```
details, see the recently published paper: Lazarus et al, "SPARKLING: variable-density k-space filling curves for accelerated T_2^* -weighted MRI", Magn Reson
         Med 2019; 81:3643:3661.

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           • Target: ISBI'19 tutorial on Recent advances in acquisition and reconstruction for Compressed Sensing MRI

    Revision: 01/06/2021 for ATSI MSc hands-on session at Paris-Saclay University.

 In [6]: #DISPLAY T2* MR IMAGE
          %matplotlib inline
         import numpy as np
         import os.path as op
         import os
         import math ; import cmath
         import matplotlib
         import matplotlib.pyplot as plt
         import sys
          from mri.operators import NonCartesianFFT
          from mri.operators.utils import convert_locations_to_mask, \
             gridded_inverse_fourier_transform_nd
          from pysap.data import get_sample_data
          from skimage import data, img_as_float, io, filters
          from modopt.math.metrics import ssim
         mri_img = get_sample_data('2d-mri')
         img_size = mri_img.shape[0]
         plt.figure()
         plt.title("T2* axial slice, size = {}".format(img_size))
         if mri_img.ndim == 2:
             plt.imshow(mri_img, cmap=plt.cm.gray)
          else:
              plt.imshow(mri_img)
         plt.show()
                                    T2* axial slice, size = 512
                       100 -
                      200 -
                      300 -
                      500
                                  100
                                           200
 In [7]: from scipy.io import loadmat
          cwd = os.getcwd()
         dirimg_2d = op.join(cwd,"..","data")
         k_spark = loadmat(op.join(cwd, "..", "data", "samples_SPARKLING_N512_nc34x3073_0S1.mat"))
         k_spark_vec = k_spark['samples']
          Kmax = np.amax(k\_spark\_vec)
          #print(Kmax)
          k_spark_vec = k_spark_vec*1/(2*np.pi*Kmax)
          #save in npz format in the outdir directory
         outdir = op.join(cwd,"..","output")
          filename_traj = "sparkling_radial_N" + str(img_size) + ".npz"
         outfile = op.join(outdir, filename_traj)
         np.savez(outfile, k_spark_vec)
          k_spark = plt.figure(figsize=(7,7))
          plt.scatter(k_spark_vec[:,0], k_spark_vec[:,1], marker = '.', s=0.1)
         plt.grid()
          #Figure layout
         unit = 1/4
          tick = np.arange(-0.5, 0.5 + unit, unit)
         label = [r"$-\frac{1}{2}$", r"$-\frac{1}{4}$", r"$0$", r"$+\frac{1}{4}$", r"$+\frac{1}{2}$"]
         plt.xticks(tick/np.pi, labels = label, fontsize = 16); plt.yticks(tick/np.pi, labels = label, fontsize = 16)
         plt.xlabel(r"$k_x$", fontsize = 22) ; plt.ylabel(r"$k_y$", fontsize = 22)
         plt.title("K-space sampling, spiral in-out initialization", fontsize = 18)
         plt.show()
         1280.0
                    K-space sampling, spiral in-out initialization
 In [9]: data=convert_locations_to_mask(k_spark_vec, mri_img.shape)
          fourier_op = NonCartesianFFT(samples=k_spark_vec, shape=mri_img.shape,
                                       implementation='cpu')
          kspace_obs = fourier_op.op(mri_img.data)
                                                    Traceback (most recent call last)
         <ipython-input-9-46089bf0f5cd> in <module>
               1 data=convert_locations_to_mask(k_spark_vec, mri_img.shape)
               2 fourier_op = NonCartesianFFT(samples=k_spark_vec, shape=mri_img.shape,
                                               implementation='cpu')
               4 kspace_obs = fourier_op.op(mri_img.data)
         ~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, implementation, n_
         coils, **kwargs)
             550
                         if implementation == 'cpu':
             551
                             self.implementation = NFFT(samples=samples, shape=shape,
          --> 552
                                                         n_coils=self.n_coils)
             553
                          elif implementation == 'cuda' or implementation == 'opencl':
             554
                              self.implementation = NUFFT(samples=samples, shape=shape,
