

(Q) INVERSION COUNT.(1) ALGORITHM - Regular:

// Input: array with choices of all the students

// Output: Total count inversions.

// This function calculates count inversions by brute force

```
def CI-brute (students [ ]):
    → for s in students:
        n, Count = len(s), 0
    → for i in range (n):
        for j in range (i+1, n):
            if S[i] > S[j]:
                Count++
    total+= Count
    → return total
```

(2) ALGORITHM - DAC with mergesort

```
def mergesort (S [ ], l, r):
    count = 0
    if left < right:
        mid = (l+r)/2
        count += mergesort (S [ ], l, mid)
        count += mergesort (S [ ], mid+1, r)
        count += merge (S [ ], l, m, r)
```

```
def merge(students s, l, m, r):
```

```
    i = l
```

```
    j = m + 1
```

```
    k = l
```

```
    inv-count = 0
```

```
    while i <= mid && j <= r:
```

```
        if s[i] <= s[j]:
```

```
            temp[k] = s[i]
```

```
            i++
```

```
        else: count += (mid - i + 1)
```

```
            temp[k] = s[j]
```

```
            j++
```

```
            k++
```

```
    while i <= mid:
```

```
        temp[k++] = s[i++]
```

```
    while j <= r:
```

```
        temp[k++] = s[j++]
```

```
    for i in (l, r + 1):
```

```
        s[i] = temp[i]
```

```
    return count
```

```
def CI_DAC():
```

```
    count = 0
```

```
    for s in students:
```

```
        for s in the students[]:
```

```
            count += mergesort(SC[], 0, n-1);
```

```
    return count
```

③ Time Complexity.

LINEAR \Rightarrow It has 3 for loops that traverse across array. Let n be the length of individual array and m be the length of the dataset.

$$\therefore T(n) = m \times n \times n$$

$$T(n) = O(m \cdot n^2)$$

DAC \Rightarrow It uses mergesort algorithm which has ' $n \log n$ ' time complexity and this process is repeated ' m ' times for m arrays in dataset.

$$\therefore T(n) = m \times n \log n$$

$$T(n) = O(mn \log n)$$

4. TEST CASES :

1. Positive Testcases

All array values are integers and no values are missing. There are 100 such arrays in the dataset. Return and Display number of Total Inversions and students with 0, 1, 2, ... inversions.

2. Negative Testcases

(1) If the array has non-int values

→ DISPLAY :- Incorrect input, choices ~~can't~~ have to be integers.

(2) If the array is empty

→ DISPLAY :- Dataset is empty

(3) Negative Values of choices

→ DISPLAY :- Choices can't be negative.

(4) If array has 'zero' as a choice.

→ DISPLAY :- Course Codes can't be zero

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(b) KARATSUBA MULTIPLICATION

```
def multiplication(x, y):  
    a, b = str(x), str(y)  
    result = 0
```

```
    for i in range(len(b):  
        dB = int(b[-1-i])
```

```
        pp = 0  
        carry = 0
```

```
        for j in range(len(a):
```

```
            dA = int(a[-j-1])
```

```
            p = (dA * dB) + carry
```

```
            carry = p // 10
```

```
            p = p % 10
```

```
            pp += p * (10 ** j)
```

```
        if carry > 0:
```

```
            pp += carry * (10 ** len(a))
```

```
        pp *= (10 ** i)
```

```
    result += pp
```

```
    return result
```


② ALGORITHM - Karatsuba (DAC)

// Input : Two n digit positive integer x and y

// Output : The product $x * y$.

// This function uses DAC (Karatsuba Algorithm) to calculate product .

Karatsuba ($\text{int } a, \text{int } b$) :

if $a < 10$ or $b < 10$:

return $a * b$

else :

~~$n = m$~~ $m = n / 2$

$a_H, a_L = \text{divmod}(a, 10^{**m})$

$b_H, b_L = \text{divmod}(b, 10^{**m})$

$z_0 = \text{karatsuba}(a_L, b_L)$

$z_1 = \text{karatsuba}(a_L + a_H, b_L + b_H)$

$z_2 = \text{karatsuba}(a_H, b_H)$

return $(z_2) * (10^{**n}) + (z_1 - z_2 - z_0) * (10^{**m}) + z_0$

③ Time Complexity

In Karatsuba algorithm, we convert problem of size 'n' into problem of size 'n/2' and recursively call the function thrice.

$$\text{Therefore : } T(n) = 3T(n/2) + O(n)$$

By master eqn:

$$a=3, b=2, d=1$$

$$b^d = 2, a=3$$

$$a > b^d$$

$$\therefore T(n) = n^{\log_b a} = n^{\log_2 3}$$

$$\therefore T(n) \approx n^{1.585}$$

In regular multiplication, we use 2 loops to iterate through every digit of each number multiplying each digit takes constant time.

$$\therefore TC = T(n) = O(n \times m) \quad \text{if } m, n \text{ are digits in the no.}$$

$$\approx O(n^2) \quad \dots \text{if } n \approx m$$

④ Testcases:

① Positive Testcases:

$$(1) a = 123456789012345$$

$$b = 987654321098765$$

$$a * b = 121932631137021034431113635425$$

$$(2) a = 987654321012345$$

$$b = 123456789098765$$

$$a * b = 121932631246761025482140780925$$

$$(3) a = 123456789876543$$

$$b = 987654321234567$$

$$a * b = 121932631333333086631194987481$$

② Negative Testcases

$$(1) a = 123456 \quad (\text{One of the number is zero})$$

$$b = 0$$

$$a * b = 0$$

$$(2) a = 123456 \quad (\text{number is not an integer})$$

$$b = ""$$

Both numbers should be integers.

$$(3) a = -1234 \quad (\text{number is negative})$$

$$b = 4567$$

$$a * b = -5635678$$

$$(4) a = 12.3 \quad b = 1234$$

DISPLAY: Both numbers should be integer.