Fourth exercice: Cartesian perodic under-sampling along parallel lines

Here the goal is to illustrate the typical artifacts of standard **deterministic regular** (or **periodic**) undersampling along the phase encoding direction (here k_y) used in parallel imaging. Below we illustrate the following cases:

1. full Cartesian sampling $R = n/m = 1$ where $n = N^2$ is the image size, N the image dimension and m the number of measurements in k-space:
2. undersampling with a factor $R = 2$
3. undersampling with a factor $R = 4$
4. undersampling with a factor $R = 8$

- Author: Philippe Ciuciu (philippe.ciuciu@cea.fr)
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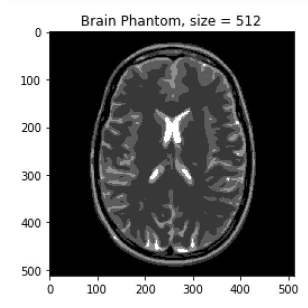
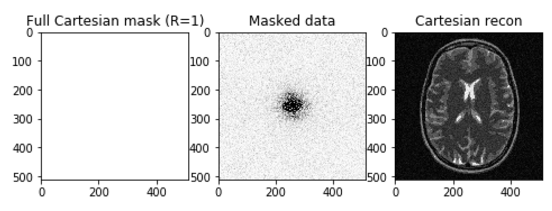
- Target: [ISBI'19 tutorial] on **Recent advances in acquisition and reconstruction for Compressed Sensing MRI**
- **Revision**: 01/06/2021 for ATSI MSc hands-on session at Paris-Saclay University.

In [1]:
```
#DISPLAY BRAIN PHANTOM
%matplotlib inline

import numpy as np
import os.path as op
import os
import math ; import cmath
import matplotlib.pyplot as plt
import sys

from skimage import data, io, filters
#get current working dir
cwd = os.getcwd()
#cwd= "/"
dirimg_2d = op.join(cwd,"..", "data")
img_size = 512   #256
FOV = 0.2 #field of view in meters
pixelSize = FOV/img_size

#load data file corresponding to the target resolution
filename = "BrainPhantom" + str(img_size) + ".png"
mri_filename = op.join(dirimg_2d, filename)
mri_img = io.imread(mri_filename, as_gray=True)
plt.figure()
plt.title("Brain Phantom, size = "+ str(img_size))
if mri_img.ndim == 2:
    plt.imshow(mri_img, cmap=plt.cm.gray)
else:
    plt.imshow(mri_img)
plt.show()
```



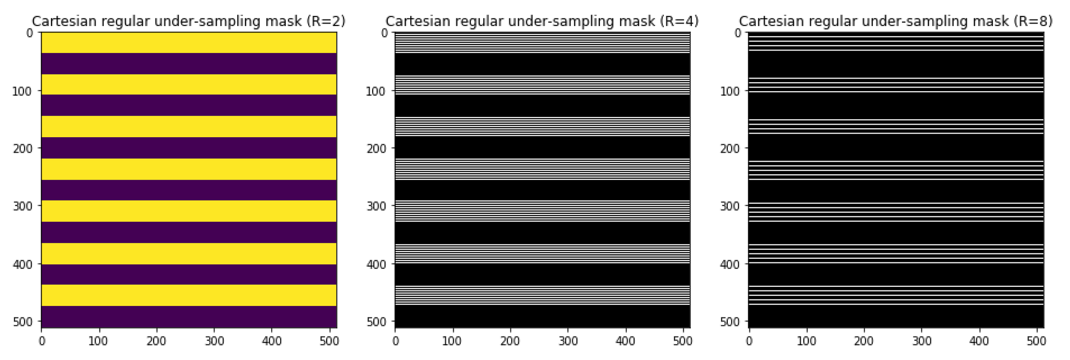
In [43]:
```
kspace_mask_full = np.ones((img_size, img_size), dtype="float64")

#import numpy.fft as fft
norm = "ortho"
def fft(x):
    return np.fft.fft2(x, norm=norm)

def ifft(x):
    return np.fft.ifft2(x, norm=norm)

# Generate the subsampled kspace with R=2
kspace_data = np.fft.fftshift(fft(mri_img))   # put the 0-freq in the middle of axes as

# Generate the kspace data: first Fourier transform the image
kspace_data = np.fft.fftshift(fft(mri_img))
#add Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Mask data to perform subsampling
kspace_data *= kspace_mask_full

# Zero order solution
image_rec0 = ifft(np.fft.ifftshift(kspace_data))

fig, axs = plt.subplots(1, 3, figsize=(8, 8) )
axs[0].imshow(kspace_mask_full, cmap='gray_r')
axs[0].set_title("Full Cartesian mask (R=1)")
axs[1].imshow(np.abs(kspace_data),  cmap='gray_r', vmax=.01*np.abs(kspace_data).max())
axs[1].set_title("Masked data")
axs[2].imshow(np.abs(image_rec0), cmap='gray')
axs[2].set_title("Cartesian recon")
```

