ARSIG TP : Discrete transformation in Wavelets

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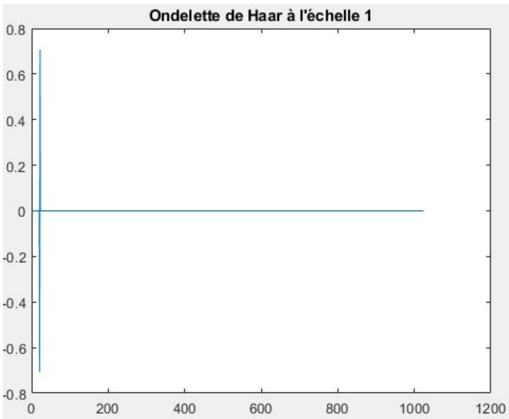
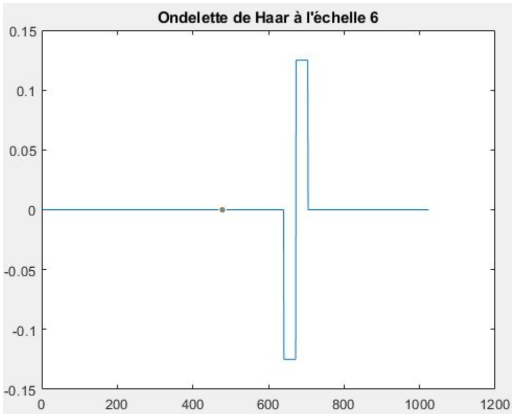
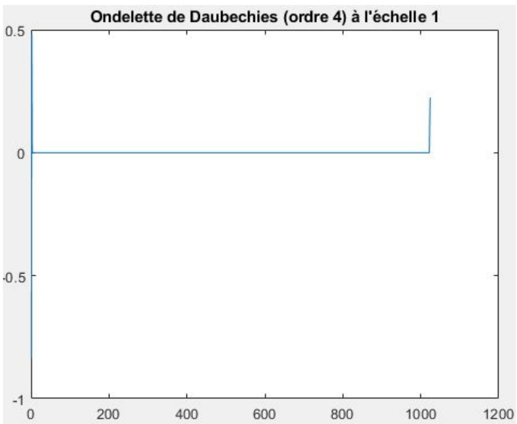
1 Reverse DWT wavelet and scale function plotting

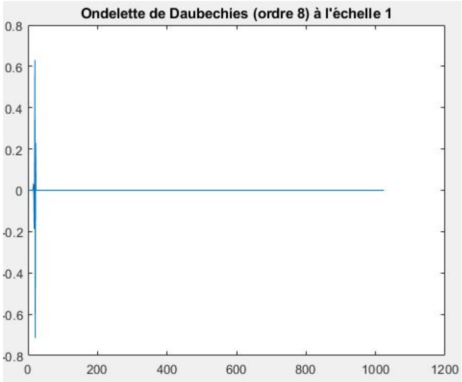
$d_j[k]$ corresponds to the DWT of x to the index $\frac{N}{2^j} + K + 1$ (if we start the indices at 1).

If we want to plot the Haar wavelet at scale 1, we therefore do the inverse DWT transform of a DWT with only the first coefficient being 1, the others being 0.

