HW2_report

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By including this in my report, I agree to abide by the Academic Integrity Policy mentioned above.

0.0.1 Problem 1

The goal of this problem is to implement a function for AHE as described in Chapter 1 of Adaptive Histogram Equalization - A Parallel Implementation.
The function has the following specifications:

- The desired function AHE() takes two inputs: the image im and the contextual region size win size.
- Using the pseudocode in Algorithm 1 as a reference, compute the enhanced image after AHE.
- You may use loops if necessary. You should not make use of any inbuilt MATLAB/Python functions for AHE or HE.
- The function returns one output: the enhanced image after AHE.

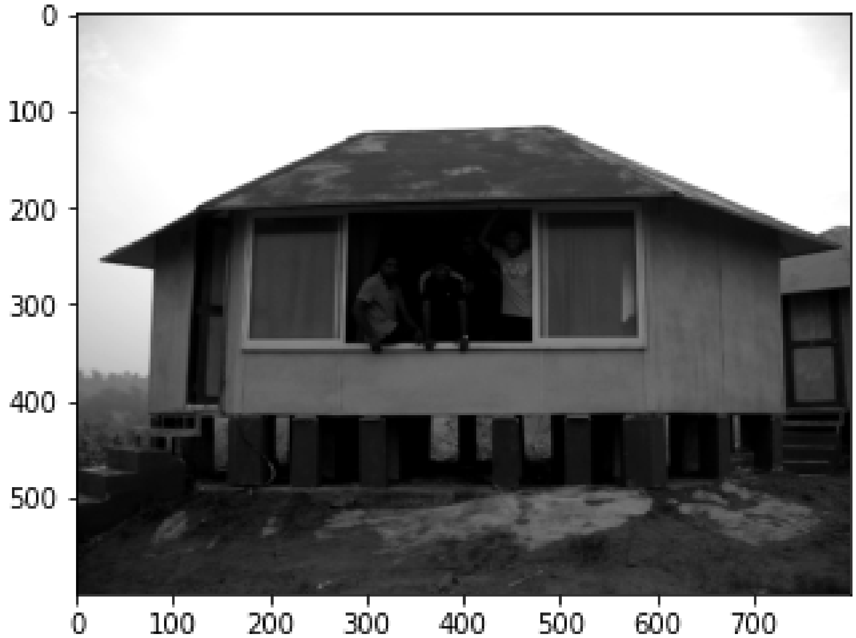
[106]:
```
from tqdm import tqdm
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as image
import cv2
import skimage.morphology as skim_morph
import scipy.ndimage.measurements as scipy_im_measure
import pandas as pd
from lloyd_python import lloyds
```

[107]:
```
beach = cv2.imread('beach.png', 0)
```

[108]:
```
type(beach)
```

[108]: numpy.ndarray

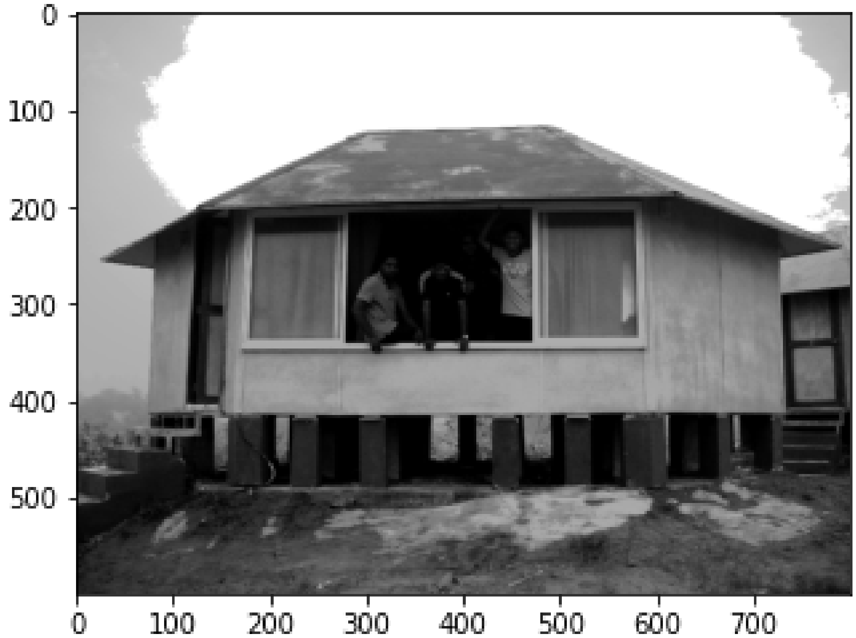
[109]:
```
fig = plt.figure()
ax = fig.add_subplot()
ax.imshow(beach, cmap = 'gray');
```



Normal histogram equalization

[110]:
```
he_beach = cv2.equalizeHist(beach)
```

[111]:
```
fig = plt.figure()
ax = fig.add_subplot()
ax.imshow(he_beach, cmap = 'gray');
```



Adaptive Histogram Equalization

[112]:
```
def AHE(im, win_size):

    edge_offset = win_size//2
    im_padded = np.pad(im, win_size//2, mode = 'symmetric')
    ahe_im_padded = np.zeros(im_padded.shape)

    # taking only the pixels which can have a legit contextual region
    for i in tqdm(range(edge_offset,im_padded.shape[0]-edge_offset)):
        for j in range(edge_offset,im_padded.shape[1]-edge_offset):

            rank = 0

            contextual_region = im_padded[i-edge_offset:
 ↪i+edge_offset,j-edge_offset:j+edge_offset]

            for m in range(contextual_region.shape[0]):
                for n in range(contextual_region.shape[1]):
                    if(im_padded[i,j]>contextual_region[m,n]):
                        rank = rank + 1
            ahe_im_padded[i,j] = rank*255/(win_size**2)

    ahe_im = ahe_im_padded[edge_offset:(im_padded.
 ↪shape[0]-edge_offset),edge_offset:(im_padded.shape[1]-edge_offset)]
```

