Third exercice: 1D Cartesian structured VDS along parallel lines

In this notebook, we perform pseudo-random variable density sampling along the phase encoding direction. A handcrafted density is designed and samples are then drawn by virtually inverting its cumulative density function. Then these samples define the selected phase encoding lines retained in the sampling mask. Low frequencies are more sampled than higher frequencies.

- Author: Philippe Ciuciu (philippe.ciuciu@cea.fr)

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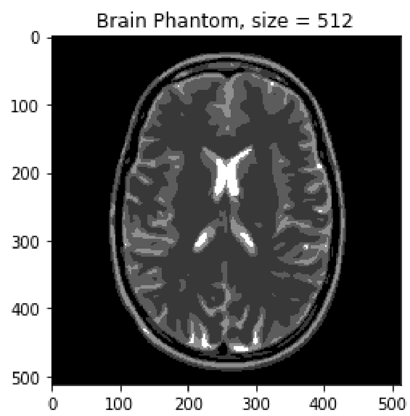
- **Revision**: 01/06/2021 for ATSI MSc hands-on session at Paris-Saclay University.

In [45]:

```
#DISPLAY BRAIN PHANTOM
%matplotlib inline

import numpy as np
import os.path as op
import os
import math ; import cmath
import matplotlib.pyplot as plt
import sys

from skimage import data, io, filters
#get current working dir
cwd = os.getcwd()
#cwd= "/"
dirimg_2d = op.join(cwd, "..", "data")
img_size = 512   #256
FOV = 0.2 #field of view in meters
pixelSize = FOV/img_size

#load data file corresponding to the target resolution
filename = "BrainPhantom" + str(img_size) + ".png"
mri_filename = op.join(dirimg_2d, filename)
mri_img = io.imread(mri_filename, as_gray=True)
plt.figure()
plt.title("Brain Phantom, size = "+ str(img_size))
if mri_img.ndim == 2:
    plt.imshow(mri_img, cmap=plt.cm.gray)
else:
    plt.imshow(mri_img)
plt.show()
```

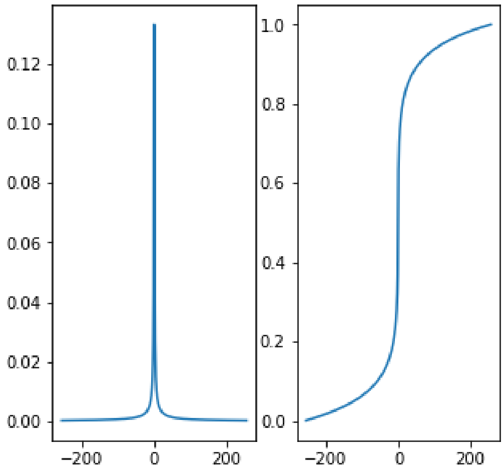


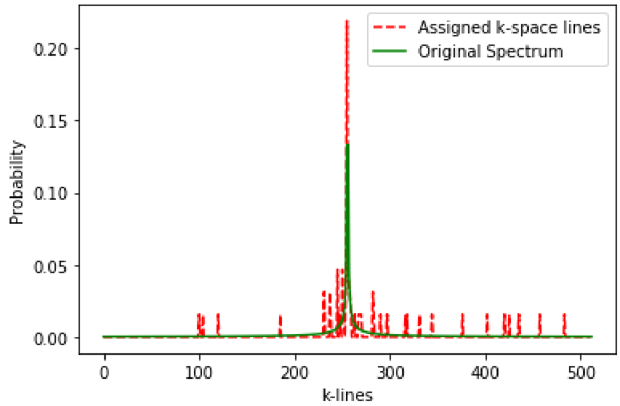
In [46]:

```
import numpy as np
import sys
import numpy.random as ra

eps = sys.float_info.epsilon
```