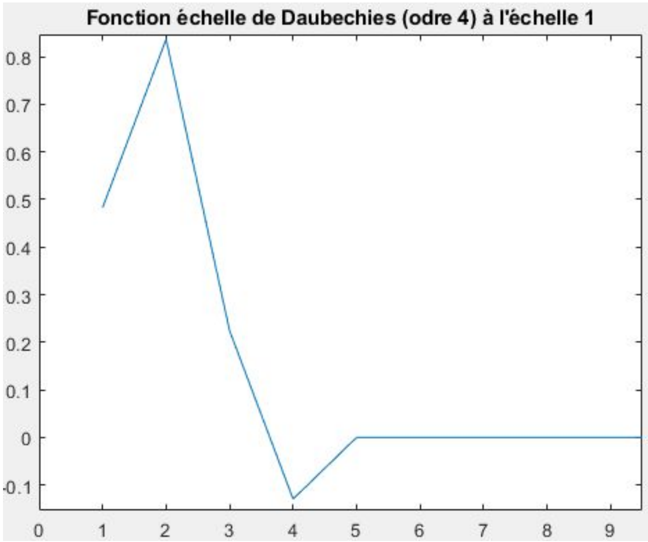


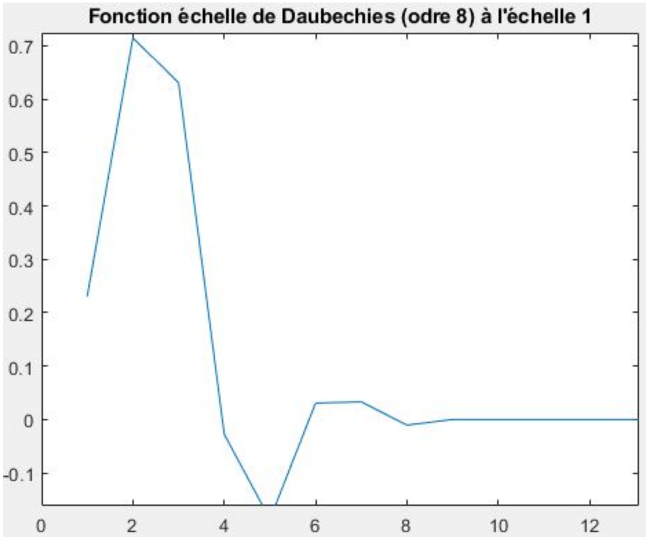


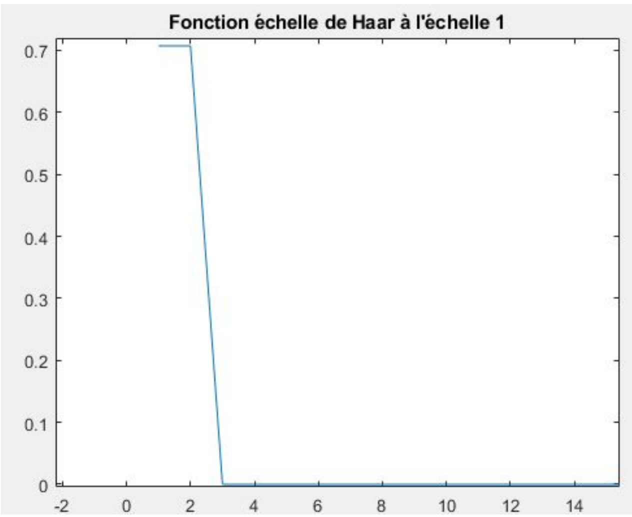




(There are 1024 coefficients)



