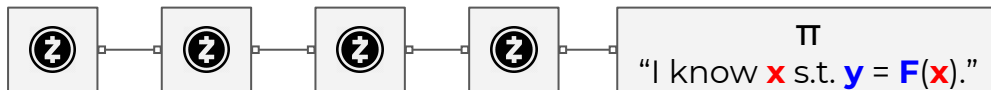


A cryptographic protocol achieving a digital currency that is:

- **Decentralized** - works when given any (ideal) ledger

Zero-Knowledge Proofs

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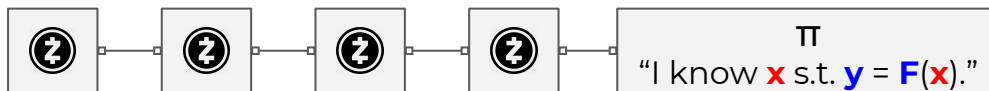


A cryptographic protocol achieving a digital currency that is:

- **Decentralized** - works when given any (ideal) ledger
- **Privacy-preserving** - anyone can post a payment transaction to anyone else, while provably hiding the sender, receiver, and amount

Zero-Knowledge Proofs

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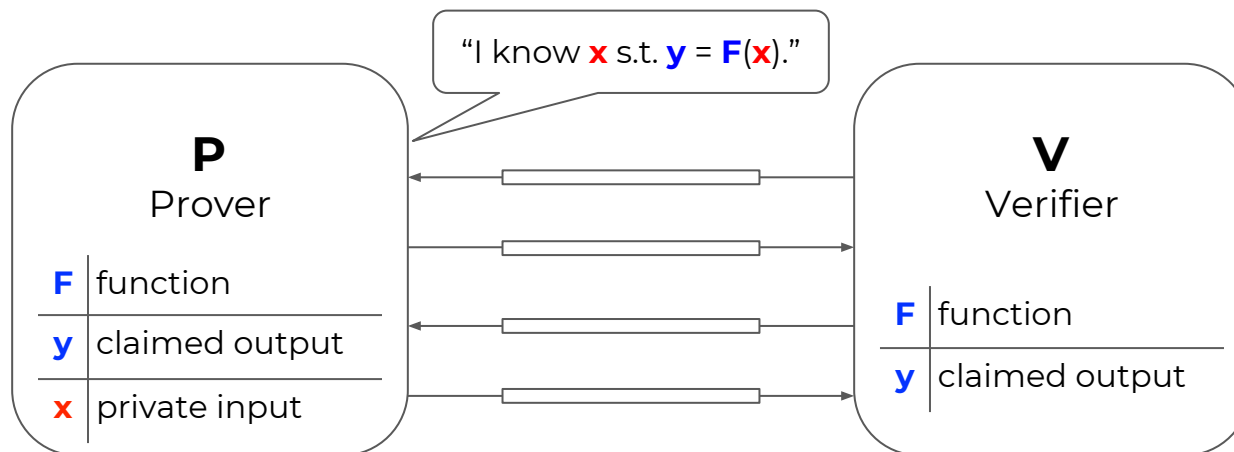


A cryptographic protocol achieving a digital currency that is:

- **Decentralized** - works when given any (ideal) ledger
- **Privacy-preserving** - anyone can post a payment transaction to anyone else, while provably hiding the sender, receiver, and amount
- **Efficient** - payment transactions take less than 1 minute to produce, are less than 1 kB in size, and take a few milliseconds to verify

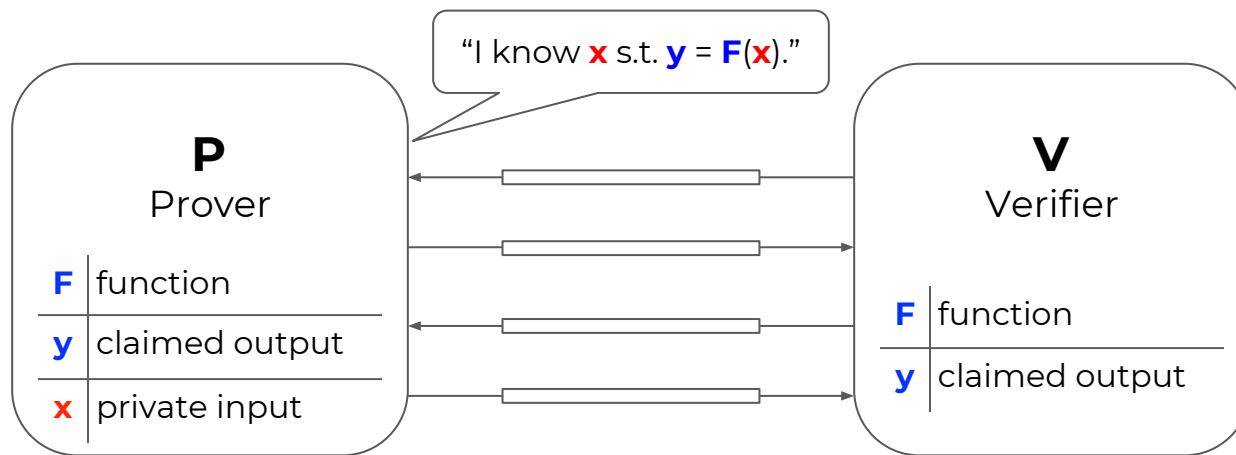
Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computation integrity.



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Privacy-preserving cryptographic proofs of computation integrity.

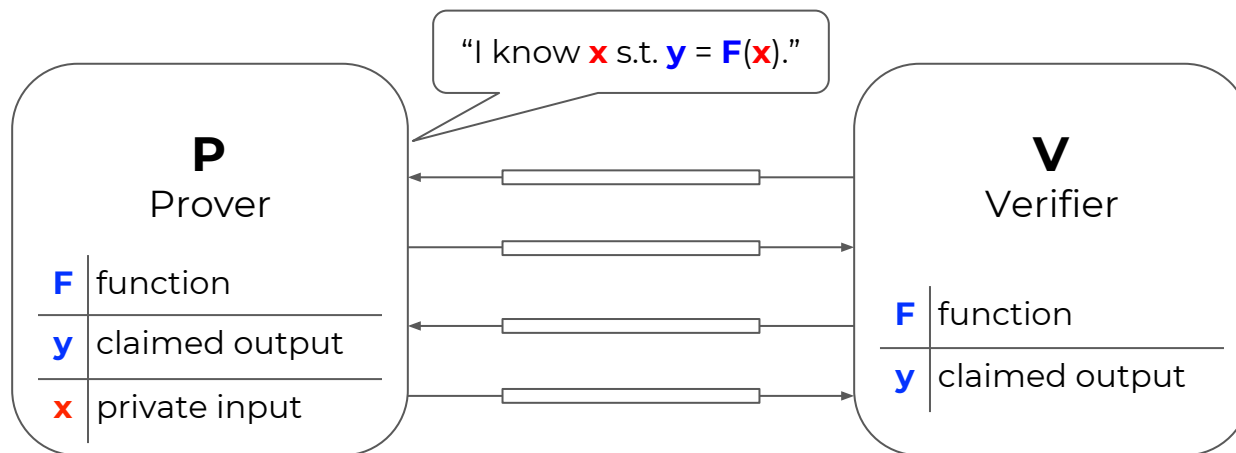


Completeness

$$\exists \mathbf{x}: \mathbf{y} = \mathbf{F}(\mathbf{x}) \rightarrow \Pr[\mathbf{P}(\mathbf{F}, \mathbf{y}, \mathbf{x}) \text{ convinces } \mathbf{V}(\mathbf{F}, \mathbf{y})] = 1$$

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computation integrity.



Completeness

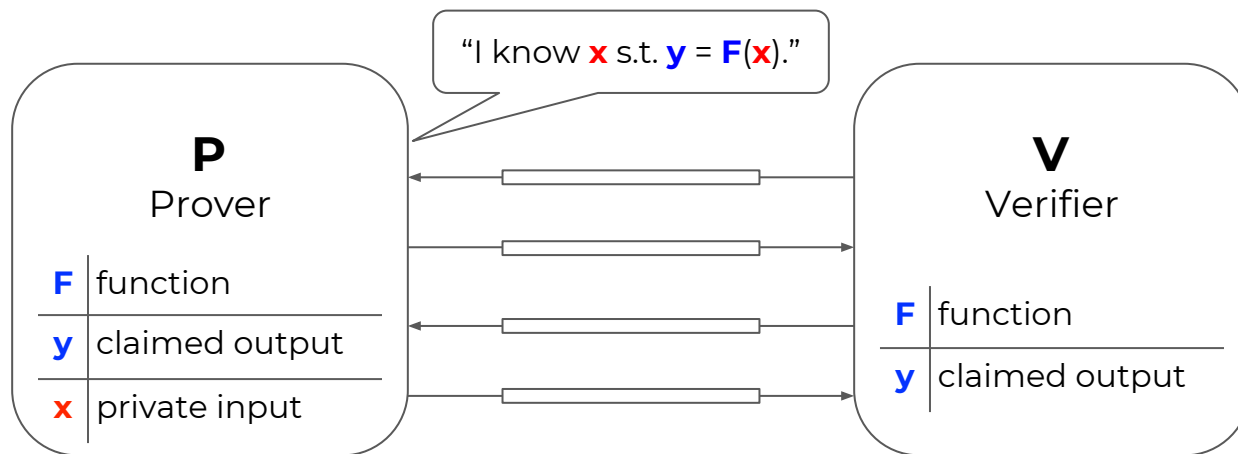
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Soundness

$\nexists \mathbf{x}: \mathbf{y} = \mathbf{F}(\mathbf{x}) \rightarrow \forall \mathbf{P}' \Pr[\mathbf{P}' \text{ convinces } \mathbf{V}(\mathbf{F}, \mathbf{y})] \approx 0$

