09.non-Cartesian_CS_image_recon

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1 Nineth exercice: non-Cartesian MR image reconstruction

In this tutorial we will reconstruct an MRI image from radial undersampled kspace measurements. Let us denote Ω the undersampling mask, the under-sampled Fourier transform now reads F_Ω.

1.1 Import neuroimaging data

We use the toy datasets available in pysap, more specifically a 2D brain slice and the radial under-sampling scheme. We compare zero-order image reconstruction with Compressed sensing reconstructions (analysis vs synthesis formulation) using the FISTA algorithm for the synthesis formulation and the Condat-Vu algorithm for the analysis formulation.

We remind that the synthesis formulation reads (minimization in the sparsifying domain):

$$\widehat{z} = \text{arg} \min_{z\in C^n_\Psi} \frac{1}{2}\|y - F_\Omega \Psi^* z\|_2^2 + \lambda\|z\|_1$$

and the image solution is given by $\widehat{x} = \Psi^*\widehat{z}$. For an orthonormal wavelet transform, we have $n_\Psi = n$ while for a frame we may have $n_\Psi > n$.

while the analysis formulation consists in minimizing the following cost function (min. in the image domain):

$$\widehat{x} = \text{arg} \min_{x\in C^n} \frac{1}{2}\|y - F_\Omega x\|_2^2 + \lambda\|\Psi x\|_1 \,.$$

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- Date: 01/06/2021
- Target: ATSI MSc students, Paris-Saclay University

[1]:
```
# Package import
from mri.operators import NonCartesianFFT, WaveletN, WaveletUD2
from mri.operators.utils import convert_locations_to_mask, \
    gridded_inverse_fourier_transform_nd
from mri.reconstructors import SingleChannelReconstructor
import pysap
from pysap.data import get_sample_data

# Third party import
from modopt.math.metrics import ssim
from modopt.opt.linear import Identity
```

```python
from modopt.opt.proximity import SparseThreshold
import numpy as np
import matplotlib.pyplot as plt
```

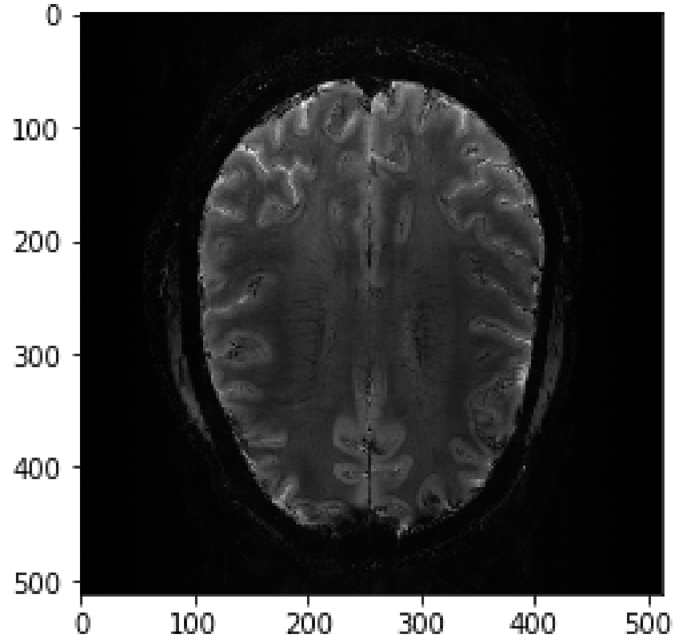
```
/home/ciuciu/anaconda3/lib/python3.7/site-
packages/mri/operators/fourier/cartesian.py:33: UserWarning: pynufft python
package has not been found. If needed use the master release. Till then you
cannot use NUFFT on GPU
  warnings.warn("pynufft python package has not been found. If needed use "
/home/ciuciu/anaconda3/lib/python3.7/site-
packages/mri/operators/fourier/non_cartesian.py:42: UserWarning: gpuNUFFT python
package has not been found. If needed please check on how to install in README
  warnings.warn("gpuNUFFT python package has not been found. If needed "
```

