Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} -2 \\ -4 \end{pmatrix}$	slope of AB
$\mathbf{m_2}$	$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	slope of BC
$\mathbf{m_3}$	$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$	slope of CA
$\|\|A-B\|\|$	4.47	length of AB
$\|\|B-C\|\|$	3.16	length of BC
$\|\|C-A\|\|$	3.16	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non-collinear
$\mathbf{n_1}$	$\begin{pmatrix} -4 \\ 2 \end{pmatrix}$	AB
c_1	10	
$\mathbf{n_2}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	BC
c_2	5	
$\mathbf{n_3}$	$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	CA
c_3	-5	
Area	5	Area of Triangle
$\angle A$	45°	Angles
$\angle B$	45°	
$\angle C$	90°	

TABLE 1: Vectors.

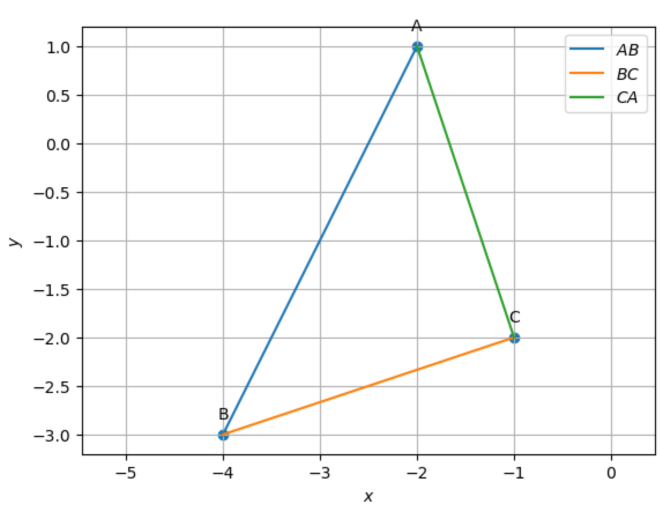

Fig. 1: triangle plotted using python

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -2.5 \\ -2.5 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -1.5 \\ -0.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} -3 \\ -1 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} -0.5 \\ -3.5 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix}$	
c_4	7.5	
$\mathbf{m_5}$	$\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 2.5 \\ -2.5 \end{pmatrix}$	
c_5	-2.5	
$\mathbf{m_6}$	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	
c_6	-5	
$\mathbf{G}$	$\begin{pmatrix} -2.33 \\ -1.33 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