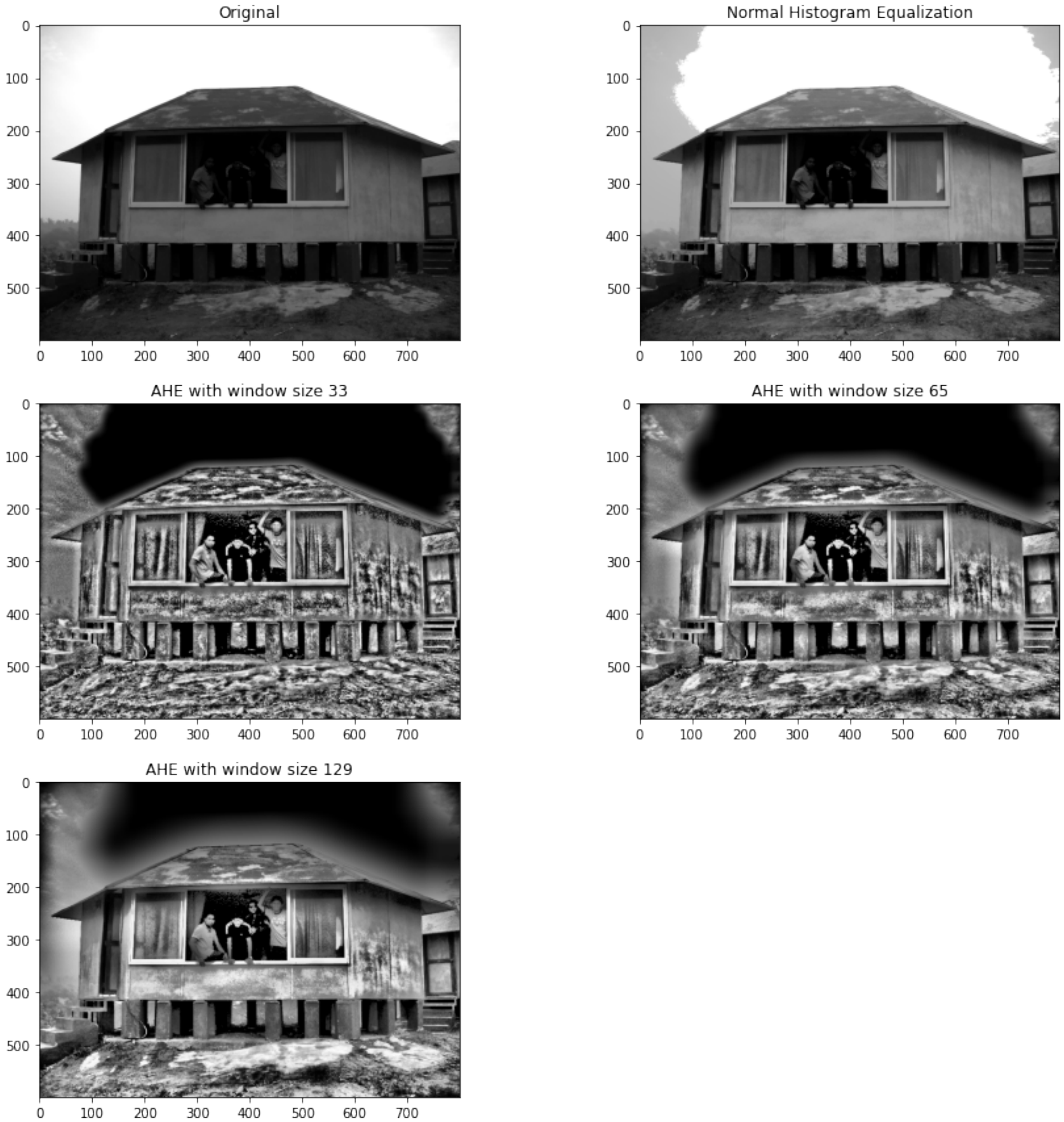
```
    return(ahe_im)
```

I have already run the below code and obtained the images.
It takes a long time to run.
For the purpose of this report I will import the images that I saved from the first time I run the below code.

[113]:
```
# ahe_im_33 = AHE(beach, 33)
# ahe_im_65 = AHE(beach, 65)
# ahe_im_129 = AHE(beach, 129)
```

[118]:
```
ahe_im_33 = image.imread('ahe_im_33.jpg')
ahe_im_65 = image.imread('ahe_im_65.jpg')
ahe_im_129 = image.imread('ahe_im_129.jpg')
```

[119]:
```
fig = plt.figure(figsize = (15,15))
ax_orig = fig.add_subplot(321)
ax_orig.imshow(beach, cmap = 'gray')
ax_orig.set_title('Original')

ax_he = fig.add_subplot(322)
ax_he.imshow(he_beach, cmap = 'gray')
ax_he.set_title('Normal Histogram Equalization')

ax_33 = fig.add_subplot(323)
ax_33.imshow(ahe_im_33, cmap = 'gray')
ax_33.set_title('AHE with window size 33')

ax_65 = fig.add_subplot(324)
ax_65.imshow(ahe_im_65, cmap = 'gray')
ax_65.set_title('AHE with window size 65')

ax_129 = fig.add_subplot(325)
ax_129.imshow(ahe_im_129, cmap = 'gray')
ax_129.set_title('AHE with window size 129');
```



How does the original image qualitatively compare to the images after AHE and HE respectively? The original image is quite similar to the image after HE. However, AHE works much better in giving us information about the details in the image. The images become more like the original with increase in the window size, but the level of detail decreases with an increase in window size.

Which strategy (AHE or HE) works best for beach.png and why? Is this true for any image in general? AHE works better for beach.png. HE gives a better image, but we still can't see the subjects that are in the window clearly even after HE.

This might not be true in general, as beach.png had a very dark region and a very bright region and when this is not true, HE might work better.

0.0.2 Binary Morphology

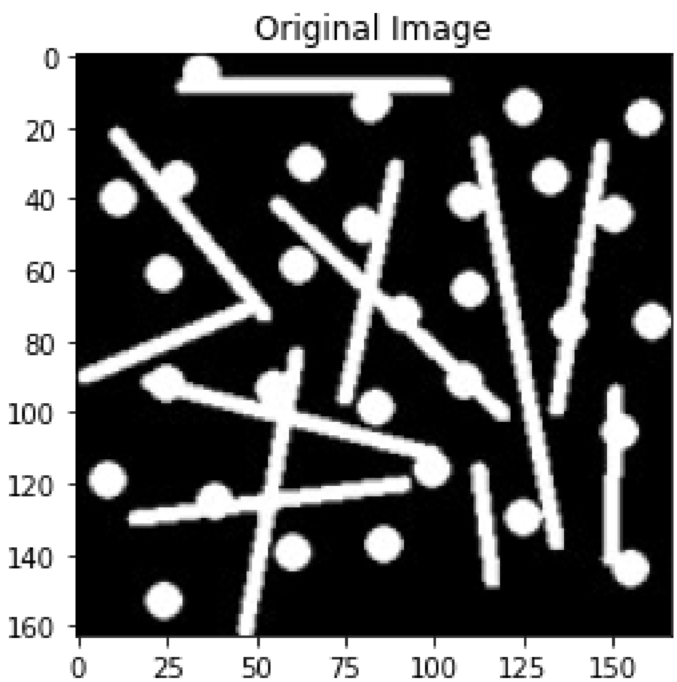
Problem 2 Part 1

[3]:
```
bin_image = image.imread('circles_lines.jpg')
```

[4]:
```
bin_im_1_chan = np.mean(bin_image, axis = 2)
```

[5]:
```
plt.imshow(bin_im_1_chan, cmap = 'gray')
plt.title('Original Image')
```

[5]:
```
Text(0.5, 1.0, 'Original Image')
```

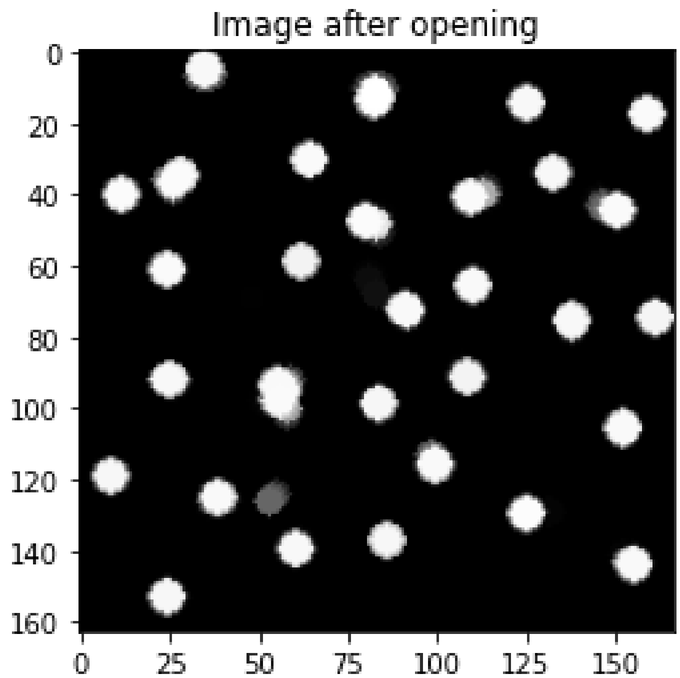


Reference: https://scikit-image.org/docs/dev/auto_examples/applications/plot_morphology.html

[9]:
```
selem = skim_morph.disk(4)
bin_opened = skim_morph.opening(bin_im_1_chan, selem)
```