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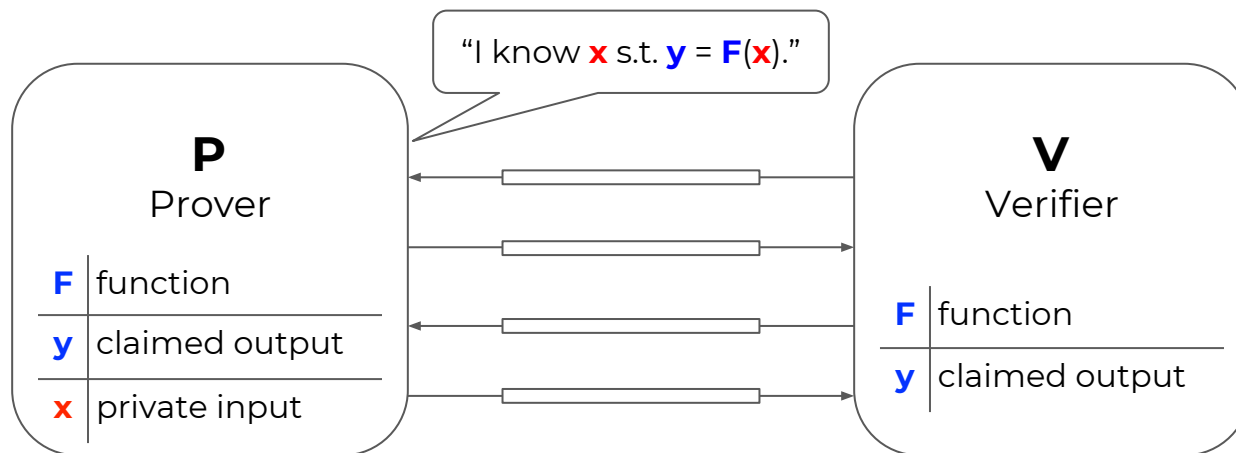
Zero-Knowledge

$\exists x: y = F(x) \rightarrow \mathbf{S}(F, y) \approx \text{view of } \mathbf{V}(F, y) \text{ after talking to } \mathbf{P}(F, y, x)$

Simulator

Zero-Knowledge Proofs

Privacy-preserving cryptographic proofs of computation integrity.



Powerful cryptographic primitive.

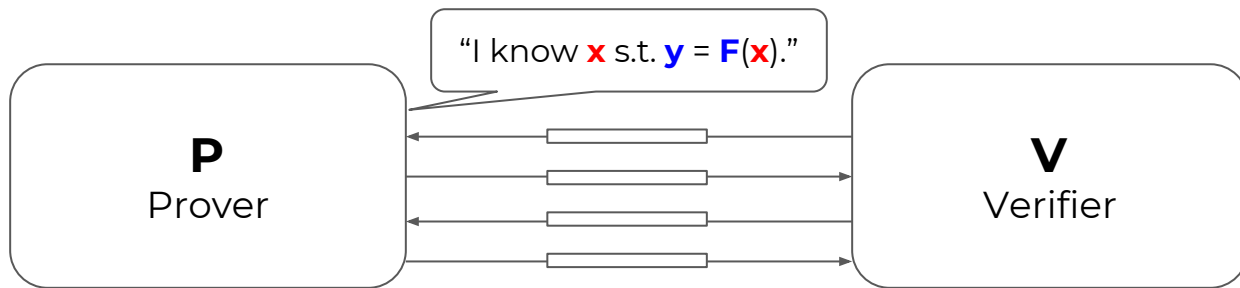
**However, it is
interactive, not succinct, and has bad concrete efficiency.**

Communication complexity
& verification complexity
are proportional to time(F)

Relies on PCPs
(probabilistically
checkable proofs)

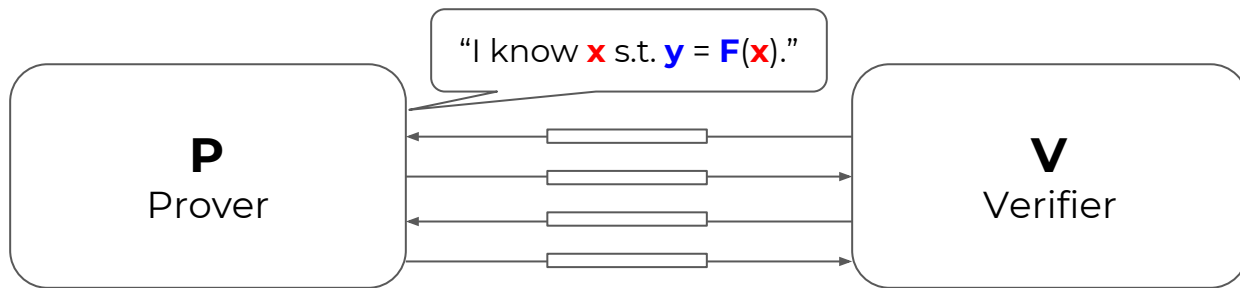
Zero-Knowledge Succinct Proofs

Privacy-preserving cryptographic **succinct** proofs of computation integrity.



Zero-Knowledge Succinct Proofs

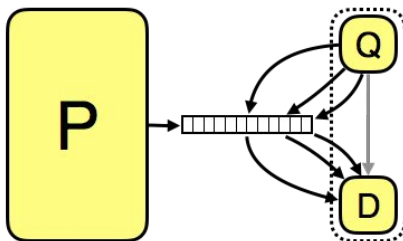
Privacy-preserving cryptographic **succinct** proofs of computation integrity.



Achieving Succinctness

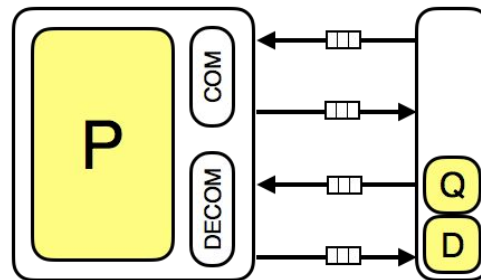
Probabilistically Checkable Proof

[BFLS91][FGLSS96][AS92][ALMSS92]



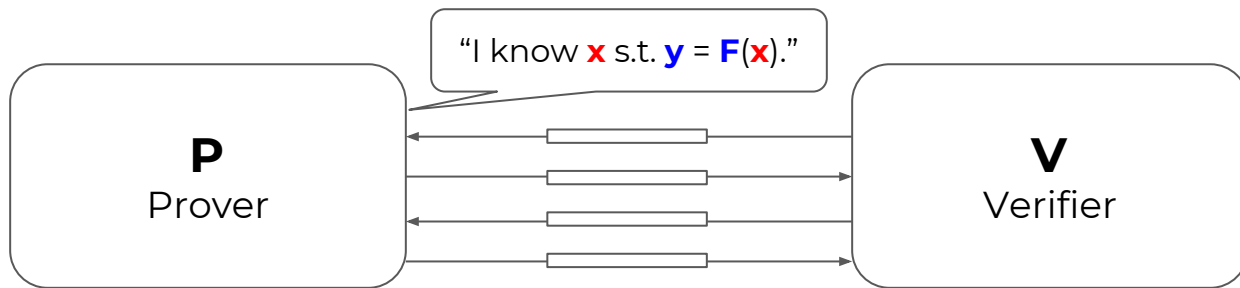
[Kilian92]

Zero Knowledge Succinct Proof



Zero-Knowledge Succinct Proofs

Privacy-preserving cryptographic **succinct** proofs of computation integrity.



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Soundness

$\nexists \mathbf{x}: \mathbf{y} = \mathbf{F}(\mathbf{x}) \rightarrow \forall \mathbf{P}' \Pr[\mathbf{P}' \text{ convinces } \mathbf{V}(\mathbf{F}, \mathbf{y})] \approx 0$

Zero-knowledge

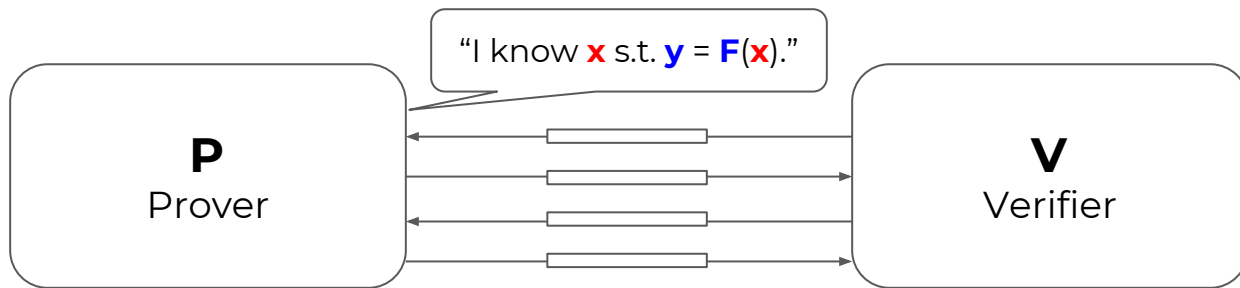
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Succinctness

$\mathbf{V}(\mathbf{F}, \mathbf{y})$ runs in time proportional to $|\mathbf{F}| + |\mathbf{y}|$ (not \mathbf{F} 's runtime)

Zero-Knowledge Succinct Proofs

Privacy-preserving cryptographic **succinct** proofs of computation integrity.