Out[43]:
```
Text(0.5, 1.0, 'Cartesian recon')
```



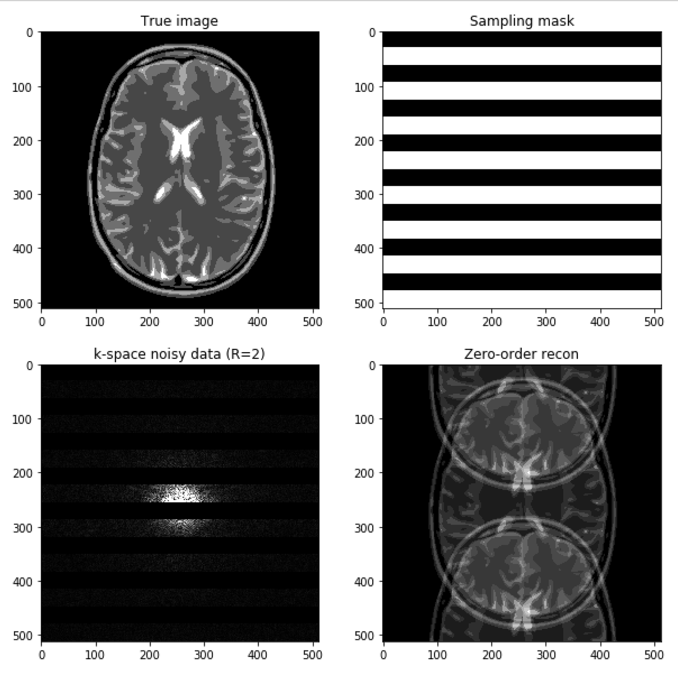
In [30]:
```
import numpy.matlib as mlib

# generate Cartesian lines in a straightforward manner
#a = (np.linspace(0,img_size,img_size+1))/img_size -0.5    # work in normalized frequency
r2 = (int)(img_size/2)
r4 = (int)(img_size/4)
r8 = (int)(img_size/8)
print("2-fold undersampling, m= ", r2)
print("4-fold undersampling, m= ", r4)
print("8-fold undersampling, m= ", r8)

selected_ksp_line = np.ones((1, img_size), dtype="float64")
skipped_ksp_line = np.zeros((1, img_size), dtype="float64")
k_space_pattern_r2 = np.concatenate((selected_ksp_line, skipped_ksp_line), axis=0)
kspace_mask_r2 = np.tile(k_space_pattern_r2, (r2, 1))

#k_space_pattern_r4 = np.concatenate((selected_ksp_line, skipped_ksp_line, skipped_ksp_line,skipped_ksp_line), axis=
0)
k_space_pattern_r4 = np.concatenate((selected_ksp_line, np.tile(skipped_ksp_line, (3,1))), axis=0)
kspace_mask_r4 = np.tile(k_space_pattern_r4, (r4, 1))

k_space_pattern_r8 = np.concatenate((selected_ksp_line, np.tile(skipped_ksp_line, (7,1))), axis=0)
kspace_mask_r8 = np.tile(k_space_pattern_r8, (r8, 1))


fig, axs = plt.subplots(1, 3, figsize=(16, 16) )
axs[0].imshow(kspace_mask_r2) #, cmap='Greys_r'
axs[0].set_title("Cartesian regular under-sampling mask (R=2)")
axs[1].imshow(kspace_mask_r4, cmap='Greys_r')
axs[1].set_title("Cartesian regular under-sampling mask (R=4)")
axs[2].imshow(kspace_mask_r8, cmap='Greys_r')
axs[2].set_title("Cartesian regular under-sampling mask (R=8)")
```

```
2-fold undersampling, m=  256
4-fold undersampling, m=  128
8-fold undersampling, m=  64
[[1. 1. 1. ... 1. 1. 1.]
 [0. 0. 0. ... 0. 0. 0.]
 [0. 0. 0. ... 0. 0. 0.]
 [0. 0. 0. ... 0. 0. 0.]
 [1. 1. 1. ... 1. 1. 1.]]
```