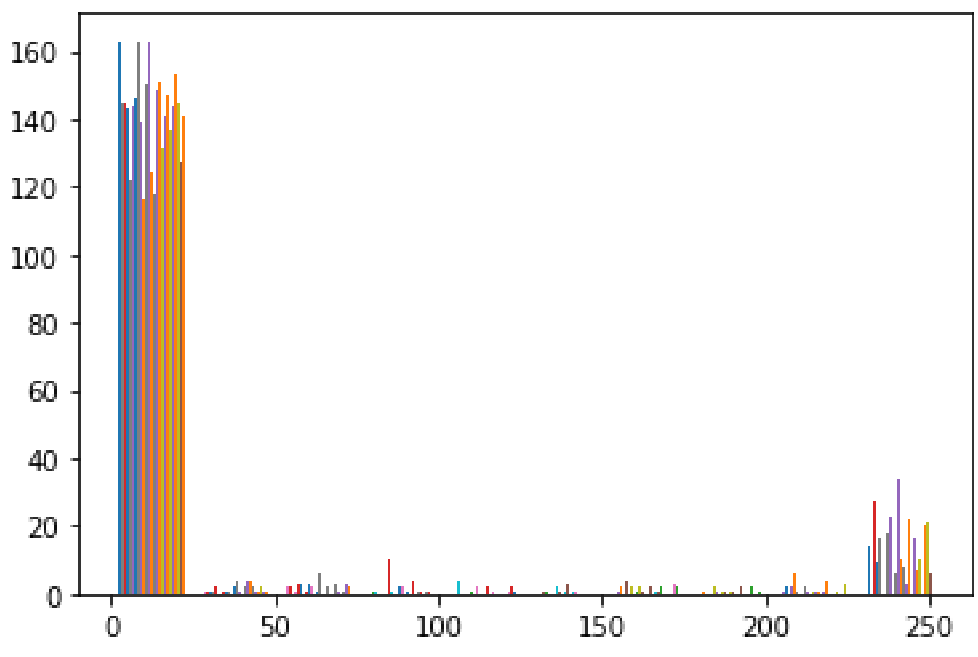
[10]:
```
plt.imshow(bin_opened, cmap = 'gray')
plt.title('Image after opening')
```

[10]:
```
Text(0.5, 1.0, 'Image after opening')
```



Thresholding

[11]:
```
plt.hist(bin_opened);
```



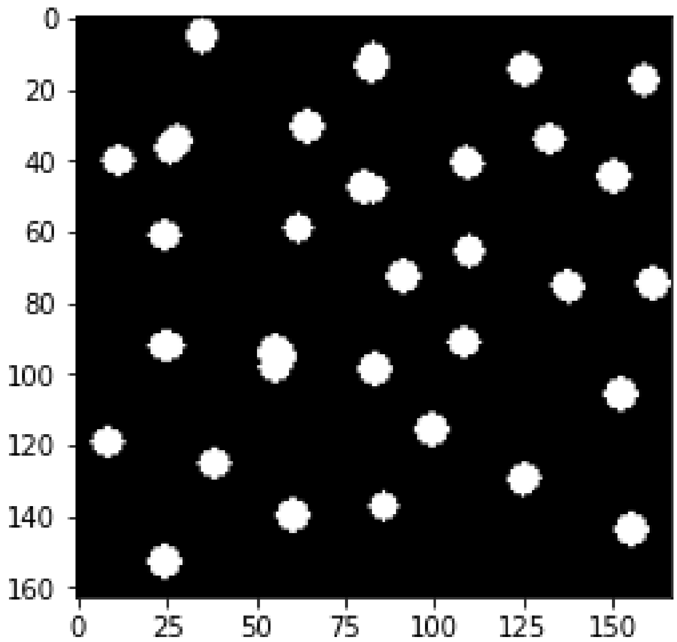
Observing the histogram above, and considering we want to remove the gray areas and convert them to black, we threshold the image with intensity as 225.

[13]:
```
thresholded_bin = np.zeros(bin_opened.shape)
```

[14]:
```
thresholded_bin = np.where(bin_opened>225.0, 1, 0)
```

[15]:
```
plt.imshow(thresholded_bin, cmap = 'gray')
```

[15]: <matplotlib.image.AxesImage at 0x7fd433620dd8>



Once we have a binary image with just the circles, the individual regions need to be labeled to represent distinct objects in the image i.e. connected component labeling. This can be done in Python using scipy.ndimage.measurements.label().

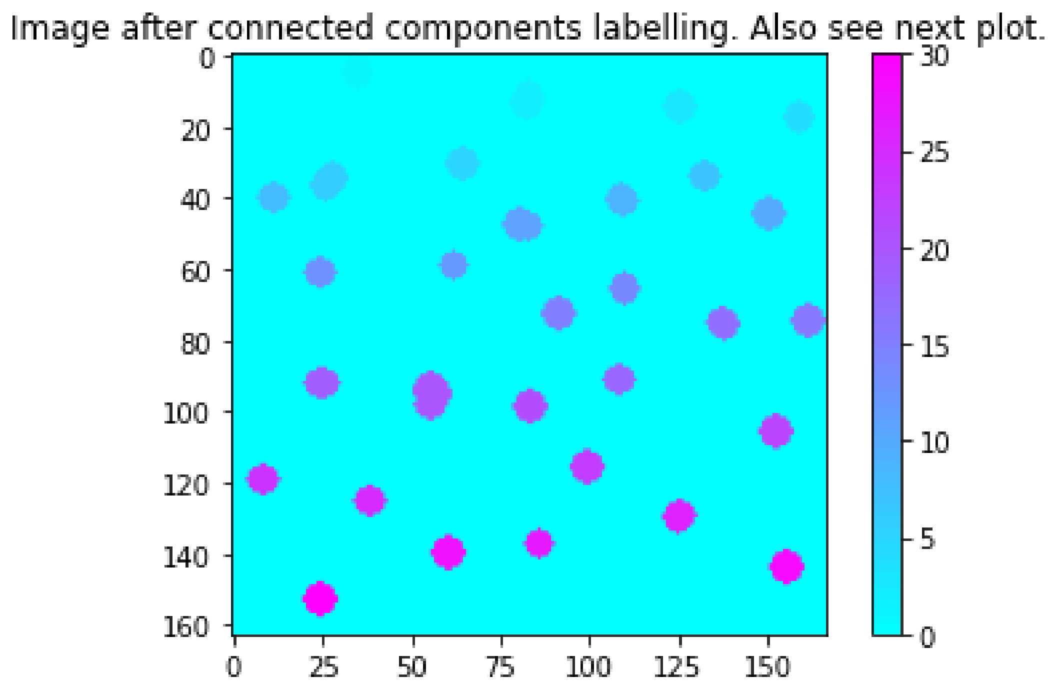
Labelling

[80]:
```
strel = np.ones((3,3))
```

[81]:
```
# input image is thresholded_bin
thresholded_bin_labeled, num = scipy_im_measure.label(thresholded_bin, strel)
```

[138]:
```
fig = plt.figure()
ax = fig.add_subplot()
mappa = ax.imshow(thresholded_bin_labeled, cmap = 'cool')
ax.set_title('Image after connected components labelling. Also see next plot.')
```

```
fig.colorbar(mappa);
```

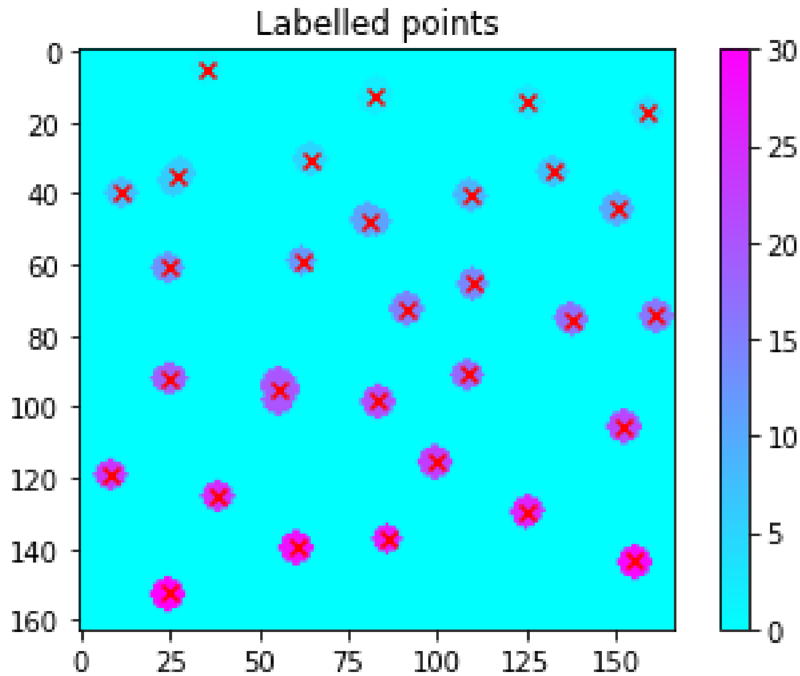


For each labeled region, calculate centroid and area.
strategy: using np.where for label == 1 extract all the indices. The indices contain information about the position of the features. Once we get the indices, we take mean to get centroid, and take count to get area in pixels.