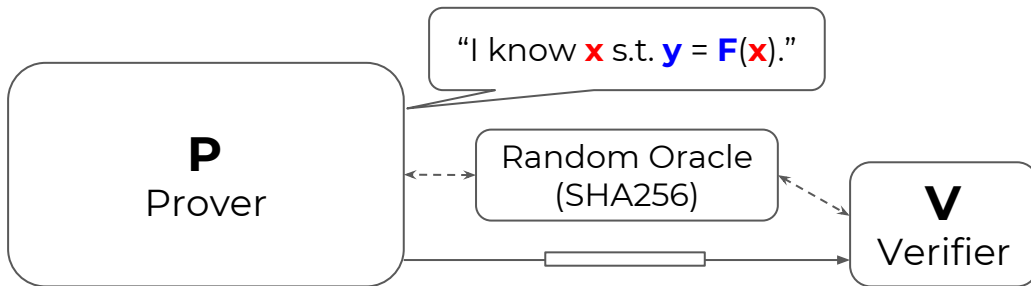
More powerful cryptographic primitive.

**However, it is still
interactive, ~~not succinct~~, and has bad concrete efficiency.**

Relies on PCPs
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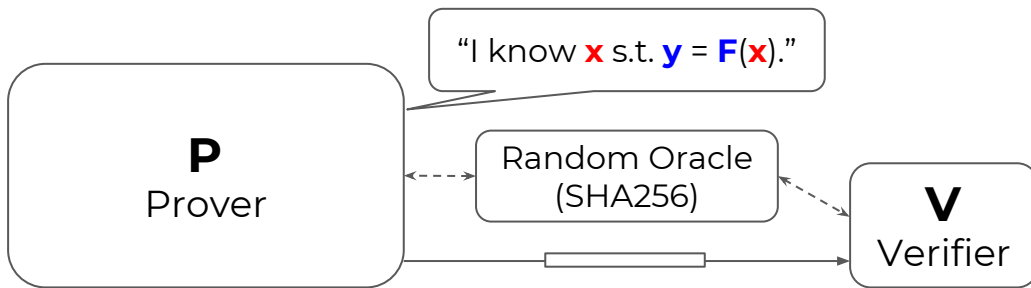
Zero-Knowledge Succinct Non-Interactive Proofs

Privacy-preserving cryptographic succinct,
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Zero-Knowledge Succinct Non-Interactive Proofs

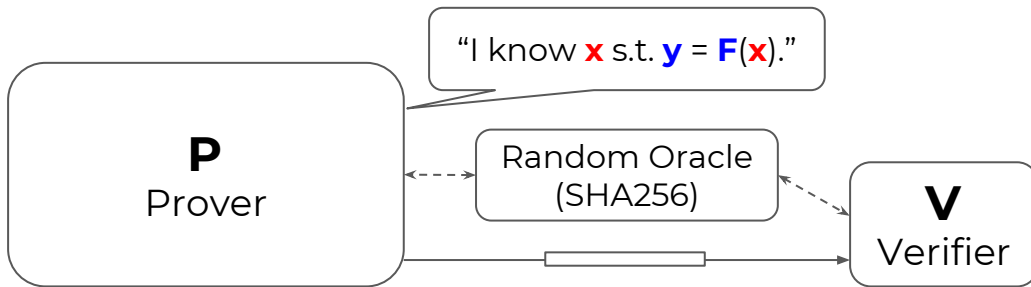
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Zero-**K**nowledge **S**uccinct **N**on-interactive **A**rgument of **K**nowledge

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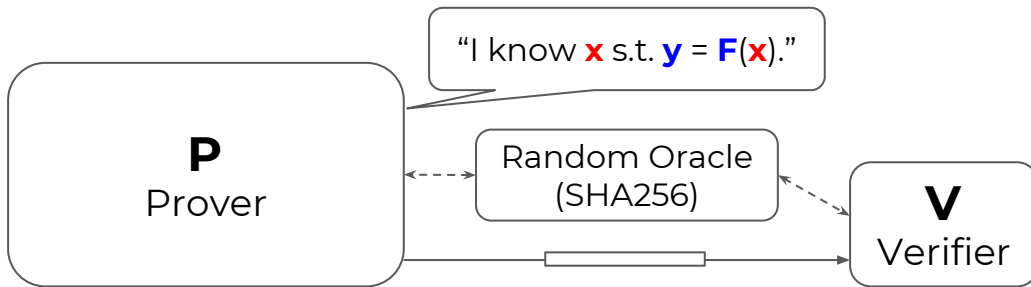


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Zero-knowledge - *Proofs do not reveal the witness*

Zero-Knowledge Succinct Non-Interactive Proofs

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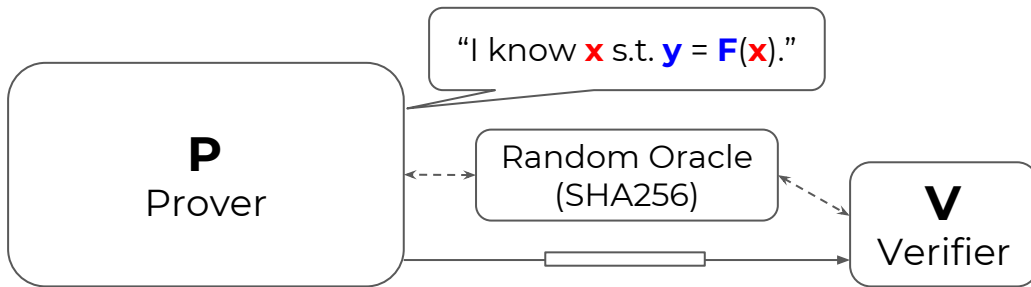
Zero-**K**nowledge **S**uccinct **N**on-interactive **A**rgument of **K**nowledge

Zero-knowledge - *Proofs do not reveal the witness*

Succinct - *Proofs are short and verification is fast*

Zero-Knowledge Succinct Non-Interactive Proofs

Privacy-preserving cryptographic succinct,
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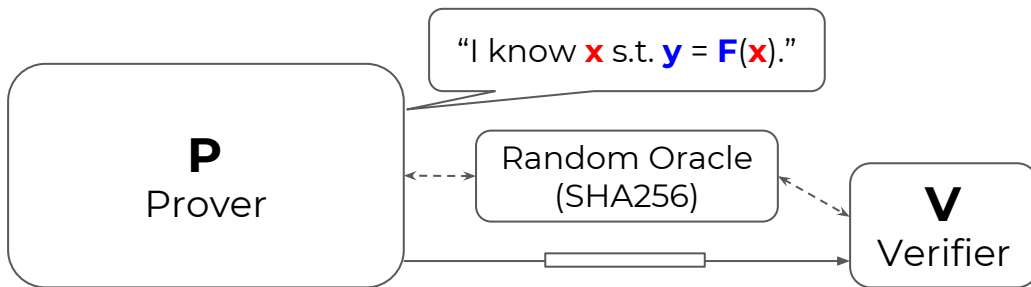
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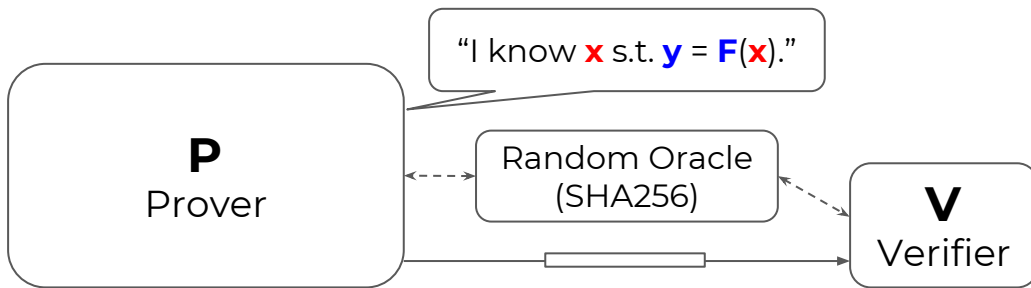
Succinct - *Proofs are short and verification is fast*

Non-interactive - *No interaction is necessary between the prover and the verifier*

Argument - *Soundness holds against a polynomially-bounded verifier*

Zero-Knowledge Succinct Non-Interactive Proofs

Privacy-preserving cryptographic succinct,
non-interactive proofs of computation integrity.

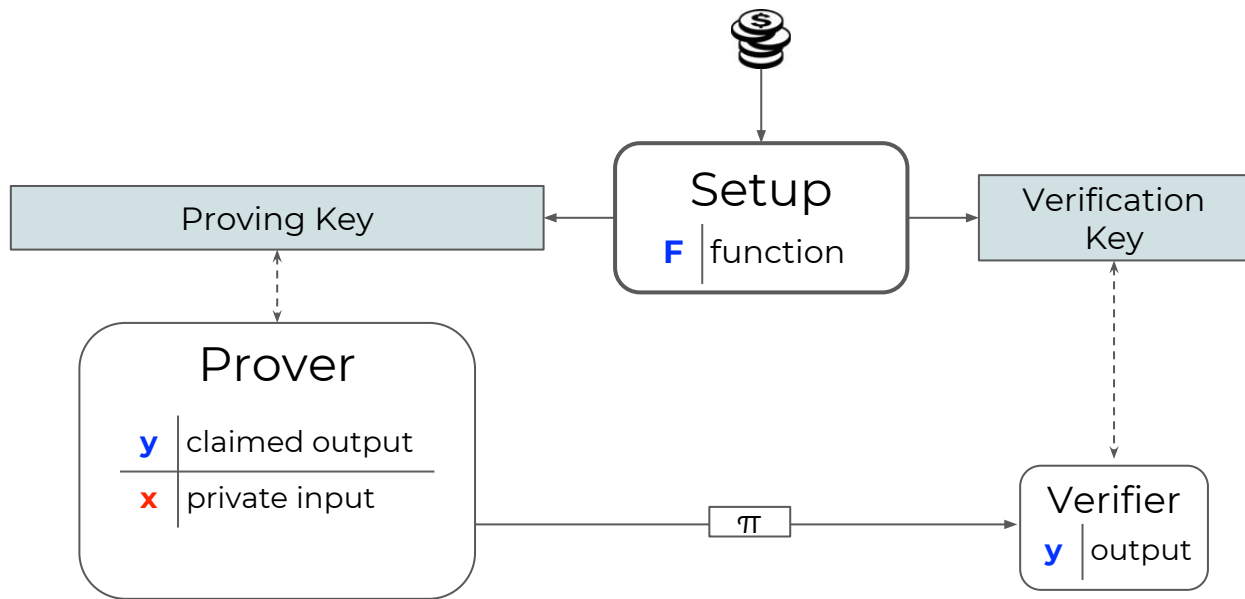


Even more powerful cryptographic primitive.