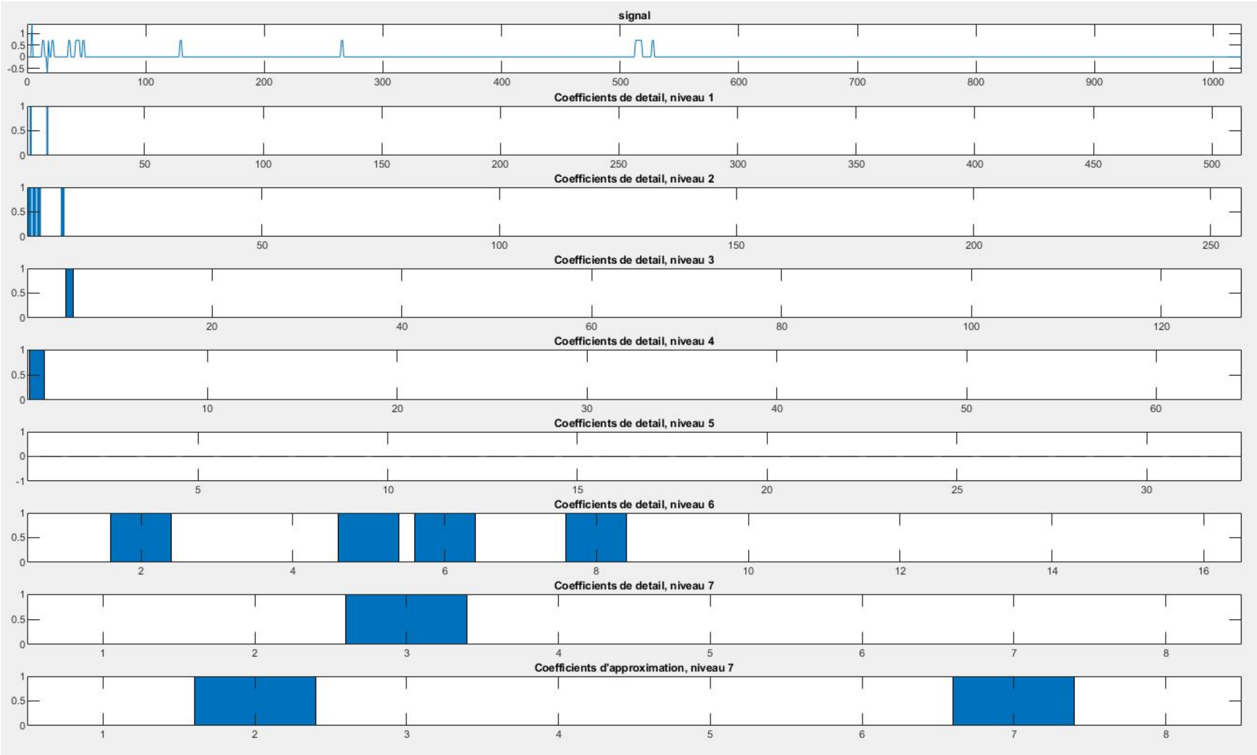




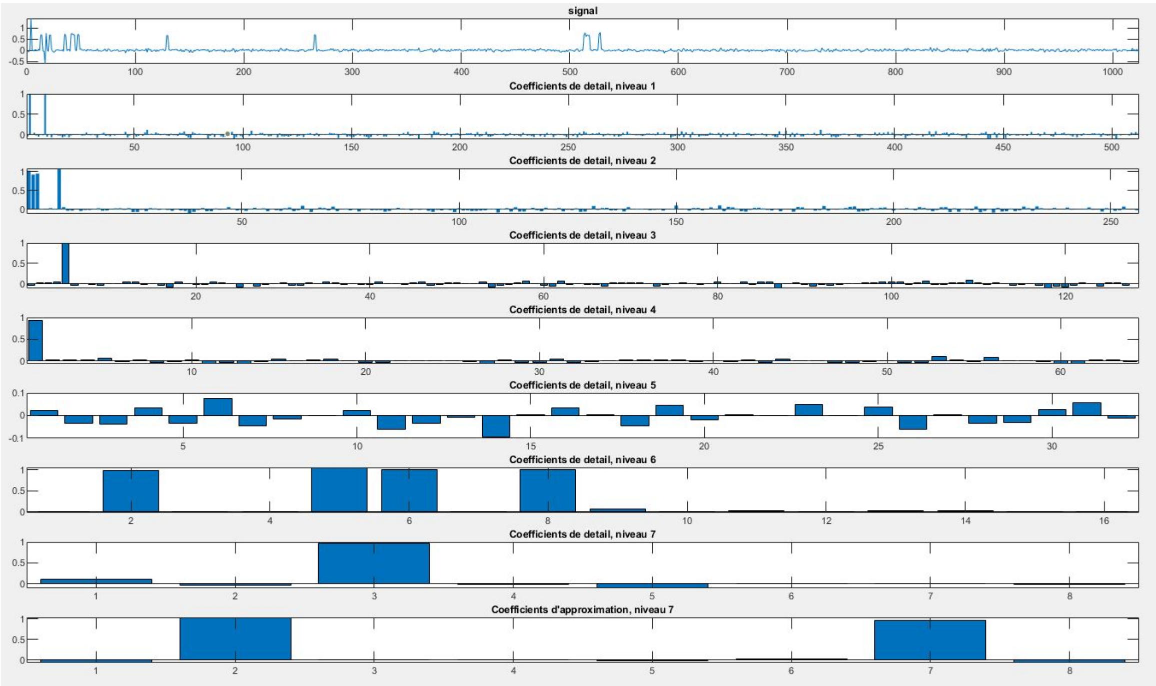
Increasing the scale corresponds to zooming in on the function. Daubechies wavelets are more complex than Haar's but are chosen to have good properties.

2 Wavelet spatial denoising

In this part, we consider transforms with the Haar wavelet.

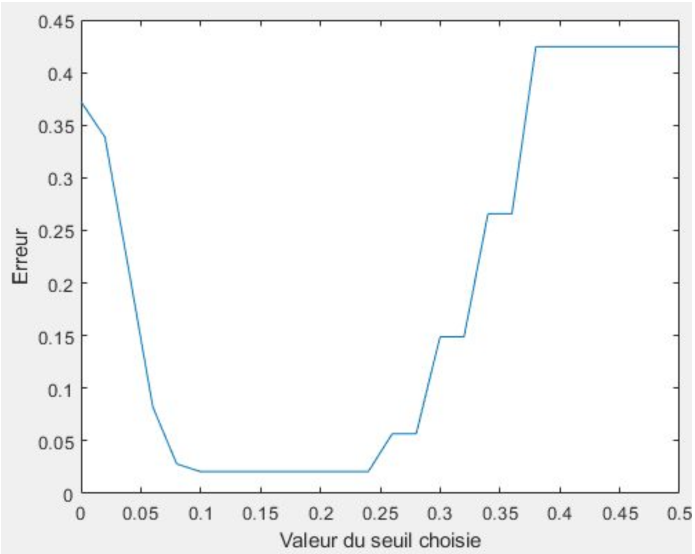


Here, a signal was generated with coefficients randomly positioned in its DWT.
Let's now add noise to this signal.



The signal appears slightly disturbed. Small coefficients appear at all levels of the DWT.

