Assignment 3

PH17B011
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1. The interpolation using N=3 points gives a lorentzian curve and is the better estimator.
Hence, the quality of data matters more that the number of data points

2. With increased N, the ~~err~~ relative error comes to a minima.
Hence larger intervals in the range give better estimate.

4. i) 3 point ~~error~~ formula:

$$f_3' = \frac{f_h - f_{-h}}{2h} - \frac{h^2 f'''}{6} + O(h^3)$$

Hence error is of the order $O(h^2)$
and approximation error is $\frac{h^2 f'''}{6}$

Machine precision: $\left| \frac{f_h - f_{-h}}{2h} \right| \leq \frac{e_m}{2h}$

Total error: $e_{Total} \leq \frac{e_m}{2h} + \frac{h^2 f'''}{6}$

Minimizing,

$$\frac{d\, e_{total}}{dh} = -\frac{e_m}{2h^2} + \frac{h f'''}{3} = 0$$

$$f'''(x)\Big|_{x=\sqrt{2}} = \frac{10}{27}$$

$$\therefore h = \left(\frac{81\, e_m}{20}\right)^{1/3}.$$

single precision : $|e_m| \leq 10^{-7}$

$\Rightarrow h \sim 0.007$

~~doubl~~ double prec : $|e_m| \leq 10^{-15}$

$\Rightarrow h \simeq 1.6 \times 10^{-5}$

2 point formula :

$$f'_2 = \frac{f_n - f_0}{h} + O(h)$$

Approx. error : $\frac{hf''}{2}$

Total error $= \frac{e_m}{h} + \frac{hf''}{2}$

$\Rightarrow h \sim \left(\frac{9}{\sqrt{2}} e_m\right)^{1/2}$ for $f''\Big|_{x=\sqrt{2}} = \frac{2\sqrt{2}}{9}$

$\therefore$ for single prec, $h \sim 0.0008$

double prec ; $h \sim .8 \times 10^{-7}$.