For visibility, we also mark the labelled points with a red x.

[83]:
```
fig_scp = plt.figure()
ax_scp = fig_scp.add_subplot()
mappa2 = ax_scp.imshow(thresholded_bin_labeled, cmap = 'cool')

centroid_list = []
centroid_areas = []

for i in range(1,num+1):
    # extract positions of label
    pos = np.asarray(np.where(thresholded_bin_labeled == i))
    # take mean of the pos
    centroid_pos = np.mean(pos, axis = 1)
    ax_scp.scatter(centroid_pos[1], centroid_pos[0], marker = 'x', c = 'r')

    centroid_list.append(centroid_pos)
    centroid_areas.append(pos.shape[1])
```

```
ax_scp.set_title('Labelled points');
fig_scp.colorbar(mappa2);
```



Tabulating these values

[143]:
```
circle_index = [i+1 for i in range(num)]
circle_index_df = pd.DataFrame(circle_index,columns=['Circle Index'])
df = pd.DataFrame(centroid_list, columns=['Row number (approx)', 'Column number␣
↪(approx)'])
df2 = pd.DataFrame(centroid_areas, columns = ['Area of circle in pixels'])
df3 = pd.concat([circle_index_df, df,df2], axis = 1)
df3
```

[143]:
```
    Circle Index  Row number (approx)  Column number (approx)  \
0              1             5.000000               35.000000
1              2            12.719512               82.731707
2              3            14.500000              125.500000
3              4            17.500000              159.000000
4              5            30.500000               64.500000
5              6            35.500000               27.000000
6              7            34.000000              132.500000
7              8            40.000000               11.500000
8              9            40.671875              109.328125
9             10            44.500000              150.500000
```

10	11	47.679012	81.271605
11	12	59.000000	62.000000
12	13	61.000000	24.500000
13	14	65.500000	110.000000
14	15	72.500000	91.500000
15	16	74.500000	161.500000
16	17	75.328125	137.671875
17	18	91.000000	108.500000
18	19	92.000000	25.000000
19	20	95.436364	55.772727
20	21	98.500000	83.500000
21	22	105.500000	152.500000
22	23	115.500000	99.500000
23	24	119.000000	8.500000
24	25	125.000000	38.500000
25	26	129.328125	125.328125
26	27	137.000000	86.000000
27	28	139.500000	60.500000
28	29	143.500000	155.500000
29	30	152.500000	24.500000

	Area of circle in pixels
0	67
1	82
2	68
3	58
4	68
5	82
6	58
7	58
8	64
9	68
10	81
11	49
12	58
13	58
14	68
15	68
16	64
17	58
18	67
19	110
20	68
21	68
22	68
23	58
24	58

25	64
26	49
27	68
28	68
29	68

The structuring element I used for opening in this problem was:

selem = skim_morph.disk(4). It is displayed in the code block below.

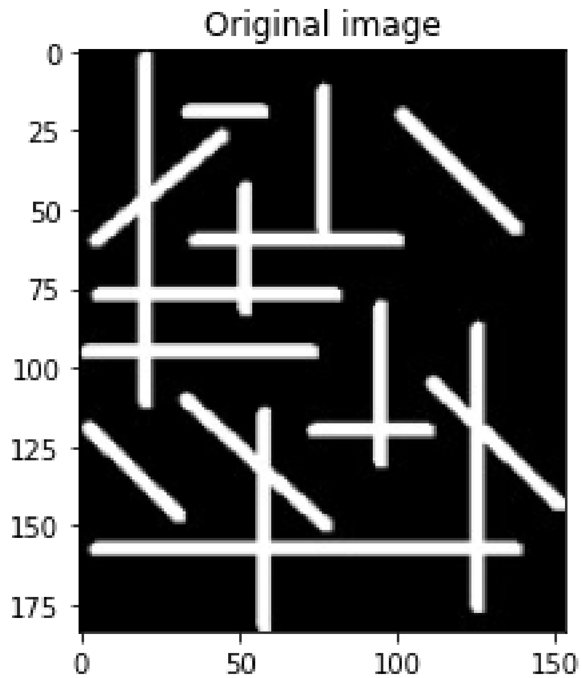
[85]:
```
selem
```

[85]:
```
array([[0, 0, 0, 0, 1, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 1, 1, 0, 0],
       [0, 1, 1, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 1, 1, 0],
       [1, 1, 1, 1, 1, 1, 1, 1, 1],
       [0, 1, 1, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 1, 0, 0, 0, 0]], dtype=uint8)
```

Problem 2 Part 2

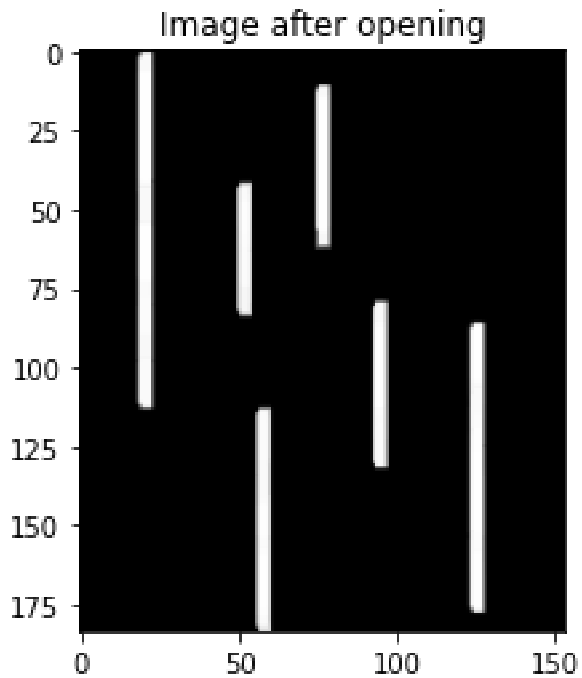
[28]:
```
lines = image.imread('lines.jpg')
lines_1_chan = np.mean(lines, axis = 2)
```

[29]:
```
plt.imshow(lines_1_chan, cmap = 'gray')
plt.title('Original image');
```



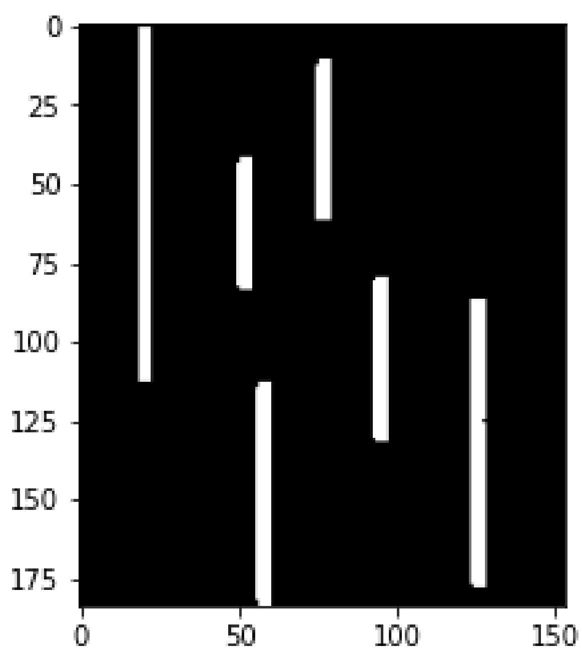
"Only the vertical lines should remain after the opening." - Piazza post @61

[43]:
```
selem_2 = skim_morph.rectangle(10,3)
bin_opened_2 = skim_morph.opening(lines_1_chan, selem_2)
plt.imshow(bin_opened_2, cmap = 'gray')
plt.title('Image after opening');
```



We threshold this.