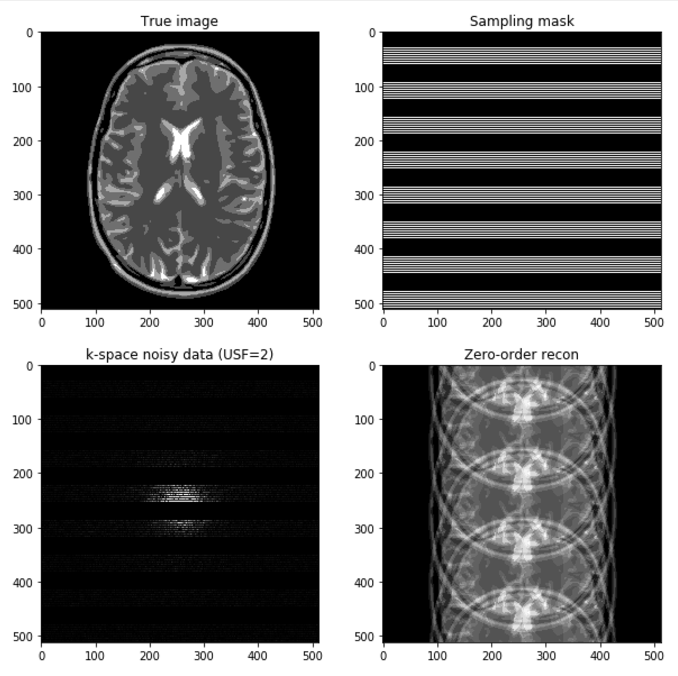
- Generate undersampled data for $R = 2$ and perform image reconstruction
- What do you observe?

In [39]:
```
# Generate the kspace data: first Fourier transform the image
kspace_data_r2 = np.fft.fftshift(fft(mri_img))
#add Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Mask data to perform subsampling
kspace_data_r2 *= kspace_mask_r2

# Zero order image reconstruction
image_rec0_r2 = ifft(np.fft.ifftshift(kspace_data_r2))

fig, axs = plt.subplots(2, 2, figsize=(10, 10) )
axs[0,0].imshow(mri_img, cmap='Greys_r')
axs[0,0].set_title("True image")
axs[0,1].imshow(kspace_mask_r2, cmap='Greys_r')
axs[0,1].set_title("Sampling mask")
axs[1,0].imshow(np.abs(kspace_data_r2),  cmap='gray', vmax=0.01*np.abs(kspace_data_r2).max())
#axs[1].imshow(np.abs(np.fft.ifftshift(kspace_data)), cmap='Greys_r')
axs[1,0].set_title("k-space noisy data (R=2)")
axs[1,1].imshow(np.abs(image_rec0_r2), cmap='gray')
axs[1,1].set_title("Zero-order recon")
plt.show()
```



