Extensive Design Calculations for Business Jet

Aerospace Engineer

1 Fuselage Sizing

1.1 Fuselage Length and Diameter

- **Fuselage Length,** L_f: 107.5 ft
- **Fuselage Diameter,** D_f: 10 ft
- **Cabin Length,** L_c: 60 ft
- **Cabin Height,** H_c: Approximately 6.1 ft (provides ample space for passenger comfort)

1.2 Cross-Sectional Area

The cross-sectional area of the fuselage can be calculated using:

$$A_f = \frac{\pi}{4} D_f^2$$

$$A_f = \frac{\pi}{4} \times (10)^2 = 78.54 \, \text{sq ft}$$

2 Wing Geometry and Sizing

2.1 Given Parameters

- **Wing Span,** b: 93.5 ft
- **Wing Area,** S: 950 sqft
- **Aspect Ratio,** AR: 9.2
- **Taper Ratio,** λ: 0.4

2.2 Chord Lengths

Root Chord (c_r) and **Tip Chord** (c_t) can be calculated using the taper ratio:

$$c_r = \frac{2S}{b(1+\lambda)}$$

$$c_r = \frac{2 \times 950}{93.5 \times (1 + 0.4)} = 14.53\,\text{ft}$$

$$c_t = \lambda \times c_r = 0.4 \times 14.53 = 5.81\,\text{ft}$$

2.3 Mean Aerodynamic Chord (MAC)

$$MAC = \frac{2}{3} c_r \cdot \frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

$$MAC = \frac{2}{3} \times 14.53 \times \frac{1 + 0.4 + 0.4^2}{1 + 0.4} = 9.7\,\text{ft}$$

2.4 Wing Sweep Angle

- **Sweep Angle,** Λ: 22° at the quarter chord, chosen for drag reduction at high subsonic speeds.

2.5 Wing Loading

$$W/S = \frac{MTOW}{S} = \frac{45000}{950} = 47.37\,\text{lb/sq ft}$$

3 Tail Sizing

3.1 Horizontal Tail

- **Tail Volume Coefficient,** V_h: 1.0

$$V_h = \frac{S_h \times l_h}{S \times MAC}$$

Where:

- l_h = Distance from wing aerodynamic center to tail = 50 ft

$$S_h = \frac{1.0 \times 950 \times 9.7}{50} = 184.3\,\text{sq ft}$$

3.2 Vertical Tail

- **Vertical Tail Volume Coefficient,** V_v: 0.06

$$V_v = \frac{S_v \times l_v}{S \times b}$$

Where:

- $l_v = 52\,\text{ft}$

$$S_v = \frac{0.06 \times 950 \times 93.5}{52} = 102.7\,\text{sq ft}$$

4 Weight Calculations

4.1 Maximum Takeoff Weight (MTOW)

$$MTOW = 45,000\,\text{lbs}$$

4.2 Weight Breakdown

- **Operating Empty Weight (OEW)**: Estimated as 55% of MTOW

$$OEW = 0.55 \times 45,000 = 24,750\,\text{lbs}$$

- **Fuel Weight**: Approximately 40% of MTOW

$$W_f = 0.40 \times 45,000 = 18,000\,\text{lbs}$$

- **Payload (Passengers and Baggage)**: 4.500 lbs

5 Mission Profile Analysis

5.1 Takeoff and Climb

- **Takeoff Speed,** V_{TO}: 160 knots
- **Climb Gradient**: 2500 ft/min to 35.000 ft
- **Fuel Burned During Climb**: 5% of total fuel weight

$$0.05 \times 18,000 = 900\,\text{lbs}$$

5.2 Cruise

- **Cruise Speed**: Mach 0.88 (510 knots or 945 km/h)
- **Cruise Altitude**: 35.000 ft

5.2.1 Range Calculation Using Breguet Equation

$$R = \frac{V}{SFC} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_i}{W_f}\right)$$

Where:

- $V = 945\,\text{km/h}$
- $SFC = 0.55\,\text{lb/lb/hr}$
- $L/D = 17$
- $W_i = 45,000\,\text{lbs}$
- $W_f = 27,000\,\text{lbs}$

$$R = \frac{945}{0.55} \times 17 \times \ln\left(\frac{45,000}{27,000}\right)$$
$$R \approx 1718.18 \times 17 \times 0.5108 \approx 14,939\,\text{km}$$

5.3 Descent and Landing

- **Descent Rate**: $1800\,\text{ft/min}$ from $35.000\,\text{ft}$
- **Fuel Burned During Descent**: 3% of total fuel

$$0.03 \times 18,000 = 540\,\text{lbs}$$

- **Approach Speed**: $150\,\text{knots}$

6 Fuel Capacity and Efficiency

6.1 Total Fuel Capacity

$$\text{Total Fuel Capacity} = W_f = 18,000\,\text{lbs}$$

6.2 Specific Fuel Consumption (SFC)

- **Cruise SFC**: $0.55\,\text{lb/lb/hr}$

7 Aerodynamic Performance

7.1 Lift-to-Drag Ratio (L/D)

- **Maximum L/D Ratio during Cruise**: 17

8 Stability and Control Analysis

8.1 Longitudinal Stability

- **Center of Gravity (CG) Location**: Positioned at 25-30% of the MAC to ensure a stable pitch moment.
- **Horizontal Tail Contribution**: The horizontal tail surface area ($S_h =$ 184.3 sq ft) provides sufficient pitch control authority to ensure stability across all phases of flight, including takeoff, cruise, and landing.
- **Elevator Authority**: Elevator deflections will be sufficient to manage changes in CG during passenger load variation and fuel burn.