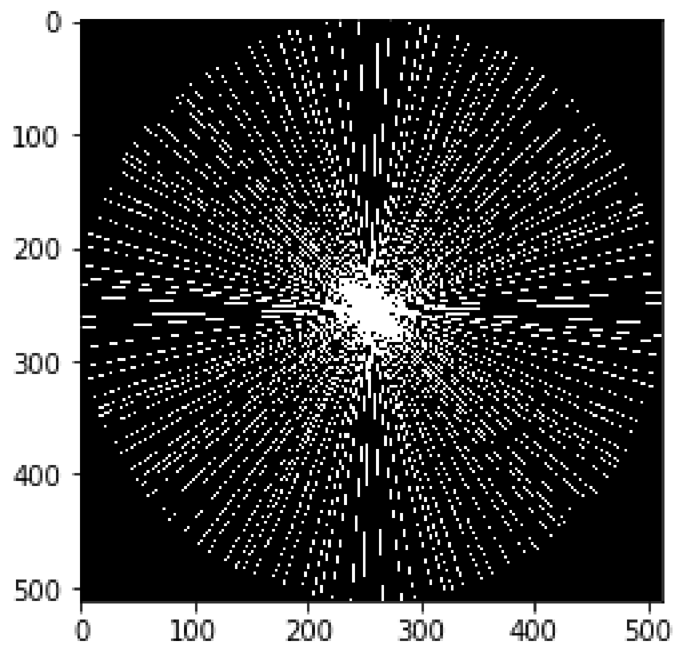
1.2 Loading input data

[2]:

```python
image = get_sample_data('2d-mri')
radial_mask = get_sample_data("mri-radial-samples")
kspace_loc = radial_mask.data
mask = pysap.Image(data=convert_locations_to_mask(kspace_loc, image.shape))
plt.figure()
plt.imshow(image, cmap='gray')
plt.figure()
plt.imshow(mask, cmap='gray')
plt.show()
```

```
/home/ciuciu/anaconda3/lib/python3.7/site-
packages/mri/operators/fourier/utils.py:76: FutureWarning: Using a non-tuple
sequence for multidimensional indexing is deprecated; use `arr[tuple(seq)]`
instead of `arr[seq]`. In the future this will be interpreted as an array index,
`arr[np.array(seq)]`, which will result either in an error or a different
result.
  mask[test] = 1
```


0
100
200
300
400
500
0
100
200
300
400
500


0
100
200
300
400
500
0
100
200
300
400
500

1.3 Generate the kspace

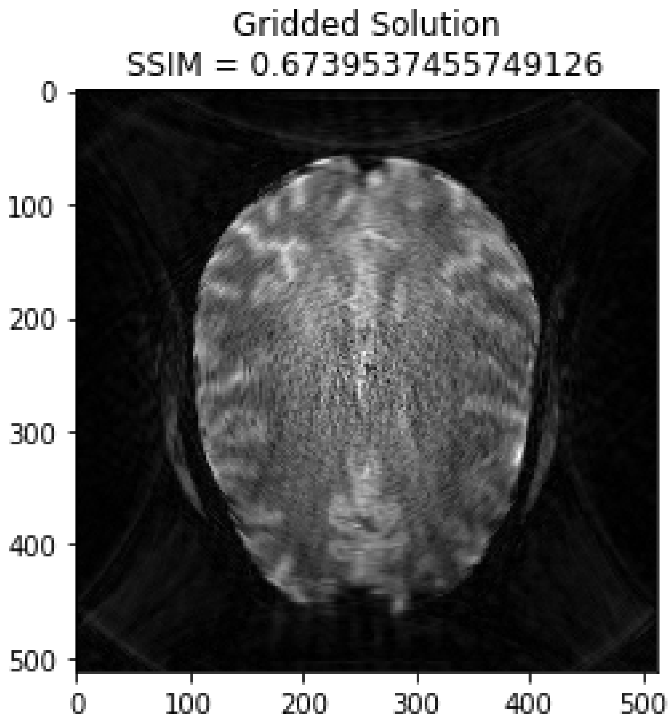
From the 2D brain slice and the acquisition mask, we retrospectively undersample the k-space using a cartesian acquisition mask We then reconstruct the zero order solution as a baseline

Get the locations of the kspace samples

[3]:
```
fourier_op = NonCartesianFFT(samples=kspace_loc, shape=image.shape,
                             implementation='cpu')
kspace_obs = fourier_op.op(image.data)
```

Gridded solution

[4]:
```
grid_space = np.linspace(-0.5, 0.5, num=image.shape[0])
grid2D = np.meshgrid(grid_space, grid_space)
grid_soln = gridded_inverse_fourier_transform_nd(kspace_loc, kspace_obs,
                                                 tuple(grid2D), 'linear')
plt.imshow(np.abs(grid_soln), cmap='gray')
# Calculate SSIM
base_ssim = ssim(grid_soln, image)
plt.title('Gridded Solution\nSSIM = ' + str(base_ssim))
plt.show()
```



