Sixth exercice: Non-Cartesian spiral under-sampling

In this notebook, you can play with the design parameters to regenerate different spiral in-out patterns (so, we draw as many spiral arches as the number of shots). You can play with the number of shots by changing the under-sampling factor.

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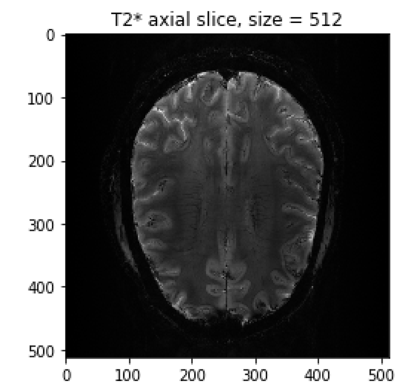
- **Revision**: 01/06/2021 for ATSI MSc hands-on session at Paris-Saclay University.

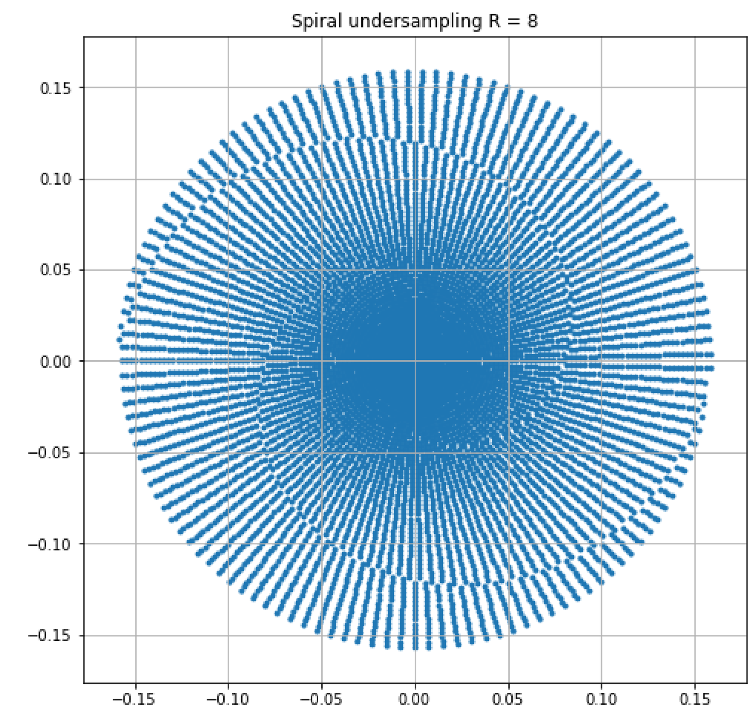
In [43]:
```
#DISPLAY T2* MR IMAGE
%matplotlib inline

import numpy as np
import os.path as op
import os
import math ; import cmath
import matplotlib.pyplot as plt
import sys
from mri.operators import NonCartesianFFT
from mri.operators.utils import convert_locations_to_mask, \
    gridded_inverse_fourier_transform_nd
from pysap.data import get_sample_data


from skimage import data, img_as_float, io, filters
from modopt.math.metrics import ssim

mri_img = get_sample_data('2d-mri')
img_size = mri_img.shape[0]

plt.figure()
plt.title("T2* axial slice, size = {}".format(img_size))
if mri_img.ndim == 2:
    plt.imshow(mri_img, cmap=plt.cm.gray)
else:
    plt.imshow(mri_img)
plt.show()
```



In [18]:
```
def complex_to_2d(points):
    X = points.real
    Y = points.imag
    return np.asarray([X, Y]).T
```