         ~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, n_coils)
             105
                          # TODO Parallelize this if possible
                          self.nb_coils = n_coils
             106
          --> 107
                          self.plan = pynfft.NFFT(N=shape, M=len(samples))
             108
                          self.plan.x = self.samples
             109
                          self.plan.precompute()
         NameError: name 'pynfft' is not defined
In [10]: grid_space = np.linspace(-0.5, 0.5, num=mri_img.shape[0])
         grid2D = np.meshgrid(grid_space, grid_space)
         grid_soln = gridded_inverse_fourier_transform_nd(k_spark_vec, kspace_obs,
                                                           tuple(grid2D), 'linear')
         plt.imshow(np.abs(grid_soln), cmap='gray')
          # Calculate SSIM
         base_ssim = ssim(grid_soln, mri_img)
         plt.title('Gridded Solution\nSSIM = ' + str(base_ssim))
         plt.show()
                                                    Traceback (most recent call last)
         <ipython-input-10-336799777e9e> in <module>
               1 grid_space = np.linspace(-0.5, 0.5, num=mri_img.shape[0])
               2 grid2D = np.meshgrid(grid_space, grid_space)
          ----> 3 grid_soln = gridded_inverse_fourier_transform_nd(k_spark_vec, kspace_obs,
                                                                   tuple(grid2D), 'linear')
                5 plt.imshow(np.abs(grid_soln), cmap='gray')
         NameError: name 'kspace_obs' is not defined
In [11]: # fista rec using PySAP (branch pogm_addition: https://github.com/zaccharieramzi/pysap/tree/pogm_addition)
          from modopt.opt.linear import Identity
          from modopt.opt.proximity import SparseThreshold, LinearCompositionProx
          from mri.numerics.fourier import NFFT
          from pysap import Image
          from mri.numerics.gradient import GradAnalysis2
          from mri.numerics.linear import WaveletN
          from mri.numerics.reconstruct import sparse_rec_pogm
          from mri.numerics.utils import convert_mask_to_locations
          from modopt.math.metrics import ssim
          ## ops init
          kspace_loc = convert_mask_to_locations(k_spark_vec)
         linear_op = WaveletN(
             nb_scale=4,
             wavelet_name="db4",
             padding_mode="periodization",
          fourier_op = NFFT(
             samples= k_spark_vec * np.pi,
             shape= mri_img.shape,
          ##compute the kspace data
          kspace_data_nfft = fourier_op.op(mri_img)
         ## now back to ops
         gradient_op = GradAnalysis2(
             data=kspace_data_nfft,
              fourier_op=fourier_op,
         # define the proximity operator
         prox_op = LinearCompositionProx(
             linear_op=linear_op,
             prox_op=SparseThreshold(Identity(), 0.05, thresh_type="soft"),
         if 1:
             ## run pogm' (ie POGM with restart)
             x_final, costs, metrics = sparse_rec_pogm(prox_op=prox_op, linear_op=Identity(), gradient_op=gradient_op,
                                             max_nb_of_iter=100, metric_call_period=20)
          pogm\_rec = np.abs(x\_final)
         img_rec = Image(data=pogm_rec)
         #img_rec.show()
         #img_rec = np.abs(x_final)
         #print(metrics)
          #SSIM
         ssim_pogm = ssim(mri_img, pogm_rec)
         ssim_pogm = float(round(abs(ssim_pogm),3))
         plt.figure()
         plt.title('Restored image (POGM) : SSIM = ' + str(ssim_pogm))
         plt.imshow(pogm_rec, cmap='gray')
         plt.show()
         WARNING: Making input data immutable.
         100% (100 of 100) |############### Elapsed Time: 0:00:34 Time: 0:00:34
                             Restored image (POGM) : SSIM = 0.937
                       100 -
                      200 -
```

300

400 -

500

0

100

200

300

400

500

Non cartesian sampling: SPARKLING imaging

We explore the performance of SPARKLING (*Spreading projection Algorithm for Rapid K-space sampLING*) as non-Cartesian imaging technique. We do not actually provide the code of this algorithm but instead upload result files containing trajectories generated from the previous *radial in-out* initialization. For