

Gaussian

Problem-9.3.22

EE22BTECH11045 - Samudrala Chaithanya

An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution: Given,
Number of trials,

$$n = 6 \quad (1)$$

$$p = 2q \quad (2)$$

Where

p = probability of success

q = probability of failure

We know,

$$p + q = 1 \quad (3)$$

$$3q = 1 \quad (4)$$

$$q = \frac{1}{3} \quad (5)$$

and

$$p = \frac{2}{3} \quad (6)$$

Number of trials	n	6
Probability of success	p	0.667(2/3)
Probability of Failure	q	0.333(1/3)

1. Binomial:

In binomial distribution, Cumulative Distribution Function is (C.D.F):

$$\Pr(X = k) = \binom{n}{k} \cdot p^k \cdot q^{n-k} \quad (7)$$

Where

$\Pr(X = k)$ = Probability of getting k successes in n trials

Now,

$$\Pr(X = 4) = \binom{6}{4} \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^{6-4} \quad (8)$$

$$\approx 0.329 \quad (9)$$

$$\Pr(X = 5) = \binom{6}{5} \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^{6-5} \quad (10)$$

$$\approx 0.263 \quad (11)$$

$$\Pr(X = 6) = \binom{6}{6} \cdot \left(\frac{2}{3}\right)^6 \cdot \left(\frac{1}{3}\right)^{6-6} \quad (12)$$

$$\approx 0.088 \quad (13)$$

The probability of atleast 4 successes is

$$\Pr(X \geq 4) = \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) \quad (14)$$

$$= 0.329 + 0.263 + 0.088 \quad (15)$$

$$= 0.680 \quad (16)$$

\therefore Probability of getting atleast 4 successes in the next 6 trials is 0.680 or 68.00%.

2. Gaussian:

Here,
Mean,

$$\mu = np \quad (17)$$

$$= 4 \quad (18)$$

Standard deviation,

$$\sigma = \sqrt{npq} \quad (19)$$

$$\approx 1.63299 \quad (20)$$

Probability of atleast 4 successes, $\Pr(X \geq 4)$ can be written as,

$$\Pr(X \geq 4) = 1 - \Pr(X < 4) \quad (21)$$

Let us take $X = 4$

Now, Z-score

$$Z = \frac{X - \mu}{\sigma} \quad (22)$$

$$= \frac{4 - 4}{1.63299} \quad (23)$$

$$= 0 \quad (24)$$

Here, we make use of a function called Q-function.

$$\Pr(X < k) = Q(Z) \quad (25)$$

Q-function is defined as,

$$Q(Z) = \frac{1}{2} \operatorname{erfc}\left(\frac{Z}{\sqrt{2}}\right) \quad (26)$$

Where,

$$erfc(Z) = 1 + erf(Z) \quad (27)$$

$$= 1 + \frac{2}{\sqrt{\pi}} \int_0^Z e^{-x^2} \cdot dx \quad (28)$$

Here,

$$\Pr(X < 4) = Q(0) \quad (29)$$

$$= \frac{1}{2}(1) \quad (30)$$

$$= 0.5 \quad (31)$$

$$\Rightarrow \Pr(X \geq 4) = 1 - \Pr(X < 4) \quad (32)$$

$$= 1 - 0.5 \quad (33)$$

$$= 0.5 \quad (34)$$

\therefore Probability of getting atleast 4 successes in the next 6 trials is approximately 0.5 or 50.00%.