Gaussian

EE22BTECH11045 - Samudrala Chaithanya

Question 9.3.22 An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.
Solution: Given,
Number of trials,

$$n = 6 \tag{1}$$

$$p = 2q \tag{2}$$

Where
p = probability of success
q = probability of failure
We know,

$$p + q = 1 \tag{3}$$

$$3q = 1 \tag{4}$$

$$q = \frac{1}{3} \tag{5}$$

and

$$p = \frac{2}{3} \tag{6}$$

Here,

Mean,

$$\mu = np \tag{7}$$
$$= 4 \tag{8}$$

Standard deviation,

$$\sigma = \sqrt{npq} \tag{9}$$
$$\approx 2 \tag{10}$$

Probabiliity of atleast 4 successes, $P(X \geq 4)$ can be written as,

$$P(X \geq 4) = 1 - P(X < 4) \tag{11}$$

Where
$P(X = k)$ = Cumulative Distribution Function (CDF) of random variable X
By using 3.5 as continuity correction,

$$P(X < 4) = P(X \leq 3.5) \tag{12}$$

Now,z-score

$$z = \frac{X - \mu}{\sigma} \tag{13}$$
$$= \frac{3.5 - 4}{2} \tag{14}$$
$$= -0.25 \tag{15}$$

From z-table

$$P(z < -0.25) \approx 0.4013 \tag{16}$$

Using gaussian approximation,

$$P(X < 4) \approx 0.4013 \tag{17}$$
$$\implies P(X \geq 4) = 1 - P(X < 4) \tag{18}$$
$$\approx 1 - 0.4013 \tag{19}$$
$$\approx 0.5987 \tag{20}$$

$\therefore$ Probability of getting atleast 4 successes in the next 6 trials is approximately 0.5987 or 59.87%.