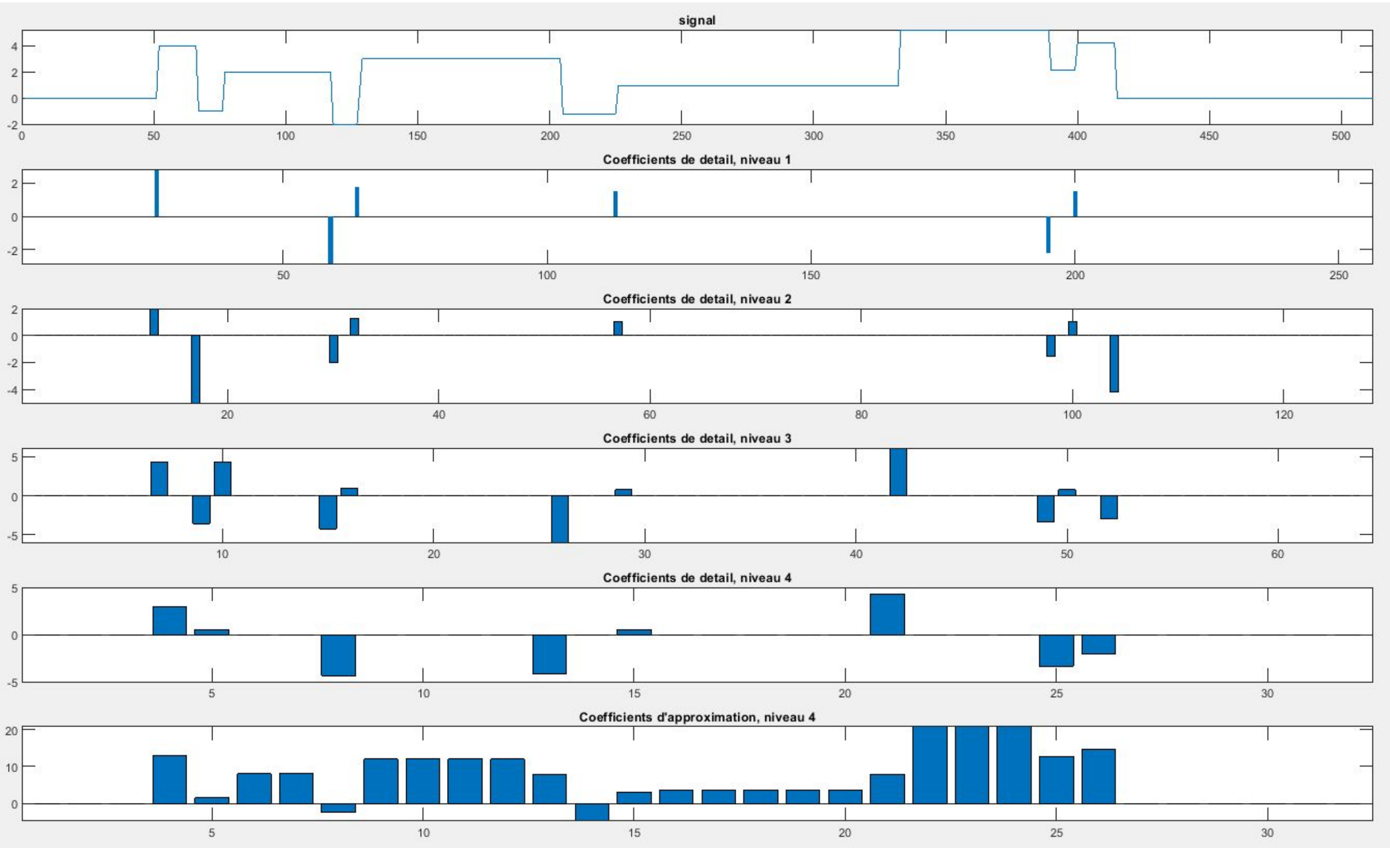
If we apply a threshold of 0.15 to this DWT, we find almost the DWT and the original signal.



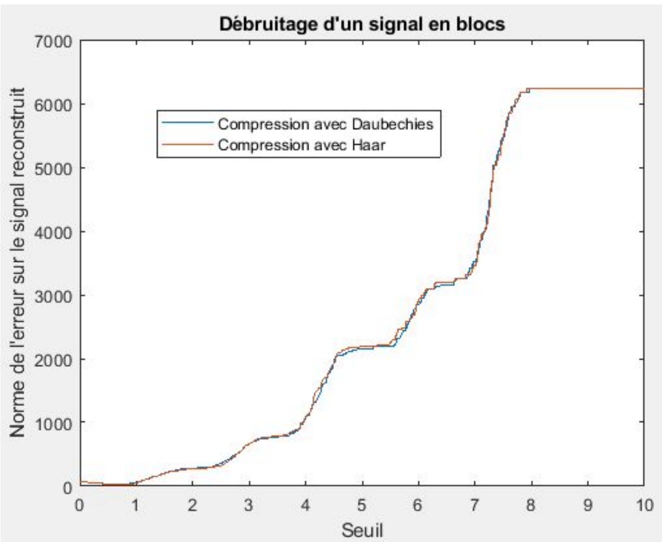
(Error in standard 2 of the difference between the reconstructed signals).