8.2 Lateral and Directional Stability

- **Vertical Tail Contribution**: The vertical tail, with an area of $S_v =$ 102.7 sq ft, ensures adequate yaw stability and prevents adverse yaw effects during flight disturbances.
- **Directional Stability**: The swept vertical tail provides an aerodynamic restoring force in case of sideslip, ensuring the aircraft naturally returns to straight flight after perturbations.

8.3 Control Surface Sizing

- **Ailerons**: Sized at 15% of the wingspan to provide efficient roll control. Located at the outboard sections of the wings for optimal roll moment generation.
- **Elevators and Rudder**:
  - **Elevators**: Positioned on the trailing edge of the horizontal stabilizer. Adequate sizing ensures effective pitch control at all speeds.
  - **Rudder**: Mounted on the vertical stabilizer, the rudder provides sufficient yaw control, especially during crosswind conditions or during an engine-out scenario.

9 Engine Selection and Performance

9.1 Engine Model

- **Engine Type**: GE Passport turbofan engines
- **Thrust Rating**: 18.000 lbs each
- **Total Thrust**: 36.000 lbs, providing a high thrust-to-weight ratio for efficient climb and short takeoff distance.

9.2 Engine Placement

- **Rear-Mounted Engines**: The engines are mounted on the rear fuselage, which reduces cabin noise and allows for a cleaner wing design by eliminating interference drag.
- **Thrust-to-Weight Ratio**:

$$\frac{T}{W} = \frac{36,000}{45,000} = 0.8$$

  This thrust-to-weight ratio ensures a good balance between takeoff performance and efficient cruise.

10 Fuel Capacity and Usage

10.1 Total Fuel Capacity

- **Fuel Weight**: 18.000 lbs, which is approximately 40% of the MTOW. This capacity supports the long-range capability of the aircraft.

10.2 Fuel Efficiency

- **Specific Fuel Consumption (SFC)**: Assumed to be 0.55 lb/lb/hr for cruise conditions, providing optimal fuel usage for long-distance travel.
- **Fuel Consumption During Mission Phases**:
  - **Climb Phase**: 5% of total fuel weight

$$0.05 \times 18,000 = 900\,\text{lbs}$$

  - **Cruise Phase**: Majority of the fuel is consumed during cruise, considering the long-range mission of 14,000 km.
  - **Descent Phase**: 3% of total fuel weight

$$0.03 \times 18,000 = 540\,\text{lbs}$$

11 Aerodynamic Performance

11.1 Lift-to-Drag Ratio (L/D)

- **Maximum L/D Ratio during Cruise**: 17, achieved through optimized wing design, appropriate sweep angle, and advanced airfoil selection.

11.2 Drag Polar Equation

- The drag polar is given by:

$$C_D = C_{D_0} + KC_L^2$$

Where:

  - C_{D_0}: Parasite drag coefficient, estimated to be 0.018 for a clean configuration.
  - K: Induced drag factor, calculated as

$$K = \frac{1}{\pi e AR}$$

  Assuming an Oswald efficiency factor $e = 0.85$,

$$K = \frac{1}{\pi \times 0.85 \times 9.2} \approx 0.0408$$

12 Mission Profile Summary

12.1 Takeoff

- **Takeoff Distance,** S_{TO}:

$$S_{TO} = \frac{W_{TO}^2}{g \cdot T \cdot \sigma \cdot C_{L_{max}}} \cdot f$$

Where:

  - $W_{TO} = 45.000\,\text{lbs}$ (Maximum Takeoff Weight)
  - $T = 36.000\,\text{lbs}$ (Total Thrust)
  - $\sigma = 1$ (Air density ratio at sea level)
  - $C_{L_{max}} = 2.2$ (Maximum lift coefficient during takeoff)
  - $g = 32.2\,\text{ft/s}^2$ (Acceleration due to gravity)
  - $f = 1.2$ (Empirical correction factor for business jets)

$$S_{TO} \approx \frac{(45,000)^2}{32.2 \times 36,000 \times 1 \times 2.2} \times 1.2 \approx 4,330\,\text{feet}$$

12.2 Cruise Range Calculation

Using the Breguet range equation:

$$R = \frac{V}{SFC} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_i}{W_f}\right)$$

Where:

- $V = 945\,\text{km/h}$
- $SFC = 0.55\,\text{lb/lb/hr}$
- $L/D = 17$
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- $W_f = 27,000\,\text{lbs}$

$$R \approx 1718.18 \times 17 \times 0.5108 \approx 14,939\,\text{km}$$

12.3 Descent and Landing

- **Descent Rate**: $1800\,\text{ft/min}$
- **Fuel Burn During Descent**: 3% of total fuel weight

$$540\,\text{lbs}$$

- **Approach Speed**: $150\,\text{knots}$

13 Conclusion

The detailed analysis and extensive calculations for the business jet design demonstrate the feasibility of achieving a range of 14.000 km with optimal aerodynamic performance and fuel efficiency. Key features such as rear-mounted engines, optimized wing geometry, and efficient weight distribution contribute to the overall success of the design.