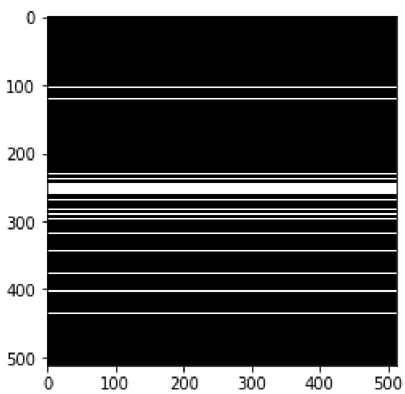
```
#img_size = 512
# generate Cartesian lines in a straightforward manner
#a = np.linspace(0,img_size,img_size)
#a = (np.linspace(0,img_size,img_size+1))/img_size -0.5    # work in normalized frequency


c = np.ones((1, img_size), dtype="float64")
kspace_mask = np.tile(c, (img_size, 1))
#kspace_lines = np.linspace(-1/2., 1/2.,img_size)
kspace_lines = np.linspace(-1/2., 1/2.,img_size)*img_size
# define the taret sampling density (ie non-iniform over k-space lines)
decay = 1.
# Define the sampling density
p_decay = np.power(np.abs(kspace_lines),-decay)
p_decay = p_decay/np.sum(p_decay)
# generate its CDF
cdf_pdecay = np.cumsum(p_decay)


pmax = p_decay.max()
pmin = p_decay.min()

# Plot the density and its cumulative distribution function (CDF) p_decay
plt.figure()
fig, axs = plt.subplots(1, 2, figsize= (5,5) )
axs[0].plot(kspace_lines, p_decay)
axs[1].plot(kspace_lines, cdf_pdecay)

# Perform pseudo-random sampling: technique used:
# draw uniform variables and invert the CDF to get back to p_decay-distributed sampled
nb_samples = (int)(img_size/8)
print(nb_samples)
samples = ra.uniform(0, 1, nb_samples)
gen_klines = [int(kspace_lines[np.argwhere(cdf_pdecay == min(cdf_pdecay[(cdf_pdecay - r) > 0]))]) f
or r in samples]

# shift the samples lines by half of the k-space (ie image size as we're in Cartesian ref)
gen_klinesb = ((np.array(gen_klines) - 1) / 1).astype(int) + (int)(img_size/2)
#gen_klinesb = ((np.array(gen_klines) - 1) / 1).astype(int)
print(gen_klinesb)   # unsorted samples
times = np.arange(1, img_size, 1)
lc = np.bincount(gen_klinesb, minlength=len(times))

# check that histogram of sample values fits the prescribed density p_decay
plt.figure()
plot1, = plt.plot(lc/float(sum(lc)), 'r--', label='Assigned k-space lines')
plot2, = plt.plot(p_decay,'g',label='Original Spectrum')
plt.xlabel('k-lines')
plt.ylabel('Probability')
plt.legend(handles=[plot1,plot2])
plt.show()


#print(p_decay.min())
sampled_klines = np.array(np.unique(gen_klinesb))
print(sampled_klines)

nblines = np.size(sampled_klines)
print(nblines)
threshold = 2. * p_decay.min()  # sys.float_info.epsilon \simeq 2e-16
kspace_mask = np.zeros((img_size,img_size), dtype="float64")
kspace_mask[sampled_klines,:] = np.ones((nblines,img_size) , dtype="float64")

plt.figure()
plt.imshow(kspace_mask, cmap='gray')
plt.show()
```

```
64
[237 331 318 250 255 254 255 270 245 250 259 282 231 257 269 245 316 120
 258 185 256 268 290 258 104 255 255 425 420 344 247 254 282 256 251 263
 483 255 255 252 402 255 255 283 100 250 457 255 255 231 267 254 237 255
 255 238 257 435 256 297 245 376 255 255]


<Figure size 432x288 with 0 Axes>
```






```
[100 104 120 185 231 237 238 245 247 250 251 252 254 255 256 257 258 259
 263 267 268 269 270 282 283 290 297 316 318 331 344 376 402 420 425 435
 457 483]
38
```



In [7]:

```
#import numpy.fft as fft
norm = "ortho"
#norm = None
def fft(x):
    return np.fft.fft2(x, norm=norm)

def ifft(x):
    return np.fft.ifft2(x, norm=norm)


# Generate the kspace data: first Fourier transform the image
kspace_data = np.fft.fftshift(fft(mri_img))
#add Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Mask data to perform subsampling
kspace_data *= kspace_mask
```

```
kspace_data *= kspace_mask

#noisy k-space data using Gaussian complex-valued random noise
signoise = 10
kspace_data += np.random.randn(*mri_img.shape) * signoise * (1+1j)
# Zero order solution
image_rec0 = ifft(np.fft.ifftshift(kspace_data))

fig, axs = plt.subplots(2, 2, figsize=(10, 10) )
axs[0,0].imshow(mri_img, cmap='Greys_r')
axs[0,0].set_title("True image")
axs[0,1].imshow(kspace_mask, cmap='Greys_r')
axs[0,1].set_title("Sampling mask")
axs[1,0].imshow(np.abs(kspace_data),  cmap='Greys_r', vmax=0.005*np.abs(kspace_data).max())
#axs[1].imshow(np.abs(np.fft.ifftshift(kspace_data)), cmap='Greys_r')
axs[1,0].set_title("k-space noisy data (1D VDS)")
axs[1,1].imshow(np.abs(image_rec0), cmap='Greys_r')
axs[1,1].set_title("Zero-order recon")
plt.show()
```