Application to more realistic signals

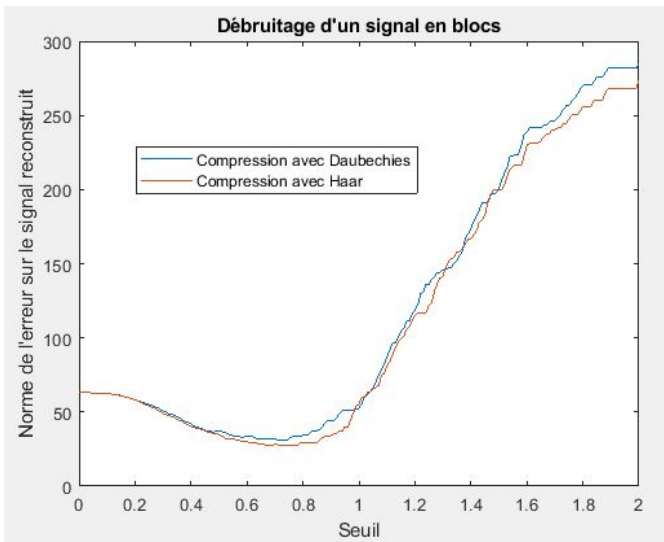
Signal in blocks :

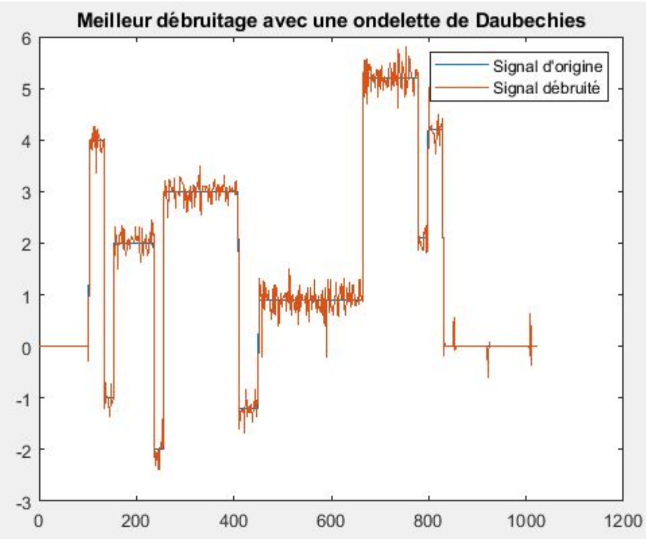


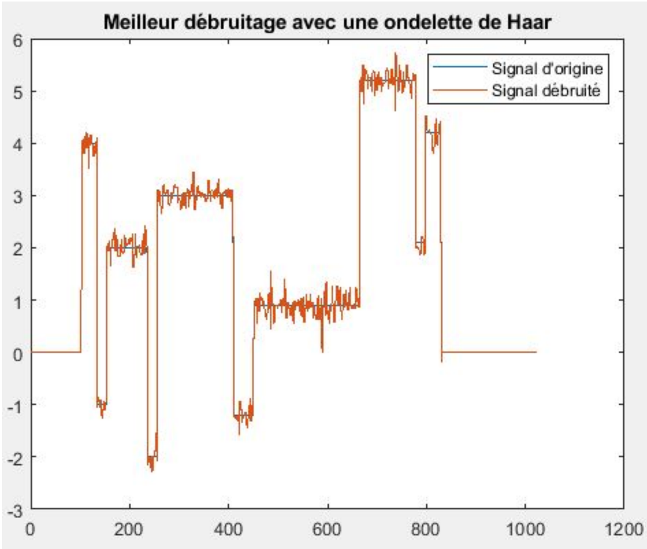
Haar wavelets are ideal for block signals.
Depending on the desired threshold, Haar or Daubechian wavelets are more suitable for signal compression.
It is important to note that Haar is the most effective in minimizing the error.