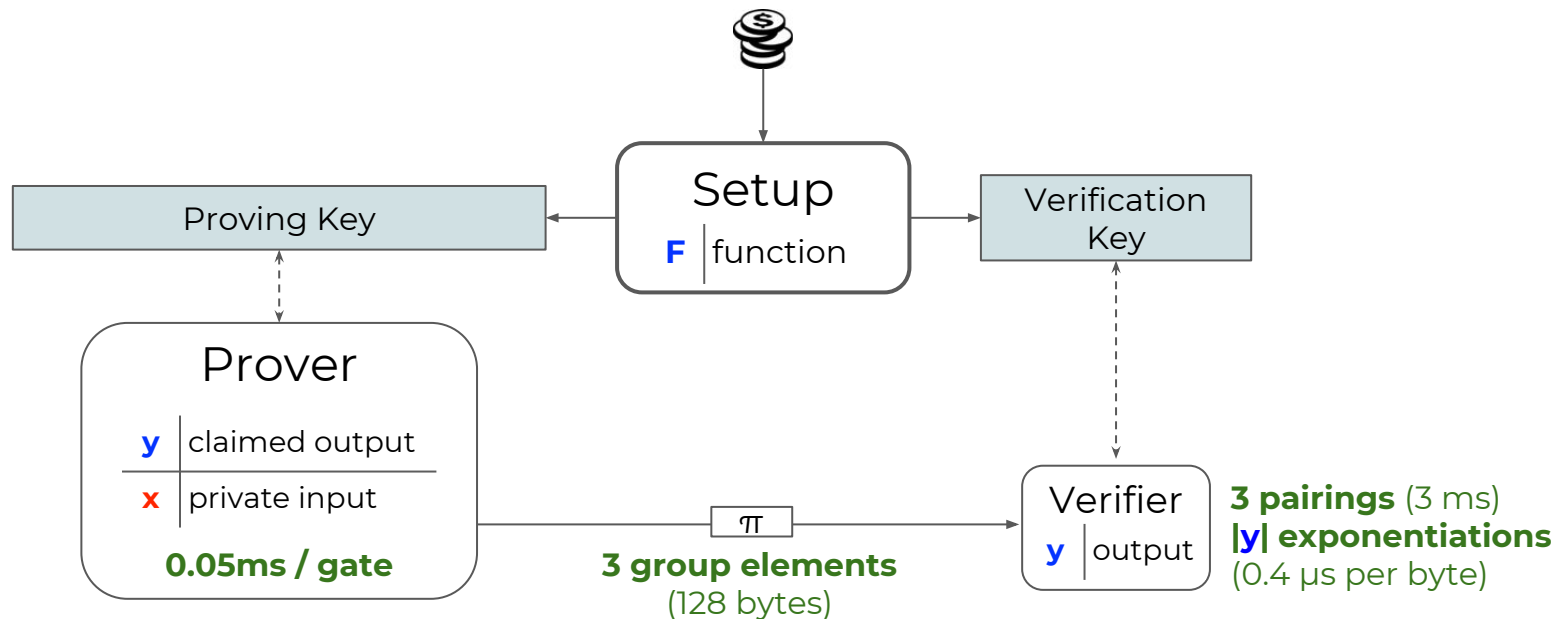
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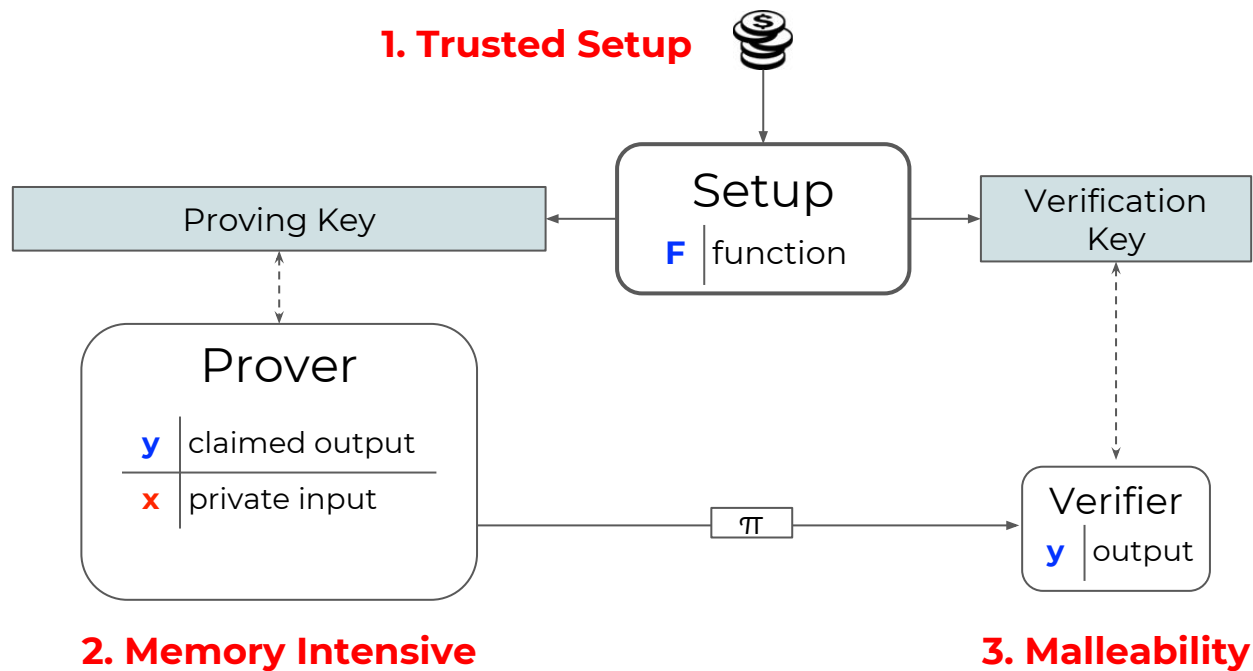
Preprocessing zkSNARKs



