Nineth exercice: non-Cartesian MR image reconstruction

In this tutorial we will reconstruct an MRI image from radial undersampled kspace measurements. Let us denote Ω the undersampling mask, the under-sampled Fourier transform now reads F_{Ω}.

Import neuroimaging data

We use the toy datasets available in pysap, more specifically a 2D brain slice and the radial under-sampling scheme. We compare zero-order image reconstruction with Compressed sensing reconstructions (analysis vs synthesis formulation) using the FISTA algorithm for the synthesis formulation and the Condat-Vu algorithm for the analysis formulation.

We remind that the synthesis formulation reads (minimization in the sparsifying domain): $\hat{z} = \text{arg}^{\overset{\min}{z\in C_\Psi^n}} \frac{1}{2} \|$ and the image solution is given by \widehat{x} = \Psi^*\widehat{z}. For an orthonormal wavelet transform, we have n_\Psi=n while for a frame we may have n_\Psi > n.

while the analysis formulation consists in minimizing the following cost function (min. in the image domain): \widehat{x} = \text{arg}\,\min_{x\in C^n} \frac{1}{2} \|y - F_\Omega x\|_2^2 + \lambda \|\Psi x\|_1 \,.

- Author: Chaithya G R & Philippe Ciuciu
- Date: 01/06/2021
- Target: ATSI MSc students, Paris-Saclay University

In [9]:

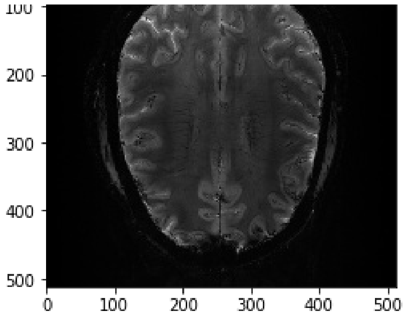
```
# Package import
#from mri.numerics.fourier import NFFT
#from mri.numerics.reconstruct import sparse_rec_fista
#from mri.numerics.utils import generate_operators
#from mri.numerics.utils import convert_locations_to_mask
#from mri.parallel_mri.extract_sensitivity_maps import \
#    gridded_inverse_fourier_transform_nd
from mri.operators import NonCartesianFFT, WaveletN, WaveletUD2
from mri.operators.utils import convert_locations_to_mask, \
    gridded_inverse_fourier_transform_nd
from mri.reconstructors import SingleChannelReconstructor
import pysap
from pysap.data import get_sample_data

# Third party import
from modopt.math.metrics import ssim
from modopt.opt.linear import Identity
from modopt.opt.proximity import SparseThreshold
import numpy as np
import matplotlib.pyplot as plt
```

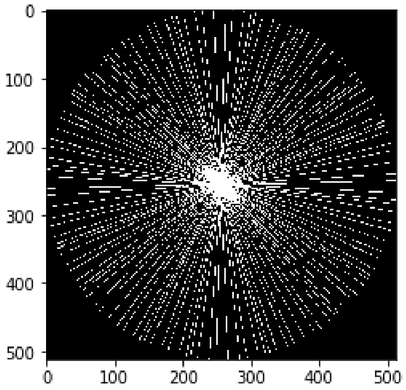
Loading input data

In [16]:

```
image = get_sample_data('2d-mri')
radial_mask = get_sample_data("mri-radial-samples")
kspace_loc = radial_mask.data
mask = pysap.Image(data=convert_locations_to_mask(kspace_loc, image.shape))
plt.figure()
plt.imshow(image, cmap='gray')
plt.figure()
plt.imshow(mask, cmap='gray')
plt.show()
```

```
<pysap.base.image.Image object at 0x7ff26d2d6750>
```





Generate the kspace

From the 2D brain slice and the acquisition mask, we retrospectively undersample the k-space using a cartesian acquisition mask We then reconstruct the zero order solution as a baseline