1.4 FISTA optimization

We now want to refine the zero order solution using a FISTA optimization. The cost function is set to Proximity Cost + Gradient Cost

[5]:
```
linear_op = WaveletN(wavelet_name="sym8", nb_scales=4)
regularizer_op = SparseThreshold(Identity(), 6 * 1e-7, thresh_type="soft")
```

2 Generate operators

[6]:
```
reconstructor = SingleChannelReconstructor(
    fourier_op=fourier_op,
    linear_op=linear_op,
    regularizer_op=regularizer_op,
    gradient_formulation='synthesis',
    verbose=1,
)
```

```
WARNING: Making input data immutable.

Lipschitz constant is 570.4522582982656
The lipschitz constraint is satisfied
```

2.1 Synthesis formulation: FISTA optimization

We now want to refine the zero order solution using a FISTA optimization. The cost function is set to Proximity Cost + Gradient Cost

[8]:
```
x_final, costs, metrics = reconstructor.reconstruct(
    kspace_data=kspace_obs,
    optimization_alg='fista',
    num_iterations=200,
)
image_rec = pysap.Image(data=np.abs(x_final))
recon_ssim = ssim(image_rec, image)
plt.imshow(np.abs(image_rec), cmap='gray')
recon_ssim = ssim(image_rec, image)
plt.title('FISTA Reconstruction\nSSIM = ' + str(recon_ssim))
plt.show()
```

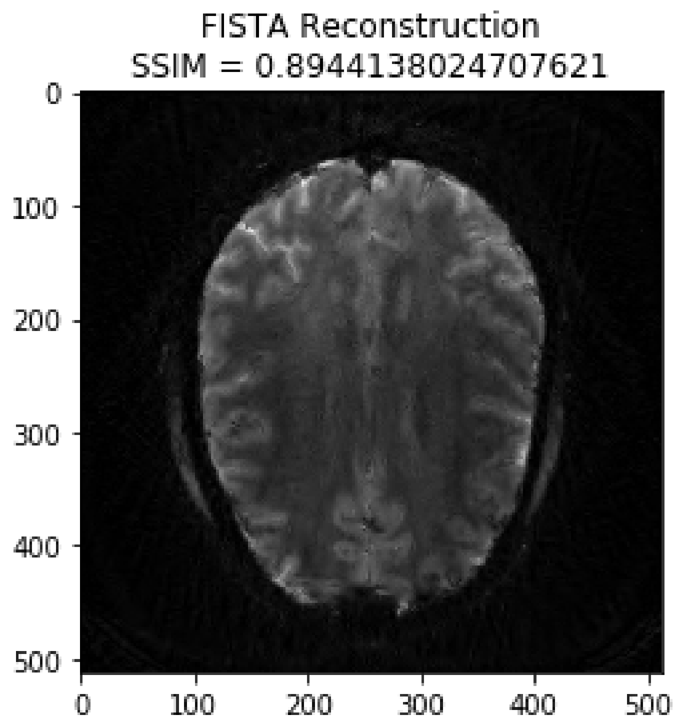
```
WARNING: Making input data immutable.
N/A% (0 of 200) |                        | Elapsed Time: 0:00:00 ETA:  --:--:--

 - mu:  6e-07
 - lipschitz constant:  570.4522582982656
 - data:  (512, 512)
```

```
 - wavelet:  <mri.operators.linear.wavelet.WaveletN object at 0x7fd8c5170ed0> -
4
 - max iterations:  200
 - image variable shape:  (512, 512)
 - alpha variable shape:  (291721,)
----------------------------------------
Starting optimization…

100% (200 of 200) |######################| Elapsed Time: 0:00:28 Time:  0:00:28

 - final iteration number:  200
 - final log10 cost value:  6.0
 - converged:  False
Done.
Execution time:  28.90419067896437  seconds
----------------------------------------
```