Preprocessing zkSNARKs

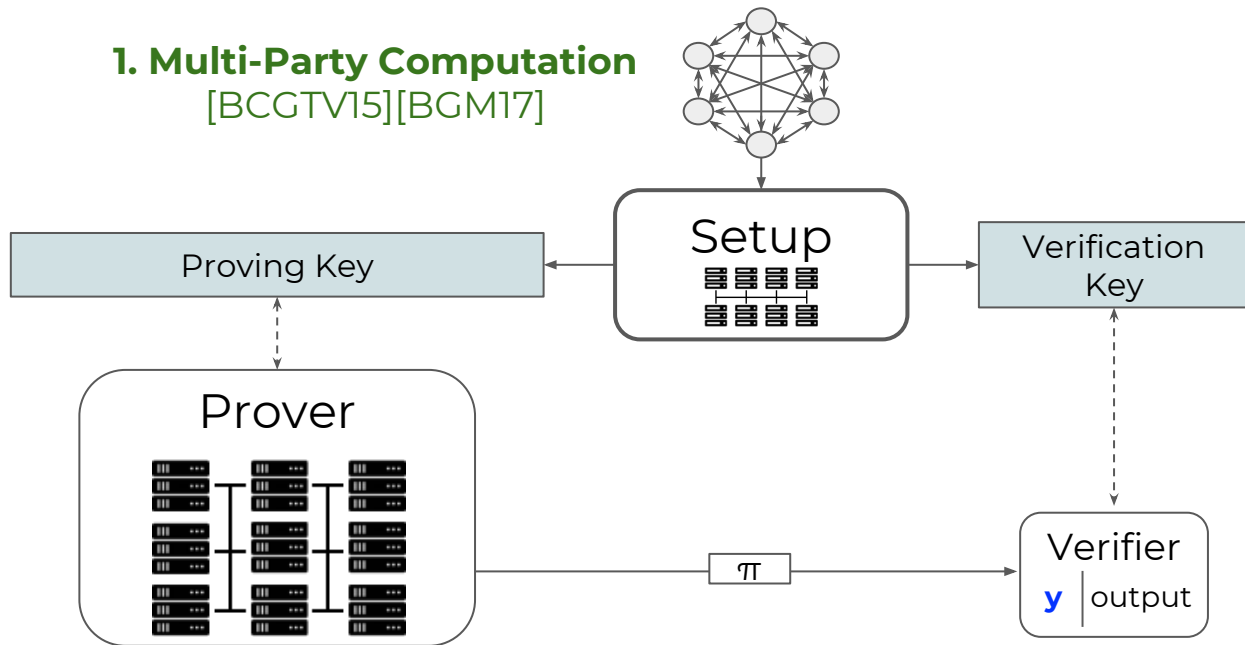


Preprocessing zkSNARKs



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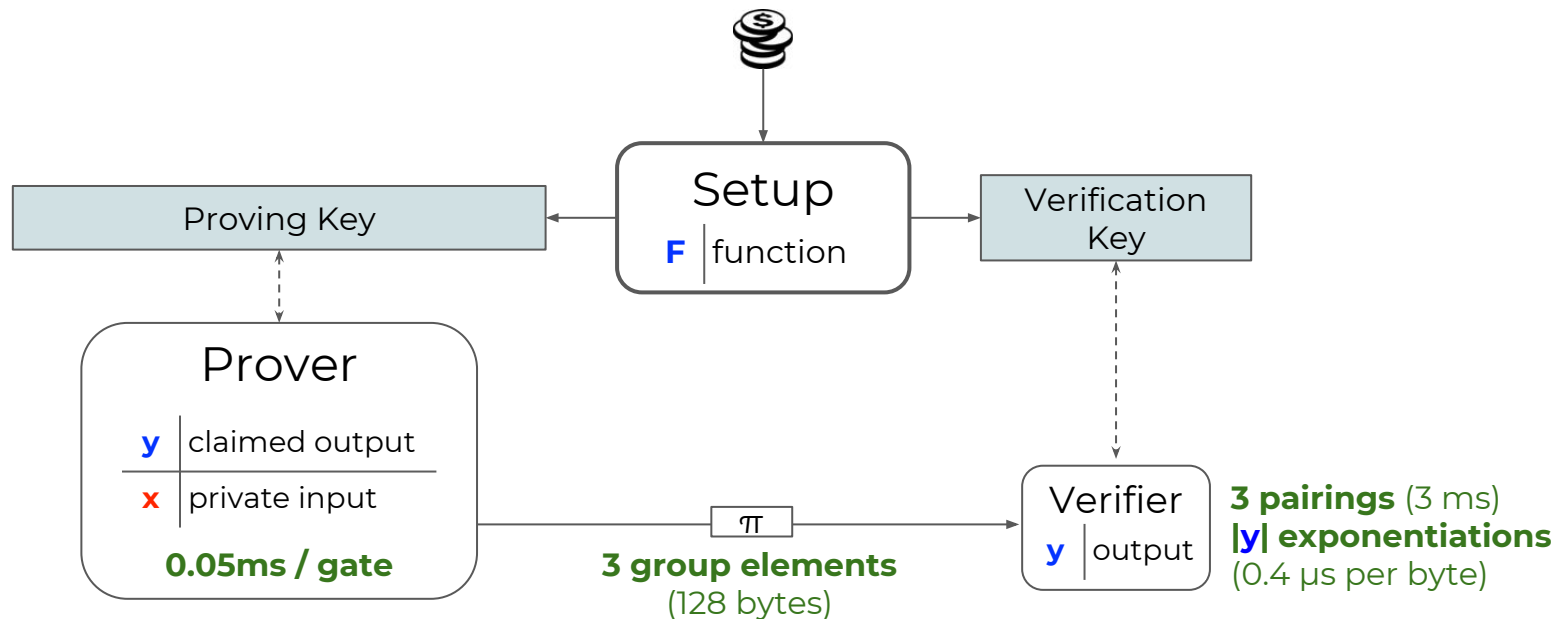
1. Multi-Party Computation [BCGTV15][BGM17]



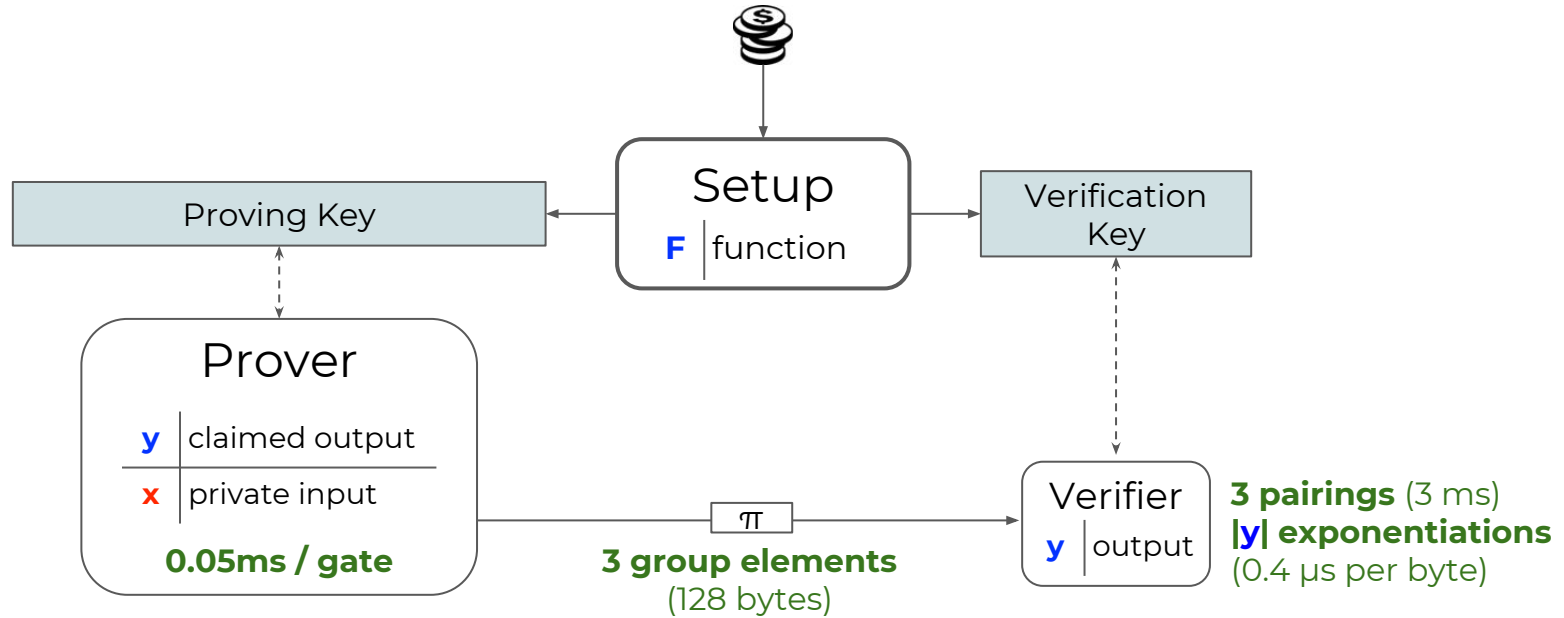
2. Distributed zkSNARKs

3. Simulation-Extractable zkSNARKs [GM17]

Preprocessing zkSNARKs



Pairing-based Preprocessing zkSNARKs



Prime-order bilinear map

G_1, G_2, G_T finite cyclic groups of prime order p

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A pairing is a map,

$$e: G_1 \times G_2 \rightarrow G_T$$

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Given g in G_1 and h in G_2 , a bilinear map ensures

$$e(g^a, h^b) = e(g, h)^{ab}.$$

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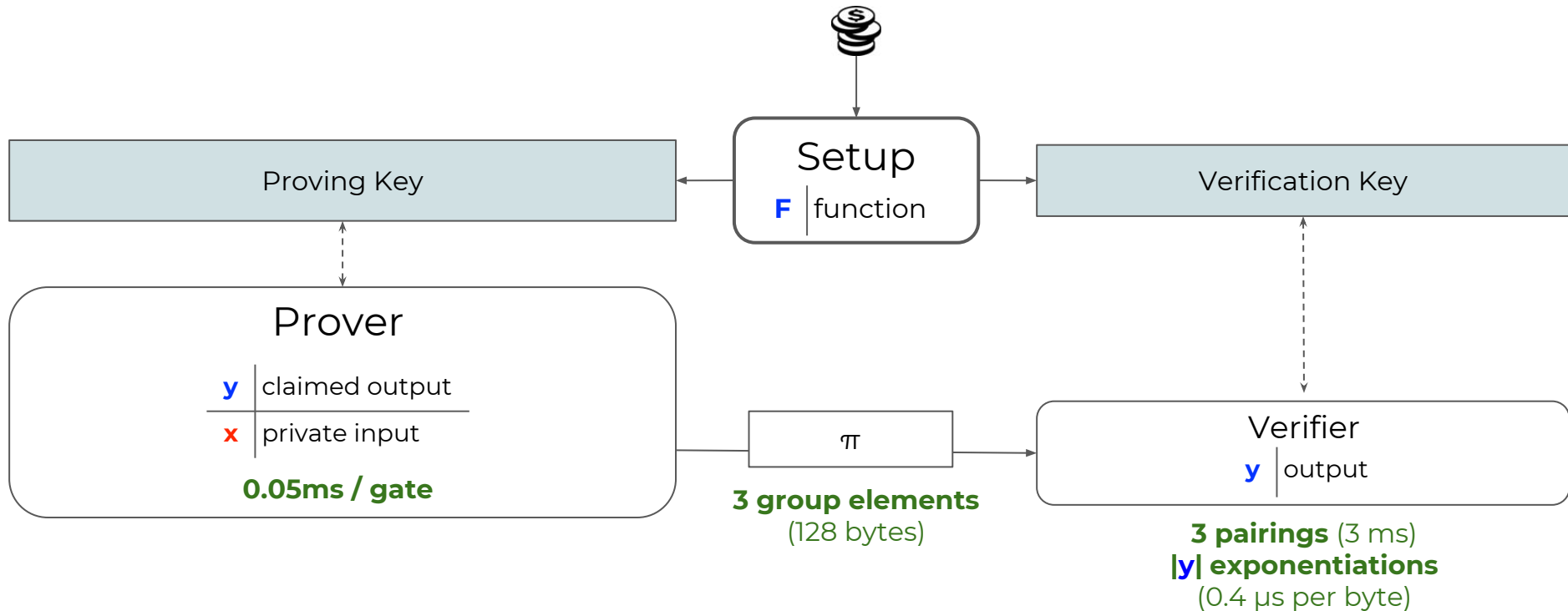
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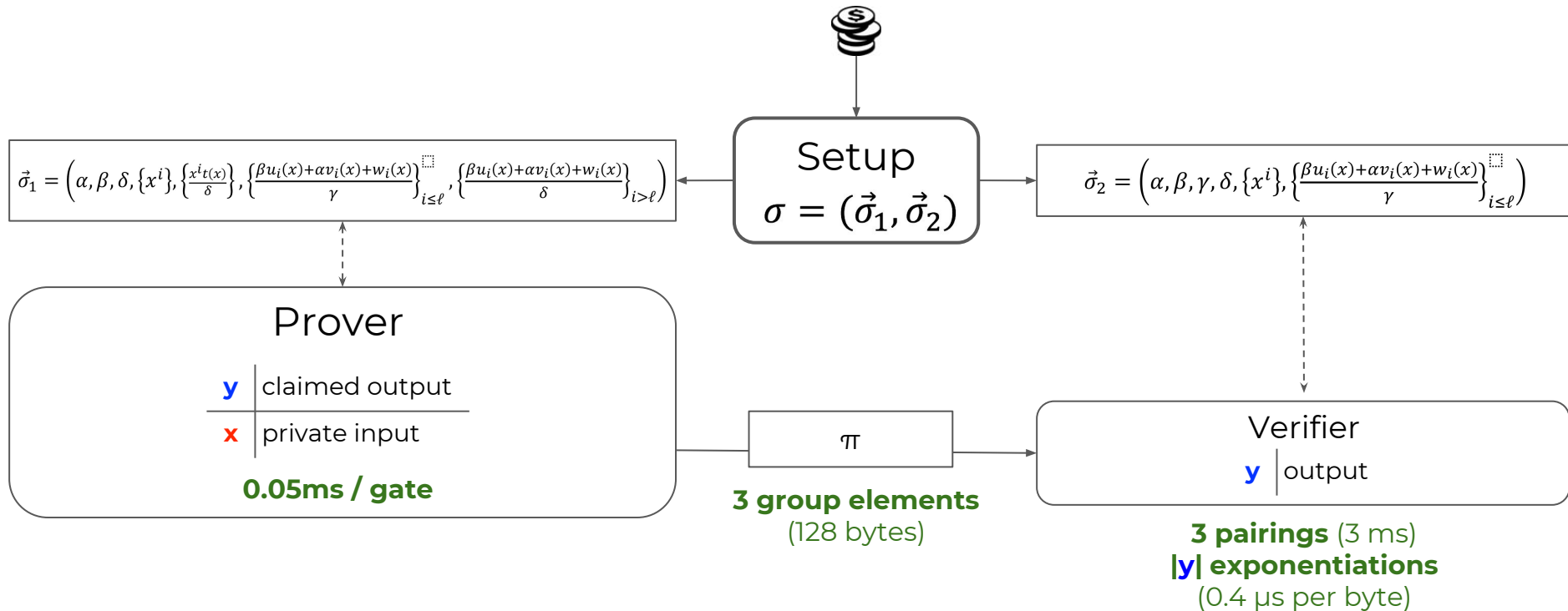
Notation:

$$[a]_1 = g^a \quad [b]_2 = h^b \quad [c]_T = e(g, h)^c$$

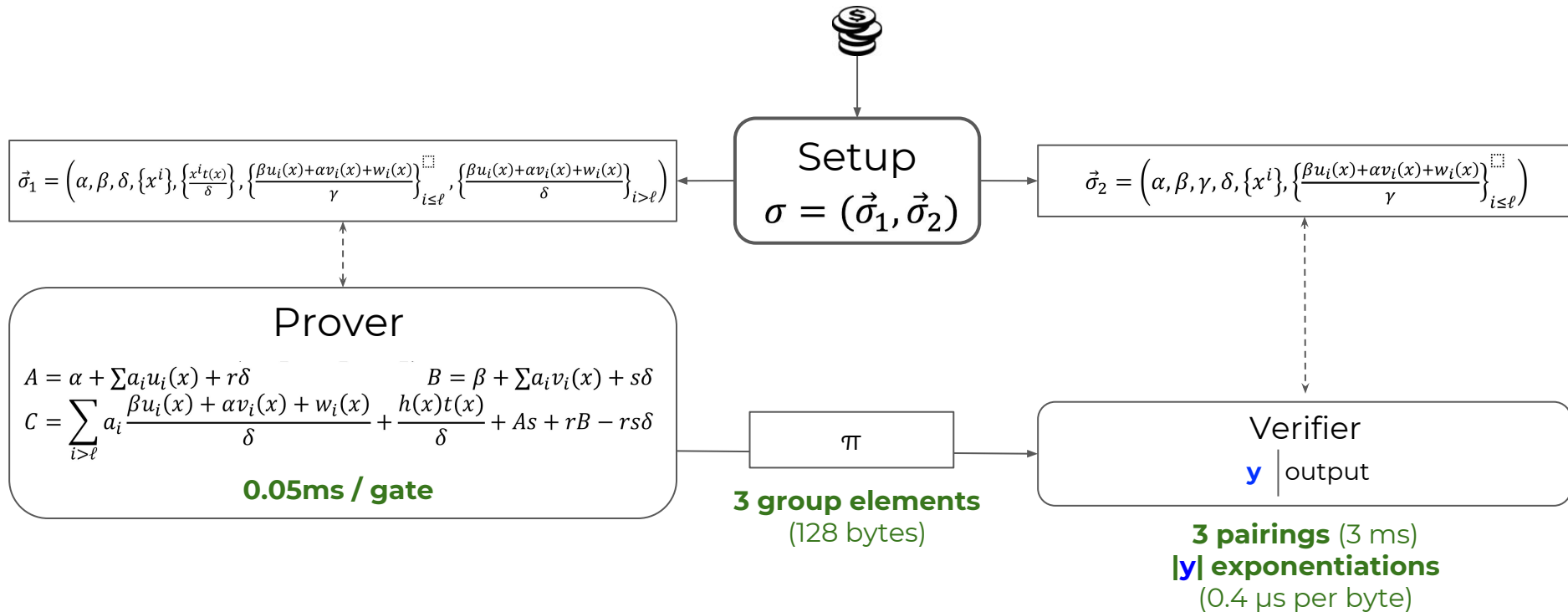
Pairing-based Preprocessing zkSNARKs



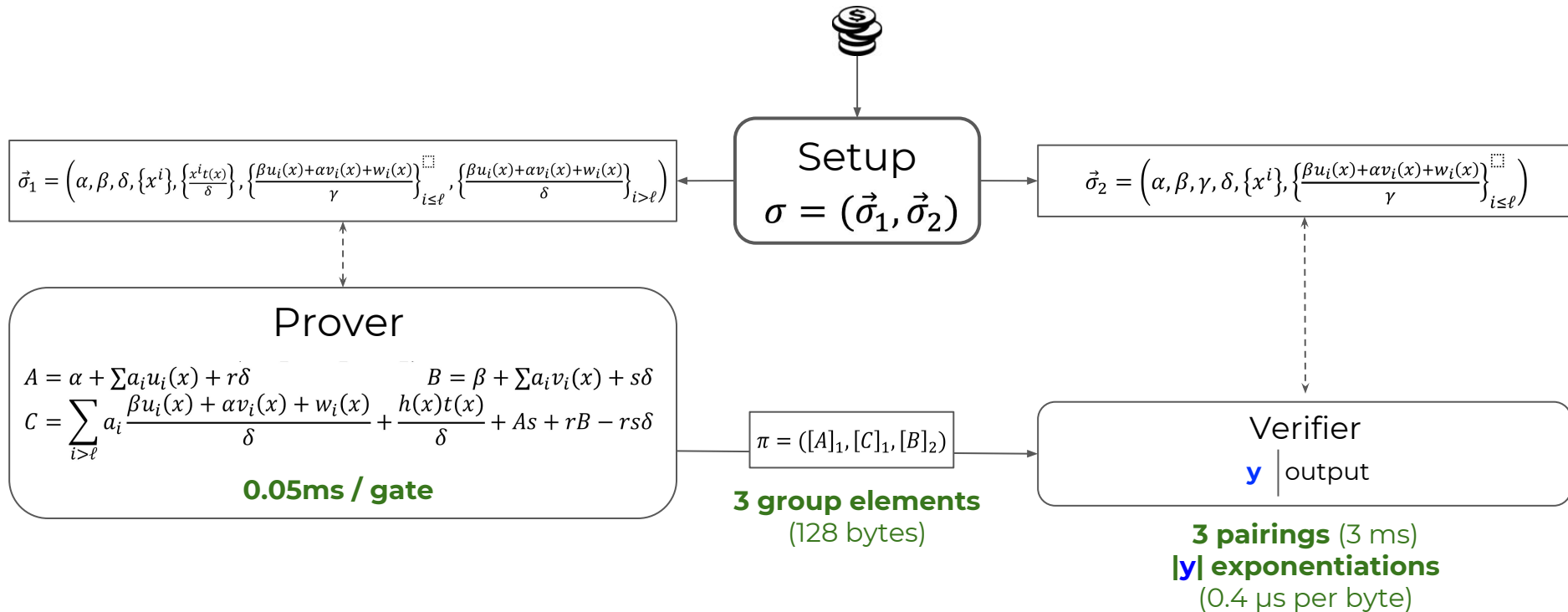
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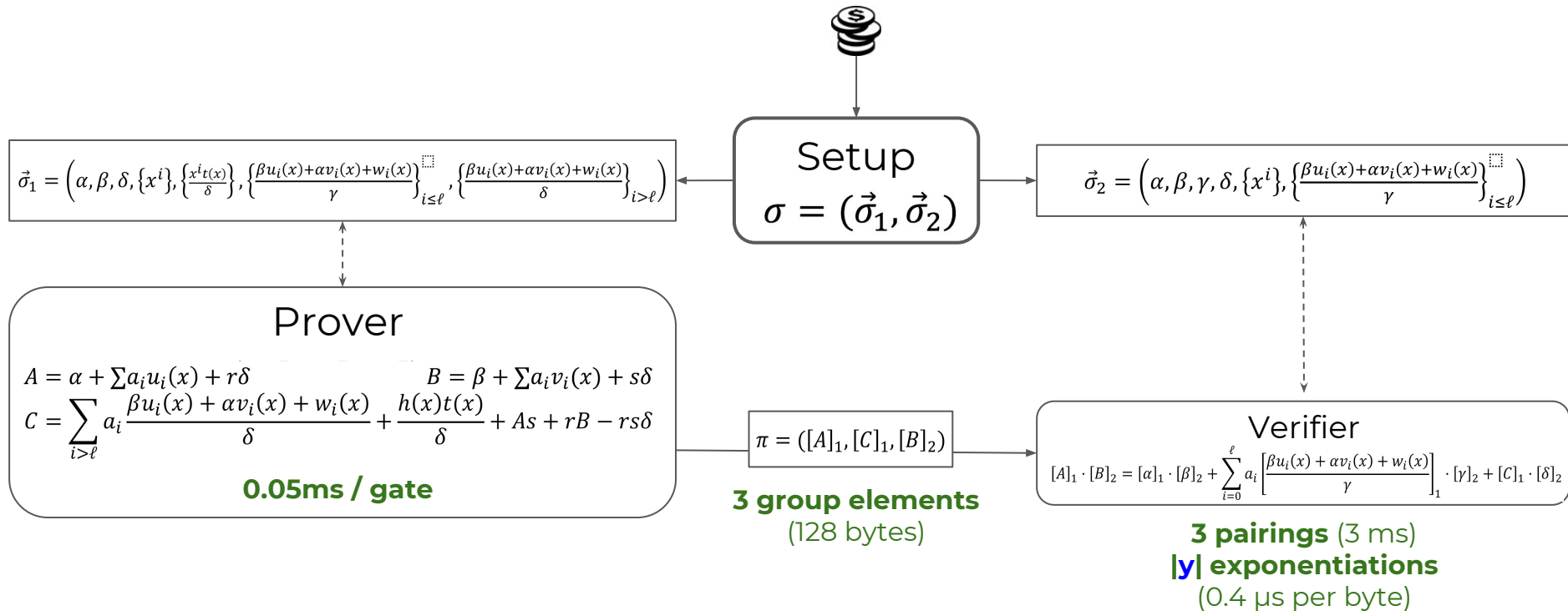
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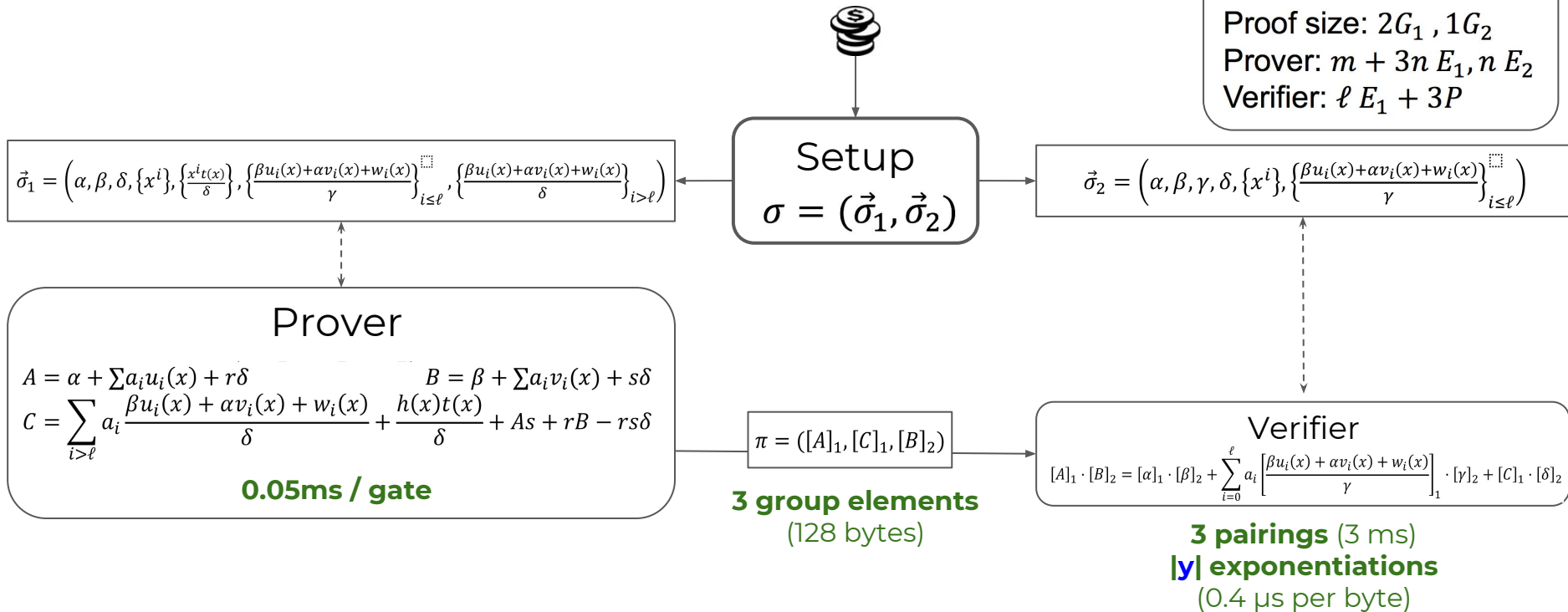
Pairing-based Preprocessing zkSNARKs



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Pairing-based Preprocessing zkSNARKs



libsnark: a C++ library for zkSNARKs
(libsnark.org)

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(libsark.org)

Algebraic Core	Polynomial Interpolation / Evaluation	Fast Fourier Transforms in Finite Fields	Lagrange-Coefficient Computations
	Finite Field Arithmetic	Bilinear Group Arithmetic	Fixed & Variable Base Multi-Exponentiation

libsark: a C++ library for zkSNARKs

(libsark.org)

- **libff**: a C++ library for Finite Fields and Elliptic Curves
(github.com/scipr-lab/libff)

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libsark: a C++ library for zkSNARKs

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- **libfqfft**: a C++ library for FFTs in Finite Fields
(github.com/scipr-lab/libfqfft)

- **libff**: a C++ library for Finite Fields and Elliptic Curves