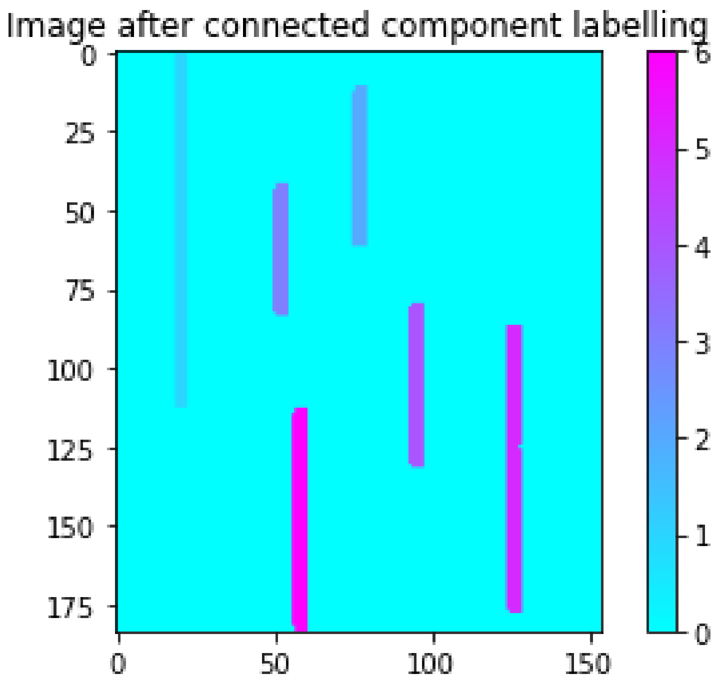
[42]:
```
thresh_temp = np.where(bin_opened_2>=100, 1, 0)
plt.imshow(thresh_temp, cmap = 'gray');
```



[44]:
```
strel = np.ones((3,3))
```

[45]:
```
thresholded_bin_labeled, num = scipy_im_measure.label(thresh_temp, strel)
```

[53]:
```
plt.imshow(thresholded_bin_labeled, cmap = 'cool')
plt.colorbar()
plt.title('Image after connected component labelling');
```



[70]:
```
line_centroid_list = []
line_length_list = []
for i in range(1,num+1):

    pos = np.asarray(np.where(thresholded_bin_labeled == i))
    centroid_pos = np.mean(pos, axis = 1)
    line_len = pos[0][-1] - pos[0][0]
    line_centroid_list.append(centroid_pos)
    line_length_list.append(line_len)
```

[77]:
```
line_index = [i+1 for i in range(6)]
df_index = pd.DataFrame(line_index, columns=['Line Index'])
df_line_centroid = pd.DataFrame(line_centroid_list, columns=['Row number',␣
 ↪'Column number'])
df2_line_centroid = pd.DataFrame(line_length_list, columns = ['Length of line'])
```

```
df3_line_centroid = pd.concat([df_index, df_line_centroid,df2_line_centroid],␣
 ↪axis = 1)
df3_line_centroid
```

[77]:

	Line Index	Row number	Column number	Length of line
0	1	56.500000	20.500000	111
1	2	36.193676	77.015810	50
2	3	62.594203	52.028986	41
3	4	105.500000	95.015504	51
4	5	131.916115	126.000000	90
5	6	148.000000	58.022792	70

The structuring element I used for opening in this problem was:

selem_2 = skim_morph.rectangle(10,3)

It is displayed in the code block below.

[86]:
```
selem_2
```

[86]:
```
array([[1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1],
       [1, 1, 1]], dtype=uint8)
```

0.0.3 Problem 3

(i) Write a function that takes as inputs a greyscale 8-bit (uint8) image, a scalar s [1, 7] and performs uniform quantization over the entire range [0, 255] so that the output is quantized to an s-bit image. You may use loops for this part if necessary

[123]:
```
def unif_quantizer(im, num_bits):

    def interval_extractor(bit):
        """returns a list of intervals given number of bits, assuming that each␣
 ↪pixel is represented by 8 bits"""
        smol = int(256/2**bit)
        list_intervals = []
        for i in range(1, 2**bit):
            list_intervals.append(smol*i)
        return(list_intervals)
```

```
    def whole_partition(partition):
        """prepends 0 and appends 256 to partition"""
        partition2 = np.insert(partition, 0, 0)
        partition2 = np.append(partition2, 256)
        return(partition2)

    def unif_codebook_generator(full_ints):
        """creates codebook from modified partition array given uniform␣
↪quantization"""
        unif_codebook = []
        for i in range(len(full_ints)-1):
            cb_element = (full_ints[i+1] + full_ints[i])//2 # to ensure this is␣
↪always an integer
            unif_codebook.append(cb_element)
        return(unif_codebook)

    partition = interval_extractor(num_bits)
    full_partition = whole_partition(partition)
    codebook = unif_codebook_generator(full_partition)
    house = np.zeros(im.shape)

    for i in range(len(codebook)):
        house = np.where((im>full_partition[i]) &␣
↪(im<=full_partition[i+1]),codebook[i],house)

    return(house)
```

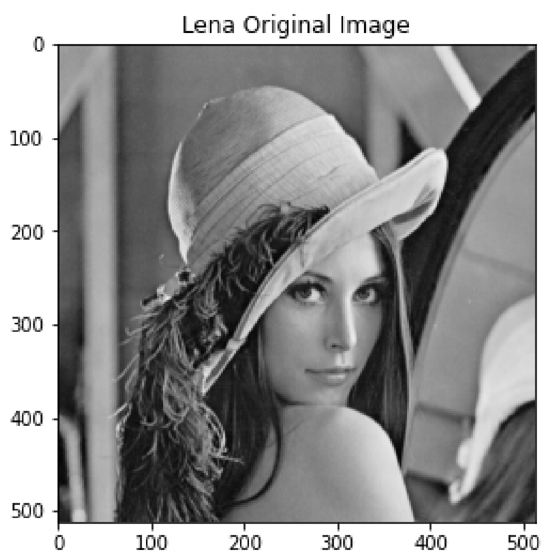
(ii) For the images lena512.tif and diver.tif, calculate the MSE values for s [1,7] using both your uniform quantizer and the Lloyd-Max quantizer (you may use loops for the Lloyd-Max quantizer as well). Plot the results (MSE versus number of bits). Show one plot for lena512.tif (with both uniform and Lloyd-Max quantization) and another plot for diver.tif. Compare the results for the different quantizers/images and explain them. That is, why does one quantizer outperform the other, and why is the performance gap larger for one image than for the other?

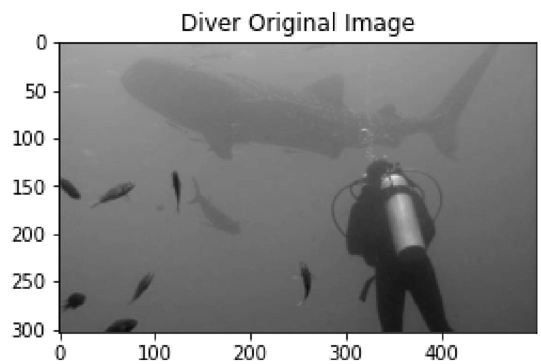
[124]:
```
def lloyd_max_quantizer(imej, s):

    [M, N] = imej.shape
    training_set = np.reshape(imej,(N*M,1))
    [partition, codebook] = lloyds(training_set, [2**(s)])

    def whole_partition(partition):
        partition2 = np.insert(partition, 0, 0)
        partition2 = np.append(partition2, 256)
        return(partition2)

    partition3 = whole_partition(partition)
```

```
    house = np.zeros(imej.shape)

    for i in range(len(codebook)):
        house = np.where((imej>partition3[i]) &
↪(imej<=partition3[i+1]),codebook[i],house)

    return(house)
```

