```
In[191]:= Remove["Global`*"]
```

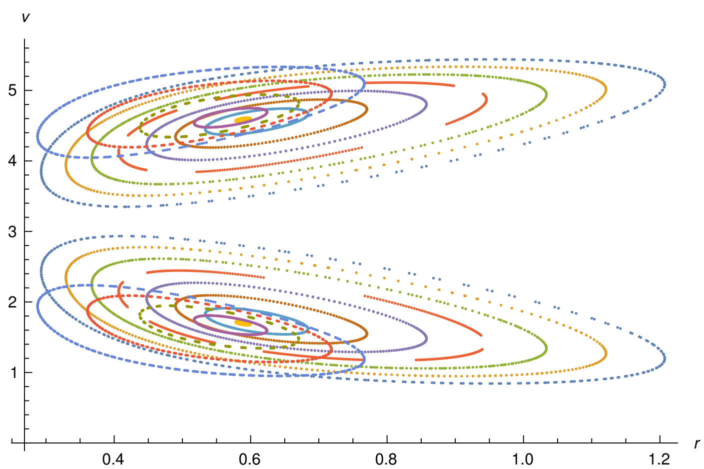
Poincare Maps

```
In[192]:= eq1[μ_] := { v '[t] == ( r[t] × ω[t]^2 + 9.8 (Cos[θ[t]] - μ )) / (1 + μ),
              ω '[t] == (- 2 v[t] × ω[t] - 9.8 Sin[θ[t]]) / r[t], r '[t] == v[t], θ '[t] == ω[t]};

In[285]:= psect[{r0_, θ0_, v0_, ω0_}] :=
           Reap[NDSolve[{eq1[3], r[0] == r0, θ[0] == θ0, v[0] == v0, ω[0] == ω0,
                WhenEvent[θ[t] == 0, Sow[{r[t], v[t]}]]}, {}, {t, 0, 1000}, MaxSteps → ∞]][[-1, 1]]

In[200]:= abcdata = Mod[Map[psect, {{1, Pi / 2, 0, 0}, {1, Pi / 2 - 0.1, 0, 0}, {1, Pi / 2 - 0.2, 0, 0},
                {1, Pi / 2 - 0.3, 0, 0}, {1, Pi / 2 - 0.4, 0, 0}, {1, Pi / 2 - 0.5, 0, 0},
                {1, Pi / 2 - 0.6, 0, 0}, {1, Pi / 2 - 0.7, 0, 0}, {1, Pi / 2 - 0.8, 0, 0},
                {1, Pi / 2 - 0.9, 0, 0}, {1, Pi / 2 - 1, 0, 0}, {1, Pi / 2 - 1.1, 0, 0}}], 2 π];
          ListPlot[abcdata, ImageSize → Medium, AxesLabel → {r, v}]
```

Out[201]=



Above is the Poincare map (r vs v) for the system, varying the initial angle $\theta(0)$

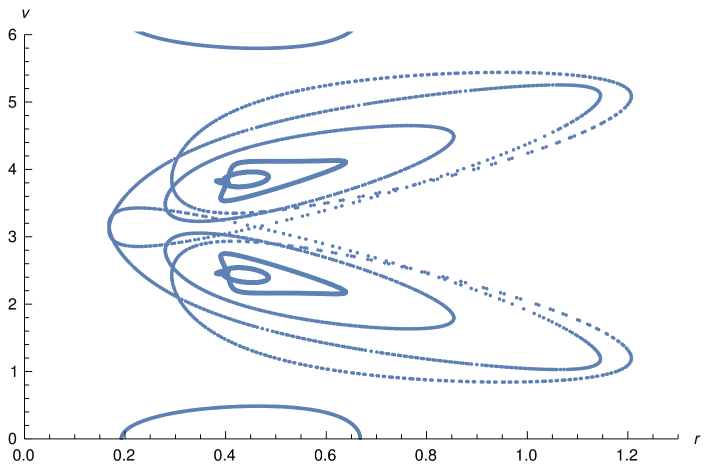
```
In[202]:= Export["/home/hp/Desktop/acads/SAM/poincare_r.jpg", %201, "JPEG"]

Out[202]= /home/hp/Desktop/acads/SAM/poincare_r.jpg

In[184]:= psect[{r0_, θ0_, v0_, ω0_}] :=
           Reap[NDSolve[{eq1[8], r[0] == r0, θ[0] == θ0, v[0] == v0, ω[0] == ω0,
                WhenEvent[θ[t] == 0, Sow[{r[t], v[t]}]]}, {}, {t, 0, 1000}, MaxSteps → ∞]][[-1, 1]]
          dat = Mod[Map[psect, {{1, Pi / 2, 0, 0}}], 2 Pi];
          p7 = ListPlot[dat, PlotRange → {{0, 1.3}, {0, 6}}, PlotLabel → {r (t), v (t)}]
```