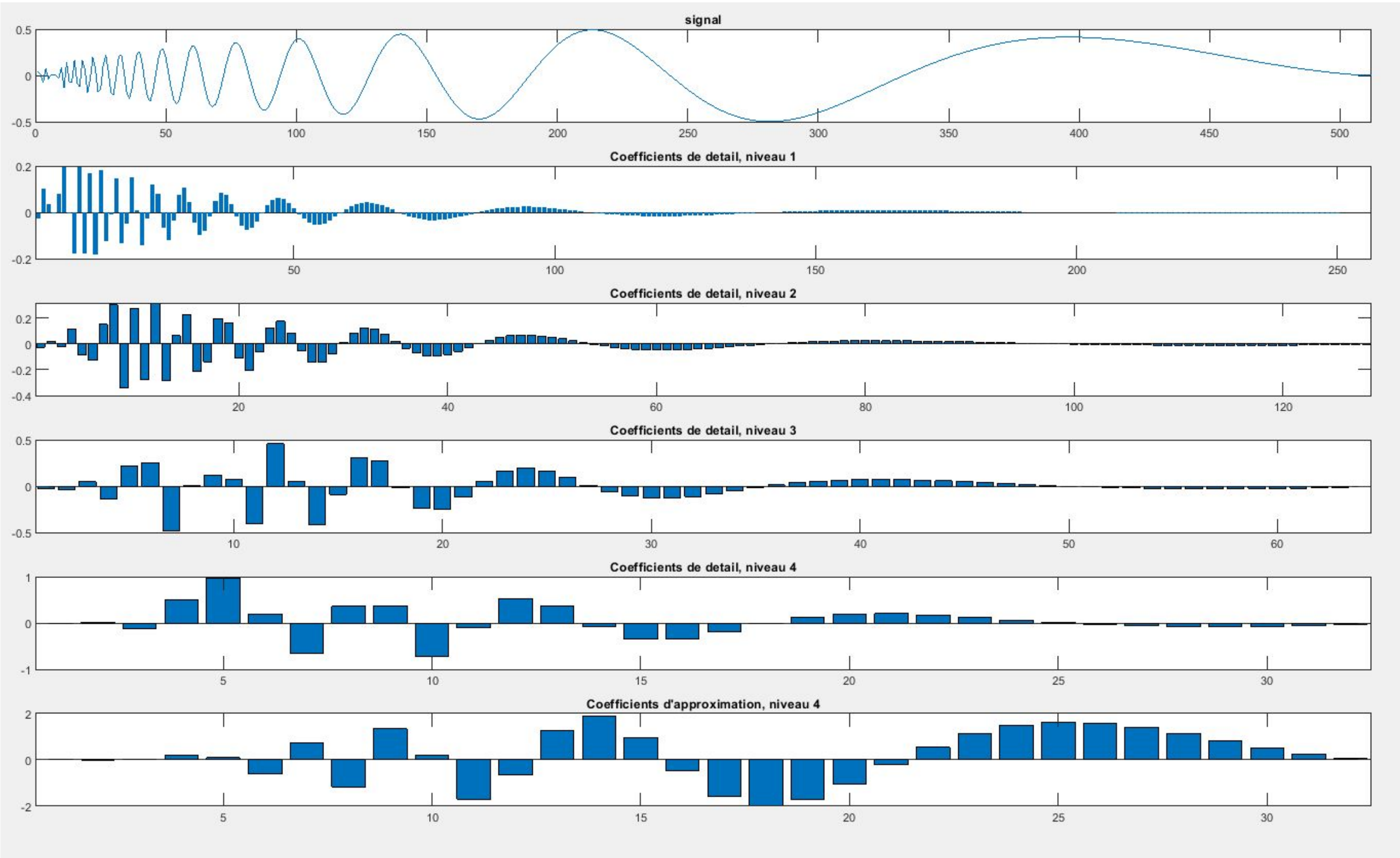
Zoom



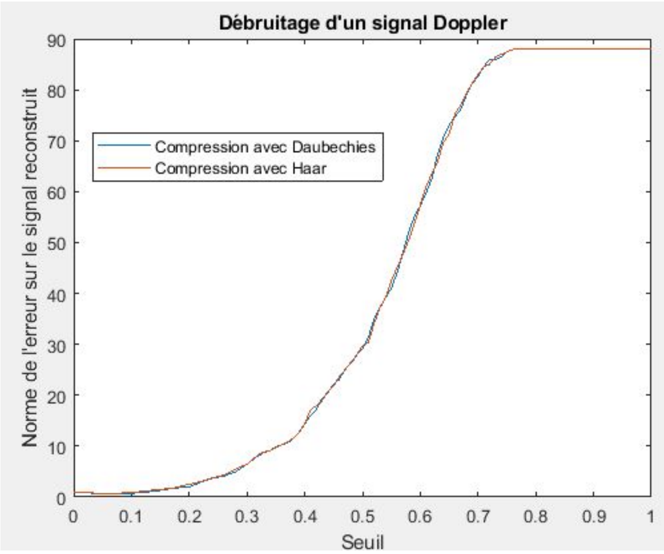




Signal Doppler :