(github.com/scipr-lab/libff)

Algebraic Core	Polynomial Interpolation / Evaluation	Fast Fourier Transforms in Finite Fields	Lagrange-Coefficient Computations
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libsnark: a C++ library for zkSNARKs

(libsnark.org)

Backend	Arithmetic Circuits		Boolean Circuits		Proof-Carrying Data
	PGHR 13	Groth 16	DFGK 14	Groth 16	BCTV14b (recursive proof composition)
Algebraic Core	Polynomial Interpolation / Evaluation		Fast Fourier Transforms in Finite Fields		Lagrange-Coefficient Computations
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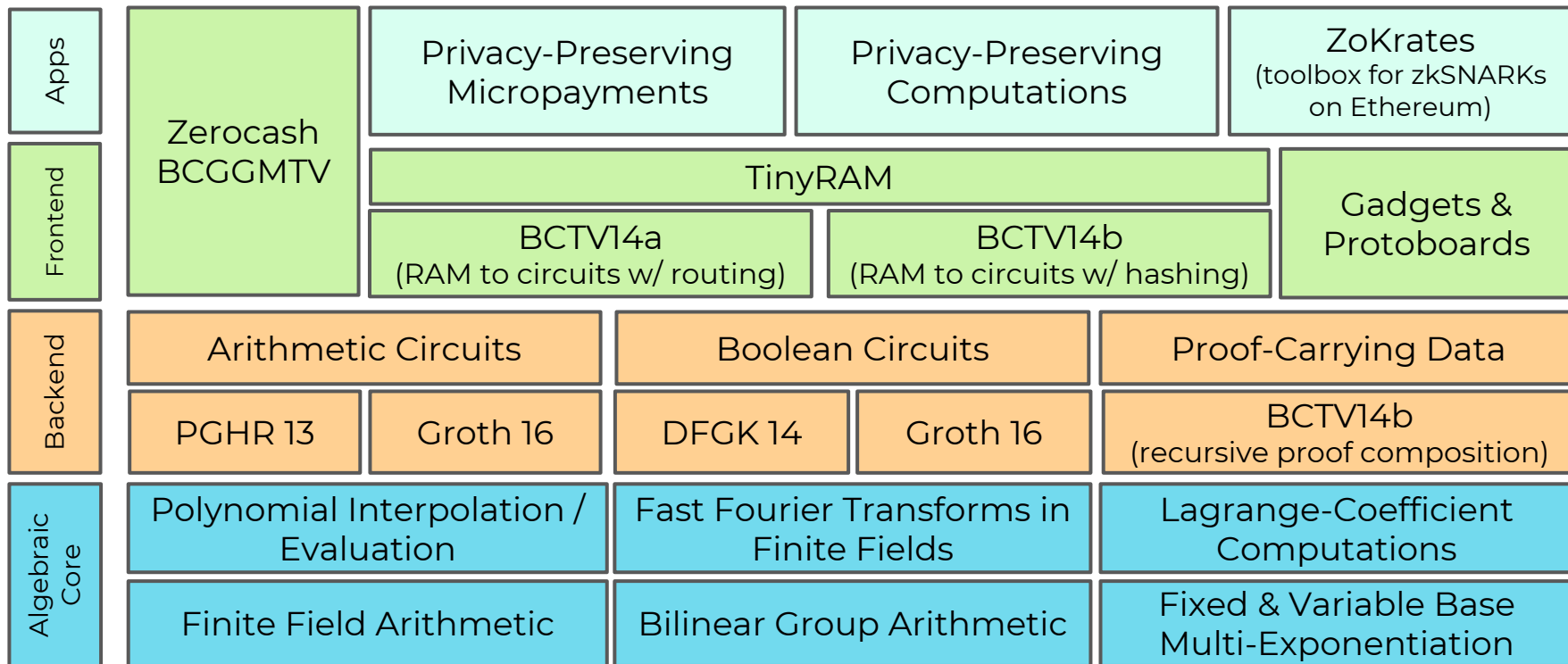
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Frontend	Zerocash BCGGMTV	TinyRAM			Gadgets & Protoboards
		BCTV14a (RAM to circuits w/ routing)		BCTV14b (RAM to circuits w/ hashing)	
Backend	Arithmetic Circuits		Boolean Circuits		Proof-Carrying Data
	PGHR 13	Groth 16	DFGK 14	Groth 16	BCTV14b (recursive proof composition)
Algebraic Core	Polynomial Interpolation / Evaluation		Fast Fourier Transforms in Finite Fields		Lagrange-Coefficient Computations
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Developing on **libsnaark**

libff

Choice of elliptic curve implementations

Barreto-Naehrig curve

(~128 bits of security)