In[189]:=
```
Show[p1, p2, p3, p4, p5, p6, p7, AxesLabel → {r, v}]
```

Out[189]=



This is the Poincare map (r vs v) for the system, varying the initial angle θ (0)

In[190]:=
```
Export["/home/hp/Desktop/acads/SAM/poincare_mu.jpg", %189, "JPEG"]
```

Out[190]= /home/hp/Desktop/acads/SAM/poincare_mu.jpg

Phase Space Plots

In[297]:=
```
μ = 3;
sol1 = NDSolve[{ v '[t] == ( r[t] × ω[t]^2 + 9.8 (Cos[θ[t]] − μ)) / (1 + μ),
   ω '[t] == (− 2 v[t] × ω[t] − 9.8 Sin[θ[t]]) / r[t], r '[t] == v[t], θ '[t] == ω[t], r[0] == 1,
   θ[0] == Pi / 2, v[0] == 0, ω[0] == 0}, {r, v, θ, ω}, {t, 0, 100}, MaxSteps → 200 000]
```

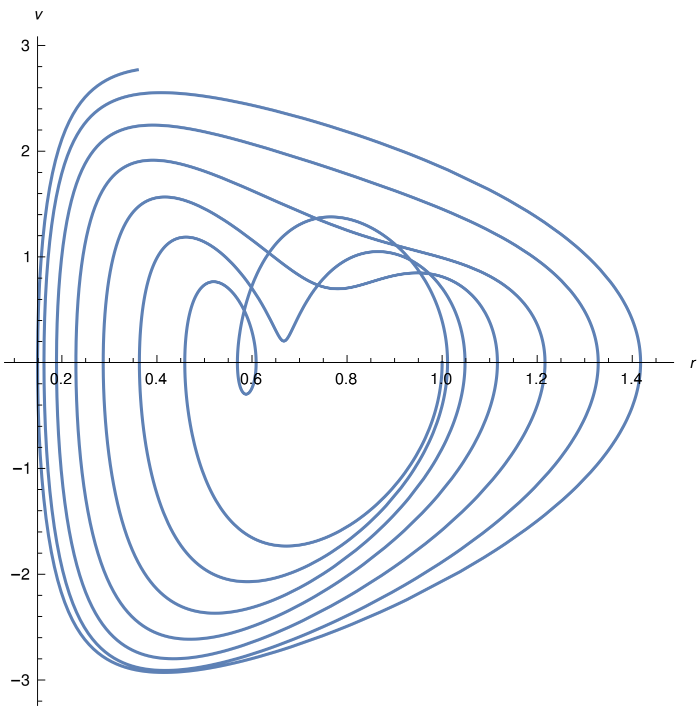
Out[298]=
```
{{r → InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ],
  v → InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ],
  θ → InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ],
  ω → InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ]}}
```

r vs v:

In[299]:=
```
a[t_] := r[t] /. sol1
b[t_] := v[t] /. sol1

ParametricPlot[{a[t][[1]], b[t][[1]]}, {t, 0, 10},
 AspectRatio → 1, ImageSize → Medium, AxesLabel → {r, v}]
```

Out[301]=



θ vs ω:

In[302]:=
```
c[t_] := θ[t] /. sol1
d[t_] := ω[t] /. sol1
ParametricPlot[{c[t][[1]], d[t][[1]]}, {t, 0, 10}, AspectRatio → 1,
 ImageSize → Medium, AxesLabel → {θ, ω}, PlotRange → Full]
```

Out[304]=