Route Optimization Algorithm

Indian Institute of Technology, Bombay

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The Problem

- Single Depot Vehicle Scheduling Problem
- It is a combinatorial optimization problem for which finding the optimal solution is NP hard. Hence generally heuristic approaches are taken according to the size of data
- Constraints:
  - Time window
  - Bus capacity; limit and minimum 85% occupancy
  - Number of buses
- The objective is to minimise the operational cost of buses

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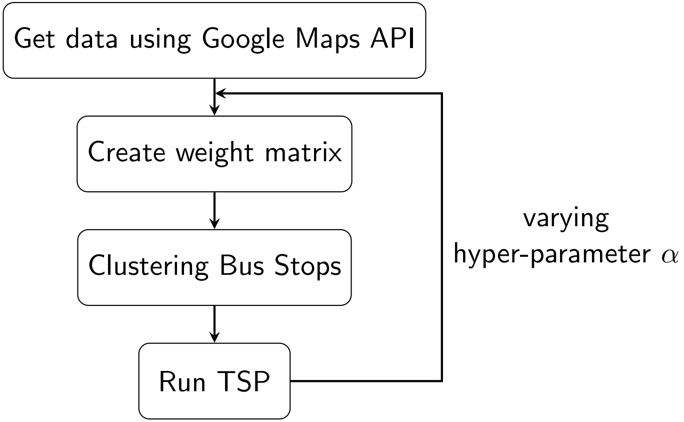
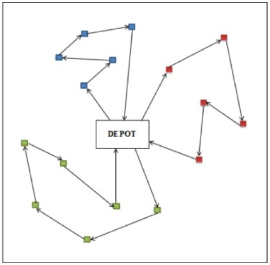
- This problem is generally approached using Integer Programming, by forming a set of equations and finding solutions and optimizing the process by using methods like ant colonization

Flow of Solving



Travelling Salesman Problem(TSP)

- What is TSP and why TSP?



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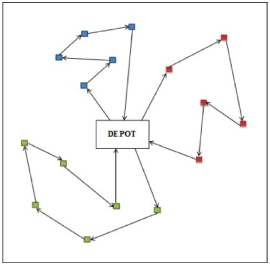

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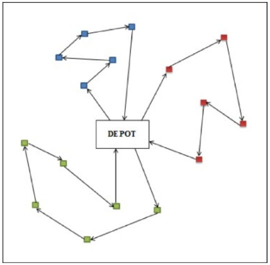

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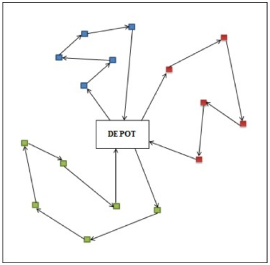
- Once the bus stops are in clusters we will assign one bus to each cluster and use TSP to determine route of the bus



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- Thus, $T_{withoutclustering} \propto n^2 2^n$ and $T_{withclustering} \propto k n_i^2 2^{n_i}$ Therefore,

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- Hence with increase in number of clusters k, the time taken for TSP decreases exponentially

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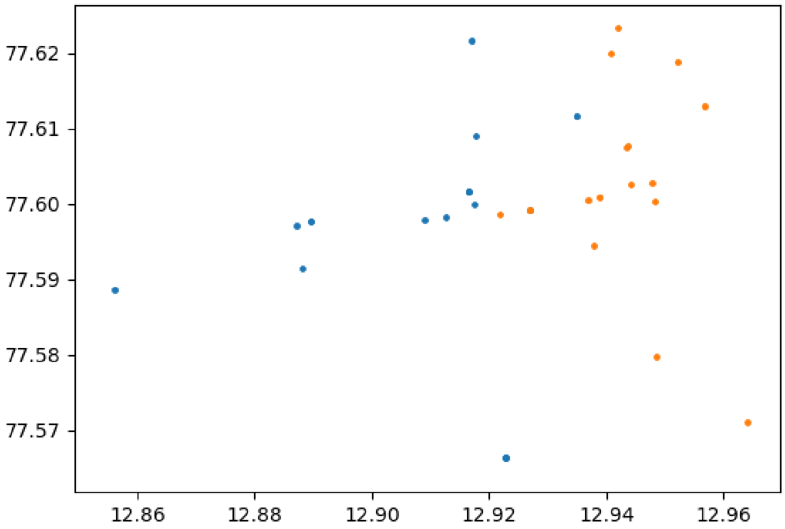
- If the size of a cluster exceeds bus capacity then treat this cluster as an independent problem and use the procedure recursively over it