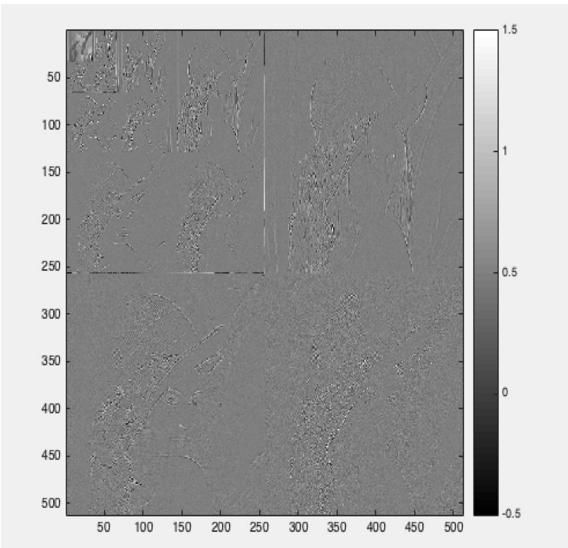
More coefficients are needed at smaller scales. Their orders of magnitude are also less important than for the block signal.



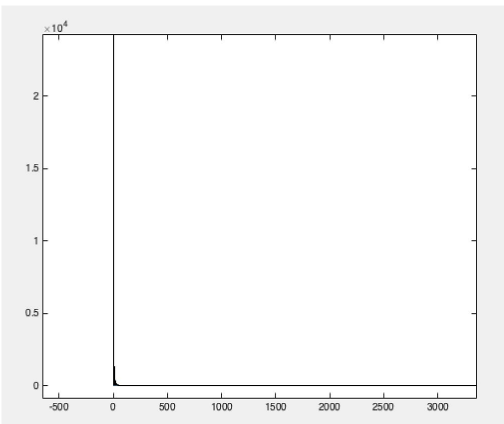
The difference is minimal.

There is relatively little difference between the two bases on these two examples

3 Image compression

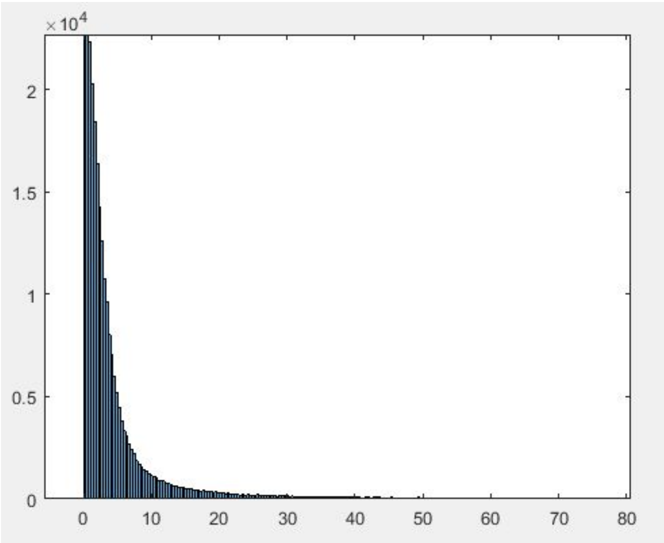


DWT of the image in order 4. The image of Lena can still be seen in the approximation of the top left corner.



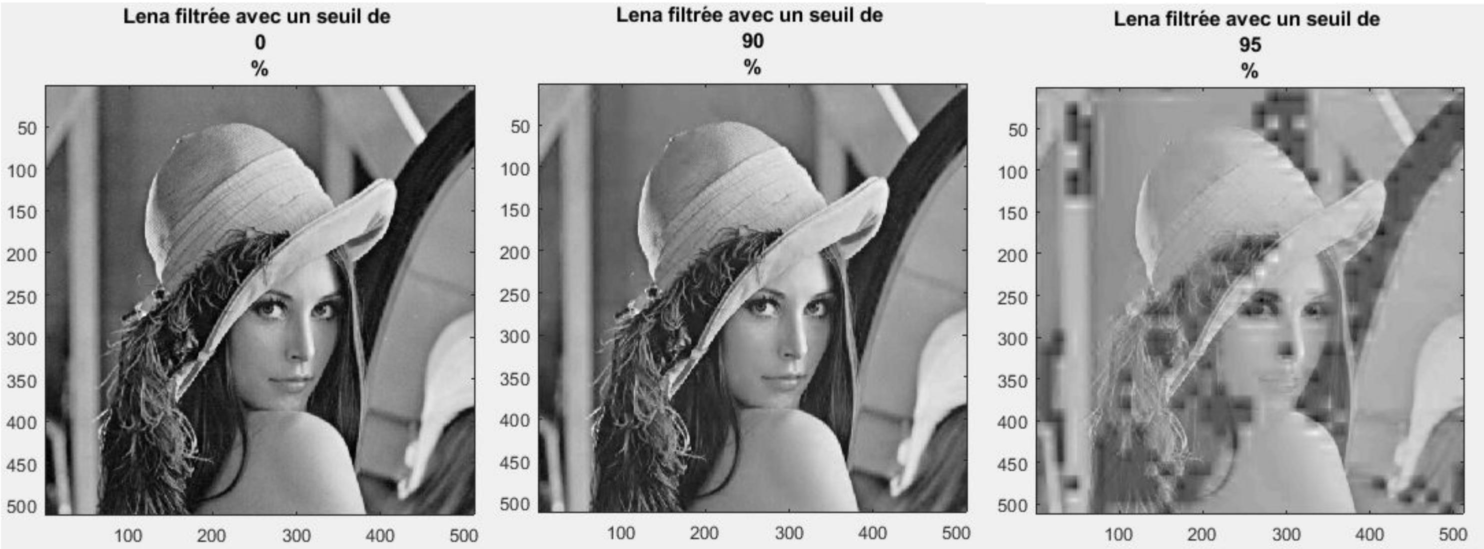
Histogram of coefficient values. The vast majority have low values (between 0 and 50). Some of them have a very high value (>3000) and are