In [47]:
```
# set up the first shot
rfactor = 8
num_shots = math.ceil(img_size/rfactor)
print("number of shots: {}".format(num_shots))

# define the regularly spaced samples on a single shot
#nsamples = (np.arange(0,img_size) - img_size//2)/(img_size)
num_samples = img_size
num_samples = (num_samples + 1) // 2
print("number of samples: {}".format(num_samples))
num_revolutions = 1

shot = np.arange(0, num_samples, dtype=np.complex_)
radius = shot / num_samples * 1 / (2 * np.pi) * (1 - np.finfo(float).eps)
angle = np.exp(2 * 1j * np.pi * shot / num_samples * num_revolutions)
# first half of the spiral
single_shot = np.multiply(radius, angle)
# add second half of the spiral
#single_shot = np.append(np.flip(single_shot, axis=0), -single_shot[1:])
single_shot = np.append(np.flip(single_shot, axis=0), -single_shot)
#print(single_shot)
print("number of samples per shot: {}".format(np.size(single_shot)))

# vectorize the nb of shots
#vec_shots = np.arange(0,nb_shots + 1)

k_shots = np.array([], dtype = np.complex_)
#for i in vec_shots:
for i in np.arange(0, num_shots):
    shot_rotated = single_shot * np.exp(1j * 2 * np.pi * i / (num_shots * 2))
    k_shots = np.append(k_shots, shot_rotated)
    #np.append(k_shots, complex_to_2d(shot_rotated))

print(k_shots.shape)
kspace_loc = np.zeros((len(k_shots),2))
kspace_loc[:,0] = k_shots.real
kspace_loc[:,1] = k_shots.imag

#Plot full initialization
kspace = plt.figure(figsize = (8,8))
#plot shots
plt.scatter(kspace_loc[::4,0], kspace_loc[::4,1], marker = '.')
plt.title("Spiral undersampling R = %d" %rfactor)

axes = plt.gca()
plt.grid()
```

```
number of shots: 64
number of samples: 256
number of samples per shot: 512
(32768,)
```



In [32]:
```
print(np.arange(0, num_shots))
```

```
[  0   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17
  18  19  20  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35
  36  37  38  39  40  41  42  43  44  45  46  47  48  49  50  51  52  53
  54  55  56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71
  72  73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89
  90  91  92  93  94  95  96  97  98  99 100 101 102 103 104 105 106 107
 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125
 126 127]
```

In [29]:
```
data=convert_locations_to_mask(kspace_loc, mri_img.shape)
fourier_op = NonCartesianFFT(samples=kspace_loc, shape=mri_img.shape,
                             implementation='cpu')
kspace_obs = fourier_op.op(mri_img.data)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-29-b3cf02563ff2> in <module>
      1 data=convert_locations_to_mask(kspace_loc, mri_img.shape)
      2 fourier_op = NonCartesianFFT(samples=kspace_loc, shape=mri_img.shape,
----> 3                              implementation='cpu')
      4 kspace_obs = fourier_op.op(mri_img.data)

~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, implementation, n_
coils, **kwargs)
    550         if implementation == 'cpu':
    551             self.implementation = NFFT(samples=samples, shape=shape,
--> 552                                        n_coils=self.n_coils)
    553         elif implementation == 'cuda' or implementation == 'opencl':
    554             self.implementation = NUFFT(samples=samples, shape=shape,

~/work/code/git/pysap-mri/mri/operators/fourier/non_cartesian.py in __init__(self, samples, shape, n_coils)
    105         # TODO Parallelize this if possible
    106         self.nb_coils = n_coils
--> 107         self.plan = pynfft.NFFT(N=shape, M=len(samples))
    108         self.plan.x = self.samples
    109         self.plan.precompute()

NameError: name 'pynfft' is not defined
```

In [17]:
```
grid_space = np.linspace(-0.5, 0.5, num=mri_img.shape[0])
grid2D = np.meshgrid(grid_space, grid_space)
grid_soln = gridded_inverse_fourier_transform_nd(kspace_loc, kspace_obs,
                                                 tuple(grid2D), 'linear')
plt.imshow(np.abs(grid_soln), cmap='gray')
# Calculate SSIM
base_ssim = ssim(grid_soln, mri_img)
plt.title('Gridded Solution\nSSIM = ' + str(base_ssim))
plt.show()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-17-e6c474cb4bbd> in <module>
      1 grid_space = np.linspace(-0.5, 0.5, num=mri_img.shape[0])
      2 grid2D = np.meshgrid(grid_space, grid_space)
----> 3 grid_soln = gridded_inverse_fourier_transform_nd(kspace_loc, kspace_obs,
      4                                                  tuple(grid2D), 'linear')
      5 plt.imshow(np.abs(grid_soln), cmap='gray')

NameError: name 'kspace_obs' is not defined
```