Get the locations of the kspace samples

In [7]:

```
#fourier_op = NFFT(samples=kspace_loc, shape=image.shape)
#kspace_obs = fourier_op.op(image.data)
fourier_op = NonCartesianFFT(samples=kspace_loc, shape=image.shape,
                             implementation='cpu')
kspace_obs = fourier_op.op(image.data)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-7-fa4814d2c13c> in <module>
      2 #kspace_obs = fourier_op.op(image.data)
      3 fourier_op = NonCartesianFFT(samples=kspace_loc, shape=image.shape,
----> 4                              implementation='cpu')
      5 kspace_obs = fourier_op.op(image.data)

~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, implementation, n_coils, **kwargs)
    550         if implementation == 'cpu':
    551             self.implementation = NFFT(samples=samples, shape=shape,
--> 552                                        n_coils=self.n_coils)
    553         elif implementation == 'cuda' or implementation == 'opencl':
    554             self.implementation = NUFFT(samples=samples, shape=shape,

~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, n_coils)
    105         # TODO Parallelize this if possible
    106         self.nb_coils = n_coils
--> 107         self.plan = pynfft.NFFT(N=shape, M=len(samples))
    108         self.plan.x = self.samples
    109         self.plan.precompute()

NameError: name 'pynfft' is not defined
```

Gridded solution

In [ ]:

```
grid_space = np.linspace(-0.5, 0.5, num=image.shape[0])
grid2D = np.meshgrid(grid_space, grid_space)
grid_soln = gridded_inverse_fourier_transform_nd(kspace_loc, kspace_obs,
                                                 tuple(grid2D), 'linear')
plt.imshow(np.abs(grid_soln), cmap='gray')
# Calculate SSIM
base_ssim = ssim(grid_soln, image)
plt.title('Gridded Solution\nSSIM = ' + str(base_ssim))
plt.show()
```

FISTA optimization

We now want to refine the zero order solution using a FISTA optimization. The cost function is set to Proximity Cost + Gradient Cost

In [11]:

```
linear_op = WaveletN(wavelet_name="sym8", nb_scales=4)
regularizer_op = SparseThreshold(Identity(), 6 * 1e-7, thresh_type="soft")
```

Generate operators

In [12]:

```
reconstructor = SingleChannelReconstructor(
    fourier_op=fourier_op,
    linear_op=linear_op,
    regularizer_op=regularizer_op,
    gradient_formulation='synthesis',
    verbose=1,
)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-12-8eee53257f67> in <module>
      1 reconstructor = SingleChannelReconstructor(
----> 2     fourier_op=fourier_op,
      3     linear_op=linear_op,
      4     regularizer_op=regularizer_op,
      5     gradient_formulation='synthesis',

NameError: name 'fourier_op' is not defined
```

Synthesis formulation: FISTA optimization

We now want to refine the zero order solution using a FISTA optimization. The cost function is set to Proximity Cost + Gradient Cost

In [ ]:

```
x_final, costs, metrics = reconstructor.reconstruct(
    kspace_data=kspace_obs,
    optimization_alg='fista',
    num_iterations=200,
)
image_rec = pysap.Image(data=np.abs(x_final))
recon_ssim = ssim(image_rec, image)
plt.imshow(np.abs(image_rec), cmap='gray')
recon_ssim = ssim(image_rec, image)
plt.title('FISTA Reconstruction\nSSIM = ' + str(recon_ssim))
plt.show()
```

Analysis formulation: Condat-Vu reconstruction

In [14]:

In [14]:

```
linear_op = WaveletUD2(
    wavelet_id=24,
    nb_scale=4,
)
```

In [15]:

```
reconstructor = SingleChannelReconstructor(
    fourier_op=fourier_op,
    linear_op=linear_op,
    regularizer_op=regularizer_op,
    gradient_formulation='analysis',
    verbose=1,
)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-15-6c81cdac1fa0> in <module>
      1 reconstructor = SingleChannelReconstructor(
----> 2     fourier_op=fourier_op,
      3     linear_op=linear_op,
      4     regularizer_op=regularizer_op,
      5     gradient_formulation='analysis',

NameError: name 'fourier_op' is not defined
```

In [ ]:

```
x_final, costs, metrics = reconstructor.reconstruct(
    kspace_data=kspace_obs,
    optimization_alg='condatvu',
    num_iterations=200,
)
image_rec = pysap.Image(data=np.abs(x_final))
plt.imshow(np.abs(image_rec), cmap='gray')
recon_ssim = ssim(image_rec, image)
plt.title('Condat-Vu Reconstruction\nSSIM = ' + str(recon_ssim))
plt.show()
```