Clustering of the newly given data



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- $\alpha = 1$ will give the minimum distance solution ignoring time window, $\alpha = 0$ will give the minimum time solution ignoring cost
- We want to have a solution such that the time taken by the buses falls within the time window. We will use a particular value for α

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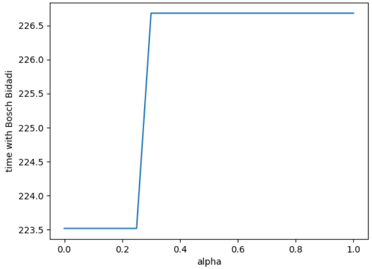
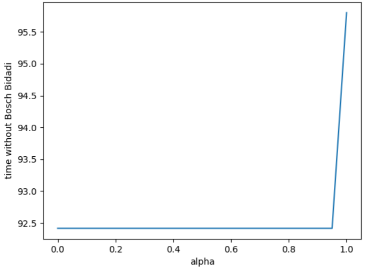
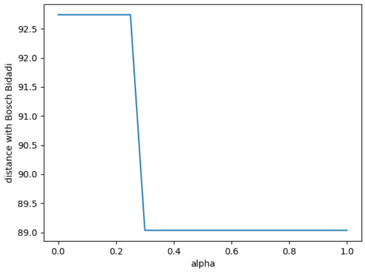
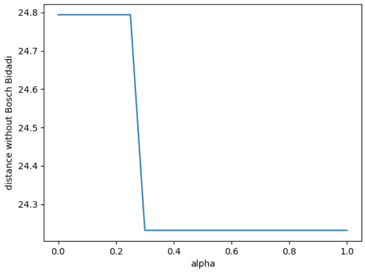




Handling the Distance constraint





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- This is solved in the way same as that for TSP with a modified equation for Dynamic Programming
- The time for this computation will be lesser than that of the TSP done initially as number of points have reduced and hence also efficient

Results

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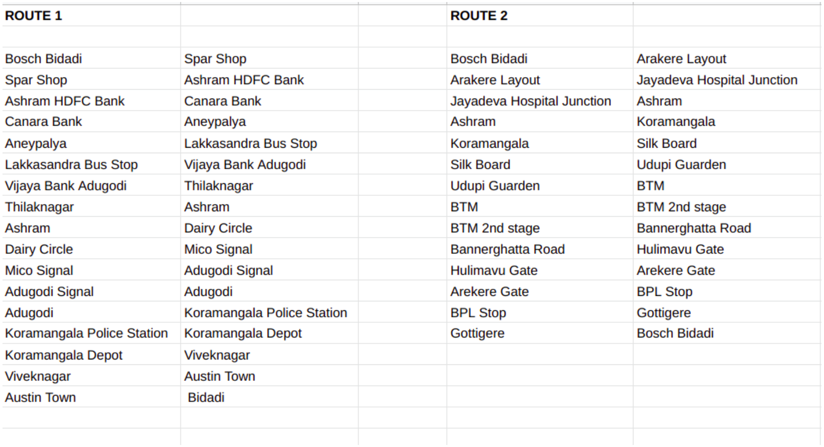
- An optimal solution is to use 2 buses following the routes :