Edwards curve

(~80 bits of security)

MNT4 curve

(~80 bits of security)

MNT6 curve

(~80 bits of security)

Developing on **libsnaark**

libff

Choice of elliptic curve implementations

Barreto-Naehrig curve

(~128 bits of security)

Edwards curve

(~80 bits of security)

MNT4 curve

(~80 bits of security)

MNT6 curve

(~80 bits of security)

Bilinear Group Arithmetic

Developing on **libsark**

libff

Choice of elliptic curve implementations

Barreto-Naehrig curve
(~128 bits of security)

Edwards curve
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MNT4 curve
(~80 bits of security)

MNT6 curve
(~80 bits of security)

libfqfft

Choice of evaluation domains

Standard radix-2
(size $m = 2^k$ & m -th roots of unity)

Extended radix-2
(size $m = 2^{k+1}$ & m -th roots of unity,
union a coset of these roots)

Step radix-2
(size $m = 2^k + 2^r$ & 2^k -th roots of unity,
union a coset of 2^r -th roots of unity)

Geometric Sequence
(size m & sampled points of $a_n = r^{(n-1)}$)

Arithmetic Sequence
(size m & sampled points of $a_i = a_1 + (i - 1)*d$)

Bilinear Group Arithmetic

Developing on **libsark**

libff

Choice of elliptic curve implementations

Barreto-Naehrig curve
(~128 bits of security)

Edwards curve
(~80 bits of security)

MNT4 curve
(~80 bits of security)

MNT6 curve
(~80 bits of security)

Bilinear Group Arithmetic

libfqfft

Choice of evaluation domains

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Arithmetic Sequence
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Polynomial Interpolation /
Evaluation

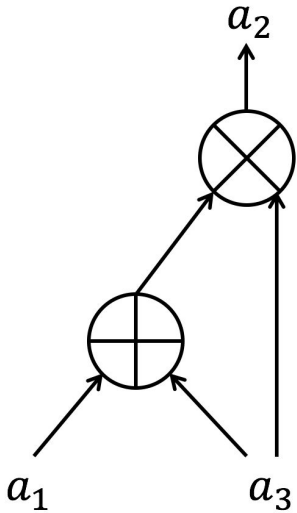
Developing on **libsnark**

libsnark arithmetic circuits

Developing on **libsark**

libsark arithmetic circuits

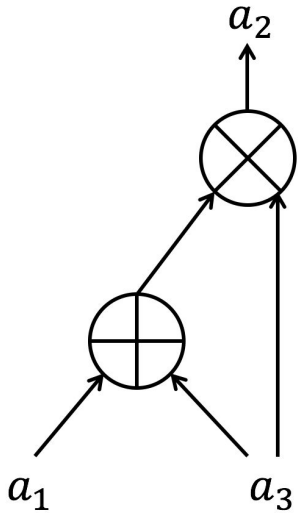
Write as quadratic equation over F_p
 $(a_1 + a_3) \cdot a_3 = a_2$



Developing on **libsark**

libsark arithmetic circuits

Write as quadratic equation over F_p
 $(a_1 + a_3) \cdot a_3 = a_2$



Arithmetic circuits can be expressed in the form,

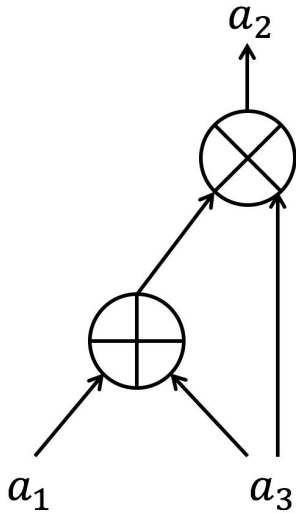
$$\sum a_i u_i \cdot \sum a_i v_i = \sum a_i w_i$$

over variables a_1, \dots, a_m , where by convention $a_0 = 1$.

Developing on **libsark**

libsark arithmetic circuits

Write as quadratic equation over F_p
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$$\sum a_i u_i \cdot \sum a_i v_i = \sum a_i w_i$$

over variables a_1, \dots, a_m , where by convention $a_0 = 1$.

An arithmetic circuit defines an *NP-language* with statements (a_1, \dots, a_ℓ) and witnesses $(a_{\ell+1}, \dots, a_m)$.

Developing on **libsnark**

libsnark gadgetlib1 & gadgetlib2

Developing on **libsnark**

libsnark gadgetlib1 & gadgetlib2

Low-level libraries which expose all features of the preprocessing zkSNARK

Gadgets &
Protoboards

Developing on **libsnark**

libsnark gadgetlib1 & gadgetlib2

Low-level libraries which expose all features of the preprocessing zkSNARK

Finite Fields

Elliptic Curves

Pairings

Multi-Exponentiation

Hash Functions (*SHA256*)

Merkle Trees (*Authentication Paths*)

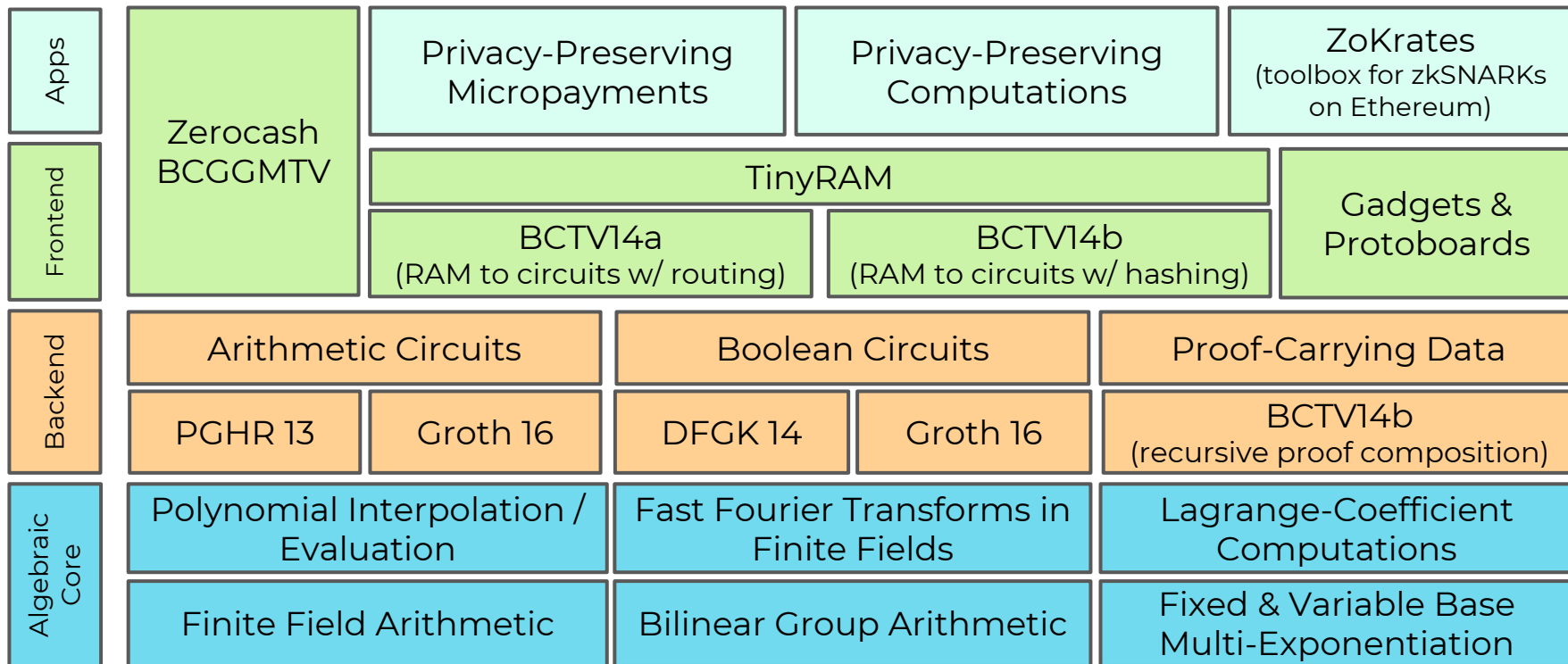
CPU Checkers (*TinyRAM*)

zkSNARK verifier

Gadgets &
Protoboards

libsnark: a C++ library for zkSNARKs

(libsnark.org)



libsnark: a C++ library for zkSNARKs

(libsnark.org)

To get started:

github.com/howardwu/libsnark-tutorial

Frontend	BCGGMTV	TinyRAM		Gadgets & Protoboards	
		BCTV14a (RAM to circuits w/ routing)	BCTV14b (RAM to circuits w/ hashing)		
Backend	Arithmetic Circuits		Boolean Circuits		Proof-Carrying Data
	PGHR 13	Groth 16	DFGK 14	Groth 16	BCTV14b (recursive proof composition)
Algebraic Core	Polynomial Interpolation / Evaluation		Fast Fourier Transforms in Finite Fields		Lagrange-Coefficient Computations
	Finite Field Arithmetic		Bilinear Group Arithmetic		Fixed & Variable Base Multi-Exponentiation

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Thanks!