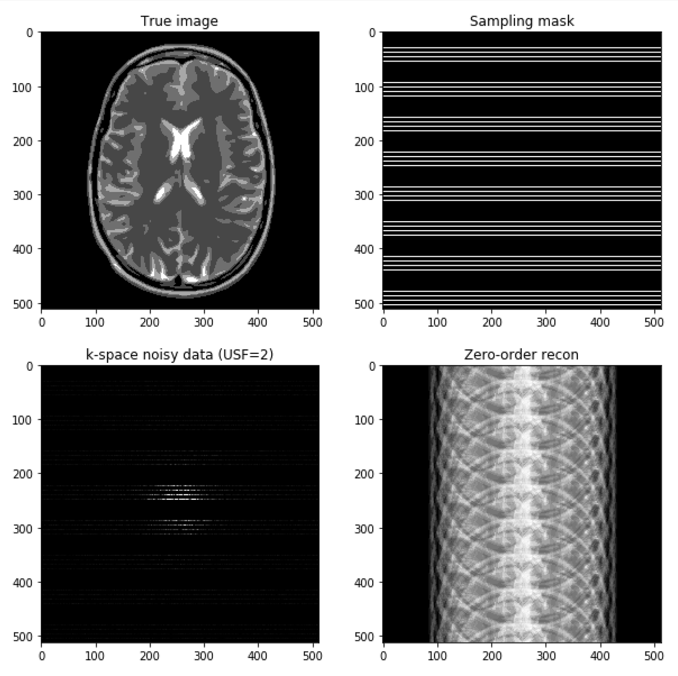
- Generate undersampled data for $R = 4$ and perform image reconstruction
- What do you observe?

In [40]:
```
# Generate the kspace data: first Fourier transform the image
kspace_data_r4 = np.fft.fftshift(fft(mri_img))
#add Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Mask data to perform subsampling
kspace_data_r4 *= kspace_mask_r4

# Zero order image reconstruction
image_rec0_r4 = ifft(np.fft.ifftshift(kspace_data_r4))

fig, axs = plt.subplots(2, 2, figsize=(10, 10) )
axs[0,0].imshow(mri_img, cmap='Greys_r')
axs[0,0].set_title("True image")
axs[0,1].imshow(kspace_mask_r4, cmap='Greys_r')
axs[0,1].set_title("Sampling mask")
axs[1,0].imshow(np.abs(kspace_data_r4),  cmap='gray', vmax=0.01*np.abs(kspace_data_r4).max())
axs[1,0].set_title("k-space noisy data (USF=2)")
axs[1,1].imshow(np.abs(image_rec0_r4), cmap='Greys_r')
axs[1,1].set_title("Zero-order recon")
plt.show()
```



- Generate undersampled data for $R = 8$ and perform image reconstruction
- What do you observe?

In [41]:
```
# Generate the kspace data: first Fourier transform the image
kspace_data_r8 = np.fft.fftshift(fft(mri_img))
#add Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Mask data to perform subsampling
kspace_data_r8 *= kspace_mask_r8

# Zero order image reconstruction
image_rec0_r8 = ifft(np.fft.ifftshift(kspace_data_r8))

fig, axs = plt.subplots(2, 2, figsize=(10, 10) )
axs[0,0].imshow(mri_img, cmap='Greys_r')
axs[0,0].set_title("True image")
axs[0,1].imshow(kspace_mask_r8, cmap='Greys_r')
axs[0,1].set_title("Sampling mask")
axs[1,0].imshow(np.abs(kspace_data_r8),  cmap='gray', vmax=0.01*np.abs(kspace_data_r4).max())
axs[1,0].set_title("k-space noisy data (USF=2)")
axs[1,1].imshow(np.abs(image_rec0_r8), cmap='Greys_r')
axs[1,1].set_title("Zero-order recon")
plt.show()
```



QUESTION:

- Do you know what key ingredient may help to recover the reference image pretty well while still using these regular under-sampling patterns?