Results



ROUTE 1		ROUTE 2	
Bosch Bidadi	Spar Shop	Bosch Bidadi	Arakere Layout
Spar Shop	Ashram HDFC Bank	Arakere Layout	Jayadeva Hospital Junction
Ashram HDFC Bank	Canara Bank	Jayadeva Hospital Junction	Ashram
Canara Bank	Aneypalya	Ashram	Koramangala
Aneypalya	Lakkasandra Bus Stop	Koramangala	Silk Board
Lakkasandra Bus Stop	Vijaya Bank Adugodi	Silk Board	Udupi Guarden
Vijaya Bank Adugodi	Thilaknagar	Udupi Guarden	BTM
Thilaknagar	Ashram	BTM	BTM 2nd stage
Ashram	Dairy Circle	BTM 2nd stage	Bannerghatta Road
Dairy Circle	Mico Signal	Bannerghatta Road	Hulimavu Gate
Mico Signal	Adugodi Signal	Hulimavu Gate	Arekere Gate
Adugodi Signal	Adugodi	Arekere Gate	BPL Stop
Adugodi	Koramangala Police Station	BPL Stop	Gottigere
Koramangala Police Station	Koramangala Depot	Gottigere	Bosch Bidadi
Koramangala Depot	Viveknagar		
Viveknagar	Austin Town		
Austin Town	Bidadi		

Results

- For the given sample data set, no solution is possible which satisfies all the given constraints
- Reason is that the depot 'Bosch Bidadi' is itself at a far off distance from the other stops, explained on maps
- 'Mantri Apartment' is itself 50 km and at a time of 1 hr from the depot, while time window is of 1hr 20 min
- We can handle such outliers
- Since number of people is 29 and 85% of bus capacity is 27, only one bus can be used. Solution for this leads to a minimum time of 4 hrs which is not acceptable
- Possible way to reduce time is to use 2 or more buses but then they will have less occupancy

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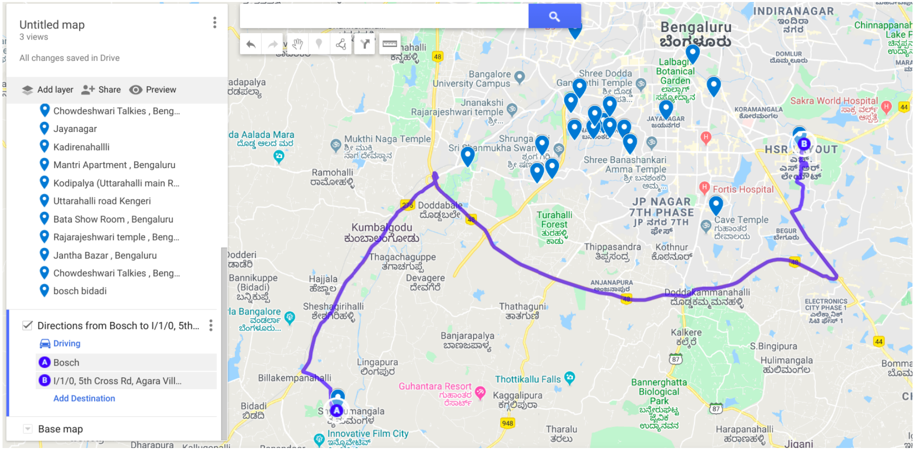
Untitled map
3 views
All changes saved in Drive
Add layer Share Preview
- Chowdeshwari Talkies , Beng...
- Jayanagar
- Kadirenahallli
- Mantri Apartment , Bengaluru
- Kodipalya (Uttarahalli main R...
- Uttarahalli road Kengeri
- Bata Show Room , Bengaluru
- Rajarajeshwari temple , Beng...
- Jantha Bazar , Bengaluru
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- bosch bidadi

Directions from Bosch to I/1/0, 5th...
Driving
A Bosch
B I/1/0, 5th Cross Rd, Agara Vill...
Add Destination
Base map

Challenges faced

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- To handle this, whenever there are such locations which have an average distance more than 100 km from the other points, we append the city name to it for the API call

Quantum Algorithm to solve the TSP

- NP hard problem in combinatorial optimization takes exponential time order for solving by classical brute force method

[1] *Karthik Srinivasan, Saipriya Satyajit, Bikash K Behera, and Prasanta K. Panigrahi*, **Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience**

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- This provides us a quadratic speedup over the classical brute force method for a large number of cities [1]

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