[125]:
```
def MSE_calc(im1, im2):

    assert(im1.shape == im2.shape)

    num_pixels = im1.shape[0]*im1.shape[1]
    err = im1 - im2
    sq_err = err**2
    sum_sq_err = np.sum(sq_err)
    mean_sq_err = sum_sq_err/num_pixels

    return(mean_sq_err)
```

[131]:
```
lena = image.imread('lena512.tif')[:,:,0]
diver = image.imread('diver.tif')

fig = plt.figure(figsize = (10,7.5))
ax1 = fig.add_subplot(121)
ax1.imshow(lena, cmap = 'gray')
ax1.set_title('Lena Original Image')
ax2 = fig.add_subplot(122)
ax2.imshow(diver, cmap = 'gray')
ax2.set_title('Diver Original Image');
```

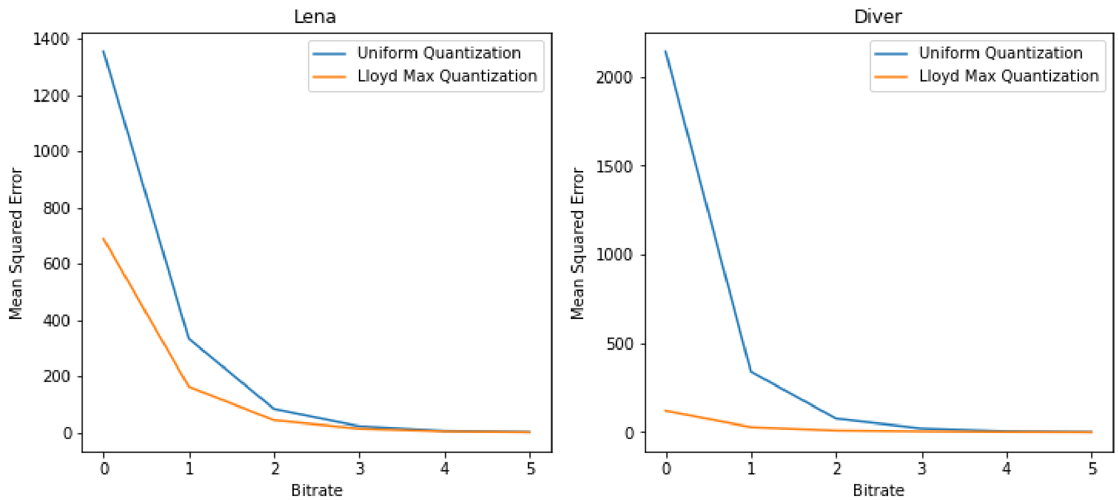




[127]:
```
unif_MSE_lena = []
lm_MSE_lena = []

for i in range(6):

    s = i + 1

    imej_sbit_unif = unif_quantizer(lena, s)
    imej_sbit_lm = lloyd_max_quantizer(lena, s)

    MSE_sbit_uniform = MSE_calc(lena, imej_sbit_unif)
    MSE_sbit_lm = MSE_calc(lena, imej_sbit_lm)


    unif_MSE_lena.append(MSE_sbit_uniform)
    lm_MSE_lena.append(MSE_sbit_lm)


unif_MSE_diver = []
lm_MSE_diver = []

for i in range(6):

    s = i + 1

    imej_sbit_unif = unif_quantizer(diver, s)
    imej_sbit_lm = lloyd_max_quantizer(diver, s)

    MSE_sbit_uniform = MSE_calc(diver, imej_sbit_unif)
    MSE_sbit_lm = MSE_calc(diver, imej_sbit_lm)


    unif_MSE_diver.append(MSE_sbit_uniform)
    lm_MSE_diver.append(MSE_sbit_lm)
```

[135]:
```
fig = plt.figure(figsize = [12,5])
ax1 = fig.add_subplot(121)
ax1.plot(unif_MSE_lena, label = 'Uniform Quantization')
ax1.plot(lm_MSE_lena, label = 'Lloyd Max Quantization')
ax1.legend()
ax1.set_title('Lena')
ax1.set_xlabel('Bitrate')
ax1.set_ylabel('Mean Squared Error')

ax2 = fig.add_subplot(122)
```

```
ax2.plot(unif_MSE_diver, label = 'Uniform Quantization')
ax2.plot(lm_MSE_diver, label = 'Lloyd Max Quantization')
ax2.legend()
ax2.set_title('Diver')
ax2.set_xlabel('Bitrate')
ax2.set_ylabel('Mean Squared Error');
```



Compare the results for the different quantizers/images and explain them. That is, why does one quantizer outperform the other, and why is the performance gap larger for one image than for the other?

1. For both images, the Lloyd Max Quantizer performs significantly better than the Uniform Quantizer.
2. The absolute difference in performance between the two decreases with increasing number of bits.
3. The performance gap for the Diver image is higher than the Lena image.

The lloyd max quantizer quantizes the image with the motive of reducing mean squared loss between the image to be quantized, and the quantized image. Hence, when we plot the mean squared error, we expect it to be less for images quantized using the Lloyd Max algorithm. Hence, the codebook and partition returned by the Lloyd max function is always better suited to minimize MSE. This is why the Lloyd Max quantizer performs better than the Uniform quantizer.

That is, the uniform quantization algorithm performs poorly on the diver image. We investigate by plotting all the uniformly quantized images against the lloyd max quantized images of the diver. For comparison, we will also plot the same for the lena image.

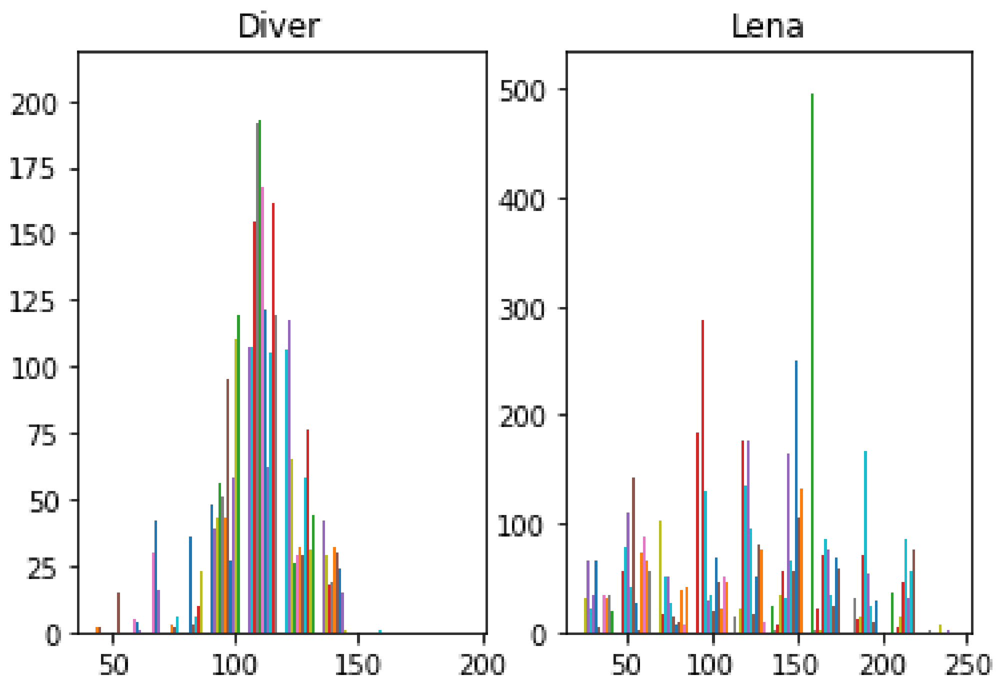
[97]:
```
fig, axs = plt.subplots(7,4, figsize = [20,25])
for i in range(7):
    s = i + 1
    imej_sbit_unif = unif_quantizer(diver, s)
```

```
    axs[i,0].imshow(imej_sbit_unif,cmap = 'gray')
    axs[i,0].set_title('Uniform Quantizer, {0} bit'.format(s))
    imej_sbit_lm = lloyd_max_quantizer(diver, s)
    axs[i,1].imshow(imej_sbit_lm, cmap = 'gray')
    axs[i,1].set_title('Lloyd Max Quantizer, {0} bit'.format(s))

    imej_sbit_unif_lena = unif_quantizer(lena, s)
    axs[i,2].imshow(imej_sbit_unif_lena,cmap = 'gray')
    axs[i,2].set_title('Uniform Quantizer, {0} bit'.format(s))
    imej_sbit_lm_lena = lloyd_max_quantizer(lena, s)
    axs[i,3].imshow(imej_sbit_lm_lena, cmap = 'gray')
    axs[i,3].set_title('Lloyd Max Quantizer, {0} bit'.format(s))
```

Further, we also plot the histograms for both images.