therefore very important, we will only use the DWT with the largest coefficients.
Let's see the histogram in more detail on the zone [0,50].

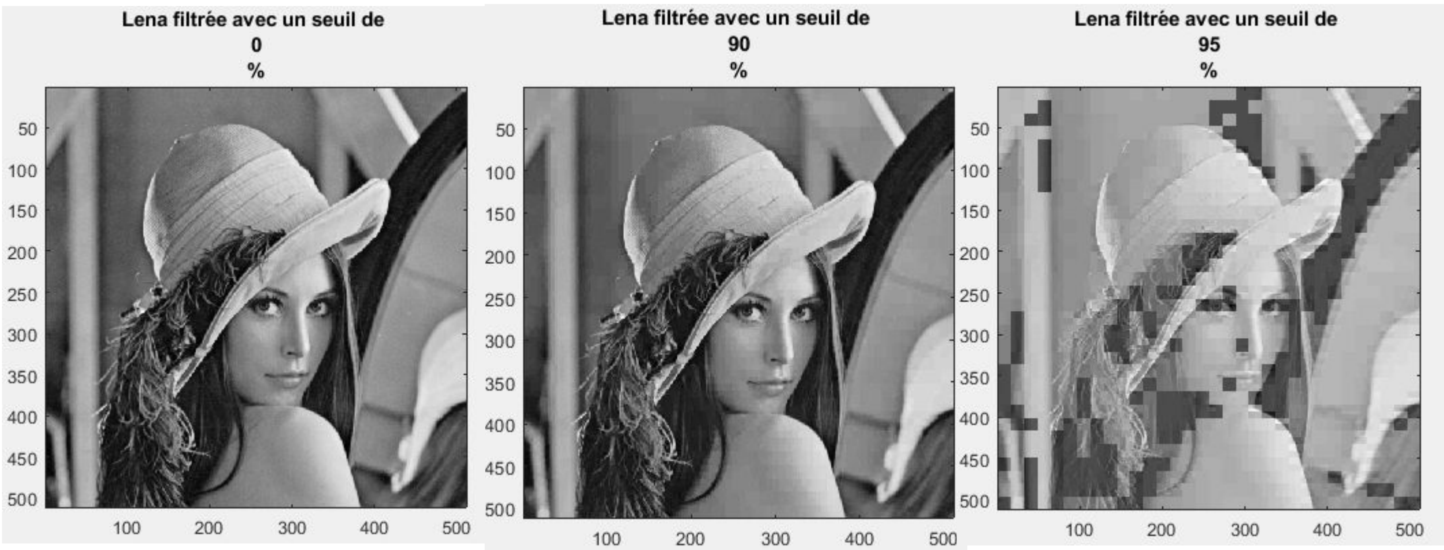


It is reasonable to set the threshold around 15 to have few coefficients but always a correct level of detail. This can be compared with a threshold of 5 and a threshold of 40.

With Daubechies wavelets



With Haar's wavelets



Threshold (%)	0	90	95
Error (standard 2) with Daubechies	0	405	$9{,}8.10^4$
Error (standard 2) with Haar	0	335	$9{,}1.10^4$

Surprisingly, the Daubechies wavelets are quite disappointing in terms of raw performance here. It should be noted, however, that they give a better visual impression, especially on the shoulder which has a squared aspect with Haar thresholded at 90% but not with Daubechies.