Gaussian
Problem-9.3.22

EE22BTECH11045 - Samudrala Chaithanya

An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution: Given,
Number of trials,

$$n = 6 \tag{1}$$

$$p = 2q \tag{2}$$

Where
p = probability of success
q = probability of failure
We know,

$$p + q = 1 \tag{3}$$

$$3q = 1 \tag{4}$$

$$q = \frac{1}{3} \tag{5}$$

and

$$p = \frac{2}{3} \tag{6}$$

Number of trials	n	6
Probability of success	p	0.667(2/3)
Probability of Failure	q	0.333(1/3)

1. Binomial:
   In binomial distribution, Cumulative Distribution Function is (C.D.F):

$$\Pr(X = k) = \binom{n}{k} \cdot p^k \cdot q^{n-k} \tag{7}$$

Where
$\Pr(X = k)$ = Probability of getting k successes in n trials
Now,

$$\Pr(X = 4) = \binom{6}{4} \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^{6-4} \tag{8}$$

$$\approx 0.329 \tag{9}$$

$$\Pr(X = 5) = \binom{6}{5} \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^{6-5} \tag{10}$$

$$\approx 0.263 \tag{11}$$

$$\Pr(X = 6) = \binom{6}{6} \cdot \left(\frac{2}{3}\right)^6 \cdot \left(\frac{1}{3}\right)^{6-6} \tag{12}$$

$$\approx 0.088 \tag{13}$$

The probability of atleast 4 successes is

$$\Pr(X \geq 4) = \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) \tag{14}$$

$$= 0.329 + 0.263 + 0.088 \tag{15}$$

$$= 0.680 \tag{16}$$

$\therefore$ Probability of getting atleast 4 successes in the next 6 trials is 0.680 or 68.00%.

2. Gaussian:

   Here,
   Mean,

$$\mu = np \tag{17}$$
$$= 4 \tag{18}$$

   Standard deviation,

$$\sigma = \sqrt{npq} \tag{19}$$
$$\approx 1.63299 \tag{20}$$

   Probabiliity of atleast 4 successes, $\Pr(X \geq 4)$ can be written as,

$$\Pr(X \geq 4) = 1 - \Pr(X < 4) \tag{21}$$

   Let us take $X = 4$
   Now,Z-score

$$Z = \frac{X - \mu}{\sigma} \tag{22}$$
$$= \frac{4 - 4}{1.63299} \tag{23}$$
$$= 0 \tag{24}$$

   Here, we make use of a fuction called Q-function.

$$\Pr(X < k) = Q(Z) \tag{25}$$

   Q-function is defined as,

$$Q(Z) = \frac{1}{2} erfc\left(\frac{Z}{\sqrt{2}}\right) \tag{26}$$

Where,

$$
\begin{aligned}
erfc(Z) &= 1 + erf(Z) && (27)\\
&= 1 + \frac{2}{\sqrt{\pi}} \int_0^Z e^{-X^2} \cdot dx && (28)
\end{aligned}
$$

Here,

$$
\begin{aligned}
\Pr(X < 4) &= Q(0) && (29)\\
&= \frac{1}{2}(1) && (30)\\
&= 0.5 && (31)
\end{aligned}
$$

$$
\begin{aligned}
\implies \Pr(X \geq 4) &= 1 - \Pr(X < 4) && (32)\\
&= 1 - 0.5 && (33)\\
&= 0.5 && (34)
\end{aligned}
$$

$\therefore$ Probability of getting atleast 4 successes in the next 6 trials is approximately 0.5 or 50.00%.

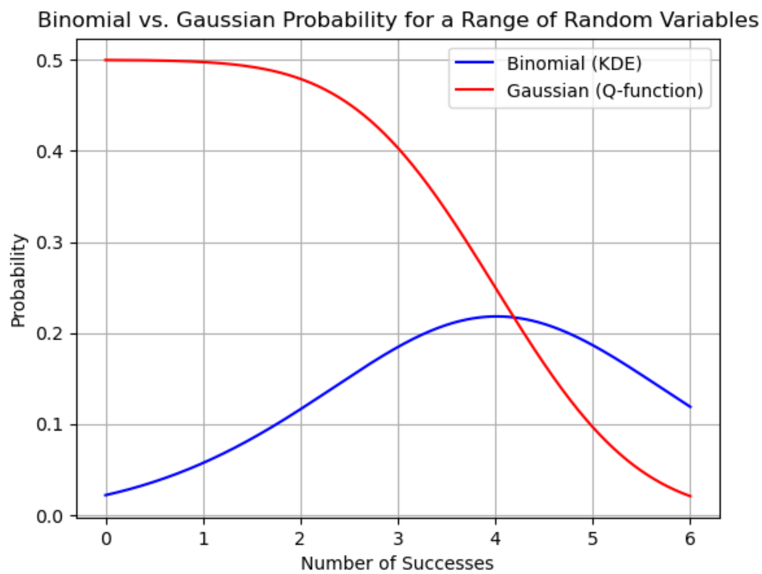

Figure 1: Comparing probabilities