[98]:
```
fig = plt.figure()
ax1 = fig.add_subplot(121)
ax1.hist(diver)
```

```
ax1.set_title('Diver')
ax2 = fig.add_subplot(122)
ax2.hist(lena)
ax2.set_title('Lena');
```



By observing the histograms of the two images, we can see that the diver histogram is less 'equalized' than lena.

Upon observing the quantization of the diver image, we see that the uniform quantizer performs badly, whereas this is not the case for lena.

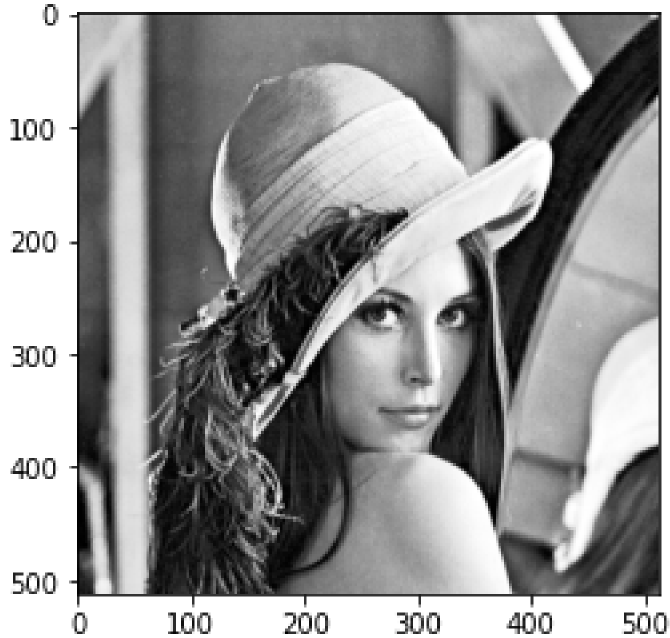
This is because lena's histogram is more equalized than diver. The uniform equalizer has hard coded partitions and codebooks, so if large parts of the image lie below or above a certain threshold, it will perform badly.

On the other hand, the Lloyd Max equalizer's partitions and codebook vary from image to image, and since it is built with the express purpose of minimizing mean squared loss, it sets the partitions and codebooks such that mean squared loss is minimized for every level of quantization.

(iii) Now use global histogram equalization on lena512.tif and on diver.tif to generate two new images, for example:

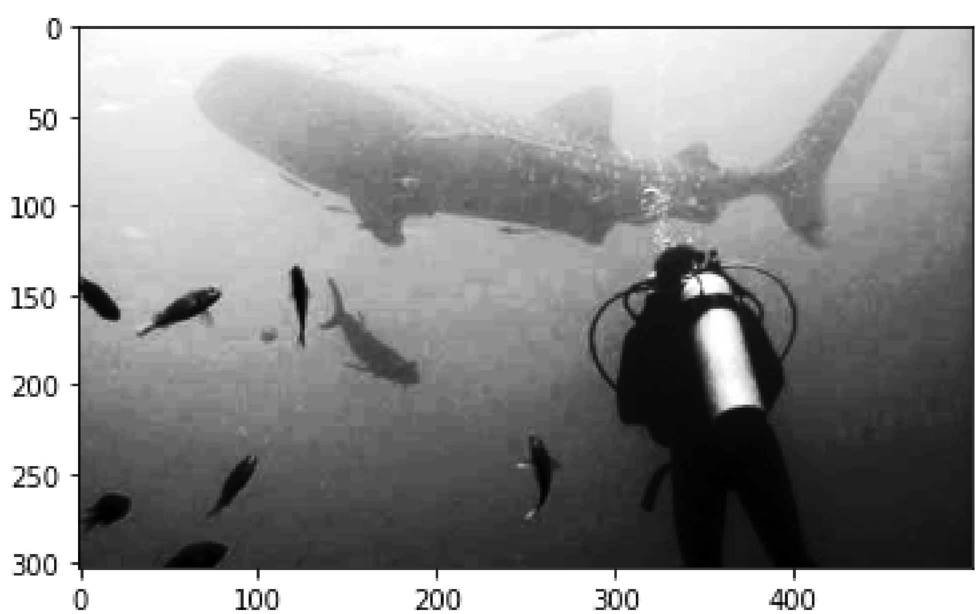
[100]:
```
he_lena = cv2.equalizeHist(lena)
plt.imshow(he_lena, cmap = 'gray')
```

[100]: <matplotlib.image.AxesImage at 0x7fd420e70eb8>




```
[101]: he_diver = cv2.equalizeHist(diver)
       plt.imshow(he_diver, cmap = 'gray')
```

[101]: <matplotlib.image.AxesImage at 0x7fd42046b630>

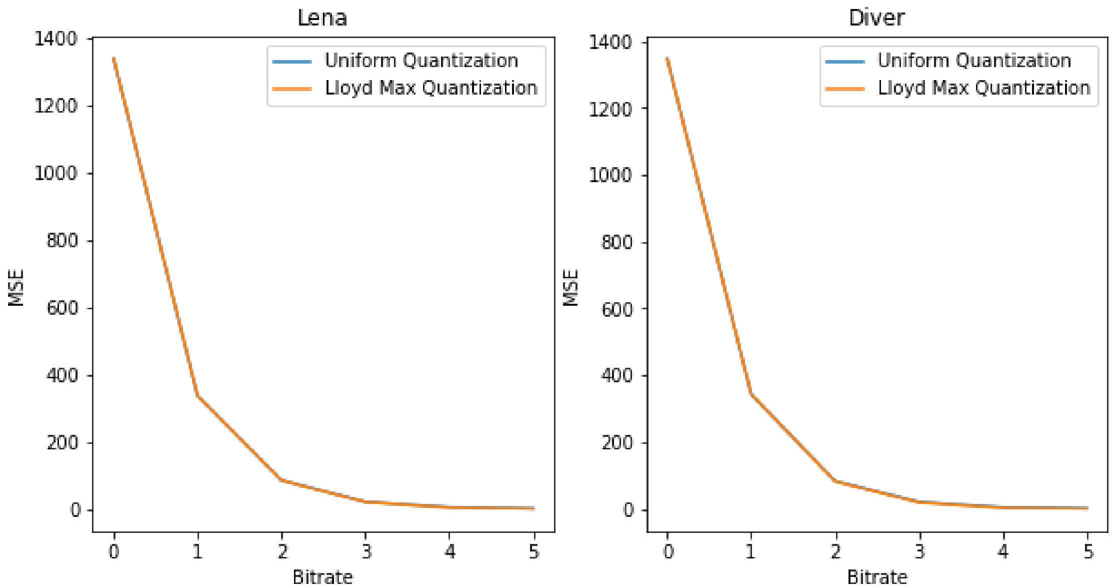


Repeat part (ii) for these two new images. Compare them with the previous set of plots. What has happened to the gap in MSE between the two quantization approaches and why?