2.2 POGM reconstruction

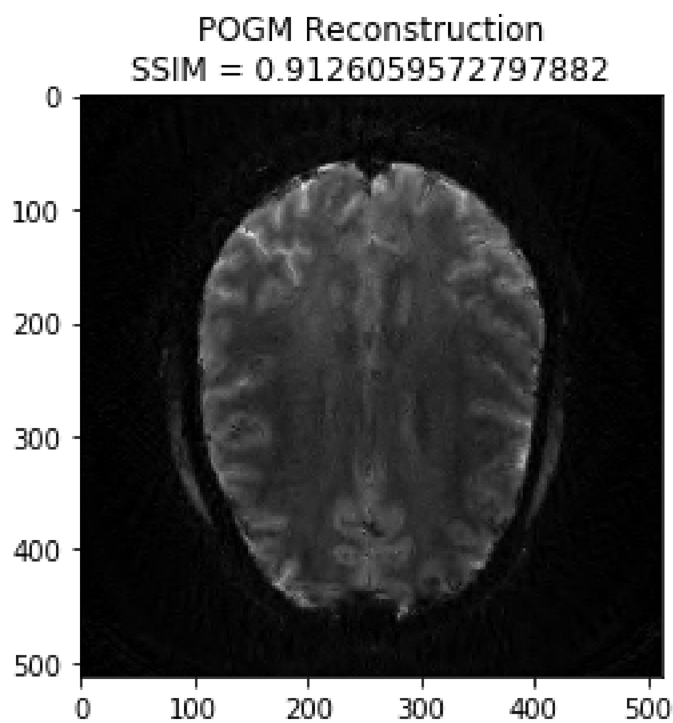
[9]:
```python
x_final, costs, metrics = reconstructor.reconstruct(
    kspace_data=kspace_obs,
    optimization_alg='pogm',
    num_iterations=200,
```

```
)
image_rec = pysap.Image(data=np.abs(x_final))
recon_ssim = ssim(image_rec, image)
plt.imshow(np.abs(image_rec), cmap='gray')
recon_ssim = ssim(image_rec, image)
plt.title('POGM Reconstruction\nSSIM = ' + str(recon_ssim))
plt.show()
```

```
 - mu:  6e-07
 - lipschitz constant:  570.4522582982656
 - data:  (512, 512)
 - wavelet:  <mri.operators.linear.wavelet.WaveletN object at 0x7fd8c5170ed0> -
4
 - max iterations:  200
 - image variable shape:  (1, 512, 512)
----------------------------------------
Starting optimization…

100% (200 of 200) |######################| Elapsed Time: 0:00:31 Time:  0:00:31

 - final iteration number:  200
 - final log10 cost value:  6.0
 - converged:  False
Done.
Execution time:  31.568309169029817  seconds
----------------------------------------
```



2.3 Analysis formulation: Condat-Vu reconstruction

[13]:
```
#linear_op = WaveletN(wavelet_name="sym8", nb_scales=4)
linear_op = WaveletUD2(
    wavelet_id=24,
    nb_scale=4,
)
```

[14]:
```
reconstructor = SingleChannelReconstructor(
    fourier_op=fourier_op,
    linear_op=linear_op,
    regularizer_op=regularizer_op,
    gradient_formulation='analysis',
    verbose=1,
)
```

```
WARNING: Making input data immutable.
```

```
Lipschitz constant is 570.452258225419
The lipschitz constraint is satisfied
```

[15]:
```
x_final, costs, metrics = reconstructor.reconstruct(
    kspace_data=kspace_obs,
    optimization_alg='condatvu',
    num_iterations=200,
)
image_rec = pysap.Image(data=np.abs(x_final))
plt.imshow(np.abs(image_rec), cmap='gray')
recon_ssim = ssim(image_rec, image)
plt.title('Condat-Vu Reconstruction\nSSIM = ' + str(recon_ssim))
plt.show()
```

```
 - mu:  6e-07
 - lipschitz constant:  570.452258225419
 - tau:  0.003499650305671012
 - sigma:  0.5
 - rho:  1.0
 - std:  None
 - 1/tau - sigma||L||^2 >= beta/2:  True
 - data:  (512, 512)
 - wavelet:  <mri.operators.linear.wavelet.WaveletUD2 object at 0x7fd8c528ba90>
- 4
 - max iterations:  200
 - number of reweights:  0
```

```
 - primal variable shape:  (512, 512)
 - dual variable shape:  (2621440,)
----------------------------------------
Starting optimization…

100% (200 of 200) |######################| Elapsed Time: 0:03:28 Time:  0:03:28

 - final iteration number:  200
 - final cost value:  1000000.0
 - converged:  False
Done.
Execution time:  210.95389512297697  seconds
----------------------------------------
```