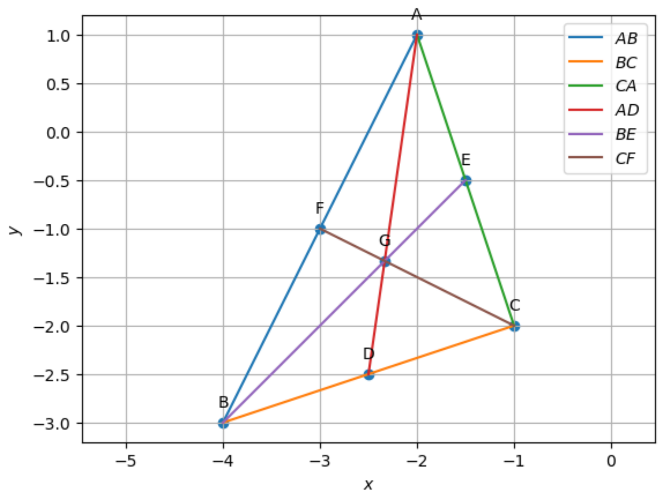

Fig. 2: medians plotted using python

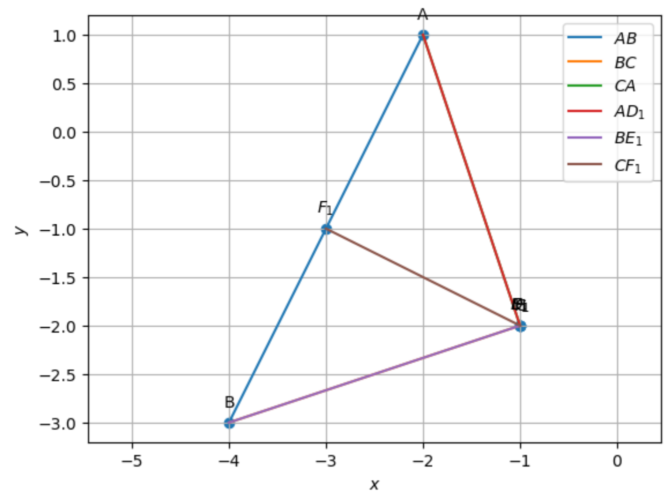

Fig. 3: altitudes plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}-1\\-2\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-1\\-2\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-3\\-1\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}1\\-3\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-3\\-1\end{pmatrix}$	
c_7	5	
$\mathbf{m_8}$	$\begin{pmatrix}3\\1\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}1\\-3\end{pmatrix}$	
c_8	5	
$\mathbf{m_9}$	$\begin{pmatrix}-2\\1\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}1\\2\end{pmatrix}$	
c_9	-5	
$\mathbf{H}$	$\begin{pmatrix}-1\\-2\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}1\\-3\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}3\\1\end{pmatrix}$	
c_{10}	-10	
$\mathbf{m_{11}}$	$\begin{pmatrix}-3\\-1\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}1\\-3\end{pmatrix}$	
c_{11}	0	
$\mathbf{m_{12}}$	$\begin{pmatrix}4\\-2\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}2\\4\end{pmatrix}$	
c_{12}	-10	
$\mathbf{O}$	$\begin{pmatrix}-3\\-1\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	2.24	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	90°	$\angle BOC = 2\angle BAC$
$\angle BAC$	45°	
$\angle AOC$	90°	$\angle AOC = 2\angle ABC$
$\angle ABC$	45°	
$\angle AOB$	180°	$\angle AOB = 2\angle BCA$
$\angle BCA$	90°	

TABLE 4: Perpendicular Bisector.

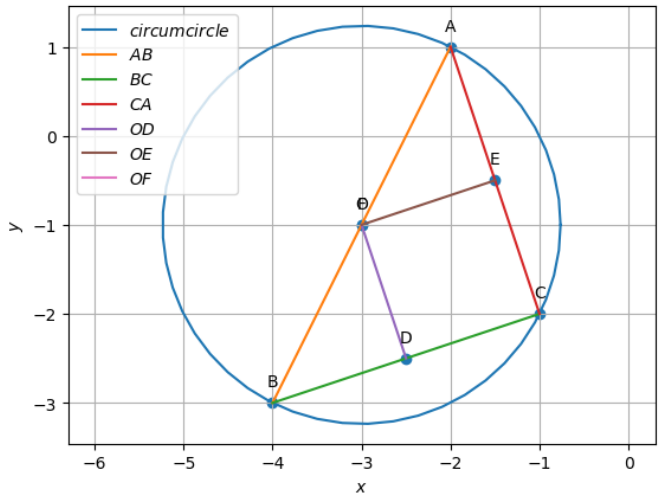

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}0.13\\1.84\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}1.84\\-0.13\end{pmatrix}$	
c_{13}	-3.82	
$\mathbf{m_{14}}$	$\begin{pmatrix}1.40\\1.21\end{pmatrix}$	BI
$\mathbf{n_{14}}$	$\begin{pmatrix}-1.21\\1.40\end{pmatrix}$	
c_{14}	0.65	
$\mathbf{m_{15}}$	$\begin{pmatrix}1.26\\-0.63\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}0.63\\1.26\end{pmatrix}$	
c_{15}	-3.16	
$\mathbf{I}$	$\begin{pmatrix}-2.17\\-1.41\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}-1.88\\-2.29\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-1.29\\-1.12\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}-3\\-1\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	0.93	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	22.5°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	22.5°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	45°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

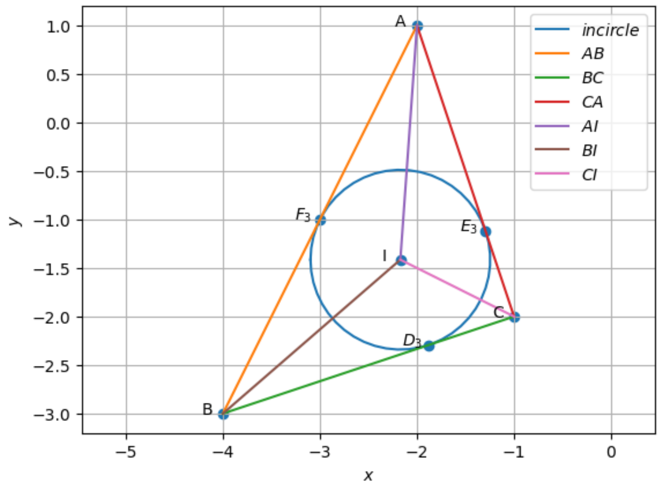

Fig. 5: Angle bisectors plotted using python