[102]:
```
unif_MSE_he_lena = []
lm_MSE_he_lena = []

for i in range(6):

    s = i + 1

    imej_sbit_unif = unif_quantizer(he_lena, s)
    imej_sbit_lm = lloyd_max_quantizer(he_lena, s)

    MSE_sbit_uniform = MSE_calc(he_lena, imej_sbit_unif)
    MSE_sbit_lm = MSE_calc(he_lena, imej_sbit_lm)


    unif_MSE_he_lena.append(MSE_sbit_uniform)
    lm_MSE_he_lena.append(MSE_sbit_lm)


unif_MSE_he_diver = []
lm_MSE_he_diver = []

for i in range(6):

    s = i + 1

    imej_sbit_unif = unif_quantizer(he_diver, s)
    imej_sbit_lm = lloyd_max_quantizer(he_diver, s)

    MSE_sbit_uniform = MSE_calc(he_diver, imej_sbit_unif)
    MSE_sbit_lm = MSE_calc(he_diver, imej_sbit_lm)


    unif_MSE_he_diver.append(MSE_sbit_uniform)
    lm_MSE_he_diver.append(MSE_sbit_lm)
```

[103]:
```
fig = plt.figure(figsize = [10,5])
ax = fig.add_subplot(121)
ax.plot(unif_MSE_he_lena, label = 'Uniform Quantization')
ax.plot(lm_MSE_he_lena, label = 'Lloyd Max Quantization')
ax.legend()
ax.set_title('Lena')
ax.set_ylabel('MSE')
ax.set_xlabel('Bitrate')
ax2 = fig.add_subplot(122)
ax2.plot(unif_MSE_he_diver, label = 'Uniform Quantization')
```

```
ax2.plot(lm_MSE_he_diver, label = 'Lloyd Max Quantization')
ax2.legend()
ax2.set_title('Diver')
ax2.set_ylabel('MSE')
ax2.set_xlabel('Bitrate');
```



The gap in the MSE of the Uniform Quantizer and the Lloyd Max Quantizer has reduced to almost 0.
Reason:
The optimal partition levels returned by the Lloyd Max quantizer came out to be the same as that of the uniform quantizer, since histogram equalization by definition changes the probability mass function of the image, and Lloyd Max quantization intervals are a function of the pmf.

In this case, histogram equalization made sure that the pmf is an even function, and more importantly, almost a constant function.

Assuming that it was constant, we can actually solve the integration in Eq 8-56 of the Gonzalez book and prove that under this assumption, the Lloyd Max quantizer will give us the following equation:

$$t_i = (s_{i-1} + s_i)/2$$

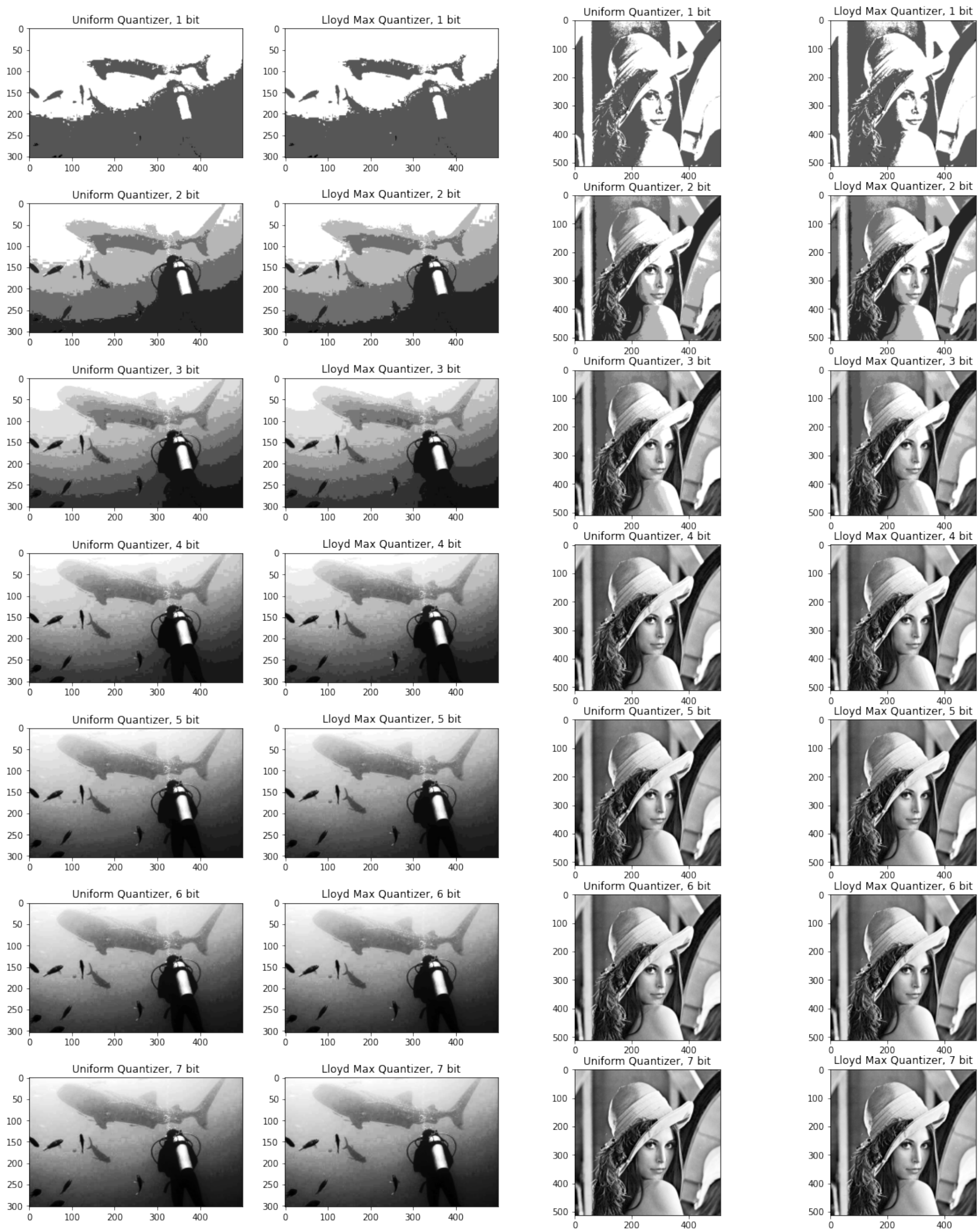
Here t_i is the reconstruction value and s_i-1 to s_i defines the interval which we assign to t_i.

This is quite similar to how we calculate the threshold values in the uniform quantizer: we assign values that lie in a certain interval to their average.

If my understanding is correct, this is why the MSE values are the same for both uniform quantization and lloyd max quantization.

[104]:

```
fig, axs = plt.subplots(7,4, figsize = [20,25])
for i in range(7):
    s = i + 1
    imej_sbit_unif = unif_quantizer(he_diver, s)
    axs[i,0].imshow(imej_sbit_unif,cmap = 'gray')
    axs[i,0].set_title('Uniform Quantizer, {0} bit'.format(s))
    imej_sbit_lm = lloyd_max_quantizer(he_diver, s)
    axs[i,1].imshow(imej_sbit_lm, cmap = 'gray')
    axs[i,1].set_title('Lloyd Max Quantizer, {0} bit'.format(s))

    imej_sbit_unif_lena = unif_quantizer(he_lena, s)
    axs[i,2].imshow(imej_sbit_unif_lena,cmap = 'gray')
    axs[i,2].set_title('Uniform Quantizer, {0} bit'.format(s))
    imej_sbit_lm_lena = lloyd_max_quantizer(he_lena, s)
    axs[i,3].imshow(imej_sbit_lm_lena, cmap = 'gray')
    axs[i,3].set_title('Lloyd Max Quantizer, {0} bit'.format(s))
```



(iv) Why is the MSE of the 7-bit Lloyd-Max quantizer zero or near zero for the equalized images? One might have thought that equalization is not to the advantage of the Lloyd-Max quantizer, because equalizing the histogram should be flattening the distribution, making it more uniform, which should be to the advantage of the uniform quantizer. Explain this phenomenon. The MSE of the 7-bit Lloyd-Max quantizer approaches 0 in the equalized images.

This is because 7 bit is very close to the actual number of bits used to encode the image (8). And given optimum quantization, which the Lloyd-Max quantizer performs, the last bit does not contribute much to the mean squared error.

The Lloyd Max quantizer always has the advantage when minimizing mean squared error. The uniform quantizer performs well here because of our assumption of an almost flat pmf. However, this might not be the case for both these images.

The Lloyd Max quantizer will adjust its intervals such that the error is always smaller than the error given by the uniform quantizer.