

- Problem:

Suppose the equations AB, BC and CA are respectively given by

$$\mathbf{n}_i^\top \mathbf{x} = c_i \quad (1)$$

$$i = 1, 2, 3$$

The equations of the respective angle bisectors are then given by

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad (2)$$

$$i \neq j$$

Substitute numerical values and find the equations of the angle bisectors of **A**, **B** and **C**.

- Solution:

Given $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

The normal form of a line is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (3)$$

where $\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{m}$ and $\mathbf{m} = (\mathbf{B} - \mathbf{A})$, where **A**, **B** are two points on the line and **m** is slope vector of the line.

From problem 1.1.5,

$$CA : (4 \quad -4) \mathbf{x} = 8 \quad (4)$$

Similarly;

$$AB : (7 \quad 5) \mathbf{x} = 2 \quad (5)$$

$$BC : (-11 \quad -1) \mathbf{x} = -10 \quad (6)$$

From eq.1, the equations AB, BC and CA are respectively,

$$n_i^\top \mathbf{x} = c_i \quad (7)$$

$$i = 1, 2, 3$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

$$\mathbf{n}_2 = \begin{pmatrix} -11 \\ -1 \end{pmatrix}$$

$$\mathbf{n}_3 = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

and

$$c_1 = 2$$

$$c_2 = -10$$

$$c_3 = 8$$

From eq 2, the equations of the respective angle bisectors are then given by,

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|}$$

$$i \neq j$$

For internal angle bisector,

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|}$$

\Rightarrow Internal angle bisector of **B** (i=1 , j=2) :

$$\frac{(7 \ 5) \mathbf{x} - 2}{\left\| \begin{pmatrix} 7 \\ 5 \end{pmatrix} \right\|} = \frac{(-11 \ -1) \mathbf{x} - 10}{\left\| \begin{pmatrix} -11 \\ -1 \end{pmatrix} \right\|}$$

$$\therefore \left\| \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\| = \sqrt{x_1^2 + x_2^2} :$$

$$\Rightarrow \frac{(7 \ 5) \mathbf{x} - 2}{\sqrt{74}} = \frac{(11 \ -1) \mathbf{x} - 10}{\sqrt{122}}$$

$$\Rightarrow \frac{(7 \ 5) \mathbf{x} - 2}{\sqrt{37}} = \frac{(11 \ -1) \mathbf{x} - 10}{\sqrt{61}}$$

$$\Rightarrow (7\sqrt{61} - 11\sqrt{37} \ 5\sqrt{61} + \sqrt{37}) \mathbf{x} = 2(\sqrt{61} - 5\sqrt{37})$$

\therefore Internal angle bisector of **B** :

$$(7\sqrt{61} - 11\sqrt{37} \ 5\sqrt{61} + \sqrt{37}) \mathbf{x} = 2(\sqrt{61} - 5\sqrt{37})$$

or

$$(-12.2386 \ 45.1304) \mathbf{x} = -45.2071$$